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FUZZY DECISION MAKING

15.1 GENERAL DISCUSSION

Making decisions is undoubtedly one of the most fundamental activities of human beings. We all are faced in our daily life with varieties of alternative actions available to us and, at least in some instances, we have to decide which of the available actions to take. The beginnings of decision making, as a subject of study, can be traced, presumably, to the late 18th century, when various studies were made in France regarding methods of election and social choice. Since these initial studies, decision making has evolved into a respectable and rich field of study. The current literature on decision making, based largely on theories and methods developed in this century, is enormous.

The subject of decision making is, as the name suggests, the study of how decisions are actually made and how they can be made better or more successfully. That is, the field is concerned, in general, with both descriptive theories and normative theories. Much of the focus in developing the field has been in the area of management, in which the decision-making process is of key importance for functions such as inventory control, investment, personnel actions, new-product development, and allocation of resources, as well as many others. Decision making itself, however, is broadly defined to include any choice or selection of alternatives, and is therefore of importance in many fields in both the "soft" social sciences and the "hard" disciplines of natural sciences and engineering.

Applications of fuzzy sets within the field of decision making have, for the most part, consisted of fuzzifications of the classical theories of decision making. While decision making under conditions of risk have been modeled by probabilistic decision theories and game theories, fuzzy decision theories attempt to deal with the vagueness and nonspecificity inherent in human formulation of preferences, constraints, and goals. In this chapter, we overview the applicability of fuzzy set theory to the main classes of decision-making problems.

Classical decision making generally deals with a set of alternative states of nature (outcomes, results), a set of alternative actions that are available to the decision maker, a relation indicating the state or outcome to be expected from each alternative action, and, finally, a utility or objective function, which orders the outcomes according to their desirability. A decision is said to be made under conditions of *certainty* when the outcome for each action can be determined and ordered precisely. In this case, the alternative that leads to the outcome yielding the highest utility is chosen. That is, the decision-making problem becomes an optimization problem, the problem of maximizing the utility function. A decision is made under conditions of *risk*, on the other hand, when the only available knowledge concerning the outcomes consists of their conditional probability distributions, one for each action. In this case, the decision-making problem becomes an optimization problem of maximizing the expected utility. When probabilities of the outcomes are not known, or may not even be relevant, and outcomes for each action are characterized only approximately, we say that decisions are made under *uncertainty*. This is the prime domain for fuzzy decision making.

Decision making under uncertainty is perhaps the most important category of decision-making problems, as well characterized by the British economist Shackle [1961]:

> In a predestinate world, decision would be *illusory*; in a world of perfect foreknowledge, *empty*; in a world without natural order, *powerless*. Our intuitive attitude to life implies non-illusory, non-empty, non-powerless decision... Since decision in this sense excludes both perfect foresight and anarchy in nature, it must be defined as choice in face of bounded uncertainty.

This indicates the importance of fuzzy set theory in decision making.

Several classes of decision-making problems are usually recognized. According to one criterion, decision problems are classified as those involving a single decision maker and those which involve several decision makers. These problem classes are referred to as *individual decision making* and *multiperson decision making*, respectively. According to another criterion, we distinguish decision problems that involve a *simple optimization* of a utility function, an *optimization under constraints*, or an *optimization under multiple objective criteria*. Furthermore, decision making can be done in *one stage*, or it can be done iteratively, in *several stages*. This chapter is structured, by and large, according to these classifications.

We do not attempt to cover fuzzy decision making comprehensively. This would require a large book fully specialized on this subject. Instead, we want to convey the spirit of fuzzy decision making, as applied to the various classes of decision problems.

15.2 INDIVIDUAL DECISION MAKING

Fuzziness can be introduced into the existing models of decision models in various ways. In the first paper on fuzzy decision making, Bellman and Zadeh [1970] suggest a fuzzy model of decision making in which relevant goals and constraints are expressed in terms of fuzzy sets, and a decision is determined by an appropriate aggregation of these fuzzy sets. A decision situation in this model is characterized by the following components:

- a set A of *possible actions*;
- a set of *goals* $G_i (i \in \mathbb{N}_n)$, each of which is expressed in terms of a fuzzy set defined on A;
- a set of *constraints* $C_j (j \in \mathbb{N}_m)$, each of which is also expressed by a fuzzy set defined on A.

It is common that the fuzzy sets expressing goals and constraints in this formulation are not defined directly on the set of actions, but indirectly, through other sets that characterize relevant states of nature. Let G'_i and C'_j be fuzzy sets defined on sets X_i and Y_j, respectively, where $i \in \mathbb{N}_n$ and $j \in \mathbb{N}_m$. Assume that these fuzzy sets represent goals and constraints expressed by the decision maker. Then, for each $i \in \mathbb{N}_n$ and each $j \in \mathbb{N}_m$, we describe the meanings of actions in set A in terms of sets X_i and Y_j by functions

$$g_i : A \to X_i,$$

$$c_j : A \to Y_j,$$

and express goals G_i and constraints C_j by the compositions of g_i with G'_i and the compositions of c_j and C'_j; that is,

$$G_i(a) = G'_i(g_i(a)), \tag{15.1}$$

$$C_j(a) = C'_j(c_j(a)) \tag{15.2}$$

for each $a \in A$.

Given a decision situation characterized by fuzzy sets A, $G_i (i \in \mathbb{N}_n)$, and $C_j (j \in \mathbb{N}_m)$, a *fuzzy decision*, D, is conceived as a fuzzy set on A that simultaneously satisfies the given goals G_i and constraints C_j. That is,

$$D(a) = \min[\inf_{i \in \mathbb{N}_n} G_i(a), \inf_{j \in \mathbb{N}_m} C_j(a)] \tag{15.3}$$

for all $a \in A$, provided that the standard operator of fuzzy intersection is employed.

Once a fuzzy decision has been arrived at, it may be necessary to choose the "best" single crisp alternative from this fuzzy set. This may be accomplished in a straightforward manner by choosing an alternative $\hat{a} \in A$ that attains the maximum membership grade in D. Since this method ignores information concerning any of the other alternatives, it may not be desirable in all situations. When A is defined on $\mathbb{R}$, it is preferable to determine $\hat{a}$ by an appropriate defuzzification method (Sec. 12.2).

Before discussing the various features of this fuzzy decision model and its possible modifications or extensions, let us illustrate how it works by two simple examples.

Example 15.1

Suppose that an individual needs to decide which of four possible jobs, a_1, a_2, a_3, a_4, to choose. His or her goal is to choose a job that offers a high salary under the constraints that the job is interesting and within close driving distance. In this case, $A = \{a_1, a_2, a_3, a_4\}$, and the fuzzy sets involved represent the concepts of *high salary*, *interesting job*, and *close driving distance*. These concepts are highly subjective and context-dependent, and must be defined by the individual in a given context. The goal is expressed in monetary terms, independent of the jobs available. Hence, according to our notation, we denote the fuzzy set expressing the goal by G'. A possible definition of G' is given in Fig. 15.1a, where we assume, for convenience, that the underlying universal set is $\mathbb{R}^+$. To express the goal in terms of set A, we need a function g: $A \to \mathbb{R}^+$, which assigns to each job the respective salary. Assume the following assignments:

$$g(a_1) = \$40{,}000,$$
$$g(a_2) = \$45{,}000,$$
$$g(a_3) = \$50{,}000,$$
$$g(a_4) = \$60{,}000.$$

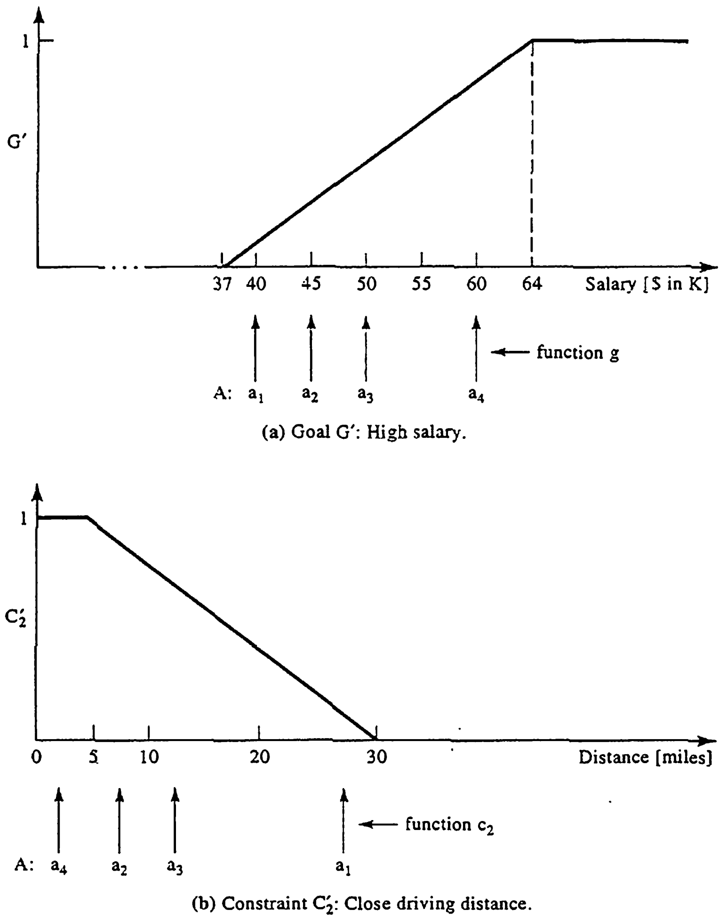

Figure 15.1 Fuzzy goal and constraint (Example 15.1): (a) goal G': high salary; (b) constraint C_2': close driving distance.

This assignment is also shown in Fig. 15.1a. Composing now functions g and G', according to (15.1), we obtain the fuzzy set

$$G = .11/a_1 + .3/a_2 + .48/a_3 + .8/a_4,$$

which expresses the goal in terms of the available jobs in set A.

The first constraint, requiring that the job be interesting, is expressed directly in terms of set A (i.e., c_1, in (15.2) is the identity function and $C_1 = C_1'$). Assume that the individual assigns to the four jobs in A the following membership grades in the fuzzy set of interesting jobs:

$$C_1 = .4/a_1 + .6/a_2 + .2/a_3 + .2/a_4.$$

The second constraint, requiring that the driving distance be close, is expressed in terms of the driving distance from home to work. Following our notation, we denote the fuzzy set expressing this constraint by C'_2. A possible definition of C'_2 is given in Fig. 15.1b, where distances of the four jobs are also shown. Specifically,

$$c_2(a_1) = 27 \text{ miles},$$
$$c_2(a_2) = 7.5 \text{ miles},$$
$$c_2(a_3) = 12 \text{ miles},$$
$$c_2(a_4) = 2.5 \text{ miles}.$$

By composing functions c_2 and C'_2, according to (15.2), we obtain the fuzzy set

$$C_2 = .1/a_1 + .9/a_2 + .7/a_3 + 1/a_4,$$

which expresses the constraint in terms of the set A.

Applying now formula (15.3), we obtain the fuzzy set

$$D = .1/a_1 + .3/a_2 + .2/a_3 + .2/a_4,$$

which represents a fuzzy characterization of the concept of *desirable job*. The job to be chosen is $\hat{a} = a_2$; this is the most desirable job among the four available jobs under the given goal G and constraints C_1, C_2, provided that we aggregate the goal and constraints as expressed by (15.3).

Example 15.2

In this very simple example, adopted from Zimmermann [1987], we illustrate a case in which A is not a discrete set. The board of directors of a company needs to determine the optimal dividend to be paid to the shareholders. For financial reasons, the dividend should be *attractive* (goal G); for reasons of wage negotiations, it should be *modest* (constraint C). The set of actions, A, is the set of possible dividends, assumed here to be the interval $[0, a_{\max}]$ of real numbers, where $a_{\max}$ denotes the largest acceptable dividend. The goal as well as the constraint are expressed directly as fuzzy sets on $A = [0, a_{\max}]$. A possible scenario is shown in Fig. 15.2, which is self-explanatory.

The described fuzzy decision model allows the decision maker to frame the goals and constraints in vague, linguistic terms, which may more accurately reflect practical problem solving situations. The membership functions of fuzzy goals in this model serve much the same purpose as utility or objective functions in classical decision making that order the outcomes according to preferability. Unlike the classical theory of decision making under constraints, however, the symmetry between the goals and constraints under this fuzzy model allows them to be treated in exactly the same manner.

Formula (15.3), based upon the standard operator of fuzzy intersection, does not allow, however, for any interdependence, interaction, or trade-off between the goals and constraints under consideration. For many decision applications, this lack of compensation may not be appropriate; the full compensation or trade-off offered by the union operation that corresponds to the logical "or" (the max operator) may be inappropriate as well. Therefore, an alternative fuzzy set intersection or an averaging operator may be used to reflect a situation in which some degree of positive compensation exists among the goals and constraints.

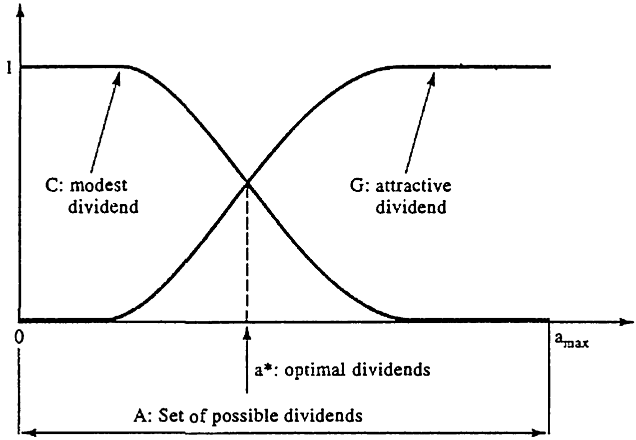

Figure 15.2 Illustration to Example 15.2.

This fuzzy model can be further extended to accommodate the relative importance of the various goals and constraints by the use of weighting coefficients. In this case, the fuzzy decision D can be arrived at by a convex combination of the n weighted goals and m weighted constraints of the form

$$D(a) = \sum_{i=1}^{n} u_i G_i(a) + \sum_{j=1}^{m} v_j C_j(a) \tag{15.4}$$

for all $a \in A$, where u_i and v_j are non-negative weights attached to each fuzzy goal $G_i (i \in \mathbb{N}_n)$ and each fuzzy constraint $C_j (j \in \mathbb{N}_m)$, respectively, such that

$$\sum_{i=1}^{n} u_i + \sum_{j=1}^{m} v_j = 1.$$

However, a direct extension of formula (15.3) may be used as well; that is,

$$D(a) = \min[\inf_{i \in \mathbb{N}_n} G_i^{u_i}(a), \inf_{j \in \mathbb{N}_m} C_j^{v_j}(a)], \tag{15.5}$$

where the weights u_i and v_j possess the above-specified properties.

15.3 MULTIPERSON DECISION MAKING

When decisions made by more than one person are modeled, two differences from the case of a single decision maker can be considered: first, the goals of the individual decision makers may differ such that each places a different ordering on the alternatives; second, the individual decision makers may have access to different information upon which to base their decision. Theories known as n-person game theories deal with both of these considerations, team theories of decision making deal only with the second, and group-decision theories deal only with the first.