Tutorial-2 (DAA)

Q1

```
void fun (int n)
{
    int j = 1, i = 0;
    while (i < n)
    { i += j; j++; }
```

Sol

for j = 1 i = 1
     j = 2 i = 1 + 2
     j = 3 i = 1 + 2 + 3

] m levels

for (i)

$1 + 2 + 3 + \cdots \alpha n$

$1 + 2 + 3 + m \alpha n$

$\frac{m(m+1)}{2} \alpha n$

$m \approx \sqrt{n}$

∴ By summation.

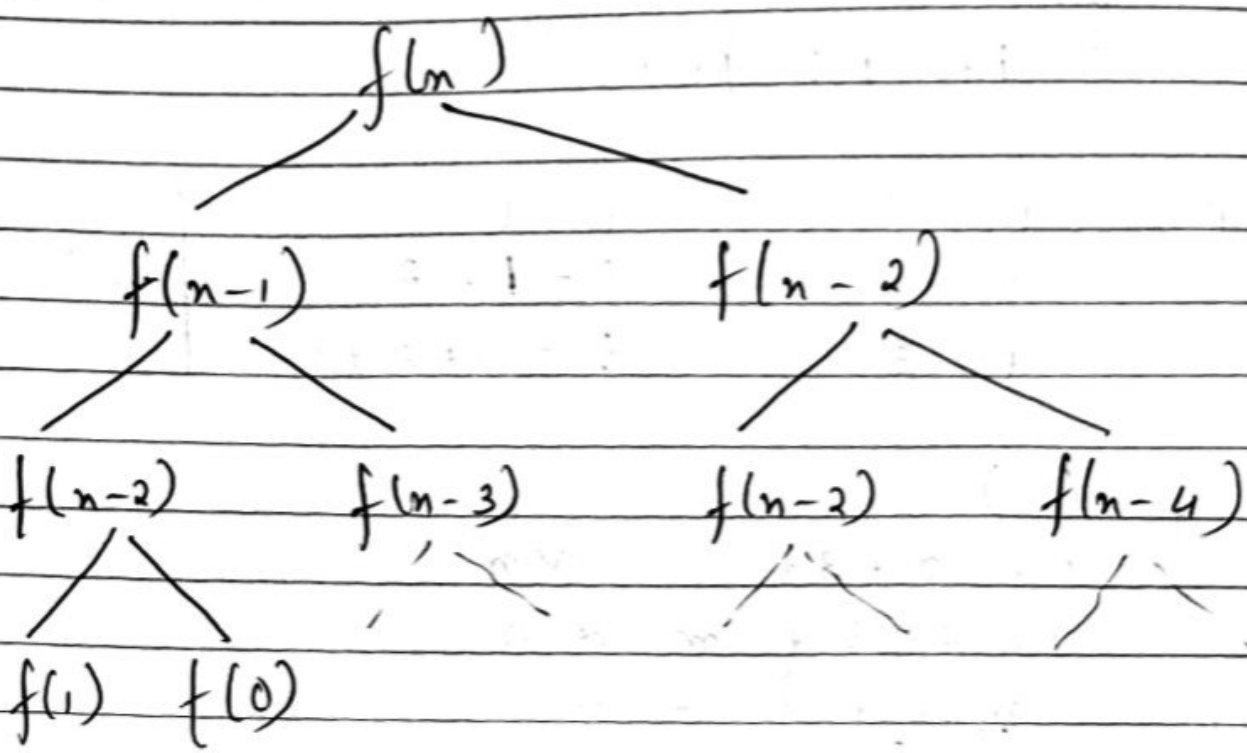
$\Rightarrow \sum_{i=1}^{m} 1 \Rightarrow 1 + 1 + 1 \cdots \sqrt{n}$ times

$$T(n) = \sqrt{n}$$

Q2) For Fibonacci series

$f(n) = f(n-1) + f(n-2)$ $\quad f(0) = 0$, $f(1) = 1$

Sol) By forming tree



∴ At every func. call we get 2 func. call.

for n levels.

$= 2 \times 2 \times \ldots$ n times

$\therefore$ $\boxed{T(n) = 2^n}$

Max Space :

Considering recursive stack

No. of calls max = n.

for each call

Space Complexity ~~(for)~~ = O(1)

$T(n) = O(n)$

Without considering recursive S

Space Complexity = O(1)

$T(n) = O(1)$

Q3

① $n \log n$, n^3, $\log(\log n)$

↓

Sol Quick Sort

```
void QuickSort (int arr[ ], int low, int high)
{
    if (low < high)
    {
        int pi = partition (arr, low, high);
        QuickSort (arr, low, pi-1);
        QuickSort (arr, pi+1, high);
    }
}
int partition( int arr[ ], int low, int high)
{
    int part = arr[high];
    int i = (low-1);

    for(int j= low ; j <=high-1 ; j++)
    {
        if (arr[j] < part)
        {
            i++;
            swap (&arr[i], &arr[j]);
        }
    }
    swap (&arr[i+1], &arr[high]);
    return (i+1);
}
```

classmate

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(2) n^3

Multiplication of 2 square matrix

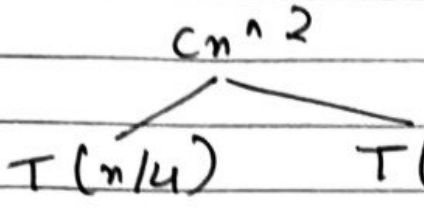
```
for(i=0; i < r1 ; i++)
    for(j=0; j < c2 ; j++)
        for (k=0; k < c1 ; k++)
        {
            res[i][j] += a[i][k] * b[k][j]
        }
```

(3) log(log n)

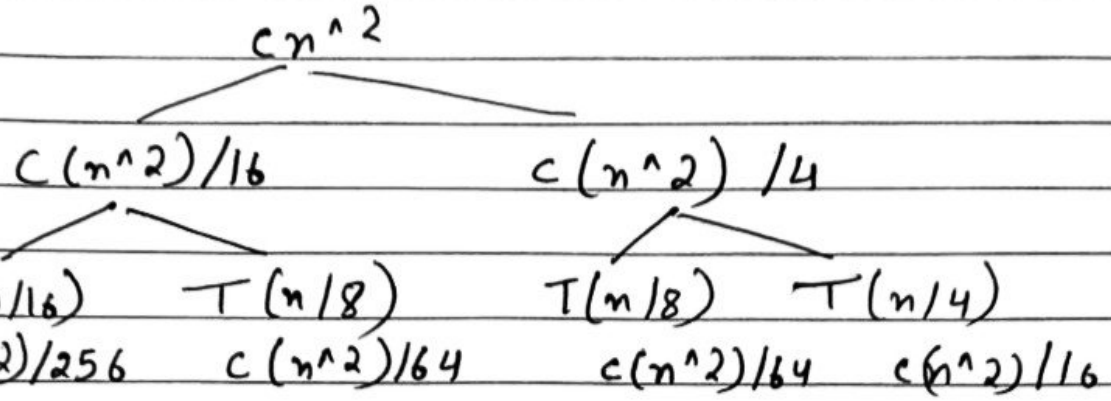
```
for (i=2; i<n; i = i*i)
{
    count++;
}
```

Q4. $T(n) = T(n/4) + T(n/2) + cn^2$

Ans →



Further breaking $T(n/4)$ & $T(n/2)$



Summation level by level.

$T(n) = c(n^2 + 5(n^2)/16 + 25(n^2)/256) +$

G.P. $\Rightarrow r = \frac{5}{16}$

To get upper bound. we can sum above series for infinite term.

$$\text{Sum} = \frac{(n^2)}{\left(1 - \frac{5}{16}\right)}$$

$T(n) \Rightarrow O(n^2)$

Q5

```
int fun (int n)
{
    for (int i=1; i<=n; i++)
    {
        for (int j=1; j<n; j+=i)
        {
            //Some O(1) task
        }
    }
}
```

Sol

For i =	Inner Loop (j)
1	n
2	$n/2$
3	$n/3$
4	$n/4$
⋮	
n	n/n

Total Time Complexity $= \left(n + n/2 + n/3 + \cdots + n/n\right)$

$= n * \left(1 + 1/2 + 1/3 + \cdots + 1/n\right)$

$\downarrow$

~~$T(n) =$~~ $\Theta(\log n)$

$T(n) = O(n \log n)$

Q6 Time Complexity:

```
for (int i=2; i<=n; i=pow(i,k))
{
    //Same O(1) exp. or statement
}.
```

i
2
2^k
2^{k^2}
2^{k^3}
.
.
$2^{k^{\log_k(\log(n))}}$

where $2^{k^{\log_k(\log n)}} \leq n$ $\quad [m = \log_k \log_2 n]$

$2^{\log n} \leq n$ $\quad$ No. of Iteration = m.

$n \leq n$ Agree

So, there are in total $\log_k(\log(n))$ many iterations & each iteration take constant amt. of time.

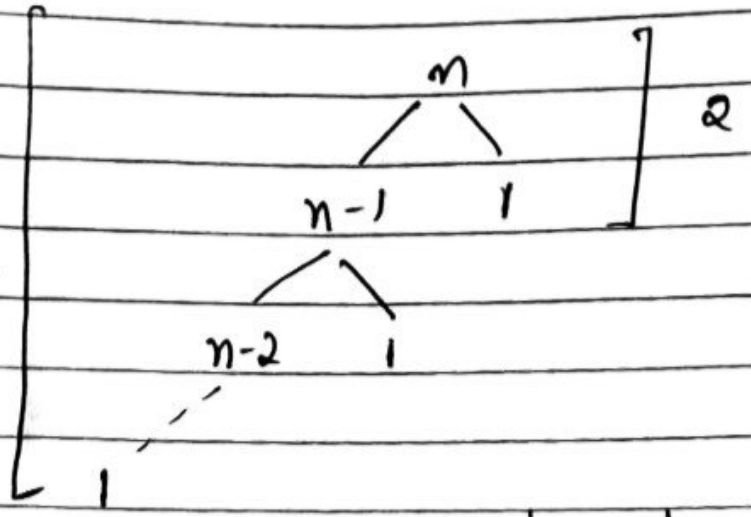
$\therefore T(n) = 1 + 1 + + \cdots \log_k \log n$ times.

$T(n) = O(\log_k(\log(n)))$

Q2

Sol Given algo divides array in 99% & 1% part (Sorting Algo).

$\therefore T(n) = T(n-1) + O(1)$



n work is done at each level for merging.

$T(n) = (T(n-1) + T(n-2) + \cdots T(1) + O(1)) \times n$

$= n \times n$

$\therefore \boxed{T(n) = O(n^2)}$

Lowest Height = 2

Highest " = n

$\therefore$ Diff = $n - 2 \quad n > 1$

The given Algo provides linear result.

Q Arrange following in increasing order of rate of growth.

Sol For large values of n.

(a) $100 < \log\log n < \log n < (\log n)^2 < \sqrt{n} < n < n\log n < \log(n!) < n^2 < 2^n < 4^n < 2^{2^n}$

(b) $1 < \log\log n < \sqrt{\log n} < \log n < \log 2n < 2\log n < n < n\log n < 2n < 4n < \log(n!) < n^2 < n! < 2^{2^n}$

(c) $96 < \log_8 n < \log 2n < 5n < n\log_6 n < n\log_2 n < \log(n!) < 8n^2 < 7n^3 < n! < 8^{2n}$