Probabilistic Model

- Uncertainty & Probability
- Baye's rule
- Choosing Hypotheses- Maximum a posteriori
- Maximum Likelihood - Baye's concept learning
- Maximum Likelihood of real valued function

From Jiawei Han's slides

Uncertainty

- Our main tool is the probability theory, which assigns to each sentence numerical degree of belief between *0* and *1*
- It provides a way of summarizing the uncertainty

Variable

- Boolean random variables: cavity might be true or false
- Discrete random variables: weather might be sunny, rainy, cloudy, snow
  - *P(Weather=sunny)*
  - *P(Weather=rainy)*
  - *P(Weather=cloudy)*
  - *P(Weather=snow)*
- Continuous random variables: the temperature has continuous values

Where do probabilities come from?

- Frequents:
  - From experiments: form any finite sample, we can estimate the true fraction and also calculate how accurate our estimation is likely to be
- Subjective:
  - Agent's believe
- Objectivist:
  - True nature of the universe, that the probability up heads with probability 0.5 is a probability of the coin

Contd…

- Before the evidence is obtained; prior probability
  - *P(a)* the prior probability that the proposition is true
  - $P(cavity)=0.1$

- After the evidence is obtained; posterior probability
  - $P(a|b)$
  - The probability of a given that all we know is b
  - $P(cavity|toothache)=0.8$

Axioms of Probability

Zur Anzeige wird der QuickTime™
Dekompressor „TIFF (Unkomprimiert)"
benötigt.

(Kolmogorov's axioms,
first published in German 1933)

- All probabilities are between 0 and 1. For any proposition a $\quad 0 \leq P(a) \leq 1$
- $P(true)=1, \; P(false)=0$
- The probability of disjunction is given by

$$P(a \vee b) = P(a) + P(b) - P(a \wedge b)$$

Contd..

- Product rule

$$P(a \wedge b) = P(a \mid b)P(b)$$

$$P(a \wedge b) = P(b \mid a)P(a)$$

Theorem of total probability

If events $A_1, \ldots, A_n$ are mutually

exclusive with then

$$\sum_{i=1}^{n} P(A_i) = 1$$

$$P(B) = \sum_{i=1}^{n} P(B|A_i)P(A_i)$$

$$P(B) = \sum_{i=1}^{n} P(B, A_i)$$

Bayes Theorem

$$P(h|D) = \frac{P(D|h)P(h)}{P(D)}$$

- *P(h)* = prior probability of hypothesis h
- *P(D)* = prior probability of training data *D*
- *P(h|D)* = probability of *h* given *D*
- *P(D|h)* = probability of *D* given *h*

Choosing Hypotheses

- Generally want the most probable hypothesis given the training data
- **Maximum a posteriori** hypothesis h_{MAP}:

$$h_{MAP} = \arg\max_{h \in H} P(h|D)$$

Contd..

$$h_{MAP} = \arg\max_{h \in H} P(h|D)$$

$$= \arg\max_{h \in H} \frac{P(D|h)P(h)}{P(D)}$$

$$= \arg\max_{h \in H} P(D|h)P(h)$$

Contd..

- If assume $P(h_i)=P(h_j)$ for all h_i and h_j, then can further simplify, and choose the

- **Maximum likelihood** (ML) hypothesis

$$h_{ML} = \arg\max_{h_i \in H} P(D|h_i)$$

Naïve Bayesian Classification

- If i-th attribute is categorical:
  $P(d_i|C)$ is estimated as the relative freq of samples having value d_i as i-th attribute in class C
- If i-th attribute is continuous:
  $P(d_i|C)$ is estimated thru a Gaussian density function
- Computationally easy in both cases

Play-tennis example: estimating $P(x_i|C)$

Outlook	Temperature	Humidity	Windy	Class
sunny	hot	high	false	N
sunny	hot	high	true	N
overcast	hot	high	false	P
rain	mild	high	false	P
rain	cool	normal	false	P
rain	cool	normal	true	N
overcast	cool	normal	true	P
sunny	mild	high	false	N
sunny	cool	normal	false	P
rain	mild	normal	false	P
sunny	mild	normal	true	P
overcast	mild	high	true	P
overcast	hot	normal	false	P
rain	mild	high	true	N

P(p) = 9/14
P(n) = 5/14

outlook	
P(sunny|p) = 2/9	P(sunny|n) = 3/5
P(overcast|p) = 4/9	P(overcast|n) = 0
P(rain|p) = 3/9	P(rain|n) = 2/5
temperature	
P(hot|p) = 2/9	P(hot|n) = 2/5
P(mild|p) = 4/9	P(mild|n) = 2/5
P(cool|p) = 3/9	P(cool|n) = 1/5
humidity	
P(high|p) = 3/9	P(high|n) = 4/5
P(normal|p) = 6/9	P(normal|n) = 2/5
windy	
P(true|p) = 3/9	P(true|n) = 3/5
P(false|p) = 6/9	P(false|n) = 2/5

Naive Bayesian Classifier (II)

- Given a training set, we can compute the probabilities

Outlook	P	N		Humidity	P	N
sunny	2/9	3/5		high	3/9	4/5
overcast	4/9	0		normal	6/9	1/5
rain	3/9	2/5				
Tempreature				Windy		
hot	2/9	2/5		true	3/9	3/5
mild	4/9	2/5		false	6/9	2/5
cool	3/9	1/5				

Play-tennis example: classifying X

- An unseen sample X = <rain, hot, high, false>

- P(X|p)·P(p) =
  P(rain|p)·P(hot|p)·P(high|p)·P(false|p)·P(p) =
  3/9·2/9·3/9·6/9·9/14 = 0.010582
- P(X|n)·P(n) =
  P(rain|n)·P(hot|n)·P(high|n)·P(false|n)·P(n) =
  2/5·2/5·4/5·2/5·5/14 = 0.018286

- Sample X is classified in class n (don't play)

The independence hypothesis…

- … makes computation possible
- … yields optimal classifiers when satisfied
- … but is seldom satisfied in practice, as attributes (variables) are often correlated.
- Attempts to overcome this limitation:
  - Bayesian networks, that combine Bayesian reasoning with causal relationships between attributes

Training dataset

Class:
C1:buys_computer='yes'
C2:buys_computer='no'

Data sample:

X =
(age<=30,
Income=medium,
Student=yes
Credit_rating=Fair)

age	income	student	credit_rating	buys_computer
<=30	high	no	fair	no
<=30	high	no	excellent	no
30…40	high	no	fair	yes
>40	medium	no	fair	yes
>40	low	yes	fair	yes
>40	low	yes	excellent	no
31…40	low	yes	excellent	yes
<=30	medium	no	fair	no
<=30	low	yes	fair	yes
>40	medium	yes	fair	yes
<=30	medium	yes	excellent	yes
31…40	medium	no	excellent	yes
31…40	high	yes	fair	yes
>40	medium	no	excellent	no

Naïve Bayesian Classifier: Example

- Compute $P(X|C_i)$ for each class

P(buys_computer=„yes“)=9/14

P(buys_computer=„no“)=5/14

P(age="<30" | buys_computer="yes") = 2/9=0.222
P(age="<30" | buys_computer="no") = 3/5 =0.6
P(income="medium" | buys_computer="yes")= 4/9 =0.444
P(income="medium" | buys_computer="no") = 2/5 = 0.4
P(student="yes" | buys_computer="yes)= 6/9 =0.667
P(student="yes" | buys_computer="no")= 1/5=0.2
P(credit_rating="fair" | buys_computer="yes")=6/9=0.667
P(credit_rating="fair" | buys_computer="no")=2/5=0.4

- X=(age<=30 ,income =medium, student=yes,credit_rating=fair)

$P(X|C_i)$: P(X|buys_computer="yes")= 0.222 x 0.444 x 0.667 x 0.0.667 =0.044

P(X|buys_computer="no")= 0.6 x 0.4 x 0.2 x 0.4 =0.019

$P(X|C_i)*P(C_i)$: P(X|buys_computer="yes") * P(buys_computer="yes")=0.028
P(X|buys_computer="no") * P(buys_computer="no")=0.007

- X belongs to class "buys_computer=yes"