Ch-11

2D Transformations

$[[x \quad y]] \rightarrow$ Row Matrix $\rightarrow [x \quad y \quad z]$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \rightarrow$ ~~Coloumn~~ Column Matrix]

$$[x \quad y] \begin{bmatrix} a & b \\ c & d \end{bmatrix} = [(ax+cy) \; (bx+dy)]$$

Case1 $a = d = 1 \qquad b = c = 0$

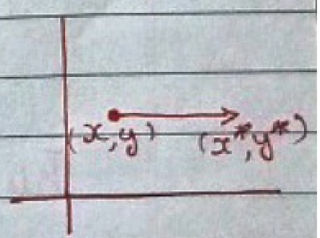
$$T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow \text{Identity}$$

$$[x \quad y] \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = [x \quad y]$$

position vector remains unchanged.

Case2 $d = 1 \qquad b = \cancel{a}\, c = 0$

$$[x \quad y] \begin{bmatrix} a & 0 \\ 0 & 1 \end{bmatrix} = [ax \quad y]$$



Note:- The point P's x-coordinate is scaled by the factor of a.

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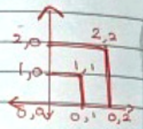
<u>Case 3</u> $b = c = 0$

$$[x \quad y]\begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix} = [ax \quad dy] = [x^* \quad y^*]$$

x is scaled by factor of a.
y is scaled by factor of d.

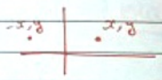


~~Case~~ if $a = d \Rightarrow$ equal scaling
$a \neq d \Rightarrow$ unequal scaling

$a > 1 \Rightarrow$ enlargement is done.

$0 < a = d < 1 \Rightarrow$ compression is done.

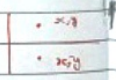
<u>Case 4</u> $b = c = 0 \qquad a = -1 \qquad d = 1$



$$[x \quad y]\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = [-x \quad y]$$

This matrix is for a reflection through y-axis
y - axis $(x = 0)$.

<u>Case 5</u> $b = c = 0 \qquad a = 1 \qquad d = -1$



$$[x, y]\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = [x \quad -y]$$

This is a reflection through the x-axis $(y = 0)$

<u>Case 6</u> Reflection through the origin.
$b = c = 0 \quad d = a < 0$

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<u>Note</u>: Both reflection & scaling involve only the diagonal terms of the transformation matrix.

<u>Case 7</u> $a = d = 1 \qquad c = 0$

$$[x \quad y]\begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix} = \cancel{[(x+b) \quad y]}$$

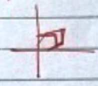
$$= [x \quad \underline{(bx + y)}]$$

~~skewing~~ Shearing

Shearing is performed in y proportional to x coordinate.

<u>Case 8</u> $$[x, y]\begin{bmatrix} 1 & 0 \\ c & 1 \end{bmatrix} = [(x + cy) \quad y]$$

Shearing is performed in x proportional to y coordinate.



$P = [0 \quad 0]$

$$[0 \quad 0]\begin{bmatrix} a & b \\ c & d \end{bmatrix} = [0 \quad 0]$$

<u>Note</u>: The origin is invariant with respect to 2×2 transformation matrix.
We can move the origin.
This is the reason why we use the homogeneous coordinates.

Que

$A = [0 \quad 1]$

$B = [2 \quad 3]$

$T = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$

Transform the line ab using the transformation matrix T.

$[A]\ [T] = [A^*]$

$[B]\ [T] = [B^*]$

$\therefore$

$$\begin{array}{c} A \\ B \end{array}\begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 11 & 7 \end{bmatrix}\begin{array}{c} A^* \\ B^* \end{array}$$

To Prove

(1) There is a one to one correspondance b/w the points on the line ab & the $a^* b^*$.

$A = [x_1 \quad y_1]$

$B = [x_2 \quad y_1]$

General T matrix = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$\begin{array}{c} A^* \\ B^* \end{array}\begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} (x_1 a + y_1 c) & (x_1 b + y_1 d) \\ (x_2 a + y_2 c) & (x_2 b + y_2 d) \end{bmatrix}_{2\times 2}\begin{array}{c} \rightarrow A^* \\ \rightarrow B^* \end{array}$$

mid point of $A^*B^* = \frac{A^* + B^*}{2}$ ~~$= ax_1 + y_1 c + a$~~

$$= \left[\frac{ax_1 + cy_1 + ax_2 + cy_2}{2} \quad \frac{bx_1 + dy_1 + bx_2 + dy_2}{2}\right] \text{ — (1)}$$

Mid-point of $AB = m = \frac{A+B}{2}$

$$= \left[\frac{x_1 + x_2}{2} \quad \frac{y_1 + y_2}{2}\right] = m$$

$$m\,[T] = \left[\frac{x_1 + x_2}{2} \quad \frac{y_1 + y_2}{2}\right]\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\Rightarrow \left[\frac{ax_1 + ax_2 + cy_1 + cy_2}{2} \quad \frac{bx_1 + bx_2 + dy_1 + dy_2}{2}\right]$$

$$\Rightarrow \left[\frac{ax_1 + ay_1 + ax_2 + cy_2}{2} \quad \frac{bx_1 + dy_1 + bx_2 + dy_2}{2}\right] \text{ — (2)}$$

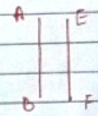
Though (1) = (2) Hence, proved.

Next statement to Prove

(2) || lines remain || even after transformation. This also implies a parallelogram remains a parallelogram even after the transformation

$[A] = [x_1 \quad y_1]$

$[B] = [x_2 \quad y_2]$



Slope of $AB = \frac{y_2 - y_1}{x_2 - x_1} = m =$ slope

$$\begin{bmatrix} A^* \\ B^* \end{bmatrix} = \begin{bmatrix} A \\ B \end{bmatrix} = [T] = \begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} ax_1 + cy_1 & bx_1 + dy_1 \\ ax_2 + cy_2 & bx_2 + dy_2 \end{bmatrix} \begin{matrix} \rightarrow A^* \\ \rightarrow B^* \end{matrix}$$

$m^* =$ slope of $A^*B^* = \frac{(bx_2 + dy_2 - bx_1 - dy_1)}{(ax_2 + cy_2 - ax_1 - cy_1)}$

$$= \frac{b(x_2 - x_1) + d(y_2 - y_1)}{a(x_2 - x_1) + c(y_2 - y_1)}$$

$\Downarrow$

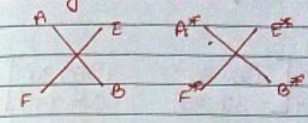
divide the num & den by $x_2 - x_1$

$$m^* = \frac{b + d\left(\frac{y_2 - y_1}{x_2 - x_1}\right)}{a + c\left(\frac{y_2 - y_1}{x_2 - x_1}\right)} = \frac{b + dm}{a + cm}$$

m^* is dependent on a, b, c, d, m (original slope of AB & EF). All these variables are same for AB & EF therefore, A^*B^* will be $\parallel$ to E^*F^*. Hence, proved.

Next

③ Intersecting lines remains intersecting even after ~~intersecting~~ intersection.

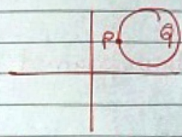


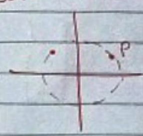
⇒ Need to be done from book.

Rotation

① This matrix we rotate the point counter clock wise about the origin.

$$T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$ → anti-clockwise.





Determinant is $= 1$; $\det[T] = 1$

All the transformation with the determinant identically equal to 1 give the pure rotation.

② $[T]^T = [T]^{-1}$

The transpose of T will be equal to -1.

③ $\cos(-\theta) = \cos\theta$
$\sin(-\theta) = -\sin\theta$

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ +\sin\theta & \cos\theta \end{bmatrix}$$
Clockwise

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Que Given a ΔABC. Rotate the Δ 90° about the origin in the ~~coun~~ anticlockwise direction

$A(3,-1)$ $B(4,1)$ $C(2,1)$

$$\begin{bmatrix} 3 & -1 \\ 4 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} \cos 90^\circ & +\sin 90^\circ \\ -\sin 90^\circ & \cos 90^\circ \end{bmatrix}$$

$$\begin{bmatrix} 3 & -1 \\ 4 & 1 \\ 2 & 1 \end{bmatrix}_{3\times 2} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}_{2\times 3}$$

$$\begin{bmatrix} 1 & 3 \\ -1 & 4 \\ -1 & 2 \end{bmatrix} \begin{matrix} A^* \\ B^* \\ C^* \end{matrix}$$

A^* B^* C^* is a transformed ~~tr~~ Δ.

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Rotation Matrix

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \qquad \theta = 180^\circ$$

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\theta = 270 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$\theta = 360 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Reflection x-axis $(y=0)$ $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

y-axis $(x=0)$ $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

$x=y$ $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$x=-y$ $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

If the determinent of transformation matrix is identically equal to '-1' then the transformation is said to be a pure reflection.

Que Given a ΔABC ; $A(4,1)$, $B(5,2)$, $C(4,3)$ reflect the Δ about x-axis & about the line $x=-y$.

$A(4,1)$, $B(5,2)$, $C(4,3)$ $\quad X = \begin{bmatrix} 4 & 1 \\ 5 & 2 \\ 4 & 3 \end{bmatrix}$

$3x + 2y = 0$

$8x + 4y = 0$ $\rightarrow \begin{bmatrix} 3 & 8 \\ 2 & 4 \end{bmatrix}$

$T_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$T_2 = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ $\quad [X'] = [X][T_1][T_2]$

$$[X'] = \begin{bmatrix} 4 & 1 \\ 5 & 2 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

$$[X'] = \begin{bmatrix} 4 & -1 \\ 5 & -2 \\ 4 & -3 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -4 \\ 2 & -5 \\ 3 & -4 \end{bmatrix} \quad (1)$$

Perform rotation of 270° anticlockwise about the origin.

$A(4,1)$, $B(5,2)$, $C(4,3)$ $\quad X = \begin{bmatrix} 4 & 1 \\ 5 & 2 \\ 4 & 3 \end{bmatrix}$

$T = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ $\quad [X'] = [X][T]$

$$= \begin{bmatrix} 4 & 1 \\ 5 & 2 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 4 \\ -2 & 5 \\ -3 & 4 \end{bmatrix} \quad (2)$$

Reflection about x-axis followed by $x = -y$ is same as rotation about 270°.

To prove : Matrix multiplication is non-commutative.

(OR)

The order of the transformations perform can't be reversed.

Let's assume matrix multiplication is commutative.

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $\quad B = \begin{bmatrix} 1 & 0 \\ 2 & 2 \end{bmatrix}$

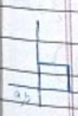
$[X] = [A][B]$ — (1)

$[Y] = [B][A]$ — (2)

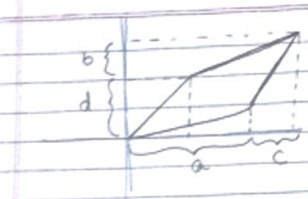
$$[X] = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 11 & 8 \end{bmatrix}$$

$$[Y] = \begin{bmatrix} 1 & 0 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 8 & 12 \end{bmatrix}$$

$[X]$ = Unit square at the origin $= \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$ $\quad T = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$



$$[X][T] = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & 0 \\ c & d \\ a & b \\ a+c & b+d \end{bmatrix}$$



The origin remains invariant with respect to 2×2 transformation matrix

a & d are scaling factors
b & c perform ~~shearing~~ shearing.

Area Property

Area of the transformed figure (A_t) = Area of the original figure × det [T]

$\triangle ABC$ $A(1,0)$ $B(0,1)$ $C(-1,0)$ $T = \begin{bmatrix} 3 & 2 \\ -1 & 2 \end{bmatrix}$

What will be the area of transformed $\triangle$.

Area of the $\triangle ABC = \frac{1}{2} \times (x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2))$

$= \frac{1}{2}(1(1-0) + 0 + (-1)(0-1))$

$= \frac{1}{2}(1+1) = \frac{1}{2} \times 2 = 1$

Area of the $\triangle ABC = 1$

$\det[T] = 8$

Area of the Transformed $\triangle = 1 \times 8 = 8$ unit sq.

$[X'] = [X][T]$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ -1 & 2 \end{bmatrix}$

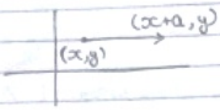
$= \begin{bmatrix} 3 & 2 \\ -1 & +2 \\ -3 & -2 \end{bmatrix}$

Area of $X' = \frac{1}{2}(3(2+2) + (-1)(-2-2) + (-3)(2-2))$

$= \frac{1}{2}(12+4) = \frac{16}{2} = 8$ unit sq.

Homogenous Coordinates

(x, y)



$$T = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$[x \quad y] \begin{bmatrix} 1 & 0 \\ a & 1 \end{bmatrix} = [x+a \quad y] \ (\times)$$

1) We can't perform translation with 2×2 transformation matrix.

2) The origin ~~is~~ is invariant, this 2×2 transformation matrix.

1) $[x \quad y \quad h]$
   $\downarrow$
   homogenous plane $(h = 1)$.

$$T = \begin{bmatrix} a & b & 0 \\ c & d & 0 \\ m & n & 1 \end{bmatrix}$$

$m, n \Rightarrow$ will perform translation.

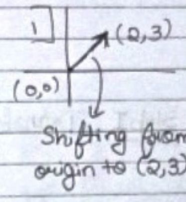
$$[x \quad y]_{1\times2} \begin{bmatrix} & & \\ & & \end{bmatrix}_{3\times3} = [x+a \quad y]$$

not possible.

$$\cancel{[x \quad y \quad 1]_{1\times3} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ a & 0 \end{bmatrix}_{3\times2} = [x+a \quad y]}$$

$$[x \quad y \quad 1]_{1\times3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & 0 & 1 \end{bmatrix}_{3\times3} = [x+a \quad y \quad 1]$$

2) $$[0 \quad 0 \quad 1] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 3 & 1 \end{bmatrix} = [2 \quad 3 \quad 1]$$



Que Which point from the given homogenous coordinate can be represented in 2-d.

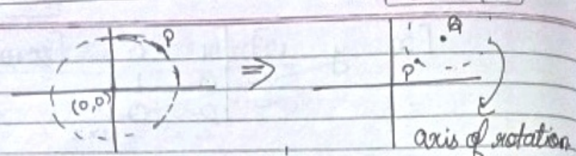
$[4 \quad 6 \quad 2] \rightarrow [2 \quad 3 \quad 1]$

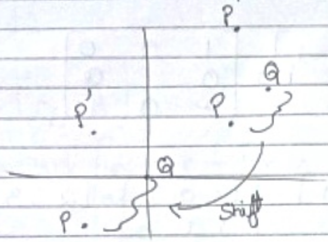
$[10 \quad 15 \quad 5] \rightarrow [2 \quad 3 \quad 1]$

$[20 \quad 30 \quad 10] \rightarrow [2 \quad 3 \quad 1]$

They represent the same point $(2, 3)$ in 2-D

Rotation about the arbitrary point 'Q'





Step I Translate the point 'Q' (m,n) to the origin.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -m & -n & 1 \end{bmatrix}$$

Step II Perform the rotation asked in the Que.

$$\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Step III Perform inverse translation.

$$\begin{array}{ccccccc} T_1 & T_2 & T_3 & \text{Rot} & T_3^{-1} & T_2^{-1} & T_1^{-1} \\ 180^\circ & -3,2 & 90^\circ & & -90^\circ & 3,2 & -180^\circ \end{array}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ m & n & 1 \end{bmatrix}$$

Que. Rotate the point (10, 13) about the point (4,3) 90° counter clockwise.

$P(10, 13) \quad Q(4,3)$

$[T_1]$ = Translate the point Q to the ~~T_2~~ origin $= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & -3 & 1 \end{bmatrix}$

$X = [10 \quad 13 \quad 1]$

$$[T_2] = \begin{bmatrix} \cos 90^\circ & \sin 90^\circ & 0 \\ -\sin 90^\circ & \cos 90^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[T_3] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 4 & 3 & 1 \end{bmatrix}$$

$$[X'] = [X][T_1][T_2][T_3]$$

$$[X'] = [10 \quad 13 \quad 1] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & -3 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} [T_3]$$

$$= [10 \quad 13 \quad 1] \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 3 & -4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 4 & 3 & 1 \end{bmatrix}$$

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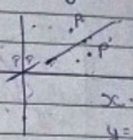
$$= \begin{bmatrix} 10 & 13 & 1 \end{bmatrix}_{1\times3} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 7 & -1 & 1 \end{bmatrix}_{3\times3}$$

$$= \begin{bmatrix} -6 & 9 & 1 \end{bmatrix}_{1\times3}$$

Reflection through an arbitrary line

[Trans][Rot][Ref][Rot]$^{-1}$[Trans]$^{-1}$



$x=0 \Rightarrow$ y axis
$y=0 \Rightarrow$ x axis

Step 1 Translate the line and the object such that the line passes through the origin.

Step 2 Rotate the line and the object until the line is coincident with one of co-ordinate axis.

Step 3 Perform reflection

Step 4 Perform inverse rotation

Step 5 Translate the line back to the original location.

Ques Given a $\triangle ABC$, $A(2,4)$, $B(4,6)$, $C(2,6)$ and a line $L \Rightarrow y = \frac{1}{2}(x+4)$. Reflect the $\triangle ABC$ through the line L.

$L = y = \frac{1}{2}(x+4)$

$x = 0 \quad y = 2$
$x = 1 \quad y = 5/2$
$x = 2 \quad y = 3$
$x = 3 \quad y = 7/2$

we can take any point.

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1) Translation

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & -3 & 1 \end{bmatrix}$$

2) ~~Reflection~~ Rotation

$y = mx + c$

$m = \frac{1}{2} \qquad \left(\tan^{-1}\frac{1}{2}\right) = 26.57$

$\cancel{\left(\tan^{-1}\left(\frac{1}{2}\right)\right)}$

$$\begin{bmatrix} \cos -26.57 & \sin -26.57 & 0 \\ -\sin -26.57 & \cos -26.57 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3) Reflection to x-axis

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

4) Rotation Inverse

$$\begin{bmatrix} \cos 26.57 & \sin 26.57 & 0 \\ -\sin 26.57 & \cos 26.57 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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5) Translation $^{-1}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 3 & 1 \end{bmatrix}$$

6) $$X = \begin{array}{c} \begin{bmatrix} 2 & 4 & 1 \\ 4 & 6 & 1 \\ 2 & 6 & 1 \end{bmatrix} \end{array} \begin{array}{l} A \\ B \\ C \end{array}$$ ~~Obb~~ Object matrix

$$X' = [X][\cancel{Rot}\ \text{Tran}][Rot][Ref][Rot^{-1}][Trans^{-1}]$$

$$X' = \begin{bmatrix} 2 & 4 & 1 \\ 4 & 6 & 1 \\ 2 & 6 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & -3 & 1 \end{bmatrix} \begin{bmatrix} \cos -26.57 & -\sin 26.57 & 0 \\ \sin 26.57 & \cos 26.57 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 26.57 & \sin 26.57 & 0 \\ -\sin 26.57 & \cos 26.57 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 3 & 1 \end{bmatrix}$$

$$X' = \begin{bmatrix} 0 & 1 & 1 \\ 2 & 3 & 1 \\ 0 & 3 & 1 \end{bmatrix} \begin{bmatrix} \cos 26.57 & -\sin 26.57 & 0 \\ \sin 26.57 & \cos 26.57 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 26.57 & \sin 26.57 & 1 \\ \sin 26.57 & -\cos 26.57 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 3 & 1 \end{bmatrix}$$

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Scaling

local scaling

$$[x \quad y \quad 1] \begin{bmatrix} s_1 & 0 & 0 \\ 0 & s_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} = [s_1x \quad s_2y \quad 1]$$

$s_1, s_2 > 1 \Rightarrow$ enlargement
$0 < s_1, s_2 < 1 \Rightarrow$ compression

$s_1 \neq s_2 \Rightarrow$ non-uniform scaling

$s_1 = 2$
$s_2 = 3$

$s_1 = s_2 \Rightarrow$ uniform scaling.

overall scaling

$$[x \quad y \quad 1] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & s \end{bmatrix} = [x \quad y \quad s] = \left[\frac{x}{s} \quad \frac{y}{s} \quad 1\right]$$

$s > 1 \Rightarrow$ Compression

$s < 1 \Rightarrow$ enlargement

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Points at infinity

Que $x + y = 1$
$2x - 3y = 0$

Check these points are intersecting or ∥ using matrix method.

$x + y - 1 = 0$
$2x - 3y = 0$

$$[x \quad y \quad 1] \underbrace{\begin{bmatrix} 1 & 2 & 0 \\ 1 & -3 & 0 \\ -1 & 0 & 1 \end{bmatrix}}_{m} = [0 \quad 0 \quad 1]$$

$$[m]^{-1} = \frac{1}{5} \begin{bmatrix} -3 & +2 & 0 \\ +1 & -1 & 0 \\ 3 & 2 & 5 \end{bmatrix}$$

~~[x y 1]~~

$$[x \quad y \quad 1] = \frac{1}{5}[0 \quad 0 \quad 1] \begin{bmatrix} 3 & 2 & 0 \\ 1 & -1 & 0 \\ 3 & 2 & 5 \end{bmatrix}$$

$$= \left[0 \quad 0 \quad \frac{1}{5}\right] \begin{bmatrix} 3 & 2 & 0 \\ 1 & -1 & 0 \\ 3 & 2 & 5 \end{bmatrix}$$

$$[x \quad y \quad 1] = \left[\frac{3}{5} \quad \frac{2}{5} \quad 1\right]$$

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Que $x + y = 1 \Rightarrow x + y - 1 = 0$

$x + y = 0$

$1 = 1$ (X) because we will not able to find the determinent

$x = x$ So we have to take any other value

$$[x \ y \ 1]\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} = [0 \ 0 \ x]$$

m

$$[x \ y \ 1][m][m^{-1}] = [0 \ 0 \ x][m]^{-1}$$

$$[x \ y \ 1] = [0 \ 0 \ x]\begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$[x \ y \ 1] = [x \ -x \ 0]$$

h is 0 . Point at infinity.

2 , 3 , 0
2 , −3 , 0
Lines are ||.

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A point at infinity on the +ve x axis

$[3 \ 0 \ 0]$

−ve x - axis = $[-3 \ 0 \ 0]$

+ve y - axis = $[0 \ 3 \ 0]$

−ve y - axis = $[0 \ -10 \ 0]$

Que Cyrus Beck

Given a clip rectangle PQRS P(5,5), Q(15,15)

E $(3, 13)$ — x_0, y_0 A $(10, 3)$
F $(7, 11)$ — x_1, y_1 B$(12, 18)$.

Clip the line using cyrus beck.

$x_{min} = 5$ $y_{min} = 5$
$x_{max} = 15$ $y_{max} = 15$

$x = x_{min}$ $t = \dfrac{-(x_0 - x_{min})}{x_1 - x_0} = \dfrac{-(3-5)}{7-3} = \dfrac{2}{4} = \dfrac{1}{2}$

Negation of a den tell that the point is potentially entering or leaving $-2 < 0 \Rightarrow PE$

$x = x_{max}$ $t = \dfrac{x_0 - x_{max}}{-(x_1 - x_0)} = \dfrac{3-15}{-(7-3)} = \dfrac{-12}{-4} = +3$ $t \notin (0,1)$ neglect

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$y = y_{min}$ $\quad t = \frac{(y_0 - y_{min})}{y_1 - y_0} = \frac{-(13-5)}{15-13} = \frac{-8}{-2} = 4 \quad t \notin (0,1)$ neglect

$y = y_{max}$ $\quad t = \frac{y_0 - y_{max}}{-(y_1 - y_0)} = \frac{13-15}{-(11-13)} = \frac{-2}{-2} = -1 \quad t \notin (0,1)$ neglect

$$P(t) = P_0 + (P_1 - P_0)t \qquad 0 \le t \le 1$$

$$P\left(\tfrac{1}{2}\right)x = 3 + (7-3)\tfrac{1}{2} = 5$$

$$P\left(\tfrac{1}{2}\right)y = 13 + (11-13)\tfrac{1}{2} = 12$$

My intersection point is (5, 12).

* Do for A(10, 3) & B(12, 18).

A (10, 3) — x_0, y_0

B (12, 18) — x_1, y_1

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Ch - 3D

[x y z h]

$$T = \begin{bmatrix} a & b & c & p \\ d & e & f & q \\ g & i & j & r \\ l & m & n & s \end{bmatrix}_{4\times4}$$

local scaling

$$\begin{bmatrix} a & 0 & 0 & 0 \\ 0 & e & 0 & 0 \\ 0 & 0 & j & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

overall scaling

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & s \end{bmatrix}$$

Que Given a unit Q at the origin perform uniform scaling by a factor of 2. Give the resultant Q.

for homogeneous coordinates.

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

overall

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1/2 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 1/2 \\ 0 & 0 & 1 & 1/2 \\ 0 & 1 & 0 & 1/2 \\ 0 & 1 & 1 & 1/2 \\ 1 & 0 & 0 & 1/2 \\ 1 & 0 & 1 & 1/2 \\ 1 & 1 & 0 & 1/2 \\ 1 & 1 & 1 & 1/2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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Divide each row by 1/2 (we want final row be 1)

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 2 & 1 \\ 0 & 2 & 0 & 1 \\ 0 & 2 & 2 & 1 \\ 2 & 0 & 0 & 1 \\ 2 & 0 & 2 & 1 \\ 2 & 2 & 0 & 1 \\ 2 & 2 & 2 & 1 \end{bmatrix}$$

Shearing → off diagnols

$$[x \quad y \quad z \quad 1] \begin{bmatrix} 1 & b & c & 0 \\ d & 1 & f & 0 \\ g & i & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow [(x+dy+gz) \quad (y+bx+iz) \quad (z+cx+fy) \quad 1]$$

$(x+dy+gz)$ ↓ shearing is done in x by a factor of y & z

$(y+bx+iz)$ ↓ shearing is in y by a factor of x & z

Rotation wrt an axis (anti-clockwise)

$$x\text{-axis} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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wrt an axis y axis

$$y\text{-axis} = \begin{bmatrix} \cos\theta & 0 & -\sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$z\text{-axis} = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For clock wise direction → change it's (θ) to $(-\theta)$

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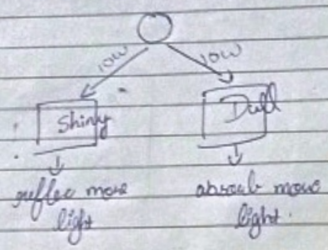
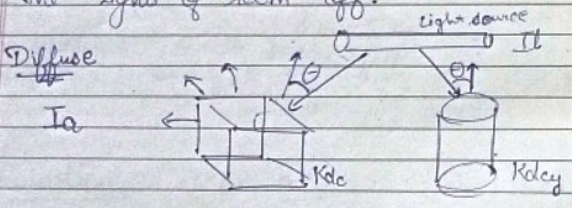
Light



Specular reflection => small area with lots of light & that area so looks too shiny.

Diffuse ~~area~~ reflection => equal division of light through which we will able to see the ~~whol~~ whole object.

Ambient light => a small ~~light~~ source of light coming into the room after all the lights of room off.



$I_{ambdiff} = I_a \times K_{dc}$	$I_{ambdiff} = I_a \times K_{dcy}$
$I_{diff} = I_a \times K_{dc} + I_l \cos\theta \times K_{dc}$	$I_{diff} = I_a \times K_{dcy} + I_l \cos\theta \times K_{dcy}$

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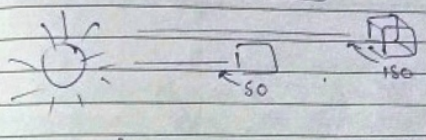
Object properties $= n_s$

Viewer's view $= \phi$.

Halfway vector $= \dfrac{L+V}{|L+V|}$

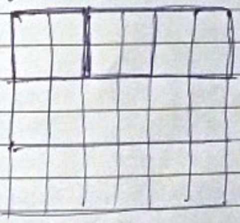
Intensity Attenuation (decrease)



Decrease in the intensity of light is depend on the distance b/w light source & of object.

~~Z~~ A buffer algo => Z buffer of a transe objects.

Half toning



$6 \times 6 = 36$

0,1

0	1
2	3

If we on 1 box.
[0] = light black

If we on 2 box
[0 | 1] = brighter than 1

If we on 3 box
[0 | 1 / 2] = brighter than 2

If we on 4 box
[0 1 / 2 3] = black spot

0	1	4	
2	3	5	

4 & 5 look grey as compare to the left box which is black in colour.

4 & 5 also a black spot but lighter than (0, 1, 2, 3) box.

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Dithering Techniques

& realization

It is just same as half toning.

Assignment

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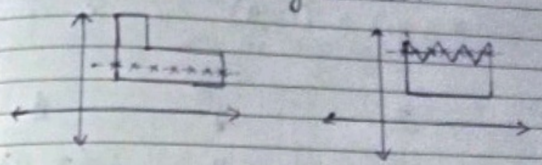
Que It states that the parity is initially even. Each intersection encountered, inverts the parity. We draw / illuminate when the parity is odd. Do not draw when parity is even.



Que2 True, 2D rotations are commutative. This means rotations doesn't affect the final result.

Example :-

$$\Rightarrow \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \quad \{R_1 \times R_2\}$$

$$\begin{bmatrix} \cos\theta\cos\phi - \sin\theta\sin\phi & -\cos\theta\sin\phi - \sin\theta\cos\phi \\ \sin\theta\cos\phi + \cos\theta\sin\phi & -\sin\theta\sin\phi + \cos\theta\cos\phi \end{bmatrix}$$

$$\cancel{\cos(\theta} \begin{bmatrix} \cos(\theta+\phi) & -\sin(\theta+\phi) \\ \sin(\theta+\phi) & \cos(\theta+\phi) \end{bmatrix} \quad — (1)$$

$$\Rightarrow \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \quad \{R_2 \times R_1\}$$

$$\begin{bmatrix} \cos\phi\cos\theta - \sin\phi\sin\theta & -\cos\phi\sin\theta - \sin\phi\cos\theta \\ \sin\phi\cos\theta + \cos\phi\sin\theta & -\sin\phi\sin\theta + \cos\phi\cos\theta \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi+\theta) & -\sin(\theta+\phi) \\ \sin(\theta+\phi) & \cos(\theta+\phi) \end{bmatrix} \quad — (2)$$

Both the operations yield the same result.
Equation 1 = 2.

Que 3 $(x_1, y_1) = (3, 4)$

$(x_2, y_2) = (13, 9)$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5}{10} = \frac{1}{2}$

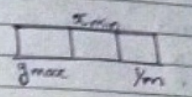
x	y	
3	4	(3, 4)
4	4+0.5 = 4.5~5	(4, 5)
5	4.5+0.5 = 5	(5, 5)
6	5+0.5 ~ 6	(6, 6)
7	5.5+0.5 = 6	(7, 6)
8	6+0.5 ~ 7	(8, 7)
9	6.5+0.5 = 7	(9, 7)
10	7+0.5 ~ 8	(10, 8)
11	7.5+0.5 = 8	(11, 8)
12	8+0.5 ~ 9	(12, 9)
13	8.5+0.5 = 9	(13, 9)

Que 4 Scan line Algorithm

There are 2 data structure used for scan line algorithm :-

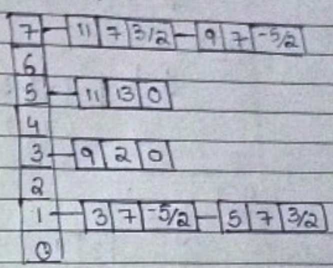
AET - Active Edge Table
ET - Edge Table / Global Edge Table.

Global Edge Table is maintain as a bucket on the y_{min} value. Each line edge has a node in the global edge table of the form y_{max}, x_{min} and $1/m$.



Explanation with an example :-

A (2,3)
B (7,1)
C (13,5)
D (13,11)
E (7,7)
F (2,9)



Bucket-sort edge Table

AB

3	7	-5/2

BC

5	7	3/2

CD

11	13	0

DE

11	7	3/2

EF

9	7	-5/2

FA

9	2	0

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Active Edge Table will be drawn at (y=c). The nodes are copied from 1 active edge table to another, new nodes are also added from the global edge table.

AET

y=0 Null

AET

y=1

| 3 | 7 | -5/2 | — | 5 | 7 | 3/2 |

(7,1) ⟷ (7,1)

AET

y=2

| 3 | 9/2 | -5/2 | — | 5 | 17/2 | 3/2 |

(5,2) ⟷ (8,2)

AET

y=3

| 5 | 10 | 3/2 | — | 9 | 2 | 0 |

arrange the nodes

| 9 | 2 | 0 | — | 5 | 10 | 3/2 |

(2,3) (10,3)

AET

y=4

| 9 | 2 | 0 | — | 5 | 23/2 | 3/2 |

(2,4) (11,4)

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AET

y=5

| 9 | 2 | 0 | — | 11 | 13 | 0 |

(2,5) (13,5)

AET

y=6

| 9 | 2 | 0 | — | 11 | 13 | 0 |

(2,6) (13,6)

AET

y=7

| 9 | 2 | 0 | — | 11 | 13 | 0 | — | 11 | 7 | 3/2 | (7,7)

(2,7) (13,7)

| 9 | 7 | -5/2 | (7,7)

arrange the nodes

| 9 | 2 | 0 | — | 9 | 7 | -5/2 | — | 11 | 7 | 3/2 | — | 11 | 13 | 0 |

(2,7) (7,7) (7,7) ~~(13,~~ (13,7)

AET

y=8 (2,8) ⟷ (4,8) (9,8) ⟷ (13,8)

AET y=9 (11,9) ⟷ (13,9)

AET y=10 (13,10) ⟷ (13,10)

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Ques In ΔABC

A (0,0)
B (1,1)
C (5,2)

$$X = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 5 & 2 & 1 \end{bmatrix}$$

$$\text{Scaling } S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/3 \end{bmatrix}$$

$$X' = [X][S]$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1/3 \\ 1 & 1 & 1/3 \\ 5 & 2 & 1/3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 3 & 3 & 1 \\ 15 & 6 & 1 \end{bmatrix}$$ (After multify each value of the matrix with 3)

Translate the point B from (3,3) to (1,1)

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$$\text{Translate Matrix}, T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & -2 & 1 \end{bmatrix}$$

$$\text{Final Matrix}, X'' = [X'][T]$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 3 & 3 & 1 \\ 15 & 6 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -2 & 1 \\ 1 & 1 & 1 \\ 13 & 4 & 1 \end{bmatrix}$$

So, final points are A(-2,-2), B(1,1), C(13,4)

Ques Depth Sort Algorithm or Painter's Algorithm

1 Sort on Z-value

2 Resolve any ambiguity or split

3 Scan convert each polygon starting from the smallest Z value.
(Nearest to the screen or farthest from the view point).

To check is the split is required or not :-

We have to do 5 test, if any test comes true we know there is an ambiguity and split is needed. If the test is false we check the next test. If all the tests fails that means there is no ambiguity and you can draw a polygon.

If we have to draw the polygon P we will check the polygon P against all the other polygon and all the 5 tests must fails for all other Q's

T-1 X extends don't overlap → T/F

T-2 Y extends don't overlap → T/F

T-3 Is P entirely on the opposite side of Q's plain ~~form~~ from the view point.

↳ Is P completely behind Q

T-4 Is Q entirely on the same side of the P's plain on the view point.

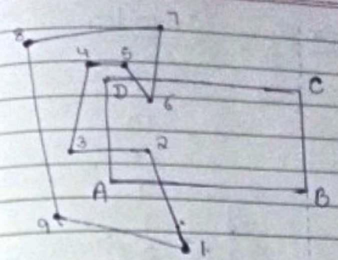
↳ Is P completely in front of Q.

T-5 3D-2D projection

Perform 3D to 2D projection and check if the projects don't overlap.

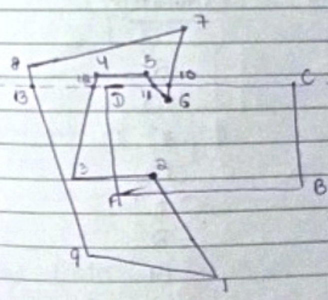
If all the 5 tests fails then split has to be perform.

Que 7



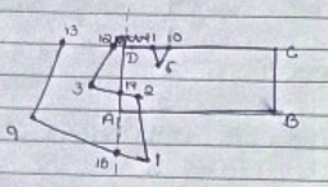
Right Edge

Invertex	Outvertex
6	6
7	7
8	8
9	9
1	1
2	2
3	3
4	4
5	5
6	6



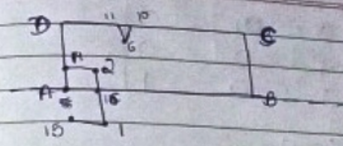
Top Edge

Invertex	Outvertex
6	6
7	10
8	13
9	9
1	1
2	2
3	3
4	12
5	11
6	6



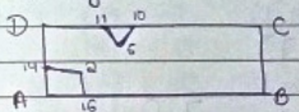
Left Edge

Invertex	Outvertex
6	6
10	10
13	[illegible]
9	15
1	1
2	2
3	14
12	
11	11
6	6



Invertex	Outvertex
6	6
10	10
~~11~~	~~11~~
15	-
1	16
2	2
14	14
11	11
6	6

Final Image



Qu9 Let $X = \begin{bmatrix} a & b & 1 \\ c & d & 1 \end{bmatrix}$

a) Rotation about origin by 90°

$$Rot = \begin{bmatrix} \cos 90^\circ & \sin 90^\circ & 0 \\ -\sin 90^\circ & \cos 90^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$X_1 = [X][Rot]$$

$$= \begin{bmatrix} a & b & 1 \\ c & d & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -b & a & 1 \\ -d & c & 1 \end{bmatrix}$$

b) Translation in x-axis by 20 units

$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 20 & 0 & 1 \end{bmatrix}$$

$$X_2 = [X_1][T]$$

$$= \begin{bmatrix} -b & a & 1 \\ -d & c & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 20 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -b+20 & a & 1 \\ -d+20 & c & 1 \end{bmatrix}$$

c) Uniform scaling by a factor of 3

$$S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/3 \end{bmatrix}$$

$$X_3 = [X_2][S]$$

$$= \begin{bmatrix} -b+20 & a & 1 \\ -d+20 & c & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/3 \end{bmatrix}$$

$$= \begin{bmatrix} -b+20 & a & 1/3 \\ -d+20 & c & 1/3 \end{bmatrix}$$

$$= \begin{bmatrix} -3b+60 & 3a & 1 \\ -3d+60 & 3c & 1 \end{bmatrix}$$

d) Reflection about line $y = -x$

$$Ref = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$X_4 = [X_3][Ref]$$

$$= \begin{bmatrix} -3b+60 & 3a & 1 \\ -3d+60 & 3c & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -3a & 3b-60 & 1 \\ -3c & 3d-60 & 1 \end{bmatrix}$$

So, if we start with a 2D matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ after all the transformation, we get

$$\begin{bmatrix} -3a & 3b-60 \\ -3c & 3d-60 \end{bmatrix}$$

Que 10 $[X] = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$

a) Translation in x-direction by 2 units

$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

$[X_1] = [X][T]$

$= \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 2 & 0 & 0 & 1 \\ 3 & 0 & 1 & 1 \\ 3 & 1 & 1 & 1 \\ 2 & 1 & 1 & 1 \\ 2 & 0 & 0 & 1 \\ 3 & 0 & 0 & 1 \\ 3 & 1 & 0 & 1 \\ 2 & 1 & 0 & 1 \end{bmatrix}$

b) Double the size of cube by 2 units

$S = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$[X_2] = [X_1][S]$

$= \begin{bmatrix} 2 & 0 & 0 & 1 \\ 3 & 0 & 1 & 1 \\ 3 & 1 & 1 & 1 \\ 2 & 1 & 1 & 1 \\ 2 & 0 & 0 & 1 \\ 3 & 0 & 0 & 1 \\ 3 & 1 & 0 & 1 \\ 2 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 4 & 0 & 0 & 1 \\ 6 & 0 & 2 & 1 \\ 6 & 2 & 2 & 1 \\ 4 & 2 & 2 & 1 \\ 4 & 0 & 0 & 1 \\ 6 & 0 & 0 & 1 \\ 6 & 2 & 0 & 1 \\ 4 & 2 & 0 & 1 \end{bmatrix}$$

c) Reflect ~~above~~ about $x-y$ plane

$$Ref = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[X_3] = [X_2]\,[Ref]$$

$$= \begin{bmatrix} 4 & 0 & 0 & 1 \\ 6 & 0 & 2 & 1 \\ 6 & 2 & 2 & 1 \\ 4 & 2 & 2 & 1 \\ 4 & 0 & 0 & 1 \\ 6 & 0 & 0 & 1 \\ 6 & 2 & 0 & 1 \\ 4 & 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 & 0 & 1 \\ 6 & 0 & 2 & 1 \\ 6 & 2 & 2 & 1 \\ 4 & 2 & 2 & 1 \\ 4 & 0 & 0 & 1 \\ 6 & 0 & 0 & 1 \\ 6 & 2 & 0 & 1 \\ 4 & 2 & 0 & 1 \end{bmatrix}$$

Ques 3

Center = $(5,5)$
Radius = 10

x	y	d
0	10	$\frac{5}{4} - 10 = \frac{-35}{4} < 0$ (E)
1	10	$\frac{-35}{4} + 3 = \frac{-23}{4} < 0$ (E)
2	10	$\frac{-23}{4} + 5 = \frac{-3}{4} < 0$ (E)
3	10	$\frac{-3}{4} + 7 = \frac{25}{4} > 0$ (SE)
4	9	$+\frac{25}{4} + 6 - 20 + 5 = \frac{-11}{4} < 0$ (E)
5	9	$\frac{-11}{4} + 13 = \frac{41}{4} > 0$ (SE)
6	8	$\frac{41}{4} + 10 - 18 + 5 = 29 > 0$ (SE)
7	7	$x \geq y$, Stop