

### 1. What is pseudocode?

Pseudocode is a high-level description of a computer program or algorithm that uses natural language constructs resembling the structure of a programming language, but is not bound by the syntax of any specific programming language. It's often used during the initial stages of software development to outline the logic and flow of a program without getting bogged down in the details of actual code implementation.

### 2. What are the types of algorithm efficiencies?

Algorithm efficiency refers to how well an algorithm utilizes computational resources, such as time and space, to solve a problem. There are several types of algorithm efficiencies:

**Time Efficiency:** This refers to how quickly an algorithm can solve a problem as a function of the size of the input. Time complexity is often measured in terms of the "Big O" notation, which provides an upper bound on the growth rate of the algorithm's running time as the input size increases.

**Space Efficiency:** This concerns the amount of memory or storage space an algorithm requires to solve a problem, again as a function of the input size. Space complexity is also commonly measured using Big O notation, indicating the maximum amount of memory the algorithm will consume.

**Worst-case Efficiency:** This describes the algorithm's performance when given the worst possible input. It provides an upper bound on the time or space requirements for any input size.

**Best-case Efficiency:** This describes the algorithm's performance when given the best possible input. It provides a lower bound on the time or space requirements for any input size.

**Average-case Efficiency:** This considers the algorithm's performance averaged over all possible inputs. It provides a more realistic assessment of an algorithm's efficiency under typical conditions but can be more challenging to analyze than worst or best cases.

### 3. What is the complexity of Quick sort?

The average-case time complexity of QuickSort is  $O(n \log n)$ , where "n" is the number of elements in the array being sorted. This average time complexity arises from the partitioning process, which divides the array into smaller subarrays, followed by recursively sorting these subarrays.

However, it's important to note that QuickSort's worst-case time complexity is  $O(n^2)$ . This worst-case scenario occurs when the pivot selection consistently leads to unbalanced partitions, such as when the smallest or largest element is always chosen as the pivot. To mitigate this, various strategies for pivot selection (like choosing the median of three randomly selected elements) or employing randomized algorithms can be used, typically reducing the likelihood of encountering the worst-case scenario.

In practice, QuickSort is often very efficient and outperforms other sorting algorithms for large datasets, especially when implemented with optimizations to handle the worst-case scenarios.

### 4. What is complexity of Binary Search?

The time complexity of binary search is  $O(\log n)$ , where "n" represents the number of elements in the sorted array being searched.

Binary search works by repeatedly dividing the sorted array in half and comparing the target value with the middle element. Based on this comparison, it either narrows down the search to the lower or upper half of the array, effectively reducing the search space by half with each iteration. This logarithmic time complexity arises from the fact that the search space is halved in each step, leading to a very efficient search process, especially for large datasets.

#### 5. Explain important properties of B-Tree.

B-Trees are balanced tree data structures that are commonly used in databases and file systems for efficient storage and retrieval of data. They possess several important properties that make them well-suited for these purposes:

**Balanced Structure:** B-Trees are balanced trees, meaning that the height of the tree remains relatively constant regardless of the number of elements stored in it. This balance is achieved by imposing specific rules on the structure of the tree and performing balancing operations (like splitting and merging nodes) during insertion and deletion.

**Ordered Data:** In a B-Tree, the keys stored in each node are ordered. This ordering facilitates efficient searching, insertion, and deletion operations, as it enables binary search-like algorithms within each node.

**Multiple Keys and Child Pointers:** Unlike binary search trees, which typically have two child pointers per node, B-Trees can have multiple keys and child pointers per node. This feature allows B-Trees to store a large number of keys in each node, reducing the height of the tree and improving access times.

**Fan-Out Factor:** B-Trees have a parameter called the "order" or "fan-out factor," denoted by "t." This parameter determines the minimum and maximum number of keys and child pointers that each node can have. Typically, a B-Tree of order "t" can have between "t-1" to "2t-1" keys and "t" to "2t" child pointers in each node.

**Efficient Disk Access:** B-Trees are optimized for disk-based storage systems. By ensuring a large number of keys and child pointers per node, B-Trees minimize the number of disk accesses required to retrieve or modify data, as each node can store a significant amount of data that fits into a single disk block.

**Self-Balancing Property:** B-Trees maintain their balance through self-balancing operations performed during insertions and deletions. These operations ensure that the tree remains balanced, preventing degradation in performance due to uneven distribution of keys.

**High Performance:** Due to their balanced structure and efficient use of disk access, B-Trees offer high-performance operations for search, insertion, deletion, and range queries. They are widely used in database management systems and file systems to provide fast and reliable data access.

Overall, the combination of these properties makes B-Trees a versatile and efficient data structure for organizing and managing large volumes of data in disk-based storage systems.

#### 6. What is the complexity of Selection sort?

The time complexity of Selection Sort is  $O(n^2)$ , where "n" represents the number of elements in the array being sorted.

In Selection Sort, the algorithm repeatedly finds the minimum (or maximum) element from the unsorted part of the array and moves it to the beginning (or end) of the sorted portion of the array. This process is repeated until the entire array is sorted.

The outer loop iterates "n" times to select each element, and for each iteration, the inner loop iterates (n-1), (n-2), (n-3), ..., 2, 1 times to find the minimum element among the remaining unsorted elements. This results in a total of  $(n-1) + (n-2) + (n-3) + \dots + 2 + 1 = (n^2 - n) / 2$  comparisons and swaps in the worst-case scenario.

Although Selection Sort is straightforward to implement, its time complexity of  $O(n^2)$  makes it inefficient for large datasets compared to more efficient sorting algorithms like Merge Sort or Quick Sort.

7. Write and explain merge sort algorithm using divide and conquer strategy. Also analyze the complexity.

Merge Sort is a popular sorting algorithm that utilizes the "divide and conquer" strategy to sort an array or list of elements efficiently. The basic idea behind Merge Sort is to recursively divide the input array into smaller subarrays until each subarray contains only one element. Then, it merges adjacent subarrays while ensuring that the elements are in sorted order. This merging process continues until the entire array is sorted.

Here's a step-by-step explanation of the Merge Sort algorithm using the divide and conquer strategy:

**Divide:** The input array is recursively divided into two halves until each half contains only one element. This is done by continuously splitting the array into smaller subarrays.

**Conquer:** Once the subarrays contain only one element, they are considered sorted by default. This step involves recursively sorting each of these smaller subarrays.

**Combine (Merge):** After all subarrays are sorted, they are merged back together in a sorted manner. This merging process combines two adjacent sorted subarrays into a single sorted subarray. During merging, elements from the two subarrays are compared and placed in the correct order.

**Repeat:** Steps 1 to 3 are repeated recursively until the entire array is sorted.

Here's a high-level overview of the Merge Sort algorithm:

```
MergeSort(arr[], left, right)

if left < right

    1. Find the middle point to divide the array into two halves:

        middle = (left + right) / 2

    2. Call MergeSort for the first half:

        MergeSort(arr, left, middle)

    3. Call MergeSort for the second half:

        MergeSort(arr, middle + 1, right)

    4. Merge the two halves sorted in step 2 and 3:

        Merge(arr, left, middle, right)
```

The Merge function takes care of merging two sorted subarrays into a single sorted array. It involves comparing elements from both subarrays and merging them into a temporary array, which is then copied back to the original array.

Merge Sort has a time complexity of  $O(n \log n)$  in all cases (worst-case, average-case, and best-case), making it one of the most efficient comparison-based sorting algorithms. Additionally, it is a stable sorting algorithm, meaning that the relative order of equal elements is preserved during sorting.

8. Write down Prim's algorithm and analyze the complexity.

Prim's algorithm is a greedy algorithm used to find the minimum spanning tree (MST) of a connected, undirected graph. The MST of a graph is a subset of its edges that forms a tree and includes all the vertices of the graph while minimizing the total weight of the edges.

Here's the pseudocode for Prim's algorithm:

```
Prim(Graph G, Vertex start):
```

```
    Initialize an empty set MST to store the edges of the minimum spanning tree
```

```
    Initialize a priority queue (minHeap) to store vertices with their respective key values
```

```
    Initialize a boolean array visited[] to keep track of visited vertices, initially all false
```

```
        Set the key value of all vertices to infinity except for start, which is set to 0
```

```
    Insert start into minHeap with key value 0
```

```
    while minHeap is not empty:
```

```
        Extract the vertex u with the minimum key value from minHeap
```

```
        Mark u as visited
```

```
            for each neighbor v of u:
```

```
                if v is not visited and the weight of edge (u, v) is less than v's key value:
```

```
                    Update v's key value to the weight of edge (u, v)
```

```
                    Insert v into minHeap with updated key value
```

```
                    Update the parent of v to u in MST
```

```
    Return MST
```

Now, let's analyze the time complexity of Prim's algorithm:

The initialization of data structures (sets, arrays, priority queue) takes  $O(V)$  time, where  $V$  is the number of vertices in the graph.

The main loop of the algorithm runs for  $O(V)$  iterations because each vertex is visited once.

Inside the loop, the extraction of the minimum key from the priority queue takes  $O(\log V)$  time.

For each vertex, the algorithm may potentially update the key value of all its adjacent vertices, which requires  $O(E)$  operations in total, where  $E$  is the number of edges in the graph.

Insertion and updating the priority queue can take up to  $O(\log V)$  time per operation.

Therefore, the overall time complexity of Prim's algorithm is  $O(V \log V + E \log V)$ , which can be simplified to  $O((V + E) \log V)$  using the fact that in a connected graph,  $E \geq V - 1$ . This complexity makes Prim's algorithm very efficient, especially for dense graphs where  $E$  is close to  $V^2$ .

9. State knapsack problem. Give an algorithm for knapsack problem using greedy strategy.

The Knapsack Problem is a classic optimization problem where you have a set of items, each with a weight and a value, and you want to maximize the total value while keeping the total weight within a certain limit (the capacity of the knapsack). The greedy strategy is not generally optimal for the knapsack problem, but it can be used in certain cases where items have a value-to-weight ratio, and the goal is to maximize the value-to-weight ratio of the items chosen. This strategy is known as the "Fractional Knapsack Problem."

Here's the algorithm for the Fractional Knapsack Problem using a greedy approach:

Calculate the value-to-weight ratio for each item:  $\text{value}/\text{weight}$ .

Sort the items in descending order of their value-to-weight ratio.

Initialize the total value of items in the knapsack,  $\text{totalValue}$ , to 0.

Initialize the total weight of items in the knapsack,  $\text{totalWeight}$ , to 0.

Iterate through the sorted items:

If adding the entire item to the knapsack doesn't exceed the capacity:

Add the entire item to the knapsack.

Update  $\text{totalValue}$  and  $\text{totalWeight}$ .

If adding the entire item would exceed the capacity:

Add a fraction of the item to the knapsack to fill the remaining capacity.

Update  $\text{totalValue}$  and  $\text{totalWeight}$ .

Break the loop since the knapsack is full.

Return  $\text{totalValue}$  as the maximum value that can be obtained.

Here's a Python implementation of the fractional knapsack problem using a greedy approach:

```
def fractional_knapsack(items, capacity):  
    # Calculate value-to-weight ratio for each item  
    for item in items:  
        item['ratio'] = item['value'] / item['weight']  
  
    # Sort items by value-to-weight ratio in descending order  
    items.sort(key=lambda x: x['ratio'], reverse=True)  
  
    total_value = 0
```

```
total_weight = 0

for item in items:
    if total_weight + item['weight'] <= capacity:
        # Add the entire item to the knapsack
        total_value += item['value']
        total_weight += item['weight']
    else:
        # Add a fraction of the item to fill the remaining capacity
        fraction = (capacity - total_weight) / item['weight']
        total_value += fraction * item['value']
        total_weight += fraction * item['weight']
        break

return total_value

# Example usage:
items = [{'weight': 10, 'value': 60}, {'weight': 20, 'value': 100}, {'weight': 30, 'value': 120}]
capacity = 50
print("Maximum value:", fractional_knapsack(items, capacity))
```

This implementation returns the maximum value that can be obtained within the given capacity.