

result of the high accumulation rate at the site, for which reason the WAIS Divide ice core provides excellent temporal resolution capabilities for trapped gas in the ice. The firn filtering timescale is much lower than our maximum sampling rate of about 110 yr and is also substantially below the 600-yr cut-off frequency that we apply in the data splining. For all these reasons the noble gas records presented here contain no intrinsic temporal dampening element such as is known to occur in other atmospheric gas records and are (within the given uncertainties and the current understanding) a direct representation of MOT. There are, however, processes that can alter the scaling between noble gases and MOT; these are discussed and quantified in Methods subsections 'Inferring atmospheric noble gas ratios from the raw data' and 'Box model to infer MOT'.

There does exist a scenario under which our noble gas data would be blind to MOT changes: if there were a large portion of the ocean that exchanges heat with the atmosphere without exchanging gases. The corresponding water masses would be characterized by disequilibrium between temperature and dissolved noble gases, with the same magnitude of disequilibrium for all noble gases. Today, such waters seem not to exist because all deeper ocean water masses found so far contain an amount of noble gases corresponding to their temperature^{15,16} (with a tendency to noble gas undersaturation, however, caused by fast cooling and not of the same magnitude for all noble gases; see also Methods subsection 'Inferring atmospheric noble gas ratios from the raw data'). The glacial ocean circulation pattern suggested in ref. 22 could have favoured the production of such 'blind' water masses during the LGM; however, it is important to note that our data would only be affected if these water masses were completely isolated from the atmosphere while exchanging heat before sinking into the deep ocean (conceivable if there were a gas-impermeable sea ice layer through which heat could be conducted, so that the waters underneath would sink into the deep ocean without any more atmospheric contact). If the sea ice were only partly or slowly permeable for noble gases or the waters had only a very short exposure time with the atmosphere (expected if polynyas (areas of open sea surrounded by ice) were as important for deep-water formation as they are today⁵⁷), the 'blindness' would no longer exist. As soon as a slight exchange of gases occurred, Kr would come closer to equilibrium than Xe because of the faster equilibration time of Kr (similar concept as behind the fast-cooling effect¹⁵). Under such a situation our data would show a discrepancy between the MOT signal in $\delta\text{Xe}/\text{N}_2$ relative to $\delta\text{Kr}/\text{N}_2$ (because we assume constant equilibration over time; see also Methods subsection 'Inferring atmospheric noble gas ratios from the raw data') and, hence, be indicative of such a process (which is not the case). This scenario of 100% decoupling for a large portion of the ocean is conceivable under a Snowball Earth scenario, but seems very unrealistic and hypothetical for the LGM situation, because there is no indication that deep waters would form in such a way. However, further studies with state-of-the-art climate models are needed to rule out these unrealistic but not-yet-excludable effects. Note that if the LGM ocean had had such a 'blind' water mass, the transition from 'blind' to 'not blind' would have needed to happen immediately because an 'in between' state should appear as a phase of discrepancy between MOT values from $\delta\text{Xe}/\text{N}_2$ and $\delta\text{Kr}/\text{N}_2$ (which is not the case).

Inferring atmospheric noble gas ratios from the raw data. The heavy noble gas ratios we obtain from the ice core samples are highly fractionated with respect to the atmospheric value, mainly owing to gravitational fractionation in the static firn air column at the top of an ice sheet, below which the air is trapped in the ice. The depth of this firn column changes over time and is influenced by the local snow accumulation rate and temperature, among other things⁵⁸. The effective firn air depth at a specific point in time can be 'measured' by analysing stable gas isotope ratios of N_2 ($\delta^{15}\text{N}$), Ar ($\delta^{40}\text{Ar}$), Kr ($\delta^{86}\text{Kr}$) and Xe ($\delta^{132}\text{Xe}$). By combining these ratios it is also possible to resolve the minor thermal and kinetic fractionation processes that might have occurred⁵⁹. The conditions required for kinetic fractionation to occur—as described in ref. 59 (very low accumulation rate, low temperature)—do not apply to the WAIS Divide ice core drill site^{13,36}, for which reason this effect is not considered in our calculations and we consider only gravitational and thermal fractionation. With the method used in this study we obtain the atmospherically stable ratios of $\delta^{15}\text{N}$ ($^{29}\text{N}_2/^{28}\text{N}_2$), $\delta^{40}\text{Ar}$ and $\delta^{86}\text{Kr}$ ($^{86}\text{Kr}/^{86}\text{Kr}$) with a precision that enables us to resolve the thermal and gravitational fractionation processes adequately¹⁰.

In theory—knowing all the air fractionation processes occurring in the firn column—the differences between the measured isotope ratios can be used to reconstruct the thermal fractionation component using the well known thermal diffusivity parameters^{59,60}. Since we have three isotope ratio pairs but only one fractionation effect that should affect these values, the system is over-determined and we can check whether it is consistent for all possible combinations. However, any combination including $\delta^{86}\text{Kr}$ to determine the thermal component results in a temperature difference of 1.5 °C to 2 °C between the top and bottom of the firn column (referred to as the 'firn thermal gradient') for the LGM and Holocene

periods, which is unrealistic because of the stable surface temperatures during these periods¹³. About the same constant offset is found during the transition period compared to the modelled firn thermal gradients of ref. 36. If $\delta^{15}\text{N}$ and $\delta^{40}\text{Ar}$ is used, the thermal component is in rough agreement with the expectations through the whole record. We have thoroughly tested our method for possible analytical artefacts that could fractionate or contaminate $\delta^{86}\text{Kr}$, without success. Also, if there were such an artefact, we would have corrected for it to a large extent given that we reference our ice sample measurements to modern air samples, which are measured on the concept of identical treatment¹⁰.

To circumvent $\delta^{86}\text{Kr}$ in a first step, we use an independent scenario of firn thermal gradient based on ref. 36. After applying this scenario to the data we follow the approach of ref. 9 and use $\delta^{40}\text{Ar}$ to obtain the gravitational correction component for all other elements. $\delta^{40}\text{Ar}$ has the smallest analytical uncertainty per mass unit—1.5 per meg (that is, $1.5 \times 0.001\%$) on average—and hence, provides the highest possible accuracy for this largest, but well defined, correction factor. The isotope data that are corrected using this approach (Extended Data Fig. 1) show clearly that $\delta^{86}\text{Kr}$ is depleted relative to $\delta^{15}\text{N}$ and $\delta^{40}\text{Ar}$ (referred to as the 'Kr anomaly'), which is the reason why the firn thermal gradients based on $\delta^{86}\text{Kr}$ mentioned above turn out wrongly. We believe this Kr anomaly is a true signal in the trapped ice, probably caused by a firn fractionation mechanism that is yet unknown. Further investigations from other sites are needed for a better understanding of the mechanism behind it.

The Kr anomaly seems mainly to consist of a fairly constant offset relative to the other isotope of −56 per meg without any obvious trends and changes over time (Extended Data Fig. 1). This indicates that the underlying mechanism is fairly stable over time, for which reason we correct the $\delta^{86}\text{Kr}$ raw data by this average offset. If we use the corrected $\delta^{86}\text{Kr}$ values and compare the firn thermal gradients based on the different isotope pairs again, the results are now consistent with each other (the gradients involving $\delta^{86}\text{Kr}$ do now provide realistic and comparable values, as do the values based on $\delta^{15}\text{N}$ and $\delta^{40}\text{Ar}$ for the whole record period).

Therefore we derived a second scenario for firn thermal gradients based on the measured isotopes (including the corrected $\delta^{86}\text{Kr}$) by averaging the gradients derived from the three possible isotope pairs (see Extended Data Fig. 1b). This data-based scenario is independent of the first model-based scenario of ref. 36, and together the scenarios represent the uncertainty range associated with the thermal-correction component for our study. We account for this uncertainty range in our final MOT record by combining the 3,000 Monte Carlo MOT realizations of each scenario and propagate this uncertainty element into our final record (see more details in Methods subsection 'Box model to infer MOT'). In general, the uncertainty associated with this thermal correction is comparable to the one originating from the analytical uncertainties. The analytical uncertainties translate into about 0.2 °C uncertainty in MOT (see Methods subsection 'Potential biases in noble gases from ice samples as a proxy for MOT') whereas the effect of the two scenarios on our MOT estimate is within about 0.25 °C (corresponding to a 1 °C firn thermal gradient difference between the scenarios).

We cannot exclude the possibility that the underlying mechanism of the Kr anomaly also affects to some extent the gas ratios we use to reconstruct MOT ($\delta\text{Kr}/\text{N}_2$, $\delta\text{Xe}/\text{N}_2$, $\delta\text{Xe}/\text{Kr}$). As seen in Extended Data Fig. 2, the reconstructed atmospheric noble gas ratios are depleted during the Holocene period, which translates into an average Holocene MOT of −0.36 °C below present values, as seen in our MOT record in the main text (Fig. 2). This Holocene MOT 'offset' is more than the observed ocean warming since the industrialization¹ and, hence, would suggest that there was substantial MOT warming already before industrialization. This 'offset', however, could also be an artefact because the mechanism behind the Kr anomaly might also deplete $\delta\text{Kr}/\text{N}_2$, $\delta\text{Xe}/\text{N}_2$ and $\delta\text{Xe}/\text{Kr}$. Since the Kr anomaly seen in Extended Data Fig. 1 is fairly constant over time, the effect on $\delta\text{Kr}/\text{N}_2$, $\delta\text{Xe}/\text{N}_2$ and $\delta\text{Xe}/\text{Kr}$ is also expected to be constant over time, for which reason we argue that the mechanism behind the Kr anomaly produces—if any at all—a constant bias to our MOT record of perhaps −0.36 °C, but does not change the relative changes within our record. Therefore, relative changes, such as the Holocene–LGM MOT difference or the MOT trends of the different periods, are not affected by this potential bias and represent the effective changes in MOT. However, the readers have to be careful in interpreting the absolute values we derive from our records, because of the potential bias described here. Nevertheless, we do not apply any offset correction to our MOT record, as we do not feel confident to do so at present.

Despite the fact that the conditions at the WAIS Divide site do not fit the conditions required for kinetic fractionation as described in ref. 59, we tested this model and interpreted the Kr anomaly as caused by kinetic fractionation and used the model to scale the anomaly to the elemental ratios. With this approach, the resulting MOT records for the Late Holocene are found to be warmer than today by about 0.25 °C and not consistent with each other for the LGM period. Accordingly, the mechanism behind our gas fractionation must be somewhat different to kinetic fractionation.