Metric Space — Literal Reading

Definition

A **metric space** is a pair ((U, D)) where

$$D : U \times U \to \mathbb{R}_{\geq 0}$$

is a *distance function* satisfying, for all $x, y, z \in U$:

1. $D(x, y) \geq 0$
2. $D(x, x) = 0$
3. $x \neq y \implies D(x, y) > 0$
4. $D(x, y) = D(y, x)$
5. $D(x, z) \leq D(x, y) + D(y, z)$

Symbols and Components

Symbol	Meaning	Type / Role
U	The **set of all objects** we can measure distances between. Examples: $\mathbb{R}$, $\mathbb{R}^n$, or a set of strings, points, images, etc.	Set
D	The **distance function**. Takes two elements of U and returns a nonnegative real number.	Function $D : U \times U \to \mathbb{R}_{\geq 0}$
x, y, z	Arbitrary **elements (points)** in U. Used universally ("for all").	Variables
$\mathbb{R}$	The set of **real numbers**.	Standard symbol for real line
$\mathbb{R}_{\geq 0}$	The set of **nonnegative real numbers**: $\{r \in \mathbb{R} \mid r \geq 0\}$.	Codomain of D
$U \times U$	The **Cartesian product** of U with itself — all ordered pairs (x, y) with $x, y \in U$.	Domain of D
"for all $x, y, z \in U$"	Universal quantifier — conditions hold **for every triple of points** in U.	Logical quantification
$D(x, z) \leq D(x, y) + D(y, z)$	**Triangle inequality** — direct path no longer than detour via y.	Core axiom

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Function Type Summary

$$D : U \times U \longrightarrow \mathbb{R}_{\geq 0}$$

- **Input:** Two elements $x, y \in U$
- **Output:** A real number $r = D(x, y) \geq 0$
- **Meaning:** "The distance between x and y"