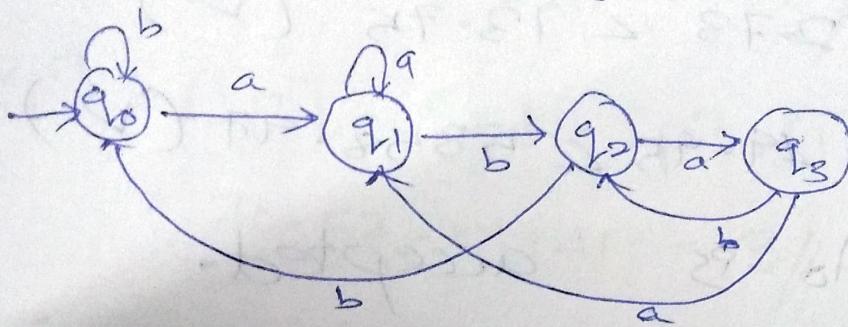


Theory of Computations

(i)  $L = \{ aba, aaba, baba \dots \}$

DFA      transition diagram



Transition Tab

$s$	a	b
$q_0$	$q_1$	$q_0$
$q_1$	$q_1$	$q_2$
$q_2$	$q_3$	$q_0$
$q_3$	$q$	$q_2$

Checking for (i) bbabab

$$f(q_0, b) = q_0$$

$$f(q_0, b) = q_0$$

$$f(q_0, a) = q_1$$

$$f(q_1, b) = q_2$$

$$f(q_2, a) = q_3$$

$$f(q_3, b) = q_2$$

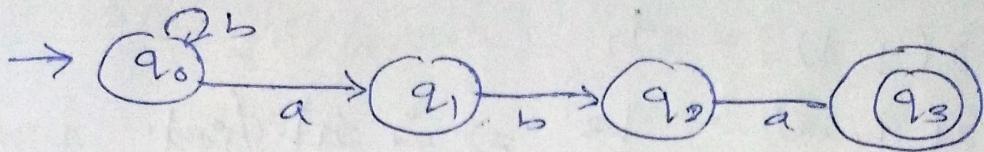
Not satisfied.

(ii) baba

$$\begin{array}{l|l} s(q_0, b) = q_0 & s(q_0, a) = q_1 \\ s(q_1, b) = q_2 & s(q_2, a) = q_2 \end{array}$$

If B satisfied.

NFA diagram:



9) bbabab

$$\delta(q_0, b) = q_0$$

$$\delta(q_0, b) = q_0$$

$$\delta(q_1, a) = \{q_0, q_3\}$$

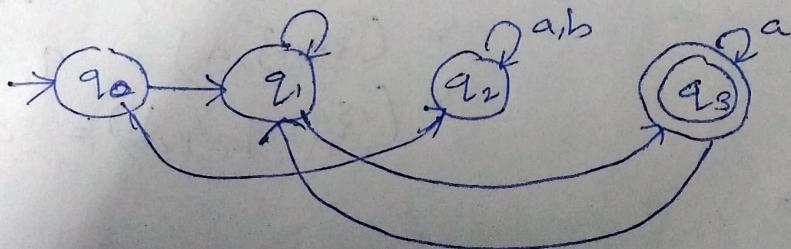
$$\delta(\{q_0, q_1\}, b) = \{q_0, q_2\}, \quad \delta(\{q_0, q_2\}, a) = \{q_0, q_1, q_3\}$$

$$\delta(\{q_1, q_2\}, a) = \{q_0, q_1, q_3\} \quad \therefore \text{satisfied.}$$

$$\delta(\{q_0, q_1, q_3\}, b) = \{q_0, q_2, \emptyset\}$$

$\therefore$  not satisfied

(2)  $L = \{ba, baa, bba, \dots\}$



$\delta$	a	b
$q_0$	$q_2$	$q_1$
$q_1$	$q_3$	$q_1$
$q_2$	$q_2$	$q_2$
$q_3$	$q_3$	$q_1$

(i) bbabab

$$\delta(q_0, b) = q_1$$

$$\delta(q_1, b) = q_1$$

$$\delta(q_1, a) = q_3$$

$$\delta(q_3, b) = q_1$$

$$\delta(q_1, a) = q_3$$

$$\delta(q_3, b) = q_1$$

$\therefore$  unsatisfied

(ii) baba

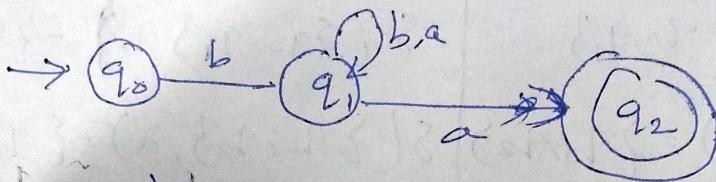
$$\delta(q_0, b) = q_1$$

$$\delta(q_1, a) = q_3 \Rightarrow \text{satisfied.}$$

$$\delta(q_3, b) = q_1$$

$$\delta(q_1, b) = q_3$$

NFA:



$\delta$	a	b
$q_0$	$\emptyset$	$\{q_1\}$
$q_1$	$\{q_1, q_2\}$	$\{q_3\}$
$q_2$	$\emptyset$	$\emptyset$

(i) bbbabab

$$\delta(q_0, b) = q_1$$

$$\delta(q_1, b) = q_1$$

$$\delta(q_1, a) = \{q_1, q_2\}$$

$$\delta(\{q_1, q_2\}, b) = q_1$$

$$\delta(\{q_1, q_2\}, b) = q_1$$

$$\delta(\{q_1, q_2\}, a) = q_1 \cup q_2$$

$$\delta(\{q_1, q_2\}, b) = q_1$$

(iii) baba

$$\delta(q_0, b) = q_1$$

$$\delta(q_0, a) = \{q_1, q_2\}$$

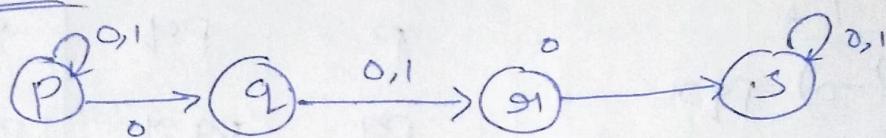
$$\delta(\{q_1, q_2\}, b) = q_1$$

$$\delta(\{q_1, q_2\}, a) = \{q_1, q_2\}$$

∴ satisfied.

$$\textcircled{3} \quad \begin{aligned} M &= \{ Q, \Sigma, F, S \} \\ Q &= \{ P, Q, R, S \} \end{aligned} \quad \left. \begin{aligned} \Sigma &= \{ 0, 1 \} \\ F &= \{ Q \} \end{aligned} \right.$$

NFA



$$s(P, 0) = \{ P, Q \}$$

$$s(P, 1) = \{ P \}$$

$$s(\{P, Q\}, 0) = \{ P, Q, R \}$$

$$s(\{P, Q\}, 1) = \{ P, R \}$$

$$s(\{P, Q, R\}, 0) = \{ P, Q, S \}$$

$$s(\{P, Q, R\}, 1) = \{ P, S \}$$

$$s(\{P, Q, S\}, 0) = \{ P, Q, R \}$$

$$s(\{P, Q, R\}, 1) = \{ P, S \}$$

$$s(\{P, S\}, 0) = P$$

$$s(\{P, Q, R, S\}, 0) = \{ P, Q, R, S \}$$

DFA:

$$q^* =$$

# DFA

$$M' = \{Q', \Sigma, P, F, S\}$$

$$\delta(P, 0) = PQ$$

$$\delta(PQ, 0) = PQS$$

$$\delta(PQS)$$

$$\delta(P) = P$$

$$\delta(PQ, 1) = PS$$

$$\delta(PQS, 0) = PQS$$

$$\delta(PQS, 1) = PS$$

$$\delta(PS, 0) = PQS$$

$$\delta(PQS, 1) = PS$$

$$\delta(PQS, 1) = PS$$

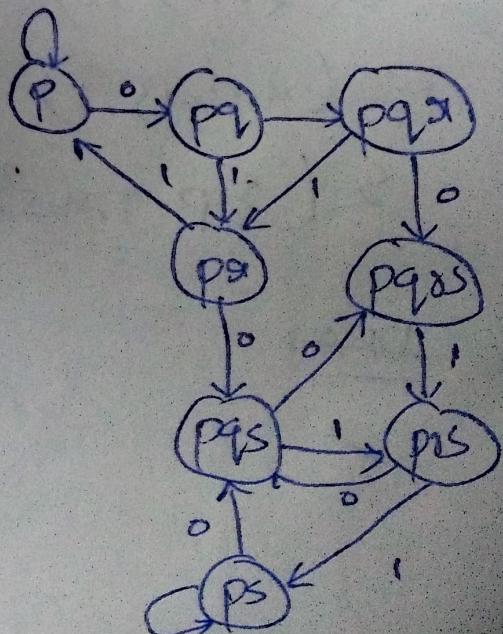
$$\delta(PS, 1) = PS$$

$$\delta(PS, 0) = PQS$$

$$\delta(PS) = PS$$

Transition table

S	0	1
P	PQ	P
PQ	PQS	PQS
PQS	PQS	PQS
PS	PS	PS



$\Omega' = \{ p, pq, pq\alpha, p\alpha, pq\alpha s, pqs, ps, ps^2 \}$   
 $F = \{ pq\alpha s, pqs, ps \} \Rightarrow L = \{ 0, 1 \}$

(ii)  $H = \{ Q, \Sigma, P, F, S \}$ ,  $F = \{ q_1, s \}$   
 $\Sigma = \{ 0, 1 \}$ ,  $Q = \{ p, q, \alpha, s \}$ .

DFA:

$$q' = \{ Q', \Sigma, S, F', S' \}.$$

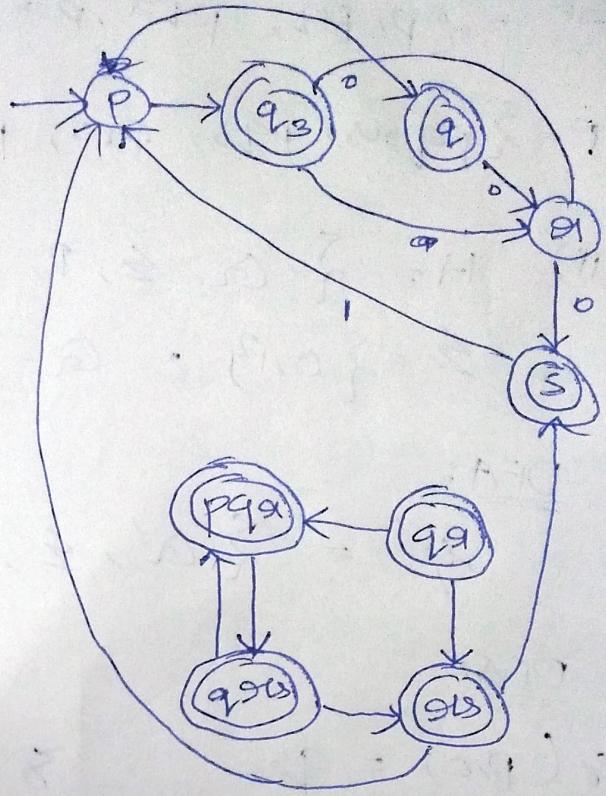
DFA:

$$\begin{aligned}
 \delta(p, 0) &= q_s \\
 \delta(p, 1) &= q \\
 \delta(q_s, 0) &= \alpha \\
 \delta(q_s, 1) &= pq\alpha \\
 \delta(q, 0) &= \alpha \\
 \delta(\alpha, 0) &= s \\
 \delta(\alpha, 1) &= p \\
 \delta(pq\alpha, 0) &= q_{\alpha s} \\
 \delta(pq\alpha, 1) &= pq\alpha \\
 \delta(p\alpha, 0) &= \alpha s \\
 \delta(q\alpha, 1) &= pq\alpha \\
 \delta(s, 0) &= \emptyset \\
 \delta(s, 1) &= p \\
 \delta(\alpha s, 0) &= \alpha s \\
 \delta(q_{\alpha s}, 1) &= \alpha s \\
 \delta(\alpha s, 0) &= s \\
 \delta(\alpha s, 1) &= p
 \end{aligned}$$

NFA:

$$\begin{aligned}
 \delta(p, 0) &= \{ q_s, s \} \\
 \delta(p, 1) &= \alpha \\
 \delta(\{ q_s, s \}, 0) &= \alpha \\
 \delta(\{ q_s, s \}, 1) &= \{ p, q, \alpha \} \\
 \delta(\{ q_s \}, 0) &= \alpha \\
 \delta(q_s, 1) &= \{ q_s, \alpha \} \\
 \delta(\alpha, 0) &= s \\
 \delta(\alpha, 1) &= p \\
 \delta(\{ p, q, \alpha \}, 0) &= \{ q_s, \alpha, s \} \\
 \delta(\{ q_s, \alpha \}, 0) &= \{ \alpha, s \} \\
 \delta(\{ q_s, \alpha \}, 1) &= \{ p, q, \alpha \} \\
 \delta(s, 0) &= \emptyset \\
 \delta(s, 1) &= p \\
 \delta(\{ q_s, \alpha, s \}, 0) &= \{ \alpha, s \} \\
 \delta(\{ q_s, \alpha, s \}, 1) &= \{ p, q, \alpha \} \\
 \delta(\{ \alpha, s \}, 0) &= s \\
 \delta(\{ \alpha, s \}, 1) &= p
 \end{aligned}$$

S	0	1
P	$q_1$	q
$q_3$	q	$pq$
q	q	$q_{13}$
a	s	p
$pq_1$	$q_{13}$	$pq_1$
$q_{13}$	$q_{13}$	$pq_{13}$
s	-	p
$q_{13}s$	q	$pq_{13}$
q	s	p



(iii)  $\mathcal{A} = (\Sigma, \delta, q_0, F, s)$ ,  $\Sigma = \{0, 1\}$   
 $Q = \{q_0, q_1, q_3\}$   
 $F = \{q_0\}$

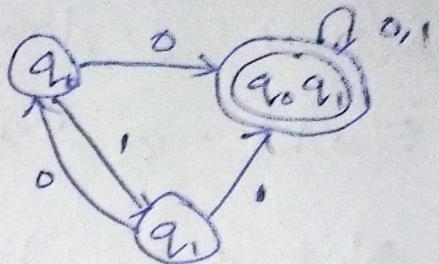
DFA:

$$\begin{aligned}\delta(q_0, 0) &= q_0 q_1 \\ \delta(q_0, 1) &= q_1 \\ \delta(q_0 q_1, 0) &= q_0 q_1 \\ \delta(q_0 q_1, 1) &= q_0 q_1 \\ \delta(q_0, q_1, 1) &= q_0 q_1 \\ \delta(q_1, 0) &= q_0 \\ \delta(q_1, 1) &= q_0 q_1\end{aligned}$$

NFA:

$$\begin{aligned}\delta(q_0, 0) &= \{q_0, q_3\} \\ \delta(q_0, 1) &= q \\ \delta(q_0 q_1, 0) &= \{q_0, q_1\} \\ \delta(q_0 q_1, 1) &= \{q_0, q_3\} \\ \delta(q_1, 0) &= q_0 \\ \delta(q_1, 1) &= \{q_0, q_3\}\end{aligned}$$

$S'$	0	1
$q_0$	$q_0 q_1$	$q_1$
$q_0 q_1$	$q_0 q_1$	$q_0 q_1$
$q_1$	$q_0$	$q_0 q_1$



④ (i)

$\epsilon$ -NFA



$$\epsilon\text{-closure}(p) = \{p, q, r\}$$

$$\epsilon\text{-closure}(q) = \{q, r\}$$

$$\epsilon\text{-closure}(r) = \{r\}$$

$$F = F \cup \{\epsilon p\} = \{p, q, r\}$$

$$S(p, q) = \epsilon\text{-closure}(S(\delta(p, \epsilon)), q)$$

$$\epsilon\text{-closure}(p) = \{p, q, r\}$$

$$S(p, b) = \epsilon\text{-closure}(S(\delta(p, \epsilon), b)) \\ = \epsilon\text{-closure}(q, b) = \{q, r\}$$

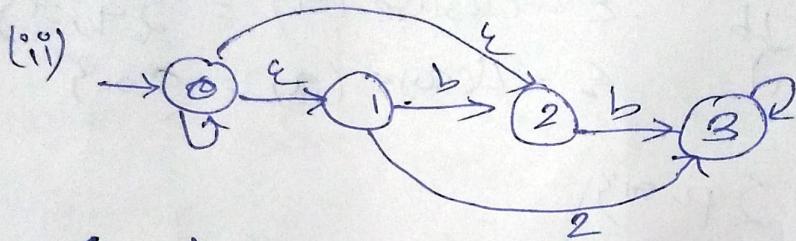
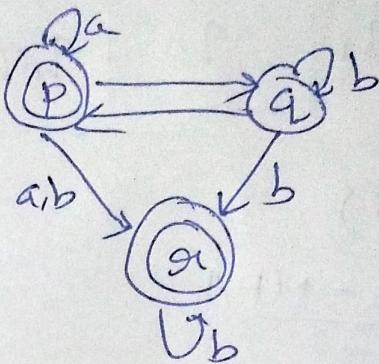
$$\delta(q, a) = \epsilon\text{-closure}(S(S(q, \epsilon)), a) \\ = \epsilon\text{-closure}(S(q, a), b) = \emptyset$$

$$S(q, b) = \epsilon\text{-closure}(S(S(q, \epsilon), b)) \\ = \{q, r\}$$

$$\delta(a, a) = \epsilon\text{-closure}(\delta(\delta(a, \epsilon), a)) = \emptyset$$

$$\delta(a, b) = \epsilon\text{-closure}(\delta(\delta(a, \epsilon), b)) = \{g_1\}$$

$\delta^1$	a	b
P	$\{\delta(p, a)\}$	$\{\delta(q, a)\}$
q	$\emptyset$	$\{\delta(q, a)\}$
r	$\emptyset$	$\{\delta(r, a)\}$



$\epsilon$ -closure of 0 = {0, 1, 2, 3}

$\epsilon$ -closure of 1 = {1, 3}

$\epsilon$ -closure of 2 = {2}

$\epsilon$ -closure of 3 = {3}

$$\delta(0, a) = \epsilon\text{-closure}(\delta(\delta(0, \epsilon), a)) = \{0, 1, 2, 3\}$$

$$\delta(0, b) = \epsilon\text{-closure}(\delta(\delta(0, \epsilon), b)) = \{2, 3\}$$

$$\delta(1, a) = \epsilon\text{-closure}(\delta(\delta(1, \epsilon), a)) = \emptyset$$

$$\delta(1, b) = \epsilon\text{-closure}(\delta(\delta(1, \epsilon), b)) = \{2\}$$

$$\delta(2, a) = \epsilon\text{-closure}(\delta(\delta(2, \epsilon), a)) = \{3\}$$

$$\delta(2, b) = \epsilon\text{-closure}(\delta(\delta(2, \epsilon), b)) = \{3\}, \quad \delta(1, c) = \{3\}, \quad \delta(2, c) = \emptyset$$

$$\delta(3, c) = \{3\}$$

