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CSE-B

TOC Assignment - 2

(1) $S \to aAS \mid b$

$A \to SbA \mid ba$

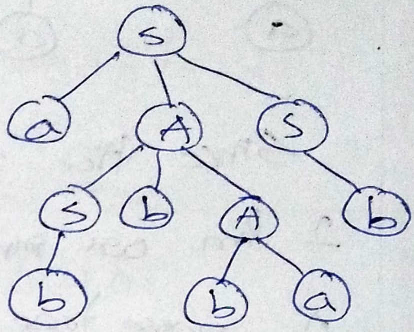
$S \xRightarrow{lm} aSbAS$

$S \xRightarrow{lm} abbAS$

$S \xRightarrow{lm} abbbaS$

$S \xRightarrow{lm} abbbab$

Parse tree



(2) $S \to SbS \mid a$, String = ababa.

Derivations.

$S \Rightarrow SbS$

$\xRightarrow{lm} SbSbS$

$\xRightarrow{lm} ababS$

$\xRightarrow{lm} ababS$

$\xRightarrow{lm} ababa$

$\xRightarrow{lm} abS$

$\xRightarrow{lm} abSbS$

$\xRightarrow{lm} ababS$

$\xRightarrow{lm} ababa$

$S \Rightarrow SbS$

$\xRightarrow{rm} Sba$

$\xRightarrow{rm} SbSba$

$\xRightarrow{rm} Sbaba$

$\xRightarrow{rm} ababa$

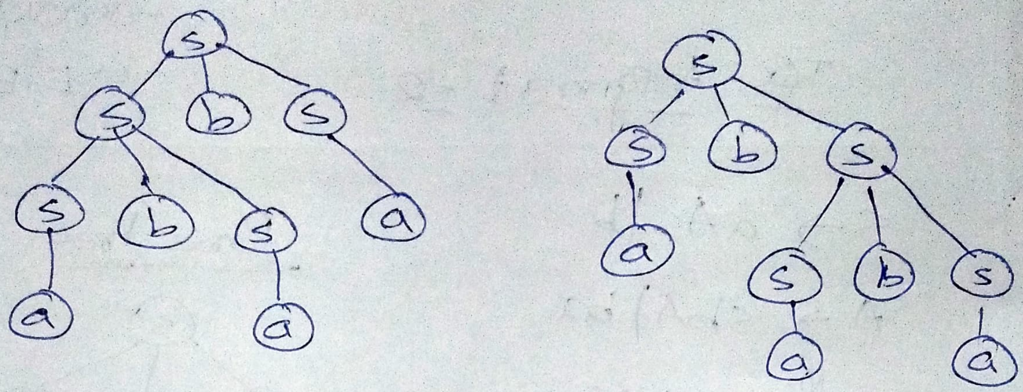
$\xRightarrow{rm} SbSbS$

$\xRightarrow{rm} SbSba$

$\xRightarrow{rm} Sbaba$

$\xRightarrow{rm} ababa$

Parse tree



Since the string ababa has more than 1 lm or rm derivation or more than 1 parse tree, then G is ambiguous grammar.

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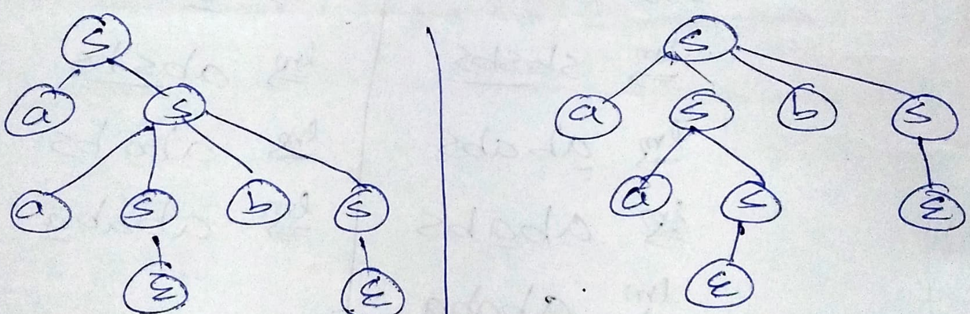
(3) $S \rightarrow aS \mid aSbS \mid \varepsilon$, string: aab

(a) parse trees



(b) lm derivation:

$S \xRightarrow{lm} aS$

$\xRightarrow{lm} a\,aSbS$

$\xRightarrow{lm} aa\varepsilon bS$

$\xRightarrow{lm} aab\varepsilon$

$\xRightarrow{lm} aab$

$S \xRightarrow{lm} aSbS$

$\xRightarrow{lm} aaSbS$

$\xRightarrow{lm} aa\varepsilon bS$

$\xRightarrow{lm} aab\varepsilon$

$\xRightarrow{lm} aab$

(c) rm derivation:

$S \overset{rm}{\Rightarrow} aS$	$S \overset{rm}{\Rightarrow} aSbS$
$\overset{rm}{\Rightarrow} aaSbS$	$\overset{rm}{\Rightarrow} aSb\varepsilon$
$\overset{rm}{\Rightarrow} aaSb\varepsilon$	$\overset{rm}{\Rightarrow} aaSb$
$\overset{rm}{\Rightarrow} aa\varepsilon b$	$\overset{rm}{\Rightarrow} aa\varepsilon b$
$\overset{rm}{\Rightarrow} aab$	$\overset{rm}{\Rightarrow} aab$

(4) $S \rightarrow A1B$; string = 00101

$A \rightarrow 0A|\varepsilon$

$B \rightarrow 0B|1B|\varepsilon$

(a) leftmost derivation	(b) rightmost derivation
$S \overset{lm}{\Rightarrow} A1B$	$S \overset{rm}{\Rightarrow} A1B$
$\overset{lm}{\Rightarrow} 0A1B$	$\overset{rm}{\Rightarrow} A10B$
$\overset{lm}{\Rightarrow} 00A1B$	$\overset{rm}{\Rightarrow} A101B$
$\overset{lm}{\Rightarrow} 00\varepsilon 1B$	$\overset{rm}{\Rightarrow} A101\varepsilon$
$\overset{lm}{\Rightarrow} 0010B$	$\overset{rm}{\Rightarrow} 0A101$
$\overset{lm}{\Rightarrow} 00101B$	$\overset{rm}{\Rightarrow} 00A101$
$\overset{lm}{\Rightarrow} 00101\varepsilon$	$\overset{rm}{\Rightarrow} 00\varepsilon 101$
$\overset{lm}{\Rightarrow} 00101$	$\overset{rm}{\Rightarrow} 00101$

(5) CFG for $L = \{a^n b^n \mid n \in Z^+\}$

$G = (\{S\}, \{a, b\}, P, S)$

Production rules P

$S \rightarrow aSb \mid ab \mid \varepsilon$

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(6) $\Sigma = \{a, b, (,), +, *, \cdot, \varepsilon\}$

CFG for Σ^* that are regular expression over alphabet $\{a, b\}$

$G = (\{S, A\}, \{a, b, (,), +, *, \cdot, \varepsilon\}, P, S)$

Production rules P

$S \rightarrow aA / bA / (S) A$

$A \rightarrow +S / \cdot S / *S / *A / \varepsilon$

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(7) $S \rightarrow a \mid aA \mid B$

$A \rightarrow aBB \mid \varepsilon$

$B \rightarrow Aa \mid b$

Step 1 : Check i) there are

ε- productions

Unit -productions

Useless symbols.

$\Rightarrow$ Eliminating ε-productions

$B \rightarrow Aa \mid a \mid b$

$S \rightarrow a \mid aA \mid a \mid B \Rightarrow S \rightarrow a \mid aA \mid B$

$\Rightarrow$ $S \rightarrow a \mid aA \mid B$

$B \rightarrow Aa \mid a \mid b$

$A \rightarrow aBB$

(ii) eliminating unit productions $S \to B$

$S \to a \mid aA \mid Aa \mid b$

$B \to Aa \mid a \mid b$

$A \to aBB$

→ eleminating useless symbols: No useless symbols.

<u>step2:</u> Eliminate terminals by introducing $C_a \to a$; $C_b \to b$

$S \to C_aA \mid AC_a \mid a \mid b$

$A \to C_aBB$; $B \to AC_a \mid a \mid b$

<u>Step3:</u> eliminate long productions.

$A \to C_aBB$; $A \to C_aD_1$; $D_1 \to BB$

→ $S \to a \mid C_aA \mid AC_a \mid b$

$B \to AC_a \mid a \mid b$; $A \to C_aD_1$; $D_1 \to BB$

$C_a \to a$; $C_b \to b$

(8)

$S \to XB \mid AA$

$A \to a \mid BA \mid AB$

$B \to b$

$X \to a$

<u>step1:</u> given grammar already in CNF form

<u>Renaming</u>. $S: A_1$; $A: A_2$; $B = A_3$; $X = A_4$

$A_1 \to A_4A_3 \mid A_2A_2$

$A_2 \to a \mid A_3A_2 \mid A_2A_3$; $A_3 \to b$; $A_4 \to a$

<u>Step2:</u> $A_i \to A_j\gamma$, $j > i$

all formulas satisfies this condition.

Step 3: Removing left recursions:

$$A_2 \to \underbrace{A_2A_3}_{\alpha} \mid \underbrace{A_3A_2}_{B_1} \mid \underbrace{a}_{B_2}$$

if $A \to A\alpha_1 \mid A\alpha_2 \ldots \mid B_1 \mid B_2 \ldots$

is converted into

$A \to B_1 \mid B_2 \mid \ldots B_n$

$A \to B_1B \mid B_2B \mid \ldots$

$B \to \alpha_1 \mid \alpha_2 \ldots$

$B \to \alpha_1 B \mid \alpha_2 B \mid \ldots$

$A_2 \to A_3A_2 \mid a$

$A_2 \to A_3A_2B_2 \mid aB_2$

$B_2 \to A_3$

$B_2 \to A_3B_2$

Step 4: Modify A_i productions.

$A_4 \to a$; $A_3 \to b$ $\quad (\because A_2 \to A_3A_2 \mid a \mid A_3A_2B_2 \mid aB_2$

$A_2 \to bA_2 \mid a \mid bA_2B_2 \mid aB_2$

$A_1 \to aA_3 \mid bA_2A_2 \mid bA_2B_2A_2 \mid aB_2A_2$

Step 5: Modify B_i productions.

$B_2 \to b \mid bB_2$ is a GNF

$A_1 \to aA_3 \mid bA_2A_2 \mid aA_2 \mid bA_2B_2A_2 \mid aB_2A_2$

$A_2 \to bA_2 \mid a \mid bA_2B_2 \mid aB_2$

$A_3 \to b$ & $A_4 \to a$.

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(9) Push down Automata

(a) $L = \{a^n b^n c^n \mid n \geq 1\}$

$L = \{abc, aabcc, \ldots\}$

Step 1: push initial symbol

$\delta(q_0, a, z) = \{(q_0, az)\}$

~~$\delta(q_0, b, a) = \{(q_0, a)\}$~~

Step 2: Push remaining input symbols

$\delta(q_0, a, a) = \{(q_0, aa)\}$

$\delta(q_0, b, a) = \{(q_0, a)\}$

Step 3: shifting state

$\delta(q_0, \varepsilon, a) = \{(q_1, a)\}$

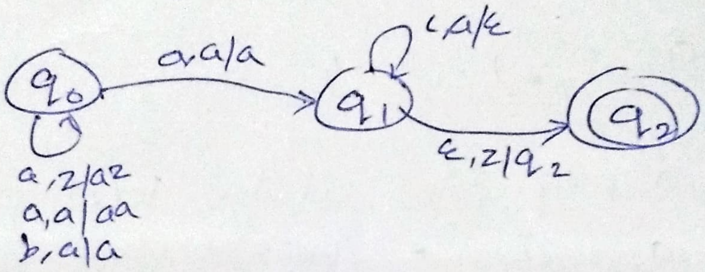
Step 4: Poping a for each input symbol c

$\delta(q_1, c, a) = \{(q_1, \varepsilon)\}$

Step 5: emptying stack

$\delta(q_1, \varepsilon, \varepsilon) = \{(q_2, z)\}$

Transition diagram



(b) $L = \{wcw^R \mid w \in (a|b)^*\}$

Step 1: push first symbol

$\delta(q_0, a, \varepsilon) = \{(q_0, az)\}$

$\delta(q_0, b, \varepsilon) = \{(q_0, bz)\}$

Step 2: pushing furthermore

$\delta(q_0, a, a) = \{(q_0, aa)\}$

$\delta(q_0, a, b) = \{(q_0, ab)\}$

$\delta(q_0, b, a) = \{(q_0, ba)\}$

$\delta(q_0, b, b) = \{(q_0, bb)\}$

Step 3: moving state for c

$\delta(q_0, a, a) = \{(q_1, a)\}$

$\delta(q_0, b, a) = \{(q_1, a)\}$

$\delta(q_0, \varepsilon, a) = \{(q_1, a)\}$

$\delta(q_0, a, b) = \{(q_1, b)\}$

$\delta(q_0, b, b) = \{(q_1, b)\}$

$\delta(q_0, \varepsilon, b) = \{(q_1, b)\}$

$\delta(q_0, a, \varepsilon) = \{(q_1, \varepsilon)\}$

$\delta(q_0, b, \varepsilon) = \{(q_1, \varepsilon)\}$

$\delta(q_0, \varepsilon, \varepsilon) = \{(q_1, \varepsilon)\}$

Step 4: Poping out while reading w^R

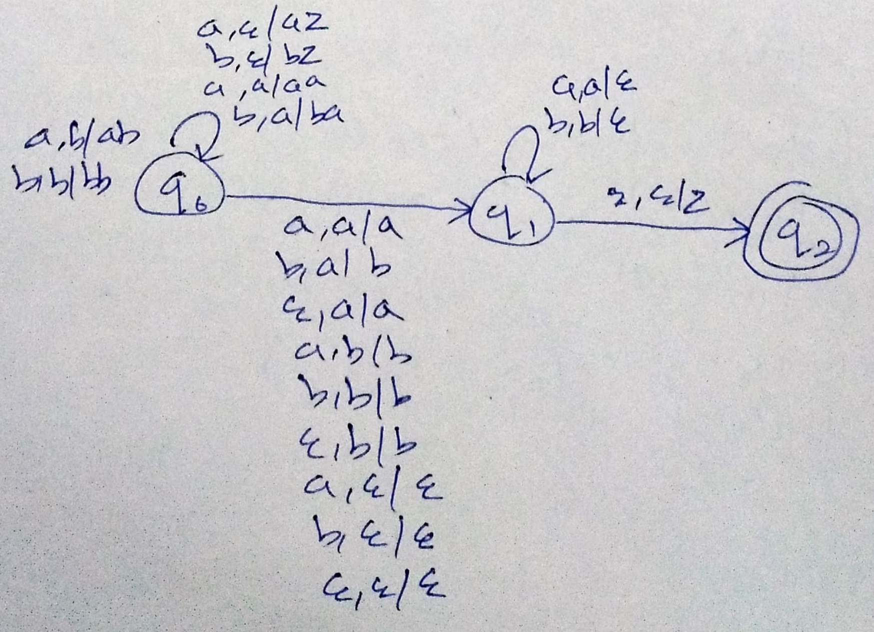
$\delta(q_1, a, a) = \{(q_1, \varepsilon)\}$

$\delta(q_1, b, b) = \{(q_1, \varepsilon)\}$

Step 5: Reaching final state

$\delta(q_1, \varepsilon, Z) = \{(q_2, Z)\}$

Transition diagram:



(iii) $L = \{a^n b^m c^m d^n \mid m, n >= 1\}$

(1) Pushing initial symbol

$$\delta(q_0, a, z) = \{(q_0, a)\}$$

(2) Pushing remaining symbols

$$\delta(q_0, a, a) = \{(q_0, aa)\}$$

$$\delta(q_0, b, a) = \{(q_0, ba)\}$$

$$\delta(q_0, b, b) = \{(q_0, bb)\}$$

(3) Shifting state

$$\delta(q_0, \varepsilon, b) = \{(q_1, b)\}$$

(4) Poping b on reading c

$$\delta(q_1, c, b) = \{(q_1, \varepsilon)\}$$

(5) Poping a on reading d

$$\delta(q_1, d, a) = \{(q_1, \varepsilon)\}$$

(6) Reaching final state

$$\delta(q_1, \varepsilon, z) = \{(q_2, \varepsilon)\}$$

Transition diagram.