model (equation (1)), in which each constituent 1D pump is characterized by its own first Chern number and therefore the corner modes manifest as the fully bound joint product of the protected topology at the boundary of the 2D pump. This in turn means that in our set-up the corner modes only weakly hybridize with the bulk and the pumping is carried by long-lived resonances. We note that topological corner modes are unique in the sense that they have two fewer dimensions than the physical dimension of the system (conventional topological modes have one fewer dimension). The appearance and demonstration of such modes has recently been reported in inversion-symmetry-protected 2D systems[32,33].

In conclusion, we have observed topological edge pumping associated with the 4D quantum Hall effect in a 2D photonic system using synthetic dimensions. These observations imply that the system is characterized by a non-zero second Chern number. Boundary phenomena provide an independent observation of the physics implied by the second Chern number of the system, in addition to the measurement of the quantized nonlinear bulk response in a similar model using cold atoms[34]. The realization of 4D quantum Hall physics opens up the possibility of realizing many new physical effects and of answering several open questions, including: whether a bulk measurement of the second Chern number can be realized in photonics via the nonlinear response to synthetic fields; whether arbitrarily high spatial dimensionality can be realized; whether interactions can lead to 4D fractional Hall physics when using synthetic dimensions; and whether there are other physical quantities that are quantized in four dimensions that can be measured directly using synthetic dimensions. Because photonic systems naturally allow for non-Hermitian Hamiltonians (which arise from gain and loss), another question is how non-Hermiticity and topological gaps associated with non-zero second Chern number interact. We expect that experimental access to 4D quantum Hall physics will open up many other directions for research.

Online Content Methods, along with any additional Extended Data display items and Source Data, are available in the online version of the paper; references unique to these sections appear only in the online paper.

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