

ASSIGNMENT - 1

(1)

$A$  - satisfiable

$A \vee B$  - satisfiable

$A \vee \neg A$  - valid

$A \wedge \neg A$  - unsatisfiable

$A = \neg A$  - satisfiable

$A = B$  - satisfiable

$A = (B = A)$  - satisfiable

$A = (A = B)$  - satisfiable

$A = \neg A$  - unsatisfiable

(2)

(i) Yes

(ii) Yes

(iii) No

(iv) Yes

(3)

$$(i) (p \rightarrow q) \rightarrow (\neg p \wedge q)$$

$$\Rightarrow (\neg p \vee q) \rightarrow (\neg p \wedge q)$$

$$\Rightarrow \neg (\neg p \vee q) \vee (\neg p \wedge q)$$

$$\Rightarrow (p \wedge \neg q) \vee (\neg p \wedge q) \Rightarrow \underline{\text{DNF}}$$

$$\Rightarrow ((p \wedge q) \vee \neg \alpha) \sim ((p \wedge q) \vee q)$$

$$\Rightarrow (p \vee \neg \alpha) \wedge (\neg q \vee \neg \alpha) \sim (p \wedge q) \wedge \top$$

$$\Rightarrow (p \vee \neg \alpha) \wedge (\neg q \vee \neg \alpha) \sim (p \wedge q)$$

CNF

(ii)  $((\neg A \rightarrow B) \vee ((A \wedge \neg C) \leftrightarrow B))$

$$\Rightarrow ((A \vee B) \vee ((A \wedge \neg C) \rightarrow B))$$
$$\sim (B \rightarrow (A \wedge \neg C))$$

$$\Rightarrow ((A \vee B) \vee [\neg (\neg A \wedge \neg C) \vee B]) \wedge [\neg B \vee (A \wedge \neg C)]$$

$$\Rightarrow ((A \vee B) \vee [(\neg A \vee C \vee B) \wedge (\neg B \wedge A)] \\ \star (\neg B \wedge \neg C))$$

$$\Rightarrow (A \vee B) \vee [(\neg A \vee B \vee C) \wedge (\neg B \wedge A) \wedge (\neg B \vee \neg C)]$$

$$\Rightarrow ((A \vee B) \vee (\neg A \vee B \vee C))$$
$$\star ((A \vee B) \vee (\neg B \wedge A)) \wedge ((A \vee B) \vee (\neg B \vee \neg C))$$

$$\Rightarrow (\top) \wedge (\top) \wedge (\top) \wedge (\top)$$

$$\Rightarrow \top$$

$$(\text{true}) \vee (\text{false}) \wedge$$

$$(\text{true}) \wedge (\text{false}) \vee (\text{false}) \wedge$$

(4)

$p$	$q$	$\alpha$	$f$
1	1	1	1
1	0	0	0
1	0	1	1
1	0	0	0
0	1	1	0
0	1	0	0
0	0	1	1
0	0	0	0

$$\begin{aligned} \underline{\text{DNF}}: & (p \wedge q \wedge \alpha) \vee (p \wedge \neg q \wedge \alpha) \\ & \quad \vee (\neg p \wedge q \wedge \alpha) \end{aligned}$$

$$\begin{aligned} \underline{\text{CNF}}: & (\neg p \vee \neg q \vee \alpha) \wedge (\neg p \vee q \vee \alpha) \\ & \wedge (p \vee \neg q \vee \neg \alpha) \wedge (p \vee q \vee \neg \alpha) \\ & \wedge (p \vee q \vee \alpha) \end{aligned}$$

(5)

Horn Formula:

$$\begin{aligned} (i) \quad & (P_5 \Rightarrow P_{11}) \wedge (P_2 \wedge P_3 \wedge P_5 \Rightarrow P_{13}) \\ & \wedge (\top \rightarrow P_5) \wedge (P_5 \wedge P_{11} \rightarrow \perp) \end{aligned}$$

$$1) \quad \top \rightarrow P_5 \quad \therefore P_5 \quad \text{PS } \top$$

$$2) \quad P_5 \rightarrow P_{11} \quad \Rightarrow \quad \top \rightarrow P_{11} \\ \therefore P_{11} \quad \text{PS } \top$$

$$3) \quad P_5 \wedge P_{11} \rightarrow \perp \quad \Rightarrow \quad \top \rightarrow \perp \\ \therefore \text{The given formula is unsatisfiable.}$$

$$\text{(ii)} \quad (T \rightarrow q) \wedge (T \rightarrow s) \wedge (w \rightarrow \perp) \\ \wedge (p \wedge q \wedge s \rightarrow \perp) \wedge (\vee \rightarrow s) \\ \wedge (T \rightarrow a) \wedge (a \rightarrow p)$$

1)  $\vdash \rightarrow q, T \rightarrow s, T \rightarrow a$   
 $\therefore q, a, s \vdash T$

2)  $a \rightarrow p \Rightarrow T \rightarrow p$   
 $\therefore p \vdash T$

3)  $p \wedge q \wedge s \rightarrow \perp$

$\Rightarrow T \not\rightarrow \perp$

( $\neg \rightarrow p \wedge q \wedge s$ )  $\vdash$  formula is unsatisfiable.

$$\text{(iii)} \quad (T \rightarrow q) \wedge (T \rightarrow s) \wedge (w \rightarrow \perp) \wedge \\ (p \wedge q \wedge s \rightarrow \vee) \wedge (\vee \rightarrow s) \wedge (T \rightarrow a) \wedge (a \rightarrow p)$$

( $\vdash T \rightarrow a, T \rightarrow q, T \rightarrow s$ ) (i)

( $\vdash \neg \rightarrow p \wedge q \wedge s \vdash T$ ) (ii)

2)  $a \rightarrow p \Rightarrow T \rightarrow p$   
 $\therefore p \vdash T$

3)  $p \wedge q \wedge s \rightarrow \vee \Rightarrow T \rightarrow \vee$

$\therefore \vee \vdash T$

$\vdash$  Formula is satisfiable

$$(iv) \neg b \wedge (\neg a \vee b \vee \neg c) \wedge a \wedge (\neg a \vee c)$$

$$\Rightarrow (T \rightarrow \neg b) \wedge (\neg (a \wedge \neg b) \vee \neg c) \wedge (T \rightarrow a)$$

$$\Rightarrow (T \rightarrow \neg b) \wedge ((a \wedge \neg b) \rightarrow \neg c) \wedge (T \rightarrow a) \wedge (a \rightarrow c)$$

1)  $T \rightarrow \neg b, T \rightarrow a$

$$\therefore a, \neg b \quad T$$

2)  $a \wedge \neg b \rightarrow \neg c$

$$\Rightarrow T \rightarrow \neg c \quad \therefore \neg c \quad T$$

3)  $a \rightarrow c \rightarrow T \rightarrow c$

Formula unsatisfiable.

⑥

$$F = \{\{A, B, \neg c\}, \{ \neg A \}, \{A, B, C\}, \{a, \neg B\}\}$$

$$F = \{A \vee B \vee \neg c\}, \{ \neg A \}, \{A \vee B \vee C\}, \{a \wedge \neg B\}$$

$$\{B \vee \neg c\}$$

$$\{B\}$$

$$\perp$$

$$\{ \neg B \}$$

$$\{B \vee C\}$$

$$\{ \neg B \}$$

$\therefore F$  ps unsatisfiable

(7)

1)  $(P \rightarrow Q) \rightarrow R$

2)  $(P \rightarrow P) \rightarrow R$

3)  $(R \rightarrow S) \rightarrow \neg(S \rightarrow Q)$

Prove R

⇒ Take negation of conclusion

A)  $\neg R$

Convert to CNF

• 1)  $\Rightarrow \neg(\neg P \vee Q) \vee \neg R$

=  $(P \wedge \neg Q) \vee \neg R$

=  $(P \vee \neg R) \wedge (\neg Q \vee \neg R) = (P \vee \neg R) \wedge (\neg Q \vee \neg R)$

2)  $(P \rightarrow P) \rightarrow R$

=  $(\neg P \vee P) \rightarrow R$

= T → R

= FVR  
= R

3)  $(R \rightarrow S) \rightarrow \neg(S \rightarrow Q)$

=  $\neg(\neg R \vee S) \vee \neg(\neg S \vee Q)$

=  $(R \wedge \neg S) \vee (S \wedge \neg Q)$

=  $(R \vee S) \wedge (R \vee \neg Q)$

=  $(R \vee S) \wedge (\neg S \vee \neg Q)$

=  $(R \vee S) \wedge (R \vee \neg Q)$

=  $\neg(\neg S \vee \neg Q)$

PVR     $\neg Q \vee R$     R    RVS     $R \vee \neg Q$      $\neg S \vee \neg Q$      $\neg R$

1  
(Contradiction)

∴ R can be proved.

$$\textcircled{8} \quad \Sigma = \{\neg(\neg(p \wedge q)), (p \wedge q)\} \vdash (p \Leftrightarrow q)$$

P	q	$\neg(\neg(p \wedge q))$	$p \wedge q$	$\{a, b\}$	$p \Leftrightarrow q$
0	0	1	0	0	1
0	1	1	0	0	0
1	0	1	0	0	0
1	1	0	1	0	1

$$\rightarrow \Sigma \not\models (p \Leftrightarrow q)$$

Since  $\{a, b\}$  and  $p \Leftrightarrow q$  doesn't match.

$$\textcircled{9} \quad P \Rightarrow Q$$

$$\Rightarrow \neg P \vee Q$$

$$\neg P \Rightarrow R$$

$$\Rightarrow P \vee R$$

$$\{ \neg P \vee Q \}, \{ P \vee R \}$$

$$\neg P \vee Q \\ \downarrow \\ Q \vee R$$

$$\neg Q \Rightarrow R \not\Rightarrow \neg Q \Rightarrow \neg R$$

(ex)

$$\neg (\neg Q \Rightarrow \neg R) \Rightarrow \neg (Q \vee \neg R)$$

$$\Rightarrow \neg Q \wedge R$$

$$\therefore \{ \neg P \vee Q \}, \{ P \vee R \}, \{ \neg Q \}, \{ R \}$$

$$\neg P \\ \downarrow \\ R \neq 1$$

∴ formula is not unsatisfiable.