CAT I

Answer key.

Part - A.

1. $L^* = \bigcup_{i=0}^{\infty} L^i = L^0 \cup L^1 \cup L^2 \cdots$

2. $00(000)^*$

3. Let Σ be an alphabet. The regular expressions over Σ and the sets that they denote are defined recursively as follows.

(i) ϕ is a RE $\rightarrow L = \{\}$

(ii) $\varepsilon \rightarrow$ RE $RL = \{\varepsilon\}$

(iii) $\forall a \in \Sigma$, RE $= a$ $L = \{a\}$

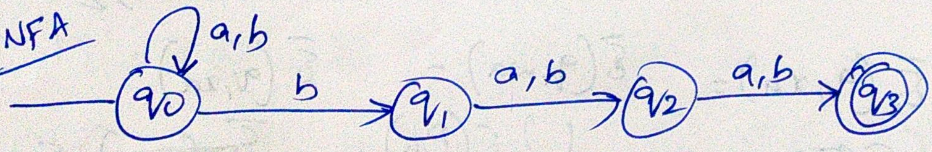
(iv) r, s are RE's $L = R, S$

(r/s) $(r+s)$ $L = R \cup S$

rs $L = RS$

r^* $L = R^*$

4. NFA



5. $00(0/1)^*11$

6. Set of all strings having 3 consecutive 0's. over $\Sigma = \{0,1\}$

Part - B

7.

ε-closure $(p) = \{p, q, r\}$

ε-closure $(q) = \{q\}$

ε-closure $(r) = \{p, q, r\}$

ε-NFA – NFA.

δ	0	1
$\rightarrow p$	$\{p, q, r\}$	$\{p, q, r\}$
q	$\{p, q, r\}$	ϕ
$* r$	$\{p, q, r\}$	$\{p, q, r\}$

8. Theorem, Proof.

9.

	DFA	NFA	ε-NFA
Transition fn. δ	$\delta: Q \times \Sigma \rightarrow Q$	$\delta: Q \times \Sigma \rightarrow 2^Q$	$\delta: Q \times \Sigma \cup \{\varepsilon\} \rightarrow 2^Q$.
Ex. TF. $\hat{\delta}$	$\hat{\delta}(q, xa) = \delta(\hat{\delta}(q, x), a)$	$\hat{\delta}(q, wa) = \bigcup_{p \in \hat{\delta}(q, w)} \delta(p, a)$	$\hat{\delta}(q, xa)$. $\cancel{\hat{\delta}(q, x) = \{p_1 p_2 \ldots p_k\}}$ ε-closure$(\{r_1, r_2 - r_m\})$ ε-closure$(\bigcup_{i=1}^{m} \delta(p_i, a))$
L	$L = \{w / \hat{\delta}(q_0, w) \in F\}$	$L = \{w / \hat{\delta}(q_0, w) \cap F \neq \phi\}$	$\Rightarrow \ldots$

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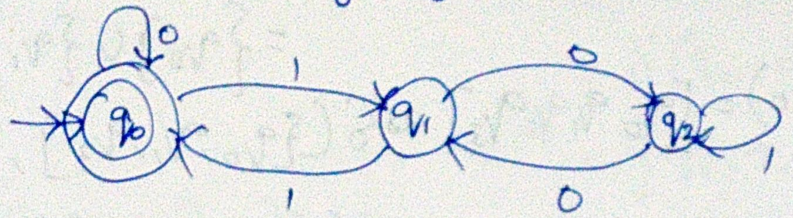
Part - C.

a. DFA divisibly by 3



0
11
110
1001

b. 15 $\rightarrow$ 1111

$\delta(q_0, 1111) = \delta(q_1, 111)$

$= \delta(q_0, 11)$

$= \delta(q_1, 1)$

$= q_0 \in F$

$\therefore$ Accepted.

4. NFA $\longrightarrow$ DFA

$M' = (Q', \Sigma, \delta', [q_0], F')$

$\delta'([q_0], 0) = [q_0 q_1]$ as $\delta(q_0, 0) = \{q_0, q_1\}$

$\delta'([q_0], 1) = [q_0]$ as $\delta(q_0, 1) = \{q_0\}$

$\delta'([q_0 q_1], 0) = [q_0 q_1 q_2]$ as $\delta(\{q_0, q_1\}, 0)$

$= \delta(q_0, 0) \cup \delta(q_1, 0)$

$= \{q_0, q_1, q_2\}$

$\delta'([q_0 q_1], 1) = [q_0 q_1]$ as $\delta(\{q_0, q_1\}, 1)$

$= \delta(q_0, \phi) \cup \delta(q_1, \phi)$

$= \{q_0\} \cup \{q_1\}$

$\delta'([q_0 q_1 q_2], 0) = [q_0 q_1 q_2 q_3]$ as $\delta(\{q_0, q_1, q_2\}, 0)$

$= \delta(q_0, 0) \cup \delta(q_1, 0) \cup \delta(q_2, 0)$

$= \{q_0, q_1, q_2, q_3\}$

$\delta'([q_0 q_1 q_2], 1) = [q_0 q_1 q_3]$ $\delta(\{q_0, q_1, q_2\}, 1)$

$= \delta(q_0, 1) \cup \delta(q_1, 1) \cup \delta(q_2, 1)$

$= \{q_0, q_1, q_3\}$

$\delta'([q_0 q_1 q_2 q_3], 0) = [q_0 q_1 q_2 q_3]$

$\delta(\{q_0, q_1, q_2, q_3\}, 0)$

$= \delta(q_0, 0) \cup \delta(q_1, 0) \cup \delta(q_2, 0) \cup \delta(q_3, 0)$

$= \{q_0, q_1, q_2, q_3\}$

$\delta'([q_0 q_1 q_2 q_3], \phi) = [q_0 q_1 q_2 q_3]$

$\delta(\{q_0, q_1, q_2, q_3\}, 1)$

$= \delta(q_0, 1) \cup \delta(q_1, 1) \cup \delta(q_2, 1) \cup \delta(q_3, 1)$

$= \{q_0, q_1, q_2, q_3\}$

$q_0 q_1 q_3], 0) = [q_0 q_1 q_2]$

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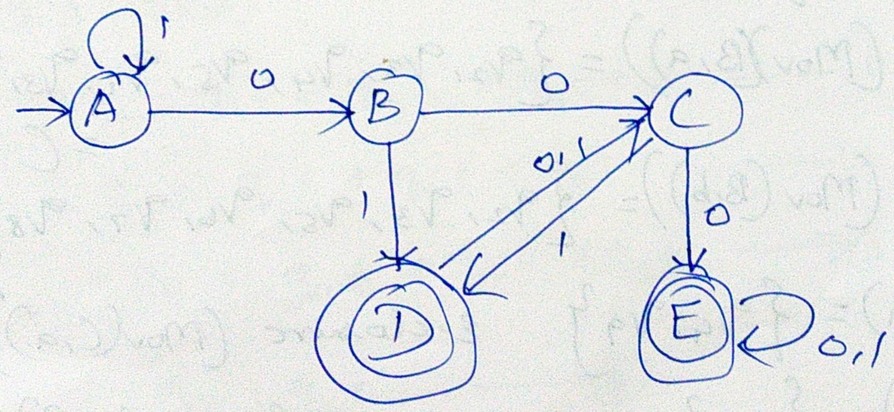
$$\delta(\{q_0, q_1, q_3\}, 0)$$
$$= \delta(q_0, 0) \cup \delta(q_1, 0) \cup \delta(q_3, 0)$$
$$= \{q_0, q_1, q_2\}$$

$$\delta'([q_0 q_1 q_3], 1) = [q_0 q_1 q_2]$$

$$\delta(\{q_0, q_1, q_3\}, 1)$$
$$= \delta(\{q_0\}, 1) \cup \delta(q_1, 1) \cup \delta(q_3, 1)$$
$$= \{q_0, q_1, q_2\}$$

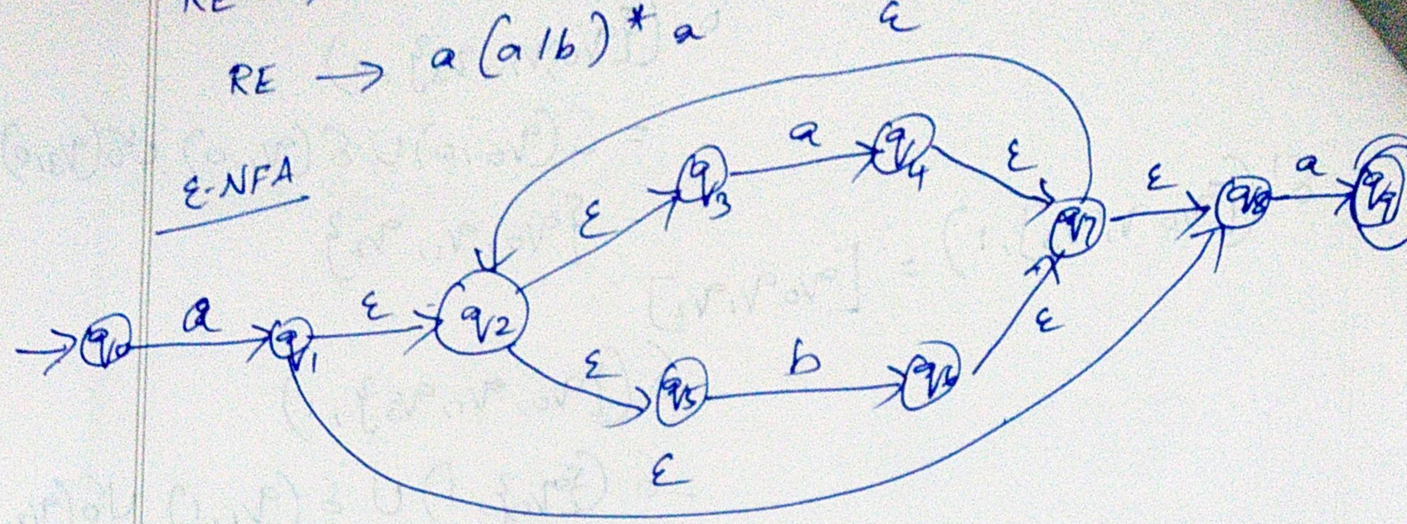
	δ'	0	1
A	$\rightarrow [q_0]$	$[q_0 q_1]$	$[q_0]$
B	$[q_0 q_1]$	$[q_0 q_1 q_2]$	$[q_0 q_1]$
C	$[q_0 q_1 q_2]$	$[q_0 q_1 q_2 q_3]$	$[q_0 q_1 \cancel{q_2} q_3]$
D	$*[q_0 q_1 q_3]$	$[q_0 q_1 q_2]$	$[q_0 q_1 q_2]$
E	$*[q_0 q_1 q_2 q_3]$	$[q_0 q_1 q_2 q_3]$	$[q_0 q_1 q_2 q_3]$



12. RE $\longrightarrow$ Minimized DFA.

RE $\longrightarrow$ $a(a|b)^*a$

ε-NFA



$\varepsilon\text{-closure}(q_0) = \{q_0\} = A.$

$mov(A, a) = \{q_1\}$

$Mov(A, b) = \{\}$

$\varepsilon\text{-closure}(Mov(A, a)) = \{q_1, q_2, q_3, q_5, q_8\} = B$

$\varepsilon\text{-closure}(Mov(A, b)) = \phi$

$Mov(B, a) = \{q_4, q_9\}$

$Mov(B, b) = \{q_6\}$

$\varepsilon\text{-closure}(Mov(B, a)) = \{q_2, q_3, q_4, q_5, q_7, q_8, q_9\} = C$

$\varepsilon\text{-closure}(Mov(B, b)) = \{q_2, q_3, q_5, q_6, q_7, q_8\} = D$

$Mov(C, a) = \{q_4, q_9\}$ $\quad \varepsilon\text{-closure}(Mov(C, a)) = C$

$Mov(C, b) = \{q_6\}$ $\quad \varepsilon\text{-closure}(Mov(C, b)) = D$

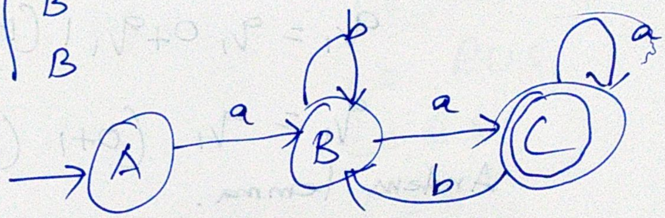
$Mov(D,a) = \{q_4, q_9\}$ $\quad$ $\varepsilon\text{-closure}(Mov(D,a)) = C$

$Mov(D,b) = \{q_6\}$ $\quad$ $\varepsilon\text{-closure}(Mov(D,b)) = D$

δ	a	b
$\rightarrow$A	B	ϕ
B	C	D
*C	C	D
D	C	D

Minimized DFA.

δ	a	b
$\rightarrow$A	B	ϕ
B	C	B
*C	C	B



13. DFA $\rightarrow$ RE.

$q_1 = q_1 0 + q_2 \phi + q_3 0 + \varepsilon$ ——— (1)

$q_2 = q_1 1 + q_2 1 + q_3 1$ ——— (2)

$q_3 = q_1 \phi + q_2 0 + q_3 \phi$ $\longrightarrow$ (3)

$q_3 = q_2 0$ ——— (4)

$q_2 = q_1 1 + q_2 1 + q_3 1$ sub. (4) in (2)

$q_2 = q_1 1 + q_2 1 \; q_2 01$ $RP + RQ = R(P+Q)$

$\underbrace{q_2}_{R} = \underbrace{q_1 1}_{Q} + \underbrace{q_2}_{R} \underbrace{(1+01)}_{P}$

$R = Q + RP$
$R = QP^*$

$q_2 = q_1 1 (1+01)^*$ —— (5)

substitute (5) in (4)

~~$q_3 = q_2 0$~~

$q_3 = q_1 1 (1+01)^* 0$ —— (6)

Substitute (6) in (1)

(1) $\rightarrow$ $q_1 = q_1 0 + q_3 0 + \varepsilon$

$q_1 = q_1 0 + q_1 1 (1+01)^* 00 + \varepsilon$

$q_1 = q_1 \; (0 + 1 \; (1+01)^* 00) + \varepsilon$

Ardens lemma.

$q_1 = \varepsilon \; (0 + 1 \; (1+01)^* 00)^*$

$\therefore$ RE $[0 + 1 (1+01)^* 00]^*$