Strassen's Matrix Multiplication

$$A=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \times B=\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = C=\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$$

```
for(i=0; i<n; i++)
{
    for(j=0; j<n; j++)
    {
        c[i,j]=0;
        for(k=0;k<n; k++)
            c[i,j]+=A[i,k]*B[k,j];
```

$O(n^3)$

Strassen's Matrix Multiplication

$$\left[\begin{array}{cc|cc} a_{11} & a_{12} & a_{13} & a_{14} \\ & A_{11} & A_{12} & \\ a_{21} & a_{22} & a_{23} & a_{24} \\ \hline a_{31} & a_{32} & a_{33} & a_{34} \\ & A_{21} & A_{22} & \\ a_{41} & a_{42} & a_{43} & a_{44} \end{array}\right] \quad B=\left[\begin{array}{cc|cc} b_{11} & b_{12} & b_{13} & b_{14} \\ & B_{11} & B_{12} & \\ b_{21} & b_{22} & b_{23} & b_{24} \\ \hline b_{31} & b_{32} & b_{33} & b_{34} \\ & B_{21} & B_{22} & \\ b_{41} & b_{42} & b_{43} & b_{44} \end{array}\right]$$

```
Algorithm MM(A,B,n)
{
  if(n ≤ 2)
  {
    C = 4 formulas
    :
  }
  else
  {
    mid .. n/2
    MM(A11,B11,n/2) + MM(A12,B21,n/2)
    MM(A11,B12,n/2) + MM(A12,B22,n/2)
    MM(A21,B11,n/2) + MM(A22,B21,n/2)
    MM(A21,B12,n/2) + MM(A22,B22,n/2)
  }
```

$$T(n)=\begin{cases} 1 & n\le 2 \\ 8\,T(n/2)+n^2 & n>2 \end{cases}$$

Strassen's Method.

$$\underset{A}{\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}} \quad \underset{B}{\begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}}$$

$$P = (A_{11} + A_{22})(B_{11} + B_{22})$$

$$Q = B_{11}(A_{21} + A_{22})$$

$$R = A_{11}(B_{12} - B_{22})$$

$$S = A_{22}(B_{21} - B_{11})$$

$$T = B_{22}(A_{11} + A_{12})$$

$$U = (A_{21} - A_{11})(B_{11} + B_{12})$$

$$V = (B_{21} + B_{22})(A_{12} - A_{22})$$

$$\begin{array}{ccc} & T \oplus & \\ \ominus\, S & \begin{bmatrix} 11 & 12 \\ 21 & 22 \end{bmatrix} & R \,\ominus \\ & Q \oplus & \end{array}$$

$B_{11}\ A_{11}\ A_{22}\ B_{22}$