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Exploring 4D quantum Hall physics with a 2D topological charge pump

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The discovery of topological states of matter has greatly improved our understanding of phase transitions in physical systems. Instead of being described by local order parameters, topological phases are described by global topological invariants and are therefore robust against perturbations. A prominent example is the two-dimensional (2D) integer quantum Hall effect[1]: it is characterized by the first Chern number, which manifests in the quantized Hall response that is induced by an external electric field[2]. Generalizing the quantum Hall effect to four-dimensional (4D) systems leads to the appearance of an additional quantized Hall response, but one that is nonlinear and described by a 4D topological invariant—the second Chern number[3,4]. Here we report the observation of a bulk response with intrinsic 4D topology and demonstrate its quantization by measuring the associated second Chern number. By implementing a 2D topological charge pump using ultracold bosonic atoms in an angled optical superlattice, we realize a dynamical version of the 4D integer quantum Hall effect[5,6]. Using a small cloud of atoms as a local probe, we fully characterize the nonlinear response of the system via *in situ* imaging and site-resolved band mapping. Our findings pave the way to experimentally probing higher-dimensional quantum Hall systems, in which additional strongly correlated topological phases, exotic collective excitations and boundary phenomena such as isolated Weyl fermions are predicted[4].

Topology, originally a branch of mathematics, has become an important concept in different fields of physics, including particle physics[7], solid-state physics[8] and quantum computation[9]. In this context, a hallmark achievement was the discovery of the 2D integer quantum Hall effect[1]. This discovery demonstrated that the Hall conductance in a perpendicular magnetic field and in response to an electric field $\boldsymbol{E}$ is quantized. In a cylindrical geometry, following Laughlin's thought experiment, $\boldsymbol{E}$ can be generated by varying the time-dependant magnetic flux $\phi_x(t)$ along the axis (x) of the cylinder[10] (Fig. 1a). The interplay between the perpendicular magnetic field and the induced electric field E_z creates a quantized Hall response in the x direction: an integer number of particles, determined by the first Chern number, is transported between the edges per quantum of magnetic flux that is threaded through the cylinder[2].

Dimensionality is crucial for topological phases and many intriguing states were recently discovered in three dimensions, such as Weyl semimetals[11,12] and three-dimensional (3D) topological insulators[13]. Ascending further in dimensions, a 4D generalization of the quantum Hall effect has been proposed in the context of astrophysics[3] and condensed-matter systems[4], and has received much attention in theoretical studies[8]. Unlike its 2D equivalent, the 4D quantum Hall effect can occur in systems with and without time-reversal symmetry[3,4]. The former constitutes the fundamental model from which many lower-dimensional time-reversal-symmetric topological insulators can be derived[8,14]. Furthermore, a 4D quantum Hall system might exhibit relativistic collective hyperedge excitations and new strongly correlated quantum Hall phases, revealing the interplay between quantum correlations and dimensionality[4]. This interest was renewed recently as a result of the unprecedented control and flexibility made possible by engineered systems such as ultracold atoms and photonics. Such systems have been used to study various topological effects[15,16], including a measurement of the second Chern number in an artificially generated parameter space[17], and offer a direct route for realizing 4D physics using synthetic dimensions[18–20].

In the simplest case, a 4D quantum Hall system can be composed of two 2D quantum Hall systems in orthogonal subspaces (Fig. 1a, b). In addition to the quantized linear response that underlies the 2D quantum Hall effect, a 4D quantum Hall system exhibits a quantized nonlinear 4D Hall response[6]. The latter arises when—simultaneously with the perturbing electric field $\boldsymbol{E}$—a magnetic perturbation $\boldsymbol{B}$ is added. The 4D geometry implies multiple possibilities for the orientation of $\boldsymbol{E}$ and $\boldsymbol{B}$; however, the resulting nonlinear response is always characterized by the same 4D topological invariant, the second Chern number. Here, we focus on the geometry depicted in Fig. 1a, b, in which the nonlinear response can be understood semi-classically as originating from a Lorentz force created by $\boldsymbol{B}$, which couples the motion in the two 2D quantum Hall systems[21]. The direction of this response is transverse to both perturbing fields. Hence, it can occur only in four or more dimensions and has therefore never been observed in any physical system.

Topological charge pumps exhibit topological transport properties that are similar to higher-dimensional quantum Hall systems and provide a way to probe 4D quantum Hall physics in lower-dimensional dynamical systems. The first example of a topological charge pump was the one-dimensional (1D) Thouless pump[5], in which an adiabatic periodic modulation generates a quantized particle transport. This modulation can be parameterized by a pump parameter and, at each point in the cycle, the 1D system constitutes a Fourier component of a 2D quantum Hall system[14,22]. The induced motion is thus equivalent to the linear Hall response and is characterized by the same 2D topological invariant, the first Chern number. Indeed, the quantum Hall effect on a cylinder can be mapped to a 1D charge pump with the threaded magnetic flux ϕ_x acting as the pump parameter[10] (Fig. 1a). Building on early condensed-matter experiments[23], topological charge pumps have recently been realized in photonic waveguides[24] and by using ultracold atoms[25,26].

A dynamical 4D quantum Hall effect can accordingly be realized by using a 2D topological charge pump[6]. Using dimensional reduction[14,22], the Fourier components of a 4D quantum Hall system can be mapped onto a 2D system. For the geometry in Fig. 1a, b, the corresponding 2D model is a square superlattice (Fig. 1c, Methods), which consists of two 1D superlattices along the x and y directions, each formed by superimposing two lattices: $V_{\mathrm{s},\mu}\sin^2(\pi\mu/d_{\mathrm{s},\mu}) + V_{\mathrm{l},\mu}\sin^2(\pi\mu/d_{\mathrm{l},\mu} - \varphi_\mu/2)$, with $\mu \in \{x, y\}$. Here, $d_{\mathrm{s},\mu}$ and $d_{\mathrm{l},\mu} > d_{\mathrm{s},\mu}$ denote the period of the short and long lattices, respectively, and $V_{\mathrm{s},\mu}$ and $V_{\mathrm{l},\mu}$ the depths of the short and long lattice potentials. The position of the long lattice is determined by the corresponding superlattice phase φ_μ.

The phase φ_x is chosen as the pump parameter; that is, pumping is performed by moving the long lattice along x. This method of pumping

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