Ex-6 – Inference Rules from KB

Inference Rules

Formal Proofs: using rules of inference to build arguments

Definition

A **formal proof** of a conclusion q given hypotheses $p_1, p_2, \ldots, p_n$ is a sequence of steps, each of which applies some inference rule to hypotheses or previously proven statements (antecedents) to yield a new true statement (the consequent).

A formal proof demonstrates that if the premises are true, then the conclusion is true.
Note that the word formal *here is not a synomym of rigorous.*
A formal proof is based simply on symbol manipulation (no need of thinking, just apply rules).
A formal proof is rigorous but so can be a proof that does not rely on symbols!

Formal proof example

Show that the hypotheses:

It is not sunny this afternoon and it is colder than yesterday.

We will go swimming only if it is sunny.

If we do not go swimming, then we will take a canoe trip.

If we take a canoe trip, then we will be home by sunset.

lead to the conclusion:

We will be home by the sunset.

Main steps:

Translate the statements into proposional logic.

Write a formal proof, a sequence of steps that state hypotheses or apply inference rules to previous steps.

Show that the hypotheses:

It is not sunny this afternoon and it is colder than yesterday. $\neg s \land c$

We will go swimming only if it is sunny. $w \to s$

If we do not go swimming, then we will take a canoe trip. $\neg w \to t$

If we take a canoe trip, then we will be home by sunset. $t \to h$

lead to the conclusion:

We will be home by the sunset. h

Step	Reason
1. $\neg s \land c$	hypothesis
2. $\neg s$	simplification
3. $w \to s$	hypothesis
4. $\neg w$	modus tollens of 2 and 3
5. $\neg w \to t$	hypothesis
6. t	modus ponens of 4 and 5
7. $t \to h$	hypothesis
8. h	modus ponens of 6 and 7

Where:
s: "it is sunny this afternoon"
c: "it is colder than yesterday"
w: "we will go swimming"
t: "we will take a canoe trip.
h: "we will be home by the sunset."

Resolution

Resolution and Automated Theorem Proving

We can build programs that automate the task of reasoning and proving theorems.

Recall that the rule of inference called **resolution** is based on the tautology:

$$((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$$

If we express the hypotheses and the conclusion as **clauses** (possible by CNF, a conjunction of clauses), we can use **resolution** as the only inference rule to build proofs!

Example (Do not confuse with the given. Its given for your understanding purpose)

Proofs that use exclusively **resolution** as inference rule

Step 1: Convert hypotheses and conclusion into clauses:

Original hypothesis	equivalent CNF	Hypothesis as list of clauses
$(p \wedge q) \vee r$	$(p \vee r) \wedge (q \vee r)$	$(p \vee r)$, $(q \vee r)$
$r \rightarrow s$	$(\neg r \vee s)$	$(\neg r \vee s)$

Conclusion	equivalent CNF	Conclusion as list of clauses
$p \vee s$	$(p \vee s)$	$(p \vee s)$

Step 2: Write a proof based on resolution:

Step	Reason
1. $p \vee r$	hypothesis
2. $\neg r \vee s$	hypothesis
3. $p \vee s$	resolution of 1 and 2

Show that the hypotheses:

$\neg s \wedge c$ translates to clauses: $\neg s$, c

$w \rightarrow s$ translates to clause: $(\neg w \vee s)$

$\neg w \rightarrow t$ translates to clause: $(w \vee t)$

$t \rightarrow h$ translates to clause: $(\neg t \vee h)$

lead to the conclusion:

h (it is already a trivial clause)

Note that the fact that p and $\neg p \vee q$ implies q (called disjunctive syllogism) is a special case of resolution, since $p \vee F$ and $\neg p \vee q$ give us $F \vee q$ which is equivalent to q.

Resolution-based proof:

Step	Reason
1. $\neg s$	hypothesis
2. $\neg w \vee s$	hypothesis
3. $\neg w$	resolution of 1 and 2
4. $w \vee t$	hypothesis
5. t	resolution of 3 and 4
6. $\neg t \vee h$	hypothesis
7. h	resolution of 5 and 6