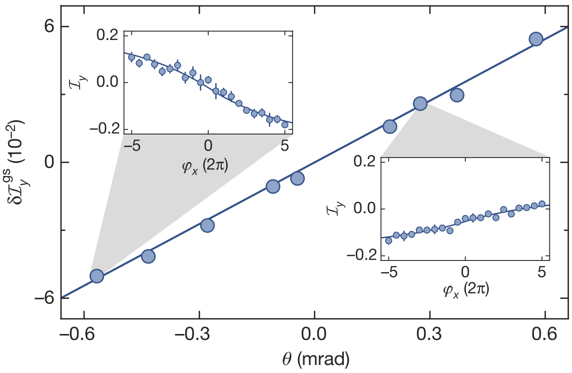

Figure 4 | Scaling of the 4D-like response with the tilt angle θ. The linear dependence on θ reveals the nonlinear character of the response, demonstrating that it is induced by two independent perturbations, $\partial\varphi_x/\partial t$ and θ. The slope $\delta\mathcal{I}_y^{gs}$ is determined as a function of θ at $\varphi_y^{(0)} = 0.500(5)\pi$ by measuring the double-well imbalance when pumping along x, as described in Fig. 3 and using the same lattice depths. The solid line shows the slope that is expected for an ideal system. The fit errors for $\delta\mathcal{I}_y^{gs}$ are smaller than the size of the data points and the insets show two examples of the measurement of $\mathcal{I}_y(\varphi_x)$ (for the values of θ indicated by the grey shading), as in Fig. 3a.

We verify this by measuring the peak slope $\delta\mathcal{I}_y^{gs}$ at $\varphi_y^{(0)} = \pi/2$ as a function of θ (Fig. 4). Doing so also provides another way of obtaining the second Chern number, by determining the slope of $\delta\mathcal{I}_y^{gs}(\theta)$ (see Methods). This linear fit gives $\nu_2^{exp} = 1.01(8)$, where the error is determined as described above. Furthermore, we confirm that the peak slope at fixed θ scales with the depth of the short y lattice $V_{s,y}$ as expected (Extended Data Fig. 1, Methods). In particular, the direction of the nonlinear response is independent of $V_{s,y}$, indicating its robustness against perturbations of the system.

In conclusion, we present an observation of a dynamical 4D quantum Hall effect, opening up a route to studying higher-dimensional quantum Hall physics experimentally. Extending our work, additional density-type nonlinear responses that are implied by the intrinsic 4D symmetry of a 2D charge pump can be measured[6]. By adding a spin-dependent Yang–Mills gauge field, a dynamical version of the time-reversal-symmetric 4D quantum Hall effect, which exhibits a ground state with SO(5) symmetry, could be realized[6]. Including interactions may yield intriguing fractional phases that originate in the 4D fractional quantum Hall effect[4], similarly to previous proposals for 1D charge pumps[28], and might enable the study of open questions in the context of Floquet engineering[15]. Going beyond the limit of weak perturbations, quantized electric quadrupole moments could be observed in spatially frustrated systems with $\theta = \pi/4$ (ref. 29). Furthermore, a quantum Hall system with four extended dimensions might be realized with cold atoms[20] using recently demonstrated techniques for creating synthetic dimensions[18,19]. In finite systems, this would permit the observation of boundary phenomena such as isolated Weyl points[30]. Ultimately, the ability to experimentally realize 4D quantum Hall systems could provide insight into lattice quantum chromodynamics models based on the Yang–Mills theory[7], and even quantum gravity[4].

We note that, simultaneously with this work, complementary results on topological edge states in 2D photonic pumps have been obtained[31].

Online Content Methods, along with any additional Extended Data display items and Source Data, are available in the online version of the paper; references unique to these sections appear only in the online paper.

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