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Photonic topological boundary pumping as a probe of 4D quantum Hall physics

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When a two-dimensional (2D) electron gas is placed in a perpendicular magnetic field, its in-plane transverse conductance becomes quantized; this is known as the quantum Hall effect[1]. It arises from the non-trivial topology of the electronic band structure of the system, where an integer topological invariant (the first Chern number) leads to quantized Hall conductance. It has been shown theoretically that the quantum Hall effect can be generalized to four spatial dimensions[2–4], but so far this has not been realized experimentally because experimental systems are limited to three spatial dimensions. Here we use tunable 2D arrays of photonic waveguides to realize a dynamically generated four-dimensional (4D) quantum Hall system experimentally. The inter-waveguide separation in the array is constructed in such a way that the propagation of light through the device samples over momenta in two additional synthetic dimensions, thus realizing a 2D topological pump[5–8]. As a result, the band structure has 4D topological invariants (known as second Chern numbers) that support a quantized bulk Hall response with 4D symmetry[7]. In a finite-sized system, the 4D topological bulk response is carried by localized edge modes that cross the sample when the synthetic momenta are modulated. We observe this crossing directly through photon pumping of our system from edge to edge and corner to corner. These crossings are equivalent to charge pumping across a 4D system from one three-dimensional hypersurface to the spatially opposite one and from one 2D hyperedge to another. Our results provide a platform for the study of higher-dimensional topological physics.

Topology manifests naturally in solid-state systems. In insulators, electrons fill electronic states below the bandgap of the system. These states can be mapped mathematically onto abstract shapes that are characterized by a topological invariant. The realization that these topological invariants manifest as quantized bulk responses, and through corresponding topologically protected boundary states, has revolutionized our understanding of material properties. These phenomena have been explored in several fields in systems beyond solid-state materials, including photonic[6,8–13] and ultracold atomic[14–17] systems.

The introduction of topology into photonics[9] has opened up many avenues of research. Much of this research has focused on the experimental observation of topologically protected edge states in systems such as photonic crystals in the microwave domain[10,13], as well as arrays of waveguides[6,8,11] and integrated ring resonators at optical frequencies[12]. In these systems, dielectric structures act as lattices for light, leading to topological 2D photonic bands. Beyond two dimensions, experiments with three-dimensional (3D) lattices have unveiled topological features[18] such as Weyl points[19,20].

The study of topological phases can be defined and understood mathematically beyond three dimensions, with a hallmark example being the 4D quantum Hall effect[2–4,7]. In 2D quantum Hall systems, energy bands are characterized by the first Chern number, which quantizes the Hall conductance and therefore counts one-dimensional (1D) chiral edge states in the system. In 4D systems, energy bands are characterized by another topological invariant—the second Chern number[2–4,7,21–24]. Similarly to the 2D case, the 4D invariant manifests through an additional quantized bulk response with 4D hypersurface phenomena. Until recently, the latter seemed only of theoretical interest because its realization requires four spatial dimensions. The flexibility of atomic and photonic systems, however, has inspired proposals to include synthetic dimensions to realize higher-dimensional topological physics[25–28].

The concept of topological pumps lends itself well to synthetic dimensions and higher-dimensional physics. Consider a family of 1D systems parameterized by a momentum in a synthetic orthogonal dimension. This momentum is the pump parameter that maps the 1D pump to the 2D quantum Hall system with a first Chern number[6,8]. The topological bulk response of the 1D pump matches that of the 2D quantum Hall effect: varying the pump parameter generates an electromotive force that pushes an integer number of charges per pump cycle across the physical dimension[5]. 1D pumps have recently been demonstrated in cold atom[16,17] and photonic[6,8] experiments.

A 2D topological pump can be subject to two pump parameters, corresponding to a 4D quantum Hall system[7]. In its simplest form, a 4D quantum Hall system is the sum of two 2D quantum Hall systems in disjoint planes[7,27,28], residing in the direct product space associated with the individual models. Correspondingly, a 2D topological pump manifests as the sum of two 1D pumps on orthogonal axes[7]. Here we consider 'off-diagonal' pumps in which the hopping is modulated as a function of the pump parameters[6,8]; that is, we study a 2D tight-binding model of particles that hop on a lattice described by the Hamiltonian (Fig. 1a)

$$H=\sum_{x,y} t_x(\phi_x)c^\dagger_{x,y}c_{x+1,y}+t_y(\phi_y)c^\dagger_{x,y}c_{x,y+1}+\text{h.c.} \tag{1}$$

where $c_{x,y}$ annihilates a particle at site (x, y); $t_i(\phi_i)=\tilde{t}_i+\lambda_i\cos(2\pi b_i i+\phi_i)$, with $i\in\{x, y\}$, are modulated hopping amplitudes in the i direction, with bare hopping $\tilde{t}_i$ and modulation λ_i amplitudes. The modulation frequencies b_i are mapped in four dimensions to two magnetic fields in the x–v and y–w planes[7]. The pump parameters ϕ_x and ϕ_y correspond to momenta in the v and w directions, respectively; that is, their modulation dynamically generates electric-field perturbations in these directions. Considering that the pump parameters correspond to additional synthetic dimensions, we characterize bandgaps of the 2D pump with non-trivial second Chern numbers that manifest as a quantized bulk response with 4D symmetry[7].

We realize such a 2D topological pump using arrays of coupled waveguides (Fig. 1b). Each array is constructed to emulate the 2D pump

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