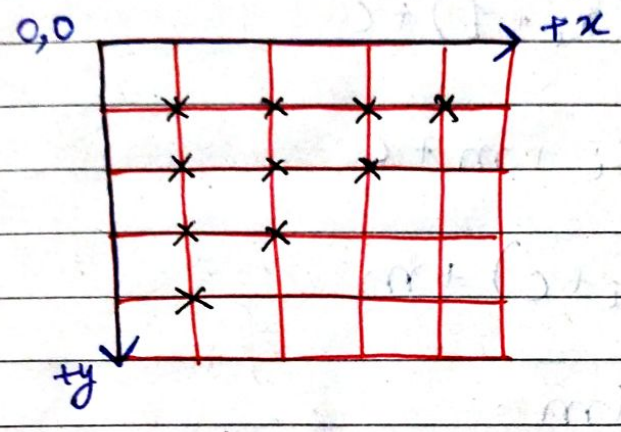
Ch-3:

Screen



(1,1) (3,4)

Pixel → on
→ off

$y=0$, $m=0$

$y=0 \Rightarrow$ x-axis , $m=0$

$x=c$, $m=\infty$

$x=0$, $m=\infty$

$x=y$, $m=1$

$x=-y$, $m=-1$

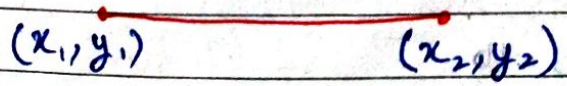
$x = y$

$x\frac{dy}{dx} = \frac{dy}{dx}$

$1 = \frac{dy}{dx} = m$

$$\boxed{y = mx + c}$$

DDA line drawing algorithm (Digital Differential Algo)



$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\boxed{y_i = m x_i + c}$$

$$y_{i+1} = m x_{i+1} + c$$

$$x_{i+1} = x_i + 1$$

So, $y_{i+1} = m(x_i + 1) + c$

$\Rightarrow y_{i+1} = m x_i + m + c$

$\Rightarrow y_{i+1} = (m x_i + c) + m$

$\Rightarrow y_{i+1} = y_i + m$

$\therefore$ This DDA line algo. only valid for $-1 \le m \le 1$

Q) Generate all the points between the end points of there line (5, 8) (9, 11)

A) $(\underset{x_1}{5}, \underset{y_1}{8})$ $(\underset{x_2}{9}, \underset{y_2}{11})$

x	y	
5	8	(5, 8)
6	$\frac{8+3}{4} = 8.75 = 9$	(6, 9)
7	$\frac{8.75+3}{4} = 9.5 = 10$	(7, 10)
8	$\frac{9.5+3}{4} = 10.25 = 10$	(8, 10)
9	$\frac{10.25+3}{4} = 11$	(9, 11)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{11-8}{9-5} = \frac{3}{4}$$

Q) Calculate all the points between the end points (1, 4) and (2, 10)

$$m = \frac{10-4}{2-1} = \frac{6}{1} = 6$$

x	y	
1	4	(1, 4)
2	4+6 = 10	(2, 10)

We can't apply DDA line algo. because slope > 1

<u>Disadvantages</u>

- It only works for the slope between $-1 \leq m \leq 1$
- There is round of error in almost all the points

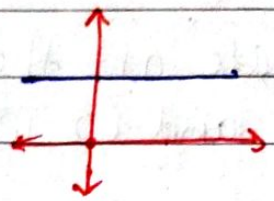
For this reason, we are moving to Bresenham's line drawing algo, which is also called mid-point

4 special cases of DDA line algo.

1. Horizontal Line

$y = c \Rightarrow m = 0$

$(1,10)$ to $(10,10)$



x	y
1	10
2	$10+0=10$
3	$10+0=10$
⋮	⋮
10	$10+0=10$

'y' remains same for all the values, just increase 'x'. Don't follow the whole process.

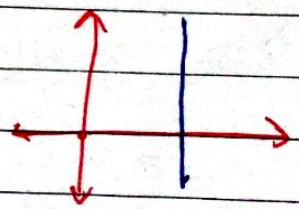
2. Vertical Line

$x = c \Rightarrow m = \infty$

$(10,3)$ to $(10,9)$

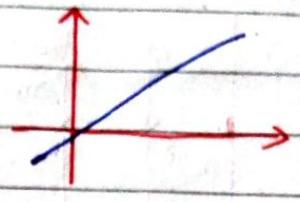
x	y
10	3
10	4
⋮	⋮
10	9

'x' remains same for all the values, just increase 'y'



3. <u>Change in x = Change in y</u>

$m = \frac{\Delta x}{\Delta y} = 1$



$(1,1)$ to $(6,6)$

x	y
1	1
2	2
3	3
⋮	⋮
6	6

4. <u>$m > 1$</u>

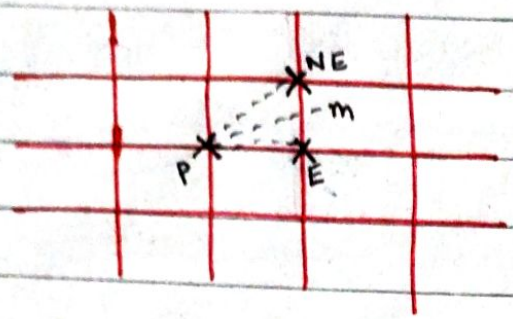
$\frac{\Delta y}{\Delta x} > 1 \Rightarrow \Delta y = \Delta x \Rightarrow y_{i+1} = y_i + 1$

$\Rightarrow x_{i+1} = x_i + \frac{1}{m}$

$(1,4)$ and $(2,10) \Rightarrow m = 6$

x	y	
1	4	(1, 4)
$1 + \frac{1}{6} = \frac{7}{6} = 1.1 = 1$	$4+1=5$	(1, 5)
$\frac{7}{6} + \frac{1}{6} = \frac{8}{6} = 1$	$5+1=6$	(1, 6)
$8/6 + 1/6 = 9/6 = 2$	$6+1=7$	(2, 7)
$9/6 + 1/6 = 2$	$7+1=8$	(2, 8)
$10/6 + 1/6 = 2$	$8+1=9$	(2, 9)
$11/6 + 1/6 = 2$	$9+1=10$	(2, 10)

Mid-Point (Bresenham's Line Drawing Algo)



$P = x_p, y_p$

(East) $E = x_p+1, y_p$

(North-East) $NE = x_p+1, y_p+1$

$$m = x, y$$

$$= \left(\frac{x_p+1 + x_p+1}{2}, \frac{y_p+1 + y_p}{2} \right)$$

$$= \left(x_p+1, y_p + \frac{1}{2} \right)$$

If line made from P is above m ⇒ NE
" " " " " " below m ⇒ E

'd' is the decision variable which is equal to the line m

$$d = F(m)$$

$$y = mx + B$$

$$\Rightarrow y = x\frac{dy}{dx} + B$$

$$\Rightarrow dx \cdot y = dy \cdot x + B \cdot dx$$

$\Rightarrow F(x,y) = x\,dy - y\,dx + B\,dx$

$\Rightarrow F(x,y) = ax - by + c$

$a = dy \quad , b = dx$

$d = F\left(x_p + 1 , y_p + \frac{1}{2}\right)$

$\Rightarrow d = a(x_p + 1) - b\left(y_p + \frac{1}{2}\right) + c$

$d > 0$, The value of 'd' is +ve and 'm' is below the line $\Rightarrow$ NE

$d < 0$, The value of 'd' is -ve and 'm' is above the line $\Rightarrow$ E

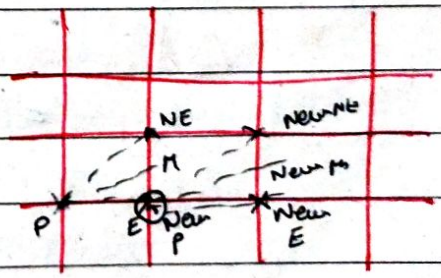
$d = 0$, The value of 'd' is 0 and 'm' is on the line $\Rightarrow$ E (either works)

Choose E

New P = $x_p + 1 , y_p$

New E = $x_p + 2 , y_p$

New NE = $x_p + 2 , y_p + 1$



New M = $\left(x_p + 2 , y_p + \frac{1}{2}\right)$

New d = $F\left(x_p + 2 , y_p + \frac{1}{2}\right)$

$$= a(x_p+2) - b\left(y_p+\frac{1}{2}\right) + c$$

~~New d = d + Δd_E~~

$$\boxed{\text{New } d = d + \Delta d_E}$$

$$\Delta d_E = a = dy \quad \text{——— (1)}$$

Similarly, when we choose NE

New P = $x_p+1,\ y_p+1$
New E = $x_p+2,\ y_p+1$
New NE = $x_p+2,\ y_p+2$

$$\text{New } M = \left(x_p+2,\ y_p+\frac{3}{2}\right)$$

$$\text{New } d = a(x_p+2) - b\left(y_p+\frac{3}{2}\right) + c$$

$$\boxed{\text{New } d_{NE} = d + \Delta d_{NE}}$$

$$\Delta d_{NE} = a - b \quad \text{——— (2)}$$
$$= dy - dx$$

Initial value of d.

(1) – x_1, y_1 = First P

ini. M = $x_1+1, y_1+\frac{1}{2}$

ini. d = F(ini. M)

$= a(x_1+1) - b\left(y_1+\frac{1}{2}\right) + c$

$= \underbrace{ax_1 - by_1 + c}_{} + a - \frac{b}{2}$

Knowing that (x_1, y_1) lies on the line we get 0.

$= a - \frac{b}{2} = dy - \frac{dx}{2}$

Q) Generate all pts. for the line (5,8) (9,11) using Bresenham's line drawing Algo.

A)

x	y	d_{ini}
5	8	$dy - \frac{dx}{2} = 3 - \frac{4}{2} = 1 > 0$ direction is NE, so increase both by 1
6	9	$1 + \Delta d_{NE} = 1 - 1 = 0$, choosing E
7	9	$0 + \Delta d_E = 3 > 0$ (NE)
8	10	$3 + \Delta d_E = 3 - 1 = 2 > 0$ (NE)
9	11	-

(Start with smallest x)

$a = dy = y_2 - y_1 = 11 - 8 = 3$

$b = dx = x_2 - x_1 = 4$

Mid Point Circle Drawing Algo / Bresenham's Circle Drawing Algo

$x^2 + y^2 = R^2 \Rightarrow y = \sqrt{R^2 - x^2}$

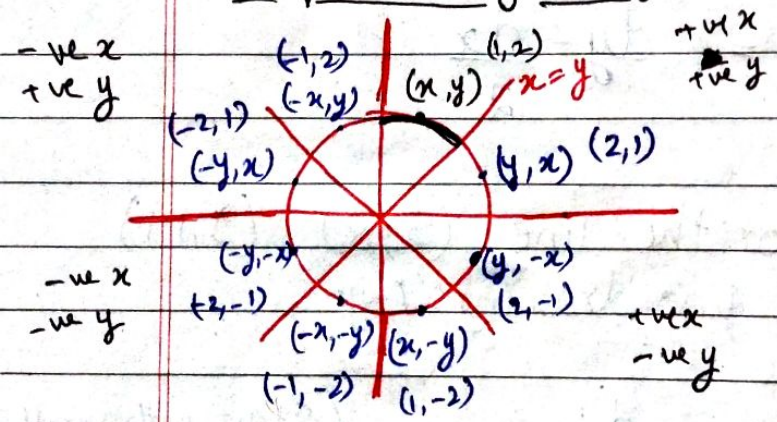
$\rightarrow f(x,y) = x^2 + y^2 - R^2$

R = Radius

Center = (0, 0)

if Center = (a, b) $\Rightarrow (x-a)^2 + (y-b)^2 = R^2$

8-point symmetry



Assume,

P = 3, 4

using 8-point symmetry

(3, 4) ⇒ -3, 4

⇒ -3, -4

⇒ 3, -4

⇒ 4, 3

⇒ -4, 3

⇒ -4, -3

⇒ 4, -3

(x, 0) on x-axis, 4 points

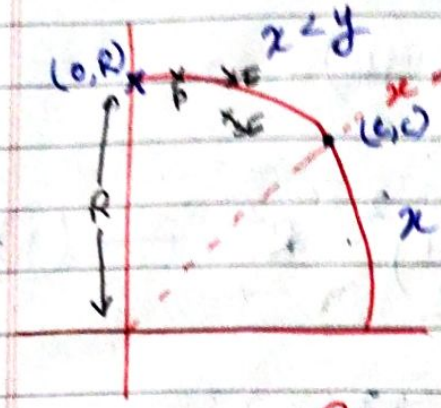
(3, 0) ⇒ (-3, 0)

⇒ (0, 3)

⇒ (0, -3)

(c, c) has 4 points ??

(c, c) ⇒ (3, 3), (-3, 3), (-3, -3), (3, -3)



Start from (0,R); stop if we get $x=y$ or $x>y$

So, stop if $x=y$.

$P = x_p, y_p$
$E = x_p+1, y_p$
$SE = x_p+1, y_p-1$

$$M = \left(x_p+1, y_p-\frac{1}{2}\right)$$

$$d = F(M)$$
$$= F\left(x_p+1, y_p-\frac{1}{2}\right)$$

$d>0$, The point M is outside the circle (SE)

$d<0$, The point M is inside the circle (E)

$d=0$, The point M is on the circle (SE)

<u>Choose E,</u>

New $P = x_p+1, y_p$
New $E = x_p+2, y_p$
New $SE = x_p+2, y_p-1$

$$M = \left(x_p+2, y_p-\frac{1}{2}\right)$$

$$\text{New } d = F\left(x_p+2,\ y_p-\frac{1}{2}\right)$$

$$= (x_p+2)^2 + \left(y_p-\frac{1}{2}\right)^2 - R^2 \quad \text{— (1)}$$

$$\text{Old } d = (x_p+1)^2 + \left(y_p-\frac{1}{2}\right)^2 - R^2$$

$$\Delta d_E = \text{New } d - \text{Old } d$$

$$= 2x_p + 3$$

Imp ★

$$\boxed{\Delta d_E = 2x_p + 3}$$

Choose SE,

New P = $x_p+1,\ y_p-1$
New E = $x_p+2,\ y_p-1$
New SE = $x_p+2,\ y_p-2$

$$\text{New } M = \left(x_p+2,\ y_p-\frac{3}{2}\right)$$

$$\text{New } d = (x_p+2)^2 + \left(y_p-\frac{3}{2}\right)^2 - R^2 \quad \text{— (2)}$$

$\Delta d_{SE} =$ New d - Old d

$= 2x_p + 3 + 2 - 2y_p$

$= 5 + 2x_p - 2y_p$

$$\boxed{\Delta d_{SE} = 2x_p - 2y_p + 5}$$

Imp

* The change in d, for the circle is a linear function in x & y, whereas for the line, it was a constant.

<u>Initial Value of d</u>

First P $\longrightarrow (0, R)$

$M_{ini} \rightarrow \left(1, R - \frac{1}{2}\right)$

$d_{ini} \rightarrow F(M_{ini})$

$= 1^2 + \left(R - \frac{1}{2}\right)^2 - R^2$

$= \cancel{R^2} + \frac{1}{4} - R + 1 - \cancel{R^2} = \frac{5}{4} - R$

$$\boxed{d_{ini} = \frac{5}{4} - R}$$

6 marks

Q) Generate all points for the circle centered at (0,0) with the radius 6.

4 marks

A) First $P = (0,6)$

$$d_{ini} = \frac{5}{4} - R = \frac{5}{4} - 6 = \frac{-19}{4}$$

$$\Delta d_E = 2(0)+3 = 3 \quad (x_{prev} = 0 \text{ initially})$$

$$\Delta d_{SE} = 2(0) - 2(6) + 5 = -7 \quad (y_{prev} = 0 \text{ initially})$$

x	y	d
0	6	$d_{ini} = \frac{-19}{4} < 0$ (E)
1	6	$\frac{-19}{4} + 3 = \frac{-7}{4} < 0$ (E)
2	6	$\frac{-7}{4} + 2(1) + 3 = \frac{13}{4} > 0$ (SE)
3	5	$\frac{13}{4} + 2(2) - 2(6) + 5 = \frac{1}{4} > 0$ (SE)
4	4	$x \geq y$, so stop

Using 8-way symetry property of the circle, we will replicate the points

Page No:

2 marks

$(0,6) \rightarrow (0,-6), (6,0), (-6,0)$

$(1,6) \rightarrow (1,-6), (-1,-6), (-1,6), (6,1),$
$(-6,1), (-6,-1), (6,-1)$

$(2,6) \rightarrow (-2,6), (-2,-6), (2,-6), (6,2),$
$(-6,2), (-6,-2), (6,-2)$

$(3,5) \rightarrow (-3,5), (-3,-5), (3,-5), (5,3),$
$(-5,3), (-5,-3), (5,-3)$

$(4,4) \rightarrow (-4,4), (-4,-4), (4,-4)$

Q) Calculate all points of the circle with the equation x^2+y^2-25.

A) First $P=(0,5)$

$d_{ini} = \frac{5}{4} - 5 = \frac{-15}{4}$

x	y	d
0	5	$\frac{-15}{4} < 0$ (E)
1	5	$\frac{-15}{4} + 3 = \frac{-3}{4} < 0$ (E)
2	5	$\frac{-3}{4} + 5 = \frac{17}{4} > 0$ (SE)
3	4	$\frac{17}{4} + 6 - 8 + 5 = \frac{13}{4} > 0$ (SE)

4 3 $x > y$, so stop

Points

$(0,5) \rightarrow (0,-5), (5,0), (-5,0)$

$(1,5) \rightarrow (1,-5), (-1,-5), (-1,5), (5,1),$
$(-5,1), (-5,-1), (5,-1)$

$(2,5) \rightarrow (2,-5), (-2,-5), (-2,5), (5,2),$
$(-5,2), (-5,-2), (5,-2)$

$(3,4) \rightarrow (3,-4), (-3,-4), (-3,4), (4,3)$
$(-4,3), (-4,-3), (4,-3)$

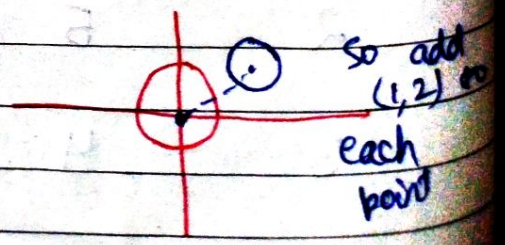
$(4,3) \rightarrow$ Same as above

Q) Calculate all points of circle with equation
$(x-1)^2 + (y-2)^2 = 16$

A) Center $= (1,2)$
Radius $= 4$

lets translate the circle to the center $(0,0)$

New eqn $\rightarrow x^2 + y^2 = 4^2$



Page No:

First $P = (0, 4)$

$d_{ini} = \frac{5}{4} - 4 = \frac{-11}{4}$

x	y	d
0	4	$\frac{-11}{4} < 0$ (E)
1	4	$\frac{-11}{4} + 3 = \frac{1}{4} > 0$ (SE)
2	3	$\frac{1}{4} + 4 - 6 + 5 = \frac{21}{4} > 0$ (SE)
3	2	$x > y$, so stop

Points

$(1, 6) \rightarrow (1, -6), (-1, -6), (-1, 6), (6, 1),$
$(-6, 1), (-6, -1), (6, -1)$

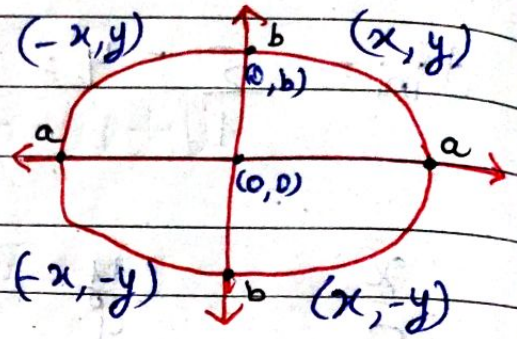
$(2, 6) \rightarrow (2, -6), (-2, -6), (-2, 6), (6, 2),$
$(6, -2), (-6, -2), (-6, 2)$

$(3, 5) \rightarrow (3, -5), (-3, -5), (-3, 5), (5, 3),$
$(-5, 3), (-5, -3), (5, -3)$

$(4, 4) \rightarrow (-4, 4), (-4, -4), (4, -4)$

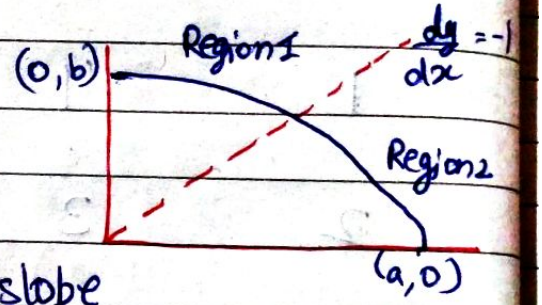
Mid-Point Ellipse Algorithm

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



Major Axis = 2a

Minor Axis = 2b



Region1 and Region 2 are seperated where the slope of the tangent is -1. When the slope of the tangent is > -1, then we are in Region 1 and the slope of the tangent < -1, then we are in Region 2.

Region 1

P = x_p, y_p

E = x_p+1, y_p

SE = x_p+1, y_p-1

M = $x_p+1, y_p-\frac{1}{2}$

d = F(M)

$$= b^2(x_p+1)^2 + a^2\left(y_p-\frac{1}{2}\right)^2 - a^2b^2$$

$d < 0$, East
$d \geq 0$, South- East

Choose E,

New P = x_p+1, y_p
New E = x_p+2, y_p
New SE = x_p+2, y_p-1

New M = $x_p+2, y_p-\frac{1}{2}$

$d_E = f(\text{new } M)$

$$= b^2(x_p+2)^2 + a^2\left(y_p-\frac{1}{2}\right)^2 - a^2b^2 \quad \text{—— (1)}$$

$$\boxed{\Delta d_E = b^2(2x_p+3)}$$

Choose SE,

New P = x_p+1, y_p-1
New E = x_p+2, y_p-1
New SE = x_p+2, y_p-2

New M = $x_p+2, y_p-\frac{3}{2}$

$d_{SE} = f(\text{new } M)$

$$= b^2(x_p+2)^2 + a^2\left(y_p-\frac{3}{2}\right) - a^2b^2 \quad \text{—— (2)}$$

$$\boxed{\Delta d_{SE} = b^2(2x_p+3) + a^2(2-2y_p)}$$

we stay in the region untill:

$$a^2\left(y_p - \frac{1}{2}\right) > b^2(x_p+1)$$

Region 2

$P = x_p, y_p$
$S = x_p, y_p - 1$
$SE = x_p+1, y_p-1$

$M = x_p + \frac{1}{2}, y_p - 1$

$d = f(M)$

$= b^2\left(x_p + \frac{1}{2}\right) + a^2(y_p-1)^2 - a^2b^2$

$d < 0$, South-Eash
$d \geq 0$, South

Choose S,

new $P = x_p, y_p - 1$
new $S = x_p, y_p - 2$
new $SE = x_p - 1, y_p - 2$

Page No:

$$\text{new } M = x_p + \frac{1}{2}, \; y_p - 2$$

$$\text{new } d_s = f(\text{new } M)$$

$$= b^2\left(x_p + \frac{1}{2}\right) + a^2(y_p - 2)^2 - a^2b^2 \quad \text{—(1)}$$

$$\boxed{\Delta d_s = a^2(3 - 2y_p)}$$

Choose SE,

$$\text{new } P = x_p + 1, \; y_p - 1$$
$$\text{new } S = x_p + 1, \; y_p - 2$$
$$\text{new } SE = x_p + 2, \; y_p - 2$$

$$\text{new } M = x_p + \frac{3}{2}, \; y_p - 2$$

$$\text{new } d_{SE} = f(\text{new } M)$$

$$= b^2\left(x_p + \frac{3}{2}\right) + a^2(y_p - 2)^2 - a^2b^2 \quad \text{—(2)}$$

$$\boxed{\Delta d_{SE} = b^2(2x_p + 2) + a^2(3 - 2y_p)}$$

We stop generating the points of region 2 when we reach, $(a, 0)$.

Initiad d

Initial $P = (0, b)$

Initial $M = \left(1, b - \frac{1}{2}\right)$

$$d_{ini} = F(M_{ini})$$

$$= F\left(1, b - \frac{1}{2}\right)$$

$$= \boxed{b^2 + a^2\left(\frac{1}{4} - b\right)}$$

Q) Calculate all the points of the ellipse $\frac{x^2}{(2a)^2} + \frac{y^2}{(10)^2} = 1$

A) Region 1

$$a^2\left(y_p - \frac{1}{2}\right) > b^2(x_p + 1)$$

$$= 400\left(y_p - \frac{1}{2}\right) > 100(x_p + 1)$$

$$= 4y_p - x_p > 3$$

x	y	d
0	10	$b^2+a^2\left(\frac{1}{4}-b\right) = -3 < 0$ (E)
1	10	$b^2(2x_p+3) = -3800 = -3500 < 0$ (E)
2	10	$-3500+100(2+3) = -3000 < 0$ (E)
3	10	$-3000+100(4+3) = -2300 < 0$ (E)
4	10	$-2300+100(6+3) = -1400 < 0$ (E)
5	10	$-1400+100(8+3) = -300 < 0$ (E)
6	10	$-300+100(10+3) = 1000 > 0$ (SE)
7	9	$100(12+3)+400(2-20)-1000 = -4200 < 0$ (E)
8	9	$-3000 < 0$ (E)
9	9	$-1100 < 0$ (E)
10	9	$100(18+3)+400(2-18)-1100 =$ (SE)
11	8	$-3100 < 0$ (E)
12	8	$-600 < 0$ (E)
13	8	$2000 > 0$ (SE)
14	7	$-600 < 0$ (E)
15	7	$2500 > 0$ (SE)
16	6	$1000 > 0$ (SE)
17	5	$3 \not> 3$; false

Move to Region 2

Region 2

x	y	d
18	4	$-2175 < 0$ (SE)
19	3	$-375 < 0$ (SE)
20	2	$2425 > 0$ (S)
20	1	$2024 > 0$ (S)
20	0	STOP

$$M = \left(x_p + \frac{1}{2}, y_p - 1\right) = \left(17 + \frac{1}{2}, 5 - 1\right)$$

$$d = F(M)$$

$$= b^2\left(17 + \frac{1}{2}\right)^2 + a^2\left(5 - 1\right)^2 - a^2b^2$$

$$= \overset{25}{\cancel{100}}\left(\frac{34}{\cancel{4}_1}\right) + 400\,(4)^2 - (20)^2(10)^2$$

$$= -2975$$

Generate all points of the ellipse using 4-way symmetry:

$(0, 10) \rightarrow (0, -10)$

$(1, 10) \rightarrow (-1, 10), (1, -10), (-1, -10)$

$(2, 10) \rightarrow (-2, 10), (2, -10), (-2, -10)$

$(3, 10) \rightarrow (-3, 10), (3, -10), (-3, -10)$

$(4, 10) \rightarrow (-4, 10), (4, -10), (-4, -10)$

$(5, 10) \rightarrow (-5, 10), (5, -10), (-5, -10)$

$(6, 10) \rightarrow (-6, 10), (6, -10), (-6, -10)$

$(7, 9) \rightarrow (-7, 9), (7, -9), (-7, -9)$

$(8, 9) \rightarrow (-8, 9), (-8, -9), (8, -9)$

$(9, 9) \rightarrow (-9, 9), (9, -9), (-9, -9)$

$(10, 9) \rightarrow (-10, 9), (10, -9), (-10, -9)$

$(11, 8) \rightarrow (-11, 8), (11, -8), (-11, -8)$

$(12, 8) \rightarrow (-12, 8), (12, -8), (-12, -8)$

$(13, 8) \rightarrow (-13, 8), (13, -8), (-13, -8)$

$(14,7) \rightarrow (-14,7), (14,-7), (-14,-7)$

$(15,7) \rightarrow (-15,7), (15,-7), (-15,-7)$

$(16,6) \rightarrow (-16,6), (16,-6), (-16,-6)$

$(17,5) \rightarrow (-17,5), (17,-5), (-17,-5)$

$(18,4) \rightarrow (-18,4), (18,-4), (-18,-4)$

$(19,3) \rightarrow (-19,3), (19,-3), (-19,-3)$

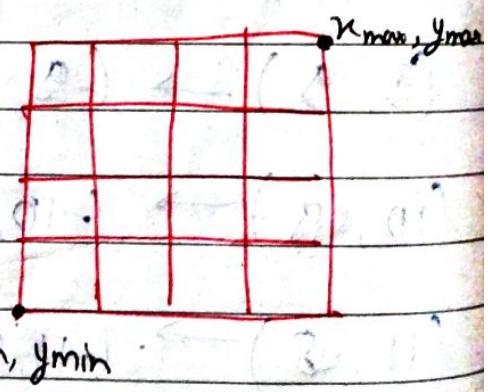
$(20,2) \rightarrow (-20,2), (20,-2), (-20,-2)$

$(20,1) \rightarrow (-20,1), (20,-1), (-20,-1)$

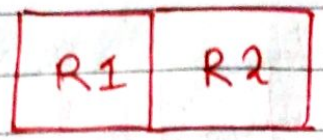
$(20,0) \rightarrow (-20,0)$

Filling Rectangle

Illuminate all pixels from (x_{min}, y_{min}) to (x_{max}, y_{max}) by:




```
for (x = xmin , x ≤ xmax , x++)
    for (y = ymin, y ≤ ymax, y++)
        put pixel (x, y, colour);
```



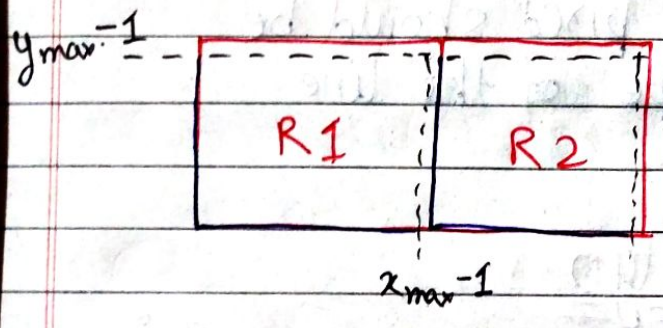
In a monochromatic func. like putpixel. If we call it twice for the same pixel, it will be off.

So, we need to call it once for the common edge of R1 & R2.

2marks

Boundary Value Rule

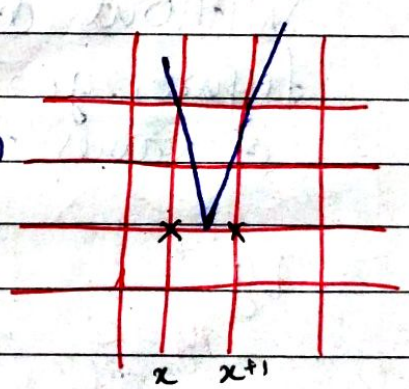
Pixels on the bottom & left will be drawn, top & right will not be drawn



Since pixels are so close together, there won't be any difference.

Sliver

- A Sliver in a very thin cone. Whichever pixel we illuminate nearest to it, it will lie outside the figure.



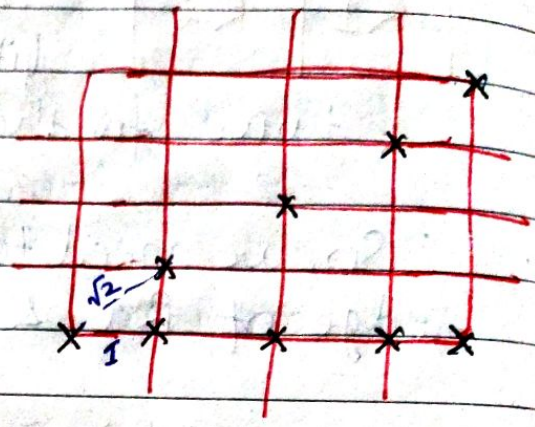
- A Slivers figure on the screen will always be distorted.

- We can never draw a sliver in the correct shape on the screen, as we can do on paper.

Varying Intensity

5 pixel → I
(side)

5 pixel → I
(diag.)



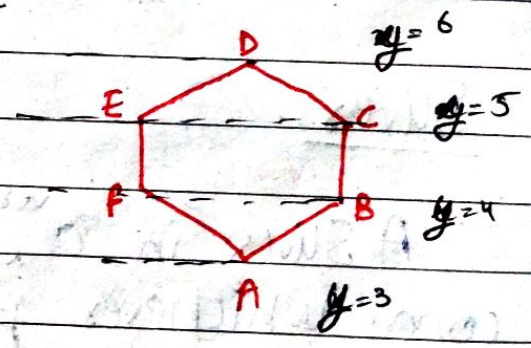
But they are not viewed as the same.

The Intensity of the pixel should be a function of slope of the line

8 marks

Filling Polygon Algo

Without drawing figure, give the range of values to illuminate



Eg:-

$y = 3$, $(3, 2)$
$y = 4$, $(4, 1) \leftrightarrow (4, 4)$
$y = 5$, $(5, 1) \leftrightarrow (5, 5)$
$y = 6$, $(6, 3)$

$y = C$, scan line
Algo gives a span or range for every scan line

It will be a 3 - step procedure.

Step 1: Find the intersection of current scan lines ($y = C$) with all the edges of the polynomial.

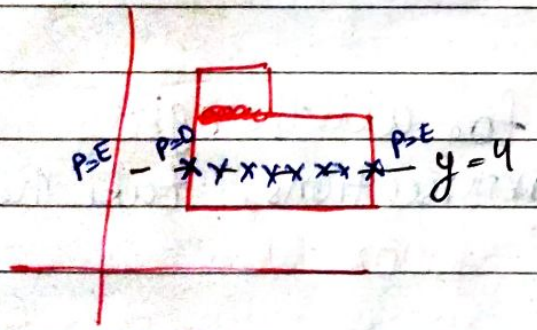
Step 2: Sort the intersections by increasing x-coordinate

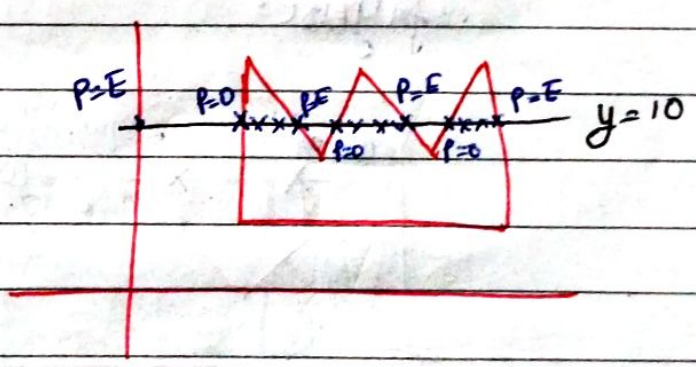
Step 3: Fill the pixels using odd-parity rule.

2-marks

Odd-Parity Rule

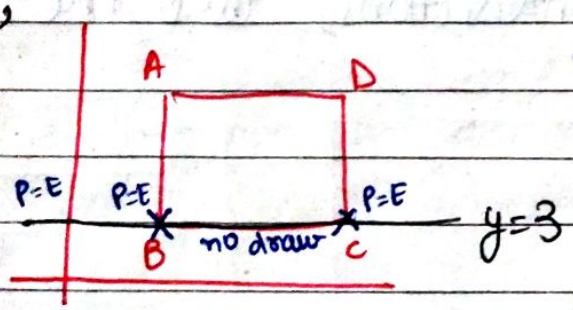
It states that the parity is initially even. Each intersection encountered, inverts the parity. We draw / illuminate when the parity is odd. Do not draw when parity is even.





But,

Case I



Vertex are considered as 2-intersections, so it has Even Parity. So it will not draw the edges.

Additional Rule,

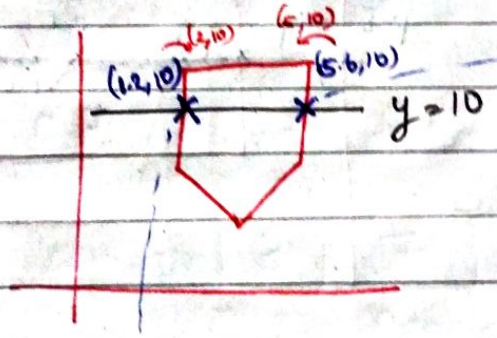
y_{min} of every edge will invert the parity but y_{max} will not.

for horizontal edges, there is no y_{min} & no y_{max}.

Why? ✓ y_{min} to draw bottom.
× y_{max} to not draw top.

Case II : Intersection with fractional co-ordinates, (Round off / floor / ceil)?

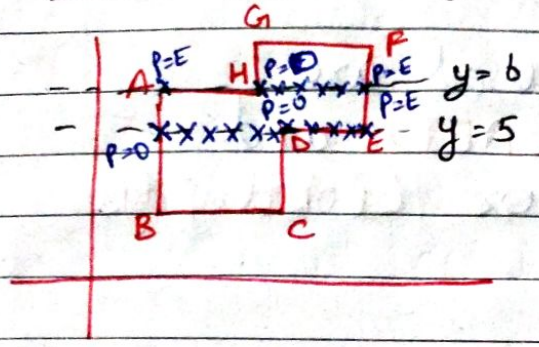
We always want to stay inside the figure.



for all the right intersections, take the floor.

for all left intersections, take the ceil.

Case 3: Horizontal Edge



① Verify, for $y=5$

At left edge, AB will make $P=0$

At point D, DE will not change parity & DC will also not change the parity, so parity $=0$

At point E, DE will not invert parity, but EF will, so parity $=E$

Check, bottom & left drawn ✓
top & right not drawn ✓

② Verify, for $y=6$

At A, AH → Horizontal, dont invert
AB → y_{max}, dont invert | $P=E$

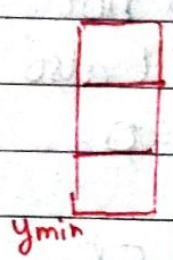
At H, AH → Horizontal, dont invert
GH → y_{min}, invert | $P=0$

At edge EF, the parity inverts and $P=E$.

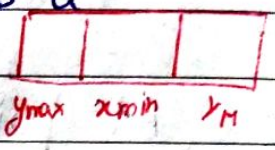
Check, bottom & left drawn ✓
R & T not drawn ✓

Scan line Algo / Polygon-Filling Algo

The 2 data structures used are Active Edge Table (AET) and Edge Table (ET) or Global Edge Table (GET).



GET is maintained as a bucket on the ymin value. Each edge has a node in GET of the form



$$y_{i+1} = y_i + 1$$
$$x_{i+1} = x_i + 1/M$$

Q) Illuminate all pixels of the polygon A(2,3); B(7,1); C(13,5); D(13,11); E(7,7); F(2,9)

ymax = 11 so go till AET, y=11

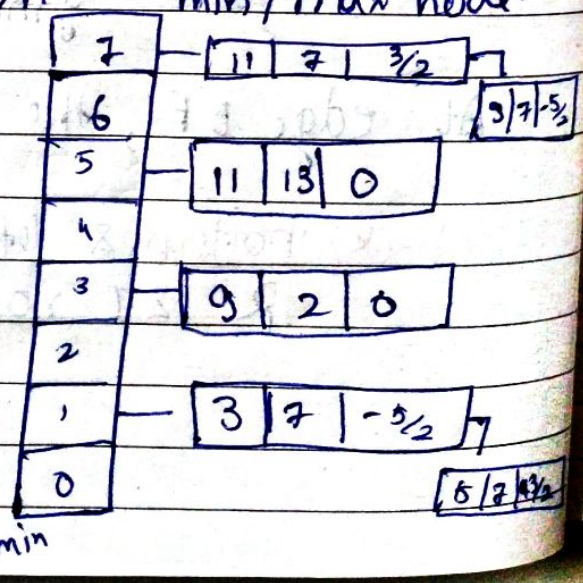
A) AB. (2,3) to (7,1) min, as 1 < 3

xmax, ymax, xmin, ymin

ymax	xmin	1/M
3	7	-5/2

Always compare the y-values & make min/max node.

$$M = \frac{3-1}{2-7} = -\frac{2}{5}$$



BC $(\overset{x_{min}}{7}, \overset{y_{min}}{1})$ & $(\overset{x_{max}}{13}, \overset{y_{max}}{5})$

5	7	3/2

$M = \frac{1-5}{7-13} = \frac{-4}{-6} = \frac{2}{3}$

CD $(13, \overset{y_{min}}{5})$ & $(13, 11)$

11	13	0

$1/M = \frac{13-13}{5-11} = 0$

DE $(13, 11)$ & $(7, \overset{y_{min}}{7})$

11	7	3/2

$1/M = \frac{13-7}{11-7} = \frac{6}{4} = \frac{3}{2}$

EF $(7, \overset{y_{min}}{7})$ & $(2, 9)$

9	7	-5/2

$\frac{1}{M} = \frac{7-2}{7-9} = \frac{5}{-2} = \frac{-5}{2}$

FA $(2, 9)$ & $(2, \overset{y_{min}}{3})$

9	2	0

$\frac{1}{M} = \frac{2-2}{9-3} = 0$

Let $G = (5, 9)$

FG $(5, 9)$ & $(2, 9)$ we cannot decide as it is a horizontal edge.

★ Tho it is a horizontal edge, there is no y_{min} & y_{max}. Hence such edges will not come in

the GET & $1/m = \infty$. Hence, we cant perform this operation, $x_{i+1} = x_i + 1/m$

NOTE:-

- AET will be drawn for every scan line ($y = c$)

- The nodes are copied from one AET to another, new nodes are also added from the GET.

(Contd Question)

AET, $y=0$ NULL

AET, $y=1$

3	7	$-5/2$	5	7	$3/2$

* The nodes should be sorted on the x- value (center value)

span at $y=1$,

$(7, 1) \longleftrightarrow (7, 1)$

center value — y stays — center value — y stays

AET, $y=2$ | y has been incremented, but x needs to be changed

| 3 | 9/2 | -5/2 | — | 5 | 17/2 | 3/2 | — No new node

span at $y=2$,

$(9/2, 2) \longleftrightarrow (17/2, 2)$

ceil floor

So, $(5,2) \longleftrightarrow (8,2)$

AET, $y=3$ | $y=3$ & $y_{max}=3$ for node 1 so dont copy it

| ~~3~~ | ~~2~~ | ~~-5/2~~ | — | 5 | 10 | 3/2 | — | 9 | 2 | 0 |

y_{max} has been reached

Sort, it based on x-value.

| 9 | 2 | 0 | — | 5 | 10 | 3/2 |

span at $y=3$,

$(2,3) \longleftrightarrow (10,3)$

★ we should always have even no. of nodes in our AET. ★

AET, $y = 4$

9	2	0	—	5	$23/2$	$3/2$

span at $y = 4$,

$$(2, 4) \longleftrightarrow (\underset{\substack{\downarrow \\ \text{floor}}}{11.5}, 4)$$

$$\Rightarrow (2, 4) \longleftrightarrow (11, 4)$$

AET, $y = 5$

9	2	0	—	11	13	0

span at $y = 5$

$$(2, 5) \longleftrightarrow (13, 5)$$

AET, $y = 6$

9	2	0	—	11	13	0

span at $y = 6$

$$(2, 6) \longleftrightarrow (13, 6)$$

AET, $y = 7$

9	2	0	11	13	0	11	7	$3/2$	9	7	$-5/2$

sort by x-values,

9	2	0	11	7	$3/2$	9	7	$-5/2$	11	13	0

illuminate dont illuminate illuminate

span at $y = 7$,

$(2,7) \longleftrightarrow (7,7)$ $(7,7) \longleftrightarrow (13,7)$

AET, $y = 8$

9	2	0	11	$7/2$	$3/2$	9	$9/2$	$-5/2$	4	13	0

Sort,

9	2	0	9	4.5	$-5/2$	11	8.5	$3/2$	11	13	0

Span at $y = 8$,

$(2,8) \longleftrightarrow (4,8)$ $(9;8) \longleftrightarrow (13,8)$

AET, $y = 9$

11	10	3/2	—	11	13	0

span at $y = 9$,

$(10, 9) \longleftrightarrow (13, 9)$

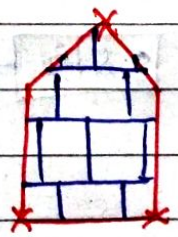
AET, $y = 10$

11	23/2	3/2	—	11	13	0

span at $y = 10$,

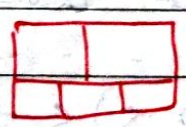
$(12, 10) \longleftrightarrow (13, 10)$

Pattern-filling Algo (fill Primitive)



1) Anchor Method

Choose an anchor along the primitive & replicate a pattern all over.



Problems

- finding an anchor for circular figures

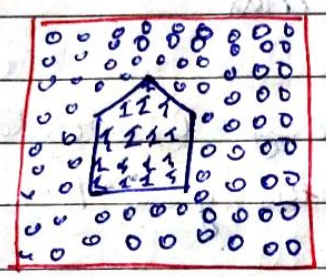
Property

The pattern remains same if the

primitive is moved from one place to another.

2) AND Method

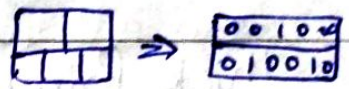
Take 2 arrays, the size of the screen.
On 1st screen, fill primitive with 1's & rest with 0's.
On 2nd screen, fill the entire screen with our pattern. Now, Bitwise AND these 2 arrays.



Pattern Array

0	0	1	0	0	1	0	0
1	0	0	1	0	0	1	0
0	1	0	0	1	0	0	1
0	0	1	0	0	1	0	0

512x512



Background is 0 & Pattern (dont care), it is 0 for 1's, all in Pattern works as X AND 1 = X.

Property

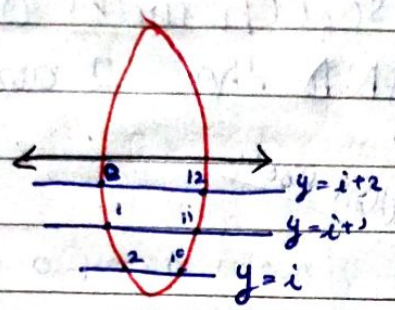
- The pattern can change if the primitive is moved from one location to another.

- There is no problem of finding an anchor in this method.

Filling Ellipse

Property of Edge - Coherence

Many points intersected at the scan line, $y = i$ are also intersected by the scan line, $y = i+1$.



Span $x = min$
$x = max$

The span (2, 10) for $y = i$, would so be included in the span for $y = i+1$.

Thick Primitive

1) Replicating Pixels

Prob 1 It is diff. to replicate & find concentric images for circular figures like S.

Eg:- Working ⇒

but if thickness = 4??

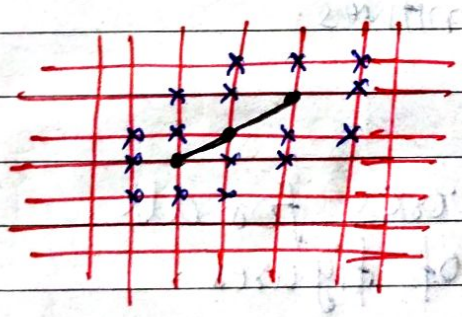
Prob 2 The width of the line should always be an odd no., otherwise we will move away from the ideal line.

Adv
It is easy to implement.

2) Moving Pen Method

If I move the pen very slowly, it will leave footprints by illuminating the pixels around it.

* When I keep my pen on a single point on the screen, because of it a cross-section of pixels are illuminated around that point are called footprints of my point.

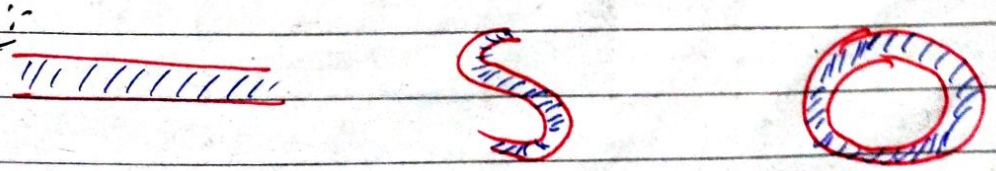


Property
There is an overlap of footprint happens in this method.
So, check pixel before illuminating

3) Filling area b/w the boundaries

User will already give 2 lines & we have to illuminate in b/w it.

Eg:-



Adv
No problem of odd or even width.

Dis

It is on the user to give 2 concentric equation which is not allways easy.

4) Approximation by Thick Polylines

Break fig into smaller images to make it as straight lines.

★ Break the primitive into smaller primitives till all the smaller primitives are straight line. Then we do piece by piece thickening of all the straight lines. Join all the small primitives.

Adv

1. This works for all types of figures.

2. It gives the best visual thickened line.

Dis

It takes too much time.

Pen Style

We can draw the entire primitive using the same pen style like:- ① — . — . — .
② — — —
③ . . . . . .

All these are saved as a bit string:

len = 6

1	1	1	0	1	0	0

— space . space

Now blindly repeat

— . — . — .

len = 4

1	1	1	0

— — —

len = 2

1	0

or

1	1	0

. . .

Eg:-

need in — . — . style.

Points	SNO
(1,1)	1
(2,2)	2
(4,4)	3
(5,6)	4
(6,7)	5
(6,8)	6
(7,7)	7
⋮	

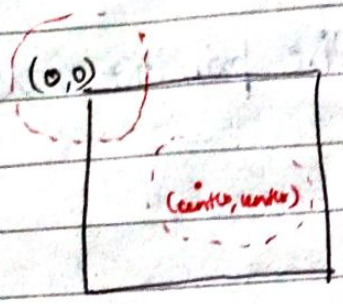
Algo

$i = SNO$

$[(i-1) \% 6]$

```
while ( i is a valid value){
    if (bitstring [(i-1)%6]){
        putpixel (x,y,color); }
    i=i+1;
}
```

Practical - Circle

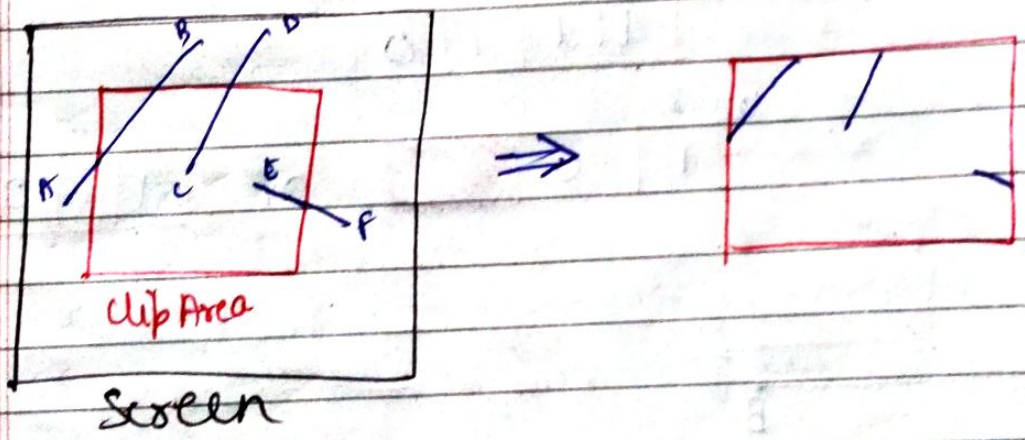


check normally, 0,0 quarter

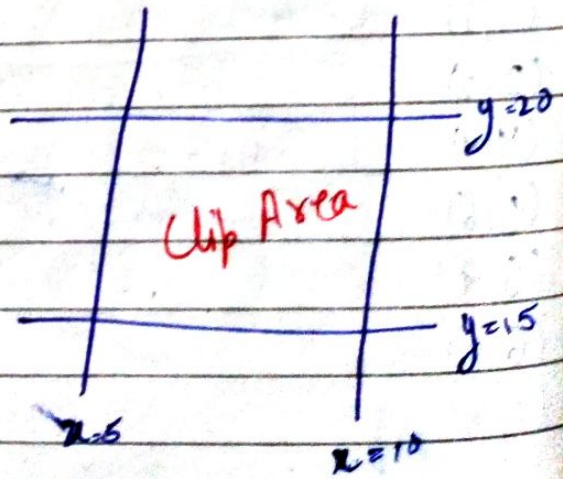
then at center 100, 100
by putpixel (x+100, y+100, color).

if center of screen,
getmaxx()/2 & use this inputpixel

Line Clipping



clip $x = x_{max}$ 10
$y = y_{max}$ 20
$x = x_{min}$ 5
$y = y_{min}$ 15



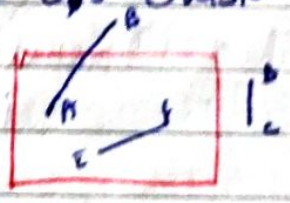
1) Clipping Individual Points

$x_{min} \le x \le x_{max}$ AND
$y_{min} \le y \le y_{max}$

All points should be checked if they are inside & then they should be displayed

Prob

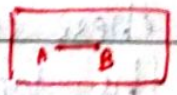
lines that are mostly outside or outside the clip are calculate wasting time like AB & CD



2) Clipping by Solving Simultaneous Eqn

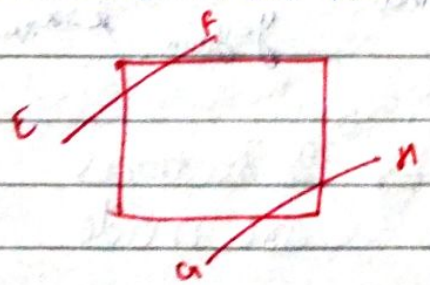
Step 1: Check for trivial acceptance - if both the end points of the line are inside the clip area, then the complete line is inside the clip area. We will illuminate all points of the line, no need to check for each point.

both points inside



Step 2: Trivial Rejection: If both the endpoints are outside the clip area, we can't say for sure that the line is completely outside

both points outside



[illegible] Line Clipping

Step 3: 1 endpoint inside, 1 outside - We need to draw only the part inside the clip area.
1 in, 1 out

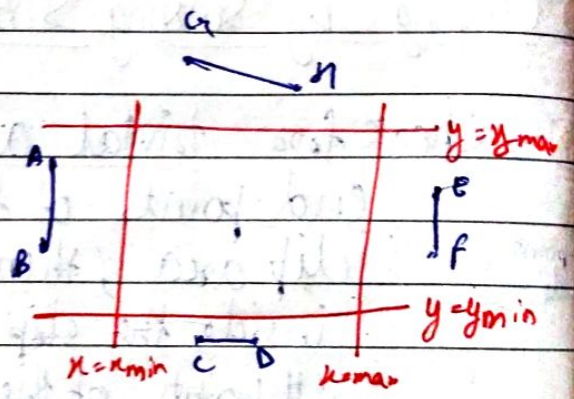
★ Imp ★

3) Cohen Sutherland line clipping

Rule 1: Trivial Acceptance: if both endpoints are inside the clip area, the line lies completely inside. So, illuminate the complete line.

Rule 2: Trivial Rejection:

$x < x_{min}$
OR $x > x_{max}$
OR $y < y_{min}$
OR $y > y_{max}$

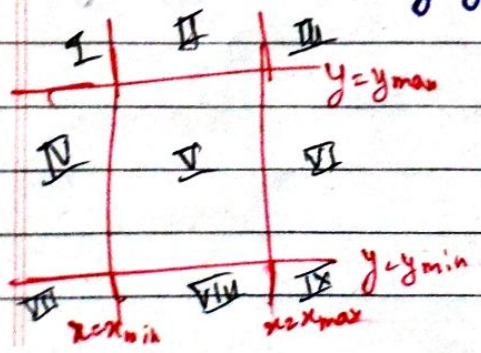


1 of these conditions need to be true for both the points. (same condition)

Rule 3: 4 bit output - a b c d (TBRL)

Top Bottom Right Left

a → $y > y_{max}$
b → $y < y_{min}$
c → $x > x_{max}$
d → $x < x_{min}$



⇒ 9 sections are made

Finding bit output in each section

1001	1000	1010
0001	0000	0010
0101	0100	0110

$y = y_{max}$

$y = y_{min}$

$x = x_{min}$ $x = x_{max}$

TA → both, all 0000.

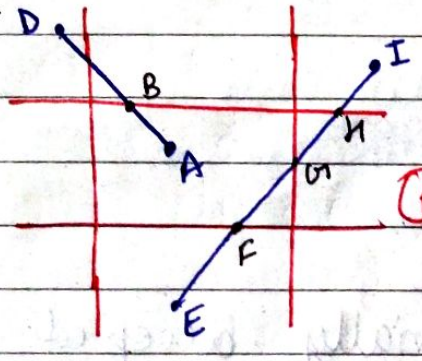
TR → both, same output but not 0000

Clip → both, diff output code.

If the output of both the endpoints is 0000, then we trivially accept the line.

If bitwise AND of both endpoints is not 0, then we trivially reject the line.

Eg:



Goal: Clip AD & EI

① AD

A → 0000

D → 1001

This line is not trivially accepted

A → 0000

D → 1001 AND

0000 → This line is not trivially rejected

Therefore, clipping is necessary.

A is already inside the clip area, so we perform clipping on D.

D → 1001 → D intersects top & left of the figure (from center)

T B R L

Calculate intersection of AD with top ($y = y_{max}$) or left ($x = x_{min}$)

Let B be the point of intersection from top,
B → 0000

So, AB is the clipped line.

② EI

I → 1010
E → 0100

This line is not trivially accepted

I → 1010
E → 0100 AND
0000 → This line is not trivially rejected

Therefore, clipping is required.

$I \rightarrow 1010$
T B R L

Calculate intersection of IE & top ($y = y_{max}$), i.e. point H

$H \rightarrow 0010$ or 1010 (use either)

Calculate intersection of EH & right ($x = x_{max}$), i.e. point G

$G \rightarrow 0000$ (stop)

$E \rightarrow 0100$
T B R L

Calc. inter. of IE & bottom ($y = y_{min}$), i.e. point F

$F \rightarrow 0000$ (stop)

So, GF is the clipped line

Q) Use Cohen Sutherland line clipping to clip between P(0,5) & Q(1,5) for points A(0,0), B(1,0), C(1,1), D(0,1)

A) $x_{min} - 0$
$x_{max} - 1$
$y_{min} - 0$
$y_{max} - 1$

P (0,5) = 1 0 0 0

Q (1,5) = 1 0 0 0

I) Trivial Acceptance

P = 1000
Q = 1000

As both points dont have opcode 0000, so it is not trivially accepted.

2) Trivial Rejection

P = 1000
Q = 1000 AND
1000 ≠ 0

As the AND is not 0000, so the line is trivially rejected.

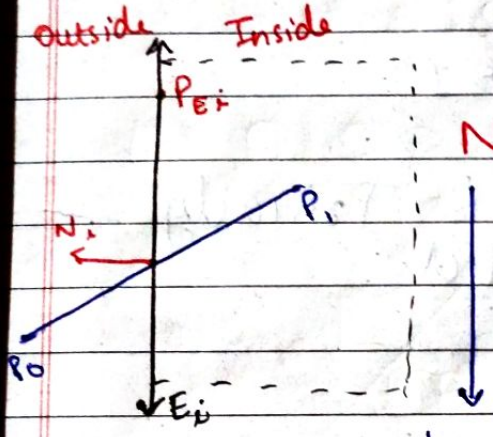
Cyrus Beck Line Clipping Algo

This uses the parametric eq^n of the line:

$$p(t) = p_0 + (p_1 - p_0)\,t \quad ; 0 \le t \le 1$$

$P(t=0) = P_0$, 1 point when t is min
$P(t=1) = P_1$, 1 point when t is max

For mid-point, $P(t=0.5) = \frac{1}{2}(P_0 + P_1)$



$$N_i \cdot (P(t) - P_{Ei}) = 0$$

$< 0 \rightarrow$ inside
$> 0 \rightarrow$ outside

$$N_i \times (P_0 + (P_1 - P_0)t - P_{Ei}) = 0$$

$$= N_i \times (P_0 - P_{Ei}) + N_i (P_1 - P_0)t = 0$$

$$\boxed{\Rightarrow t = \frac{N_i \cdot (P_0 - P_{Ei})}{-N_i \cdot D}, \quad D = P_1 - P_0}$$

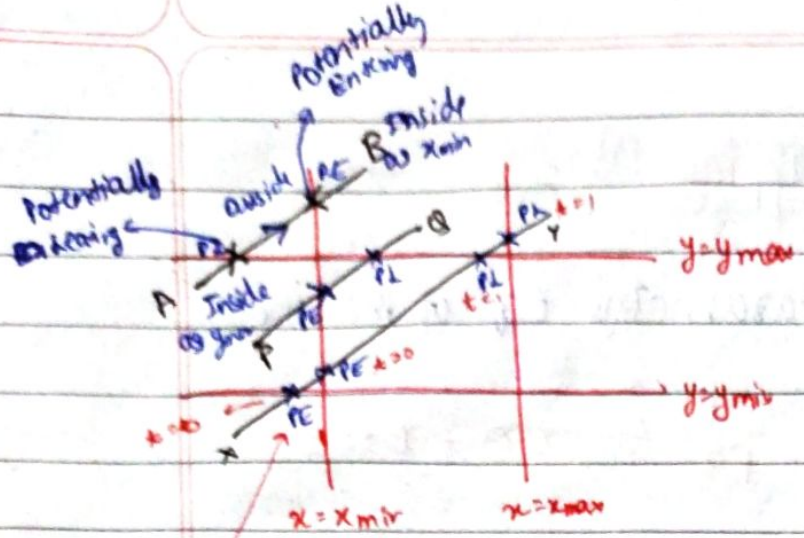
Denominator should not be 0,

So, $D \neq 0$ and $N_i \cdot D \neq 0$

$\Rightarrow P_1 - P_0 \neq 0$ $\Rightarrow P_1P_0$ not $\parallel E_i$

$\Rightarrow P_1 \neq P_0$

Should not be same point



1. PE then PL possible
2. PL then PE possible
3. Multiple PE & PL possible

A line can have at best 4 intersections

if $N_i \cdot D < 0 \Rightarrow PE$
$N_i \cdot D > 0 \Rightarrow PL$

If $\forall PE_i$, $\max PE = t_E$
If $\forall PL_i$, $\min PL = t_L$
$0 \le t_E, t_L, PE, PL \le 1$

line XY.

* $PL < PE \Rightarrow$ The line is Trivially Rejected.
(line AB)

Past Year Paper

Q) What is the condition where the line between points P(0,5) & Q(5,0) is trivially rejected by using Cohen & Sutherland line clipping algo using the clip area A(0,0), B(1,0), C(1,1) & D(0,1)?
3 marks

A) A(0,0)
B(1,0)
C(1,1)
D(0,1)

$x_{min} = 0$
$y_{min} = 0$
$x_{max} = 1$
$y_{max} = 1$

Clip Edge	Normal (N_i)	P_{E_i}	$P_0 - P_{E_i}$	$t = \frac{N_i \cdot (P_0 - P_i)}{-N_i \cdot D}$ imp
$x = x_{min}$	$-1, 0$	(x_{min}, y)	$(x_0 - x_{min}, y_0 - y)$	$\frac{-(x_0 - x_{min})}{x_1 - x_0}$
$x = x_{max}$	$1, 0$	(x_{max}, y)	$(x_0 - x_{max}, y_0 - y)$	$\frac{x_0 - x_{max}}{-(x_1 - x_0)}$
$y = y_{min}$	$0, 1$	(x, y_{min})	$(x_0 - x, y_0 - y_{min})$	$\frac{-(y_0 - y_{min})}{y_1 - y_0}$
$y = y_{max}$	$0, 1$	(x, y_{max})	$(x_0 - x, y_0 - y_{max})$	$\frac{y_0 - y_{max}}{-(y_1 - y_0)}$

P = (0,5) → 1000
Q = (1,5) → 1000

```
     1000     (P)
AND  1000     (Q)
     1000 ≠ 0
```

Hence, the line is trivially rejected

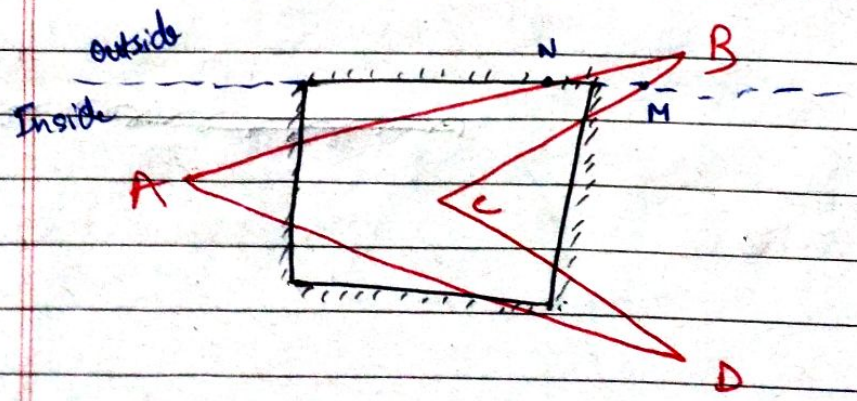
Liang-Barsky Line Clipping Algo (Extension of Cyrus Beck)

Use same table

As soon as you get PE & PL, you test the condition PL < PE.

Sutherland Hodgeman Polygon Clipping Algo

The data structure to be used in this algo: In-Vertex Array & Out-Vertex Array
We move in counter clockwise direction.
There are 4 cases:

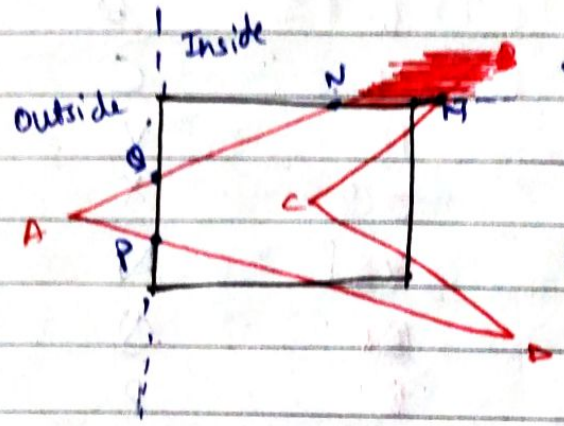


Taken a edge Top, & find inside & outside

In Vertex	Out Vertex
A	A
D	D
C	C
B	M
A	N
	A

anti-clock

since B is out

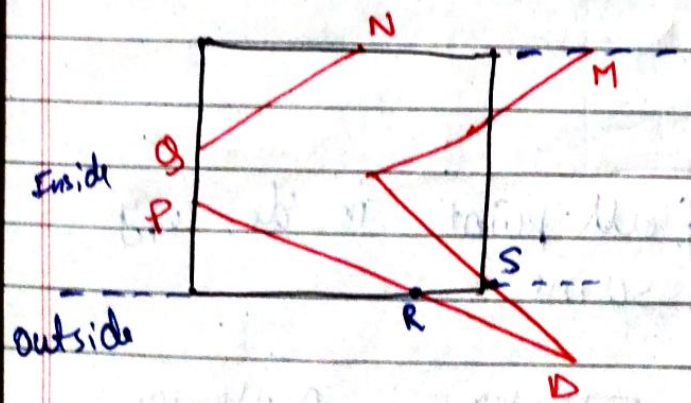


Taking left edge,

(out vertex of prev)

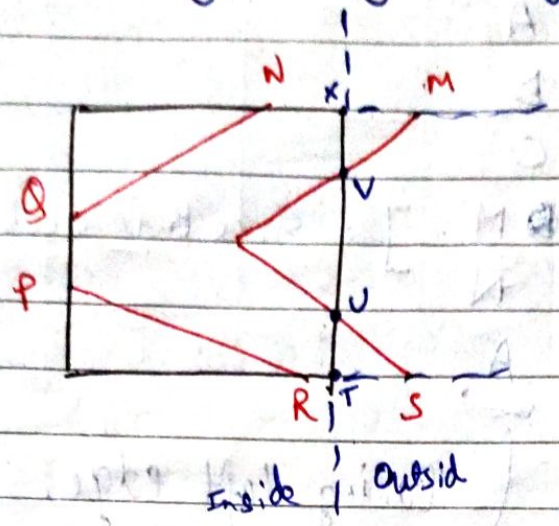
In Vertex	Out Vertex
A	P
D	D
C	C
M	M
N	N
A	Q
	P

After this step take the next edge & repeat



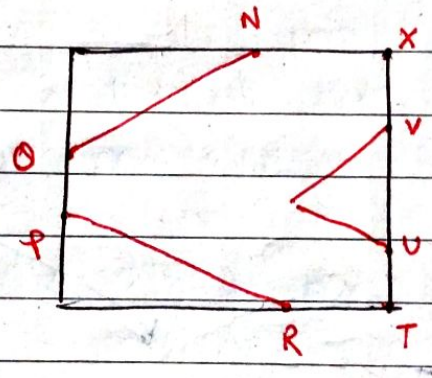
In Vertex	Out Vertex
P	P
D	R
C	S
M	C
N	M
Q	N
P	Q
	P

Taking last right edge,

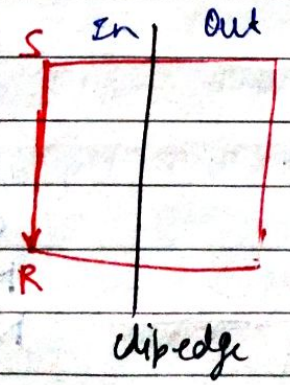


In vertex	
P	P
R	R
S	T
C	U
M	C
N	V
Q	X
P	N
	Q
	P

The final figure, will be:



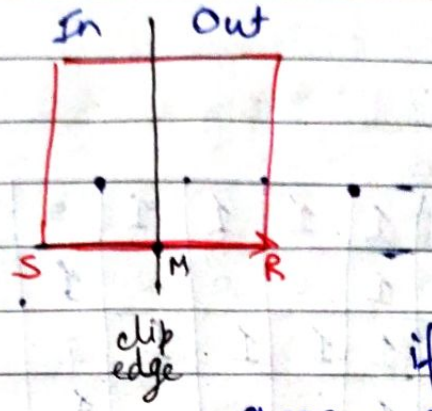
Case 1:



If all point inside, copy same

In Vertex	Out Vertex
S	S
R	R

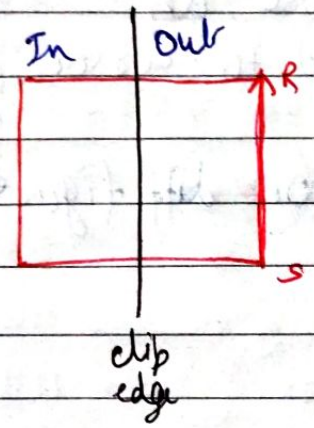
Case 2 :



InVertex	OutVertex
S	S
R	M

if one vertex is out, we add a new edge at the clip edge.

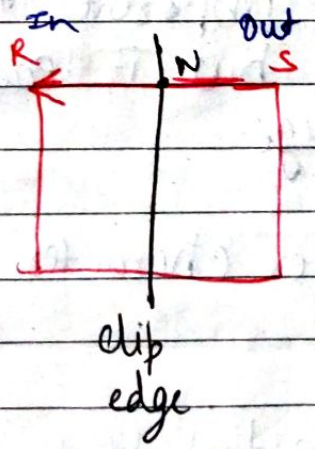
Case 3:



If both points are out, copy nothing.

InVertex	OutVertex
S	NULL
R	

Case 4:

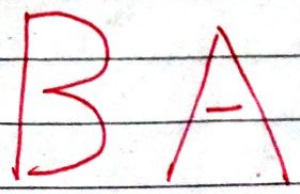


If initial is outside, create new vertex at clip edge

InVertex	OutVertex
S	N
R	R

2-3 marks

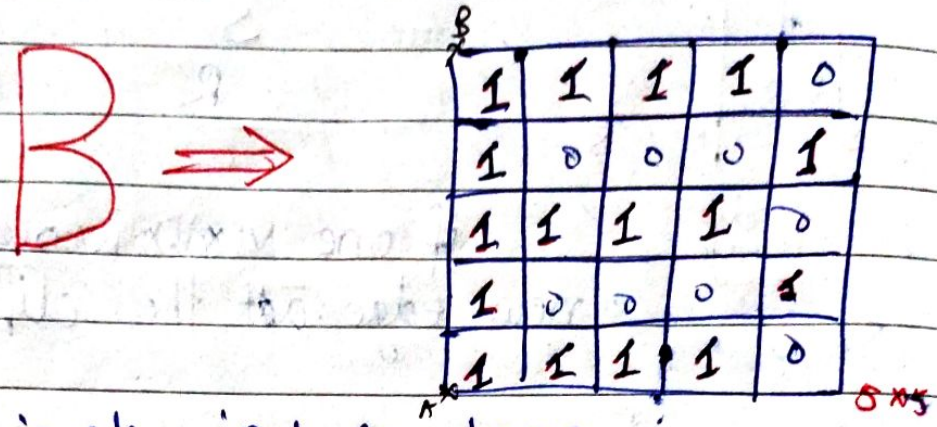
Generating Characters



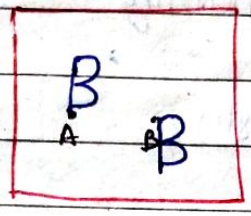
Methods:

1) Use a Bitmap
2) Using splines

1) <u>Use a bitmap</u>



It is also imp to choose an anchor as it is mapped to cursor



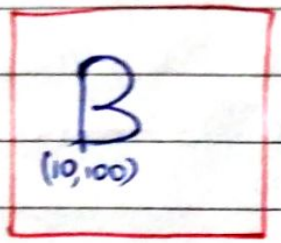
Diff. anchor, diff figure

1) Increasing size of bitmap, improves the shape of the character, but it leads to higher memory usage.

2) Saving the bitmap for ever character takes a lot of memory space.

3) Seperate bitmap is needed to be saved for all 4 faces of the character i.e. normal, bold, italic, bold italic

2) Using splines → higher order math functions or order 4, 5, 6 etc..



Splines (eq^ns)

$B(x,y) =$ _____

$A(x,y) =$ _____

$X(x,y) =$ _____

⋮

1) A lot of calculation is needed for generating the character on screen.

2) Complex to impliment.

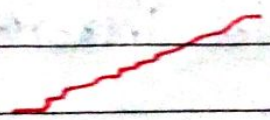
3) Difficult to draw with other 3 faces i.e. bold, italic, bold-italic.

4) It takes less memory space as compared to bitmap method.

5-6 mark

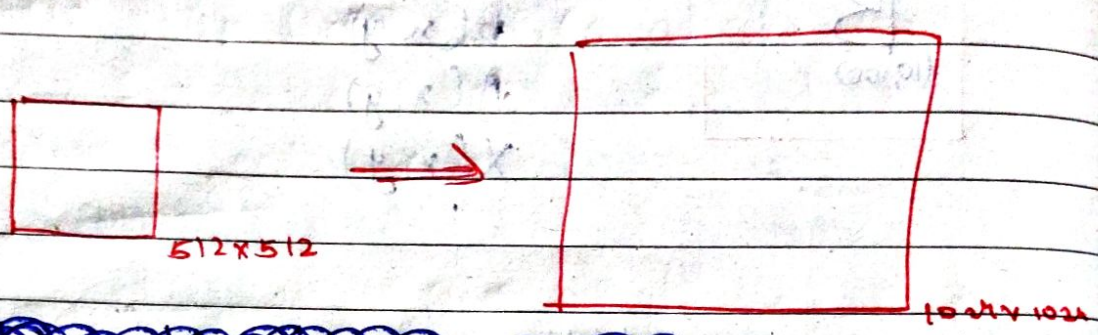
Jaggies → Zig-Zag or Staircase Effect also called Aliasing

Sometime when we draw, we get:



Methods to deal with Aliasing are called Anti-Aliasing techniques:

Method 1: Increase Resolution



Increasing resolution only reduces the aliasing effect, as more pixels mean better precision.

Improvement in zig-zag effect is at the cost of higher memory usage and increased scan-converting time. (drawing)

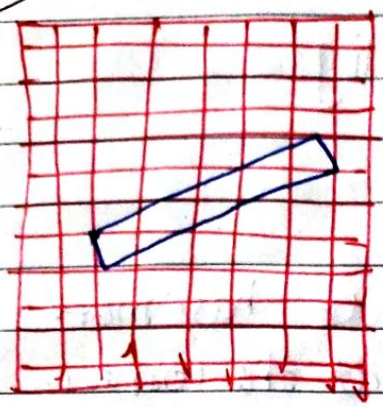
Method 2: Unweighted Area Sampling

2-3 marks

Properties

1) Intensity of pixel intersected by a line H, decreases as the distance between the line H & primitive increases

Understanding of Algo



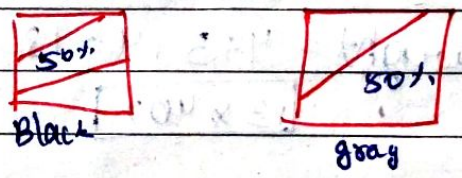
Now we use many gradients of color:

0 - White (Background)
1 - 10% coverage
2 - 20% shade
3 - 30% grey
4 - 40%
5 - 50%
6 - 60%
7 - 70%
8 - 80%
9 - 90%
10 - Black (Foreground)

Put these gradients in the pixels that have less coverage.

Choose color based on % of area covered, so it looks like it slowly blends in, instead of a abrupt contrast b/w white & black.

But,



we should have diff. colour for these 2.

we should consider position too.

2) Primitive (line to be drawn) cannot influence the intensity, if the primitive does not intersect the pixel.

3) Equal areas contribute equally, regardless of distance from center.

Method3 : Weighted Area Sampling

Properties (1 & 2 are same)

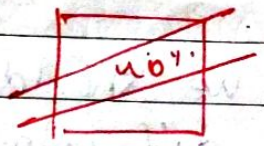
3. Area, nearer to the center has more effect than area covered at the corners.

weighing func

one possibility

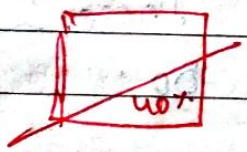
4	2	4
3	5	3
4	2	4

if we give weight to center more than the rest



weight = 4+5+4 = 13

$\boxed{13 \times 40\%}$



weight = 2+3+4 = 9

$\boxed{9 \times 40\%}$

we can also have a weight such as

weighing func

2	3	2
1	3	1
2	1	2

NOTE:- The intensity of the pixel will be decided on the basis of % of pixel covered & its weight.

Ch-15:

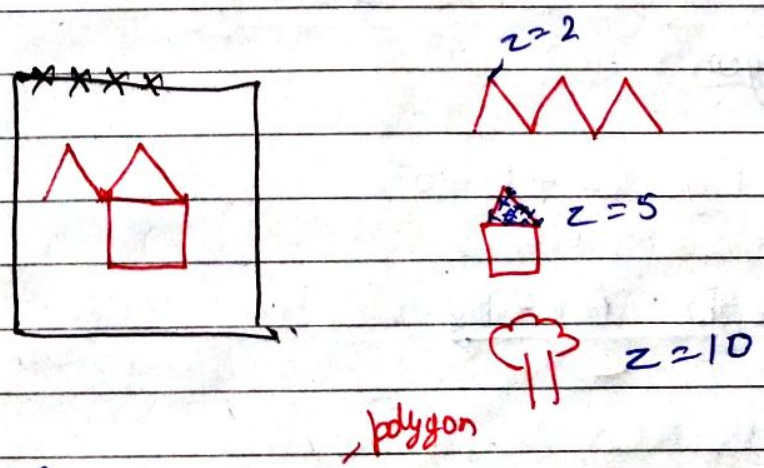
Z-Buffer Algo

We use 2 data structures zbuffer and framebuffer with a size of x * y.

zbuffer-

Pseudo

```
for (x=0; x<xmax; x++)
    for (y=0; y<ymax; y++)
    {
        zbuffer[x][y] = 0;
        framebuffer[x][y] = Bg color;
    }
```



larger z-value means object is in front.

polygon

```
for each object in the scene
    for each x,y of the current object
    {   pz = z value of object for x,y
        if (pz > zbuffer[x][y])
        {   zbuffer[x][y] = pz;
            framebuffer[x][y] = color of x,y;
        }
    }
```

2-marks Total

Advantages

1. Used to draw any object of any shape
2. Easy to implement
3. Only 1 z-value comparision is done for each objects x, y.

Problems

1. It requires a large amt of space.
2. Might face Aliasing (zig-zag effect)

2-3 marks

Eqn of Polygon

$$Ax + By + Cz + D = 0$$

$$\text{So, } z_1 = \frac{-D - Ax - By}{C}$$

Only $x \uparrow$ by 1

if $x_1 = x_1$ & $x_2 = x_1 + 1$

$$z_1 = \frac{-D - Ax_1 - By_1}{C}$$

$y_1 = y_1$ & $y_2 = y_1$

$$\text{So, } z_2 = \frac{-D - Ax_2 - By_2}{C}$$

$$\Rightarrow z_2 = \frac{-D - A(x_1+1) - By_1}{C}$$

$$\Rightarrow z_2 = \frac{-D - Ax_1 - By_1}{C} - \frac{A}{C}$$

$$\Rightarrow z_2 = z_1 - \frac{A}{C}$$

Only y ↑ by 1

$x_1 = x_1$
& $x_2 = x_1$

$y_1 = y_1$
& $y_2 = y_1 + 1$

So, $z_2 = \frac{-D - Ax_2 - By_2}{C}$

$$\Rightarrow z_2 = \frac{-D - Ax_1 - B(y_1+1)}{C}$$

$$\Rightarrow z_2 = z_1 - \frac{B}{C}$$

Q) We have 3 z-value objects as

5	3	3	5	1
1	0	2	1	
4	3	3		
1	2			
8				

5
2
3
0
6

0	0	0	0	0
0	3	4	5	6
1	2	3	4	5
8	9	6	5	4
3	2	1	0	0

A) Superimpose, compare z-value, Keep max.

5	3	3	5	1
2	3	4	5	6
4	3	3	4	5
8	9	6	5	4
8	2	1	0	0

Def for Viva only

A-buffer Algo

It is z-buffer algo for transparent & translucent objects.

Depth Sort Algo / Painter's Algo

Step 1: Sort on z-value.

Step 2: Resolve any ambiguity or split.

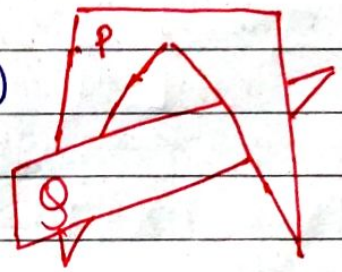
Step 3: Scan convert each polygon starting from the smallest z-value.
(nearest to screen or farthest from viewer)

NOTE:- If we ignore step 2 of splitting the object, the algo is called painters algo.

To check if a split is required or not, we have 5 tests with ↑ order of complexity. If any test comes true, we know there is an ambiguity & a split is needed. If the test is false, we check the next test & so on...

If all the tests fail, you can draw the polygon & there is no ambiguity.

NOTE:- If we have to draw the polygon 'p', we will check the polygon 'p' against all the other polygons 'q' & all the 5 tests must fail for every 'q'.



Test 1 (if true check next, if false stop & split)

x-extends (x_{min} - x_{max}) do not overlap?

Test 2

y-extends (y_{min} - y_{max}) do not overlap?

Test 3

Is 'P' entirely on the opposite side of 'Q' plane from the viewpoint? (P behind Q)

Test 4

Is 'Q' entirely on the same side of 'P' plane as the viewpoint? (P infront of Q)

Projection- 3D obj on 2D board

Test 5

Perform 3D to 2D projection & check if the projections do not overlap.

If all the 5 tests fail, we must split.

Q) Write rotations matrix when $\theta = 180°, 270°, 360°$

A) Matrix $= \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}_{\theta=180}$

$= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}_{\theta=270°}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{\theta=360°}$

Reflexion

x-axis: $(y=0)$ $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

y-axis: $(x=0)$ $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

$x=y$: $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$x = -y$: $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

<u>NOTE</u>: If the determinent of transformation matrix is identically equal to -1 then the transformation is said to be a pure reflection.

Q) Given a $\triangle ABC$, $A(4,1)$, $B(5,2)$, $C(4,3)$, reflect a $\triangle$ about x-axis & then about $x = -y$.

A) $\triangle ABC$, Matrix $X = \begin{bmatrix} 4 & 1 \\ 5 & 2 \\ 4 & 3 \end{bmatrix}$

if $3x + 2y = 0$
$3x + 4y = 0$
then $\begin{bmatrix} 3 & 8 \\ 2 & 4 \end{bmatrix}$ coeff

reflection matrix for x-axis, $T_1 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

T_2 $(x=-y)$ $= \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

Final Matrix, $X' = [X][T_1][T_2]$

$$= \begin{bmatrix} 4 & 1 \\ 5 & 2 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 1 \\ -5 & 2 \\ -4 & 3 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 4 \\ -2 & 5 \\ -3 & 4 \end{bmatrix} \quad \text{——— (1)}$$

Q) Perform rotation of 270° anti-clockwise about origin of $\triangle ABC$ above.

A) $X = \begin{bmatrix} 4 & 1 \\ 5 & 2 \\ 4 & 3 \end{bmatrix}$

$T_1 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$X' = [X][T]$

$$= \begin{bmatrix} 4 & 1 \\ 5 & 2 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -4 \\ 2 & -5 \\ 3 & -4 \end{bmatrix} \quad \text{——— (2)}$$

<u>NOTE</u>:- Reflection abt x-axis followed by reflection about $x = -y$ is same as ~~reflection~~ rotation about 270°.

TP - Matrix Mul. is non-commutative

OR

Order of transformations performed can not be reversed

Lets assume matrix multiplication is commutative

Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 2 & 2 \end{bmatrix}$

$$X = [A][B] = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 11 & 8 \end{bmatrix}$$

$$Y = [B][A] = \begin{bmatrix} 1 & 0 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 8 & 12 \end{bmatrix}$$

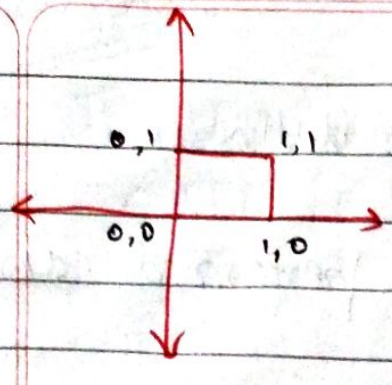
So, $\begin{bmatrix} 5 & 4 \\ 11 & 8 \end{bmatrix} \neq \begin{bmatrix} 1 & 2 \\ 8 & 12 \end{bmatrix}$

$\Rightarrow X \neq Y$

$\Rightarrow [A][B] \neq [B][A]$

$\therefore$ This is a contradiction, so our initial assumption is incorrect.

Hence, Matrix. Mul is non-commutative.



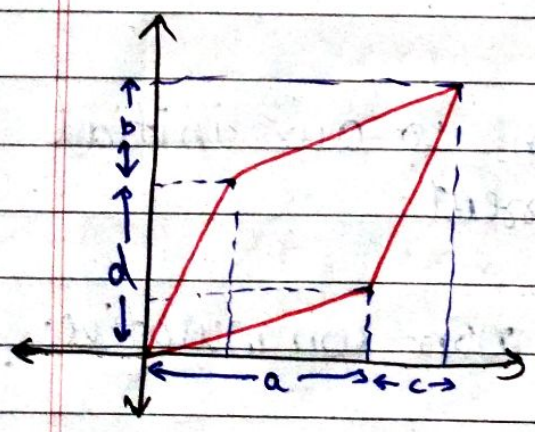
Unit sq. at origin $= \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$

$[X][T]$ for any $T = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ c & d \\ a & b \\ a+c & b+d \end{bmatrix}$$

- Origin remains invariant wrt a 2x2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$



Transformed unit square

a & d will perform scaling (↑/↓ size)

b & c perform shearing (italic / tilted)

Area Property

determinant Transformation

Ar. of transformed figure (A_t) = Ar. of Orginal figure $\times$ det [T]

Q) Given $\triangle ABC$, $A(1,0)$, $B(0,1)$, $C(-1,0)$... & transformation matrix $T = \begin{bmatrix} 3 & 2 \\ -1 & 2 \end{bmatrix}$, what will be area of transformed $\triangle$.

A) Area of $\triangle ABC = \frac{1}{2}\left[1(1-0) + 0 - 1(0-1)\right]$

$= \frac{1}{2}\left[1+1\right] = \frac{1}{2} \times 2 = 1$

$\det[T] = |T| = (3 \times 2) - (2 \times -1)$

$= 6 + 2$

$= 8$

Using Area Property,

Area of transformed $\triangle ABC$ = Area of original $\triangle ABC$ $\times$ det [T]

$= 1 \times 8$

$= 8$ units^2

To check

Transformed $\triangle ABC$ $[x'] = [x][T]$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ -1 & 2 \end{bmatrix}$$

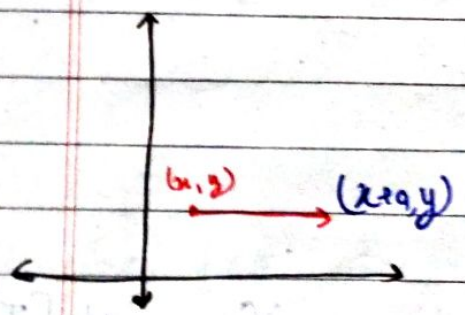
$$= \begin{bmatrix} 3 & 2 \\ -1 & 2 \\ -3 & -2 \end{bmatrix}$$

Area of $x' = \frac{1}{2}\left[3(2+2) - 1(-2-2) - 3(2-2)\right]$

$$= \frac{1}{2}\left[3 \times 4 - 1 \times (-4) - 0\right]$$

$$= \frac{1}{2}(12+4) = \frac{\cancel{16}^{8}}{\cancel{2}_{1}} = 8 \text{ units}^2$$

Homogeneous Co-ordinates



shifting a point by adding a constant $x+a$ is called Translation

So, there is no $T = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

such that $[x \;\; y][T] = [x+a, \; y]$

Page No.

Why Homogeneous?

1. We can not perform Translation with 2x2 matrix $T = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

2. The origin is invariant wrt this 2x2 transformation matrix.

Now, $\begin{bmatrix} x & y & h \end{bmatrix}$

h → homogeneous plane (usually, $h = 1$)

Now, $T = \begin{bmatrix} a & b & 0 \\ c & d & 0 \\ m & n & 1 \end{bmatrix}$

m → x, n → y, m, n → Translation

last col of $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Prob 1 solved

$$\begin{bmatrix} x & y & 1 \end{bmatrix}_{1\times3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & 0 & 1 \end{bmatrix}_{3\times3} = \begin{bmatrix} x+a & y & \end{bmatrix}$$

write $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \boxed{0} & \boxed{0} & 1 \end{bmatrix}$

change for change in x $x+a$ so put a

change for change in y $y+b$ so put b

Prob 2 solved

Shift origin from (0,0) to (2,3)

$$\begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \boxed{2} & \boxed{3} & 0 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 1 \end{bmatrix}$$

Translation x by 2
$x+2$

Translation y by 3
$y+3$

Q) Which Points do these homogeneous points represent in 2-D:

a) [4 6 2]

b) [10 15 5]

c) [20 30 10]

Div by h

a) [2 3 1] (Div by 2)

b) [2 3 1] (Div by 5)

c) [2 3 1] (Div by 10)

So, they represent (2,3) in 2-D

Rotation abt abitrary point Q (m,n)

1. Shift P & Q such that Q = (0,0)
2. Then rotate P to P'
3. Shift P' & Q back to original Q

Step 1: Translate the point Q to the origin.

Using, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -m & -n & 1 \end{bmatrix}$, ∴ Translate by $-m$ & $-n$ to get origin

Step 2: Perform the rotation asked in the question

Using, $\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Step 3: Perform inverse translation

T_1	T_2	T_3	Rotation	T_3^{-1}	T_2^{-1}	T_1^{-1}
-2	-3	-90°		90°	3	2

Using, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ m & n & 1 \end{bmatrix}$ ∴ Translating back to Q(m,n)

Q) Rotate the point P(10, 13) about the point Q(4,3) by 90° counter clockwise.

A) $X = [10 \quad 13 \quad 1]$

$T_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & -3 & 1 \end{bmatrix}$ (Translate to origin)

$T_2 = \begin{bmatrix} \cos 90 & \sin 90 & 0 \\ \sin 90 & \cos 90 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (Rotate by 90°)

$T_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 4 & 3 & 1 \end{bmatrix}$ (Translate to Q(4,3))

$X' = [X][T_1][T_2][T_3]$

$= [10 \quad 13 \quad 1] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & -3 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 4 & 3 & 1 \end{bmatrix}$

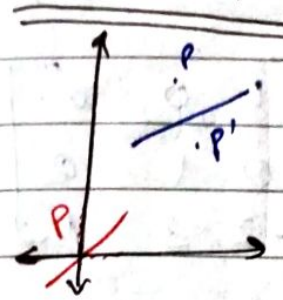
$= [6 \quad 10 \quad 1] \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 4 & 3 & 1 \end{bmatrix}$

$= [-10 \quad 6 \quad 1] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 4 & 3 & 1 \end{bmatrix}$

$= [-6 \quad 9 \quad 1]$

So, rotating $P(10, 13)$ about $Q(4, 3)$ by $90°$ counter-clockwise gives us point $P'(-6, 9)$.

Reflection through an arbitary line



$x = 0 \Rightarrow$ y-axis
$y = 0 \Rightarrow$ x-axis

Step 1: Translate the line and the object such that the line passes through the Origin

Step 2: Rotate the line & the object untill the line is coincident with one of the co-ordinate axis.

Step 3: Perform the reflection

Step 4: Perform inverse rotation

Step 5: Translate the line back to the original location

$$[Trans]\,[Rot]\,[Ref]\,[Rot]^{-1}\,[Trans]^{-1}$$

Q) Give a $\triangle ABC$ where $A(2,4)$, $B(4,6)$ & $C(2,6)$ & a line $l \Rightarrow y = \frac{1}{2}(x+4)$.
Reflect $\triangle ABC$ through line l.

A) Let $x = 2$, – any point on the line

$$y = \frac{1}{2}(2+4) = 3$$

1. Translation Matrix $\Rightarrow T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & -3 & 1 \end{bmatrix}$

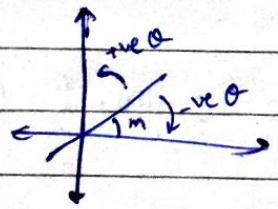
[translate (2,3) to origin (0,0)]

2. ~~Reflection~~ Rotation Matrix, $R = \begin{bmatrix} \cos(26.57) & \sin(26.57) & 0 \\ -\sin(26.57) & \cos(26.57) & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$y = mx + c$
$y = \frac{1}{2}x + 2$

so, slope $= m = \frac{1}{2}$

so, $\theta = \tan^{-1}\left(\frac{1}{2}\right) = 26.57^\circ$



Reflect to x-axis $\Rightarrow -26.57^\circ$ → as matrix works only in anticlock direction

Reflect to y-axis $\Rightarrow 90 - 26.57^\circ$
$= 63.43^\circ$

3. Reflection through x-axis, $R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

4. $R^{-1} = \begin{bmatrix} \cos 26.57 & \sin 26.57 & 0 \\ -\sin 26.57 & \cos 26.57 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

5. $T^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 3 & 1 \end{bmatrix}$

6. Object Matrix, $X = \begin{bmatrix} 2 & 4 & 1 \\ 4 & 6 & 1 \\ 2 & 6 & 1 \end{bmatrix}$ Point A, Point B, Point C

Final Matrix, $X' = [X][T][R][R_x][R^{-1}][T^{-1}]$

$$= \begin{bmatrix} 2 & 4 & 1 \\ 4 & 6 & 1 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & -3 & 1 \end{bmatrix} \begin{bmatrix} \cos(26.57) & \sin(26.57) & 0 \\ -\sin(-26.57) & \cos(26.57) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 26.57 & \sin 26.57 & 0 \\ -\sin 26.57 & \cos 26.57 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 3 & 1 \end{bmatrix}$$

Scaling

local scaling

$$[x \quad y \quad 1]\begin{bmatrix} s1 & 0 & 0 \\ 0 & s2 & 0 \\ 0 & 0 & 1 \end{bmatrix} = [xs1 \quad ys2 \quad 1]$$

$s2, s1 > 0 \Rightarrow$ enlargment

$0 < s2, s1 < 1 \Rightarrow$ compression

$s1 \neq s2 \Rightarrow$ non-uniform scaling

Overall scaling

$$[x \quad y \quad 1]\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & s \end{bmatrix} = [x \quad y \quad s] = \left[\frac{x}{s} \quad \frac{y}{s} \quad 1\right]$$

$s > 1 \Rightarrow$ compression

$s < 1 \Rightarrow$ enlargment

Q) Find point of intersection of lines
$x + y = 1$ and
$2x - 3y = 0$ & check if they intersect or parallel using matrix method.

A) $m = \begin{bmatrix} 1 & 2 \\ 1 & -3 \\ -1 & 0 \end{bmatrix}$

Line 1 Line 2

$L1: x + y - 1 = 0$

$L2: 2x - 3y + 0 = 0$

$$[x \quad y \quad 1]_{1\times3}\begin{bmatrix}1 & 2\\ 1 & -3\\ -1 & 0\end{bmatrix}_{3\times2} = [0 \quad 0]_{1\times2}$$

So, $[x \quad y \quad 1][m][m]^{-1} = [0 \quad 0][m]^{-1}$

Now we need another line that would help us solve this eqn.
Take $L3: 1 = 1$

Now, $m = \begin{bmatrix}1 & 2 & 0\\ 1 & -3 & 0\\ -1 & 0 & 1\end{bmatrix}$

$$[m]^{-1} = \frac{1}{5}\begin{bmatrix}3 & 2 & 0\\ 1 & -1 & 0\\ 3 & 2 & 5\end{bmatrix}$$

So, $[x \quad y \quad 1] = \frac{1}{5}[0 \quad 0 \quad 1]\begin{bmatrix}3 & 2 & 0\\ 1 & -1 & 0\\ 3 & 2 & 5\end{bmatrix}$

$$\Rightarrow [x \quad y \quad 1] = \begin{bmatrix}\frac{3}{5} & \frac{2}{5} & 1\end{bmatrix}$$

So, point of intersection is $(x, y) \Rightarrow \left(\frac{3}{5}, \frac{2}{5}\right)$

Page No:

Q) Find point of intersection of
$x+y=1$ & $x+y=0$

A) $M = \begin{array}{c} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \\ \begin{array}{ccc} l1 & l2 & l3 \end{array} \end{array}$

$l1: x+y-1=0$
$l2: x+y=0$
$l3: 1=1$

$$[x \quad y \quad 1] \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} = [0 \quad 0 \quad 1]$$

M^{-1} is not possible so we need a different line such that R1 & R2 are not same,
so let $l3: x=x$

Now,
$$[x \quad y \quad 1] \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} = [0 \quad 0 \quad x]$$

So, $M^{-1} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 1 \\ 1 & -1 & 0 \end{bmatrix}$

$$[x \quad y \quad 1] = [0 \quad 0 \quad x]\begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$\Rightarrow [x \quad y \quad 1] = [x \quad -x \quad 0]$$

can never be 0.

Since homogeneous co-ordinate is 0, so it is an arbitrary point at infinity, ∞.

So, both the lines are parallel lines.

NOTE:-

A point at infinity on the +ve x-axis

$$[\underset{\text{any +ve}}{3} \quad \underset{\text{x-axis}}{0} \quad \underset{\text{at infinity}}{0}]$$

but on -ve x-axis $\Rightarrow$ $[\underset{\text{any -ve}}{-3} \quad \underset{\text{x-axis}}{0} \quad \underset{\text{infinity}}{0}]$

+ve y-axis $\Rightarrow$ $[0 \quad 10 \quad 0]$

-ve y-axis $\Rightarrow$ $[0 \quad -10 \quad 0]$

Page No:

Cyrus Beck

Q) Give a clip rectangle PQRS as P(5,5), R(15,15). We have 2 lines EF & AB as E(3,13), F(7,11), A(0,3) & B(12,18).

A) $x = x_{min}$, $t = \frac{-(x_0 - x_{min})}{x_1 - x_0} = \frac{-(3-5)}{7-3} = \frac{1}{2}$ den = -2 < 1, PE

$x = x_{max}$, $t = \frac{x_0 - x_{max}}{-(x_1 - x_0)} = \frac{3-15}{-(7-3)} = \frac{-12}{-3} = 3$ [$t \notin (0,1)$ neglect]

$y = y_{min}$; $t = \frac{-(y_0 - y_{min})}{y_1 - y_0} = \frac{-(13-15)}{11-13} = \frac{-2}{-2} = 4$ [$t \notin (0,1)$ neglect]

$y = y_{max}$, $t = \frac{y_0 - y_{max}}{-(y_1 - y_0)} = \frac{(13-15)}{-(11-13)} = \frac{-2}{2} = -1$ [$t \notin (0,1)$ neglet]

$x_{min} = 5$, $x_{max} = 15$, $y_{min} = 5$, $y_{max} = 15$

In line EF, $E \rightarrow (3,13)$ $[x_0, y_0]$
$F \rightarrow (7,11)$ $[x_1, y_1]$

$P(t) = P_0 + (P_1 - P_0)t$

$P\left(\frac{1}{2}\right)_x = 3 + (7-3)\frac{1}{2} = 3 + \frac{4}{2} = 3+2 = 5$

$P\left(\frac{1}{2}\right)_y = 13 + (11-13)\frac{1}{2} = 13 - \frac{2}{2} = 13 - 1 = 12$

so, intersection point is (5, 12)

Page No:

In line AB, A $\rightarrow$ $(\overset{x_0}{10}, \overset{y_0}{3})$

B $\rightarrow$ $(\underset{x_1}{12}, \underset{y_1}{8})$

$x = x_{min}, t =$

$x = x_{max}, t =$

$y = y_{min}, t =$

$y = y_{max}, t =$

In 3-D

Homogeneous eq^n → $[x \quad y \quad z \quad h]$

$$T = \begin{bmatrix} a & b & c & p \\ d & e & f & q \\ g & i & j & r \\ \boxed{l \quad m \quad n} & & & s \end{bmatrix}_{4\times 4}$$

Translation

Local scaling ⇒ $\begin{bmatrix} a & 0 & 0 & 0 \\ 0 & e & 0 & 0 \\ 0 & 0 & j & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

overall scaling ⇒ $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & s \end{bmatrix}$

Q) Given a unit cube at the origin (0,0). Perform uniform scaling by a factor of 2. Give the resultant cube.

A) Overall Scaling, S ⇒ $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1/2 \end{bmatrix}$

Page No:

Original cube, $X = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$ — 8 edges of cube

homogeneous

Final cube, $X' = [X][S]$

$$= \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1/2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 & 1/2 \\ 0 & 0 & 1 & 1/2 \\ 0 & 1 & 0 & 1/2 \\ 0 & 1 & 1 & 1/2 \\ 1 & 0 & 0 & 1/2 \\ 1 & 0 & 1 & 1/2 \\ 1 & 1 & 0 & 1/2 \\ 1 & 1 & 1 & 1/2 \end{bmatrix}$$

[Converting h to 1 by multiplying entire matrix by 2]

So, $\Rightarrow \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 2 & 1 \\ 0 & 2 & 0 & 1 \\ 0 & 2 & 2 & 1 \\ 2 & 0 & 0 & 1 \\ 2 & 0 & 2 & 1 \\ 2 & 2 & 0 & 1 \\ 2 & 2 & 2 & 1 \end{bmatrix} = X'$

<u>Shearing</u> → off diagnot elements

$$[x \ y \ z \ 1] \begin{bmatrix} 1 & b & c & 0 \\ d & 1 & f & 0 \\ g & i & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$= [(x + dy + gz) \quad (y + bx + iz) \quad (z + cx + fy) \quad 1]$$

Shearing is done in x by a factor of y & z

done in y by factor of x & z

done in z by factor of x & y

Rotation Matrix → [wrt line (axis) in 3D, wrt Pt in 2D]

(anti-clockwise rotation)

Rot. wrt x-axis ⇒

leave row 1 & col 1

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rot wrt y-axis ⇒

leave row 2 & col 2

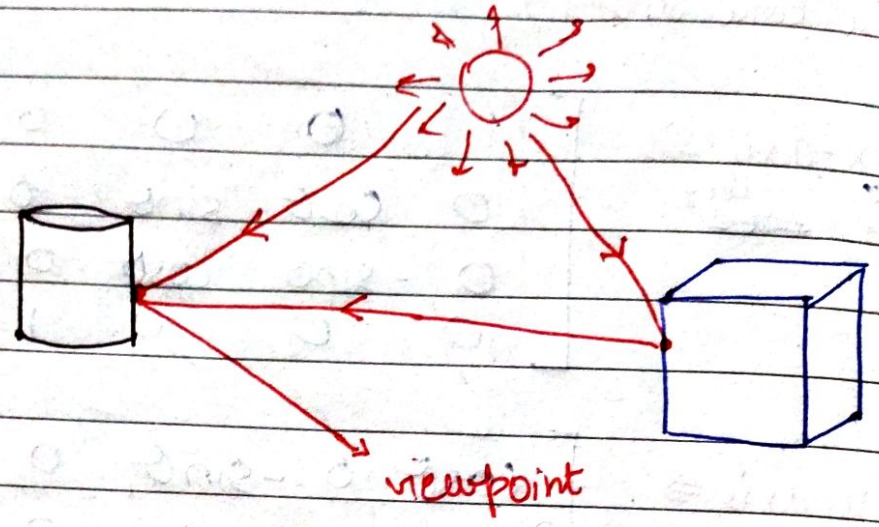
$$\begin{bmatrix} \cos\theta & 0 & -\sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rot wrt z-axis ⇒

leave row 3 & col 3

$$\begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Ch-14: Illuminated



Only Theory Questions

Q) Describe the cosine law?

Q) Describe the Phong model?

Q) Explain Fresnel's law?