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SSN COLLEGE OF ENGINEERING
RECORD SHEET

Sheet No.............

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Reg No: 205001085   Assignment - 3

(1) Find PDA for given CFG.

(a) $S \rightarrow aAS \mid b$

$A \rightarrow SbA \mid ba$

$\delta(q_0, \varepsilon, S) = \{(q_1, SZ)\}$

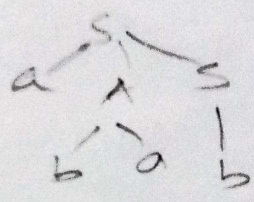
$\delta(q_1, \varepsilon, S) = \{(q_1, aAS), (q_1, b)\}$

$\delta(q_1, \varepsilon, A) = \{(q_1, SbA), (q_1, ba)\}$

$\delta(q_1, a, a) = \{(q_1, \varepsilon)\}$

$\delta(q_1, b, b) = \{(q_1, \varepsilon)\}$

$\delta(q_1, \varepsilon, Z) = \{(q_2, Z)\}$



Validation:

$w = abab$

$(q_0, abab, Z)$

$\vdash (q_1, abab, SZ)$

$\vdash (q_1, abab, aASZ)$

$\vdash (q_1, bab, ASZ)$

$\vdash (q_1, bab, baSZ)$

$\vdash (q_1, ab, aSZ)$

$\vdash (q_1, b, SZ)$

$\vdash (q_1, b, bZ)$

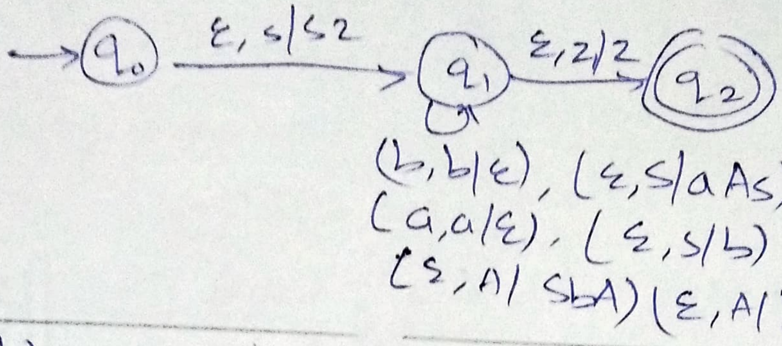
$\vdash (q_2, \varepsilon, Z)$

$S \xRightarrow{lm} aAS$

$\xRightarrow{lm} abaS$

$\xRightarrow{lm} abab$

$q_2 \in F$ accepted



(b) $S \to A$; $A \to BC$; $B \to ba$; $C \to ac$

$\delta(q_0, \varepsilon,) = \{(q_1, sz)\}$

$\delta(q_1, \varepsilon,) = \{(q_1, \cancel{B}A)\}$

$\delta(q_1, \varepsilon,) = \{(q_1, BC)\}$

$\delta(q_1, \varepsilon,) = \{(q_1, ba)\}$

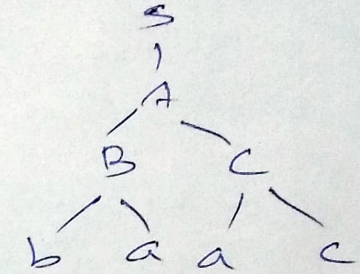
$\delta(q_1, \varepsilon,) = \{(q_1, ac)\}$

$\delta(q_1, a) = \{(q_1, \varepsilon)\}$

$\delta(q_1, b) = \{(q_1, \varepsilon)\}$

$\delta(q_1, c) = \{(q_1, \varepsilon)\}$

$\delta(q_1, z, z) = \{(q_2, z)\}$



$S \overset{lm}{\Rightarrow} A$
$\overset{lm}{\Rightarrow} BC$
$\overset{lm}{\Rightarrow} baC$
$\overset{lm}{\Rightarrow} baac$

<u>Validation:</u>

$w = baac$

$(q_0, baac, z)$
$\vdash (q_1, baac, sz)$
$\vdash (q_1, baac, Az)$
$\vdash (q_1, baac, BCz)$
$\vdash (q_1, baac, baCz)$
$\vdash (q_1, aac, aCz)$
$\vdash (q_1, ac, Cz)$
$\vdash (q_1, ac, acz)$
$\vdash (q_1, c, cz)$
$\vdash (q_2, \varepsilon, z)$

$q_2 \in F$ accepted//

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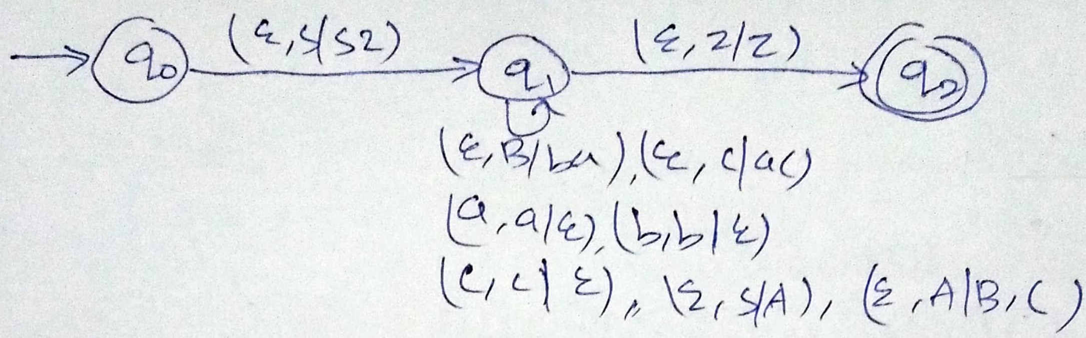

(2) Construct CFG from PDA:

$A = (\{q_0, q_1\}, \{a, b\}, \{z_0, z\}, \delta, q_0, z_0)$

$\delta(q_0, b, z_0) = \{(q_0, zz_0)\}$

$\delta(q_0, \varepsilon, z_0) = \{(q_0, \varepsilon)\}$

$\delta(q_0, b, z) = \{(q_0, zz)\}$

$\delta(q_0, a, z) = \{(q_1, z)\}$

$\delta(q_1, b, z) = \{(q_1, \varepsilon)\}$

$\delta(q_1, a, z_0) = \{(q_0, z_0)\}$

(i) Find non terminals (N)

A $[q_0 z_0 q_0] \rightarrow \varepsilon \mid b[q_0 z q_0][q_0 z_0 q_0] \mid b[q_0 z q_1][q_1 z_0 q_0]$

B $[q_0 z_0 q_1] \rightarrow b[q_0 z q_0][q_0 z_0 q_1] \mid b[q_0 z q_1][q_1 z_0 q_1]$

C $[q_1 z_0 q_0] \rightarrow a[q_0 z_0 q_0]$

D $[q_1 z_0 q_1] \rightarrow a[q_0 z_0 q_1]$

E $[q_0 z q_0] \rightarrow b[q_0 z q_0][q_0 z q_0]$

F $[q_0 z q_1] \rightarrow b[q_0 z q_0][q_0 z q_1] \mid b[q_0 z q_1][q_1 z q_1]$)

$a[q_1 z q_1]$

G $[q_1 z q_1] \rightarrow b$

(iii) Find productions (P):

→ Find start of production

$S \rightarrow [q_0 z_0 q_0] \mid [q_0 z_0 q_1]$

→ Find pop operation in productions

$\delta(q_0, \varepsilon, z_0) = \{(q_0, \varepsilon)\}$

$[q_0 z_0 q_0] \rightarrow \varepsilon$

$\delta(q_1, b, z) = \{(q_1, \varepsilon)\}$

$[q_1, z, q_1] \rightarrow b$

→ Find push operation

$\delta(q_0, b, z_0) = \{(q_0, zz_0)\}$

$[q_0 z_0 s_2] \rightarrow b [q_0 z s_1][s_1 z_0 s_2]$

$s_2 = q_0 \quad ; \quad s_1 = q_0$

$[q_0 z_0 q_0] \rightarrow b [q_0 z q_0][q_0 z_0 q_0]$

$s_2 = q_0 \quad ; \quad s_1 = q_1$

$[q_0 z_0 q_0] \rightarrow b [q_0 z q_1][q_1 z_0 q_0]$

$s_2 = q_1 \quad ; \quad s_1 = q_0$

$[q_0 z_0 q_1] \rightarrow b [q_0 z q_0][q_0 z_0 q_1]$

$s_2 = q_1 \quad ; \quad s_1 = q_1$

$[q_0 z_0 q_1] \rightarrow b [q_0 z q_1][q_1 z_0 q_1]$

$\delta(q_0, b, z) = \{(q_0, zz)\}$

$[q_0 z s_2] \to b[q_0 z s_1][s_1 z, s_2]$

$s_2 = q_0$; $s_1 = q_0$

$[q_0 z q_0] \to b[q_0 z q_0][q_0 z q_0]$

$s_2 = q_0$; $s_1 = q_1$

$[q_0 z q_0] \to b[q_0 z q_1][q_1 z q_0]$

$s_2 = q_1$; $s_1 = q_0$

$[q_0 z q_1] \to b[q_0 z q_0][q_0 z q_1]$

$s_2 = q_1$; $s_1 = q_1$

$[q_0 z q_1] \to b[q_0 z q_1][q_1 z q_1]$

$\delta(q_0, a, z) = \{(q_1, z)\}$

$[q_0 z s_1] \to a[q_1 z s_1]$

$s_1 = q_0$

$[q_0 z q_0] \to a[q_1 z q_0]$

$s_1 = q_1$

$[q_0 z q_1] \to a[q_1 z q_1]$

$\delta(q_1, a, z_0) = \{(q_0, z_0)\}$

$[q_1 z_0 s_1] \to a[q_0 z_0 s_1]$

$s_1 = q_0$

$[q_1 z_0 q_0] \to a[q_0 z_0 q_0]$

$s_1 = q_1$

$[q_1 z_0 q_1] \to a[q_0 z_0 q_1]$

(iii) Rename production

$S \to A \mid B$

$A \to \epsilon \mid bAA \mid bBC$

$B \to bAB \mid bFD$

$C \to aA$; $D \to aB$; $E \to bEE$

$F \to bEF \mid bEG \mid aG$; $G \to b$

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(3) Show $L = \{a^i b^j c^i d^j \mid i, j >= 1\}$ is not CFL.

$|w| \geq n$

$w = uvxyz$

$w = a^p b^q c^p d^q$

$|vxy| \leq n$

$u = a^p$

$vxy = b^q$

$|vy| > 0$

$vy = b^{q-r}$

$z = c^p d^q$

$uv^ixy^iz = uvxy(vy)^{i-1}z = a^p b^q (b^{q-r})^{i-1} c^p d^q$

$\Rightarrow i = 0$

$a^p b^q (b^{q-r})^{-1} c^p d^q \Rightarrow a^p b^r c^p d^q \notin L$

$\Rightarrow i = 1$

$a^p b^q (b^{q-r})^0 c^p d^q \Rightarrow a^p b^q c^p d^q \in L$

$\Rightarrow i = 2$

$a^p b^q (b^{q-r})^1 c^p d^q \Rightarrow a^p b^{2q-r} c^p d^q \notin L$

$\therefore$ Language is not a CFL.

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(6) Obtain code for $\langle M, 1011 \rangle$ where $M = (\{q_1, q_2, q_3\}, \{0,1\}, \{0,1\}, \{0,1,B\}, \delta, q_1, B, \{q_2\})$

$\delta(q_1, 1) = (q_3, 0, R)$

$\delta(q_3, 0) = (q_1, 1, R)$

$\delta(q_3, 1) = (q_2, 0, R)$

$\delta(q_3, B) = (q_3, 1, L)$

$L \to 0$; $R \to 00$ | $Q = \{q_1, q_2, q_3\}$ | $\Sigma = \{0, 1\}$
0, 00, 000 | 0, 00

$\Gamma = \{0, 1, B\}$
0 00 000

$\delta(q_1, 1) = (q_3, 0, R)$ 0100100010100

$\delta(q_3, 0) = (q_1, 1, R)$ 0001010100100

$\delta(q_3, 1) = (q_2, 0, R)$ 00010010010100

$\delta(q_3, B) = (q_3, 1, L)$ 0001000100010010

$w = 1011$

$\langle M, 1011 \rangle$

$\Rightarrow$ 01001000101001100010101001001 1
0001001001010011000100010001001011 1,
0010100100

(7) Busy Beaver problem

* $n \to$ number of states
* $M(n) \to$ set of TM's with n states and binary alphabet
* $[k]$: configuration of having a block of n consecutive 1's on an otherwise blank tape
* e : empty tape (all 0's)
* $\Sigma(M) = k$ of machine M, when started on e, halts with $[k]$, otherwise $= 0$.

* Busy Beaver problem: Find $\Sigma(n) = \max\{\Sigma(M) \mid M \in M(n)\}$

General Version

⇒ In general, busy beaver problem is to find the 'most productive' Turing machine with 'n' states and 'm' symbols

⇒ Productivity of Turing machine defined as,
(i) Number of steps taken (time)
(ii) Number of symbols written ($f(n)$)
(iii) Number of cell moved away from starting cell

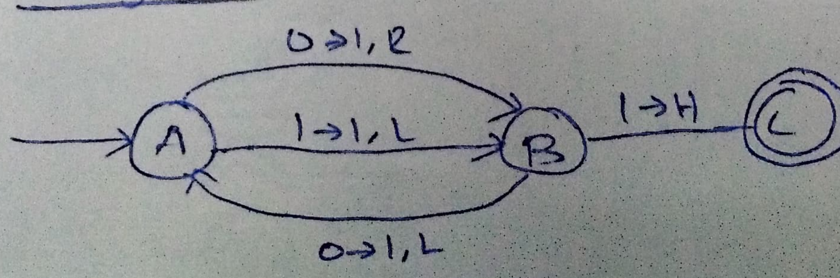
⇒ For any particular problem you can show this either by direct proof or by reducing it into another problem.

⇒ Once you discard all non-halters, you can simply run all halters to completion to figure out any kind of busy Beaver function.

Connecting Halting problem & Busy Beaver problem

If the halting problem is solvable, then any Busy Beaver problem is solvable. That is $\Sigma(n)$ can be computed if Halting problem is solvable.

State diagram



⇒ The n^{th} busy beaver number, denoted by BB(n), is maximum finite no. of state shifts undergone by halt-state.

(4) Construct Turing Machine

(a) $L = \{a^i b^j c^i d^j \mid i,j >= 1\}$

$Q = \{q_0, q_1 \ldots q_7\}$; $\Sigma = \{a,b,c,d\}$, $\Gamma = \{a,b,x,y\}$

ac forward

$\delta(q_0, a) = (q_1, x, R)$
$\delta(q_1, a) = (q_2, a, R)$
$\delta(q_1, b) = (q_1, b, R)$
$\delta(q_1, Y) = (q_1, Y, R)$
$\delta(q_1, c) = (q_2, Y, L)$

ac backword

$\delta(q_2, y) = (q_2, Y, L)$
$\delta(q_2, b) = (q_2, b, L)$
$\delta(q_2, c) = (q_2, a, L)$
$\delta(q_2, x) = (q_0, x, R)$

bd forword

$\delta(q_0, b) = (q_3, b, L)$
$\delta(q_3, b) = (q_4, P, R)$
$\delta(q_4, b) = (q_4, b, R)$
$\delta(q_4, y) = (q_4, Y, R)$
$\delta(q_4, d) = (q_5, Q, L)$

bd backword

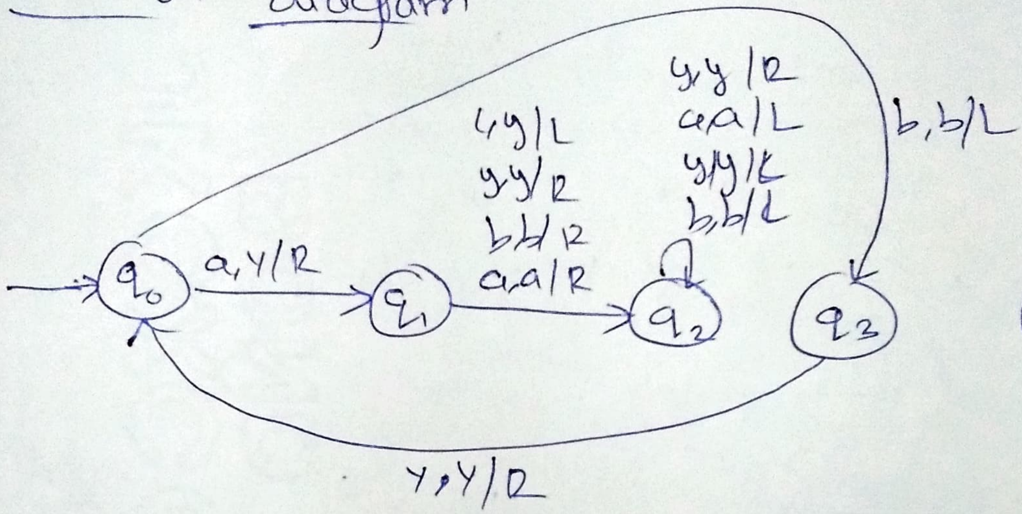
$\delta(q_5, Q) = (q_5, Q, L)$
$\delta(q_5, Y) = (q_5, Y, L)$
$\delta(q_5, b) = (q_5, b, L)$
$\delta(q_5, P) = (q_5, P, R)$

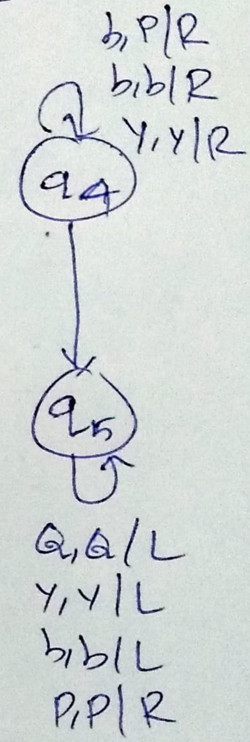
Termination

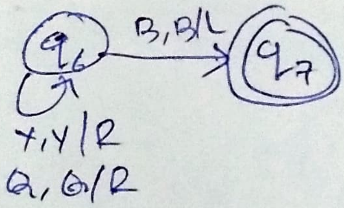
$\delta(q_3, Y) = (q_0, Y, R)$ | $\delta(q_6, Q) = (q_6, Q, R)$

$\delta(q_6, X) = (q_6, Y, R)$ | $\delta(q_6, B) = (q_7, B, L)$

Transition diagram







7 tape notation

$M = (Q, \Sigma, \Gamma, \delta, q_0, F, B)$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7\}$

$\Sigma = \{a, b, c, d\}$; $\Gamma = \{a, b, c, d, X, Y, P, Q, B\}$

$F = \{q_7\}$.

(b) $L = \{wcw^R \mid w \in (a/b)^*\}$

$L = \{aabcbaa, \ldots\}$

$d = a$ or b

$e = a$ or b

Forward Transition

$\delta([q_0, B], d) = ([q_1, d], B, R)$
$\delta([q_1, d], e) = ([q_1, d], e, R)$
$\delta([q_1, d], c) = ([q_2, d], e, R)$
$\delta([q_2, d], e) = ([q_2, d], e, R)$
$\delta([q_2, d], B) = ([q_3, d], B, L)$
$\delta([q_3, d], d) = ([q_4, B], B, L)$

Backward Transition

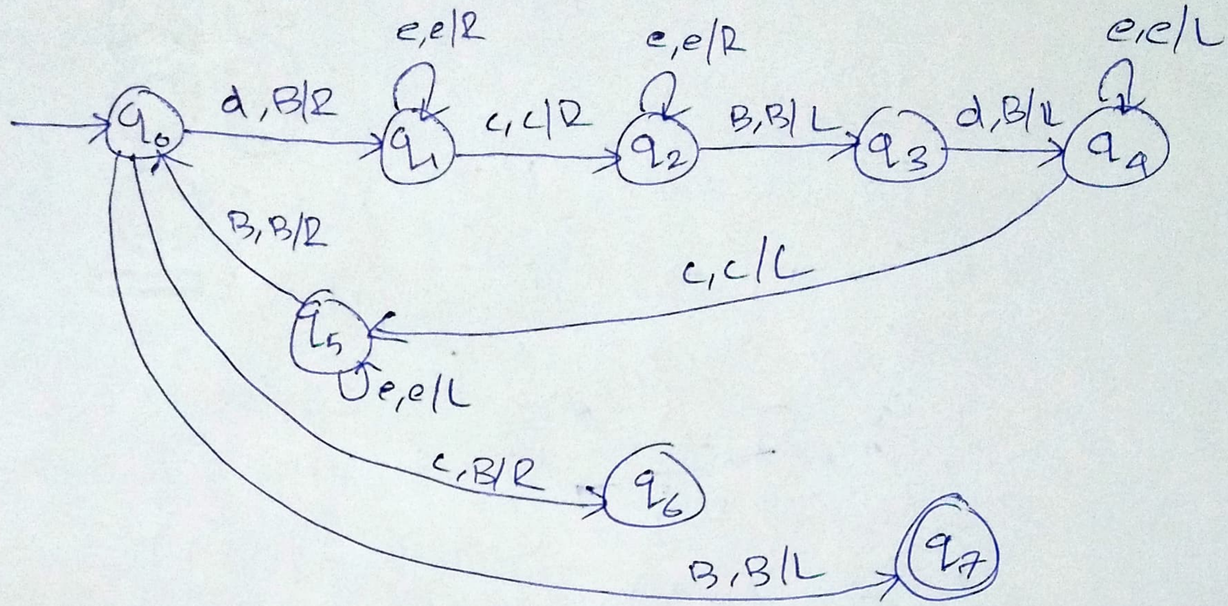
$\delta([q_4, B], e) = ([q_4, B], e, L)$
$\delta([q_5, B], e) = ([q_5, B], e, L)$
$\delta([q_5, B], e) = ([q_5, B], e, L)$
$\delta([q_5, B], B) = ([q_0, B], B, R)$

Termination:

$\delta([q_0, B], c) = ([q_6, B], B, R)$
$\delta([q_0, B], B) = ([q_7, B], B, L)$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7\}$
$\Sigma = \{a, b, c\}$; $\Gamma = \{a, b, B\}$
$F = \{q_7\}$; q_0 = Start state.

Transition diagram:



(5) Construct Turing machine

(a) $L = \{ w w^R \mid w \in (a/b)^* \}$

$L = \{ aba,\ aabaa, \ldots \}$; d = a or b
e = a or b

Forward Transition

$\delta([q_1, B], [B, d]) = ([q_2, d], [v, d], R)$

$\delta([q_2, d], [B, e]) = ([q_2, d], [B, e], R)$

$\delta([q_2, d], [B, d]) = ([q_2, d], [B, d], R)$

$\delta([q_2, d], [B, B]) = ([q_3, d], [B, B], L)$

Backward Transition

$\delta([q_3, d], [B, d]) = ([q_4, B], [v, d], L)$

$\delta([q_4, B], [B, d]) = ([q_4, B], [B, d], L)$

$\delta([q_4, B], [v, d]) = ([q_4, B], [v, d], R)$

I

Termination:

$\delta([q_4, B], [v, d]) = ([q_5, B], [v, d], R)$

Tuples

$M = (Q, \Sigma, T, \delta, q_0, B, F)$

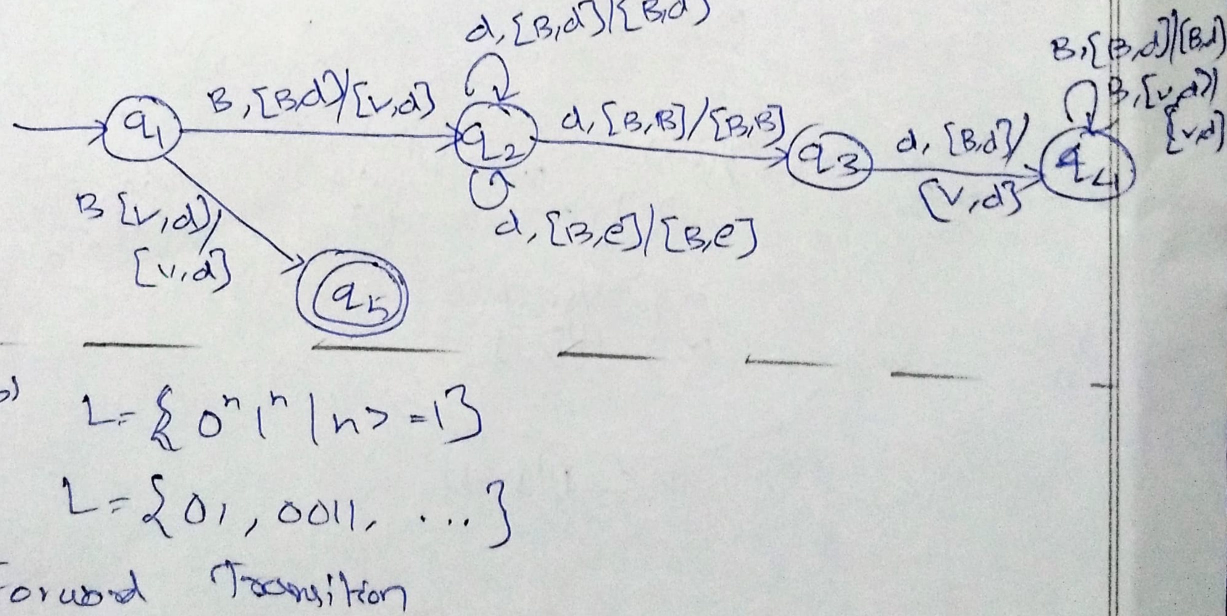
$Q = \{[q_i, d] \mid q = q_1, q_2, q_3, q_4, q_5$ & $d = a, b, B\}$

$F = \{[q_5, B]\}$

$\Sigma = \{[B, d] \mid d = a$ or $b\}$

$q_0 = [q_1, B]$

Transition diagram



(b) $L = \{0^n 1^n \mid n >= 1\}$

$L = \{01, 0011, \ldots\}$

Forward Transition

$\delta([q_0, B], [B, 0]) = ([q_1, B], [v, 0], R)$

$\delta([q, 0], [B, 0]) = ([q_1, 0], [B, 0], R)$

$\delta([q_1, 0], [v, 1]) = ([q_1, 0], [v, 1], R)$

$\delta([q_1, 0], [B, 1]) = ([q_2, B], [v, 1], L)$

Backward Transitions

$\delta([q_2, B], [V, 1]) = ([q_2, B], [V, 1], L)$

$\delta([q_2, B], [B, 0]) = ([q_2, B], [B, 0], L)$

$\delta([q_2, B], [V, 0]) = ([q_0, B], [V, 0], R)$

Termination

$\delta([q_0, B], [V, 1]) = ([q_3, B], [V, 1], R)$

$\delta([q_3, B], [V, 1]) = ([q_3, B], [V, 1], R)$

$\delta([q_3, B], [B, B]) = ([q_4, B], [V, B], L)$

Transition diagram: