UNIVERSITÀ DELLA CALABRIA

Master Degree in Telecommunications Engineering

"Mobile Radio Networks" Class

2 – Frequency reuse and planning basics

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Assignment of channels to cells

- The multiple access technique in cellular systems allows not only to create sub-channels to different flows/calls…..
- ….but also to assign channels to different cells in the network
- If you think about the thousands of cells covering the service area it is quite surprising that resources can be divided into so many small pieces

Frequency reuse

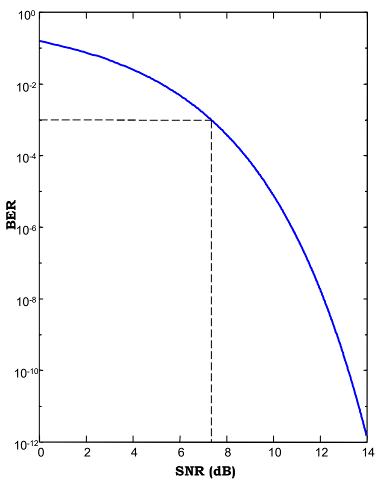
- Actually, considering the total number of channels available for different technologies there are not enough channels even assigning one channel per cell

Solution: use the same channel more times for different cells

- Channel reuse generates interference among cells using the same channels
- Reuse, if possible, only with cells that are sufficiently apart each other
- Frequency is the fundamental characteristic of cellular networks that makes its dimensioning different from other systems

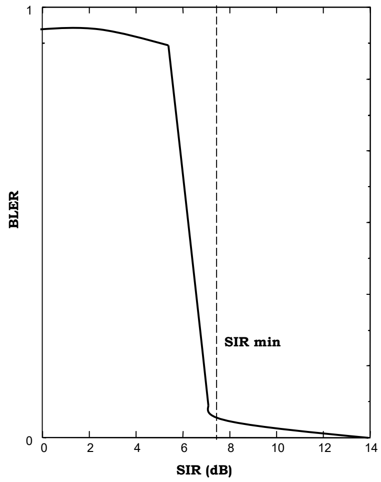
Quality

- In normal communication systems, the connection quality (in terms of BER - Bit Error Rate) depends on the *SNR (Signal-to-Noise Ratio)*
- In mobile radio systems, the ratio between signal power and interference power *SIR (Signal-to-Interference Ratio)* is considered



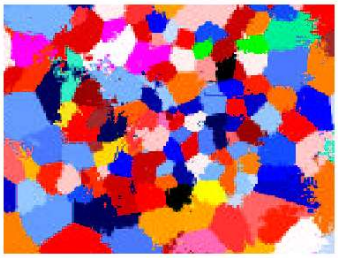
Quality

- In reality, often what matters is the probability of error on the information unit (BLER - Block Error Rate)
- Usually as a quality parameter it is required that the SIR is greater than a threshold SIR_{min}



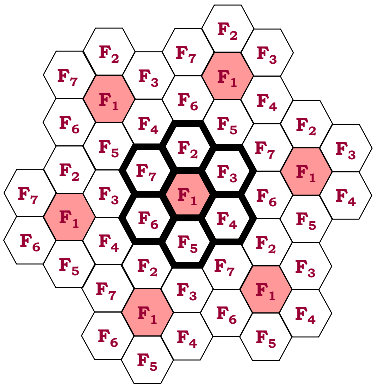
Cell shape

- Traditionally for describing in a simplified way the structure of cellular systems, the shape of cells is depicted as hexagonal
- Obviously, due to base station positions and non uniform propagation of signals due to obstacles, the real shape of cells is usually much different
- The use of the regular hexagonal shape is however a good approach to make a rough dimensioning of the system and for us to understand the basic principles of reuse



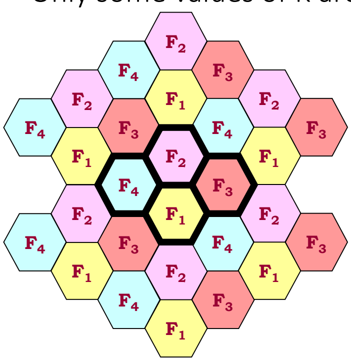
Cluster

- After coverage planning, capacity planning is in charge of defining which radio resources can be used by each cell
- The number of resources (frequencies) assigned to cells determines system capacity
- Frequencies can be reused, but SIR (quality) constraints must be enforced
- A simple 'didactical' model considers hexagonal cells and homogeneous traffic
- Frequencies are divided into K groups and assigned to a group of K cells, named cluster.
- The cluster is repeated in the area in a regular fashion

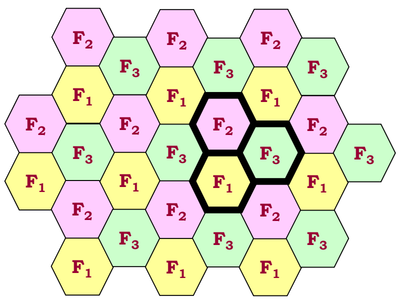


Cluster

- The number of channels per cell depends on the dimension K of the Cluster
- *Reuse efficiency= 1/K*
- Only some values of K are admissible=1,3,4,7,9,12,13, …



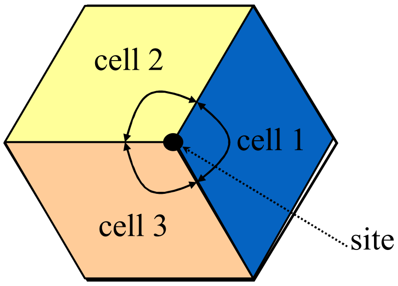
K = 4



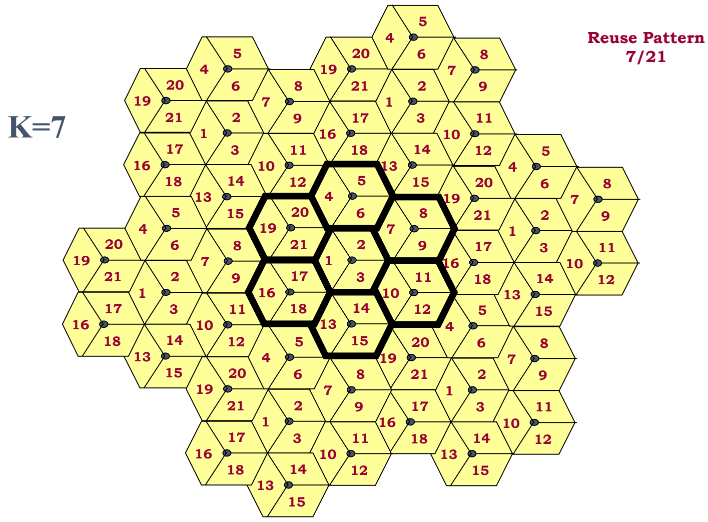
K = 3

Sectorial antennas

- The use of directive antennas allows to modify the cellular layout and reduce interference received
- In cellular systems the use of directive antennas with a 120° angle of the main lobe is quite common

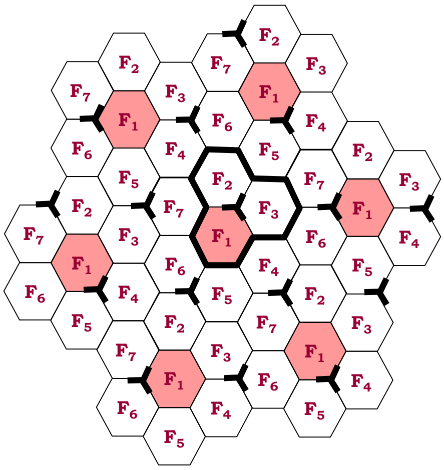


Reuse with sectors



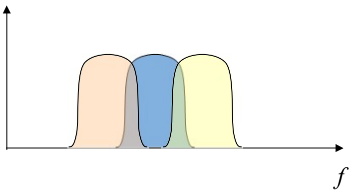
Reuse with sectors

- In this scheme the reuse scheme is the same but the base station sites are different



Assignment constraints

- Once cluster size is selected, the assignment of channels to cells is usually subject to additional constraints
- Adjacent frequencies have often slightly overlapped spectrum and therefore can generate mutual interference (*adjacent channel interference*)



- The problem can be more complex due to sectors that usually have secondary lobes in the antenna diagram that generate interference in the neighboring cells
- As a results it is <u>not usually possible to assign adjacent frequencies to cells of the same site</u>

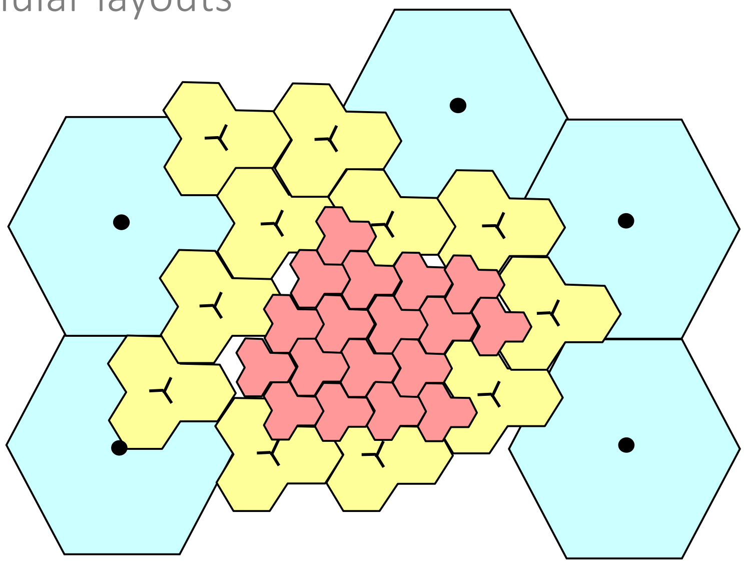
Cellular layouts

- Important observation:

  The simplified formula for cluster dimensioning does not depend on the cell radius but only on the distance ratios
  (you will see it better during the exercise lessons)

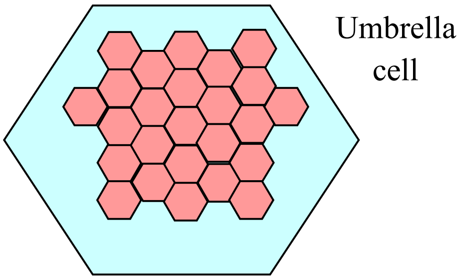
- Varying the cell radius we can vary the number of channels available per unit area
- This gives us the freedom to plan the cellular layout (cell sizes) based on the traffic density estimated in different areas

Cellular layouts



Cellular layouts: Heterogeneous networks

- We need however to consider that with small cells some of the assumptions do not hold
  - With small distances the propagation formula may change
- Moreover, with small cells the number of handovers increases
- In some cases, the coverage and mobility management can be guaranteed with an "umbrella" macro-cell



Mobile Radio Networks

❑Algorithms and Methods of static reuse

Graph based models

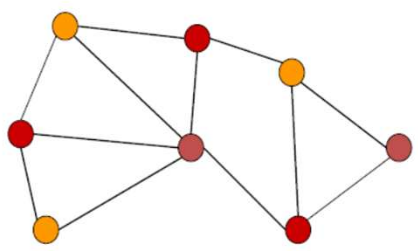
- Even if the vision of frequency reuse based on the cluster concept is simplified, it has also been used in practical cases to plan the frequency assignment in real systems at the beginning of the diffusion of such systems
- Unfortunately, cells are not hexagonal, and traffic is not homogeneous …
- Other models have been proposed for practical cases
- Some popular models are based on graph coloring problems

Graph based models

- Models which, at least in some respects, represent an evolution with respect to the cluster model
- Such models allow:
  - to consider cells with any shape
  - to assign a number of channels to each cell depending on the traffic
  - to take into account the constraints of the cells served by the same site in the case of sector antennas

Graph based models

- *Compatibility graph* G(V,E)
  - Vertices are base stations
  - Two vertices are connected by an edge if the two base stations cannot reuse the same frequencies

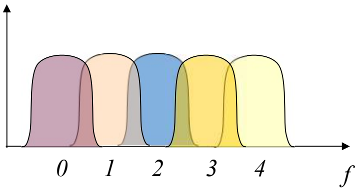


Graph based models

- Any coloring of the vertices of G (i.e., assignment of colors such that adjacent vertices have different colors) is an assignment of frequencies to the network such that no mutual interfering BSs receive the same frequency.
- A minimum cardinality coloring of G is a minimum cardinality non-interfering frequency assignment of the network.
- Graph coloring problem is NP-hard and several exact algorithms and heuristics have been proposed.
- This simple model assumes:
  - One frequency per BS
  - Two distinct frequencies do not interfere

Graph based models

- NOTE: Also, in the case of TDMA systems the cells are assigned frequencies (radio carriers), so we are still talking about frequency assignment
- The available frequencies are numbered in an orderly and sequential manner



- The set of available frequencies is

$$F = \{1, \ldots, N\}$$

- while that of the cells is:

$$S = \{1, \ldots, K\}$$

Graph based models: Frequency Assignment Problem (FAP)

- An assignment is defined by the sets:

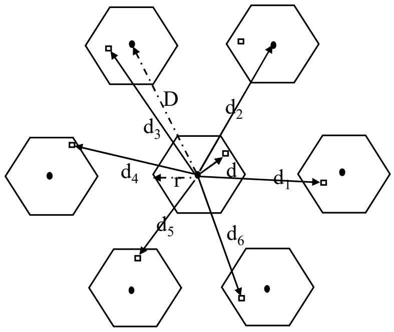
$$\{F_i\}, \quad F_i \subset F$$

- which represent the sets of frequencies assigned to the cells
- Depending on the traffic offered, it is possible to establish the minimum number m_i of channels required for each cell i (for example using Erlang B formula)
  - a *traffic constraint* on the assignment thus results:

$$|F_i| = m_i \quad \forall i \in S$$

Graph based models: Frequency Assignment Problem (FAP)

- As you will see better during the exercises, in calculating the cluster size the value of SIR_{min} generates a minimum reuse distance R



Cells that are further apart than R_{min} can use the same frequencies, while those that are at a shorter distance must use different frequencies

Graph based models: Frequency Assignment Problem (FAP)

- Simplifying, the concept of minimum reuse distance can be extended to the more general case of cells with any shape
- Furthermore, the concept of distance can be improved by considering to what extent one cell can generate interference for another
- Based on the evaluation of this mutual interference of the cells, a compatibility matrix can be generated which states for each pair of cells *(i,j)* whether they can have the same frequencies assigned
- *<u>NOTE</u>: Obviously in this way only pairs of cells are considered while the SIR actually depends on all the interferers*

Graph based models: Frequency Assignment Problem (FAP)

- The *compatibility matrix* of dimensions NxN:

$$C=\{c_{ij}\} \quad i,j \in S$$

- defines *compatibility constraints* on the assignment of frequencies:

$$|f_i - f_j| \geq c_{ij} \quad i,j \in S, f_i \in F_i, f_j \in F_j$$

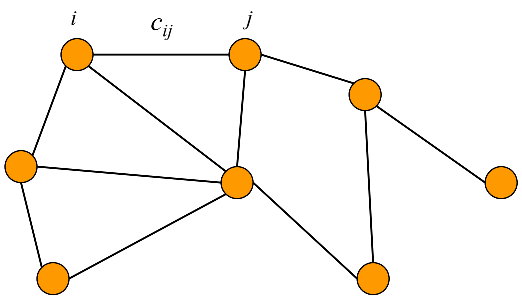
- For example
  - if $c_{ij}=0$ cells *i* and *j* can use the same frequencies
  - if $c_{ij}=1$ cells *i* and *j* cannot use the same frequencies
  - if $c_{ij}=2$ cells *i* and *j* cannot use the same frequencies nor adjacent frequencies

Graph based models: Frequency Assignment Problem (FAP)

- The c_{ij} coefficients and corresponding compatibility constraints are a simplification of the SIR constraints we used with clusters
- but the model allows more general cases to be treated with cells of any shape and with a different number of channels required for each cell
- How are the c_{ij} coefficients calculated?
  - there is no single criterion, the potential interference generated by a base station on the coverage area of the other is calculated and the c_{ij} is derived on the basis of this and the thresholds
  - Usually, $c_{ij}= c_{ji}$ is assumed

Graph based models:
Frequency Assignment Problem (FAP)

- The representation of the problem can be obtained by means of a weighted graph *G(N,A)* where the nodes are the cells, and the edges connect the nodes if $c_{ij} \neq 0$ and have weight equal to c_{ij}



Solution to FAP problem

- A compatible assignment is one which assigns to each cell the required number m_i of frequencies, and which respects all compatibility constraints
- Finding one means finding a solution to the FAP (*Frequency Assignment Problem*)
- Obviously, due to compatibility constraints, traffic constraints or the number of available frequencies, a compatible assignment may not exist!
- In this case it is necessary to relax the constraints and find an optimization goal

Optimization objective: Max-FAP

- One possible approach is to relax traffic constraints

$$|F_i| \le m_i \quad \forall i \in S$$

- and assign each vertex as many frequencies as possible:

$$\max \sum_{i \in S} |F_i|$$

- this variant of the problem is called *Maximum Service – FAP* (Max-FAP)

Optimization objective: MS-FAP

- Another approach keeps traffic and compatibility constraints tight but assumes you can use as many frequencies as you like
- The set F of available frequencies becomes a variable
- and the goal becomes to minimize the number of frequencies

$$\min |F|$$

- this variant of the problem is called *Maximum Spam – FAP* (MS-FAP)

How to find a solution?

- FAP is an NP-complete problem
- For small instances ILP solvers can provide the optimum solutions in reasonable time
- For large instances we need heuristics to get a good quality solution in short time
- Among the simplest heuristics we have a *greedy algorithm*:
  - The solution is built sequentially ordering cells and assigning frequency according to a simple merit function

Greedy algorithm for FAP

- The greedy algorithm we consider a single frequency per cell must be assigned
- therefore, to consider the general case with m_i>1 it is necessary to separate the cell i into m_i copies
- Example: $M = \{m_i\} = \{1,2,2,1,1\}$

$$C = \{c_{ij}\} = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

Greedy algorithm for FAP

$$M = \{m_i\} = \{1,1,1,2,1,1\}$$

- … example:

$$\begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ & & & & \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

Greedy algorithm for FAP

$$M = \{m_i\} = \{1,1,1,1,1,1,1\}$$

- … example:

$$\begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & 1 & 1 & & 1 & 0 & 0 \\ 1 & 0 & 0 & & 0 & 1 & 1 \\ 1 & 0 & 0 & & 0 & 1 & 1 \\ & & & & & & \\ 1 & 0 & 0 & & 0 & 1 & 1 \\ 0 & 1 & 1 & & 1 & 0 & 0 \\ 0 & 1 & 1 & & 1 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

Greedy algorithm for FAP

- For every cell i the *degree* is computed :

$$g_i = \sum_{\substack{j=1 \\ j \neq i}}^{N} c_{ij}$$

- cells are sorted in descending order in a list L
- being $m_i=1\ \forall i$, we indicate with f_i the frequency assigned to cell i (if no frequency is assigned $f_i=0$)

Greedy algorithm for FAP

- In summary, the algorithm:
  - For every cell i in L and for every frequency f in F
    - for each of the neighbors j verifies that the difference between the frequency considered and that assigned to the neighbor is less than c_{ij}
    - calculates a cost function

$$\left(c_{ij} - \left|f - f_j\right|\right) \cdot c_{ij}$$

    - the cost function is cumulated for all neighbors
    - Note that if f is compatible the cost will be zero!
  - assigns to cell i the frequency f_i with the minimum cost function

Greedy algorithm for FAP

- In a more rigorous form:

$$
\begin{array}{l}
\text{BestCost} = \infty \\
\text{For } i \in L \text{ do} \\
\quad \text{For } f \in F \text{ do} \\
\qquad \text{cost} := 0 \\
\qquad \text{For } j \in N_i \text{ do} \\
\qquad\quad \text{If } \left(f_j \neq 0\right) \text{AND} \left(\left|f\text{-}f_j\right| < c_{ij}\right) \text{then} \\
\qquad\qquad \text{cost} := \text{cost} + \left(c_{ij} - \left|f\text{-}f_j\right|\right) \cdot c_{ij} \\
\qquad\quad \text{End} \\
\qquad \text{End} \\
\qquad \text{If BestCost} \geq \text{cost then} \\
\qquad\quad \text{BestCost} := \text{cost} \\
\qquad\quad f_i := f \\
\qquad \text{End} \\
\quad \text{End} \\
\text{End}
\end{array}
$$

Greedy algorithm for FAP

Some remarks:

- the cost of compatible frequencies is zero
- therefore if at least one compatible frequency exists this is assigned to the cell
- and, therefore, the algorithm tends to provide a compatible assignment
- if no compatible frequency exists, the one with the constraint to be violated which has the least weight is chosen

Greedy algorithm for FAP

Example

$$M = \{m_i\} = \{1,1,1,1,1,1,1\}$$
$$F = \{1,\dots,3\}$$
$$C = \{c_{ij}\} = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$
$$L = \{1,2,4,7,3,6,5\}$$

Greedy algorithm for FAP

Example

If $(f_j \neq 0)$ AND $(|f\text{-}f_j| < c_{ij})$ then $(c_{ij} - |f - f_j|) \cdot c_{ij}$

- First cell: *i=1*
  - all frequencies *j* have cost 0
  - then $f_1=3$
- Second cell: *i=2*
  - costs=(0,0,1)
  - then $f_2=2$
- Third cell: *i=4*
  - costs=(0,1,1)
  - then $f_4=1$

$$M = \{m_i\} = \{1,1,1,1,1,1,1\}$$
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Greedy algorithm for FAP

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$$L = \{1,2,4,7,3,6,5\}$$

- Fourth cell: *i=7*
  - costs=(1,1,0)
  - then $f_7=3$
- Fifth cell: *i=3*
  - costs =(0,0,1)
  - then $f_3=2$
- Sixth cell: *i=6*
  - costs =(1,1,0)
  - then $f_6=3$
- Seventh cell: *i=5*
  - costs =(0,0,1)
  - then $f_5=2$