TURING MACHINES TM

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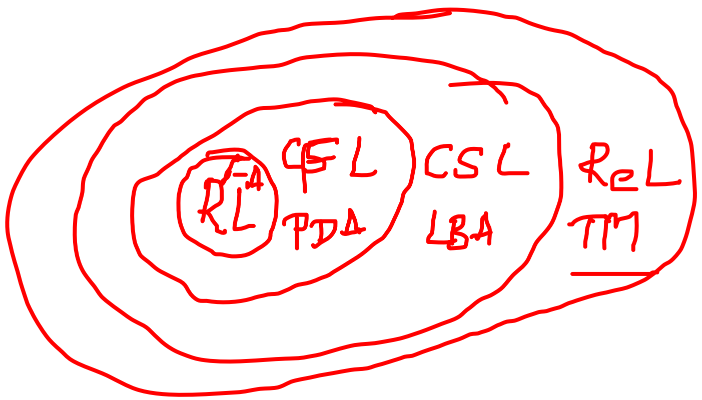
AP/CSE

LEARNING OBJECTIVE

- To Design Turing machines for any Languages (K3)
  - To Understand the concept of Turing Machine



TM X
undecidable/
unsolvable

$RL \subseteq CFL \subseteq CSL \subseteq \underline{ReL}$

INTRODUCTION

- Very powerful (abstract) machines that could simulate any modern day computer.

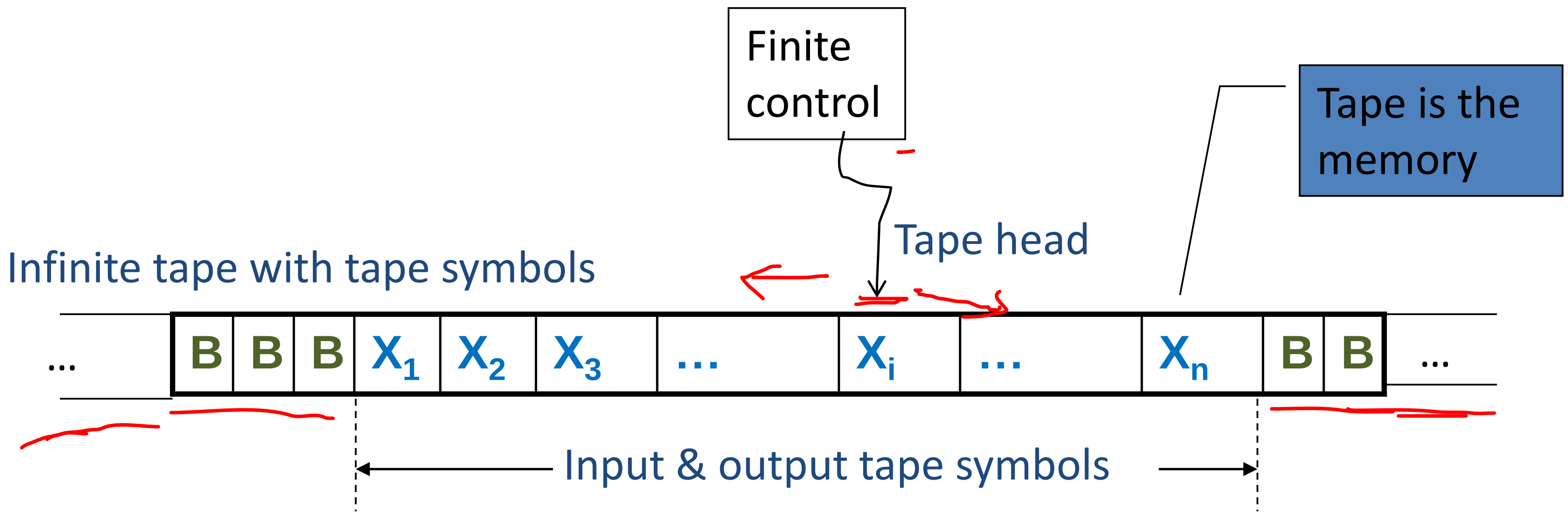
- If a problem cannot be "solved" even using a TM, then it implies that the problem is ***undecidable.***

DEVICES OF INCREASING COMPUTATIONAL POWER

- So far:
  - Finite Automata – good for devices with small amounts of memory, relatively simple control
  - Pushdown Automata – stack-based automata
- But both have limitations for even simple tasks, too restrictive as general purpose computers
- Enter the **Turing Machine**
  - More powerful than either of the above
  - Essentially a finite automaton but with unlimited memory
  - Although theoretical, can do everything a general purpose computer of today can do
    - If a TM can't solve it, neither can a computer (Undecidable problems)

TURING MACHINE

- A TM consists of a finite control (i.e. a finite state automaton) that is connected to an infinite tape.



B: blank symbol (special symbol reserved to indicate data boundary)

FORMAL DEFINITION

- A Turing machine (TM) is a 7-tuple

  $M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$ where

  - Q – A finite set of states
  - Σ – A finite set of input symbols
  - Γ – A set of tape symbols, with Σ being a subset $\Sigma \subseteq \Gamma$
  - q_0 - The start state, in Q
  - B - The blank symbol in Γ, not in Σ (should not be an input symbol $B \in \Gamma$
  - F - The set of final or accepting states

FORMAL DEFINITION

- δ: a transition function $Q \times \Gamma \rightarrow Q \times \Gamma \times \{L,R\}$
- Example $\delta(q, X) = (p, Y, L)$

δ(c/s, tape sy) = (n.s , replace sym , {L, R})

  – q - The current state, in Q

  – X - A tape symbol being scanned

  – p - The next state, in Q

  – Y - The tape symbol written on the cell being scanned, used to replace X

  – L – Left move

NOTION FOR THE TURING MACHINE

- A move of Turing machine includes:
  - Change state;
  - Write a tape symbol in the cell scanned;
  - Move the tape head left or right.

REPRESENTATION OF TM

- Turing Machines are represented in 3 ways
  - Instantaneous Descriptions
  - Transition Table
  - Transition Diagram

PDA

(q, ~~ta~~tape con, stackcont)

INSTANTANEOUS DESCRIPTIONS

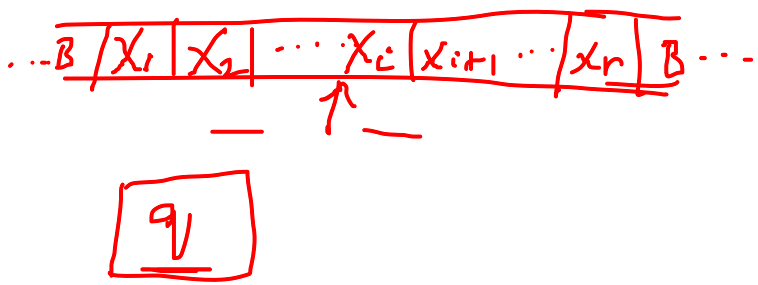
- The *instantaneous description* (ID) of a TM is represented by

  $\boldsymbol{X_1 X_2 \ldots X_{i-1}\, q\, X_i X_{i+1} \ldots X_n}$ in which

  – q is the current state

  – The tape head is scanning the i^{th} symbol from the left

  – $X_1X_2\ldots X_n$ is the portion of the tape between the leftmost and the rightmost nonblank symbols



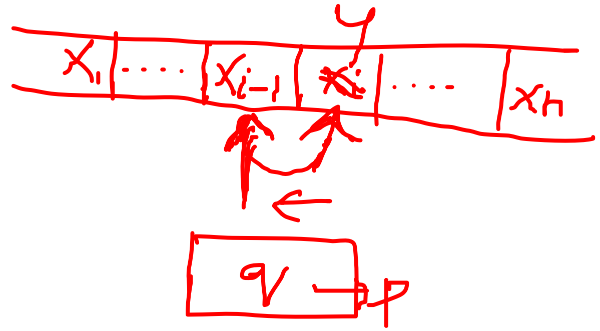
INSTANTANEOUS DESCRIPTIONS

- Moves of a TM *M* denoted by $\vdash_M$ or $\vdash$ as follows:

If $\delta(q, X_i) = (p, Y, L)$

$ID_1 \vdash ID_2$

PDA ⇒ TM



$$X_1X_2\ldots X_{i-1}qX_iX_{i+1}\ldots X_n \vdash$$

$$X_1X_2\ldots X_{i-2}pX_{i-1}Y\,X_{i+1}\ldots X_n$$

$X_1 X_2 \cdots p\, X_{i-1} Y \cdots X_n$

p

- Right moves are defined similarly.

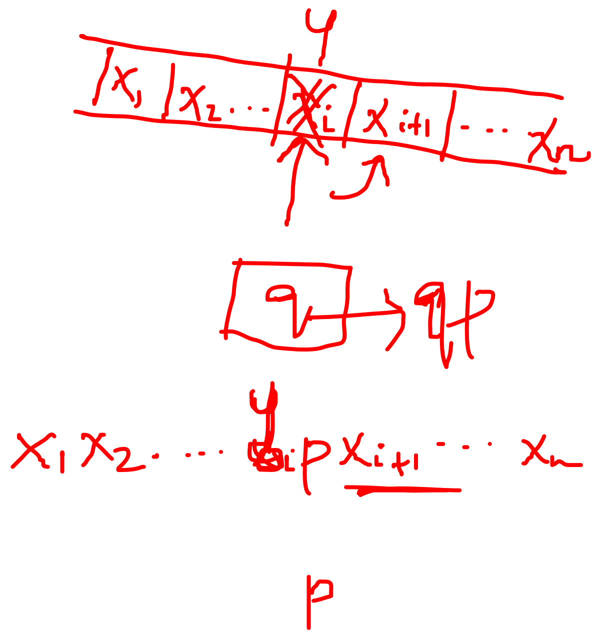
INSTANTANEOUS DESCRIPTIONS

- Moves of a TM M denoted by $\vdash_M$ or $\vdash$ as follows:

  If $\delta(q, X_i) = (p, Y, R)$

$$X_1X_2\ldots X_{i-1}qX_iX_{i+1}\ldots X_n \vdash$$

$$X_1X_2\ldots X_{i-2}X_{i-1}Yp\,X_{i+1}\ldots X_n$$



TRANSITION TABLE

- $L=\{0^n1^n \mid n \geq 1\}$

CFL ⊆ ReL

PDA TM

FAX

0011

X0Y1

XXYYB

	0	1	X	Y	B
q_0	(q_1, X, R)	-	-	-	-
q_1	$(q_1, 0, R)$	(q_2, Y, L)	-	(q_1, Y, R)	-
q_2	$(q_2, 0, L)$	-	(q_3, X, R)	(q_2, Y, L)	-
q_3	(q_1, X, R)	-	-	(q_4, Y, R)	-
q_4	-	-	-	(q_4, Y, R)	(q_5, B, N)
q_5	-	-	-	-	-

- : undefined and the machine halts.

EXAMPLE

- 0011

$q_0 0011 \vdash Xq_1 011$

$\vdash X0q_1 11$

$\vdash Xq_2 0Y1$

$\vdash q_2 X0Y1$

$\vdash Xq_3 0Y1$

$\vdash XXq_1 Y1$

$\vdash XXYq_1 1$

$\vdash XXq_2 YY$

$\vdash Xq_2 XYY$

$\vdash XXq_3 YY$

$\vdash XXYq_4 Y$

$\vdash XXYYq_4 B$

$\vdash XXYYq_5 B$

$q_5 \in F$ ∴ The string is accepted

	0	1	X	Y	B
q_0	(q_1, X, R)	-	-	-	-
q_1	$(q_1, 0, R)$	(q_2, Y, L)	-	(q_1, Y, R)	-
q_2	$(q_2, 0, L)$	-	(q_3, X, R)	(q_2, Y, L)	-
q_3	(q_1, X, R)	-	-	(q_4, Y, R)	-
q_4	-	-	-	(q_4, Y, R)	(q_5, B, N)
q_5	-	-	-	-	-

- : undefined and the machine halts.

EXAMPLE

- 000111

$q_0 000111 \vdash Xq_1 00111$

$\vdash X0q_1 0111$

$\vdash X00q_1 111$

$\vdash X0q_2 0Y11$

$\vdash Xq_2 00Y11$

$\vdash q_2 X00Y11$

$\vdash Xq_3 00Y11$

$\vdash XXq_1 0Y11$

$\vdash XX0q_1 Y11$

$\vdash XX0Yq_1 11$

$\vdash XX0q_2 YY1$

$\vdash XXq_2 0YY1$

$\vdash Xq_2 X0YY1$

$\vdash XXq_3 0YY1$

$\vdash XXXq_1 YY1$

$\vdash XXXYq_1 Y1$

$\vdash XXXYYq_1 1$

$\vdash XXXYq_2 YY$

$\vdash XXXq_2 YYY$

$\vdash XXq_2 XYYY$

$\vdash XXXq_3 YYY$

$\vdash XXXYq_4 YY$

$\vdash XXXYYq_4 Y$

$\vdash XXXYYYq_4 B$

$\vdash XXXYYYq_5 B$

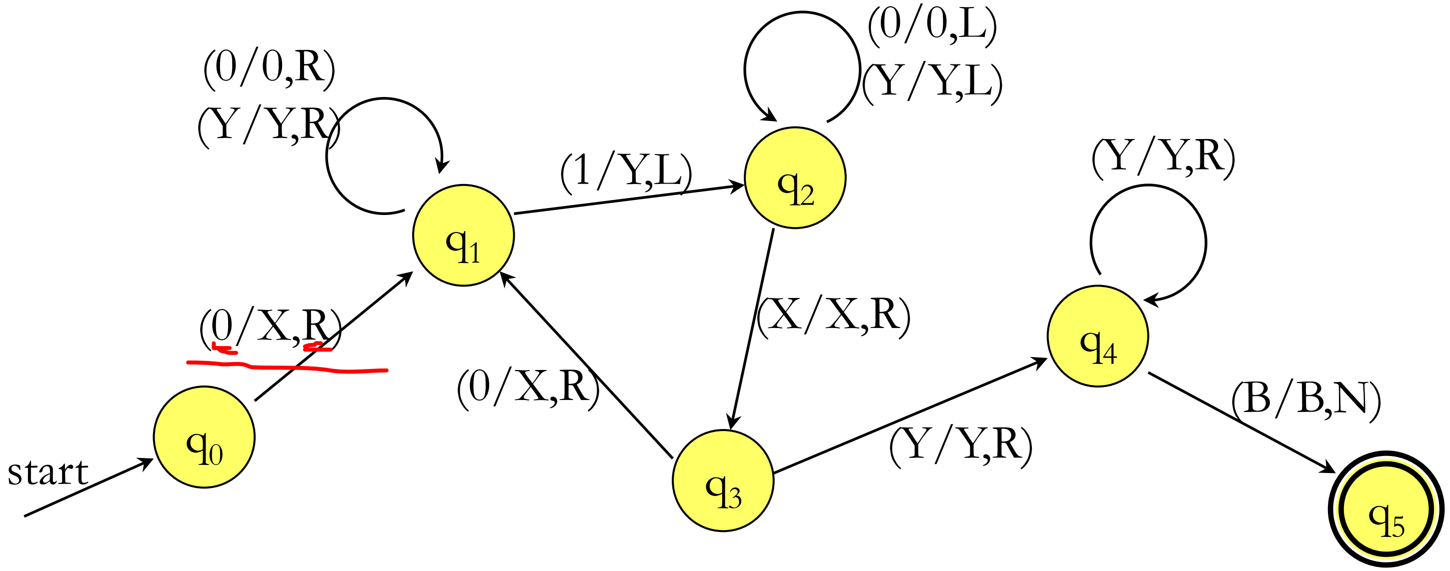
$q_5 \in F$

$\therefore$ String accepted.

	0	1	X	Y	B
q_0	(q_1, X, R)	-	-	-	-
q_1	$(q_1, 0, R)$	(q_2, Y, L)	-	(q_1, Y, R)	-
q_2	$(q_2, 0, L)$	-	(q_3, X, R)	(q_2, Y, L)	-
q_3	(q_1, X, R)	-	-	(q_4, Y, R)	-
q_4	-	-	-	(q_4, Y, R)	(q_5, B, N)
q_5	-	-	-	-	-

- : undefined and the machine halts.

TRANSITION DIAGRAM



LANGUAGE ACCEPTANCE OF TM

- Let $M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$ be a TM.
- The language accepted by M is

$$L(M) = \{w \mid w \in \Sigma^* \text{ and } q_0 w \vdash^* \alpha p \beta \text{ with } p \in F,$$
$$\alpha, \beta \in \Gamma^* \}$$

- Turing machine can accept the string by entering accepting state
- TM can reject the string by entering non-accepting state.
- TM can enter an infinite loop so that it never halts.

DESIGNING A TM

- The fundamental objective in scanning a symbol by R/W head is to ‘know’ what to do in the future.
- The machine must remember the past symbols scanned.
- Change the states only when there is a change in the written symbol or when there is a change in the movement of R/W head.

EXAMPLE

- $L=\{0^n1^n \mid n \geq 1\}$

(i) Forward

$\delta(q_0, 0) = (q_1, X, R)$

$\delta(q_1, 0) = (q_1, 0, R)$

$\delta(q_1, Y) = (q_1, Y, R)$

$\delta(q_1, 1) = (q_2, Y, L)$

(ii) Backward

$\delta(q_2, Y) = (q_2, Y, L)$

$\delta(q_2, 0) = (q_2, 0, L)$

$\delta(q_2, X) = (q_0, X, R)$

← i)

(ii) Table

(iii) T. Diagram

$Q = \{q_0, \ldots q_4\}$

$\Sigma = \{0, 1\}$

$\Gamma = \{0, 1, X, Y, B\}$

q_0

B

$F = \{q_4\}$

(iii) Terminate

$\delta(q_0, Y) = (q_3, Y, R)$

$\delta(q_3, Y) = (q_3, Y, R)$

$\delta(q_3, B) = (q_4, B, R)$

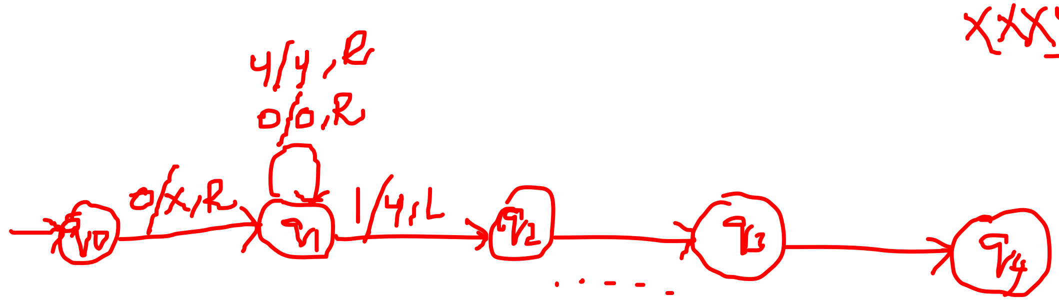
000111

X00Y11

XX0YY1

XXXYYY

XXXYYYB



q_0 0011

X011 → X0Y1 → XXYY → XXYYB

EXAMPLE

$L = \{1^n 2^n 3^n \,/\, n \geq 1\}$

Forward

$\delta(q_0, 1) = \{(q_1, X, R)\}$

$\delta(q_1, 1) = \{(q_1, 1, R)\}$ $\quad \delta(q_1, Y) = \{(q_1, Y, R)\}$

$\delta(q_1, 2) = \{(q_2, Y, R)\}$ $\quad \delta(q_2, Z) = \{(q_2, Z, R)\}$

$\delta(q_2, 2) = \{(q_2, 2, R)\}$

$\delta(q_2, 3) = \{(q_3, Z, L)\}$

~~$\delta(q_3, 3) = \{$~~

Backward

$\delta(q_3, Z) = \{(q_3, Z, L)\}$

$\delta(q_3, 2) = \{(q_3, 2, L)\}$

$\delta(q_3, Y) = \{(q_3, Y, L)\}$

$\delta(q_3, X) = \{(q_0, X, R)\}$

$\delta(q_3, 1) = \{(q_3, 1, L)\}$

Terminate

$\delta(q_0, Y) = \{(q_4, Y, R)\}$

$\delta(q_4, Z) = \{(q_4, Z, R)\}$

$\delta(q_4, B) = \{(q_5, B, N)\}$

$\delta(q_4, Y) = \{(q_4, Y, R)\}$

112233

$\vdash q_0 112233$

$\vdash X q_1 12233$

$\vdash X1 q_1 2233$

$\vdash X1Y q_2 233$

$\vdash X1Y2 q_2 33$

$\vdash X1Y2 q_3 Z3$

$\vdash X1Y q_3 2Z3$

$\vdash X1 q_3 Y2Z3$

$\vdash X q_3 1Y2Z3$

$\vdash q_3 X1Y2Z3$

$\vdash X q_0 1Y2Z3$

$\vdash XX q_1 Y2Z3$

$\vdash XXY q_1 2Z3$

$\vdash XXYY q_2 Z3$

$\vdash XXYYZ q_2 3$

$\vdash XXYYZ q_3 Z$

$\vdash XXYY q_3 ZZ$

$\vdash XXY q_3 YZZ$

$\vdash XX q_3 YYZZ$

$\vdash X q_3 XYYZZ$

$\vdash XX q_0 YYZZ$

$\vdash XXY q_4 YZZ$

$\vdash XXYY q_4 ZZ$

$\vdash XXYYZ q_4 Z$

$\vdash XXYYZZ q_4 B$

$\vdash XXYYZZ q_5 B$

$q_5 \in F$

$\therefore$ It is accepted

SUMMARY

- Definition of Turing Machine
- Model of Turing Machine
- ID of Turing Machine
- Language of a Turing Machine

TEST YOUR KNOWLEDGE

- Which of the following statements is/are FALSE?

1. For every non-deterministic Turing machine, there exists an equivalent deterministic Turing machine.
2. Turing recognizable languages are closed under union and complementation.
3. Turing decidable languages are closed under intersection and complementation.
4. Turing recognizable languages are closed under union and intersection.

A. 1 and 4 only

B. 1 and 3 only

C. 2 only

D. 3 only

TEST YOUR KNOWLEDGE

- Which of the following is true for the language

$$L=\{a^p / \ p \text{ is prime}\}$$

A. It is not accepted by a Turing Machine

B. It is regular but not context-free

C. It is context-free but not regular

D. It is neither regular nor context-free, but accepted by a Turing machine

REFERENCE

- Hopcroft J.E., Motwani R. and Ullman J.D, “Introduction to Automata Theory, Languages and Computations”, Second Edition, Pearson Education, 2008