**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

A: Given Data –

The data is normally distributed

μ = 45 min

σ = 8 min

Let X be the time its takes to complete the repair on customer’s car.

Probability (X > 50) = 1 - Probability (X ≤ 50)

Z = (X – μ) / σ

= (X – 45) /8

Probability (X ≤ 50) = Probability (Z ≤ (50 – 45) /8)

= Probability (Z ≤ 0.625)

= 73.4%

Probability that the servicemen will not meet this demand = 100 – 73.4

=26.6% or 0.2676

Or In python –

Probability that the servicemen will meet this demand -

(stats.norm.cdf(50,45,8)

= 0.7340

Probability that the servicemen will not meet this demand -

round(1 -stats.norm.cdf(50,45,8),5)

= 0.2659

Option B

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1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

A: a) Given Data –

The data is normally distributed

μ = 38 min

σ = 6 min

Let X be the no. of employees

Probability (X > 44) = 1 - Probability (X ≤ 44)

Z = (X – μ) / σ

= (X – 38) /6

Probability (X ≤ 44) = Probability (Z ≤ (44 – 38) /6)

= Probability (Z ≤ 1)

= 84.1345%

Probability that the employee will be greater than age of 44 = 100 – 84.1345

= 15.86 %

Probability (X < 38) = 0.5

Probability that the employee will be between 38 to 44 years of age = Probability (X < 44) - 0.5 = 0.84134 – 0.5 = 0.3413 = 34.13%

Or In python –

Probability that the no. of employees with age between 38 to 44 years of age

a = 1-stats.norm.cdf (38,38,6)

b = stats.norm.cdf (44,38,6)

c= b-a

c

= 0.3413

As the Probability of employees between 38 to 44 years of age is more as compared to Probability of employees greater than 44 years of age.

Therefore, More employees at the processing center are older than 44 than between 38 and 44 is **TRUE.**

b) Probability of employees less than age of 30 = Probability (X < 30)

Z = (X – μ) / σ

= (X – 38) /6

Probability (X ≤ 30) = Probability (Z ≤ (30 - 38)/6) = Probability (Z ≤ -1.333) = 9.12%

The number of employees with probability 0.0912 of them being under age 30 =

0.0912\*400 =36.48 or 36 employees

Or In python –

Probability that the no. of employees with less than 30 of age

stats.norm.cdf (30,38,6)

= 0.0912

No. of employees with age less than 30 years of age who attend training program

= 0.0912\*400

=36.48

Therefore, A training program for employees under the age of 30 at the center would be expected to attract about 36 employees is **TRUE.**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

A: If X ∼ N(µ1, σ1^2 ), and Y ∼ N(µ2, σ2^2 ) are two independent random variables then X + Y ∼ N (µ1 + µ2, σ1^2 + σ2^2 ) and

X − Y ∼ N (µ1 − µ2, σ1^2 + σ2^2)

Similarly if Z = aX + bY, where X and Y are as defined above, i.e. Z is linear combination of X and Y , then Z ∼ N(aµ1 + bµ2, a^2σ1^2 + b^2σ2^2 ).

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| Therefore, |
|  | 2X1~ N (2 u,4 σ^2) and |
|  | X1+X2 ~ N (µ + µ, σ^2 + σ^2) ~ N (2 u, 2σ^2 ) |
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Then the difference between

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

A: Since we need to find out the values of a and b, which are symmetric about the mean, such that the probability of random variable taking a value between them is 0.99, we have to work out in reverse order.

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|  | The Probability of getting value between a and b should be 0.99. |
|  | So the Probability of going wrong, or the Probability outside the a and b area is 0.01 (i.e. 1-0.99). |
|  | The Probability towards left from a = -0.005 (i.e. 0.01/2). |
|  | The Probability towards right from b = +0.005 (i.e. 0.01/2). |
|  | So since we have the probabilities of a and b, we need to calculate X, the random variable at a and b which has got these probabilities. |
|  | By finding the Standard Normal Variable Z (Z Value), we can calculate the X values. |
|  | Z=(X- μ) / σ |
|  | For Probability 0.005 the Z Value is -2.57 (from Z Table). |
|  | Z \* σ + μ = X |
|  | Z(-0.005)\*20+100 = -(-2.57)\*20+100 = 151.4 |
|  | Z(+0.005)\*20+100 = (-2.57)\*20+100 = 48.6 |
|  | So, option D is correct. |

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?