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UCS 1503 - Theory of Computation

Assignment - 2

1. Consider the grammar $G = (\{S, A\}, \{a, b\}, P, S)$ where P consists of

$S \rightarrow aAS \mid b$

$A \rightarrow SbA \mid ba$

Write the derivation and draw its equivalent Parse tree for $w = abbab$.

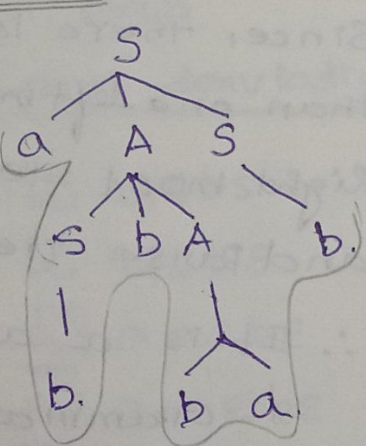
**Left most derivation**

$S \overset{lm}{\Rightarrow} aAS$
$\overset{lm}{\Rightarrow} aSbAS$
$\overset{lm}{\Rightarrow} abbAS$
$\overset{lm}{\Rightarrow} abbbaS$
$\overset{lm}{\Rightarrow} abbbab$

**Right most derivation**

$S \overset{rm}{\Rightarrow} aAS$
$\overset{rm}{\Rightarrow} aAb$
$\overset{rm}{\Rightarrow} aSbAb$
$\overset{rm}{\Rightarrow} aSbbab$
$\overset{rm}{\Rightarrow} abbbab$

**Parse Tree**




$\Rightarrow w = abbbab$

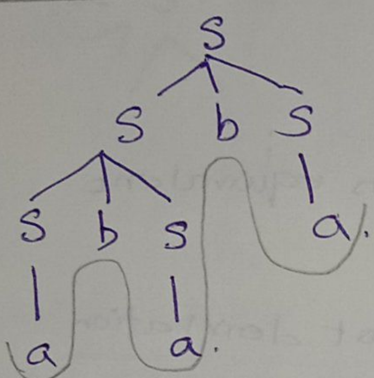
2. If G is a grammar $S \rightarrow SbS \mid a$ prove that G is ambiguous.

$G \Rightarrow S \rightarrow SbS \mid a$.

1. $S \overset{lm}{\Rightarrow} SbS$
$\overset{lm}{\Rightarrow} SbSbS$
$\overset{lm}{\Rightarrow} abSbS$
$\overset{lm}{\Rightarrow} ababS$
$\overset{lm}{\Rightarrow} ababa$.

$S \overset{rm}{\Rightarrow} SbS$
$\overset{rm}{\Rightarrow} SbSbS$
$\overset{rm}{\Rightarrow} SbSba$
$\overset{rm}{\Rightarrow} Sbaba$
$\overset{rm}{\Rightarrow} ababa$.

Parse Tree

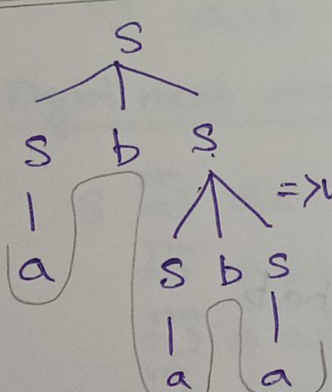


$\Rightarrow w = ababa$

2. $S \overset{lm}{\Rightarrow} SbS$
$\overset{lm}{\Rightarrow} abS$
$\overset{lm}{\Rightarrow} abSbS$.
$\overset{lm}{\Rightarrow} ababS$
$\overset{lm}{\Rightarrow} ababa$

$S \overset{rm}{\Rightarrow} SbS$
$\overset{rm}{\Rightarrow} Sba$
$\overset{rm}{\Rightarrow} SbSba$
$\overset{rm}{\Rightarrow} Sbaba$
$\overset{rm}{\Rightarrow} ababa$

Parse Tree



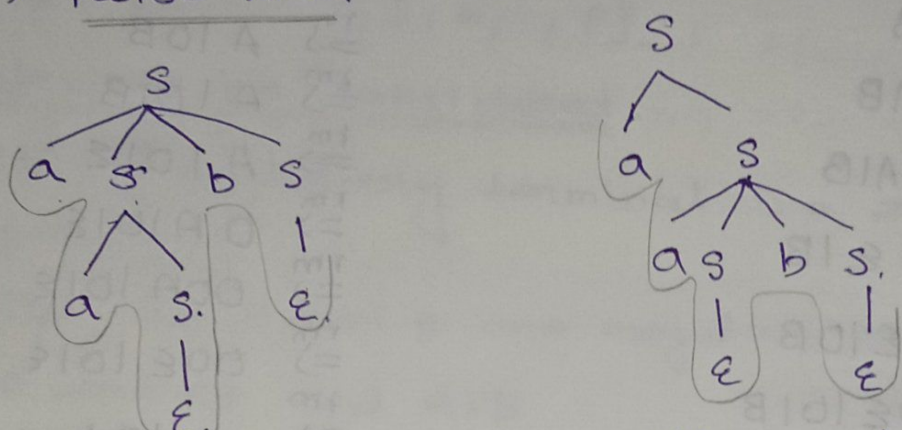
$\Rightarrow w = ababa$

Since, there is more than one left most and Right most derivation and parse Tree.

$\therefore$ It is a ambiguous Grammar.

3. Consider a grammar $S \rightarrow aS \mid aSbS \mid \varepsilon$. This grammar is ambiguous. Show that the string aab has two (a) Parse tree (b) Left most derivation (c) Right most derivation.

(a) Parse Tree:



$\Rightarrow w = aab$

$\Rightarrow w = aab$

(b) Left most derivation:

$S \overset{lm}{\Rightarrow} aSbS$
$\overset{lm}{\Rightarrow} aaSbS$
$\overset{lm}{\Rightarrow} aa\varepsilon bS$
$\overset{lm}{\Rightarrow} aa\varepsilon b\varepsilon$
$\overset{lm}{\Rightarrow} aab$

$S \overset{lm}{\Rightarrow} aS$
$\overset{lm}{\Rightarrow} aaSbS$
$\overset{lm}{\Rightarrow} aaSb\varepsilon$
$\overset{lm}{\Rightarrow} aa\varepsilon b\varepsilon$
$\overset{lm}{\Rightarrow} aab$

(c) Right most derivation:

$S \overset{rm}{\Rightarrow} aSbS$
$\overset{rm}{\Rightarrow} aSb\varepsilon$
$\overset{rm}{\Rightarrow} aaSb\varepsilon$
$\overset{rm}{\Rightarrow} aa\varepsilon b\varepsilon$
$\overset{rm}{\Rightarrow} aab$

$S \overset{rm}{\Rightarrow} aS$
$\overset{rm}{\Rightarrow} aaSbS$
$\overset{rm}{\Rightarrow} aaSb\varepsilon$
$\overset{rm}{\Rightarrow} aa\varepsilon b\varepsilon$
$\overset{rm}{\Rightarrow} aab$

4. For the grammar

$S \rightarrow A1B$

$A \rightarrow 0A \mid \varepsilon$

$B \rightarrow 0B \mid 1B \mid \varepsilon$

Give left most and Right most derivation for the string 00101.

**Left most derivation**

$$S \overset{lm}{\Rightarrow} A1B \overset{lm}{\Rightarrow} 0A1B \overset{lm}{\Rightarrow} 00A1B \overset{lm}{\Rightarrow} 00\varepsilon 1B \overset{lm}{\Rightarrow} 00\varepsilon 10B \overset{lm}{\Rightarrow} 00\varepsilon 101B \overset{lm}{\Rightarrow} 00\varepsilon 101\varepsilon \overset{lm}{\Rightarrow} 00101$$

**Rightmost derivation.**

$$S \overset{rm}{\Rightarrow} A1B \overset{rm}{\Rightarrow} A10B \overset{rm}{\Rightarrow} A101B \overset{rm}{\Rightarrow} A101\varepsilon \overset{rm}{\Rightarrow} 0A101\varepsilon \overset{rm}{\Rightarrow} 00A101\varepsilon \overset{rm}{\Rightarrow} 00\varepsilon 101\varepsilon \overset{rm}{\Rightarrow} 00101$$

5. construct CFG to generate $\{a^n b^n \mid n \in Z^+\}$

$L = \{a^n b^n \mid n \in Z^+\}$.

context free grammar.

$L = \{ab, aabb, aaabbb, \ldots\}$.

$G = S \rightarrow aSb \mid \varepsilon$.

Eg:

$w = aabb$.

$S \rightarrow aSb$.

$\Rightarrow aaSbb$.

$\Rightarrow aa\varepsilon bb$.

$\Rightarrow aabb$

The grammar is $S \rightarrow aSb \mid \varepsilon$

6. consider the alphabet $\Sigma = \{a, b, (, ), +, *, ., \epsilon\}$. Construct a context free grammar that generate all string in $\Sigma^*$ that are regular expressions over the alphabet $\{a, b\}$.

$\Sigma = \{a, b, C, ), +, *, ., \epsilon\}$.

Rules to be considered:

(a) $\phi$ and every terminal is a regular expression.

(b). If $\alpha$ and $\beta$ are regular expression, so are $\alpha\beta$ and $\alpha \vee \beta$

(c) If $\alpha$ is regular expression so is $\alpha^*$

(d) If $\alpha$ is a regular expression so is $(\alpha)$.

$G = (V, \Sigma, P, S)$ where

$V = \{S\}$

$\Sigma = \{a, b, C, ), \vee, *, \phi\}$.

$P = S \to \phi$ (Rule a)

$S \to a$ (Rule a)

$S \to b$ (Rule a)

$S \to SS$ (Rule b)

$S \to S \vee S$ (Rule b)

$S \to S^*$ (Rule c)

$S \to (S)$ (Rule d)

7. convert the given CFG to CNF.

$S \to a | aA | B$

$A \to aBB | \varepsilon$

$B \to Aa | b$

Rules to be followed :

1. No $\varepsilon$ - production.
   No unit production.
   No useless symbols.
2. Eliminate terminals from RHS for production.
3. Eliminate production with long RHS.

1. $A \to \varepsilon$ is allowed here because A doesn't appear on RHS of any other productions & unit production.

$S \to B$

$S \to Aa | b$.

Finally ① step.

$S \to a | aA | Aa | b$

$A \to aBB | \varepsilon$

$B \to Aa | b$.

2) $a \to C_a$ , $b \to C_b$ , $c \to C_c$

$S \to a | C_aA | AC_a | b$

$A \to C_aBB | \varepsilon$

$B \to AC_a | b$

3. $A \to C_aBB$

$S \to C_aD_1$

$D_1 \to BD_2$

$D_2 \to BB$

Finally ③ step.

$S \rightarrow a \mid caA \mid ACa \mid b$

$A \rightarrow BB \mid \varepsilon$

$B \rightarrow ACa \mid b$

8. convert the given CFG to GNF.

$S \rightarrow XB \mid AA$

$A \rightarrow a \mid BA \mid AB$

$B \rightarrow b$

$X \rightarrow a.$

1. Need to check the grammar is in CNF.

It is in CNF only.

2. Replace all the variables of the CFG as $A_1, A_2, \ldots A_n$:

$S \rightarrow A_1$ , $X \rightarrow A_2$ , $B \rightarrow A_3$ , $A \rightarrow A_4$.

$A_1 \rightarrow A_2 A_3 \mid A_4 A_4$

$A_4 \rightarrow a \mid A_3 A_4 \mid A_4 A_3.$

$A_3 \rightarrow b$

$A_2 \rightarrow a.$

3. $A_i \rightarrow A_j \ni j > i$

$A_4 \rightarrow a \mid A_3 A_4 \mid A_4 A_3$ $i > j$

$A_4 \rightarrow b A_4 \mid a \mid A_4 A_3.$

Finally

$A_1 \rightarrow A_2 A_3 \mid A_4 A_4$

$A_4 \rightarrow b A_4 \mid a \mid A_4 A_3$

$A_3 \rightarrow b$

$A_2 \rightarrow a$

4. Eliminate Left recursions.

$A_4 \rightarrow bA_4 \mid A_4 A_3 \mid a$

$B_4 \rightarrow A_3 \mid A_3 B_4$

$A_4 \rightarrow bA_4 \mid a \mid bA_4B_4 \mid aB_4$

Finally,

$A_1 \rightarrow A_2A_3 \mid A_4 A_4$

$A_4 \rightarrow bA_4 \mid a \mid bA_4 B_4 \mid a B_4$

$A_3 \rightarrow b$

$A_2 \rightarrow a$

$B_4 \rightarrow A_3 \mid A_3 B_4$

5. Modify A1-productions:

$A_1 \rightarrow aA_3 \mid bA_4A_4 \mid aA_4 \mid bA_4B_4A_4 \mid aB_4A_4$

6. Modify B1-productions:

$B_4 \rightarrow b \mid bB_4$

Finally, the GNF is

$A_1 \rightarrow aA_3 \mid bA_4A_4 \mid aA_4 \mid bA_4B_4A_4 \mid aB_4A_4.$

$A_2 \rightarrow a$

$A_3 \rightarrow b$

$A_4 \rightarrow bA_4 \mid a \mid bA_4B_4 \mid aB_4$

$B_4 \rightarrow b \mid bB_4.$

9. construct PAD

(a) $L = \{a^n b^m c^n / m, n > 1\}$ Acceptance by empting the stack.

(b) $L = \{WCW^R / W \in (a|b)^*, C \in a|b|epsilion\}$. Acceptance by reaching the final state

(c) $L = \{a^n b^m c^m d^n / m, n \geq 1\}$ Acceptance by empting the stack and reaching the final state.

Solutions:

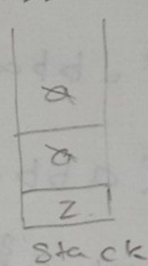
a. Given:

$L = \{a^n b^m c^n / m, n \geq 1\}$. Acceptance by empting stack.

$L = \{aabbbcc, aaabbbbccc, \ldots\}$.

aabbbcc

(i) PUSH word to the Stack.




$\delta(q0, a, z) = \{(q0, az)\}$.

$\delta(q0, a, a) = \{(q0, aa)\}$.

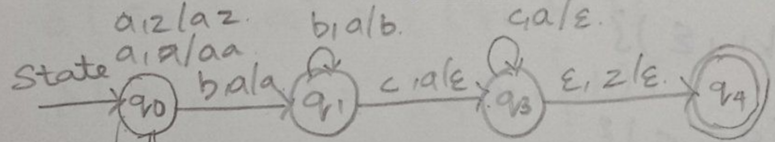
(ii) PUSH remaining symbols of w onto the Stack.

$\delta(q_1, c, a) = \{(q_2, \varepsilon)\}$

$\delta(q_2, c, a) = \{(q_2, \varepsilon)\}$.

$\delta(q_2, \varepsilon, z) = \{(q_3, \varepsilon)\}$ => Reaching the final states $\delta(q_2, \varepsilon, Z) = \{(q_3, z)\}$.

Transition Diagram:-



PDA:

$Q = \{q_0, q_1, q_2, q_3\}$.

$\Sigma = \{a, b, c\}$

$q_0$ = Initial State

$Z$ = Initial Stack Symbol.

$\Gamma = \{a, z\}$

$F = \{q_3\}$

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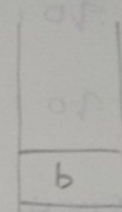
b. Given:

$L = \{wcw^R / w \in (a|b)^+, c \in a|b|epsilion\}$ Acceptance by Reaching the final state.

$w = abb$ $\quad wcc^R$ $\quad w = baa$ $\quad wcw^R$.

$w^R = bba$ $\quad abbcbba$ $\quad w^R = aab$ $\quad baaccab$

baacaab




$L = \{abbcbba, baacaab \ldots\}$

$\delta(q_0, a, z) = \{(q_0, az)\}$ Initial
$\delta(q_0, b, z) = \{(q_0, bz)\}$

$\delta(q_0, a, a) = \{(q_0, aa)\}$. Remainings are
$\delta(q_0, b, a) = \{(q_0\ ba)\}$ PUSH in this manner
$\delta(q_0, a, b) = \{(q_0, ab)\}$
$\delta(q_0, b, b) = \{(q_0, bb)\}$

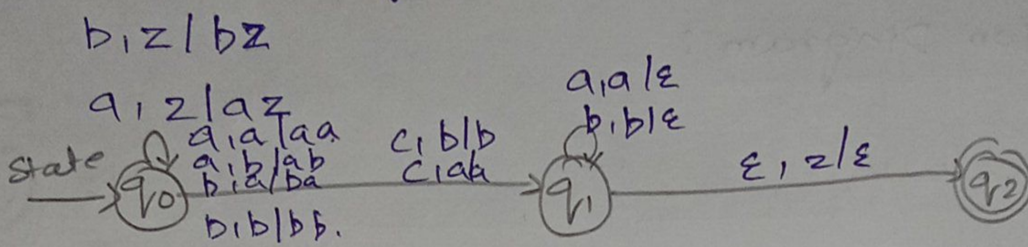
$\delta(q_0, c, a) = \{(q_1, a)\}$ Ignore it
$\delta(q_0, c, b) = \{(q_1, b)\}$.

$\delta(q_1, a, a) = \{(q_1, \varepsilon)\}$ POP
$\delta(q_1, b, b) = \{(q_1, \varepsilon)\}$
$\delta(q_1, \varepsilon, z) = \{(q_2, z)\}$

We need to more to a final State

$\delta(q_1, \varepsilon, z) = \{(q_2, \varepsilon)\}$

Transition Diagram :




PDA :

$Q = \{q_0, q_1, q_2\}$

$\Sigma = \{a, b\}$

$T = \{a, b, z\}$

$q_0$ = Initial State

$z$ = Initial Stack Symbol.

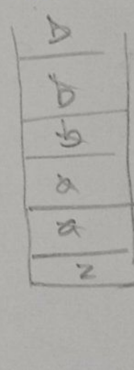
$\delta \Rightarrow$ All the 11 transitions.

$F = \{q_2\}$

6. Given:

$L = \{a^n b^m c^m d^n \mid m, n >= 1\}$ Acceptance by empting the stack and reading the final State.

$L = \{aabbbcccdd, aaabbbbccccdddd, \ldots\}$
$n = 2, m = 3$




$\delta(q_0, a, z) = \{(q_0, az)\}$ aabbbcccdd

$\delta(q_0, a, a) = \{(q_0, aa)\}$.

$\delta(q_0, b, a) = \{(q_1, ba)\}$

$\delta(q_1, b, b) = \{(q_1, bb)\}$

$\delta(q_1, c, b) = \{(q_2, \varepsilon)\}$

$\delta(q_2, c, b) = \{(q_2, \varepsilon)\}$

$\delta(q_2, d, a) = \{(q_3, \varepsilon)\}$

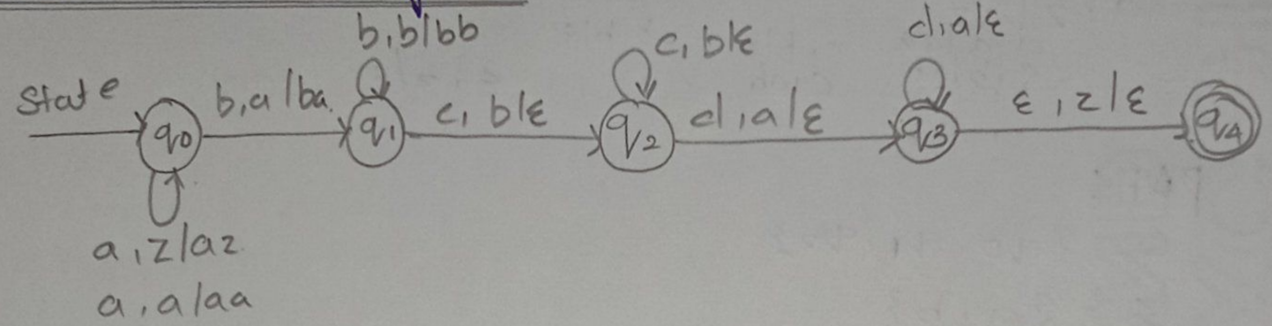
$\delta(q_3, d, a) = \{(q_3, \varepsilon)\}$

$\delta(q_3, \varepsilon, z) = \{(q_4, \varepsilon)\}$.

we need to more to a Final State

$\delta(q_3, \varepsilon, z) = \{(q_4, z)\}$

Transistion Diagram :




PDA :

$Q = \{q_0, q_1, q_2, q_3, q_4\}$

$\Sigma = \{a, b\}$

$\Gamma = \{a, b, z\}$.

$q_0$ = Initial State

$z$ = Initial Stack Symbol

$\delta$ = All a transistions.

$F = \{q_4\}$.