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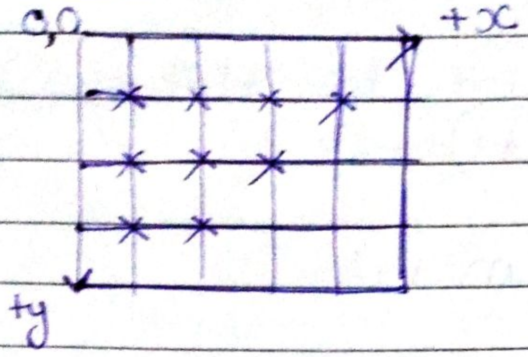
Graphics

Screen



(1,1) (3,4)

Pixcel → on
→ off

$y = c$, $m = 0$

$y = 0 \Rightarrow x\text{-axis} \Rightarrow m = 0$

$x = C$, $m = \infty$

$x = 0$, $m = \infty$

$x = y$, $m = 1$

$x = -y$, $m = -1$

$x = y$

$x \frac{dy}{dx} = \frac{dy}{dx}$

$1 = \frac{dy}{dx} = m$

$y = mx + c$

DDA line drawing Algorithm

(Digital Differential algo)

(x_1, y_1) (x_2, y_2)

$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

$y_i = mx_i + c$

$$y_{i+1} = mx_{i+1} + C$$

$$x_{i+1} = x_i + 1$$

$$y_{i+1} = m(x_i + 1) + C$$

$$y_{i+1} = mx_i + m + C$$

$$y_{i+1} = (mx_i + C) + m$$

$$y_{i+1} = y_i + m$$

This DDA line algo only valid for $-1 \le m \le 1$

Q1 Generate all the points between the end points of there line (5,8) (9,11).

(5, 8) (9, 11)

x_1, y_1 x_2, y_2

x	y	
5	8	(5, 8)
6	$8 + \frac{3}{4} = 8.75 \Rightarrow 9$	(6, 9)
7	$8.75 + \frac{3}{4} \Rightarrow 9.5$	(7, 10)
8	$9.5 + \frac{3}{4} \Rightarrow 10.25$	(8, 10)
9	$10.25 + \frac{3}{4} \Rightarrow 11$	(9, 11)

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$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{11-5}{9-5} = \frac{3}{4}$$

Q2 Calculate all the points between the end points (1, 4) and (2, 10)

$$m = \frac{10-4}{2-1} = \frac{6}{1} = 6$$

x	y	
1	4	(1, 4)
2	4+6	(2, 10)

* We cann't apply DDA line algo because $\underbrace{\text{slope}}_{[-1,\,1]}$ is > 1.

Disadvantages :-

-> It only works for the slope between $-1 \leq m \leq 1$.

-> There is round of error in almost all the points.

For this we are moving to Bresenham's line drawing algo. Which is also called mid point.

4 special cases of DDA line algo

1) Horizontal line

$y = c \Rightarrow m = 0$

(1,10) to (10,10)

x	y
1	10
2	10+0 = 10
3	10+0 = 10
.	
.	
10	10+0 = 10

'y' remain same for all the values, just increase 'x'. Don't follow the whole process.

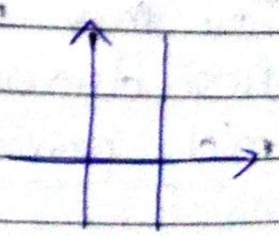
2) vertical line

$x = c \Rightarrow m = \infty$

(10, 3) to (10, 9)

'x' remains same for all the values, just increase 'y'.

x	y
10	3
10	4
10	9



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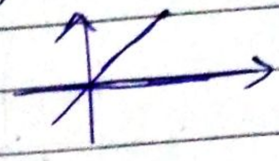
3) Change in x = Change in y

$$m = \frac{\Delta x}{\Delta y} = 1$$



(1, 1) to (6, 6)

x	y
1	1
2	2
3	3
⋮	
6	6

4) $m > 1$

$$\frac{\Delta y}{\Delta x} > 1$$

$$\Delta y > \Delta x$$

$$y_{i+1} = y_i + 1$$

$$x_{i+1} = x_i + \frac{1}{m}$$

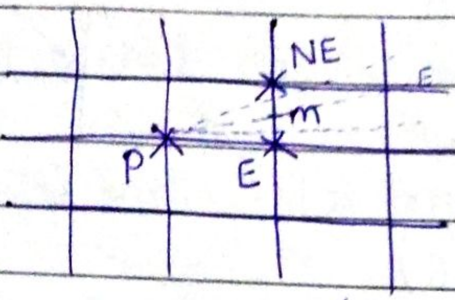
(1, 4) and (2, 10)

$m = 6$ $\frac{1}{m} = \frac{1}{6}$

4^{th} special case of DDA

x	y	
1	4	(1,4)
$1 + \frac{1}{6} = \frac{7}{6} = 1.1 = 1$	$4+1 = 5$	(1,5)
$\frac{7}{6} + \frac{1}{6} = \frac{8}{6} = 1$	$5+1 = 6$	(1,6)
$\frac{8}{6} + \frac{1}{6} = 1.5 = 2$	$6+1 = 7$	(2,7)
$\frac{9}{6} + \frac{1}{6} = 2$	8	(2,8)
$\frac{10}{6} + \frac{1}{6} = 2$	9	(2,9)
$\frac{11}{6} + \frac{1}{6} = 2$	10	(2,10)

Mid Point



$P = x_p, y_p$

(East) $E = x_p + 1, y_p$

(North East) $NE = x_p + 1, y_p + 1$

$$m = x, y$$

$$= \left(\frac{x_p + 1 + x_p + 1}{2}, \frac{y_p + 1 + y_p}{2} \right)$$

$$= \left(\frac{2 + 2x_p}{2}, \frac{2y_p + 1}{2} \right)$$

$$= \left(x_p + 1, y_p + \frac{1}{2} \right)$$

If line made from point P is above m ⇒ NE
" " " " " " " below " ⇒ E

'd' is the ~~deci~~ decision variable which is equal to the line m.

$$d = F(m)$$

$$y = mx + B$$

$$y = \frac{dy}{dx} x + B$$

$$dx \cdot y = dy \cdot x + B \cdot dx$$

$$F(x, y) = dy \cdot x - dx \cdot y + B(dx)$$

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$F(x,y) = ax - by + c$

$a = dy \qquad b = dx$

$d = F\left(x_p+1, y_p + \frac{1}{2}\right)$

$d = a(x_p+1) - b\left(y_p + \frac{1}{2}\right) + c$

$d > 0$ The value of 'd' is positive and 'm' is below the line $\Rightarrow$ NE

$d < 0$ The value of 'd' is less than zero, the 'm' is above the line $\Rightarrow$ E

$d = 0$ The value of 'd' $= 0$ then 'm' is on the line $\Rightarrow$ E

Choose E

New $P = x_p+1, y_p$

New $E = x_p+2, y_p$

New $NE = x_p+2, y_p+1$

$$\text{New } m = \left(\frac{x_p+2+x_p+2}{2}, \frac{y_p+y_p+1}{2}\right)$$

$$= \left(\frac{2x_p+4}{2}, \frac{2y_p+1}{2}\right)$$

$$= \left(x_p+2, y_p+\frac{1}{2}\right)$$

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$$\text{New } d = F(\text{New } M)$$
$$= a(x_p+2) - b\left(y_p+\frac{1}{2}\right) + c \quad \text{— (1)}$$

$$\text{Old } d = a(x_p+1) - b\left(y_p+\frac{1}{2}\right) + c \quad \text{— (2)}$$

$$\Delta d_E = \text{New } d - \text{Old } d$$
$$= (1) - (2)$$
$$= a = dy$$

Choose NE

$$\text{New } P = x_p+1,\ y_p+1$$

$$\text{New } E = x_p+2,\ y_p+1$$

$$\text{New } NE = x_p+2,\ y_p+2$$

$$\text{New } M = \left(\frac{2x_p+4}{2},\ \frac{2y_p+3}{2}\right)$$
$$= \left(x_p+2,\ y_p+\frac{3}{2}\right)$$

$$\text{New } d = F(\text{new } M)$$
$$= a(x_p+2) - b\left(y_p+\frac{3}{2}\right) + c \quad \text{— (1)}$$

$$\text{Old } d = a(x_p+1) - b\left(y_p+\frac{1}{2}\right) + c \quad \text{— (2)}$$

$$\Delta d_{NE} = \text{New } d - \text{Old } d$$
$$= a - b$$
$$\Delta d_{NE} = dy - dx$$

$$\Delta d_E = a = dy$$

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Initial value of d

(1) → x_1, y_1 = First P

Initial m = $x_1 + 1, y_1 + \frac{1}{2}$

Initial d = F(Initial m)

$= a(x_1 + 1) - b(y_1 + \frac{1}{2}) + C$

$= \underbrace{ax_1 - by_1 + C}_{} + a - \frac{b}{2}$

This is for sure on the line, So $ax_1 - by_1 + C = 0$

Initial d = $a - \frac{b}{2} = dy - \frac{dx}{2}$

Ques Generate all the points for the line A(5,8), B(9,11), using Bresenham's line drawing algorithm.

$a = dy = y_2 - y_1$
$a = 11 - 8$
$a = 3$

$b = dx = x_2 - x_1$
$b = 9 - 5 = 4$

x	y	d
5	8	Initial d = $dy - \frac{dx}{2} = 3 - \frac{4}{2}$ (NE) $= 1 > 0$
6	9	$d = \text{Old } d + \Delta d_{NE} = 1 + \Delta d_{NE}$ $= 1 + dy - dx$ $= 1 + 3 - 4 = 0$ (E)
7	9	$d = \text{old } d + \Delta d_E = 0 + dy$ $= 0 + 3 = 3 > 0$ (NE)
8	10	$d = \text{Old } d + \Delta d_{NE} = 3 + dy - dx$ $= 3 + 3 - 4 = 6 > 0$ (NE)

x	y
9	11

Mid Point Circle Line Algo / Bresenham's Circle line Algo

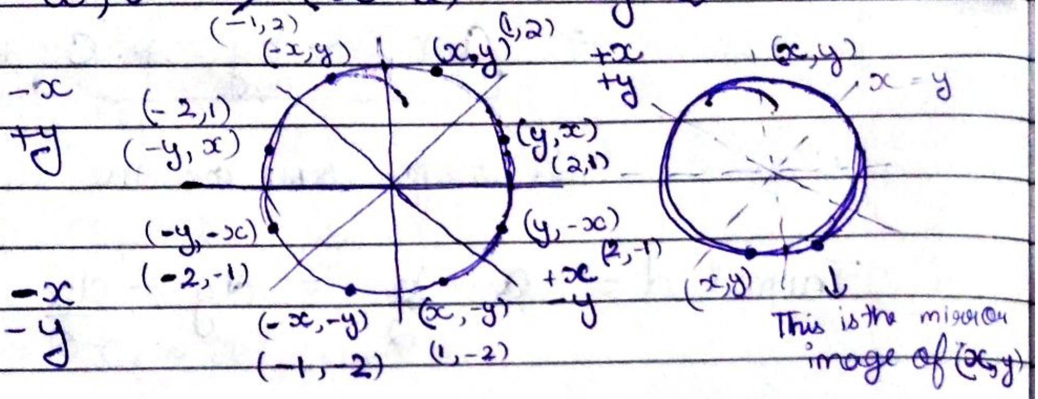
$$x^2 + y^2 = R^2 \Rightarrow y = \sqrt{R^2 - x^2}$$

$$F(x,y) = x^2 + y^2 - R^2$$

R = Radius

Centre = 0,0

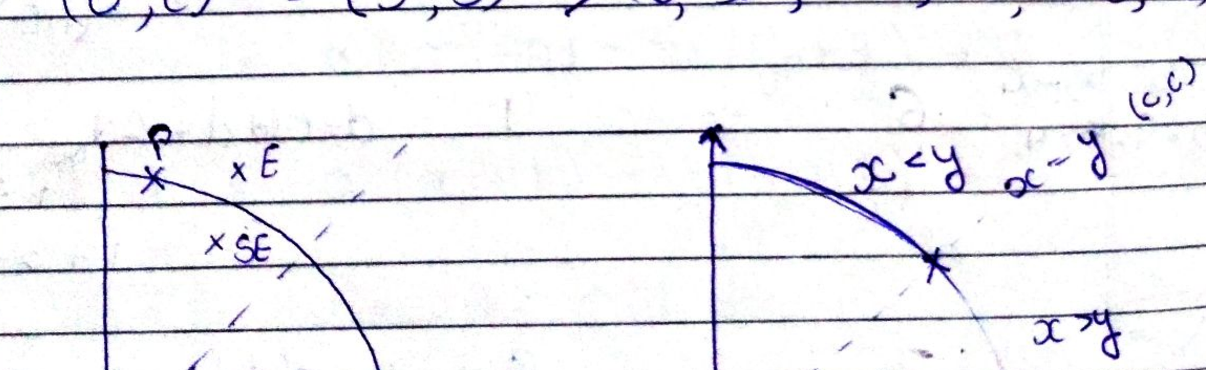
Centre = a,b $\Rightarrow (x-a)^2 + (y-b)^2 = R^2$



$(x,0) = (3,0) \Rightarrow (-3,0), (0,3), (0,-3)$

$(0,y) = (\quad) \Rightarrow$

$(c,c) = (3,3) \Rightarrow (3,-3), (-3,3), (-3,-3)$



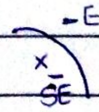
P = x_p, y_p ; E = x_p+1, y_p ; SE = x_p+1, y_p-1 ;

M = $x_p+1, y_p - \frac{1}{2}$

$d = F(m)$

$d = F(x_p+1, y_p - \frac{1}{2})$

$d > 0 \Rightarrow$ The point m is outside the circle
$\Rightarrow SE$



$d < 0 \Rightarrow$ The point m is inside the circle $\Rightarrow E$

$d = 0 \Rightarrow$ The point m is on the circle $\Rightarrow SE$.

Choose E

New $P = x_p+1, y_p$

New $E = x_p+2, y_p$

New $SE = x_p+2, y_p-1$

New $m = (x_p+2, y_p - \frac{1}{2})$

New $d = F(x_p+2, y_p - \frac{1}{2})$

New $d = (x_p+2)^2 + (y_p - \frac{1}{2})^2 - R^2$ — (1)

Old $d = F(x_p+1, y_p - \frac{1}{2})$

$= (x_p+1)^2 + (y_p - \frac{1}{2})^2 - R^2$ — (2)

$\Delta d_E = d_{new} - d_{old}$

$=$ (1) $-$ (2)

$\Delta d_E = 2x_p + 3$

Choose SE

New P = $x_p + 1$, ~~x_p~~ $y_p - 1$

New E = $x_p + 2$, $y_p - 1$

New SE = $x_p + 2$, $y_p - 2$

New M = $x_p + 2$, $y_p - \frac{3}{2}$

New d = $F(x_p + 2, y_p - \frac{3}{2})$

$= (x_p + 2)^2 + (y_p - \frac{3}{2})^2 - R^2$ — 1

Old d = $F(x_p + 1, y_p - \frac{1}{2})$

$= (x_p + 1)^2 + (y_p - \frac{1}{2})^2 - R^2$ — 2

$\Delta d_{SE} = d_{new} - d_{old}$

$= 1 - 2$

$(x_p+2)^2 - (x_p+1)^2$

$\cancel{x_p^2} + 4 + 4x_p - \cancel{x_p^2} - 1 - 2x_p$

$2x_p + 3$

$\cancel{y_p^2} + \frac{9}{4} - 3y_p - \cancel{y_p^2} - \frac{1}{4} + y_p$

$-2y_p - 2$

$\Delta d_{SE} = 2x_p + 3 \mp 4y_p - 2$

$= 2x_p \mp 2y_p + 5$

$\cancel{x_p^2} + 4 + 4x_p + \cancel{y_p^2} + \frac{9}{4} - 3y_p - \cancel{R^2} - \cancel{x_p^2} - 1 - 2x_p - \cancel{y_p^2} - \frac{1}{4} + y_p + \cancel{R^2}$

$= 2x_p - 2y_p + 5$

The change in d for the circle is a linear function in x and y, where as for the line it was a constant.

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First P = 0, R

Initial $M = \left(0+1,\ R-\frac{1}{2}\right) = \left(1, R-\frac{1}{2}\right)$

Initial d = F(Initial M)

$= F\left(1, R-\frac{1}{2}\right)$

$= 1^2 + \left(R-\frac{1}{2}\right)^2 - R^2$

Initial $d = \frac{5}{4} - R$

Que 1 Generate all the points for the circle centre at (0, 0) with the radius 6. Using Bresenham's circle line algorithm.

x	y	d
0	6	Initial $d = \frac{5}{4} - R = \frac{5}{4} - 6 = \frac{-19}{4} < 0$ (E)
1	6	$d_{new} = d_{old} + \Delta d_E$
		$= \frac{-19}{4} + 2x_p + 3$
		$= -\frac{19}{4} + 3 = -\frac{7}{4} < 0$ (E)
2	6	$d_{new} = d_{old} + \Delta d_E$
		$= -\frac{7}{4} + 2x_p + 3$
		$= -\frac{7}{4} + 2 + 3 = -\frac{7}{4} + 5 = \frac{13}{4}$ (SE)
3	5	~~$d_{new} = \frac{13}{4} + 4 + 3 = \frac{13+7}{4} = \frac{41}{4}$~~
		$d_{new} = \frac{13}{4} + 4 - 2(6) + 5 = \frac{1}{4} > 0$ (SE)
4	4	

$20 - \frac{7}{4} = \frac{13}{4}$

$\frac{20+13}{41}$

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Using 8 way symmetry property of circle we will replicate the points.

$(0,6) = (6,0), (0,-6), (-6,0)$

$(1,6) = (1,-6), (-1,-6), (-1,6), (6,1), (6,-1), (-6,1), (-6,-1)$

$(2,6) = (2,-6), (-2,-6), (-2,6), (6,2), (6,-2), (-6,2), (-6,-2)$

$(3,5) = (3,-5), (-3,5), (-3,-5), (5,3), (5,-3), (-5,3) (-5,-3)$

$(4,4) = (-4,-4), (-4,4), (4,-4)$.

Que 2 Calculate all the points of the circle whose $\cancel{x^2}$ equation is $x^2 + y^2 - 25 = 0$.

$R = 5 \qquad R \neq 25$

Que 3 Calculate all the points of the circle whose equation is $(x-1)^2 + (y-2)^2 = 16$

$R = 4 \qquad (1, 2) = \text{centre}$

Lets translate the ~~circle~~ center of the circle to the point (0,0)

$\rightarrow x^2 + y^2 = 4^2$

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Ans 2 centre (0, 0) Radius = 5

x	y	d
0	5	new d $= \frac{5}{4} - 5 = \frac{5-20}{4} = \frac{-15}{4} < 0$ (E)
1	5	new d $= \frac{-15}{4} + 2x_p + 3 = \frac{-15}{4} + 3 = \frac{-3}{4} < 0$ (E)
2	5	new d $= \frac{-3}{4} + 2x_p + 3 = \frac{-3}{4} + 2 + 3 = \frac{17}{4} > 0$ (SE)
3	4	new d $= \frac{17}{4} + 2x_p - 2y_p + 5 = \frac{17}{4} + 4 - 10 + 5 = \frac{17}{4} - 1 = \frac{13}{4} > 0$ (SE)
4	3	new d $= \frac{13}{4} + 6 - 8 + 5 = \frac{13}{4} + 3 = \frac{25}{4} > 0$ (SE)
5	2	new d $= \frac{25}{4} + 8 - 6 + 5 = \frac{25}{4} + 7 = \frac{53}{4} > 0$ (SE)

(0,5) → (0,-5), (5,0), (-5,0)

(1,5) → (1,5), (-1,-5), (-1,5), (5,1), (-5,1), (-5,-1), (5,-1)

(2,5) → (2,-5), (-2,-5), (-2,5), (5,2), (-5,2), (-5,-2), (5,-2)

(3,4) → (3,-4), (-3,-4), (-3,4), (4,3), (-4,3), (-4,-3), (4,-3)

(4,3) → same as above.

Que 3 $(x-1)^2 + (y-2)^2 = 16$

$\therefore$ centre $= (1,2)$ Radius $= 4$

Lets translate the centre of the circle to the point $(0,0)$

$x^2 + y^2 = 4^2$

x	y	d
0	4	$d_{new} = \frac{5}{4} - 4 = \frac{-11}{4} < 0$ (E)
1	4	$new\ d = \frac{-11}{4} + 2x_p + 3 = \frac{-11}{4} + 3 = \frac{1}{4} >$ (SE)
2	3	$new\ d = \frac{1}{4} + 2 - 8 + 5 = \frac{1}{4} - 1 = \frac{-3}{4} < 0$ (E)
3	3	$new\ d = \frac{-3}{4} + 4 + 3 = \frac{-3}{4} + 7 = \frac{25}{4} < 0$ (E)

$(1,6) \rightarrow (1,-6), (-1,-6), (-1,6), (6,1), (-6,1), (-6,-1), (6,-1)$

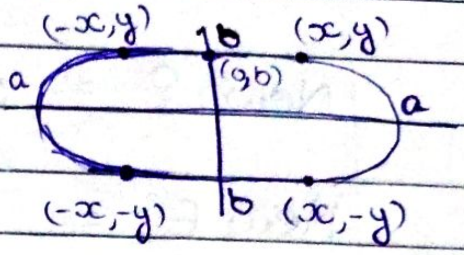
$(2,6) \rightarrow (2,-6), (-2,-6), (-2,6), (6,2), (6,-2), (-6,-2), (-6,2)$

$(3,5) \rightarrow (3,-5), (-3,-5), (-3,5), (5,3), (-5,3), (-5,-3), (5,-3)$

$(4,4) \rightarrow (-4,4), (-4,-4), (4,-4)$.

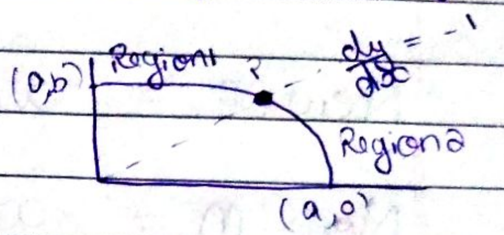
Mid Point Ellipse Algorithm

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



Major axis = $2a$

Minor axis = $2b$



Region 1 and Region 2 are separated where the slope of the tangent is (-1). When the slope of the tangent is $> (-1)$ then we are in region1. And the slope of the tangent $< (-1)$ then we are in region2.

Region 1

$P = x_p, y_p$

$E = x_p + 1, y_p$

$SE = x_p + 1, y_p - 1$

$m = x_p + 1, y_p - \frac{1}{2}$

$d = F(m) = b^2 (x_p + 1)^2 + a^2 \left(y_p - \frac{1}{2}\right)^2 - a^2 b^2$

if $d < 0$ then East

if $d \geq 0$ then South East.

Choose E

New P = $x_p + 1, y_p$

New E = $x_p + 2, y_p$

New SE = $x_p + 2, y_p - 1$

New m = $x_p + 2, y_p - \frac{1}{2}$

$d_{new} = F(\text{New m})$

$= b^2 (x_p + 2)^2 + a^2 (y_p - \frac{1}{2})^2 - a^2 b^2$ — (1)

$\Delta d_E = d_{new} - d_{old}$
$=$ (1) $-$ (2)

$\Delta d_E = b^2 (2x_p + 3)$

Choose SE

New P = $x_p + 1, y_p - 1$

New E = $x_p + 2, y_p - 1$

New SE = $x_p + 3, y_p - 2$

New m = $x_p + 2, y_p - \frac{3}{2}$

$d = F(\text{New m})$

$= b^2 (x_p + 2)^2 + a^2 (y_p - \frac{3}{2}) - a^2 b^2$ — (1)

$\Delta d_{SE} = d_{new} - d_{old}$
$= ① - ②$

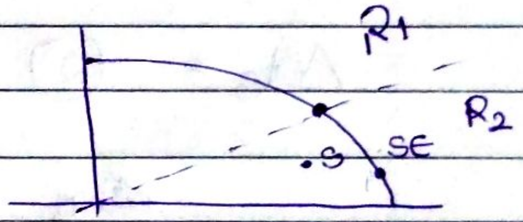
$\Delta d_{SE} = b^2(2x_p+3) + a^2(2-2y_p)$

$b^2x_p^2 + 4b^2 + 4x_pb^2 + \cancel{a^2y_p^2} + \frac{9}{4}a^2 - 3y_pa^2 - \cancel{a^2b^2} - \cancel{b^2x_p^2} - b^2 - 2x_pb^2 - \cancel{a^2y_p^2} - \frac{a^2}{4} + y_pa^2 + \cancel{a^2b^2}$

$3b^2 + 2x_pb^2 + 2a^2 - 2y_pa^2$

We stay in region1 ~~until~~ till this condition is true

$a^2(y_p - \frac{1}{2}) > b^2(x_p+1)$



Region 2

$P = x_p, y_p$

$S = x_p, y_p - 1$

$SE = x_p + 1, y_p - 1$

$m = x_p + \frac{1}{2}, y_p - 1$

$d = F(m) = b^2(x_p + \frac{1}{2}) + a^2(y_p - 1)^2 - a^2b^2$ — ①

$d < 0 \Rightarrow SE$
$d > 0 \Rightarrow S$

Choose S

New P = $x_p, y_p - 1$

New S = $x_p, y_p - 2$

New SE = $x_p + 1, y_p - 2$

New M = $x_p + \frac{1}{2}, y_p - 2$

d_{new} = F(New M)

$= b^2 (x_p + \frac{1}{2}) + a^2 (y_p - 2)^2 - a^2 b^2$ — 2

Δd_S = 2 − 1

$= a^2 (3 - 2y_p)$

$a^2 y_p^2 + 4a^2 - 4y_p a^2 - a^2 y_p^2 - a^2 + 2a^2 y_p = 3a^2 - 2a^2 y_p$

Choose SE

New P = $x_p + 1, y_p - 1$

New S = $x_p + 1, y_p - 2$

New SE = $x_p + 2, y_p - 2$

New M = $x_p + \frac{3}{2}, y_p - 2$

d_{new} = F(New M)

$= b^2 (x_p + \frac{3}{2})^2 + a^2 (y_p - 2)^2 - a^2 b^2$ — 2

$$\Delta d_{SE} = (2) - (1)$$

$$= b^2(2x_p + 2) + a^2(3 - 2y_p)$$

$$\cancel{b^2 x_p^2} + b^2\frac{9}{4} + 3x_p b^2 + \cancel{a^2 y_p^2} + a^2 4 - 4y_p a^2 - \cancel{b^2 x_p^2} - \frac{b^2}{4} - x_p b^2 - \cancel{a^2 y_p^2} - a^2 + 2y_p$$

$$2b^2 + 2x_p b^2 + 3a^2 - 2y_p a^2$$

We stop generating the points of ~~reog~~ region 2 when I reach ~~a, a~~ $(a, 0)$.

<u>Initial d</u>

Initial $P = 0, b$

Initial $m = \left(0+1,\ b - \frac{1}{2}\right)$

d Initial $= F(\text{Initial } m)$

d Initial $= F\left(1,\ b - \frac{1}{2}\right)$

d Initial $= b^2(1)^2 + a^2\left(b - \frac{1}{2}\right)^2 - a^2b^2$

d Initial $= b^2 + a^2\left(\frac{1}{4} - b\right)$

Que 1 Calculate all the points of the ellipse

$$\frac{x^2}{(20)^2} + \frac{y^2}{(10)^2} = 1$$

$a = 20 \qquad b = 10$

Region 1

$$a^2\left(y_p - \frac{1}{2}\right) > b^2(x_p+1)$$

$$400\left(y_p - \frac{1}{2}\right) > 100(x_p+1)$$

$$400y_p - 200 > 100x_p + 100$$

$$400y_p - 100x_p > 300$$

$$4y_p - x_p > 3$$

x	y	d
0	10	$b^2 + a^2\left(\frac{1}{4} - b\right) \Rightarrow 100 + 400\left(\frac{1}{4} - 10\right) \Rightarrow -3800 < 0$ (E)
1	10	$b^2(2x_p+3) - 3800 \Rightarrow -3800 + 100(3) \Rightarrow -3500 < 0$ (E)
2	10	$-3500 + b^2(2x_p+3) = -3000 < 0$ (E)
3	10	$-3000 + b^2(2x_p+3) = -2300 < 0$ (E)
4	10	$-2300 + b^2(2x_p+3) = -1400 < 0$ (E)
5	10	$-1400 + b^2(2x_p+3) = -300 < 0$ (E)
6	10	$-300 + b^2(2x_p+3) = 1000 > 0$ (SE)
7	9	$-4700 < 0$ (E)
8	9	$-3000 < 0$ (E)
9	9	$-1100 < 0$ (E)

x	y	d
11	8	-3100 < 0 (E)
12	8	-600 < 0 (E)
13	8	2100 > 0 (SE)
14	7	-600 < 0 (E)
15	7	2500 > 0 (SE)
16	6	1000 > 0 (SE)
17	5	$3 \ngtr 3$; False

300<0 (E)

Now we move to region 2

Region 2

x	y	d
18	4	-2175 < 0 (SE)
19	3	-375 < 0 (SE)
20	2	2425 > 0 ~~(E)~~ (S)
20	1	2024 > 0 (S)
20	0	Stop.

$R_2 \quad m = (x_p + \frac{1}{2}, y_p - 1)$ (17, 5)

$d = F(m)$

$d = b^2 (17 + \frac{1}{2})^2 + a^2 (5-1)^2 - a^2 b^2$

$34 = 17 \times 2$

$d = 100 \left(\frac{34}{4}\right) + 400 (4)^2 - (20)^2 \times (10)^2$ (25; 400, 100)

25
34
850

$= 30625 + 6400 - 40000$

$= -2975$

Generate all the points of the ellipse using 4 way symmetry :-

$(0,10) \rightarrow (0,-10)$

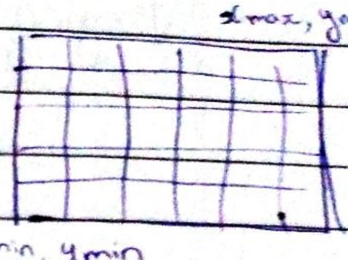
$(1,10) \rightarrow (-1,10), (1,-10), (-1,-10)$

$(2,10) \rightarrow (-2,10), (2,-10), (-2,-10)$

$(3, 10) \rightarrow$

Filling Rectangle

coordinates = x_{min}, y_{min}
x_{max}, y_{max}



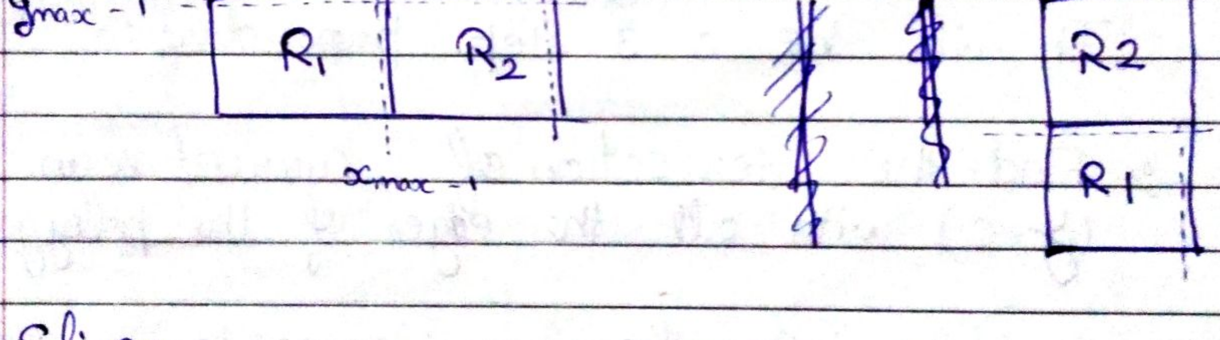
for ($x = x_{min}$; $x \le x_{max}$; x++)

for ($y = y_{min}$; $y \le y_{max}$; y++)

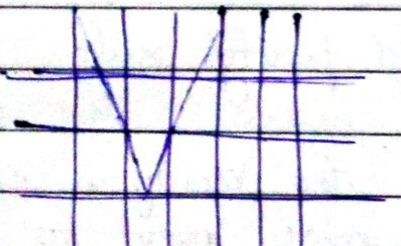
putpixel (x, y, colour);

Boundary Value Rule / Boundary Pixel Rule

Pixel on the bottom on the left will be drawn
Top and Right will not be drawn.

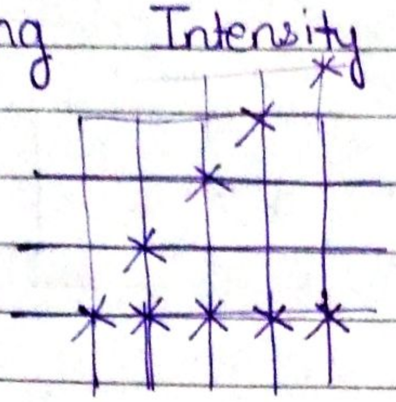


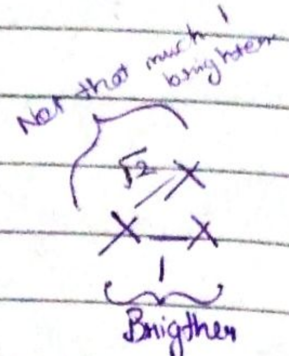
Sliver



Sliver is a very thin cone. Whichever pixel we will ~~put~~ illuminate near it, it will lie outside the actual figure. A sliver's figure on the screen will always be distorted. We ~~cann~~ can never draw a sliver in the correct shape on the screen as we do on paper.

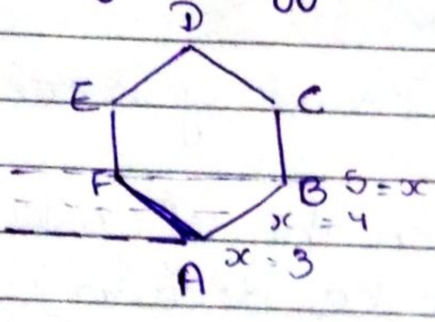
Varying Intensity





The intensity of the pixel should be a function of slope of the line.

Filling Polygon

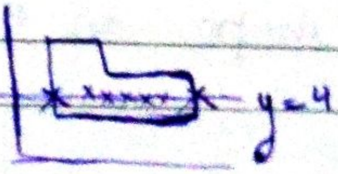


(3, 2)
(4, 1) → (4, 4)
(5, 0) → (5, 5)

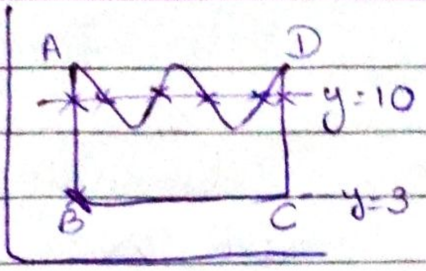
It will be a 3 step procedure :-

- Find the intersection of, current scan line. (y = c) with all the edges of the polygon.
- Sort the intersections by increasing x - coordinates
- Fill the pixels using odd parity rule

Odd parity rule states the parity is initialy even. Each intersection encounter invert the parity we draw/illuminate when the parity is odd don't draw when the parity is even.

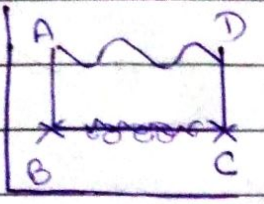


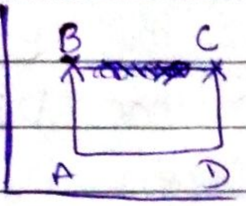
P = O
all points not counted
Then P = E



P = O
all points illuminated
P = E

P = O
B✓ X T✓ ✓
all points illuminated
P = E

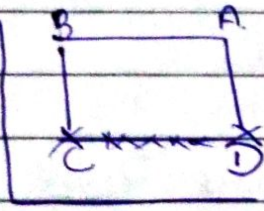




P = O
all points illuminated
P = E

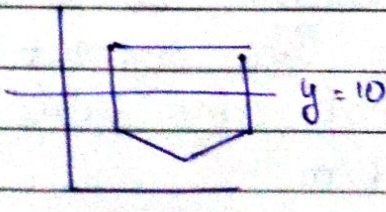
~~To~~ Additional rule for odd parity rule:

y_{min} of every edge will invert the the parity y_{max} will not. For horizontal edges there is no y_{min} and no y_{max}.



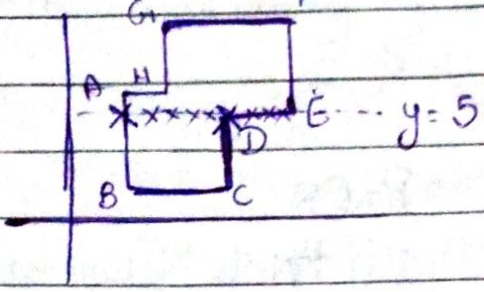
Intersection with fractional coordinates, what should be done - roundoff / floor/ceiling?

We ~~alto~~ always want to stay inside the figure



For all the left intersection we take ceiling & for right intersection we take floor.

Horizontal edges

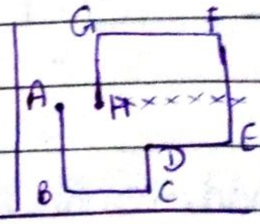


P = O Point D
illuminate Points
DE X
DC - ymax X

P = O Point E
EF ymin ✓
DE X

P = E Point A
illuminate points
AH X
AB - ymax X

⇒



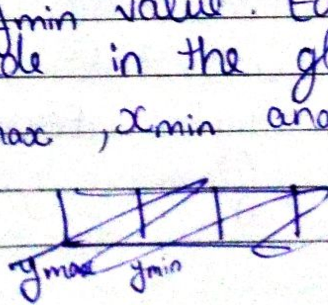
P = E Point H
AH X (horizontal)
GH - ymin ✓

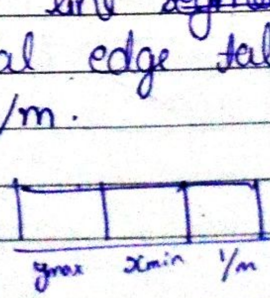
Scan line algorithm

There are 2 data structure used for scan line algo / polygon filling algo

AET → Active edge table
ET → Edge table / also called global edge table

Global Edge Table is maintain as a bucket on the ymin value. Each line ~~segment~~ edge has a node in the global edge table of the form ymax, xmin and 1/m.





Illuminate all the pixels of the polygon.

A (2,3)
B (7,1)
C (13,5)
D (13,11)
E (7,7)
F (2,9)

AB A (2,3) (7,1) min

xmax ymax — xmin ymin

3	7	-5/2
ymax	xmin	1/m

$\frac{3-1}{2-7} = \frac{2}{-5}$

GET

4
3
2
1 → [3 | 7 | -5/2]
0
ymin

We will compare the y values and make min/max.

BC (13,5) (7,1)
B C

1	13	+ 3/2

$\frac{5-1}{13-7} = \frac{-4}{6} = \frac{+2}{3}$

CD min (13,5) max (13,11)

11	13	0

~~min~~ ~~max~~

$\frac{11-5}{13-13} = \frac{6}{0} = 0$

DE max (13,11) min (7,7)

11	7	3/2

$\frac{11-7}{13-7} = \frac{4}{6} = \frac{2}{3}$

EF min (7,7) max (2,9)

9	7	-5/2

$\frac{9-7}{2-7} = \frac{2}{-5}$

ymin		
7	DE [11 | 7 | 3/2]	EF [9 | 7 | -5/2]
6		
5	FA [11 | 13 | 0]	
4		
3	FA [9 | 2 | 0]	
2		
1	AB [3 | 7 | -5/2]	BC [5 | 7 | 3/2]
0		

ymin

FA min (2,3) max (2,9)

9	2	0

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F (2, 9)
G (5, 9)

If there is a horizontal edge there is no y_{max} and no y_{min}. Hence, these kind of edge not count in global table. $y = \infty$ and hence, we can't perform this operation.

Active edge table will be drawn at (y=c). The nodes are copied from 1 active edge table to another, new nodes are are also added from the global edge table.

AET

y=0 NULL

AET

y=1

3	7	-5/2	→	5	7	3/2

(7, 1) ⟷ (7, 1)

The nodes ~~the~~ should be sorted by x value.

AET

y=2

3	9/2	-5/2	→	5	17/2	3/2

(5, 2) ⟷ (8, 2)

Date : / /

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Draw the active edge table

$y = 3$

| 5 | 10 | 3/2 | — | 9 | 2 | 0 |

(10, 3) (2, 3)

arrange the node

| 9 | 2 | 0 | — | 5 | 10 | 3/2 |

(2, 3) ⟷ (10, 3)

We should always have even no. of nodes in active edge table.

$y = 4$

| 9 | 2 | 0 | — | 5 | 23/2 | 3/2 |

(2, 4) ⟷ (11, 4)

$y = 5$

| 5 | 18 | 3/2 | ?

0

| 9 | 2 | 0 | → | 11 | 13 | 0 |

(2, 5) ⟷ (13, 5)

$y = 6$

| 9 | 2 | 0 | — | 11 | 13 | 0 |

(2, 6) ⟷ (13, 6)

$y = 7$

(7, 7)

| 9 | 2 | 0 | — | 11 | 13 | 0 | — | 11 | 7 | 3/2 |

(2, 7) (13, 7)

(7, 7) | 9 | 7 | -5/2 |

| 9 | 2 | 0 | — | 9 | 7 | -5/2 | — | 11 | 7 | 3/2 | — | 11 | 13 | 0 |

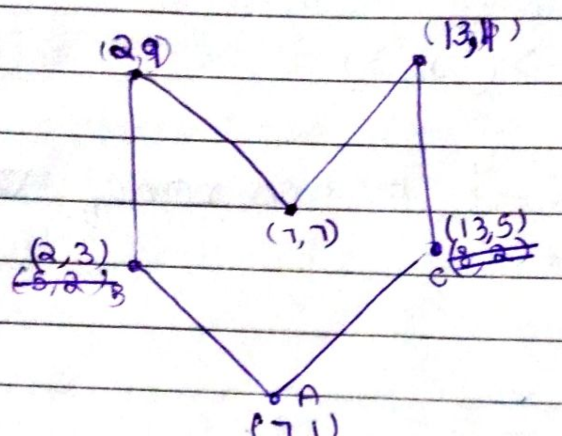
| 11 | 9 | 3/2 | — | 9 | 7 | -5/2 |

~~(7, 7)~~ ⟷ (13, 7)

y = 8 (2, 8) ⟷ (4, 8) (9, 8) ⟷ (13, 8)

y = 9 (11, 9) ⟷ (13, 9)

y = 10 (13, 10) ⟷ (13, 10)



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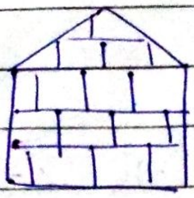
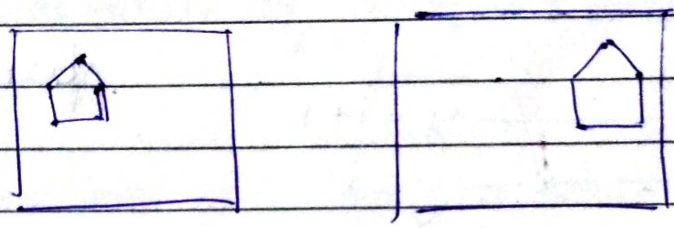
Patter Filling



primitive- y=C

1) Anchor Method

The pattern remain same, if the primitive is move from one place to another.



2) AND Method

Primitive array

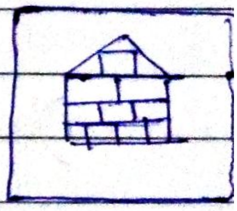
0	0	0	0	0	0
0	0	1	0	0	0
0	1	1	1	0	0
0	1	1	1	0	0
0	1	1	1	0	0
0	0	0	0	0	0

512 X 512

Pattern Array.

0	1	0	0	1	0	0
0	0	1	0	0	1	0
0	0	1	0	0	1	0
0	1	0	0	1	0	0
0	1	0	0	1	0	0
0	0	1	0	0	1	0

512 X 512



The pattern can change, if the primitive is moved from one location to another.

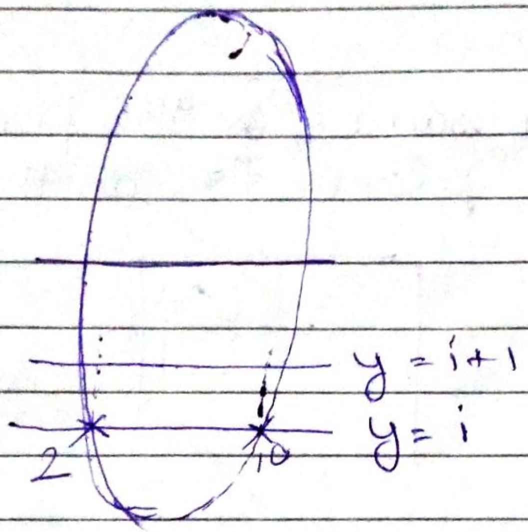
There is no problem in finding an ~~ac~~ anchor in this method.

One problem :- Take too much memory.

Filling ellipse

Edge - Coherence

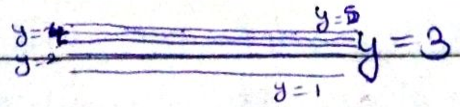
Many points intersective at the scan line $y = i$ are also intersected by the scan line $y = i+1$.



span, $x = \min$
$x = \max$

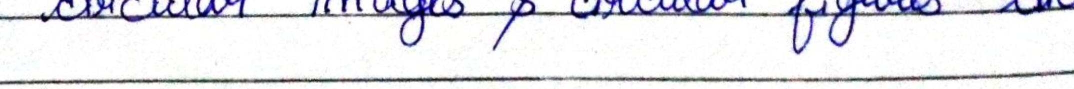
Thick Primitives.

1) Replicating Pixels.



} Thickness is 5.

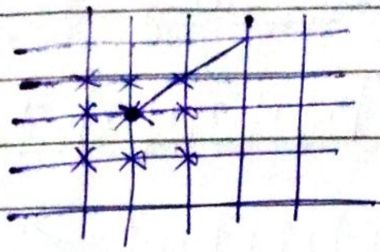
It is diff to replicate and find concentric images for circular images & circular figures like 'S'.



The width of the line should always be odd otherwise you will move away from ~~to~~ your ideal line.

Advantage -> easy to draw.

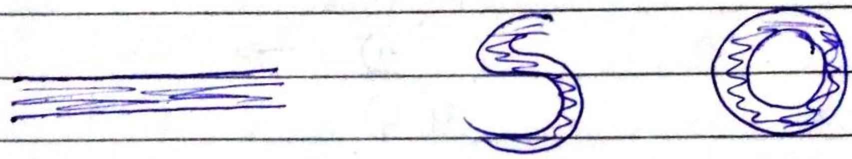
2) Moving Pen method



Footprint -> Keep my pen at a single point on the screen and because of that the cross section of pixels that are ~~elte~~ illuminated is called footprint of the pen.

There is a overlap of footprint happens in this method.

3) Filling area between the boundries

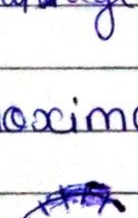


Advantage -> No problem of odd & even width.

Disadvantage -> User have ~~to~~ give 2 equations.

4) Approximation by thick polylines

Problem
↓
Take too much time.



I will break my primitives into ~~ma~~ smaller pieces such that each piece is a straight line & I will do pieces wise thickning of each straight line. Finally I join all the smaller primitives.

Advantages -> Works for all tipes of figures.
-> It gives best visual thickened line.

Pen Style

~~bit String~~

bit String

— · — · — · — → | 1 | 1 | 1 | 0 | 1 | 0 |

— — — — — → | 1 | 1 | 1 | 0 |

· · · · · · · · · · → | 1 | 0 |

(i is a valid value)

while { if (bitstring [i-1 % 6])
putpixel (x, y, colour) }
i = i+1; }

A ———— B

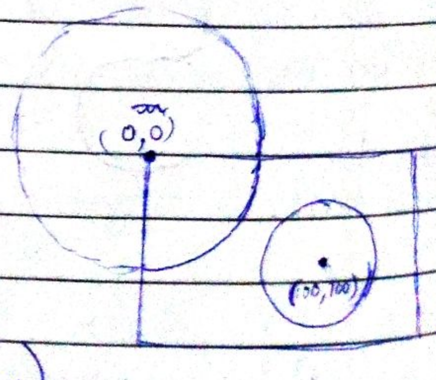
(1, 1) → 1
(2, 2) → 2
(4, 4) → 3
(5, 6) → 4
(6, 7) → 5
(6, 8) → 6
(7, 7) → 7

| 1 | 1 | 1 | 0 | 1 | 0 |
0

·:· · ·

Practical

getmaxx ( )/2
getmaxy ( )/2



(x+100, y+100, colour)

Shifting the coordinates is called Translation → 2D or 3D

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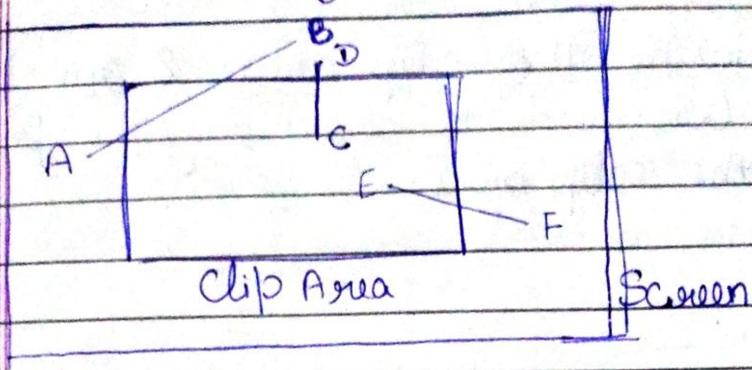
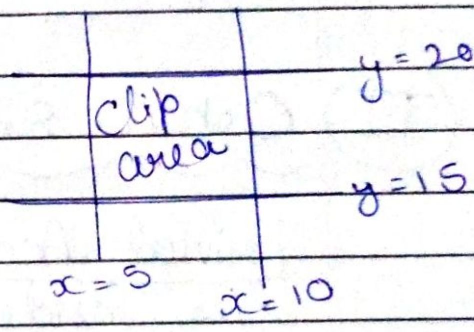
Line clipping



clip $x = x_{max} = 10$
$y = y_{max} = 20$
$x = x_{min} = 5$
$y = y_{min} = 15$



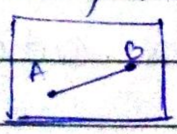
(I) Clipping individual points :

AB $x_{min} \le x \le x_{max}$ and
CD $y_{min} \le y \le y_{max}$
EF

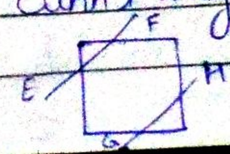
In this we do not waste time to calculate, lines outside the clip area ~~f~~ or major part is outside the CA.

(II) Clipping by solving simultaneous equation

Check for trivial acceptance -> If both the end points are inside the clip area then the complete line is inside the clip area. We will illuminate all the points of the line, no need to ~~ck~~ check the condition of the line.

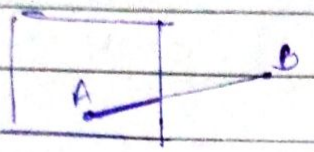


Trivial rejection -> If both the end points of the line are outside the clip area, we cann't say forsure that the line is complotely outside.



(II) ~~Cohen~~

One point is inside the clip area & one point is outside $\Rightarrow$ We need to draw only the part which is inside the clip area.



(III) Cohen Sutherland Line Clipping

Trivial Acceptance $\rightarrow$ If both the end points lies inside the clip area then the line lies completely inside the clip area. Then we illuminate the points of the line.

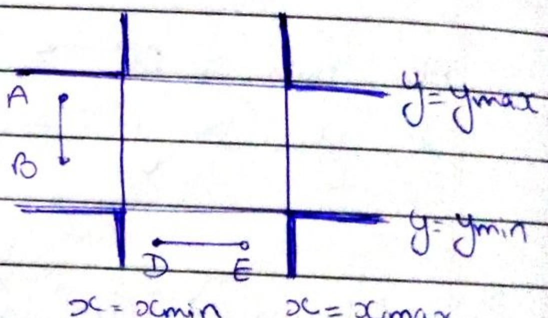
Trivial Rejection $\rightarrow$

$x < x_{min}$

or $x > x_{max}$

or ~~x~~ $y > y_{max}$

or $y < y_{min}$



One condition ~~for~~ has to be true for both end points.

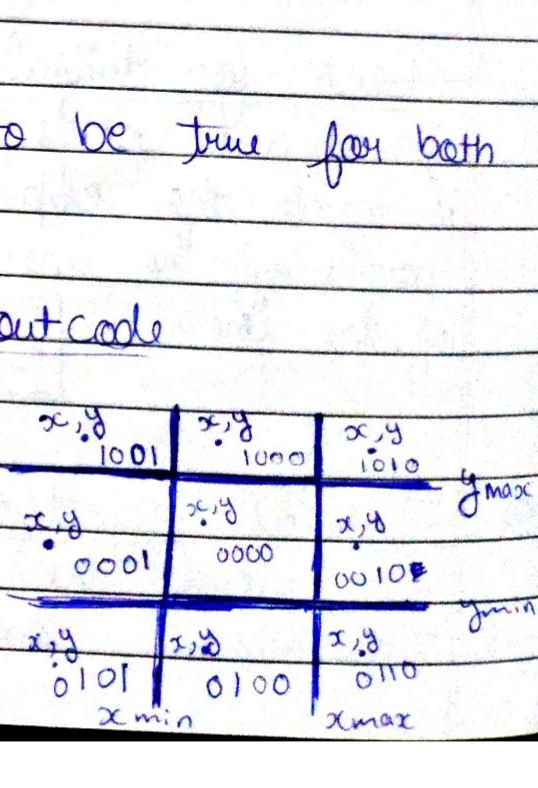
~~4 bit~~ Clipping - 4 bit outcode

a $\downarrow$ $y > y_{max}$

b $\downarrow$ $y < y_{min}$

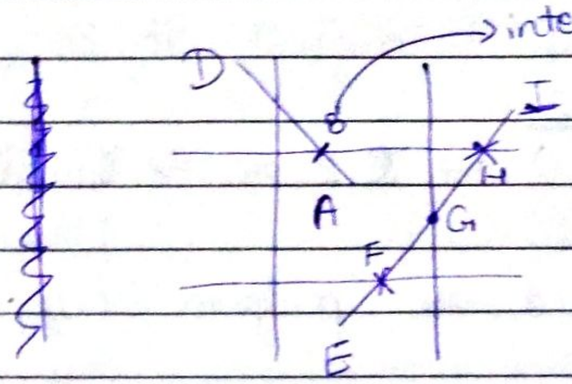
c $\downarrow$ $x > x_{max}$

d $\downarrow$ $x < x_{min}$



The condition of trivial acceptance → if the outcode of ~~all~~ both the end points is 0000 ~~so~~ we trivially accept the line.

If bitwise AND of both end points is non zero then the line is trivially rejected.



AD

A → 0000

D → 1001

TA ⇒ This line is not trivially accepted.

TR ⇒ 0000
1001
0000 ⇒ Zero ⇒ This line is not TR.

Therefore clipping needs to be done.

A is ~~about~~ already inside the clip area.

D → 1 0 0 1   AD with $y = y_{max}$

TBRL → Left, Right, Top, Bottom

T (True) B (False) R (False) L (True)

B → 0000

$\boxed{\text{AB is the clipped line.}}$

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IE

I → 1010
E → 0100

TA ⇒ The line is not TA

TR ⇒ 1010
0100
0000 ⇒ Zero ⇒ This line is not TR.

Hence, we need to perform clipping.

I →	T	B	R	L
	1	0	1	0

intersection of EI with Top edge ($y = y_{max}$)

We get H → 0010

intersection of EH with right edge ($x = x_{max}$)

We get G → 0000 → Stop

E →	T	B	R	L
	0	1	0	0

intersection of EI with the bottom edge ($y = y_{min}$)

We get F → 0000

$\boxed{\text{GF is the final clip line}}$

Que Use Cohen Sutherland Line Clipping algo to clip the line segment ~~p,q~~ p (0,8) q (1,5) by the rectangle window define by the vertices ABCD.
A (0,0), B(1,0), C(1,1), D(0,1)
Show all the steps:-

Ans Unit sq at the origin

$x_{min} - 0$ $\quad$ $x_{max} - 1$
$y_{min} - 0$ $\quad$ $y_{max} - 1$

	$y > y_{max}$	$y < y_{min}$	$x > x_{max}$	$x < x_{min}$
P (0,5)	$5 > 1$	$5 < 0$	$0 > 1$	$0 < 0$
	1	0	0	0

Output of P = 1000

Q (1,5)	$5 > 1$	$5 < 0$	$1 > 1$	$1 < 0$
	1	0	0	0

Output of Q = 1000

1) Test for Trivial Acceptance

The output should be 0000 & 0000.
So not true, hence the line is not TA.

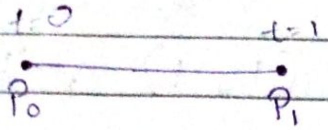
2) Test for Trivial rejection

$$\begin{array}{r} P \quad 1000 \\ \& \; Q \quad 1000 \\ \hline 1000 \end{array} \neq 0$$

'AND' of both end points is non-zero then the line is TR.

Cyrus Beck Line Clipping Algo

$$P(t) = P_0 + (P_1 - P_0)\,t \qquad 0 \le t \le 1$$

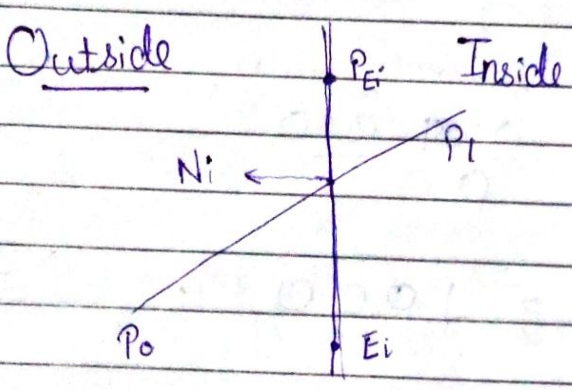


$$P_x(t) = P_{0x} + (P_{1x} - P_{0x})\,t$$

$$P_y(t) = P_{0y} + (P_{1y} - P_{0y})\,t$$

$$P(t=0) = P_0$$

$$P(t=1) = P_1 \qquad P(0.5)$$



$$N_i\,(P(t) - P_{Ei}) = 0$$
$< 0 \Rightarrow$ Inside
$> 0 \Rightarrow$ Outside

$$N_i\,(P(t) - P_{Ei}) = 0$$

$$N_i\,(P_0 + (P_1 - P_0)t - P_{Ei}) = 0$$

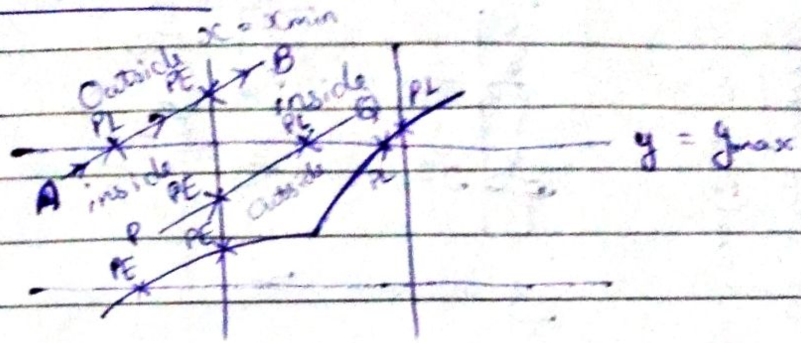
$$N_i\,(P_0 - P_{Ei}) + N_i\,(P_1 - P_0)t = 0$$

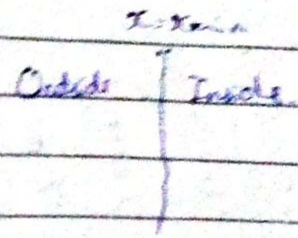
$$t = \frac{N_i \cdot (P_0 - P_{Ei})}{-N_i \cdot D} \qquad D = P_1 - P_0$$

$D \neq 0$
$\Rightarrow P_1 - P_0 \neq 0$
$\Rightarrow P_1 \neq P_0$

$N_i \cdot D \neq 0$
$\Rightarrow P_1 P_0$ not $\parallel$ E_i

PE & PL





$N_i \cdot D < 0 \Rightarrow PE$

$N_i \cdot D > 0 \Rightarrow PL$

If $\forall PE$ max $PE = t_E$

$\forall PL$ min $PL = t_L$

\# $PL < PE \Rightarrow TR$

\# $0 \le t_E, t_L, PE, PL \le 1$

$t_E < t_L$

$t_E > t_L \Rightarrow$ Reject

$0 \le t_E, t_L \le 1$

Clip Edge	Ni (Normal)	P_{Ei}	$P_0 - P_{Ei}$	$t = \frac{N_i (P_0 - P_{Ei})}{-N_i \cdot D}$, $D = P_1 - P_0$
$x = x_{min}$	$-1, 0$	(x_{min}, y)	$(x_0 - x_{min}, y_0 - y)$	$t = \frac{-(x_0 - x_{min})}{x_1 - x_0}$
$x = x_{max}$	$1, 0$	(x_{max}, y)	$(x_0 - x_{max}, y_0 - y)$	$t = \frac{(x_0 - x_{max})}{-(x_1 - x_0)}$
$y = y_{min}$	$0, -1$	(x, y_{min})	$(x_0 - x, y_0 - y_{min})$	$t = \frac{-(y_0 - y_{min})}{y_1 - y_0}$
$y = y_{max}$	$0, 1$	(x, y_{max})	$(x_0 - x, y_0 - y_{max})$	$t = \frac{y_0 - y_{max}}{-(y_1 - y_0)}$

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