TOC CAT 3
Answer key
Part – A (6×2 = 12 Marks)

1. List the different types of language acceptance by PDA and write its mathematical representation.

Acceptance by final state

– Let $M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$ be a PDA.

– The language accepted by final state is denoted by $L_F(M)$

$$L_F(M) = \{w \mid (q_0, w, z_0) \mid\!-^* (p, \varepsilon, \gamma) \text{ for some } p \text{ in } F \text{ and } \gamma \text{ in } \Gamma^*\}$$

Acceptance by empty stack

– Let $M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$ be a PDA.

– The language accepted by empty stack is denoted by $L_E(M)$

$$L_E(M) = \{w \mid (q_0, w, z_0) \mid\!-^* (p, \varepsilon, \varepsilon) \text{ for some } p \text{ in } Q\}$$

Acceptance by final state and empty stack

– Let $M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$ be a PDA.

– The language accepted by empty stack and final state is denoted $L(M)$

$$L(M) = \{w \mid (q_0, w, z_0) \mid\!-^* (p, \varepsilon, \varepsilon) \text{ for some } p \text{ in } F\}$$

2. Define the necessary rules of DPDA.

A PDA $M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$ is said to be deterministic if

– $\delta(p, a, \beta) = (q, \gamma)$
  - ie. To be deterministic, **there must be at most one choice** of move for any state p, input symbol *a*, and stack symbol β.

– $\delta(p, \varepsilon, \beta)$ is not empty then $\delta(p, a, \beta)$ must be empty for every $a \in \Sigma$, $p \in Q$, $\beta \in \Gamma$.
  - ie. **there must not be a choice between using input ε or real input.**
  - Formally, $\delta(p, \varepsilon, \beta)$ and $\delta(p, a, \beta)$ cannot both be nonempty.

3. Explain the ***Move*** interpretation of a Turing machine with an example.

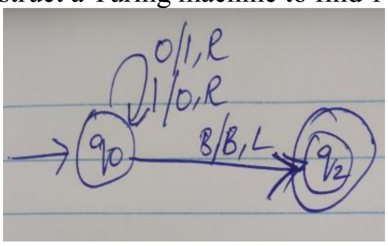
If $\delta(q, X_i) = (p, Y, L)$

$$X_1X_2\ldots X_{i-1}qX_iX_{i+1}\ldots X_n \mid\!- X_1X_2\ldots X_{i-2}pX_{i-1}Y X_{i+1}\ldots X_n$$

If $\delta(q, X_i) = (p, Y, R)$

$$X_1X_2\ldots X_{i-1}qX_iX_{i+1}\ldots X_n \mid\!- X_1X_2\ldots X_{i-2}X_{i-1}Yp X_{i+1}\ldots X_n$$

4. Construct a Turing machine to find 1's complement of a binary number.



5. What is meant by Recursive and Recursively enumerable languages? Give an example for each language.

A Language L is Recursive if and only if there is a TM that decides L.

– Let $M=(Q, \Sigma, \Gamma, \delta, q_0, B, H)$ such that
  - $H= \{h_a,h_r\}$
  - $L \subseteq \Sigma^*$ Is a language
  - Assume that the initial configuration of the TM is (q_0,w)
  - M decides L if, for all strings $w \in \Sigma^*$
    - Either $w \in L$, in which case M accepts w
    - Or $w \notin L$, then M rejects w

A Language L is Recursive Enumerable if and only if there is a TM that semidecides L.

– Let $M =(Q, \Sigma, \Gamma, \delta, q_0, B, H)$ such that
  - $H= \{h_a,h_r\}$
  - $L \subseteq \Sigma^*$ Is a language
  - Assume that the initial configuration of the TM is (q_0,w)
  - M semidecides L if, for all strings $w \in \Sigma^*$
    - Either $w \in L$, in which case M accepts w
    - Or $w \notin L$, then M does not halt

6. Identify the languages under decidable and undecidable problems.

- Regular language - decidable
- Recursive language - decidable
- Context Free Language - decidable
- Non-Recursively enumerable language - undecidable
- Context Sensitive Language - decidable
- Recursively enumerable language - undecidable

Part – B (3×6 = 18 Marks)

6. State the Pumping Lemma for Context Free Languages (CFL) and show that the language L = $\{a^n b^{n+1} c^{n+2} \mid n>=1\}$ is not a CFL.

For every context-free language L
There is an integer n, such that
For every string w in L of length $\geq$ n
There exists w = uvxyz such that:

1. $|vxy| \leq n$.
2. $|vy| > 0$.
3. For all $i \geq 0$, uv^ixy^iz is in L.

$L = \{a^n b^{n+1} c^{n+2} / n \geq 1\}$ not a CFL.

$w = a^p b^{p+1} c^{p+2}$

$|w| \geq n$.

$w = a^p$

$xy = b^{p+1}$ $\quad |vxy| < n$

$vy = b^{p-r+1}$ $\quad |vy| > 0$.

$z = c^{p+2}$

$uv^ixy^iz = uvxy(vy)^{i-1}z$

$= a^p b^{p+1} (b^{p-r+1})^{i-1} c^{p+2}$

$i=0 \quad a^p b^{p+1} (b^{p-r+1})^{-1} c^p$

$= a^p b^r c^{p+2} \notin L$

$i=1 \quad a^p b^{p+1} c^{p+2} \in L$

$i=2 \quad a^p b^{2p-r+2} c^{p+2} \notin L$.

$\therefore$ L is not a CFL.

8. Apply the procedure of CFG to PDA, to find an equivalent PDA that accepts on empty stack.
E → E+E | E*E | (E) | I
I → a | b | Ia | Ib | I0 | I1

i) push start symbol.

$\delta(q_0, \varepsilon, z) = \{(q_1, Ez)\}$

(ii) $\forall N \Rightarrow A \to \alpha$.

$\delta(q_1, \varepsilon, E) = \{(q_1, E+E), (q_1, E*E), (q_1, (E)), (q_1, I)\}$

$\delta(q_1, \varepsilon, I) = \{(q_1, a), (q_1, b), (q_1, Ia), (q_1, Ib), (q_1, I0), (q_1, I1)\}$

(iii) $\forall$ T

$\delta(q_1, +, +) = \{(q_1, \varepsilon)\}$

$\delta(q_1, *, *) = \{(q_1, \varepsilon)\}$

$\delta(q_1, (, () = \{(q_1, \varepsilon)\}$

$\delta(q_1,),)) = \{(q_1, \varepsilon)\}$

δ .... similarly for a, b, 0, 1

(iv) final state.

$\delta(q_1, \varepsilon, z) = \{(q_2, z)\}$

7. Construct a Turing Machine which accepts the language 01* + 10* using storage in finite control.

Consider a turing machine M which accepts the language 01* + 10*

Let M = (Q, {0,1}, {0,1,B}, δ, {q_0,B},B,F)

Q = {q_0,q_1} x {0,1,B}

= ([q_0,0], [q_0,1], [q_0,B], [q_1,0], [q_1,1], [q_1,B])

F = {[q_1,B]}

$\delta([q_0,B],0) = ([q_1,0], 0,R)$

$\delta([q_0,B],1) = ([q_1,1], 1, R)$

$\delta([q_1,0],1) = ([q_1,0], 1, R)$

$\delta([q_1,1],0) = ([q_1,1], 0, R)$

$\delta([q_1,0],B) = ([q_1,B],0,L)$

$\delta([q_1,1], B) = ([q_1,B], 1, L)$

Part – C (2×10 = 20 Marks)

10. (a). Construct a Turing Machine to subtract 2 numbers. (7 + 3)

f(m, n) = m-n if m >n

f(m, n) = B if m <=n

10. (b). Find the solution for ***2-1*** using the constructed Turing Machine.

$\partial(q0,0) = (q1,B,R)$ Begin. Replace the leading 0 by B.

$\partial(q1,0) = (q1,0,R)$ Search right looking for the first 1.
$\partial(q1,1) = (q2,1,R)$

$\partial(q2,1) = (q2,1,R)$ Search right past 1' s until encountering a 0. Change that 0 to 1.
$\partial(q2,0) = (q3,1,L)$

$\partial(q3,0) = (q3,0,L)$ Move left to a blank. Enter state q0 to repeat the cycle.
$\partial(q3,1) = (q3,1,L)$
$\partial(q3,B) = (q0,B,R)$

If in state q2 a B is encountered before a 0, we have situation i described above. Enter state q4 and move left, changing all 1' s to B' s until encountering a B. This B is changed back to a 0, state q6 is entered and M halts.

$\partial(q2,B) = (q4,B,L)$
$\partial(q4,1) = (q4,B,L)$
$\partial(q4,0) = (q4,0,L)$
$\partial(q4,B) = (q6,0,R)$

If in state q0 a 1 is encountered instead of a 0, the first block of 0' s has been exhausted, as in situation (ii) above. M enters state q5 to erase the rest of the tape, then enters q6 and halts.

$\partial(q0,1) = (q5,B,R)$
$\partial(q5,0) = (q5,B,R)$
$\partial(q5,1) = (q5,B,R)$
$\partial(q5,B) = (q6,B,R)$

11. (a). Construct a Turing Machine for the following language

L = $\{0^n 1^n 2^n \mid n >= 1\}$ (7+3)

11. (b). Show the validation of w = 001122 and w = 0012

Forward

$\delta(q_0, 0) = (q_1, X, R)$
$\delta(q_1, 0) = (q_1, 0, R)$
$\delta(q_1, 1) = (q_2, Y, R)$
$\delta(q_1, Y) = (q_1, Y, R)$
$\delta(q_2, 1) = (q_2, 1, R)$
$\delta(q_2, Z) = (q_2, Z, R)$
$\delta(q_2, 2) = (q_3, Z, L)$

Backward

$\delta(q_3, Z) = (q_3, Z, L)$
$\delta(q_3, 1) = (q_3, 1, L)$
$\delta(q_3, Y) = (q_3, Y, L)$
$\delta(q_3, 0) = (q_3, 0, L)$
$\delta(q_3, X) = (q_0, X, R)$

Termination

$\delta(q_0, Y) = (q_4, Y, R)$
$\delta(q_4, Y) = (q_4, Y, R)$
$\delta(q_4, Z) = (q_5, Z, R)$
$\delta(q_5, Z) = (q_5, Z, R)$
$\delta(q_5, B) = (q_6, B, R)$

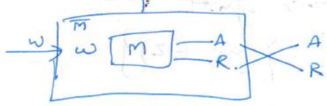
Validate

001122 0012

12. Explain the closure properties of Recursive(R) and Recursively Enumerable (RE) languages with appropriate examples.

Recursive language.

i) Recursive languages are closed under complementation.



(ii) Recursive languages are closed under union.
(iii) " " " " intersection.

Recursive enumerable languages.

i) ReL are not closed under complementation.
(ii) ReL are closed under union
(iii) " " " intersection.

w = a

13. Explain Universal Turing Machine (UTM) with encoding principle, algorithm, and neat diagram.

Concepts of UTM