Chapter 18: Contingency chi-square, Fisher's and McNemar's tests

TXCL7565/PHSC7565

What This Lecture Covers

- Contingency chi-square test (traditional)
- Fisher's exact test (small expected counts)
- McNemar's test (paired data)

CONTINGENCY CHI-SQUARE TEST

Goodness of fit vs contingency

- **Goodness of fit** - determines whether there is a convincing discrepancy between observed and theoretical proportions.

- **Contingency** - determines whether there is a convincing difference between one set of observed proportions and another set of observed proportions.

Contingency tables

A table where both the columns and rows are based upon categorization.

	Control design	Test design
Not expelled	1732 (86.6%)	1778 (88.9%)
expelled	268 (13.4%)	222 (11.1%)

Table 18.1 A contingency table showing the effect of IUD design upon the number of cases where the device was expelled.

Contingency chi-square test

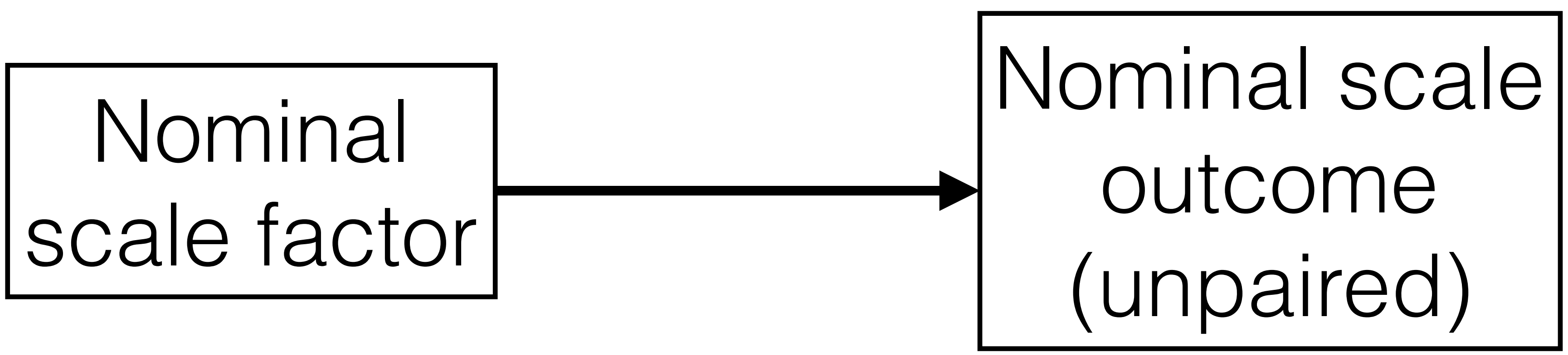


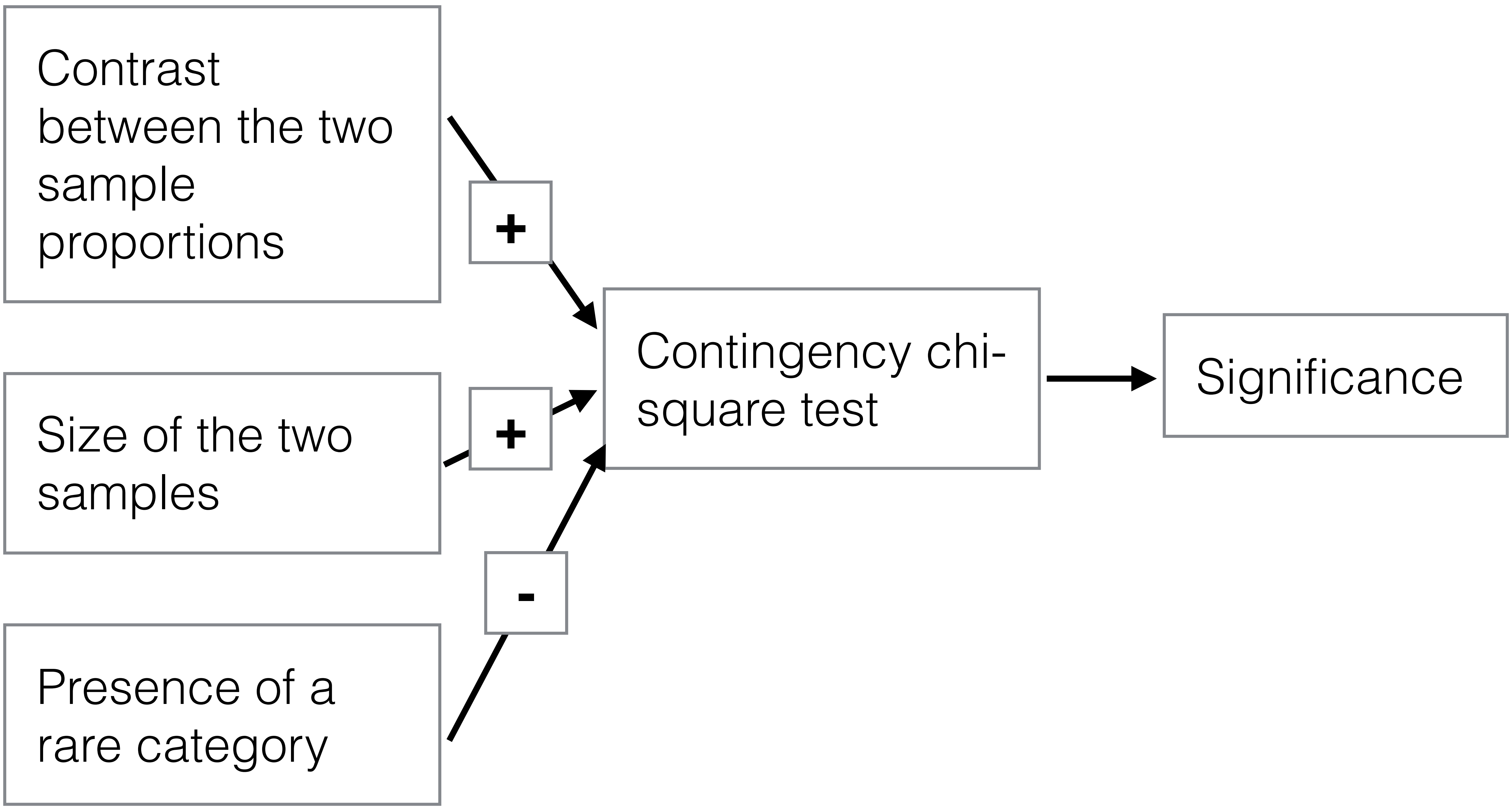
Figure 18.1 Summary of the circumstances in which a contingency chi-square test is appropriate.

Re-orienting contingency tables

	Control design	Test design
Not expelled	1732 (86.6%)	1778 (88.9%)
expelled	268 (13.4%)	222 (11.1%)

	Not expelled	expelled
Control design	1732 (49.3%)	268 (54.7%)
Test design	1778 (50.7%)	222 (45.3%)

Influences on contingency chi-square test outcome



Likelihood of significance with a contingency chi-square test

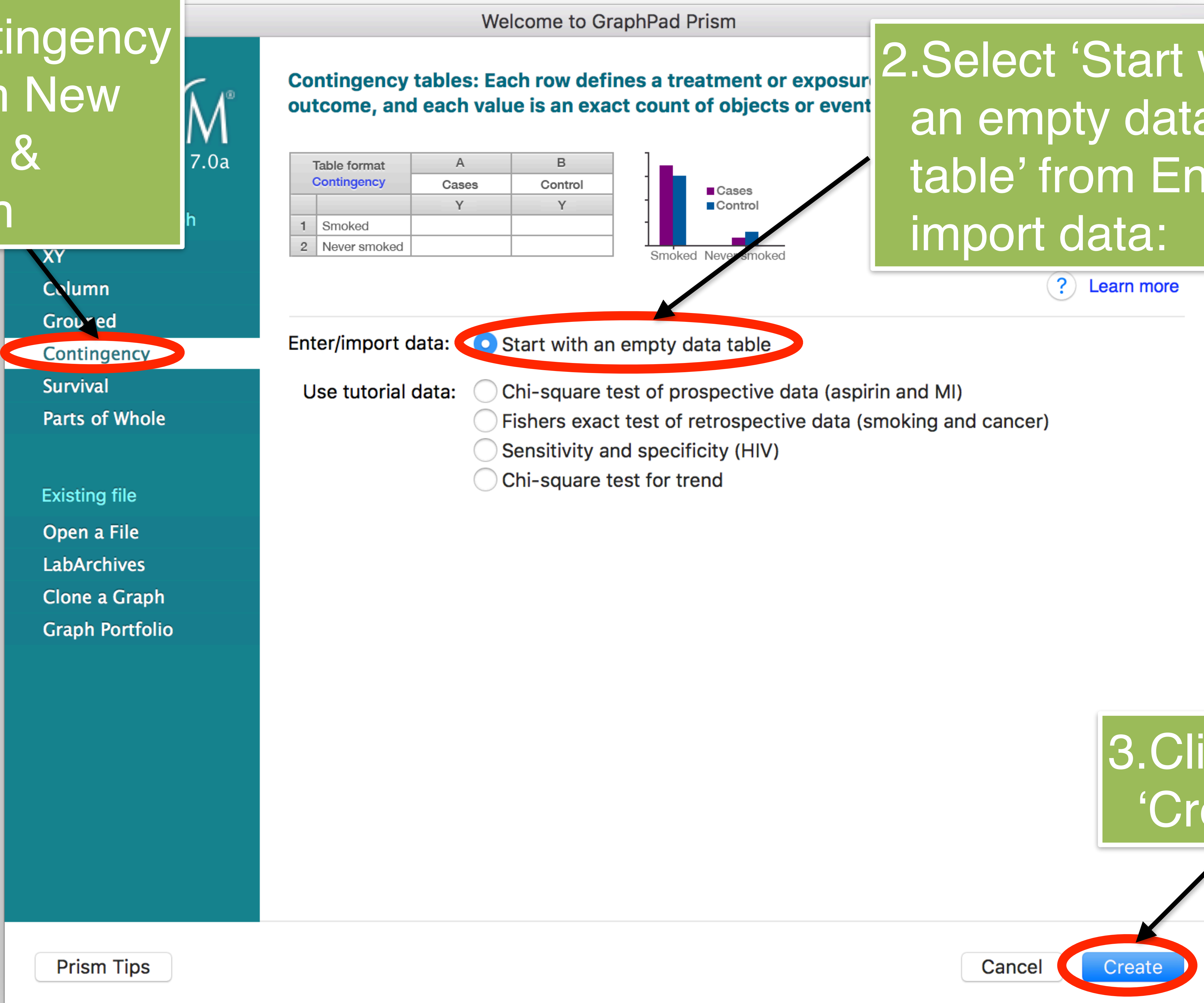
- *Most likely* - A large difference between the proportions in the two groups being compared, large sample sizes, and all categories reasonably well represented.

- *Least likely* - A small difference between the proportions in the two groups being compared, small sample sizes, and rarity of one of the categories within the samples.

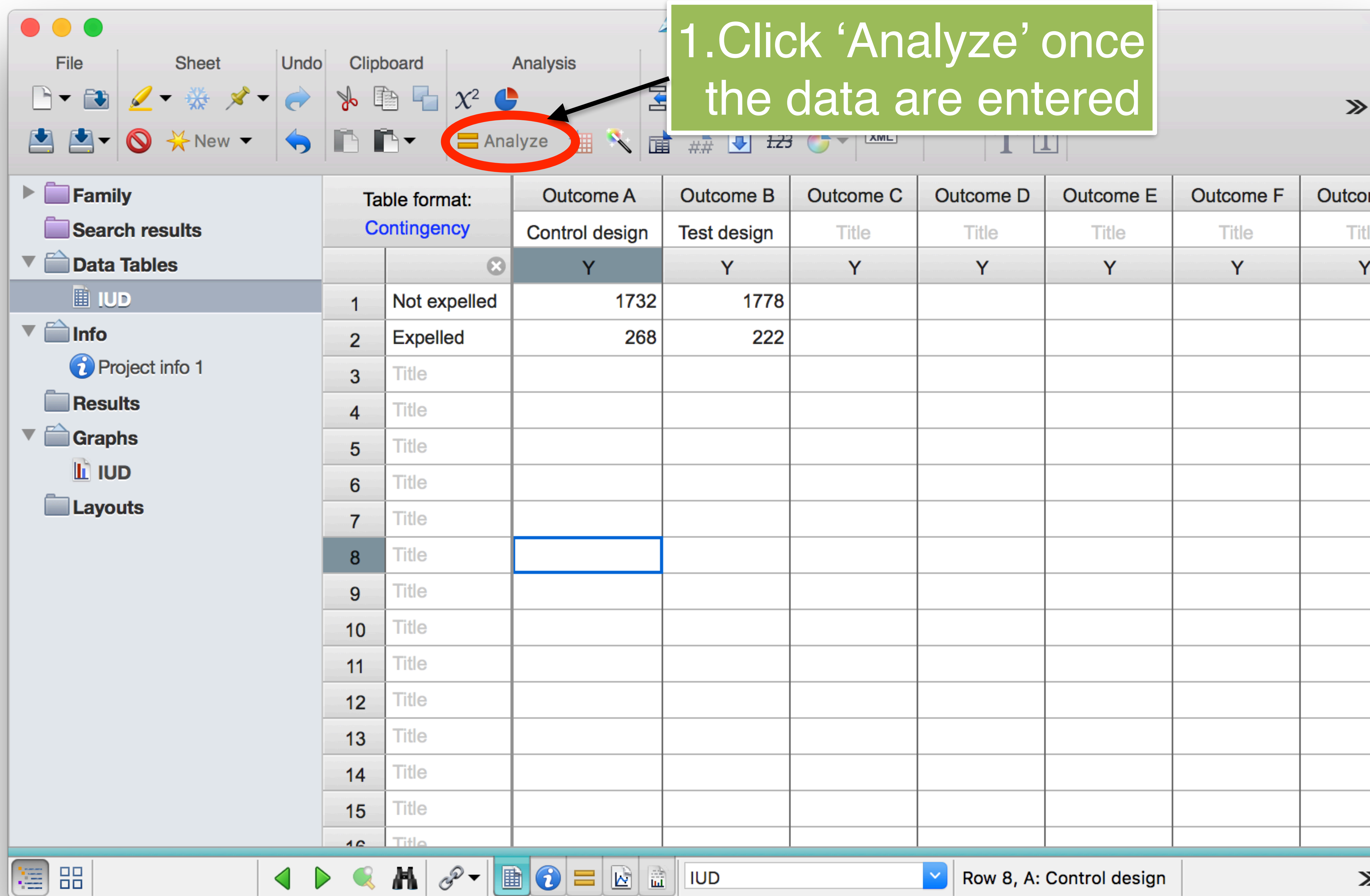
CONTINGENCY CHI-SQUARE IN GRAPHPAD

1. Select ‘Contingency’ from New table & graph

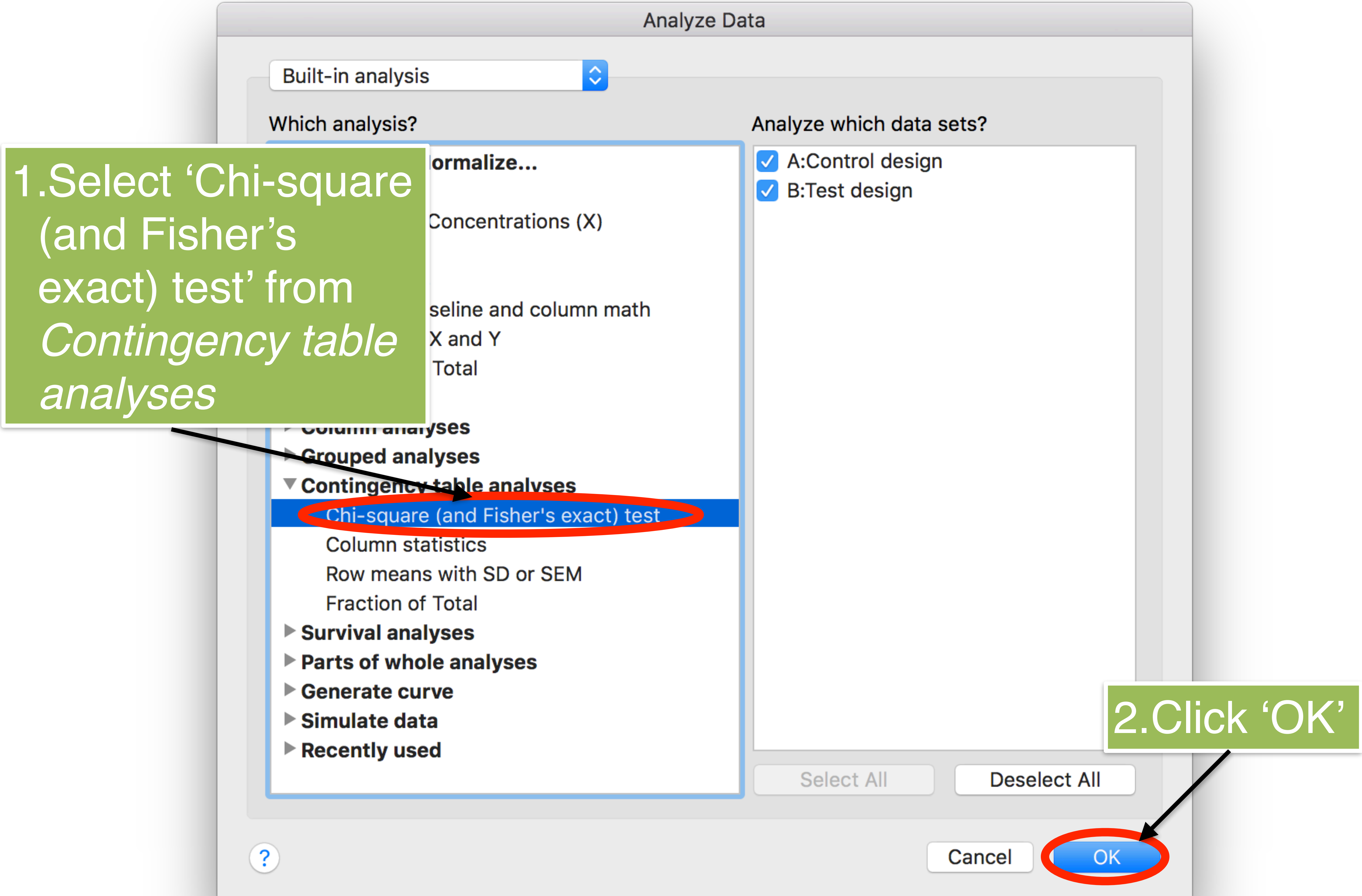
2. Select ‘Start with an empty data table’ from Enter/ import data:



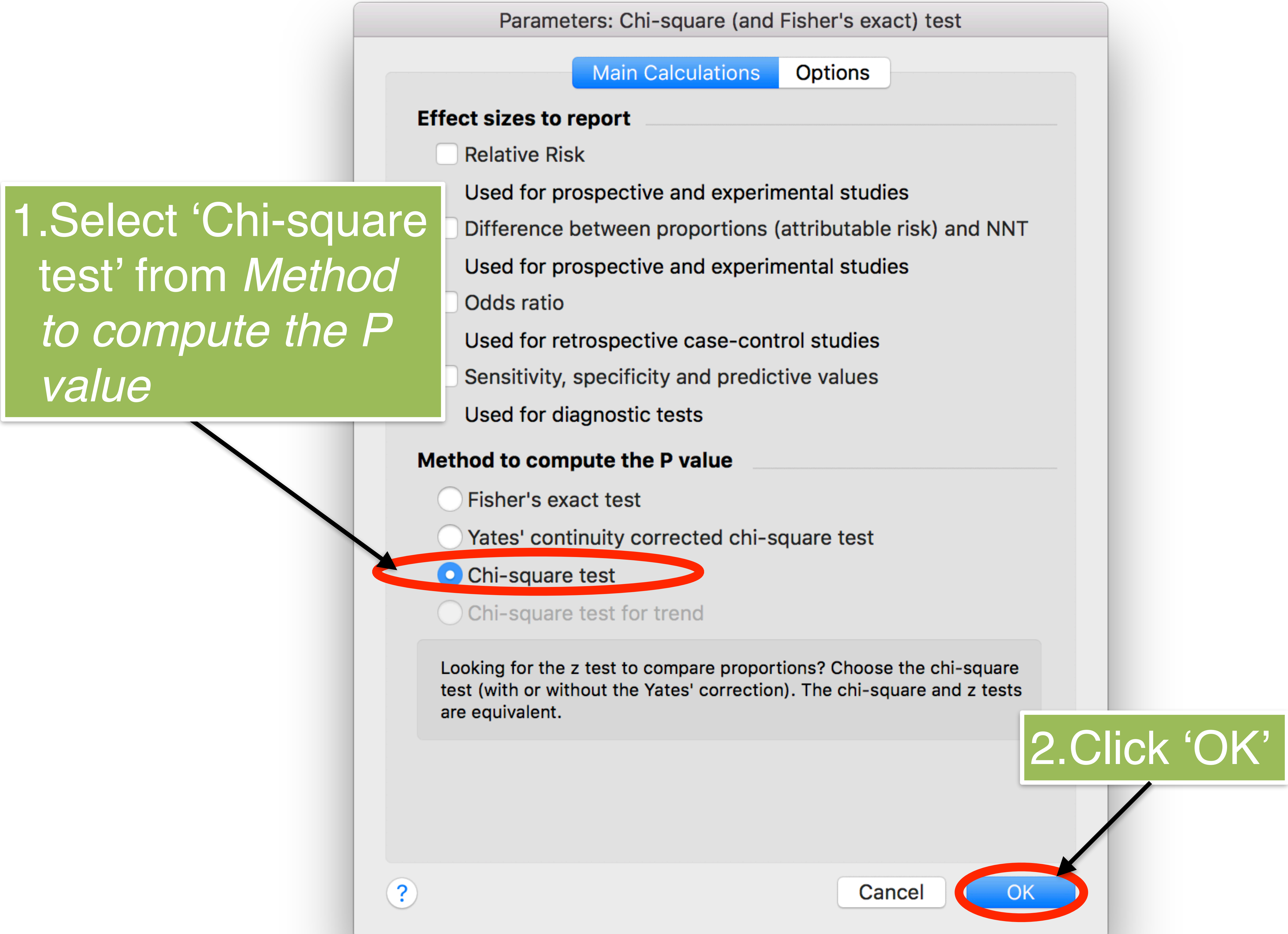
3. Click ‘Create’

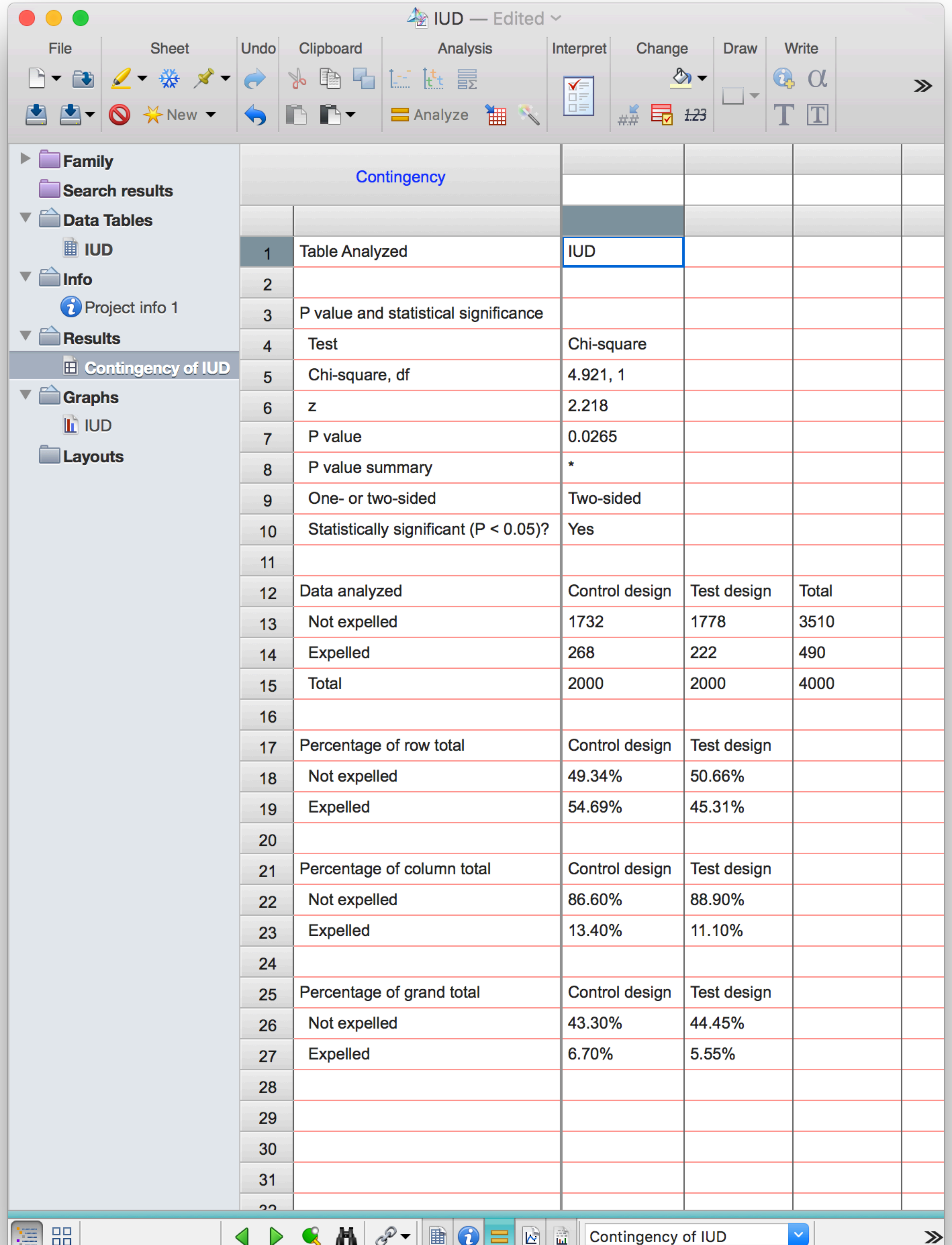


Enter count information into the table. It does not matter which variable is used to define the columns and which variable is used to describe the rows.



Analyze Data
Built-in analysis
Which analysis?
ormalize...
Concentrations (X)
seline and column math
X and Y
Total
Grouped analyses
Contingency table analyses
Chi-square (and Fisher's exact) test
Column statistics
Row means with SD or SEM
Fraction of Total
Survival analyses
Parts of whole analyses
Generate curve
Simulate data
Recently used
Analyze which data sets?
A:Control design
B:Test design
Select All
Deselect All
?
Cancel
OK
1.Select 'Chi-square (and Fisher's exact) test' from *Contingency table analyses*
2.Click 'OK'


Parameters: Chi-square (and Fisher's exact) test
Main Calculations
Options
Effect sizes to report
Relative Risk
Used for prospective and experimental studies
Difference between proportions (attributable risk) and NNT
Used for prospective and experimental studies
Odds ratio
Used for retrospective case-control studies
Sensitivity, specificity and predictive values
Used for diagnostic tests
Method to compute the P value
Fisher's exact test
Yates' continuity corrected chi-square test
Chi-square test
Chi-square test for trend
Looking for the z test to compare proportions? Choose the chi-square test (with or without the Yates' correction). The chi-square and z tests are equivalent.
?
Cancel
OK
1.Select 'Chi-square test' from Method to compute the P value
2.Click 'OK'



	Contingency			
1	Table Analyzed	IUD		
2				
3	P value and statistical significance			
4	Test	Chi-square		
5	Chi-square, df	4.921, 1		
6	z	2.218		
7	P value	0.0265		
8	P value summary	*		
9	One- or two-sided	Two-sided		
10	Statistically significant (P < 0.05)?	Yes		
11				
12	Data analyzed	Control design	Test design	Total
13	Not expelled	1732	1778	3510
14	Expelled	268	222	490
15	Total	2000	2000	4000
16				
17	Percentage of row total	Control design	Test design	
18	Not expelled	49.34%	50.66%	
19	Expelled	54.69%	45.31%	
20				
21	Percentage of column total	Control design	Test design	
22	Not expelled	86.60%	88.90%	
23	Expelled	13.40%	11.10%	
24				
25	Percentage of grand total	Control design	Test design	
26	Not expelled	43.30%	44.45%	
27	Expelled	6.70%	5.55%	

Methods:

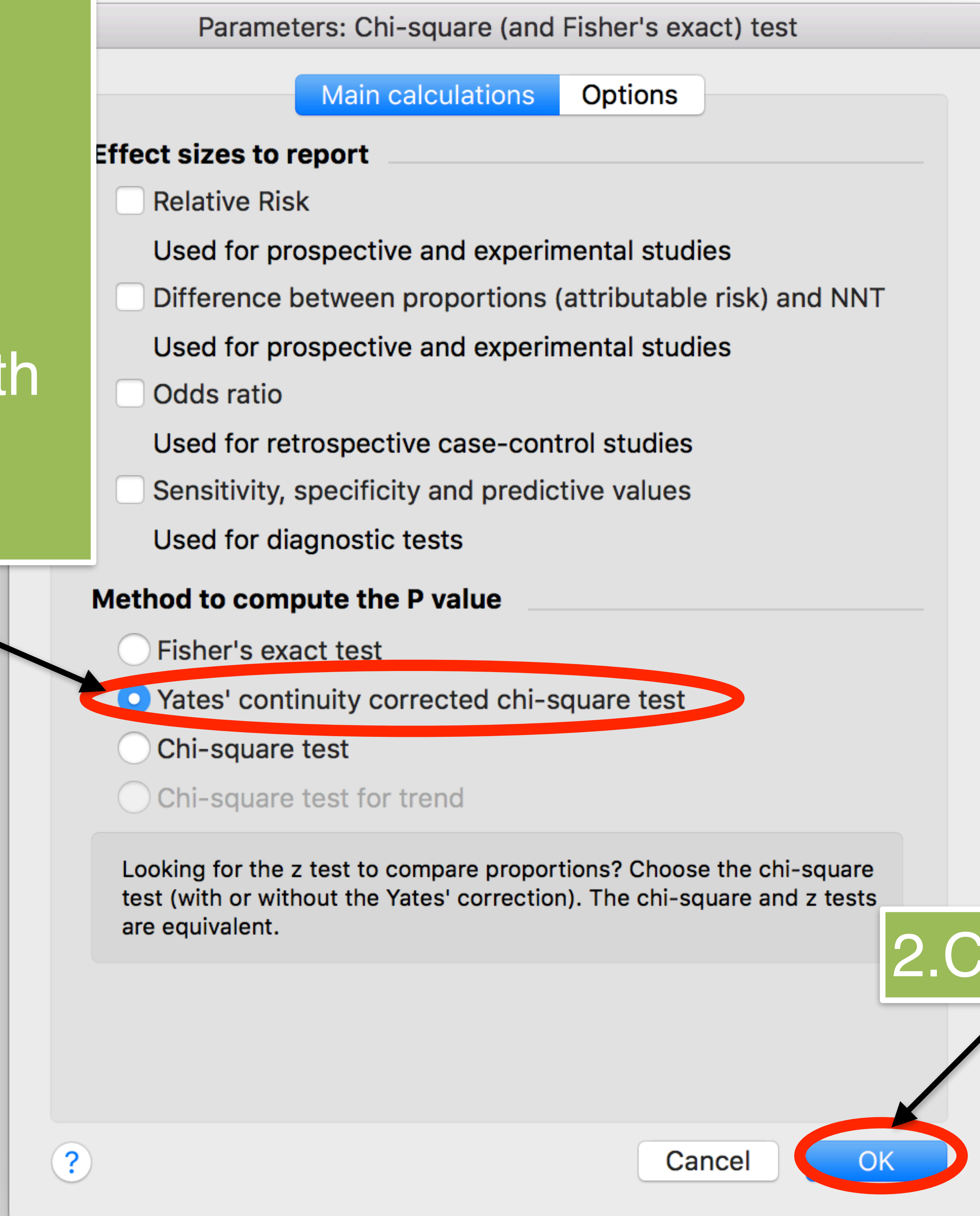
A contingency Chi-square test was used to determine the difference in the proportion of patients whose IUD was expelled between the group of patients that received the control design and the group of patients that received the test design. All statistical analyses were conducted in GraphPad Prism (v7.0a).

Results:

There was a significantly higher proportion (chi-square=4.92, df=1, p=0.0265) of patients whose IUD was expelled in the patients that received the control design (13.4% of patients with the control design) compared to the patients that received the test design (11.1% of patients with the test design).

P value and statistical significance	
Test	Chi-square
Chi-square, df	4.921, 1
z	2.218
P value	0.0265
P value summary	*
One- or two-sided	Two-sided
Statistically significant (P < 0.05)?	Yes

1. Click ‘Yates’ continuity corrected chi-square test’ to get the chi-square test with the continuity correction.



2. Click ‘OK’

P value and statistical significance	
Test	Chi-square with Yates' correction
Chi-square, df	4.71, 1
z	2.17
P value	0.0300
P value summary	*
One- or two-sided	Two-sided
Statistically significant (P < 0.05)?	Yes

Often, analysts use a continuity correction when they are analyzing a 2 by 2 table. In this case, the two tests yield similar results.

Larger contingency tables

	None	Verbal emphasis	Letter	Phone call
Attended	49 (65.3%)	53 (71.6%)	61 (83.6%)	65 (86.7%)
Did not attend	26 (34.7%)	21 (28.4%)	12 (16.4%)	10 (13.3%)

Table 18.3 Attendance at a diabetes clinic following various additional reminders (with column percentages).

- Can often make interpretation difficult because the p-value is that any of the proportions across a row (column proportions) or across a column (row proportions) are different
- Can sub-divide large tables to ask specific hypothesis but this is a slippery slope with respect to multiple testing
- Keep it simple, keep it clear

Planning experimental size

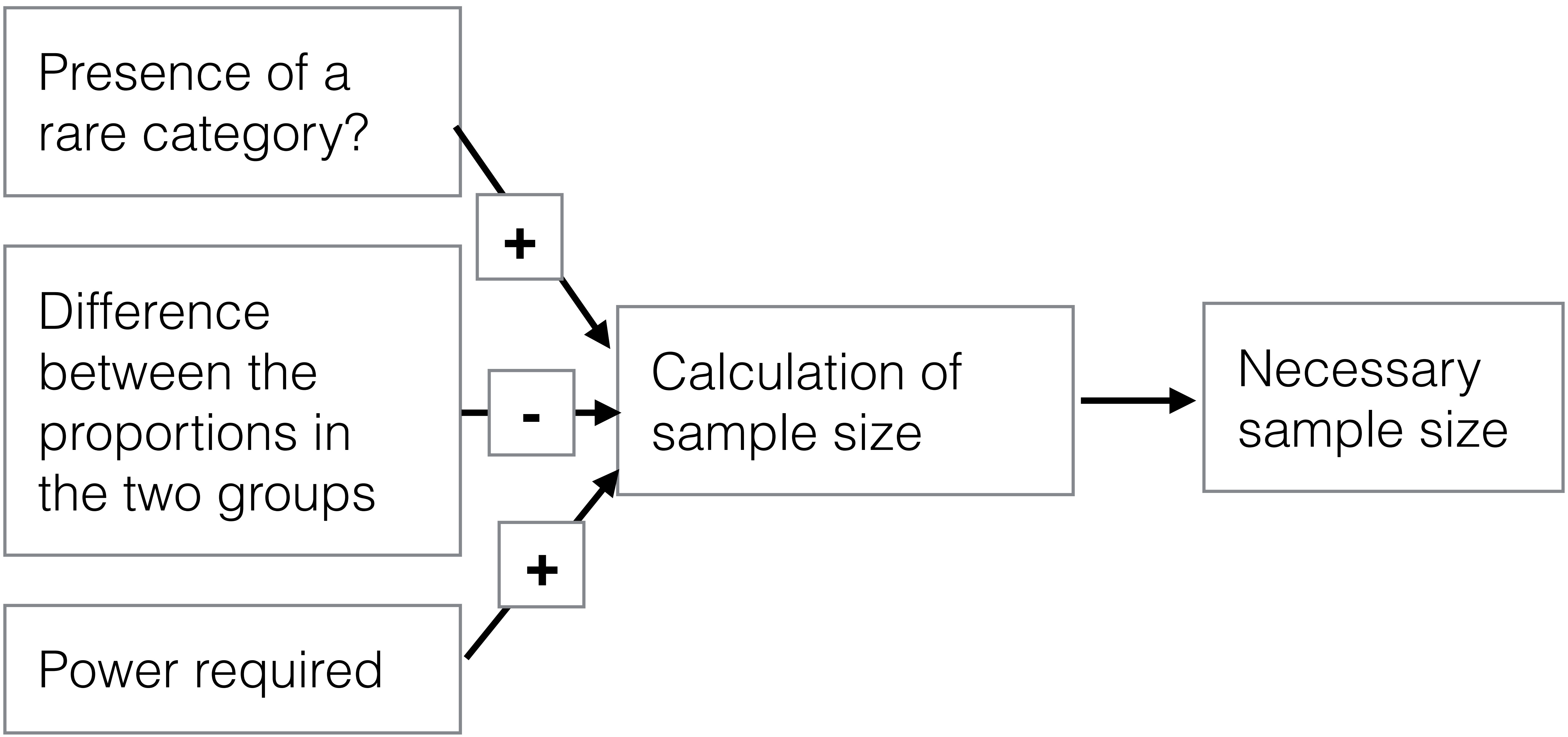


Figure 18.3 Calculation of necessary sample size for a contingency chi-square test

Generic output of sample size calculation

Sample Size for Contingency chi-square test	
Assumed proportion for group 1:	0.15
Target proportion for group 2:	0.11
Target power:	0.90
Sample size (each group):	1484
Achieved power:	0.9001

FISHER’S EXACT TEST

The problem of low expected frequencies

- The calculation of the contingency chi-square test involves some approximations that work satisfactorily for large counts but biases can creep in with lower expected counts.

- Low counts are not a problem, only low ‘expected’ counts.

Fisher's exact test

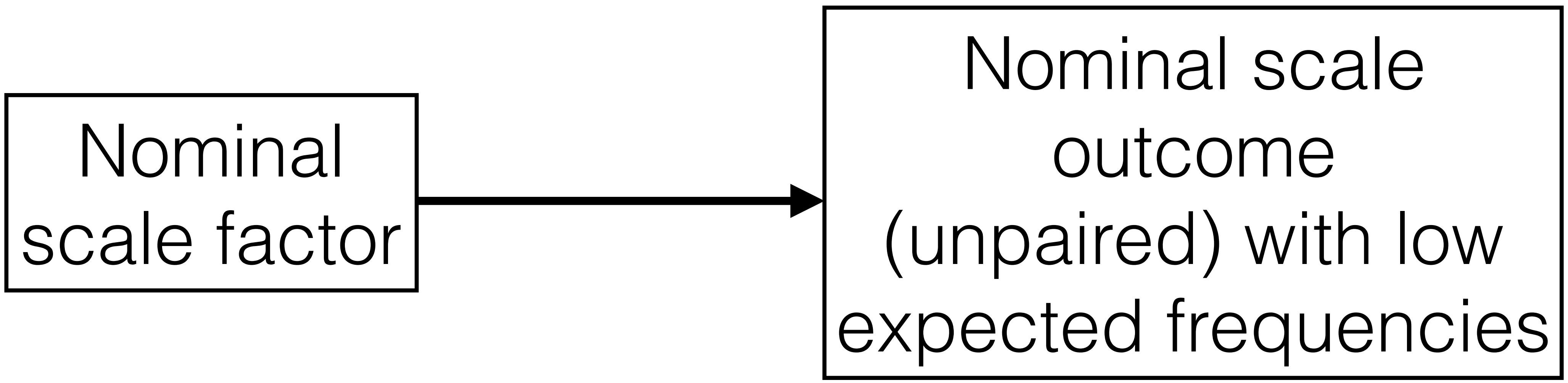


Figure 18.4 Summary of the circumstances in which a Fisher's test is appropriate.

What is a ‘low’ expected frequency

- No consensus on what ‘low’ stands for.
- Many statistical packages use an expected count of less than 5 in any cell or two cells.
- Prior to powerful computers, the Fisher’s test was computationally intensive to calculate, but now, it is not a problem. Therefore, if in doubt, use Fisher’s exact test.

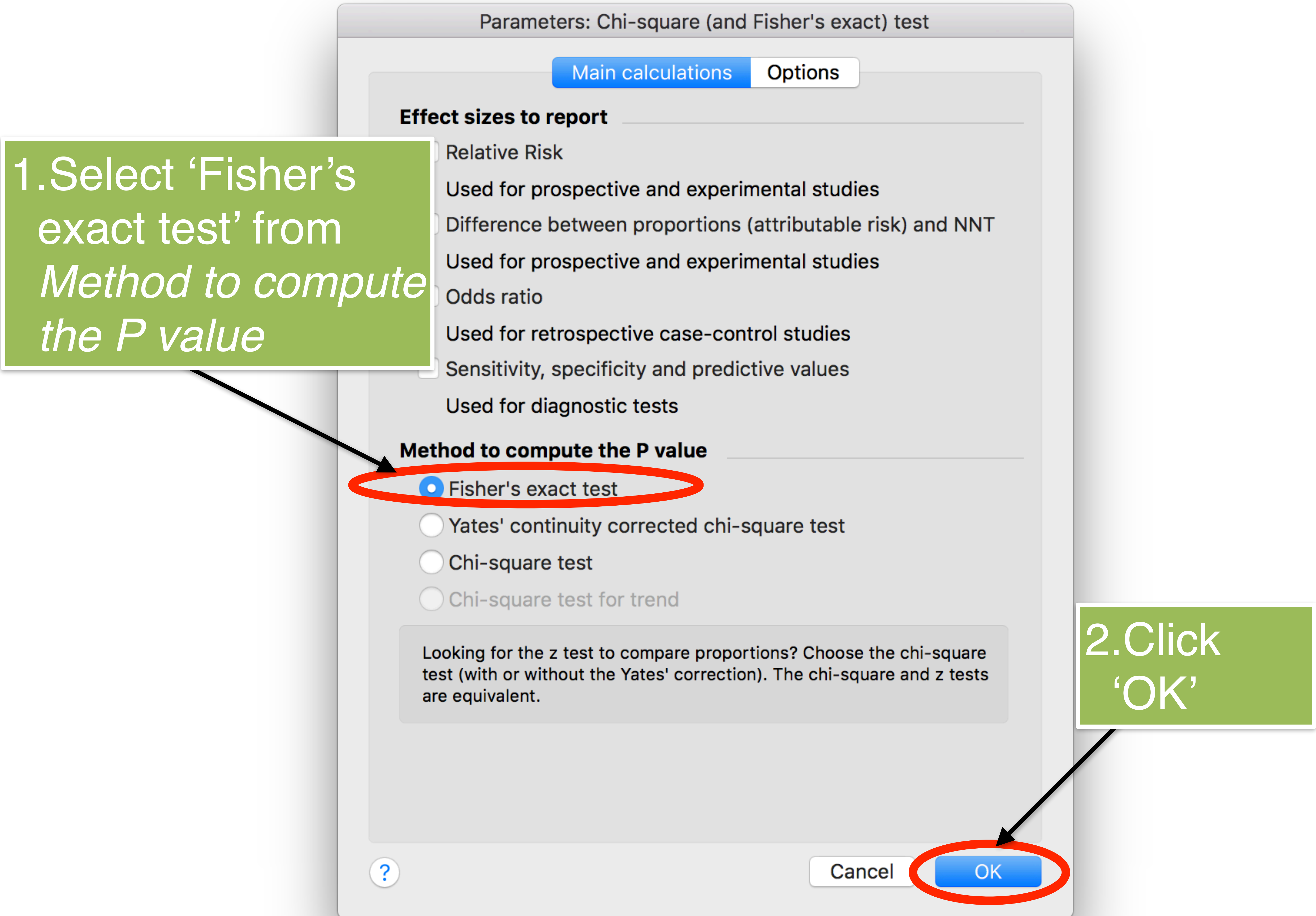
Comparison of all three methods for IUD
no low expected frequencies

Statistical Test	Chi-Square	p-value
Uncorrected chi-square	4.921	0.027
Yates' corrected chi-square	4.170	0.030
Fisher's exact test		0.030

Comparison of all three methods *with low expected frequencies*

	Placebo	Vitamin C
No cold	1 (25.0%) [2]	3 (75.0%) [2]
Cold	3 (75.0%) [2]	1 (25.0%) [2]
Statistical Tests		
	chi-square	p-value
uncorrected chi-square	2.00	0.157
Yates' corrected chi-square	0.50	0.480
Fisher's Exact test		0.486

FISHER'S EXACT TEST IN GRAPHPAD



Parameters: Chi-square (and Fisher's exact) test
Main calculations
Options
Effect sizes to report
Relative Risk
Used for prospective and experimental studies
Difference between proportions (attributable risk) and NNT
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Method to compute the P value
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Yates' continuity corrected chi-square test
Chi-square test
Chi-square test for trend
Looking for the z test to compare proportions? Choose the chi-square test (with or without the Yates' correction). The chi-square and z tests are equivalent.
?
Cancel
OK
1.Select ‘Fisher’s exact test’ from *Method to compute the P value*
2.Click ‘OK’

P value and statistical significance	
Test	Fisher's exact test
P value	0.0299
P value summary	*
One- or two-sided	Two-sided
Statistically significant (P < 0.05)?	Yes

P value and statistical significance	
Test	Chi-square with Yates' correction
Chi-square, df	4.71, 1
z	2.17
P value	0.0300
P value summary	*
One- or two-sided	Two-sided
Statistically significant (P < 0.05)?	Yes

P value and statistical significance	
Test	Fisher's exact test
P value	0.0299
P value summary	*
One- or two-sided	Two-sided
Statistically significant (P < 0.05)?	Yes

P value and statistical significance	
Test	Chi-square
Chi-square, df	4.921, 1
z	2.218
P value	0.0265
P value summary	*
One- or two-sided	Two-sided
Statistically significant (P < 0.05)?	Yes

McNEMAR’S TEST

McNemar's test

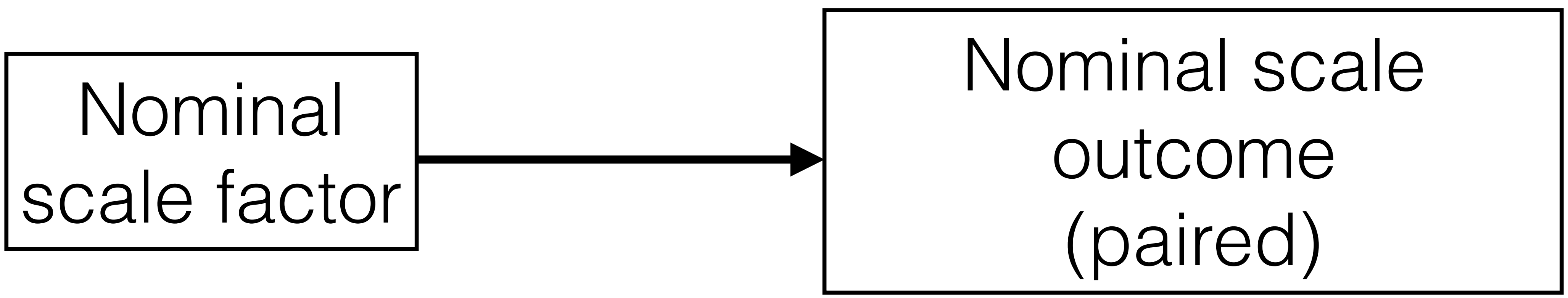


Figure 18.5 Use of McNemar's test for categorical outcomes with a ***paired*** data structure.

		Pre-training	
		Unsatisfactory	Satisfactory
Post-training	Unsatisfactory	9	1
	Satisfactory	7	8

Table 18.7 Effects of training on the quality of inhaler use among asthma patients.

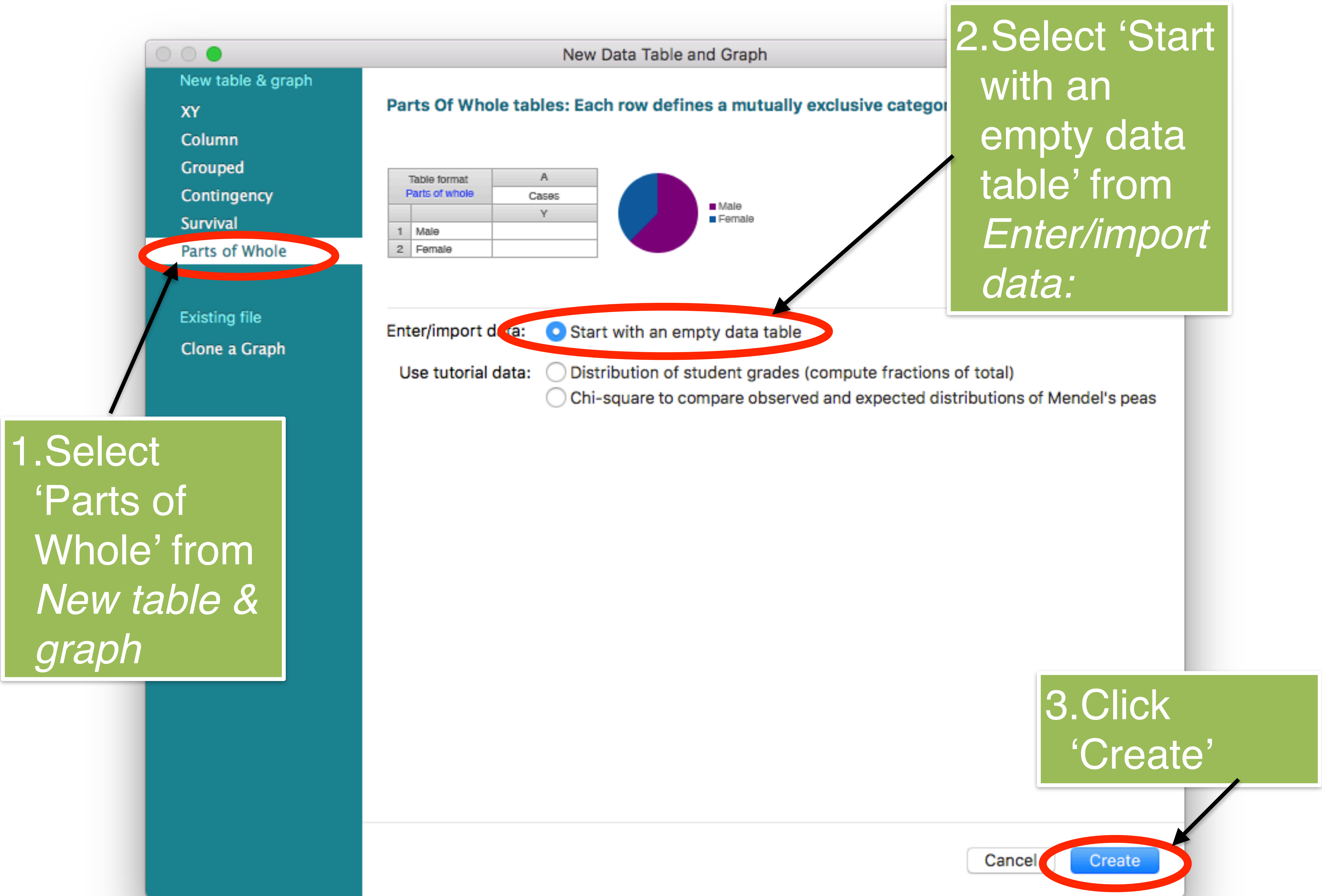
Table 18.8 Calculation of the McNemar test for the effect of training on inhaler technique

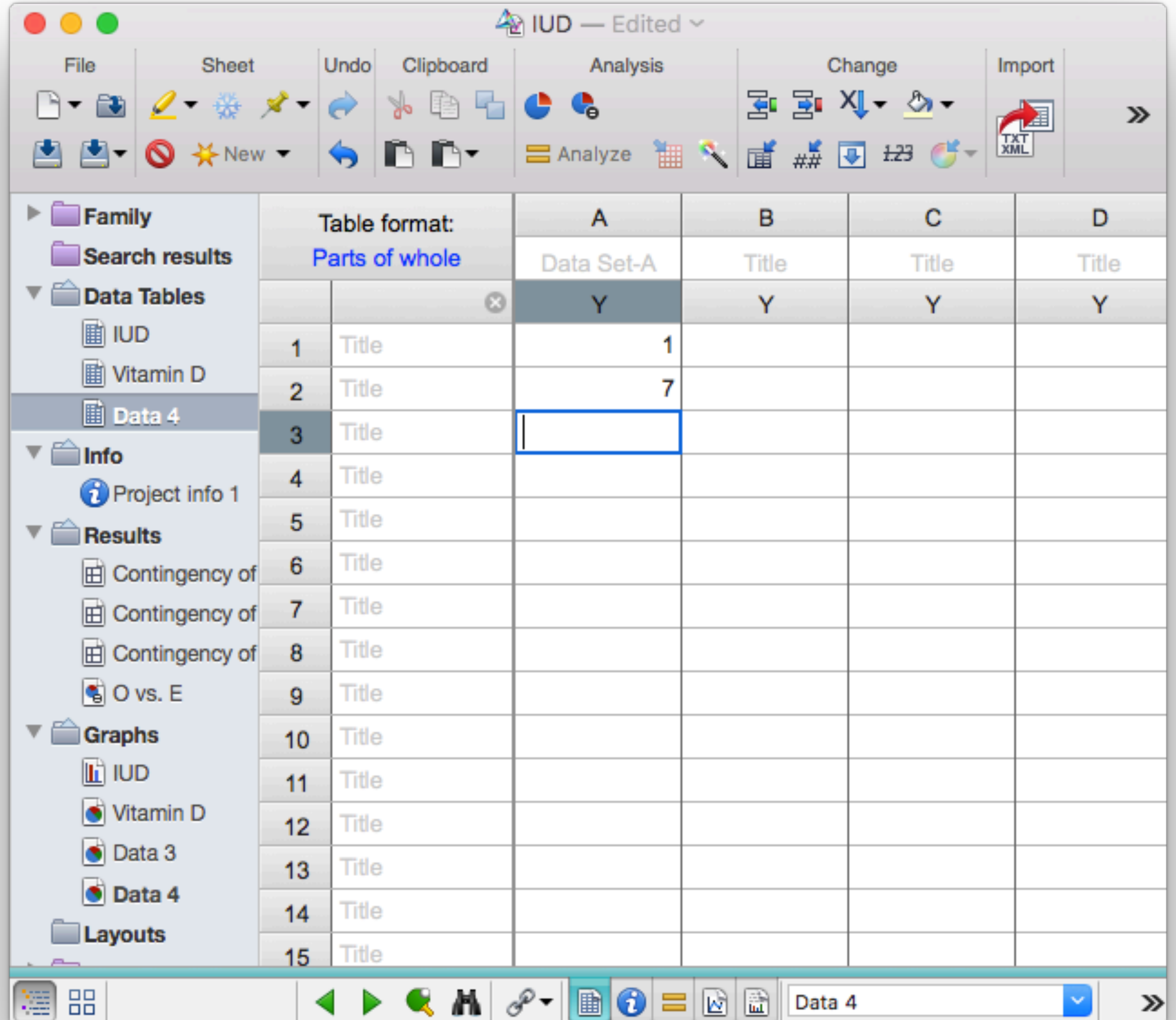
	Unsatisfactory changing to satisfactory	Satisfactory changing to unsatisfactory
Observed	7	1
Expected	(7+1)/2 = 4	(7+1)/2 = 4
Obs - Exp	7 - 4 = 3	1 - 4 = -3
Obs - Exp (Yates corr)	2.5	-2.5
$(\text{Obs - Exp})^2$	6.25	6.25
$(\text{Obs - Exp})^2$ / Exp	1.5625	1.5625
Chi-square	1.5625 + 1.5625 = 3.125	

McNEMAR’S TEST IN GRAPHPAD

GraphPad will not calculate a McNemar’s test using a chi-square statistics.

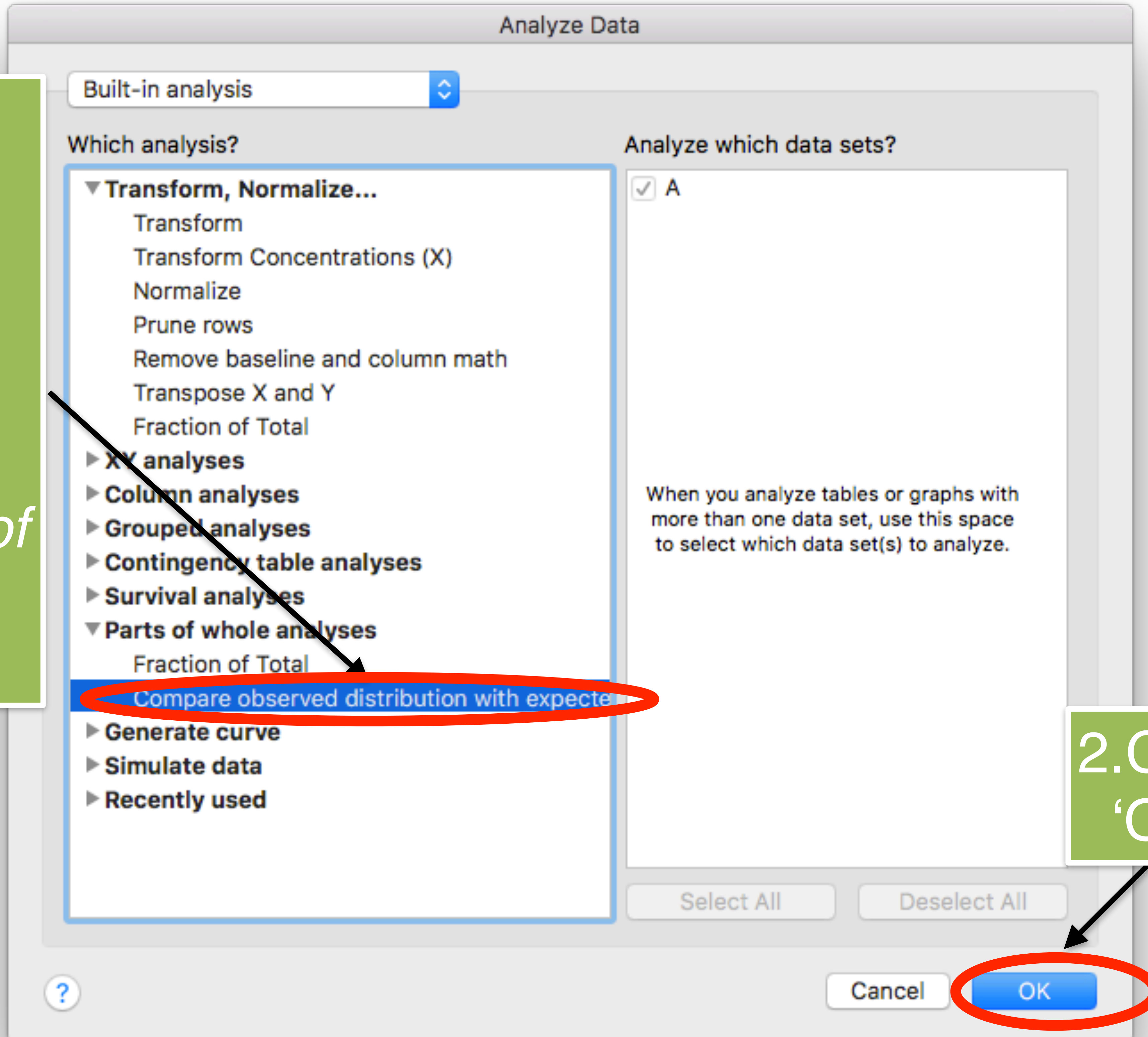
However, you can get ‘trick’ it to run a binomial version of the McNemar’s test.


New Data Table and Graph
New table & graph
XY
Column
Grouped
Contingency
Survival
Parts of Whole
Existing file
Clone a Graph
Parts Of Whole tables: Each row defines a mutually exclusive categor
Table format
Parts of whole
A
Cases
Y
1 Male
2 Female
Male
Female
Enter/import data:
Start with an empty data table
Use tutorial data:
Distribution of student grades (compute fractions of total)
Chi-square to compare observed and expected distributions of Mendel's peas
Cancel
Create
1.Select 'Parts of Whole' from *New table & graph*
2.Select 'Start with an empty data table' from *Enter/import data:*
3.Click 'Create'

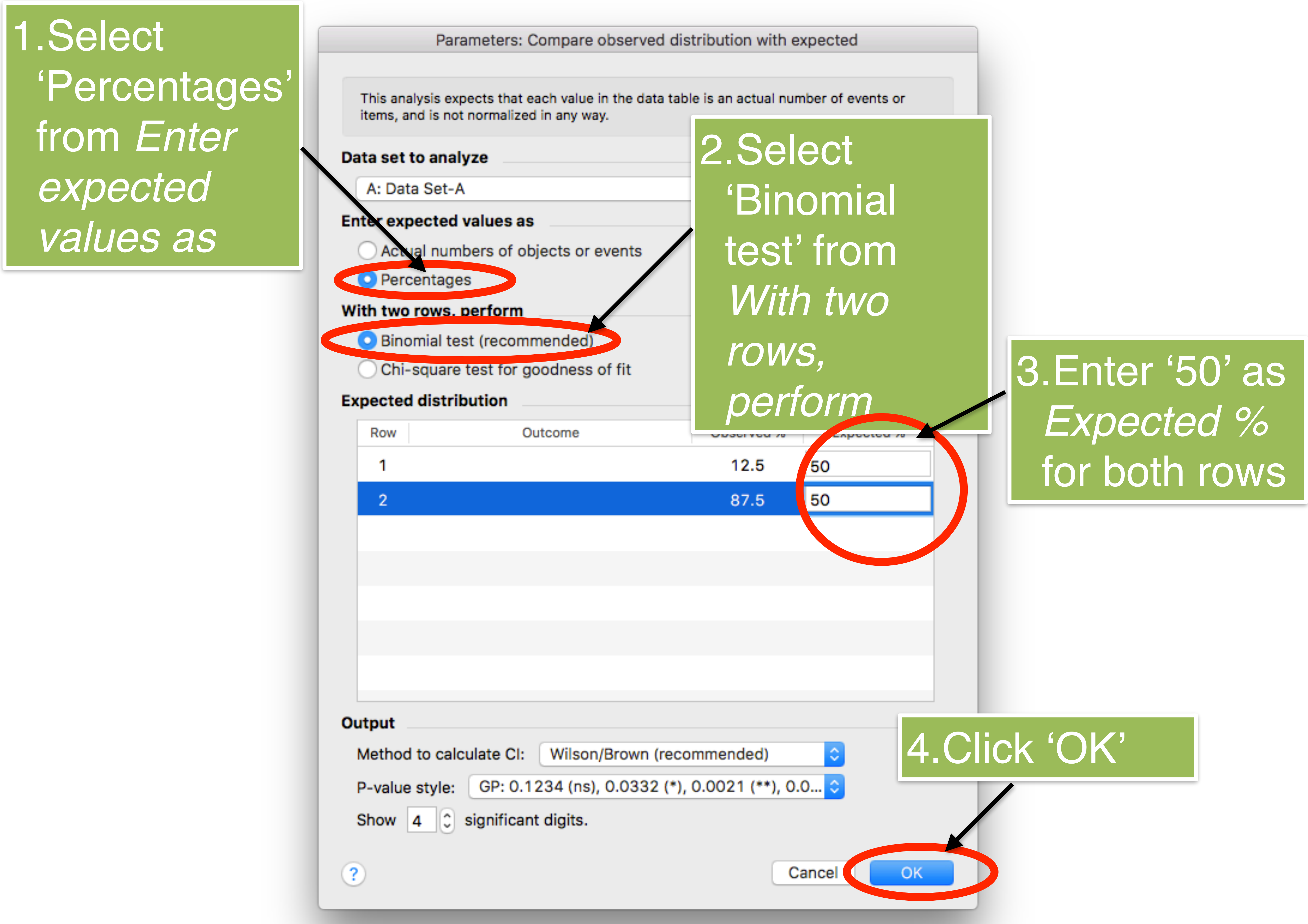


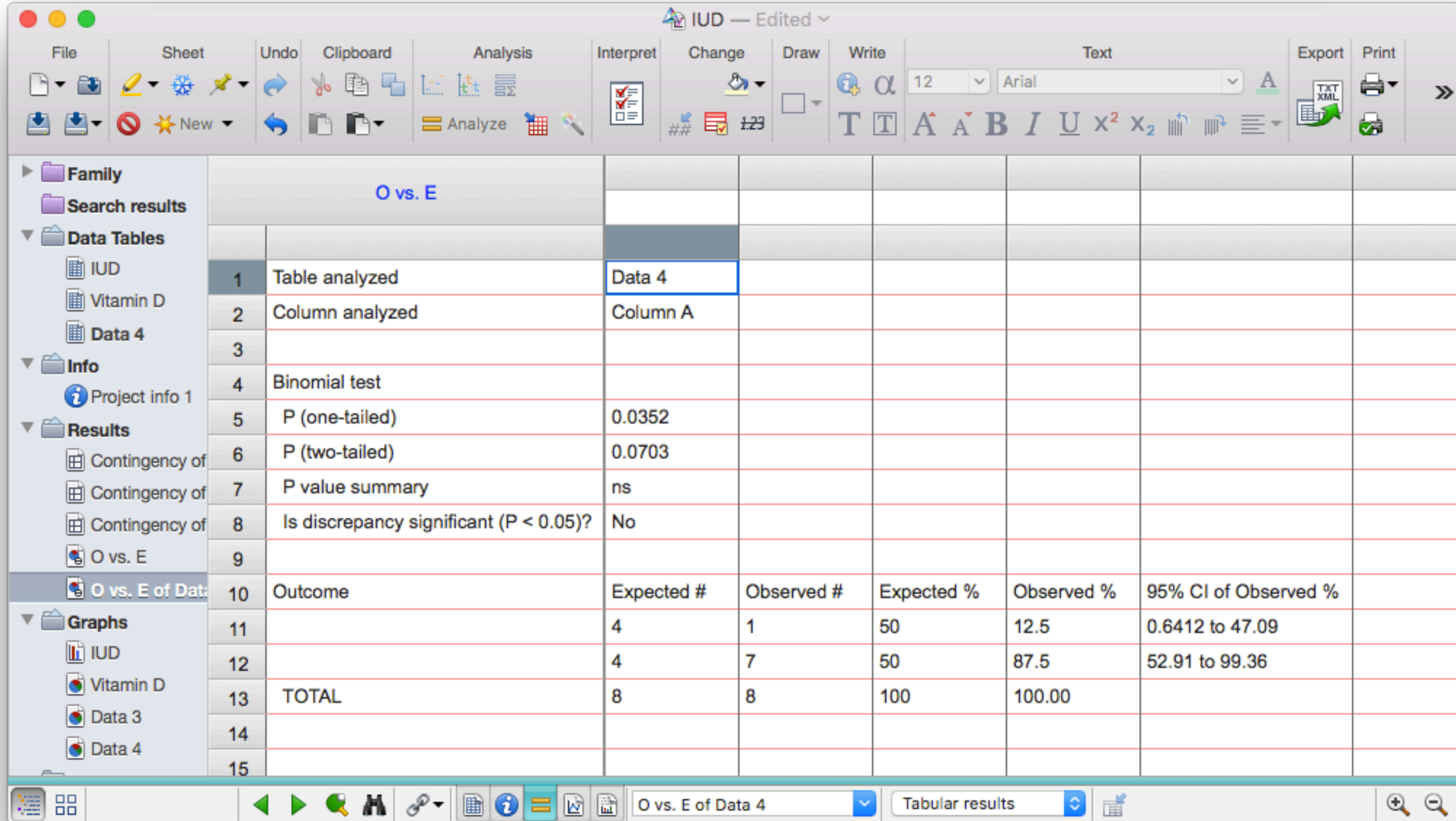
Enter the discordant number of observations only.

1. Select 'Compare observed distribution with expected' from *Parts of whole analyses*



2. Click 'OK'


1.Select 'Percentages' from *Enter expected values as*
Parameters: Compare observed distribution with expected
This analysis expects that each value in the data table is an actual number of events or items, and is not normalized in any way.
Data set to analyze
A: Data Set-A
Enter expected values as
Actual numbers of objects or events
Percentages
With two rows, perform
Binomial test (recommended)
Chi-square test for goodness of fit
Expected distribution
Row
Outcome
1
12.5
50
2
87.5
50
2.Select 'Binomial test' from *With two rows, perform*
3.Enter '50' as *Expected %* for both rows
Output
Method to calculate CI:
Wilson/Brown (recommended)
P-value style:
GP: 0.1234 (ns), 0.0332 (*), 0.0021 (**), 0.0...
Show
4
significant digits.
Cancel
OK
4.Click 'OK'



1	Table analyzed	Data 4				
2	Column analyzed	Column A				
3						
4	Binomial test					
5	P (one-tailed)	0.0352				
6	P (two-tailed)	0.0703				
7	P value summary	ns				
8	Is discrepancy significant (P < 0.05)?	No				
9						
10	Outcome	Expected #	Observed #	Expected %	Observed %	95% CI of Observed %
11		4	1	50	12.5	0.6412 to 47.09
12		4	7	50	87.5	52.91 to 99.36
13	TOTAL	8	8	100	100.00	
14						
15						

McNEMAR’S TEST IN R

```r
asthma = matrix(c(9,1,7,8),nrow=2,byrow=TRUE)
mcnemar.test(asthma)
```

```
	McNemar's Chi-squared test with continuity correction

data:  asthma
McNemar's chi-squared = 3.125, df = 1, p-value = 0.0771
```

What did we learn?

- A contingency table presents data that is based entirely upon categorization.
- Contingency chi-square test is used to determine whether the proportion of individuals falling into a particular category changes according to circumstances.
- Yates correction should be applied to the contingency chi-square test when the contingency table is 2 by 2.
- When expected counts are low, use Fisher's exact test.
- Use McNemar's test when data are paired.