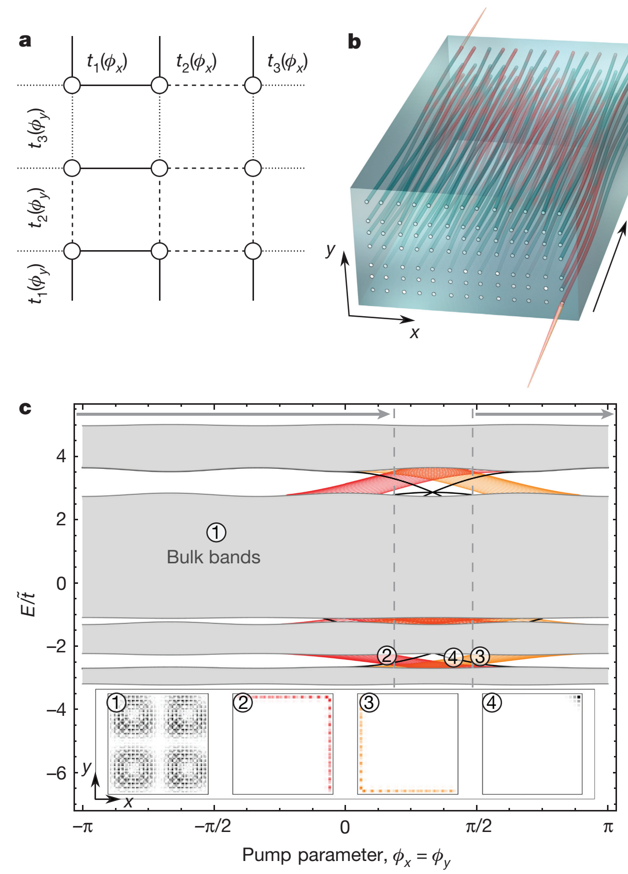

Figure 1 | 2D topological pump and its band structure. a, Schematic of the lattice model (equation (1)) with a 3 × 3 unit cell, that is, $b_x = 1/3$ and $b_y = 1/3$, resulting in three different hopping amplitudes (solid, dashed and dotted lines) in each direction, which can be modulated using the pump parameters ϕ_x and ϕ_y. **b**, Illustration of the 2D (7 × 13) array of waveguides with z-dependent spacing. Light is injected into the input facet, is pumped across the array during its propagation (owing to the topological nature of the 2D pump) and is collected on the other side using an InGaAs CCD camera. **c**, Calculated band structure for a similar device, consisting of a 70 × 70 array of coupled waveguides, where energy E is plotted along the path $\phi_x = \phi_y$ (larger dimensions chosen for clarity) at a wavelength of 1,550 nm, normalized by the bare hopping amplitude $\tilde{t}$. Bulk modes are shown in grey, edge modes in red and orange, and corner modes in black. The insets show representative wavefunctions for each type of mode. For our choice of pump parameters, the edge modes (red and orange) form wedges owing to their degeneracy. The corner modes vanish into the bulk bands along their pump path and weakly hybridize with bulk modes. We perform pumping experiments to study the properties of these boundary states, in which ϕ_x and ϕ_y are scanned between 0.477π and 2.19π (vertical dashed lines; arrows indicate the pumping direction); see Figs 2 and 3.

(equation (1) with $b_x = 1/3$ and $b_y = 1/3$), using 7 rows and 13 columns. The inter-waveguide separation is such that the evanescent coupling between nearest-neighbour waveguides is modulated according to equation (1), with $\lambda_x = \lambda_y = 1.06\,\mathrm{cm}^{-1}$ and $\tilde{t}_x = \tilde{t}_y = 1.94\,\mathrm{cm}^{-1}$ (at a wavelength of 1,550 nm). Nevertheless, the evanescent coupling is a function of both separation and wavelength (Methods, Extended Data Fig. 1). Therefore, the resulting structure has coupling between waveguides beyond its nearest neighbours and the emulated model does not decompose into two disjoint 1D pumps. Despite this, the spectrum for the device demonstrates gap-traversing boundary states, with both edge and corner states (Fig. 1c, Methods, Extended Data Fig. 2).

The appearance of such edge phenomena results from the non-trivial 4D topology of the 2D pump. The 4D symmetry of the second Chern number bulk response generates two types of response: density-type and Lorentz-type[27–29]. The edge bands support the former and the corner modes the latter (Methods). For clarity, we explain the appearance of the topological boundary modes by studying the structure of the model described in equation (1). Because this model can be decomposed into decoupled 1D pumps, each having gaps with non-trivial first Chern numbers[5,6,8], we have the following: (a) the spectrum of the 2D pump is a Minkowski sum of the spectra of the two 1D pumps, $E = E_x + E_y$; (b) the states of the model are product states of the two independent models; and (c) the product bands are associated with second Chern numbers that are equal to the product of the individual first Chern numbers[7]. The third result leads to non-trivial bulk phenomena only when gaps remain open in the summed Minkowski spectra. Importantly, the second Chern number and the corresponding 4D symmetry of its associated bulk responses imply that pumping will occur in response to a scan of either or both pump parameters ϕ_i (Methods).

Let us now consider these properties of the model in equation (1) in an open geometry. Because each 1D pump has 1D bulk modes and zero-dimensional (0D) boundary modes, (a) and (b) above imply that the 2D pump states are grouped into three categories: (i) 2D bulk modes composed of products of 1D bulk modes; (ii) edge modes composed of products of 1D bulk modes with a 0D boundary; and (iii) corner modes that are a product of 0D boundaries. The boundary modes (cases (ii) and (iii)) support the quantized second Chern number response (Methods). The 1D edge states of the 2D system are pumped in response to a single pump parameter and map onto 3D hypersurface states in four dimensions. The 0D corner states are pumped in response to one or both pump parameters and map to 2D hypersurface states. These states highlight the hypersurface phenomena that are associated with the second Chern number.

Our device does not decompose perfectly into two 1D pumps, owing to longer-ranged hopping. Nevertheless, the bulk gaps remain open. As a result, the characterization of these gaps by non-trivial second Chern numbers implies that the bulk response must remain unchanged. The appearance of edge states that traverse the gaps as a function of the pump parameters ϕ_i supports this response in a finite-sized system. Here we probe the behaviour of these states experimentally.

The waveguide array (Fig. 1b) is fabricated using femtosecond-laser writing[30,31] in such a way that each single-mode waveguide couples evanescently to its neighbours. When light is injected into the array, it excites eigenmodes according to their spatial overlap with the input beam. The diffraction of light through the array is governed by the paraxial Schrödinger equation, $i\partial_z\psi = H(z)\psi$, in which the time-evolution coordinate t in the usual Schrödinger equation is replaced by the distance of propagation z; ψ represents the tight-binding wavefunction and $H(z)$ is the Hamiltonian. Therefore, the diffraction of light through the array mimics the time evolution of the wavefunction of a quantum particle. Consequently, time-dependent pumping means adiabatically varying ϕ_i along the waveguide axis[6,8]: $\phi_i \to \phi_i(z)$.

We demonstrate experimentally the appearance of edge modes in the structure and their behaviour under scans of the pump parameters. We start by studying a structure with straight waveguides, which is therefore invariant in z. We inject light into two different waveguides in the array: one along the left edge and one along the bottom edge. The output light is collected after a diffraction length of 15 cm. Light stays confined largely to the injected edge (it mostly excites the topological localized edge bands; Fig. 2a, b). Additionally, it spreads across the whole edge, implying dispersive bands of edge modes (such as the bands that cross the gaps in Fig. 1c), in accordance with the expected density-type response (Methods). The light stays confined to a single edge as a result of the weak coupling between states on adjoining edges; that is, the long-range coupling does not break the orientation that is associated with the two orthogonal 1D topological pumps embedded in the system. Some of the edge states (case (ii) above) that we excite have the same energies as bulk states in the open system geometry (Methods). These long-lived resonances further demonstrate that the