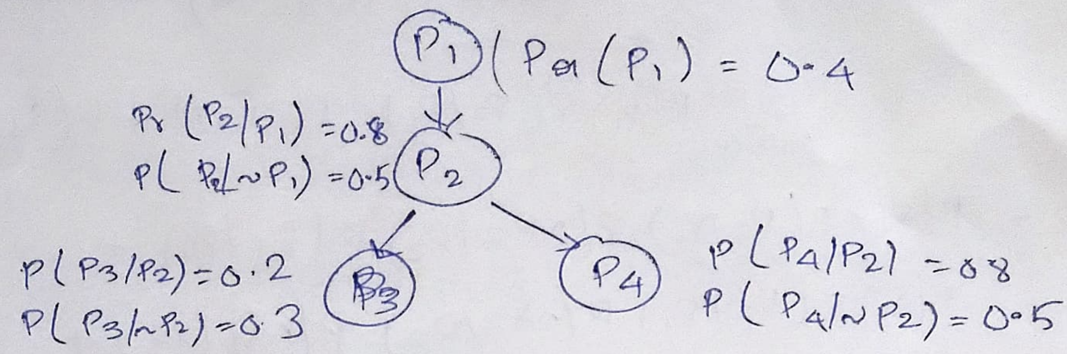
V-Sabarivasan
205001085
CSE-B

AI- Assignment -2



(i) $P(\sim P_3) = \sum_{P_1 P_2 P_3} P(P_1, P_2, \sim P_3, P_4)$

$= \sum_{P_1 P_2 P_4} P(P_1)\, P(P_2/P_1)\, P(\sim P_3/P_2)\, P(P_4/P_2)$

$= P(P_1)\, P(P_2/P_1)\, P(\sim P_3/P_2)\, P(P_4/P_2) +$
$P(\sim P_1)\, P(P_2/\sim P_1)\, P(\sim P_3/P_2)\, P(P_4/P_2) +$
$P(\sim P_1)\, P(\sim P_2/\sim P_1)\, P(\sim P_3/\sim P_2)\, P(P_4/\sim P_2) +$
$P(\sim P_1)\, P(P_2/\sim P_1)\, P(\sim P_3/P_2)\, P(P_4/\sim P_2) +$
$P(P_1)\, P(\sim P_2/P_1)\, P(\sim P_3/\sim P_2)\, P(P_4/\sim P_3) +$
$P(P_1)\, P(P_2/P_1)\, P(\sim P_3/P_2)\, P(\sim P_4/P_2) +$
$P(P_1)\, P(\sim P_2/P_1)\, P(\sim P_3/\sim P_2)\, P(\sim P_4/\sim P_2) +$
$P(\sim P_1)\, P(\sim P_2/\sim P_1)\, P(\sim P_3/\sim P_2)\, P(\sim P_4/\sim P_2)$

$= (0.4 \times 0.8 \times 0.8 \times 0.8) + (0.6 \times 0.5 \times 0.8 \times 0.8) +$
$(0.6 \times 0.5 \times 0.7 \times 0.5) + (0.6 \times 0.5 \times 0.8 \times 0.2) +$
$(0.4 \times 0.2 \times 0.7 \times 0.5) + (0.4 \times 0.2 \times 0.7 \times 0.5) +$
$(0.4 \times 0.8 \times 0.8 \times 0.2) + (0.6 \times 0.5 \times 0.7 \times 0.5)$

$= 0.762$

(ii) $P(P_2|\sim P_3) = \dfrac{P(P_2, \sim P_3)}{P(\sim P_3)}$

$P(P_2, \sim P_3) = \sum_{P_1, P_4} P(P_1, P_2, \sim P_3, P_4)$

$= \sum_{P_1 P_4} P(P_1) P(P_2/P_1) \; P(\sim P_3/P_2) \; P(P_4/P_2)$

$= P(P_1) P(P_2/P_1) P(\sim P_3/P_2) P(P_4/P_2) +$
$P(P_1) P(P_2/P_1) P(\sim P_3/P_2) \; P(\sim P_4/P_2) +$
$P(\sim P_1) P(P_2/\sim P_1) \; P(\sim P_3/P_2) \; P(P_4/P_2) +$
$P(\sim P_1) P(P_2/\sim P_1) \; P(\sim P_3/P_2) \; P(\sim P_4/P_2)$

$= (0.4 \times 0.8 \times 0.8 \times 0.8) + (0.4 \times 0.8 \times 0.8 \times 0.2) +$
$(0.6 \times 0.5 \times 0.8 \times 0.8) + (0.6 \times 0.5 \times 0.8 \times 0.2).$

$= 0.496$ //

$\therefore P(P_2/\sim P_3) = \dfrac{P(P_2, \sim P_3)}{P(\sim P_3)} = \dfrac{0.496}{0.762} = 0.6509$ /

(iii) $P(P_1/(P_2, \sim P_3)) = \dfrac{P(P_1, P_2, \sim P_3)}{P(P_2, \sim P_3)}$

$\Rightarrow P(P_1, P_2, \sim P_3) = \sum_{P_4} P(P_1, P_2, \sim P_3, P_4)$

$= \sum_{P_4} P(P_1) P(P_2/P_1) \; P(P_3/P_2) \; P(P_4/P_2)$

$= P(P_1) P(P_2/P_1) P(\sim P_3/P_2) \; P(P_4/P_2) +$
$P(P_1) P(P_2/P_1) P(\sim P_3/P_2) \; P(\sim P_4/P_2)$

$= (0.4 \times 0.8 \times 0.8 \times 0.8) + (0.4 \times 0.8 \times 0.8 \times 0.2)$

$= 0.256$ //

$\therefore P(P_1/(P_2, \sim P_3)) = \dfrac{P(P_1, P_2, \sim P_3)}{P(P_2, \sim P_2)} = \dfrac{0.256}{0.496} = 0.5161$

(iv) $P(P_1 | (\sim P_3, P_4)) = \dfrac{P(P_1, \sim P_3, P_4)}{P(\sim P_3, P_4)}$

$\Rightarrow P(P_1, \sim P_3, P_4) = \sum_{P_2} P(P_1, P_2, \sim P_3, P_4)$

$= \sum_{P_2} P(P_1) P(P_2|P_1) P(\sim P_3|P_2) P(P_4|P_2)$

$= P(P_1) P(P_2|P_1) P(\sim P_3|P_2) P(P_4|P_2) +$
$P(P_1) P(\sim P_2|P_1) P(\sim P_3|\sim P_2) P(P_4/\sim P_2)$

$= (0.4 \times 0.5 \times 0.8 \times 0.8) + (0.4 \times 0.2 \times 0.7 \times 0.5) \cdot 0.2328$

$\Rightarrow P(\sim P_3, P_4) = P(P_1, \sim P_3, P_4) + P(\sim P_1, \sim P_3, P_4)$

$\Rightarrow P(\sim P_1, \sim P_3, P_4) = \sum_{P_2} P(\sim P_1, P_2, \sim P_3, \sim P_4)$

$= \sum_{P_2} (P(P_1) P(P_2|\sim P_1) P(\sim P_3|P_2) P(P_4|P_2)$

$= P(\sim P_1) P(P_2|\sim P_1) P(\sim P_3|P_2) P(P_4|P_2) +$
$P(\sim P_1) P(\sim P_2|\sim P_1) P(\sim P_3|\sim P_2) P(P_4|\sim P_2)$

$= (0.6 \times 0.5 \times 0.8 \times 0.8) + (0.6 \times 0.5 \times 0.7 \times 0.5) = 0.297$

$\therefore P(\sim P_3, P_4) = P(P_1, \sim P_3, P_4) + P(\sim P_1, \sim P_3, P_4)$

$= 0.2328 + 0.297 = 0.5298$

---

2 When calculating $P(\sim P_3)$ and $P(P_2, \sim P_3)$, we can remove P_4 from expression since P_4 is leaf that is not a query or evidence.

$$P(\sim P_3) = \sum_{P_1 P_2} P(P_1, P_2, \sim P_3)$$

$$= \sum_{P_1 P_2} P(P_1) P(P_2 | P_1) P(\sim P_3 | P_2)$$

$$= P(P_1) P(P_2 | P_1) P(\sim P_3 | P_2) + P(P_1) P(\sim P_2 | P_1) P(\sim P_3 | \sim P_2)$$
$$+ P(\sim P_1) P(\sim P_2 | \sim P_1) P(\sim P_3 | \sim P_2) + P(\sim P_1) \cdot P(P_2 | \sim P_1) P(\sim P_3 | P_2)$$

$$= (0.4 \times 0.8 \times 0.8) + (0.4 \times 0.2 \times 0.7) + (0.6 \times 0.5 \times 0.7)$$
$$+ (0.6 \times 0.5 \times 0.8) = 0.762$$

Illy,

$$P(P_2, \sim P_3) = \sum_{P_1} (P(P_1, P_2, \sim P_3)) = \sum_{P_1} P(P_1) P(P_2 | P_1) P(\sim P_3 | P_2)$$
$$= P(P_1) P(P_2 | P_1) P(\sim P_3 | P_2) + P(\sim P_1) P(P_2 | \sim P_1) P(\sim P_3 | P_2)$$
$$= (0.4 \times 0.8 \times 0.8) + (0.6 \times 0.5 \times 0.8) = 0.496$$

$$\therefore P(P_2 | \sim P_3) = \frac{P(P_2, \sim P_3)}{P(\sim P_3)} = \frac{0.496}{0.762} = 0.6509$$

$P(P_1 | (P_2, \sim P_3))$ simplifies to $P(P_1 | P_2)$ as P_3, P_4 become unqueried and unobserved leaves

$$\therefore P(P_1 | (P_2, \sim P_3)) = P(P_1 | P_2) = \frac{P(P_1, P_2)}{P(P_2)}$$
$$= \frac{P(P_1) P(P_2 | P_1)}{P(P_2)} = \frac{0.4 \times 0.8}{0.62} = 0.5161$$

$\Rightarrow P(P_1 | (\sim P_3, P_4)) = 0.5298$ as calculated the same way as it cannot be simplified further.