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CSE - B

ASSIGNMENT - 1

(1)

A - satisfiable

$A \vee B$ - satisfiable

$A \vee \sim A$ - valid

$A \wedge \sim A$ - unsatisfiable

$A - \sim A$ - satisfiable

$A - B$ - satisfiable

$A - (B - A)$ - satisfiable

$A - (A - B)$ - satisfiable

$A - \sim A$ - unsatisfiable.

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(2) (i) Yes (ii) Yes

(iii) No (iv) Yes

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(3) (i) $(p \rightarrow q) \rightarrow (\sim r \wedge q)$

$\Rightarrow (\sim p \vee q) \rightarrow (\sim r \wedge q)$

$\Rightarrow \sim(\sim p \vee q) \vee (\sim r \wedge q)$

$\Rightarrow (p \wedge \sim q) \vee (\sim r \wedge q) \Longrightarrow$ DNF

$\Rightarrow ((p \wedge \sim q) \vee \sim r)) \wedge ((p \wedge \sim q) \vee q)$

$\Rightarrow (p \vee \sim r) \wedge (\sim q \vee \sim r) \wedge (p \vee q) \wedge T$

$\Rightarrow (p \vee \sim r) \wedge (\sim q \vee \sim r) \wedge (p \vee q)$

$\hookrightarrow$ CNF

(ii) $((\sim A \rightarrow B) \vee ((A \wedge \sim C) \leftrightarrow B))$

$\Rightarrow ((A \vee B) \vee [((A \wedge \sim C) \rightarrow B) \wedge (B \rightarrow (A \wedge \sim C))$

$\Rightarrow ((A \vee B) \vee ([\sim (A \wedge \sim C) \vee B] \wedge [\sim B \vee (A \wedge \sim C)])$

$\Rightarrow ((A \vee B) \vee ((\sim A \vee C \vee B) \wedge ((\sim B \vee A) \wedge (\sim B \vee \sim C)))$

$\Rightarrow (A \vee B) \vee [(\sim A \vee B \vee C) \wedge (\sim B \vee A) \wedge (\sim B \vee \sim C)]$

$\Rightarrow ((A \vee B) \vee (\sim A \vee B \vee C)) \wedge ((A \vee B) \vee (\sim B \vee A)) \wedge ((A \vee B) \vee (\sim B \vee \sim C))$

$\Rightarrow (T) \wedge (T) \wedge (T) \wedge (T)$

$\Rightarrow T$ //

(4)

p	q	r	f
1	1	1	1
1	1	0	0
1	0	1	1
1	0	0	0
0	1	1	0
0	1	0	0
0	0	1	1
0	0	0	0

DNF : $(p \wedge q \wedge r) \vee (p \wedge \sim q \wedge r)$
$\vee (\sim p \wedge \sim q \wedge r)$

CNF : $(\sim p \vee \sim q \vee r) \wedge (\sim p \vee q \vee r)$
$\wedge (p \vee \sim q \vee \sim r) \wedge (p \vee \sim q \vee r)$
$\wedge (p \vee q \vee r)$

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(5) Horn Formula :

(i) $(P_5 \Rightarrow P_{11}) \wedge (P_2 \wedge P_3 \wedge P_5 \rightarrow P_{13})$
$\wedge (T \rightarrow P_5) \wedge (P_5 \wedge P_{11} \rightarrow \perp)$

1) $T \rightarrow P_5$ $\therefore$ P_5 is T

2) $P_5 \rightarrow P_{11}$ $\Rightarrow$ $T \rightarrow P_{11}$
$\therefore$ P_{11} is T

3) $P_5 \wedge P_{11} \rightarrow \perp$ $\Rightarrow$ $T \rightarrow \perp$
$\therefore$ The given formula is unsatisfiable.

(ii) $(T \to q) \wedge (T \to s) \wedge (w \to \bot)$
$\wedge (p \wedge q \wedge s \to \bot) \wedge (v \to s)$
$\wedge (T \to r) \wedge (r \to p)$

1) $T \to q$, $T \to s$, $T \to r$

$\therefore q, r, s$ T

2) $r \to p \Rightarrow T \to p$

$\therefore p$ T

3) $p \wedge q \wedge s \to \bot$

$\Rightarrow T \to \bot$

$\therefore$ formula is unsatesfiable.

(iii) $(T \to q) \wedge (T \to s) \wedge (wV \to \bot) \wedge$
$(p \wedge q \wedge s \to v) \wedge (v \to s) \wedge (T \to r) \wedge (r \to p)$

1) $T \to r$, $T \to q$, $T \to s$

r, q, s T

2) $r \to p \Rightarrow T \to p$

$\therefore p$ T

3) $p \wedge q \wedge s \to v \Rightarrow T \to v$

$\therefore v$ T

$\therefore$ Formula is satesfiable

(iv) $\sim b \wedge (\sim a \vee b \vee \sim c) \wedge a \wedge (\sim a \vee c)$

$\Rightarrow (T \rightarrow \sim b) \wedge (\sim (a \wedge \sim b) \vee \sim c) \wedge (T \rightarrow a) \wedge (a \rightarrow c)$

$\Rightarrow (T \rightarrow \sim b) \wedge ((a \wedge \sim b) \rightarrow \sim c) \wedge (T \rightarrow a) \wedge (a \rightarrow c)$

1) $T \rightarrow \sim b$, $T \rightarrow a$

$\therefore a, \sim b$ T

2) $a \wedge \sim b \rightarrow \sim c$

$\Rightarrow T \rightarrow \sim c$ $\therefore \sim c$ T

3) $a \rightarrow c \Rightarrow T \rightarrow c$

Formula unsatisfiable.

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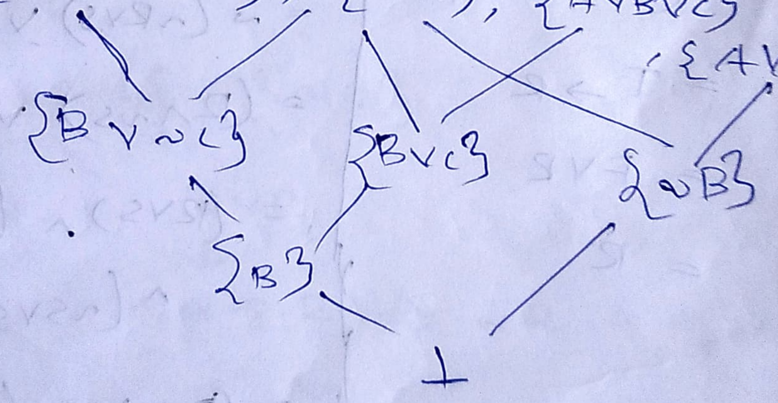
(6) $F = \{\{A, B, \sim c\}, \{\sim A\}, \{A, B, c\}, \{A, \sim B\}\}$

$F = \{A \vee B \vee \sim c\}, \{\sim A\}, \{A \vee B \vee c\}, \{A \vee \sim B\}$



$\therefore F$ is unsatisfiable

⑦

1) $(P \to Q) \to R$

2) $(P \to P) \to R$

3) $(R \to S) \to \sim(S \to Q)$

Prove R

$\Rightarrow$ Take negation of conclusion

4) $\sim R$

Convert to CNF,

1) $\Rightarrow \sim(\sim P \vee Q) \vee R$

$= (P \wedge \sim Q) \vee R$

$= (P \vee R) \wedge (\sim Q \vee R) = (P \vee R) \wedge (\sim Q \vee R)$

2) $(P \to P) \to R$

$= (\sim P \vee P) \to R$

$= T \to R$

$= F \vee R$

$= R$

3) $(R \to S) \to \sim(S \to Q)$

$= \sim(\sim R \vee S) \vee \sim(\sim S \vee Q)$

$= (R \wedge \sim S) \vee (S \wedge \sim Q)$

$= (R \vee S) \wedge (R \vee \sim Q) \wedge (\sim S \vee S) \wedge (\sim S \vee \sim Q)$

$= (R \vee S) \wedge (R \vee \sim Q) \wedge (\sim S \vee \sim Q)$

$P \vee R$ $\quad \sim Q \vee R$ $\quad R$ $\quad R \vee S$ $\quad R \vee \sim Q$ $\quad \sim S \vee \sim Q$ $\quad \sim R$

$\perp$

(Contradiction)

$\therefore$ R can be proved

(8) $\Sigma = \{ (\sim(p\wedge q)), (p\wedge q) \} \models (p \Leftrightarrow q)$

p	q	$\sim(p\wedge q)$	$p\wedge q$	$\{a,b\}$	$p\Leftrightarrow q$
0	0	1	0	0	1
0	1	1	0	0	0
1	0	1	0	0	0
1	1	0	1	0	1

$\Rightarrow \Sigma \not\models (p \Leftrightarrow q)$

since $\{a,b\}$ and $p \Leftrightarrow q$ doesn't match.

(9) $P \rightarrow Q$

$\Rightarrow \sim P \vee Q$

$\sim P \rightarrow R$

$\Rightarrow P \vee R$

$\{\sim P \vee Q\}, \{P \vee R\}$

$Q \vee R$

$\Downarrow$

$\sim Q \rightarrow R \not\Rightarrow \sim Q \rightarrow \sim R$

(or)

$\sim(\sim Q \rightarrow \sim R) \Rightarrow \sim(Q \vee \sim R)$

$\Rightarrow \sim Q \wedge R$

$\therefore \{\sim P \vee Q\}, \{P \vee R\}, \{\sim Q\}, \{R\}$

$\sim P$

$R \neq \bot$

$\therefore$ Formula is not unsatisfiable.