Parameter Estimation :

SANNVI SHARMA
102103699
3L025.

① given :- random sample ($x_1 \dots x_n$)

$$L(\theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} \times e^{-\left(\frac{x_i - \mu}{2\sigma^2}\right)}$$

natural log of likelihood function

$$\ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \left(\frac{-(x_i - \mu)^2}{2\sigma^2} - \frac{1}{2} \ln (2\pi\sigma^2) \right)$$

to find MLE, diff. log likelihood w.r.t. θ_1, θ_2

$$\frac{\partial}{\partial \theta_1} \ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \left(\frac{x_i - \mu}{\sigma^2} \right) = 0$$

$$\sum_{i=1}^{n} x_i - n\mu = 0.$$

$$\frac{\theta_1}{\mu} = \frac{1}{n} \sum_{i=1}^{n} x_i.$$

for θ_2 : $\frac{\partial}{\partial \theta_2} \ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \left(\frac{-(x_i - \theta_i)^2}{2(\theta_2)^2} + \frac{1}{2\theta_2} \right)$

$$\sum_{i=1}^{n} \left(\frac{x_i - \theta_i}{\theta_2} \right)^2 - \frac{n}{\theta_2} = 0$$

$$\frac{\theta_2^2}{\theta_2} = \frac{1}{n} \sum_{i=1}^{n} (x_i - \theta_i)^2$$

$$\theta_2 = 1/n \cdot \sum_{i=1}^{n} (x_i - \theta_i)^2$$

↳ sample variance

Find MLE of θ for binomial distribution $b(m, \theta)$ where m is known unlike

$$L(\theta) = \prod_{i=1}^{n} \binom{m}{x_i} \theta^{x_i} (1-\theta)^{m-x_i}$$

taking ln

$$\ln L(\theta) = \sum_{i=1}^{n} \left(\ln \binom{m}{x_i} + x_i \ln(\theta) + (m-x_i) \ln(1-\theta) \right)$$

$$\frac{\partial}{\partial \theta} \ln(L(\theta)) = \sum_{i=1}^{n} \left(\frac{x_i}{\theta} - \frac{m-x_i}{1-\theta} \right) = 0$$

solving for θ,

$$\sum_{i=1}^{n} \frac{x_i}{\theta} = \sum_{i=1}^{n} \frac{m-x_i}{1-\theta}$$

$$\sum_{i=1}^{n} x_i (1-\theta) = \sum_{i=1}^{n} (m-x_i)\theta$$

$$\theta \sum_{i=1}^{n} x_i = m \sum_{i=1}^{n} \theta$$

$$\theta = \frac{1}{m} \sum_{i=1}^{n} x_i$$

∴ MLE of θ is sample mean of obs.