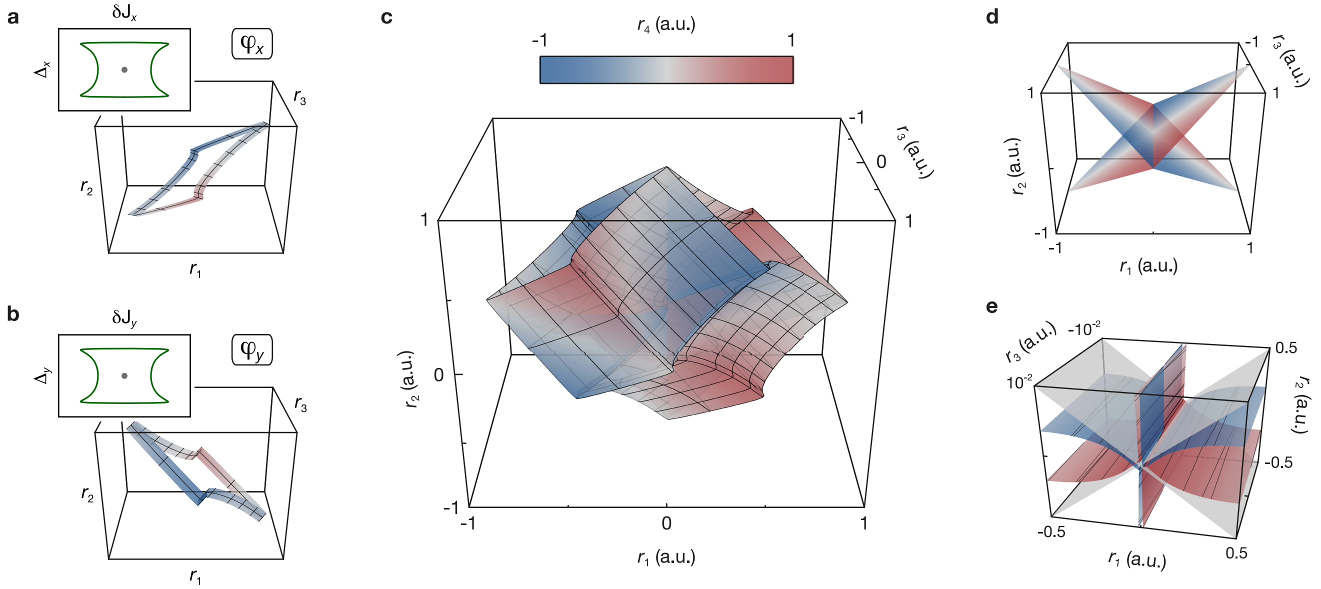

Extended Data Figure 2 | Pump cycle of the 2D topological charge pump. The 4D tight-binding parameter space (δJ_x, Δ_x, δJ_y, Δ_y) is visualized using the transformation in equation (4). **a**, Changing the pump parameter φ_x leads to a periodic modulation of δJ_x and Δ_x along a closed trajectory, as shown in the inset for a full pump cycle $\varphi_x = 0 \to 2\pi$. This pump path (green) encircles the degeneracy point at the origin (grey), at which the gap between the two lowest subbands of the Rice–Mele model closes. The surface in the main plot shows the same trace transformed according to equation (4) and with $\varphi_y \in [0.46\pi, 0.54\pi]$. The spacing of the mesh grid illustrating φ_x is $\pi/10$. **b**, For a given φ_x, a large system simultaneously samples all values of φ_y. This corresponds to a closed path in δJ_y–Δ_y parameter space, in which a singularity also occurs at the origin (inset). The main plot shows the transformed path for $\varphi_x \in [0.46\pi, 0.54\pi]$. **c**, In a full pump cycle, such a system therefore covers a closed surface in the 4D parameter space by translating the path shown in **b** along the trajectory from **a**. **d**, In the transformed parameter space, the singularities at ($\delta J_x = 0$, $\Delta_x = 0$) and ($\delta J_y = 0$, $\Delta_y = 0$) correspond to two planes that touch at the origin. **e**, Cut around $r_3 = 0$ showing both the pump path from **c** (red/blue) and the singularities from **d** (grey). Whereas they intersect in the 3D space (r_1, r_2, r_3), the value of r_4 is different on both surfaces and the 4D pump path thus fully encloses the degeneracy planes.