LU-20: Resolution

LU-16 Resolution **Period: 2**

LU Outcomes	**Level:**	A	**CO Number:**	2

1 Apply resolution to FOL

2 Convert any sentences in to CNF and BNF

Resolution

- Resolution was invented in the year 1965 by a Mathematician called Alan Robinson

Definition:

Resolution yields a complete inference algorithm when coupled with any complete search algorithm

- Resolution makes use of the inference rules. Resolution performs deductive inference.
- One can perform Resolution from a Knowledge Base (collection of sentences i.e facts and rules)
- Resolution uses proof by contradiction

Steps for Resolution

- Convert the given statements in Predicate/Propositional Logic
- Convert these statements into Conjunctive Normal Form (CNF)
- Negate the Conclusion (Proof by Contradiction)
- Resolve using a Resolution Tree (Unification)

Steps to Convert to CNF (Conjunctive Normal Form)

Eliminate implication →

$a \rightarrow b = \neg a \vee b$

Move ¬ inwards:

$\neg \forall x\ p$ becomes $\exists\ x\ \neg p$

$\neg \exists x\ p$ becomes $\forall\ x\ \neg p$

Standardize variables:

For sentences like $(\exists x P(x)) \vee (\exists x Q(x))$ which use the same variable name twice, change the name of one of the variables. This avoids confusion later when we drop the quantifiers.

∃

Steps to Convert to CNF (Conjunctive Normal Form)

Eliminate Existential Quantifier

To eliminate an Existential Quantifier, replace the variable by a independent skolem constant. This process is called as **Skolemization**

Example: ∃y: President (y)

Here **'y'** is an independent quantifier so we can replace **'y'** by any name (say – George Bush).

So, **∃y: President (y)** becomes **President (George Bush)**.

To eliminate a dependent Existential Quantifier we replace its variable by Skolem Function that accepts the value of 'x' and returns the corresponding value of 'y.'

Example: ∀x : ∃y : father_of (x, y)

Here **'y'** is dependent on **'x',** so we replace **'y'** by **S(x).**

So, **∀x : ∃y : father_of (x, y)** becomes **∀x : ∃y : father_of (x, S(x)).**

Steps to Convert to CNF (Conjunctive Normal Form) …contd

- Eliminate Universal Quantifier
- Eliminate AND
  - To eliminate ‘^’ break the clause into two, if you cannot break the clause, distribute the OR ‘v’ and then break the clause

Example - 1

Problem Statement:

1. Ravi likes all kind of food.
2. Apples and chicken are food
3. Anything anyone eats and is not killed is food
4. Ajay eats peanuts and is still alive
5. Rita eats everything that Ajay eats

Prove by resolution that **Ravi likes peanuts** using resolution

Proof by Resolution

Step 1: Converting the given statements into Predicate/Propositional Logic

i. $\forall x : \text{food}(x) \rightarrow \text{likes}(\text{Ravi}, x)$

ii. food (Apple) ^ food (chicken)

iii. $\forall a : \forall b: \text{eats}(a, b) \wedge \neg \text{killed}(a) \rightarrow \text{food}(b)$

iv. eats (Ajay, Peanuts) ^ alive (Ajay)

v. $\forall c : \text{eats}(\text{Ajay}, c) \rightarrow \text{eats}(\text{Rita}, c)$

vi. $\forall d : \text{alive}(d) \rightarrow \sim\text{killed}(d)$

vii. $\forall e: \sim\text{killed}(e) \rightarrow \text{alive}(e)$

$\neg$

Step 2: Convert into CNF

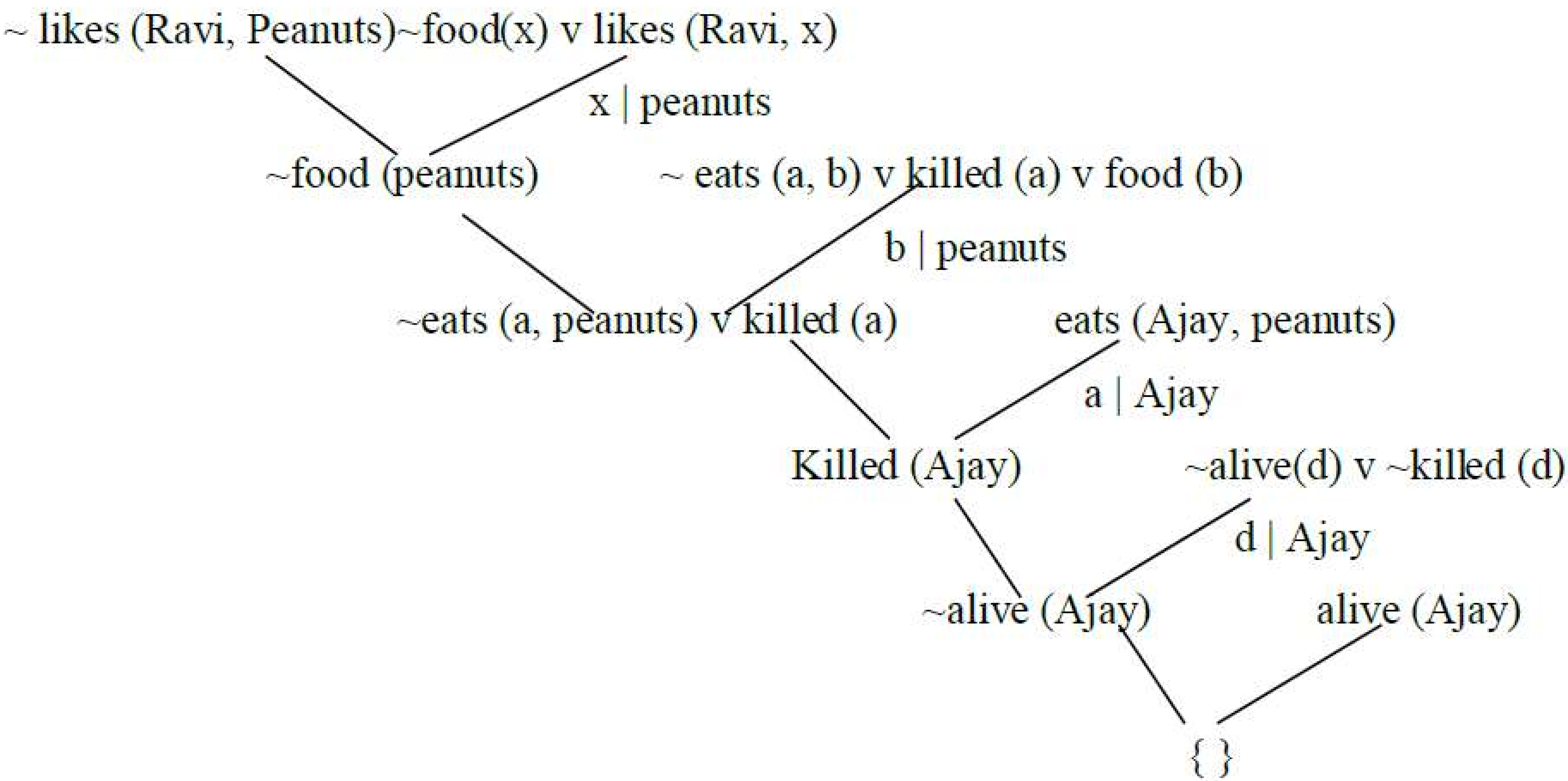
i. ~food(x) v likes (Ravi, x)
ii. Food (apple)
iii. Food (chicken)
iv. ~ eats (a, b) v killed (a) v food (b)
v. Eats (Ajay, Peanuts)
vi. Alive (Ajay)
vii. ~eats (Ajay, c) V eats (Rita, c)
viii. ~alive (d) v ~ killed (d)
ix. Killed (e) v alive (e)

We want to say "**likes (Ravi, Peanuts)**"

Step 3: **Negate** the conclusion

~ likes (Ravi, Peanuts)

Step 4:Resolve using a resolution tree



Example2

The sentences in Predicate Logic are

- $American(x) \wedge Weapon(y) \wedge Sells(x, y, z) \wedge Hostile(z) \Rightarrow Criminal(x)$.
- $Missile(x) \wedge Owns(Nono, x) \Rightarrow Sells(West, x, Nono)$
- $Enemy(x, America) \Rightarrow Hostile(x)$
- $Missile(x) \Rightarrow Weapon(x)$
- Owns(Nono,M1)
- Missile(M1)
- American(West)
- Enemy(Nono,America)

Example2

The sentences in CNF are

- $\neg$American(x) $\vee$ $\neg$Weapon(y) $\vee$ $\neg$Sells(x, y, z) $\vee$ $\neg$Hostile(z) $\vee$ Criminal (x)
- $\neg$Missile(x) $\vee$ $\neg$Owns(Nono, x) $\vee$ Sells(West, x, Nono)
- $\neg$Enemy(x,America) $\vee$ Hostile(x)
- $\neg$Missile(x) $\vee$Weapon(x)
- Owns(Nono,M1)
- Missile(M1)
- American(West)
- Enemy(Nono,America)

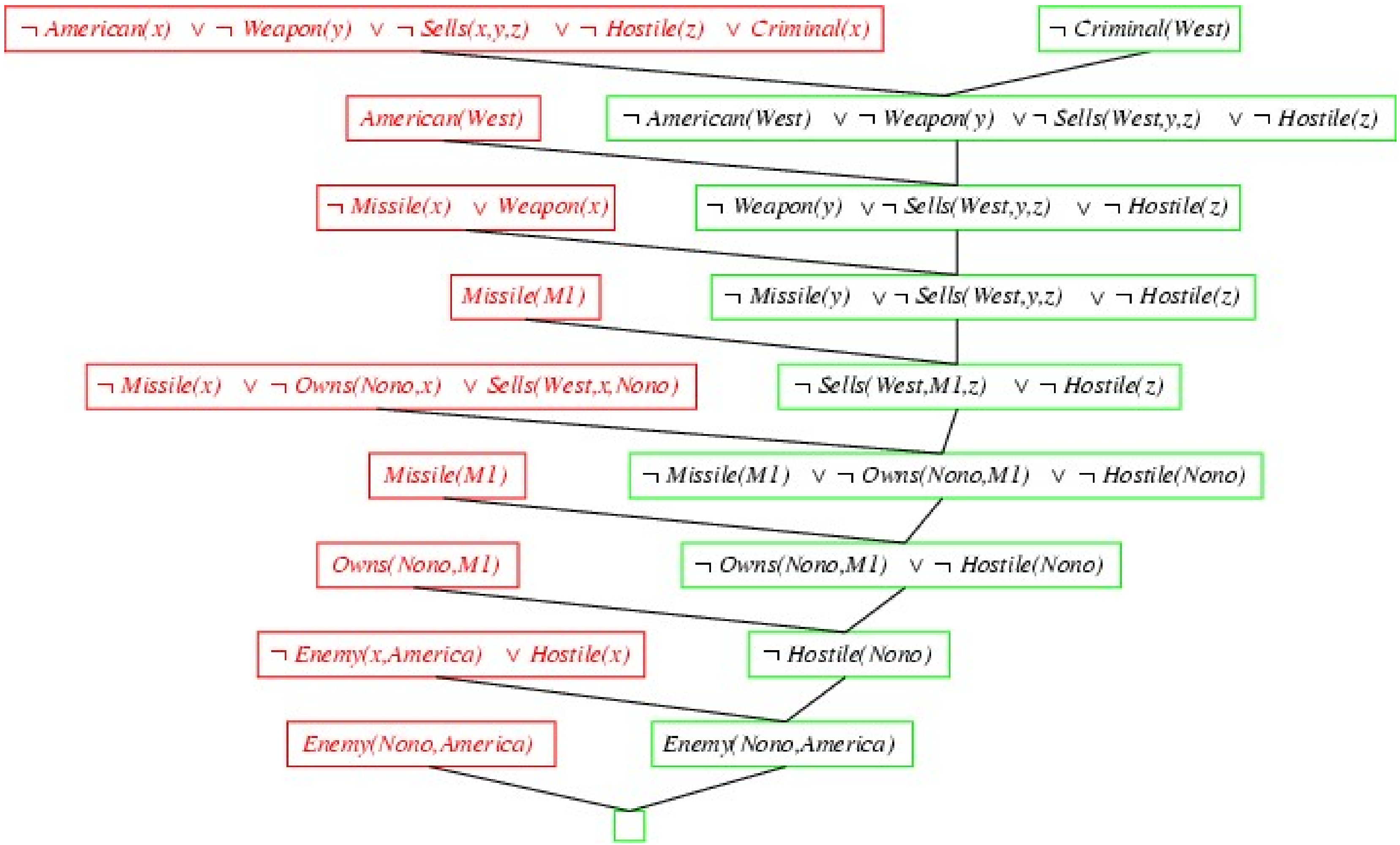
Example2

To Prove Criminal(west)

Negate the Goal as

¬Criminal (West).

Resolution proof: definite clauses



Exercise

- Everyone who loves all animals is loved by someone.
- Anyone who kills an animal is loved by no one.
- Jack loves all animals.
- Either Jack or Curiosity killed the cat, who is named Tuna.
- Did Curiosity kill the cat? – Proof by resolution