Assignment 5

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1. a) $A = \begin{bmatrix} 1 & 5 \\ 1 & -3 \end{bmatrix}$

Find E.val = $\lambda = 2, -4$ $= \begin{bmatrix} 5 & -1 \\ 1 & 1 \end{bmatrix}$

$$e^{At} = \begin{bmatrix} 5 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} e^{2t} & 0 \\ 0 & e^{-4t} \end{bmatrix} \begin{bmatrix} \frac{3}{8} & \frac{5}{8} \\ \frac{1}{8} & -\frac{1}{8} \end{bmatrix}$$

$\begin{matrix} x' \\ y' \end{matrix} = \begin{bmatrix} 1 & 5 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

$x' = x + 5y$ $\qquad x' = 0$ @ $(0,0)$, $y = \frac{-x}{5}$

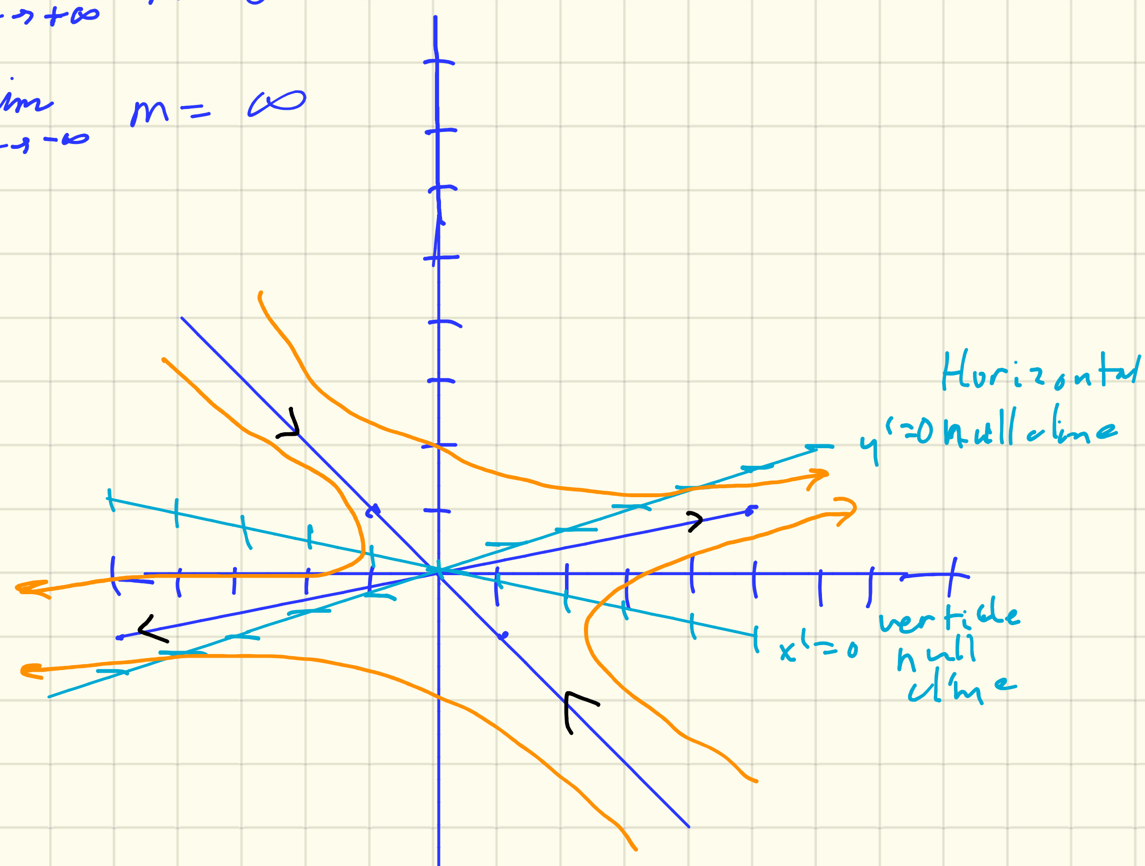
$y' = x - 3y$ $\qquad y' = 0$ @ $(0,0)$, $y = \frac{x}{3}$

Slopes $\quad \lambda_1 > 0 > \lambda_2$

$$m = \frac{y'}{x'} = \frac{e^{-4t}}{e^{2t}} = e^{-4t-2t}$$

$\lim_{t \to +\infty} m = 0$

$\lim_{t \to -\infty} m = \infty$



The equilibrium at (0,0) is unstable and repelling.

$A = \begin{bmatrix} 3 & -2 \\ 1 & 1 \end{bmatrix}$

Find E.val = $\lambda = 2+i,\ 2-i$

$e^{At} = e^{\alpha t}\begin{bmatrix} \cos \beta t & \sin \beta t \\ -\sin \beta t & \cos \beta t \end{bmatrix}$

$p = \begin{pmatrix} 1+i \\ 1 \end{pmatrix}$

So $P = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

$= e^{2t}\begin{bmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{bmatrix}$

$\begin{matrix} x' \\ y' \end{matrix} = \begin{bmatrix} 3 & -2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$

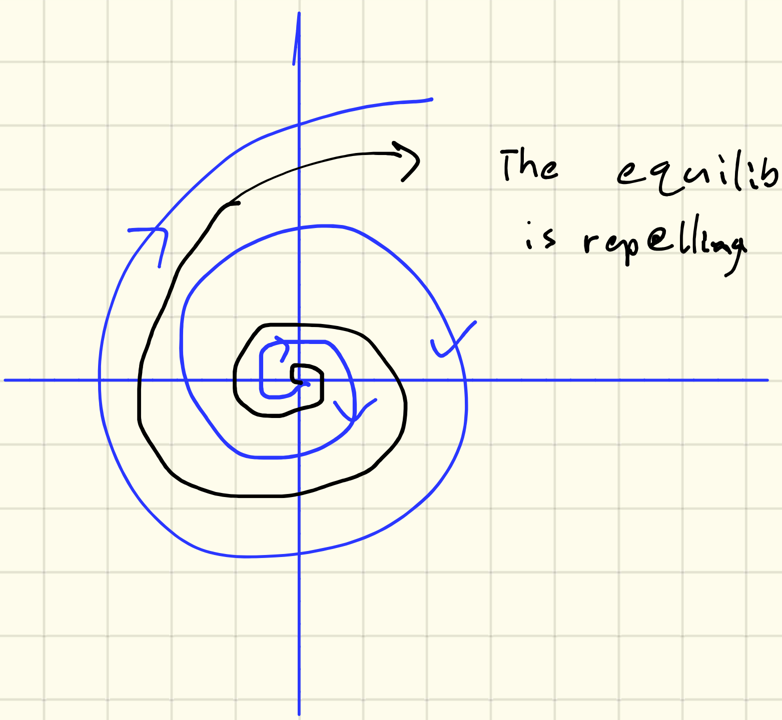
$x' = 3x - 2y$ $\qquad$ $x' = 0$, @ $(0,0)$, $y = \frac{2}{3}x$

$y' = x \quad y$ $\qquad$ $y' = 0$ @ $(0,0)$ $\quad y = -x$

<u>Slopes</u>

$m = \frac{y'}{x'}$

$\theta' = -\beta = -1$



The equilibrium at $(0,0)$ is repelling and unstable

$A = \begin{bmatrix} -3 & -5 \\ 5 & 3 \end{bmatrix}$

Find E.val $= 4i, -4i$, $\begin{bmatrix} -\frac{3}{5} + \frac{4}{5}i \\ 1 \end{bmatrix}$, $\begin{bmatrix} -\frac{3}{5} - \frac{4}{5}i \\ 1 \end{bmatrix}$

$e^{At} = \begin{bmatrix} \cos 4t & \sin 4t \\ -\sin 4t & \cos 4t \end{bmatrix}$

$\begin{bmatrix} \cos\alpha & \sin\beta \\ -\sin\beta & \cos\alpha \end{bmatrix}$

$\begin{matrix} x' \\ y' \end{matrix} = \begin{bmatrix} -3 & -5 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

$x' = -3x - 5y$

$y' = 5x + 3y$

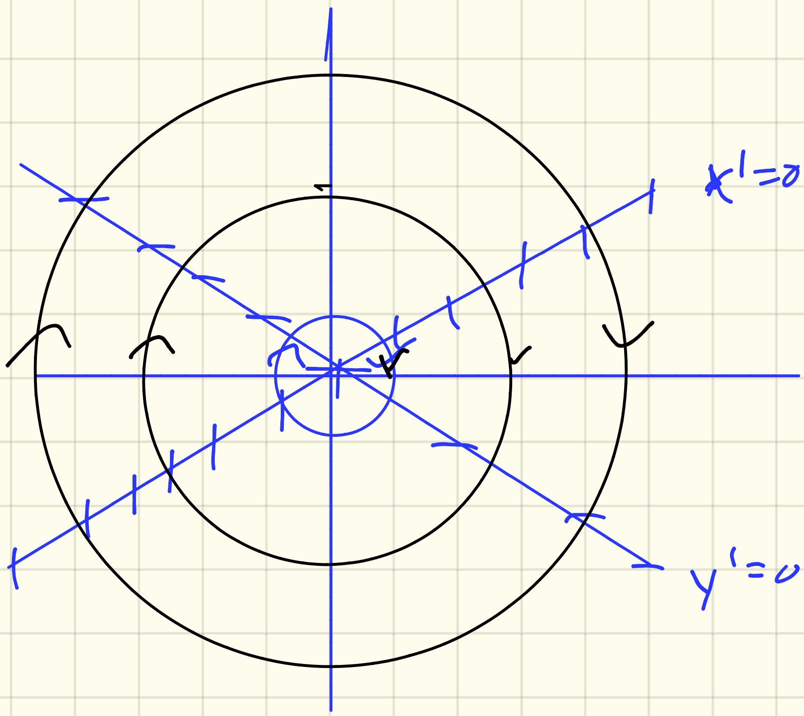
$x' = 0$ @ $(0,0)$, $y = \frac{3}{5}x$

$y' = 0$ @ $(0,0)$, $y = -\frac{3}{5}x$

Slopes

$m = \frac{y'}{x'} =$

$\theta' = .4$



equilibrium at $(0,0)$ is stable and not attracting

2) $A = \begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix}$

$\begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{matrix} 2 \\ 1 \end{matrix}$

Find E.val $= \lambda = 0$, $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$, Jord $\lambda=0$, $\begin{bmatrix} -1 \\ -1 \end{bmatrix}$ $\qquad \begin{bmatrix} -1 \\ -1 \end{bmatrix}$

$Pe^{tJ}P^{-1} = \begin{bmatrix} 2 & -1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 2t+1 & -4t \\ t & -2t+1 \end{bmatrix}$

$Pe^{tJ} = \begin{bmatrix} 2 & -1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 2 & 2t-1 \\ 1 & t-1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \begin{bmatrix} c_1 + tc_2 \end{bmatrix}$

$+ \begin{bmatrix} -1 \\ -1 \end{bmatrix} \begin{bmatrix} c_2 \end{bmatrix}$

$x(t) = 2c_1 + (2t-1)c_2$

$y(t) = c_1 + (t-1)c_2$

$\lim_{t\to\infty} x(t) = \pm\infty$ (depend on c_2)

$\lim_{t\to\infty} y(t) = \pm\infty$ (depends on c_2)

$\begin{matrix} x' \\ y' \end{matrix} = \begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

$x' = 2x - 4y \qquad x' = 0$ if $(0,0)$, $y = \frac{1}{2}x$

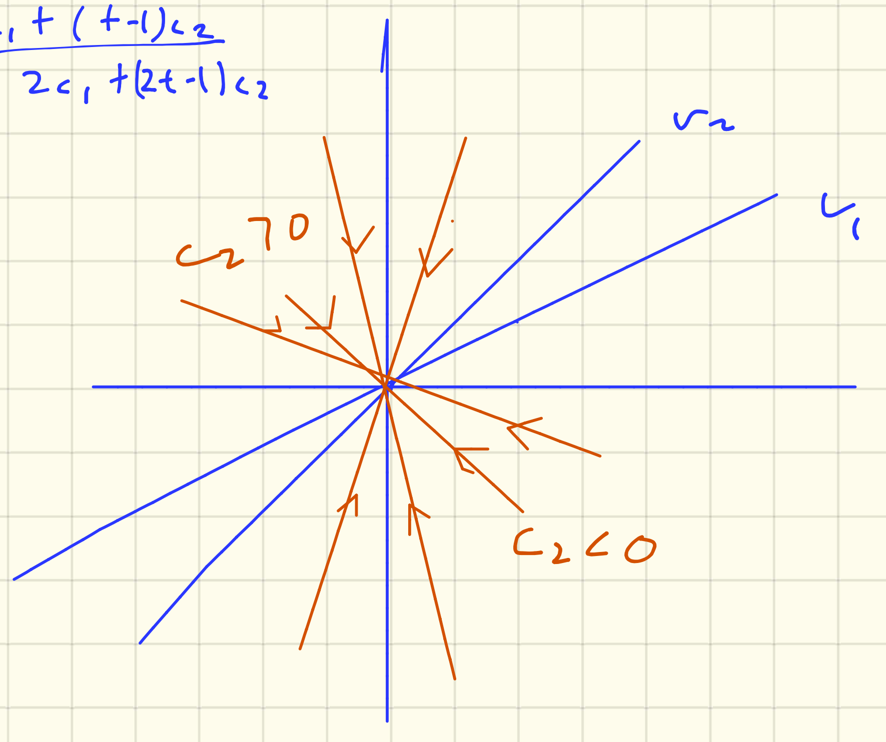
$y' = x - 2y \qquad y' = 0$ if $(0,0)$, $y = \frac{1}{2}x$

Slopes

$m = \dfrac{y'}{x'} = \dfrac{c_1 + (t-1)c_2}{2c_1 + (2t-1)c_2}$

$\lim_{t\to+\infty} m = 0$

$\lim_{t\to-\infty} m = 0$



2 ii) $A = \begin{bmatrix} 4 & -8 \\ -2 & 4 \end{bmatrix}$

Find E.val $= 0, 8,$ $\begin{bmatrix} 2 & -2 \\ 1 & 1 \end{bmatrix}$

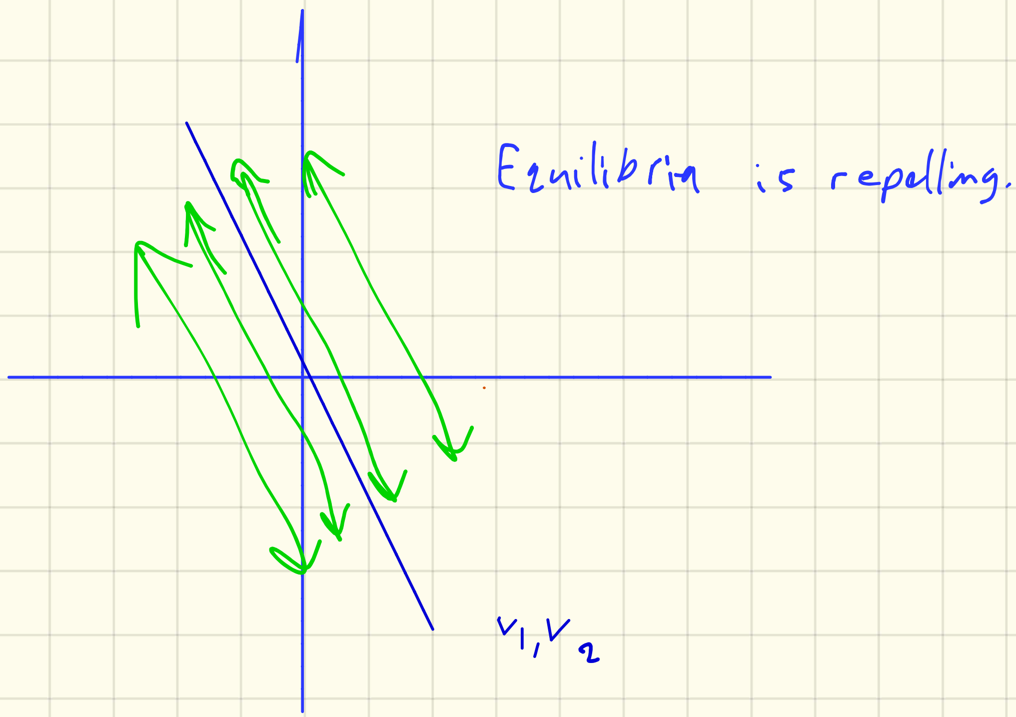
$= \begin{bmatrix} 2 & -2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} e^0 & 0 \\ 0 & e^8 \end{bmatrix} c = \begin{bmatrix} 2 & -2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & e^{8t} \end{bmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$

$\begin{matrix} x' \\ y' \end{matrix} = \begin{matrix} 4x - 8y \\ -2x + 4y \end{matrix}$

$= \begin{bmatrix} 2 \\ 1 \end{bmatrix} [c_1] + \begin{bmatrix} -2 \\ 1 \end{bmatrix} e^{8t} c_2$

$x' = 0$ at $(0,0)$, $y = \frac{1}{2}x$

$y' = 0$ at $(0,0)$, $y = \frac{1}{2}x$



3.) $x' = 6y - 3xy - 3y^2 = 3y(2-x-y)$ 5, -1

$y' = 3 - x - 3y + xy = (1-y)(3-x)$

$x' = 0$ if $3xy = 6y - 3y^2$, $x = 2 - y$, $y = 0$

$y' = 0$ if $x = 3$, $y = 1$,

Possible Equilibrium at $\begin{cases} \boxed{x = 1, y = 1} & \boxed{x = 3, y = -1} \\ \boxed{x = 3 \quad y = 0} \end{cases}$

local stability:

$$J = \begin{bmatrix} \frac{fx_1}{dx} & \frac{fx_1}{dy} \\ \frac{fx_2}{dx} & \frac{fx_2}{dy} \end{bmatrix} = \begin{bmatrix} -3y & 6 - 3x - 6y \\ -1 + y & -3 + x \end{bmatrix}$$

stability at (1, 1),

$$\begin{bmatrix} -3 & -3 \\ 0 & -2 \end{bmatrix}$$, eval are both negative real

asymptotic stable

stability at (3, -1)

$$= \begin{bmatrix} 3 & 3 \\ -2 & 0 \end{bmatrix}$$ one eval is positive, the other is zero

It is unstable and can't be determined without HOT.

stability at [3, 0]

$$= \begin{bmatrix} 0 & -3 \\ -1 & -6 \end{bmatrix}$$ one eval is negative, the other is zero.

it is unstable and can't be determined unless HO?

null cline

$x' = 0$ if $y = 0$, $y = 2 - x$

$y' = 0$ if $x = 3$, $y = 1$

stability at $(1, 1)$,

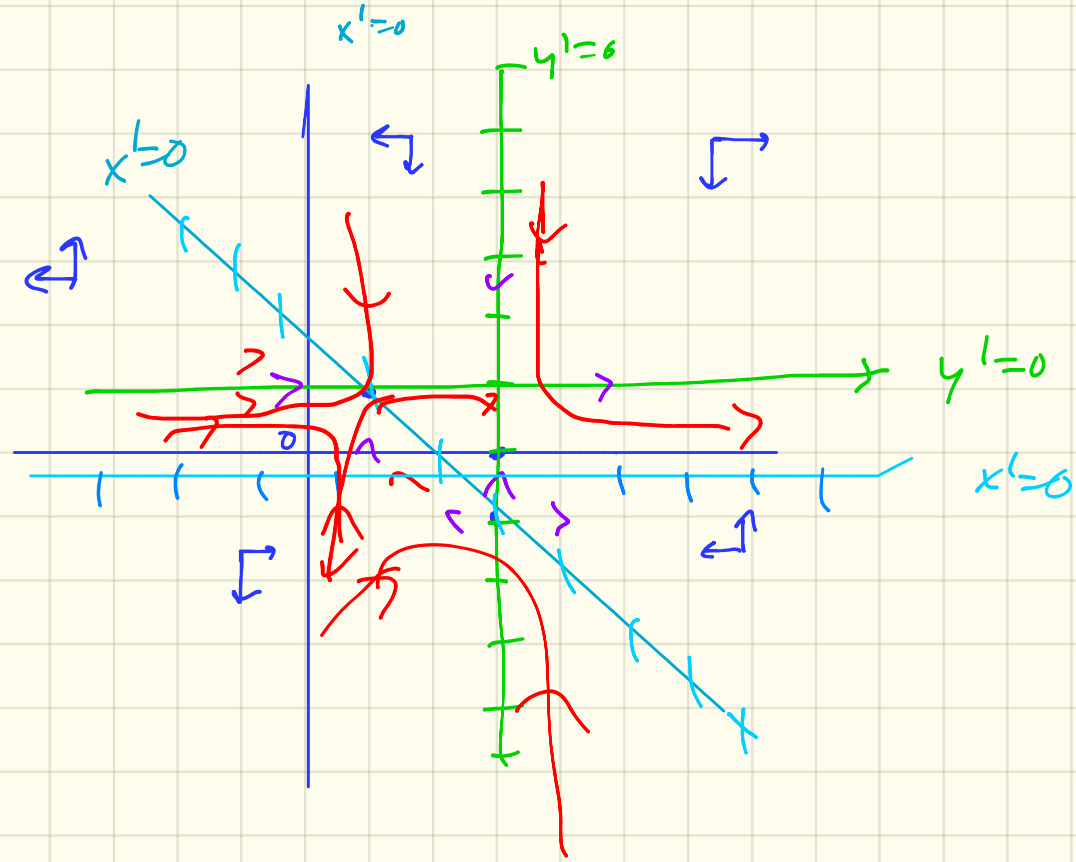
$$\begin{bmatrix} -3 & -3 \\ 0 & -2 \end{bmatrix}$$

stability at $(3, -1)$

$$= \begin{bmatrix} 3 & 3 \\ -2 & 0 \end{bmatrix}$$

stability at $[3, 0]$

$$= \begin{bmatrix} 0 & -3 \\ -1 & -6 \end{bmatrix}$$



D) unsure about what happens at $(3,-1)$ and $(3,0)$.