ME 609: Programming Project Phase 3

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Optimization Methods

Multi-variable Method : Conjugate Gradient Method.

Unidirectional Search :

- **Bracketing Method :** Bounding Phase
- **Accurate Method :** Bisection Method

Penalty Function: Bracket-Operator Penalty method

Algorithm: Conjugate Gradient Method

Step 1 Choose $x^{(0)}$ and termination parameters $\epsilon_1,\ \epsilon_2,\ \epsilon_3$. Set $k = 0$ and M.

Step 2 Find $\nabla f(x^{(0)})$ and set $s^{(0)} = -\nabla f(x^{(0)})$.

Step 3 Perform unidirectional search such that $f(x^{(0)} + \lambda^{(0)} s^{(0)})$ is minimum with termination parameter ϵ_1.
Set $x^{(1)} = x^{(0)} + \lambda^{(0)*} s^{(0)}$ and $k = 1$.
Calculate $\nabla f(x^{(1)})$.

Step 4 Set $s^{(k)} = -\nabla f(x^{(k)}) + \frac{||\nabla f(x^{(k)})||^2}{||\nabla f(x^{(k-1)})||^2} s^{(k-1)}$

Step 5 Find $\lambda^{*(k)}$ such that $f(x^{(k)} + \lambda^{(k)} s^{(k)})$ is minimum with termination parameter ϵ_1.
Set $x^{(k+1)} = x^{(k)} + \lambda^{*(k)} s^{(k)}$.
Check linear independence between $s^{(k)}$ and $s^{(k-1)}$.

Step 6 Is $\frac{||x^{(k+1)} - x^{(k)}||}{||x^{(k)}||} \le \epsilon_2$ or $||\nabla f(x^{(k+1)})|| \le \epsilon_3$? If yes, **Terminate**;
Else set $k = k + 1$ and go to Step 4.

Algorithm: Bounding Phase Method

Step 1 Choose an initial guess $x^{(0)}$ and an increment Δ. Set $k = 0$.

Step 2 If $f(x^{(0)} - |\Delta|) \geq f(x^{(0)}) \geq f(x^{(0)} + |\Delta|)$, then Δ is positive;
Else if $f(x^{(0)} - |\Delta|) \leq f(x^{(0)}) \leq f(x^{(0)} + |\Delta|)$, then Δ is negative;
Else go to Step 1.

Step 3 Set $x^{(k+1)} = x^{(k)} + 2^k \Delta$, (other exponent can be used).

Step 4 If $f(x^{(k+1)}) < f(x^{(k)})$, set $k = k + 1$ and go to Step 3;
Else the minimum lies in the interval $(x^{(k-1)}, x^{(k+1)})$ and **Terminate**.

Algorithm: Bisection Method

Step 1 Choose two points a and b such that $f'(a) < 0$ and $f'(b) > 0$. Also, choose a small number ϵ.
Set $x_1 = a$ and $x_2 = b$.

Step 2 Calculate $z = (x_2 + x_1)/2$ and evaluate $f'(z)$.

Step 3 If $|f'(z)| \leq \epsilon$, **Terminate**;
Else if $f'(z) < 0$ set $x_1 = z$ and go to Step 2;
Else if $f'(z) > 0$ set $x_2 = z$ and go to Step 2.

Algorithm: Penalty Function Method

Step 1 Choose two termination parameters ϵ_1, ϵ_2, an initial solution $x^{(0)}$, a penalty term Ω, and an initial penalty parameter $R^{(0)}$. Choose a parameter c to update R such that $0 < c < 1$ is used for interior penalty terms and $c \geq 1$ is used for exterior penalty terms. Set $t = 0$.

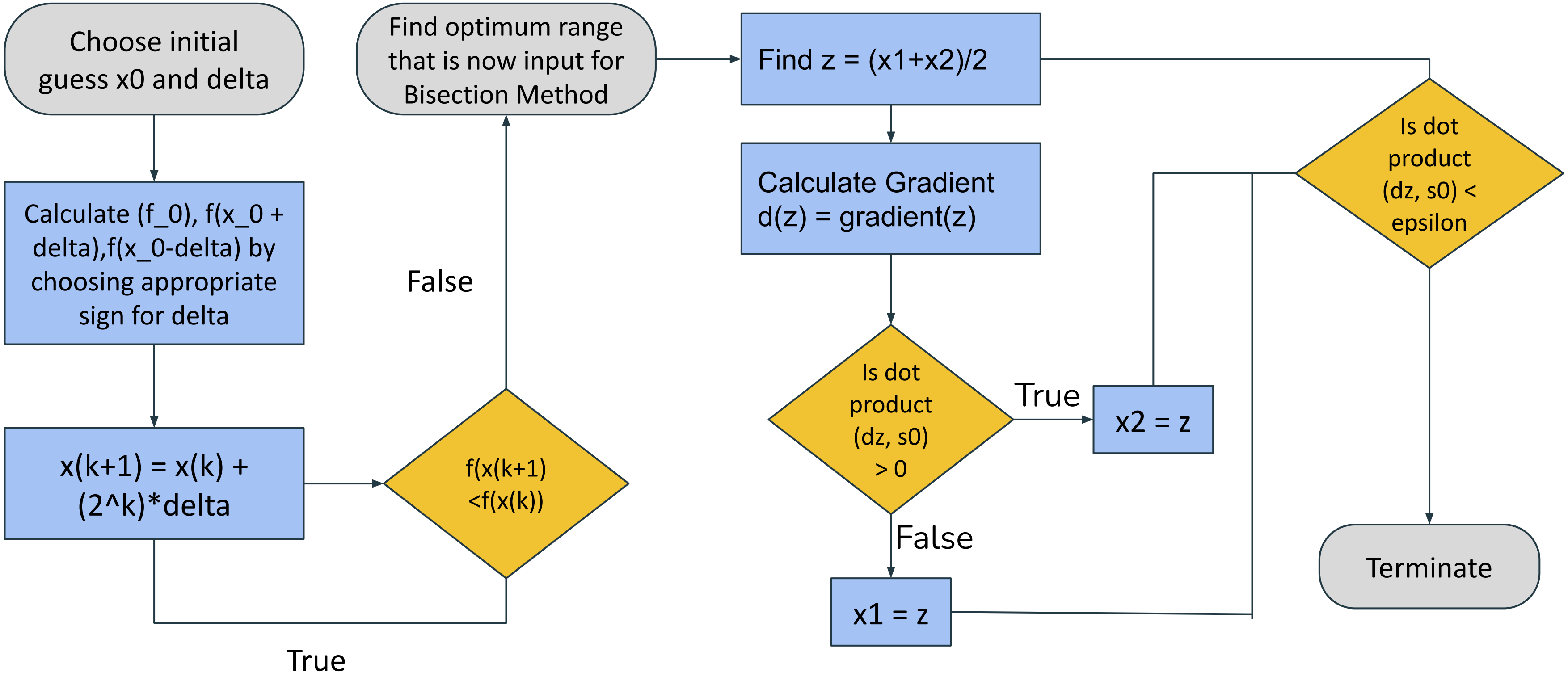
Step 2 Form $P(x^{(t)}, R^{(t)}) = f(x^{(t)}) + \Omega(R^{(t)}, g(x^{(t)}), h(x^{(t)}))$.

Step 3 Starting with $x^{(t)}$, find $x^{(t+1)}$ such that $P(x^{(t)}, R^{(t)})$ is minimum for a fixed value of $R^{(t)}$. Use ϵ_1 to terminate the unconstrained search.

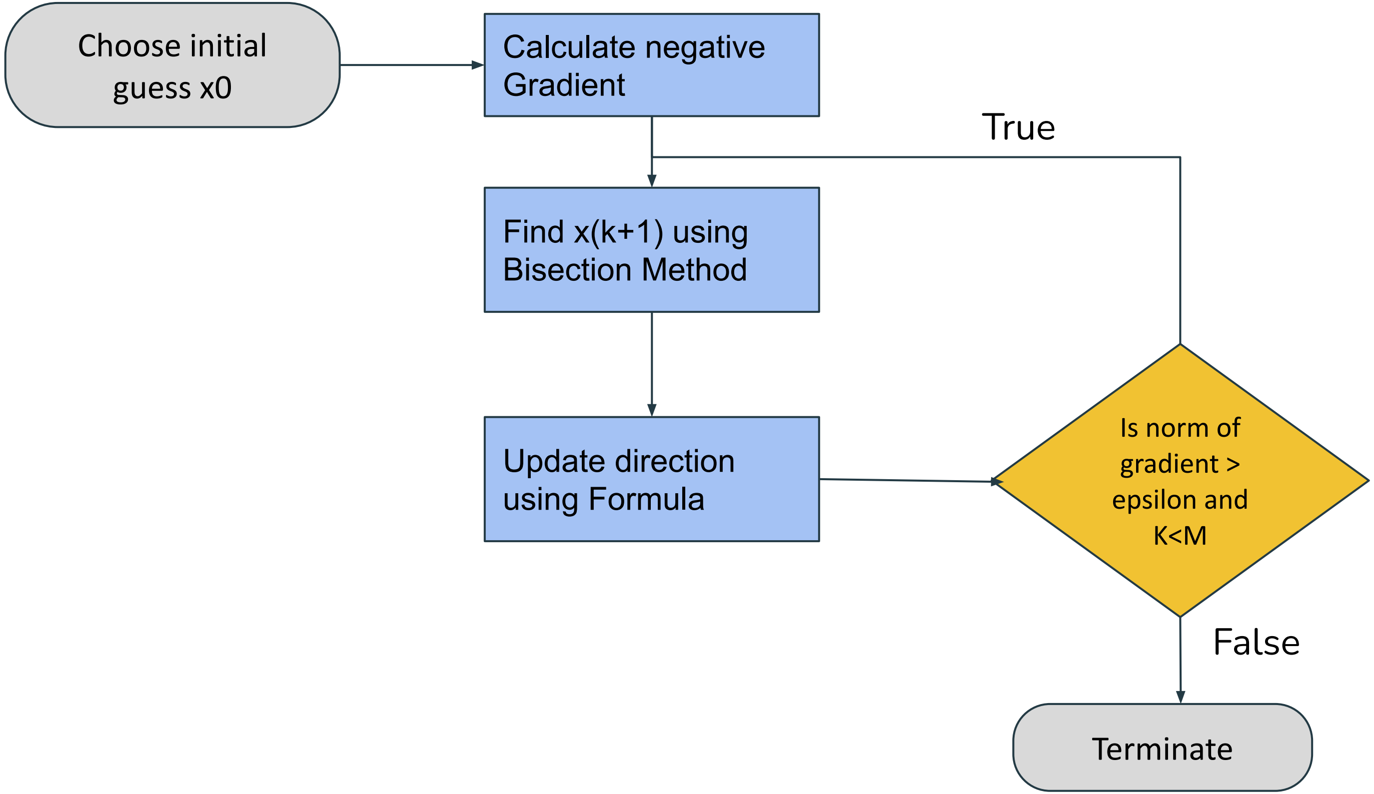
Step 4 Is $\left|P(x^{(t+1)}, R^{(t)}) - P(x^{(t)}, R^{(t-1)})\right| \leq \epsilon_2$?
If yes, set $x^{(T)} = x^{(t+1)}$ and **terminate**;
Else go to Step 5.

Step 5 Choose $R^{(t+1)} = c\, R^{(t)}$. Set $t = t + 1$ and go to Step 2.

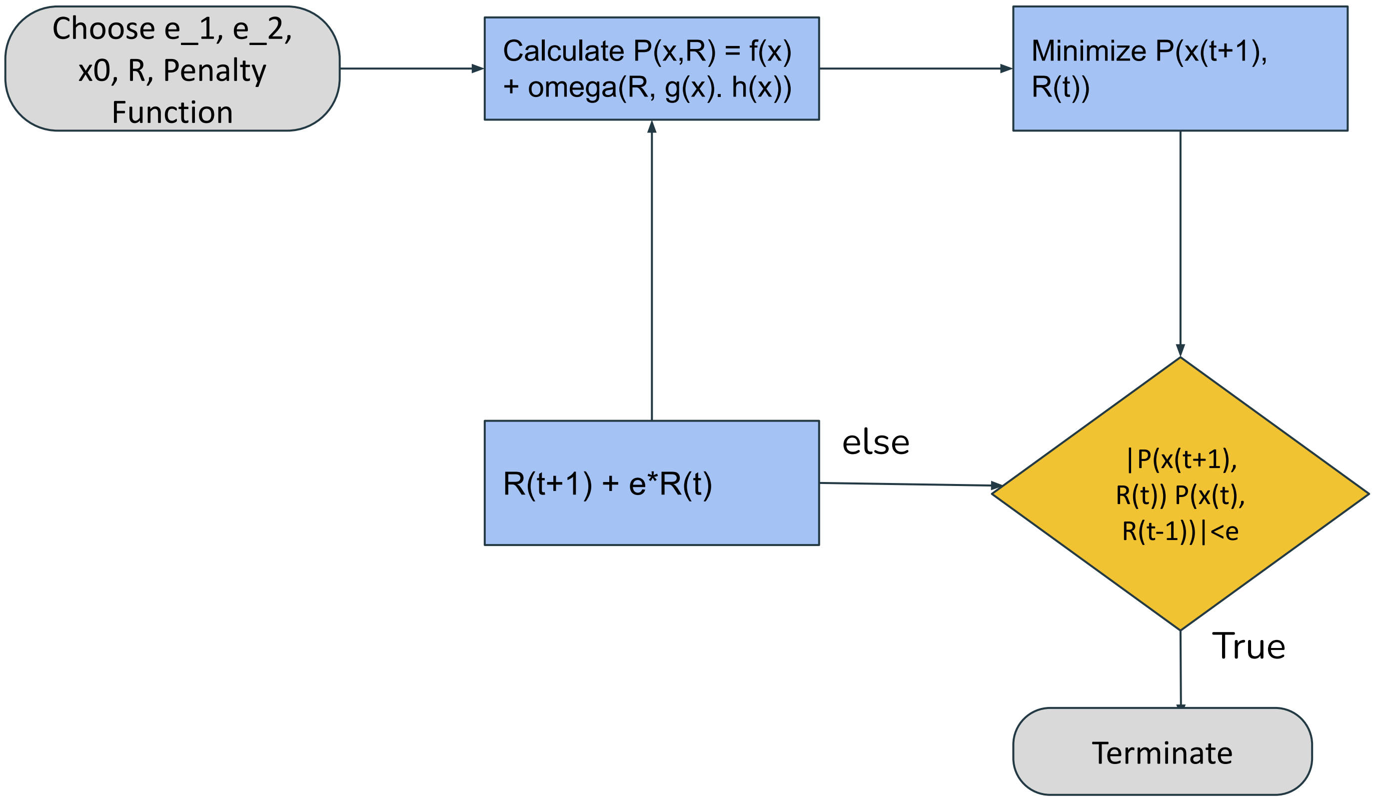
Flowchart: Bounding Phase and Bisection Method



Flowchart: Conjugate Gradient Method



Flowchart: Penalty Function Method



Problem 1:

$$\min \boldsymbol{f}(\boldsymbol{x}) = (\boldsymbol{x}_1 - \mathbf{10})^3 + (\boldsymbol{x}_2 - \mathbf{20})^3,$$

$$\textit{subject to } \boldsymbol{g}_1(\boldsymbol{x}) = (\boldsymbol{x}_1 - \mathbf{5})^2 + (\boldsymbol{x}_2 - \mathbf{5})^2 - 100 \geq 0,$$

$$\boldsymbol{g}_2(\boldsymbol{x}) = (\boldsymbol{x}_1 - \mathbf{6})^2 + (\boldsymbol{x}_2 - \mathbf{5})^2 - 82.81 \leq 0,$$

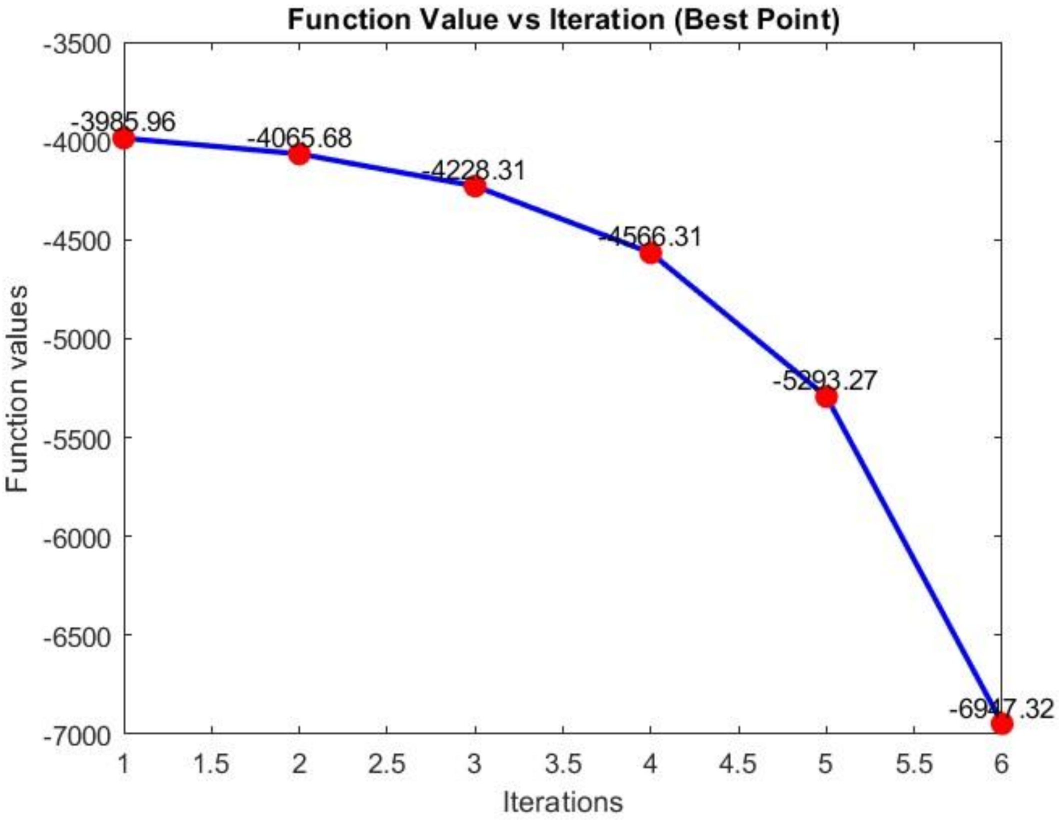
$$\mathbf{13} \leq \boldsymbol{x}_1 \leq \mathbf{20}, \qquad \mathbf{0} \leq \boldsymbol{x}_2 \leq \mathbf{4}.$$

Results for 10 different Initial Guess

Initial Guess	Function Value	Solution points
(13,2)	-6090.8965	[13.020662820562922, 1.6506104640284938]
(13,4)	-6947.319357	[13.34038640253919, 0.868444939496285]
(13,6)	-6275.515141	[13.930187343109697, 0.28150073682384624]
(14,2)	-6108.866946	[13.995732023037396, 1.650026023292236]
(14,3)	-6314.701978	[14.010579950190333, 1.4500361085967293]
(15,4)	-6893.180528	[14.693839232989921, 0.8649137835230432]
(16,5)	-6302.409676	[14.391386815037816, 0.1428709591823458]
(17,4)	-6481.267365	[15.768891899841343, 1.100539214660121]
(18,4)	-6091.448321	[16.264723425338886, 1.3710619616597277]
(18,5)	-7091.936561	[14.291113453468029, 0.1542856522465006]

Best	-6947.319357
Worst	-7091.936561
Mean	-6459.754237
Median	-6308.555827
Standard Deviation	360.5917475

Question 1 : For Best point(13,4)



Problem 2:

$$max\ \boldsymbol{f}(\boldsymbol{x}) = \frac{\boldsymbol{sin^3(2\pi x_1)\, sin(2\pi x_2)}}{\boldsymbol{x_1^3(x_1 + x_2)}},$$

$$subject\ to\ \boldsymbol{g_1(x) = x_1^2 - x_2 + 1 \le 0},$$

$$\boldsymbol{g_2(x) = 1 - x_1 + (x_2 - 4)^2 \le 0},$$

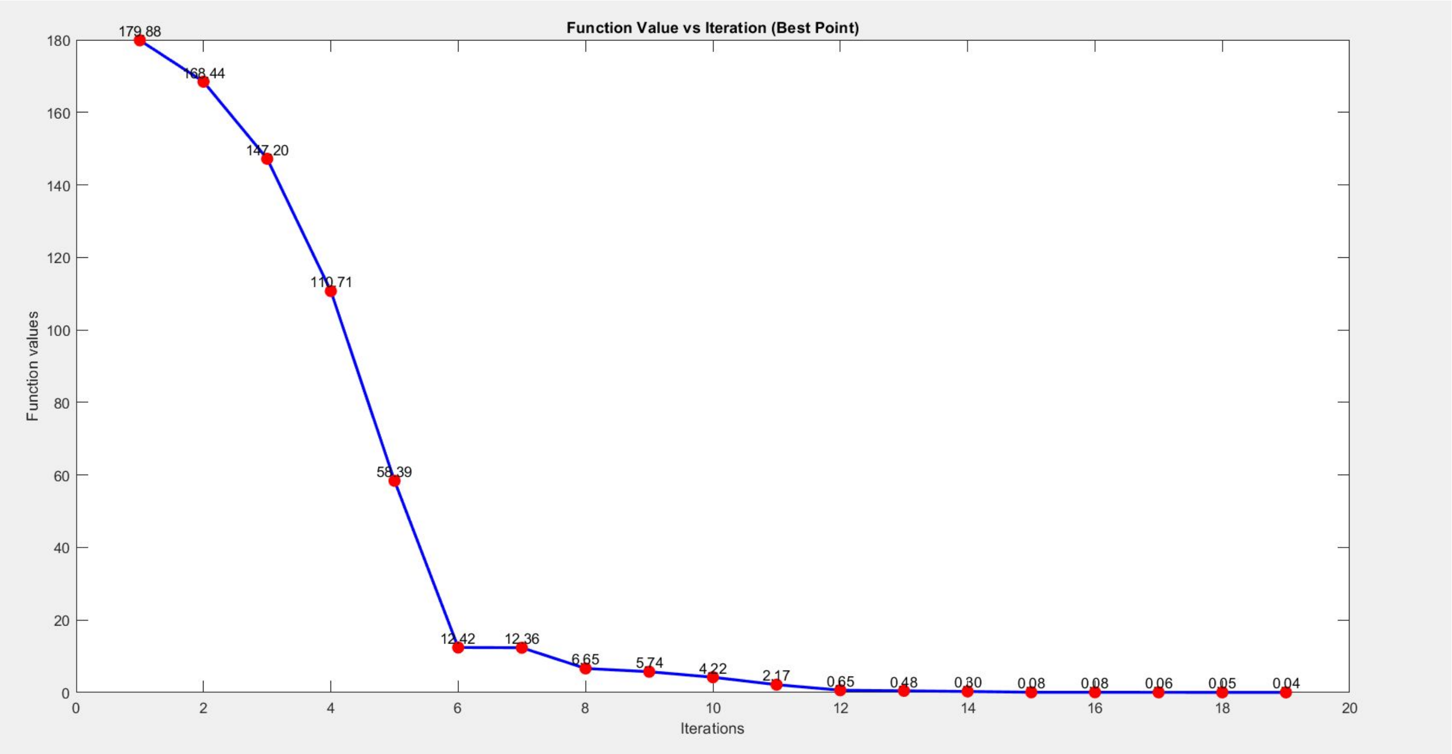
$$\boldsymbol{0 \le x_1 \le 10, \qquad 0 \le x_2 \le 10}$$

Results for 10 different Initial Guess

Initial Guess	Function Value	Solution points
(1,1)	-0.094123854	[1.2274839433478206, 4.275450390039435]
(2,2)	-0.05144141	[0.8748213047971365, 3.5847183176486586]
(3,3)	-0.105209845	[1.2239268134763213, 3.2453490129098634]
(4,4)	-1.285086061	[-0.18830203801803316, 4.456710029346844]
(5,5)	-0.028185561	[1.7158348595414878, 4.772071161847873]
(6,6)	-0.013299309	[1.8334177657939965, 4.840411851201562]
(7,7)	0.044301429	[1.2788939432726758, 5.080593880398601]
(8,8)	-0.023970236	[1.7312425086803018, 4.842756641030849]
(9,9)	-0.02848253	[1.7525157099952897, 4.758521169152821]
(10,10)	-0.500352316	[0.40472911689005264, 4.185259352341292]

Best	0.044301429
Worst	-1.285086061
Mean	-0.208584969
Median	-0.03996197
Standard Deviation	0.386473298

Question 2 : For Best point(7,7)



Results and Conclusions

- We have used bracket-operator penalty method for the constrained optimization problems provided to us. The bracket-operator penalty method penalizes the violation of constraints and guides the search towards the feasible solutions.
- This code takes input of no. of variables and the initial points after which it computers the direction for unidirectional search by calculating the negative gradient of the function at input points.
- The result of bounding phase is passed to the bisection method for accurate computation of lambda so that we obtain the most optimum point in the unidirectional search direction. Here, we ensure that the point that we want follows all the constraints and the necessary bounds.
- Upon finding the optimum value of lambda, the conjugate gradient function finds the next point in the search space, updates the search direction and again the whole process is repeated.
- When the distance of the new generated point and previous point is less than the preset epsilon, the algorithm terminates.
- The direction of maximum reduction in objective function value is the negative gradient of the point in n-dimensional space.