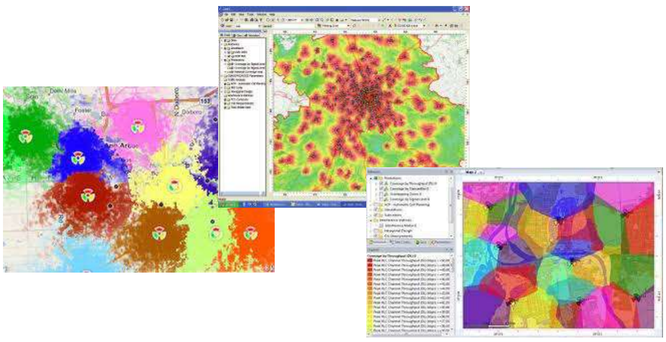
Mobile Radio Networks

❑Radio planning

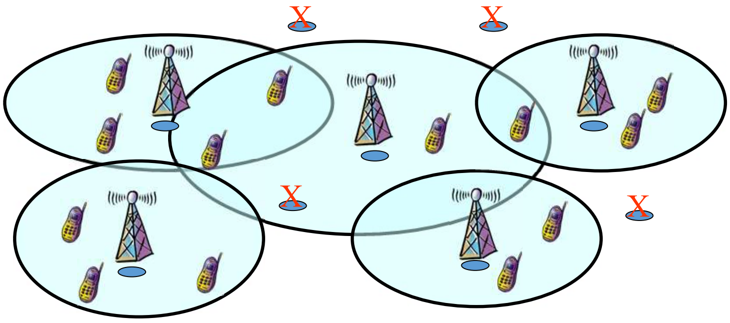
What is Radio Planning?

- When we have to install a new wireless network or extend an existing one into a new area, we need to design the fixed and the radio parts of the network. This last phase is called radio planning.



What is Radio Planning?

- The basic decisions that must be taken during the radio planning phase are:
  - Where to install base stations (or access points, depending on the technology)
  - How to configure base stations (antenna type, height, sectors orientation, tilt, maximum power, device capacity, etc.)



Radio Planning

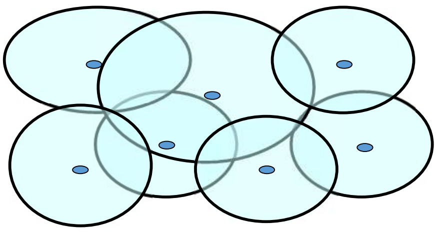
- When planning and optimizing a cellular system, a number of aspects must be considered, including
  - signal propagation,
  - traffic estimation,
  - antenna positioning,
  - antenna configuration,
  - interference.
- Here we'll focus on the decision problems that give rise to interesting and challenging mathematical programming models which must account for the peculiarities of the specific network technology.

Modelli di pianificazione radio

- In optimization models for base station planning, it is assumed to have:
  - a set of candidate sites *J*
  - traffic distribution in the area
  - the propagation characteristics

Propagation Prediction

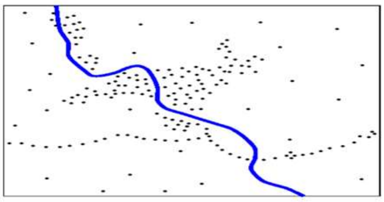
- One of the key elements for the radio planning is propagation prediction that allows to estimate the area covered by each base station
- It is the task of propagation experts to provide forecasting tools based on radio channel models (empirical and statistical models, ray tracing) and on local knowledge



- The covered area is the area where the received signal strength is above a threshold
- Received signal strength depends on emitted power and path loss

Traffic Estimation

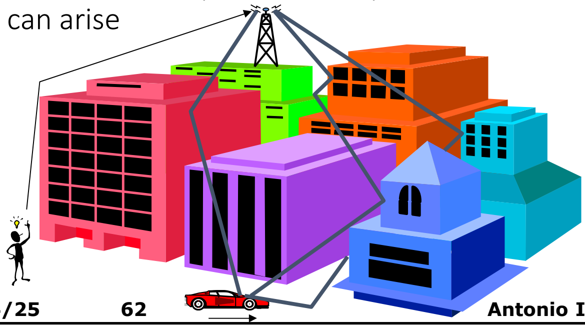
- Traffic distribution in the service area is usually hard to predict in the radio planning phase since it depends on several issues including area population, buildings, market penetration of the considered service, etc.
- Traffic distribution is usually provided using a discrete Set of points I, *test points* (TP), that are considered as centroids of traffic



- It is assumed a known radio channel attenuation a_{ij} (or the gain $g_{ij}=1/a_{ij}$) between each candidate site $j \in J$ and each test point $i \in I$

Antenna positioning

- The selection of possible antenna sites depends on several technical (traffic density and distribution, ground morphology, etc.) and non-technical (electromagnetic pollution, local authority rules, agreements with building owners, etc.) issues
- Quite often the number of possible sites is so small that there are no possible choices
- If, on the other hand, the choice is possible, the problem of an optimized planning can arise



Coverage planning

- The goal of the coverage planning phase is to:
  - Select where to install base stations
  - Select antenna configurations
- In order to guarantee that the signal level in all Test Points is high enough to guarantee a good communication quality
- Note that interference is not considered in this phase

Let us first consider a simple model where decisions are only on where to install base stations

Set covering problem (SCP)

- The *decision variables* of the problem are:

$$x_j = \begin{cases} 1 & \text{if a BS is installed in CS j} \\ 0 & \text{otherwise} \end{cases}$$

- to which a *cost of installation* c_j is associated
- a simple formulation of the problem assumes that $i \in I$ is covered by $j \in J$ if attenuation a_{ij} is below a threshold t
- Let's define the sets
  - N_i: $$N_i = \{ j \mid a_{ij} \le t \}$$ **set of sites that cover point *i***
  - P_j: $$P_j = \{ i \mid a_{ij} \le t \}$$ **set of test points covered by site *j***

Set covering problem (SCP)

- The problem turns out to be:

$$Minimize\ \ Z = \sum_{j \in J} c_j x_j$$

$$s.t.$$

$$\sum_{j \in N_i} x_j \geq 1 \qquad \forall i \in I$$

the constraint ensures that all points are covered

- in practice the variables x_j define a sub-set $J^* \subseteq J$ of sites covering all points:

$$\bigcup_{j \in J^*} P_j = I$$

Solutions to the coverage problem

- This problem is NP-hard (means that it is practically impossible to find an efficient way to solve it)
- Also in this case heuristic algorithms are used which often provide a good sub-optimal solution
- We also in this case see one of the simplest greedy algorithms
  - which sequentially adds a station at a site until full coverage is achieved

Greedy algorithm for SCP

- Step 0
  - set $J^*=\varnothing$
- Step 1
  - if $P_j = \varnothing\ \forall j$ then STOP
  - otherwise find k $\in (J\text{-}J^*)$ such that $\frac{|P_j|}{c_j}$ is maximum
- Step 2
  - add k a J^* ($J^*:=J^* \cup \{k\}$)
  - remove points P_k from the other sets ($P_j:=P_j\text{-}P_k\ \forall j$)
  - go back to Step 1.

Greedy algorithm: Example (1)

- To simplify the description of the problem we consider a coverage matrix $V= \{v_{ji}\}$, where:

$$v_{ji} = \begin{cases} 1 & \text{if } i \text{ is covered by } j \\ 0 & \text{otherwise} \end{cases}$$

- the coverage vector $\Pi=\{\pi_j\}$

$$\pi_j = \sum_{i \in I} v_{ji} = |P_j|$$

- and the cost vector $C = \{c_j\}$

Greedy algorithm: Example (2)

- *Step 0: $J^*=\varnothing$*

$$V = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix} \quad \Pi = \begin{bmatrix} 7 \\ 5 \\ 8 \\ 8 \\ 9 \end{bmatrix} \quad C = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Greedy algorithm: Example (3)

- *Step 1: k=5*

$$V = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix} \quad \Pi = \begin{bmatrix} 7 \\ 5 \\ 8 \\ 8 \\ 9 \end{bmatrix} \quad C = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Greedy algorithm: Example (4)

- *Step 2:*
  - J^{*}= {5},
  - ricalculate V and Π

$$V = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix} \quad \Pi = \begin{bmatrix} 7 \\ 5 \\ 8 \\ 8 \\ 9 \end{bmatrix} \quad C = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Greedy algorithm: Example (5)

- *Step 2:*
  - ... ricalculate V and Π

$$V = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \Pi = \begin{bmatrix} 3 \\ 3 \\ 2 \\ 2 \\ 0 \end{bmatrix}$$

Greedy algorithm: Example (6)

- *Step 1:*
  - *k=1*
- *Step 2:*
  - $J^* = \{5,1\}$,
  - ricalculate V and Π

$$V = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \Pi = \begin{bmatrix} 3 \\ 3 \\ 2 \\ 2 \\ 0 \end{bmatrix}$$

Greedy algorithm: Example (7)

- … ricalculate V and Π

$$V = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \Pi = \begin{bmatrix} 0 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

Greedy algorithm: Example (8)

- *Step 1:*
  - *k=2*
- *Step 2:*
  - J^*= {5,1,2},
  - ricalculate V and Π

$$V = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \Pi = \begin{bmatrix} 0 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

Greedy algorithm: Example (9)

- … ricalculate V and Π

$$V = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \Pi = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

Greedy algorithm: Example (10)

- *Step 1:*
  - *k=3*
- *Step 2:*
  - *J*= {5,1,2,3},*
  - ricalculate V and Π

$$V = \begin{bmatrix} 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \end{bmatrix} \quad \Pi = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

Greedy algorithm: Example (11)

- … ricalculate V and Π
- STOP

$$V = \begin{bmatrix} 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \end{bmatrix} \quad \Pi = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Greedy algorithm: Example (12)

- In this simple example it's easy to observe that the solution obtained by the greedy algorithm $J^{*}=\{5,1,2,3\}$ is sub-optimal
- In fact, this solution has a lower cost:

$$V=\begin{bmatrix} 1&1&0&1&1&0&0&1&0&0&0&0&1&0&1\\ 1&1&1&0&0&1&0&0&0&0&0&1&0&0&0\\ 1&0&0&0&0&0&1&1&1&1&1&0&0&1&1\\ 1&1&1&1&1&0&1&0&0&0&0&0&0&1&1\\ 1&0&1&0&1&0&1&0&1&0&1&0&1&1&1 \end{bmatrix} \quad \Pi=\begin{bmatrix}7\\5\\8\\8\\9\end{bmatrix} \quad C=\begin{bmatrix}1\\1\\1\\1\\1\end{bmatrix}$$