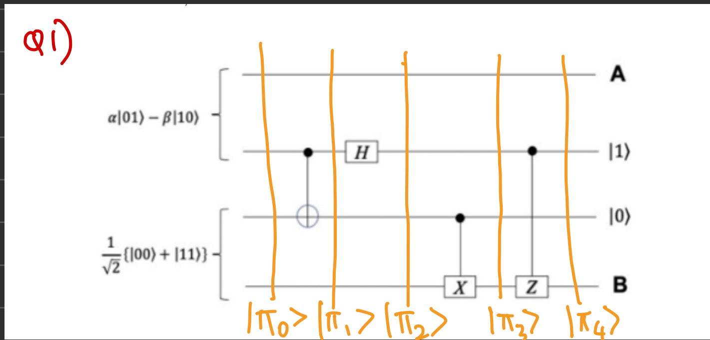
Q1)



$$|\pi_0\rangle = \left(\alpha|01\rangle - \beta|10\rangle\right)\frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\left(\alpha|0100\rangle + \alpha|0111\rangle - \beta|1000\rangle - \beta|1011\rangle\right)$$

$$|\pi_1\rangle = \frac{1}{\sqrt{2}}\left[\alpha\left(|0110\rangle + |0101\rangle\right) - \beta\left(|1000\rangle + |1011\rangle\right)\right]$$

$$|\pi_2\rangle = \frac{1}{\sqrt{2}}\left[\alpha\left(|0\rangle|-\rangle|10\rangle + |0\rangle|-\rangle|01\rangle\right) - \beta\left(|1\rangle|+\rangle|00\rangle + |1\rangle|+\rangle|11\rangle\right)\right]$$

$$= \frac{1}{2}\left[\alpha\left(|0\rangle\{|0\rangle - |1\rangle\}|10\rangle + |0\rangle\{|0\rangle - |1\rangle\}|01\rangle\right) - \beta\left(|1\rangle\{|0\rangle + |1\rangle\}|00\rangle + |1\rangle\{|0\rangle + |1\rangle\}|11\rangle\right)\right]$$

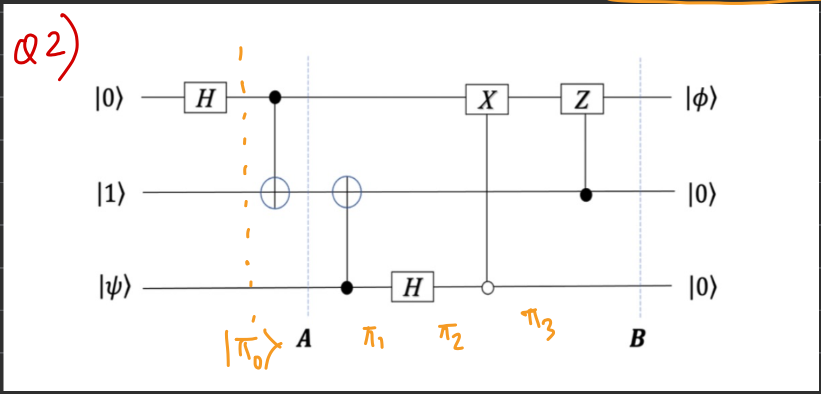
$$= \frac{1}{2}\left[\alpha\left(|0010\rangle - |0110\rangle + |0001\rangle - |0101\rangle\right) - \beta\left(|1000\rangle + |1100\rangle + |1011\rangle + |1111\rangle\right)\right]$$

$$|\pi_3\rangle = \frac{1}{2}\left[\alpha\left(|0011\rangle - |0111\rangle + |0001\rangle - |0101\rangle\right) - \beta\left(|1000\rangle + |1100\rangle + |1010\rangle + |1110\rangle\right)\right]$$

$$|\pi_4\rangle = \frac{1}{2}\left[\alpha\left(|0011\rangle + |0111\rangle + |0001\rangle + |0101\rangle\right) - \beta\left(|1000\rangle + |1100\rangle + |1010\rangle + |1110\rangle\right)\right]$$

$$|\pi_4\rangle = \frac{1}{2}\Big[\alpha|0011\rangle - \beta|1010\rangle\Big] \Rightarrow \text{Final state at A and B:}$$

$$\boxed{\frac{1}{2}\Big[\alpha|01\rangle - \beta|10\rangle\Big]}$$



$$|\pi_0\rangle = |+\rangle|1\rangle|\psi\rangle = \left(|0\rangle + |1\rangle\right)\frac{1}{\sqrt{2}}|1\rangle\left(\alpha|0\rangle + \beta|1\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\left(|01\rangle + |11\rangle\right)\left(\alpha|0\rangle + \beta|1\rangle\right)$$

$$A \Rightarrow \frac{1}{\sqrt{2}}\left(|01\rangle + |10\rangle\right)\left(\alpha|0\rangle + \beta|1\rangle\right)$$

$$|\pi_1\rangle = \frac{1}{\sqrt{2}}\left(\alpha|010\rangle + \beta|001\rangle + \alpha|100\rangle + \beta|111\rangle\right)$$

$$\langle\pi_2\rangle = \frac{1}{\sqrt{2}}\Big[\alpha\big(|01\rangle|+\rangle + |10\rangle|+\rangle\big) + \beta\big(|00\rangle|-\rangle + |11\rangle|-\rangle\big)\Big]$$

$$= \frac{1}{2}\Big[\alpha\left(|01\rangle + |10\rangle\right)\left(|0\rangle + |1\rangle\right) + \beta\left(|00\rangle + |11\rangle\right)\left(|0\rangle - |1\rangle\right)\Big]$$

$$|\pi_3\rangle = \frac{1}{2}\Big[\alpha\left(|010\rangle + |111\rangle + |100\rangle + |001\rangle\right)$$

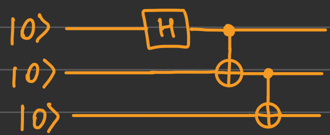
$$+ \beta\left(|000\rangle - |101\rangle + |110\rangle - |011\rangle\right)\Big]$$

$$B \Rightarrow \frac{1}{2}\Big[\alpha\left(|010\rangle - |111\rangle + |100\rangle + |001\rangle\right)$$

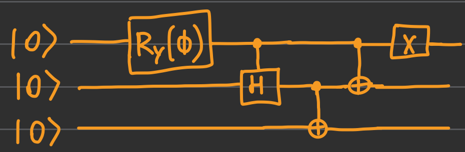
$$+ \beta\left(|000\rangle - |101\rangle - |110\rangle - |011\rangle\right)\Big]$$

$$= \frac{1}{\sqrt{2}}\Big[\alpha|100\rangle + \beta|000\rangle\Big] = \overbrace{\boxed{\frac{1}{\sqrt{2}}\Big[\alpha|1\rangle + \beta|0\rangle\Big]}}^{\phi}|00\rangle$$

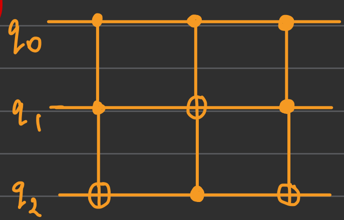
Q3) $|\psi\rangle_{GHZ} = \frac{1}{\sqrt{2}}\left(|000\rangle + |111\rangle\right)$



$|\psi\rangle_{W} = \frac{1}{\sqrt{3}}\left(|001\rangle + |010\rangle + |100\rangle\right)$



Q4)



← control qubit

} Target qubits

Truth table :

(ON) q_0	q_1	q_2	⇒	output $(\lvert q_2 q_1 q_0\rangle)$
1	0	0		$\lvert 001\rangle$
1	0	1		$\lvert 011\rangle$
1	1	0		$\lvert 101\rangle$
1	1	1		$\lvert 111\rangle$