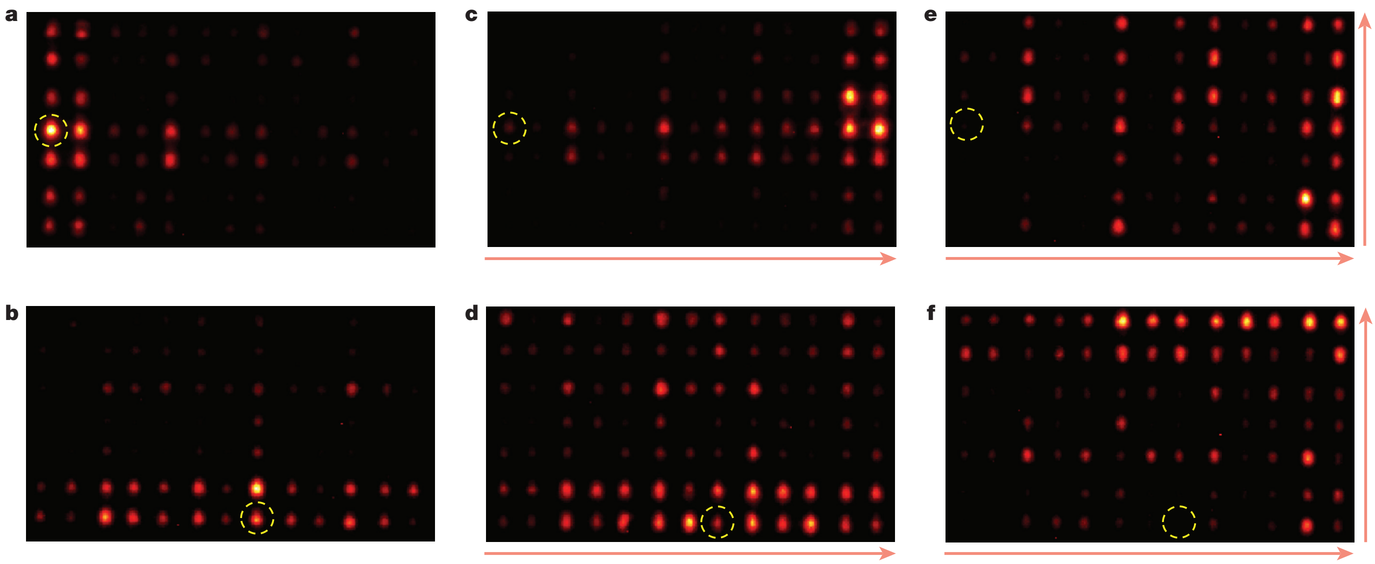

Figure 2 | Edge-to-edge pumping. Images of the output facet of waveguide arrays after $z = 15$ cm of propagation are shown. **a**, **b**, Device with no pumping, corresponding to a model with $\phi_x = \phi_y = 0.477\pi$ (see Fig. 1). Light that is injected at the centre of the left (**a**) or bottom (**b**) edges excites the topological edge bands and spreads out along the edge. **c**, **d**, Pumping of ϕ_x (from 0.477π to 2.19π while ϕ_y is held constant at 0.477π) causes the light injected at the left edge to be pumped to the right (**c**); no such pumping is observed when light is injected at the bottom edge (**d**). **e**, **f**, When ϕ_x and ϕ_y are simultaneously pumped (from 0.477π to 2.19π), light injected at the left (**e**) and bottom (**f**) edges pumps from left to right and bottom to top, respectively. Light in the bulk arises from imperfect coupling to edge states and from deviations from adiabaticity. The yellow dashed circles indicate the injection sites at the input facet ($z = 0$) and the red arrows indicate the direction of pumping. These results demonstrate that edge bands exist in the structure and appear on opposite sides of the device as a function of the pump parameters, in accordance with the density-type bulk response that is implied by the 4D Hall-type band structure of the system.

long-range coupling is a small perturbation of the decoupled model in equation (1).

Having established that we can excite the edge modes of the 2D pump, we demonstrate their behaviour under scans of the pump parameters ϕ_i. We implement edge pumping by allowing the positions of the waveguides to 'wiggle' by varying ϕ_i as a function of z (Fig. 1b). We vary these pump parameters within the range $[0.477\pi, 2.19\pi]$ because localized edge modes exist at these values (a full pumping cycle is not necessary to observe edge pumping from one side of the system to the other). We fabricate separate arrays that correspond to two scenarios: (1) pumping in only the x direction; and (2) pumping in both the x and y directions. In case (1), we see that when light is injected at the left edge, it is pumped to the right edge (Fig. 2c); however, when it is injected at the bottom, it is not pumped to the top because ϕ_y is not pumped (Fig. 2d). In case (2), we observe that the edge states pump both from left to right (Fig. 2e) and bottom to top (Fig. 2f). We injected light with several different input wavefunctions along the edge in question (including single and double waveguide inputs), which resulted in different amounts of overlap of the input wavefunction with each of the edge bands; clear pumping was observed in each case. These results show that an electromotive force applied in the v and w directions induces pumping of edge bands from one 3D (v, w, y) hyperplane to the opposite one in the x direction, and from one 3D (v, w, x) hyperplane to the opposite one in the y direction, as implied by the 4D Hall bulk density-type response (Methods).

We examine the pumping of states at the corners of the arrays for the same range of ϕ_x and ϕ_y as for edge states. The presence of the corner modes (black in Fig. 1c) support the Lorentz-type bulk response (Methods). Depending on the values of ϕ_x and ϕ_y, the corner modes can either be in the bandgap or overlap with bulk modes where they can hybridize to form long-lived resonances. In the experiment, the bottom-left-corner mode is directly excited and pumped along the bottom edge, in conjunction with it being the boundary mode of the 1D pump that crosses edge to edge (Fig. 3a, b). Interestingly, when we scan ϕ_x and ϕ_y simultaneously, the bottom-left-corner mode is pumped mostly to the top-right corner (Fig. 3c) despite any hybridization with bulk modes. Such diagonal pumping under a concurrent ϕ_i scan agrees with the 4D symmetry of the second Chern number bulk response, that is, with the Lorentz-type transverse response (Methods). The photonic diagonal pumping through bulk bands is expected in the decoupled

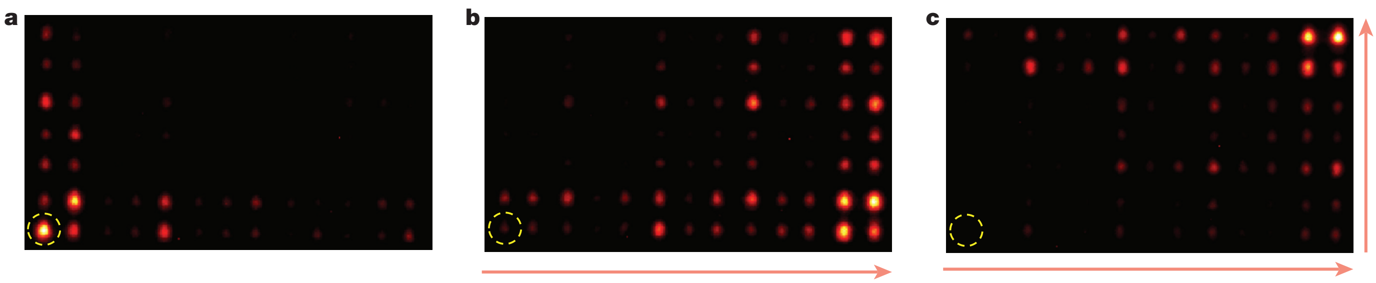

Figure 3 | Corner-to-corner pumping. Images and devices are similar to those described in Fig. 2. **a**, With no pumping, so light stays confined to the corner. **b**, Light is pumped from the bottom-left corner to the bottom-right corner via ϕ_x. **c**, When ϕ_x and ϕ_y are both pumped, the corner state is pumped from bottom-left to top-right. The corner state passes through the bulk band and remains localized because it is a long-lived resonance, not in the bandgap (Methods). Its appearance on the diagonally opposite corner is in accordance with the Lorentz-type response that is implied by the 4D Hall-type band structure of the system.