UCS1503
THEORY OF COMPUTATION
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CSE-A

TUTORIAL

1. Grammar $G = (\{S, A\}, \{a, b\}, P, S)$

$$\left.\begin{array}{l} S \rightarrow aAS \mid b \\ A \rightarrow SbA \mid ba. \end{array}\right\} P$$

Given word $w = abbbab$

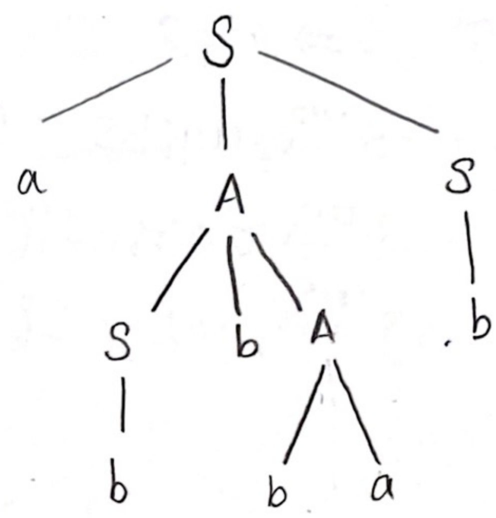
$S \overset{LMD}{\Longrightarrow} aSbAS \quad (A \rightarrow SbA)$

$S \overset{LMD}{\Longrightarrow} abbAS \quad (S \rightarrow b)$

$S \overset{LMD}{\Longrightarrow} abbbaS \quad (A \rightarrow ba)$

$S \overset{LMD}{\Longrightarrow} abbbab \quad (S \rightarrow b)$

Equivalent parse tree



2. Grammar G
with $S \rightarrow SbS \mid a$.

Let us consider a string $w = ababa$

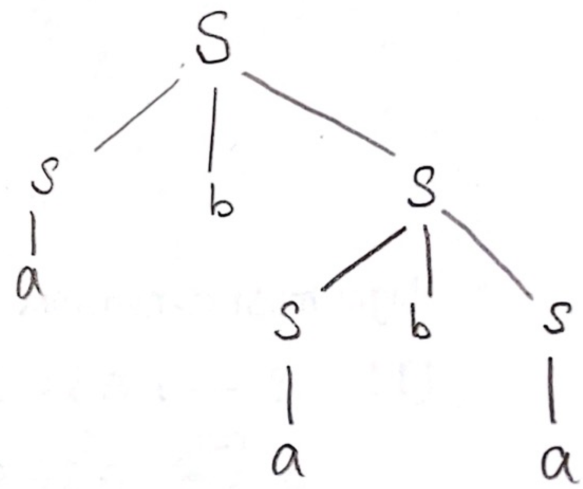
I: $S \overset{LMD}{\Longrightarrow} abS \quad (S \rightarrow a)$

$S \overset{LMD}{\Longrightarrow} abSbS \quad (S \rightarrow SbS)$

$S \overset{LMD}{\Longrightarrow} ababS \quad (S \rightarrow a)$

$S \overset{LMD}{\Longrightarrow} ababa \quad (S \rightarrow a)$

parse tree $\Longrightarrow$



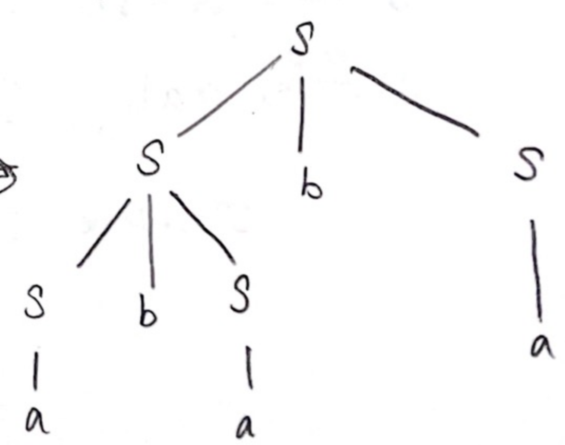
II: $S \overset{LMD}{\Longrightarrow} SbSbS \quad (S \rightarrow SbS)$

$S \overset{LMD}{\Longrightarrow} abSbS \quad (S \rightarrow a)$

$S \overset{LMD}{\Longrightarrow} ababS \quad (S \rightarrow a)$

$S \overset{LMD}{\Longrightarrow} ababa \quad (S \rightarrow a)$

$\Longrightarrow$



Since 2 different LMDS and parse trees exist for string $w = ababa$, grammar G is said to be ambiguous.

3) for grammar,

$$S \rightarrow aS \mid aSbS \mid \varepsilon$$

String $w = aab$.

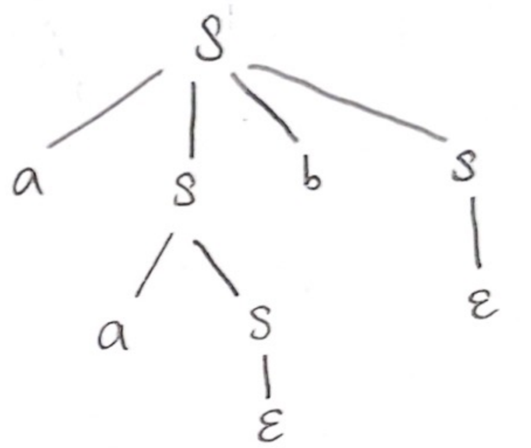
Leftmost derivations.

(I) $S \Rightarrow aSbS$

$S \xRightarrow{LMD} aaSbS \quad (S \rightarrow aS)$

$S \xRightarrow{LMD} aabS \quad (S \rightarrow \varepsilon)$

$S \xRightarrow{LMD} aab \quad (S \rightarrow \varepsilon)$

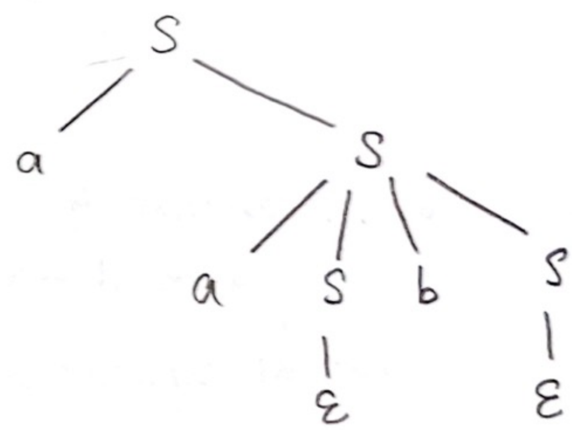


(II) $S \rightarrow aS$

$S \xRightarrow{LMD} aaSbS \quad (S \rightarrow aSbS)$

$S \xRightarrow{LMD} aabS \quad (S \rightarrow \varepsilon)$

$S \xRightarrow{LMD} aab \quad (S \rightarrow \varepsilon)$



Therefore proved that 2 LMDS &
2 different parse trees exist.

Right-most derivations

(I) $S \rightarrow aSbS$

$S \xRightarrow{RMD} aSb\varepsilon \quad (S \rightarrow \varepsilon)$

$S \xRightarrow{RMD} aaSb \quad (S \rightarrow aS)$

$S \xRightarrow{RMD} aab \quad (S \rightarrow \varepsilon)$.

(II) $S \longrightarrow aS$

$S \xrightarrow{RMD} aaSbS \quad (S \rightarrow aSbS)$

$S \xRightarrow{RMD} aaSb\varepsilon \quad (S \rightarrow \varepsilon)$

$S \xRightarrow{RMD} aab \quad (S \rightarrow \varepsilon)$.

Therefore, string $w = aab$ has 2 right most derivations.

4) for grammar,

$$S \rightarrow A1B$$
$$A \rightarrow 0A \mid \varepsilon$$
$$B \rightarrow 0B \mid 1B \mid \varepsilon$$

String $w = 00101$.

Leftmost derivation.

$S \rightarrow A1B$

$S \xrightarrow{LMD} 0A1B \quad (A \rightarrow 0A)$

$S \xrightarrow{LMD} 00A1B \quad (A \rightarrow 0A)$

$S \xrightarrow{LMD} 00\varepsilon 1B \quad (A \rightarrow \varepsilon)$

$S \xrightarrow{LMD} 0010B \quad (B \rightarrow 0B)$

$S \xrightarrow{LMD} 00101B \quad (B \rightarrow 1B)$

$S \xrightarrow{LMD} 00101\varepsilon \quad (B \rightarrow \varepsilon)$

$S \Rightarrow 00101$.

Rightmost derivation

$S \rightarrow A1B$

$S \xrightarrow{RMD} A10B \quad (B \rightarrow 0B)$

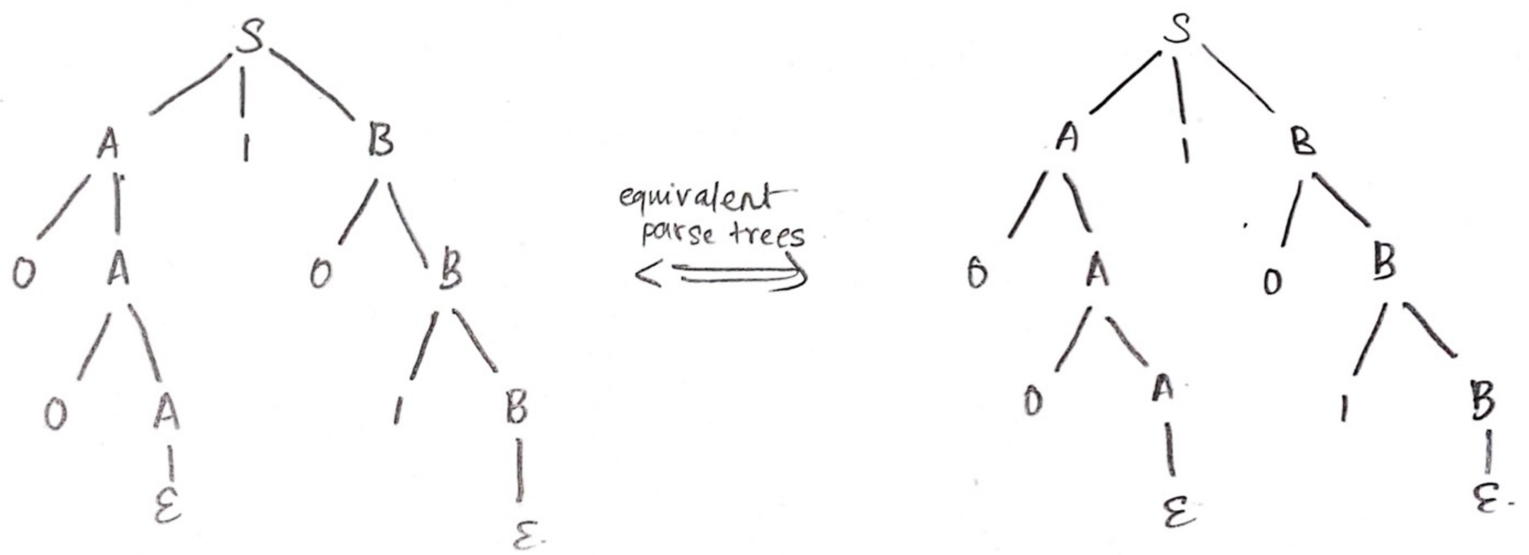
$S \xrightarrow{RMD} A101B \quad (B \rightarrow 1B)$

$S \xrightarrow{RMD} A101\varepsilon. \quad (B \rightarrow \varepsilon)$

$S \xrightarrow{RMD} 0A101 \quad (A \rightarrow 0A)$

$S \xrightarrow{RMD} 00A101 \quad (A \rightarrow 0A)$

$S \xrightarrow{RMD} 00101 \quad (A \rightarrow \varepsilon)$.



5). CFG to generate $\{a^n b^n \mid n \in Z^+\}$

$L = \{ab, aabb, aaabbb \ldots\}$.

Context-free grammar

$$S \rightarrow aSb \mid ab$$

6) Alphabet $\Sigma = \{a, b, (,), +, *, ., \varepsilon\}$.

Context-free grammar

$$G = (\{S\}, \{a, b, (,), +, *, ., \varepsilon\}, P, S\}$$

P consists of

$S \rightarrow S$

$S \rightarrow SS$

$S \rightarrow a$

$S \rightarrow b$

$S \rightarrow \varepsilon$

$S \rightarrow S*$

$S \rightarrow S+S$

7) $S \to a|aA|B$
$A \to aBB|\varepsilon$
$B \to Aa|b$

Step1: Check if there are ε-productions/unit production

- No useless symbols.
- Elimination of ε-productions
  - $B \to Aa|b$ Since $A \to \varepsilon$, A is nullable.

    $B \to Aa|a|b$.

  - $S \to a|aA|a|B$.

  A has other productions.

  $\therefore$ $S \to a|aA|B$
  $A \to aBB$
  $B \to Aa|a|b$

- Unit production elimination

  $S \to a|aA|Aa|b$.
  $A \to aBB$.
  $B \to Aa|a|b$.

Step2: Eliminate terminals by introducing

$C_a \to a$
$C_b \to b$

~~$S \to C_a|C_aA|AC_a|C_b$.~~ $S \to C_aA|AC_a|a|b$.

~~$A \to C_aBB$.~~ $A \to C_aBB$. $C_a \to a$

~~$B \to AC_a|C_a|C_b$.~~ $B \to AC_a|a|b$. $C_b \to b$.

~~Is in~~. chomsky normal form.

where every production is of the form $A \to a$ or $A \to BC$ and $S \to \varepsilon$ if $\varepsilon \in L(G)$. $A, B, C \in NT$ symbols.

Step3: Long RHS production elimination

$A \to C_aBB$. $D_a \to BB$
$A \to C_aD_a$

$\therefore$ $S \to C_aA|AC_a|a|b$.
$A \to C_aD_a$.
$D_a \to BB$
$B \to AC_a|a|b$. is in CNF.

8) Context free grammar.

$S \to XB \mid AA$.
$A \to a \mid BA \mid AB$
$B \to b$.
$X \to a$

STEP1:
It's already in CNF. Therefore, there are no ε-productions, unit productions or useless productions.

Renaming $S = A_1$, $A = A_2$, $B = A_3$, $X = A_4$.

$A_1 \to A_4A_3 \mid A_2A_2$.
$A_2 \to a \mid A_3A_2 \mid A_2A_3$.
$A_3 \to b$.
$A_4 \to a$.

Step 2: $A_i \to A_j\gamma$ $j > i$

$A_1 \to A_4A_3 \mid A_2A_2$ ✓ $\quad A_2 \to a \mid A_3A_2$ ✓.
$A_3 \to b$ $\quad A_2 \to A_2A_3$ $i = j$
$A_4 \to a$.

Step 3: Eliminating left recursions in $A_2 \to A_2\underbrace{A_3}_{\alpha_1} \mid \underbrace{A_3A_2}_{\beta_1} \mid \underbrace{a}_{\beta_2}$.

If $A \to A\alpha_1 \mid A\alpha_2 \ldots \mid \beta_1 \mid \beta_2 \ldots$
is converted to.

$A \to \beta_1 \mid \beta_2 \ldots$
$A \to \beta_1 B \mid \beta_2 B \ldots$
$B \to \alpha_1 \mid \alpha_2 \ldots$
$B \to \alpha_1 B \mid \alpha_2 B \ldots$

$A_2 \to A_3A_2 \mid a$.
$A_2 \to A_3A_2B_2 \mid aB_2$.
$B_2 \to A_3$.
$B_2 \to A_3B_2$.

Step 4: modify A_i productions.

~~$A_1 \to aA_3 \mid$~~ $\quad (\because A_2 \to A_3A_2 \mid a \mid A_3A_2B_2 \mid aB_2)$

$A_4 \to a$ $\quad A_3 \to b$.
$A_2 \to bA_2 \mid a \mid bA_2B_2 \mid aB_2$.
$A_1 \to aA_3 \mid bA_2A_2 \mid aA_2 \mid bA_2B_2A_2 \mid aB_2A_2$.

Step 5: modify B_i productions.

$B_2 \to b \mid bB_2$,

Is in Griebach normal form

$A_1 \to aA_3 \mid bA_2A_2 \mid aA_2 \mid bA_2B_2A_2 \mid aB_2A_2$ $\quad A_2 \to bA_2 \mid a \mid bA_2B_2 \mid aB_2$.
$A_3 \to b$ $\quad A_4 \to a$

9) Push Down Automata construction:

(a) $L = \{a^n b^m c^n / m, n >= 1\}$

Final state: q_3.
Initial state: q_0
$\Sigma = \{a, b, c\}$.

$L = \{abc, aabcc, abbc, aabbcc, aaabccc, \ldots\}$.

Step 1: Push first symbol onto stack

$\delta(q_0, a, z) = \{(q_0, az)\}$.

Step 2: push remaining ones into stack.

$\delta(q_0, a, a) = \{(q_0, aa)\}$.

Step 3: Switch state when first b is read.

$\delta(q_0, b, a) = \{(q_1, a)\}$.

Step 4: Reading consecutive bs.

$\delta(q_1, b, \varepsilon) = \{(q_1, \varepsilon)\}$

Step 5: switch state when all b's are read.

$\delta(q_1, \varepsilon, b) = \{(q_2, \varepsilon)\}$

Step 6: popping ~~all the~~.

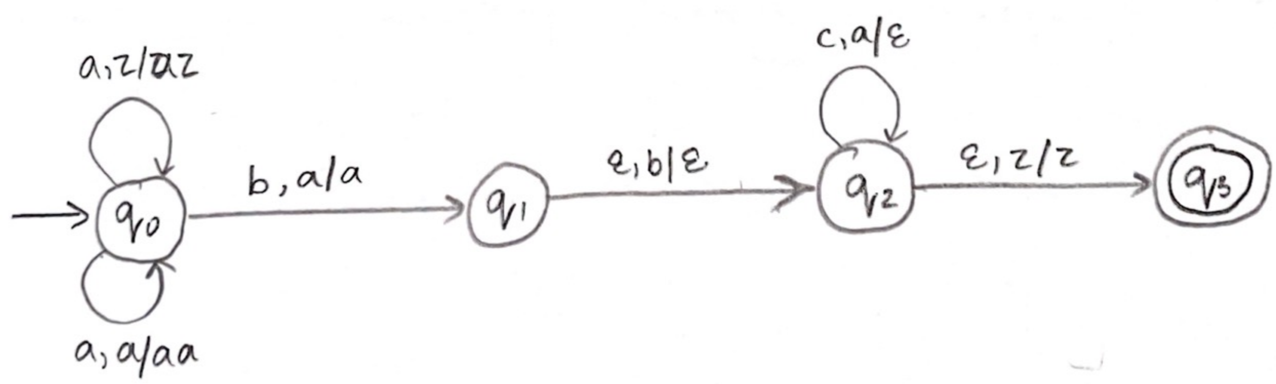
$\delta(q_2, c, a) = \{(q_2, \varepsilon)\}$.

Step 7: Final state

$\delta(q_2, \varepsilon, Z) = \{(q_3, z)\}$.

$\Gamma = \{a, z\}$.

PDA $M = (\{q_0, q_1, q_2, q_3\}, \{a, b, c\}, \Gamma, q_0, z_0, q_3)$.



(b) $L = \{ wcw^R \mid w \in (a|b)^*, c \in a|b|\varepsilon \}$.

$L = \{ ababa, abba, abbba, \varepsilon, aa, bb, aba \ldots \}$.

(a) pushing onto stack.

$\delta(q_0, a, z) = \{(q_0, az)\}$ $\delta(q_0, b, z) = \{(q_0, bz)\}$.

(b) push remaining symbols.

$\delta(q_0, a, a) = \{(q_0, aa)\}$

$\delta(q_0, a, b) = \{(q_0, ab)\}$

$\delta(q_0, b, a) = \{(q_0, ba)\}$

$\delta(q_0, b, b) = \{(q_0, bb)\}$.

(c) moving states

$\delta(q_0, a, \varepsilon) = \{(q_1, \varepsilon)\}$.

$\delta(q_0, b, \varepsilon) = \{(q_1, \varepsilon)\}$

$\delta(q_0, \varepsilon, z) = \{(q_1, z)\}$.

(d) popping symbols.

$\delta(q_1, a, a) = \{(q_1, \varepsilon)\}$

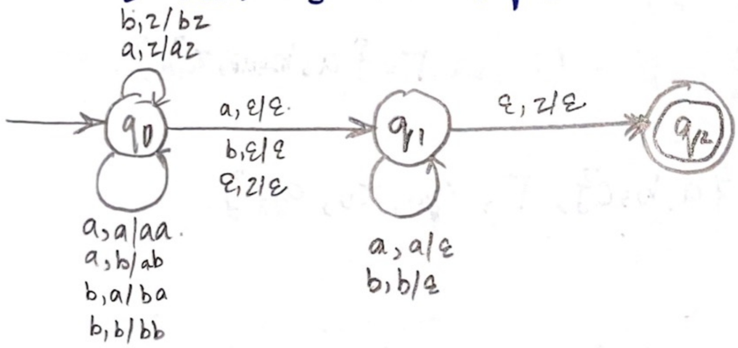
$\delta(q_1, b, b) = \{(q_1, \varepsilon)\}$

(e) Reaching final state.

$\delta(q_1, \varepsilon, z) = \{(q_2, z)\}$.

$M = (Q, \Sigma, \Gamma, q_0, z_0, F, \delta)$. $Q = \{q_0, q_1, q_2\}$ $q_0 \to q_0$.

$\Gamma = \{a, b, z_0\}$. $F = \{q_2\}$.



(c) $L = \{ a^n b^m c^m d^n \mid m, n >= 1 \}$

$L = \{ abcd, abbccd, aabcdd, aabbccdd \ldots \}$

(a) Initial symbol.

$\delta(q_0, a, z) = \{(q_0, az)\}$

(b) remaining a's

$\delta(q_0, a, a) = \{(q_0, aa)\}$.

(c) Reading b → state shift

$\delta(q_0, b, a) = \{(q_1, ba)\}$.

(d) Reading bs.

$\delta(q_1, b, b) = \{(q_1, bb)\}$.

(d) State shift

$\delta(q_1, \varepsilon, b) = \{(q_2, b)\}$

(e) Popping bs - reading c.

$\delta(q_2, c, b) = \{(q_2, \varepsilon)\}$.

(f) State shift.

$\delta(q_2, \varepsilon, a) = \{(q_3, a)\}$.

(g) Popping as - reading d.

$\delta(q_3, d, a) = \{(q_3, \varepsilon)\}$.

(h) Final state

$\delta(q_3, \varepsilon, z) = \{(q_4, z)\}$.

PDA $M = (Q, \Sigma, \Gamma, q_0, F, z_0, \delta)$

$F = \{q_4\}$. $\Sigma = \{a, b, c, d\}$.

$Q = \{q_0, q_1, q_2, q_3, q_4\}$. $q_0 \to q_0$.

$\Gamma = \{z_0, a, b\}$.