slot = 20 μs

FHSS → 28 μs

SIFS = 28 μs

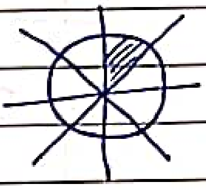
PIFS = 28+20 μs = 48 μs

DIFS = 28 + 2 × 20

CIRCLE DRAWING ALGORITHM:

$p_k < 0$:- $p_{k+1} = p_k + 2(x_k + 1) + 1$

$p_k \geq 0$:- $p_{k+1} = p_k + 2x_{k+1} - 2y_{k+1} + 1$



$p_0 = \frac{5}{4} - r \approx 1 - r$

Only for centres (0,0)
otherwise finish with (0,0) the ~~otherwise~~ add
$(x_c + x_{k+1},\ y_c + y_{k+1})$
column

$p_k < 0$ $\quad y_{k+1} = y_{k-1}$

$p_k > 0$ $\quad y_{k+1} = y_k$

$r = 10$, centre = (0,0)

k	P_k	(x_{k+1}, y_{k+1})
0	$P_0 = 1 - r$ $= -9$	$P_k < 0 \Rightarrow y_{k+1} = y_k =$ $x_{k+1} = x_k + 1$ $= 1$ (1, 10)
1	$\cancel{P_{k+1}} \leq P_k < 0$ $P_{k+1} = P_k + 2(x_k + 1) + 1$ $= -6$	$y_{k+1} = 10$ $y_{k+1} = 2$ (2, 10)
2	$P_2 = P_1 + 2(x_k + 1) + 1$ $= -6 + 2 \times 2 + 1$ $= -1$	(3, 10)
3	$P_3 = -1 + 2 \times 3 + 1 = 6$	$P_k > 0 \Rightarrow y_{k+1} = y_k -$ $= 9$ (4, 9)

~~Taking both~~,

$S_x = 0.5, \quad S_y = 3$

Representing the points in homogeneous coordinates ~~as~~

Taking $h = 1$,

$P = (2, 2, 1)$

$Q = (2, 6, 1)$

$R = (6, 6, 1)$

$S = (6, 2, 1)$

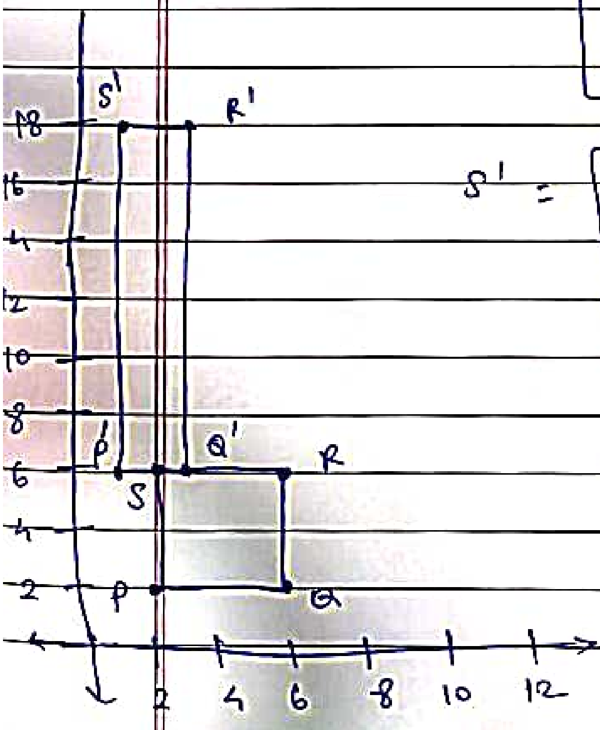
Scaling wrt origin $S = \begin{bmatrix} S_x & 0 & 1 \\ 0 & S_y & 1 \\ 0 & 0 & 1 \end{bmatrix}$

$$P' = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \\ 1 \end{bmatrix}$$

$$Q' = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 6 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 18 \\ 1 \end{bmatrix}$$

$$R' = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 6 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 18 \\ 1 \end{bmatrix}$$

$$S' = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \\ 1 \end{bmatrix}$$



The points ~~are~~ have shifted closer along x-axis since $S_x < 1$ and farther along y-axis since $S_y > 1$

Homogeneous representation of points

$P' = SP$

$$P' = \begin{bmatrix} 4 & 0 & -30 \\ 0 & 4 & -12 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -30 \\ -12 \\ 1 \end{bmatrix}$$

$$Q' = \begin{bmatrix} 4 & 0 & -30 \\ 0 & 4 & -12 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -22 \\ -4 \\ 1 \end{bmatrix}$$

$$R' = \begin{bmatrix} 4 & 0 & -30 \\ 0 & 4 & -12 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 4 \\ 1 \end{bmatrix}$$

New points => $(-30,-12), (8,8), (40,16)$

(or)

$$\begin{bmatrix} 4 & 0 & -30 \\ 0 & 4 & -12 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 2 & 10 \\ 0 & 2 & 4 \\ 1 & 1 & 1 \end{bmatrix}$$

A square has diagonal vertices $(2,2), (6,6)$. Apply scaling with scaling factor $S_x = 0.5$, $S_y = 1$ on the square and discuss the position of the transformed square. wrt the coordinate origin.

Diagonal end points of the square are $(2,2)$ and $(6,6)$. The other points are $(2,6), (6,2)$

Consistent Global State (CGS)
Inconsistent Global State (InCGS)
Strongly Consistent ~~State~~ Global State (SCGS)

time space diagram

Cuts divide ~~events~~ into past and future events.

CGS → sending was recorded but reception was not recorded but message was recorded in the channel

In transit messages.

ICGS → not sent but recorded as received.

SCGS → send and receive both recorded.

Triangle: (0,0) (2,2) (10,4)

~~Homogeneous coordinates matrix representation.~~

Scaling to 4 times the size wrt (10,4)

Translate to origin, scale, translate back.

$x_f = 10,\ y_f = 4$

$\underbrace{T(-x_f, -y_f)}_{M_3} \cdot \underbrace{S(S_x, S_y)}_{M_2} \cdot \underbrace{T(x_f, y_f)}_{M_1}$

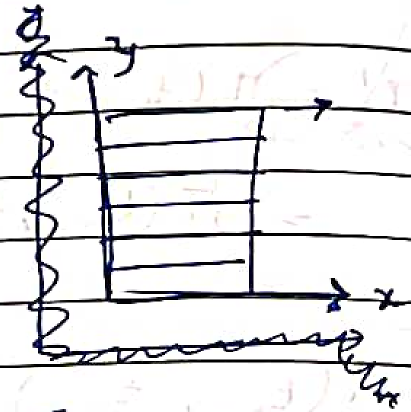
Homogeneous matrix

$$= \begin{bmatrix} 1 & 0 & -10 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 & 10(1-4) \\ 0 & 4 & 4(1-4) \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 & -30 \\ 0 & 4 & -12 \\ 0 & 0 & 1 \end{bmatrix}$$

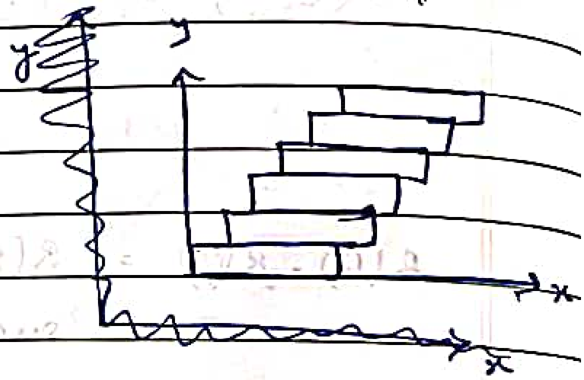
2D shear

$$\begin{bmatrix} 1 & sh_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



fixed point

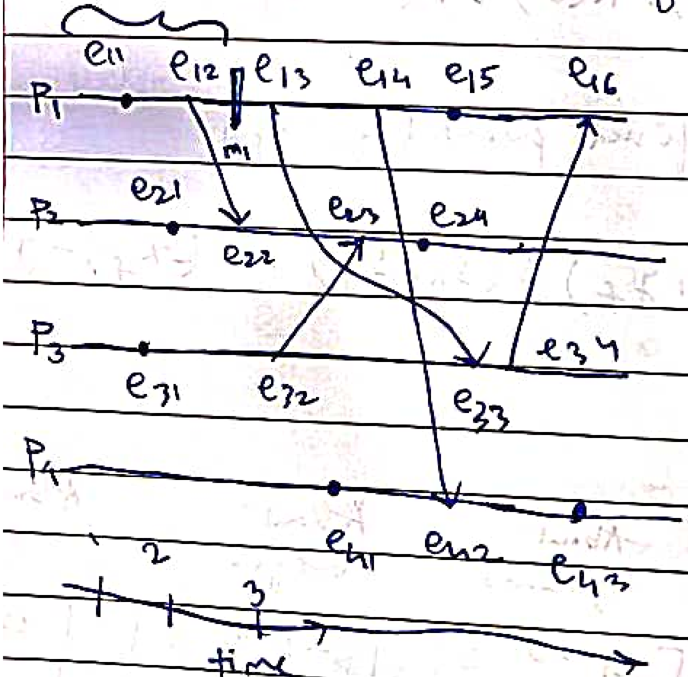
$$\begin{bmatrix} 1 & sh_x & -sh_x \cdot y_{ref} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Affine Transformation

{ unit 1 Basic transformations in unit 2 }

Local & Global states of Distributed Systems



LS_i Local State of P_i

$i \rightarrow$ process id

$LSC_{ij} \rightarrow$ communication channel connecting i^{th} process to j^{th} process

$$GS_{DS} = \sum LS_i \cup \sum LSC$$

At $t=3$, only ~~LS~~ e_{11} and e_{12} will be recorded

Two successive translations are additive.

$$P' = T(t_{2x}, t_{2y}) \cdot \{T(t_{1x}, t_{1y}) \cdot P\}$$
$$= \{T(t_{2x}, t_{2y}) \cdot T(t_{1x}, t_{1y})\} \cdot P$$

$$T(t_{2x}, t_{2y}) \cdot T(t_{1x}, t_{1y}) = \begin{bmatrix} 1 & 0 & t_{2x}+t_{1x} \\ 0 & 1 & t_{2y}+t_{1y} \\ 0 & 0 & 1 \end{bmatrix}$$

$$R(\theta) \cdot R(\phi) = R(\theta+\phi)$$

Two successive rotations are also additive

Two successive scalings are multiplicative.

Rotation w.r.t a point (not origin) (x_r, y_r)

$$P' = T(x_r, y_r) \cdot R(\theta) \cdot T(-x_r, -y_r) \cdot P$$

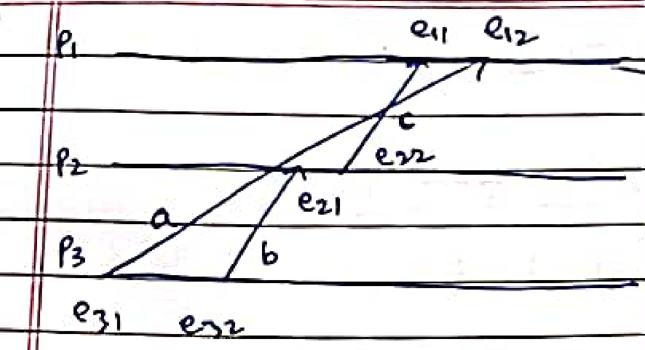
Scaling wrt a fixed point (x_f, y_f)

$$P' = T(x_f, y_f) \cdot S(s_x, s_y) \cdot T(-x_f, -y_f)$$

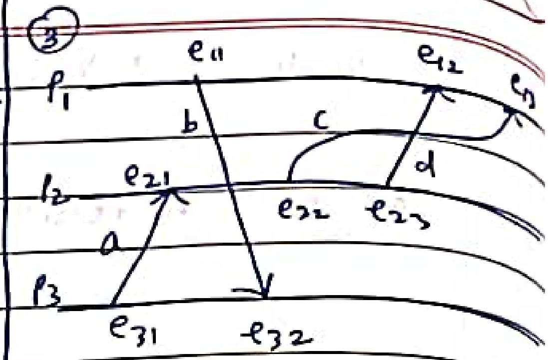
Reflection:

about x axis	About y-axis	About origin	About $x=y$
$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

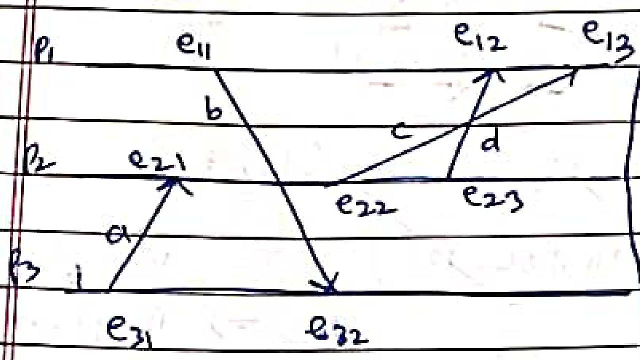
1



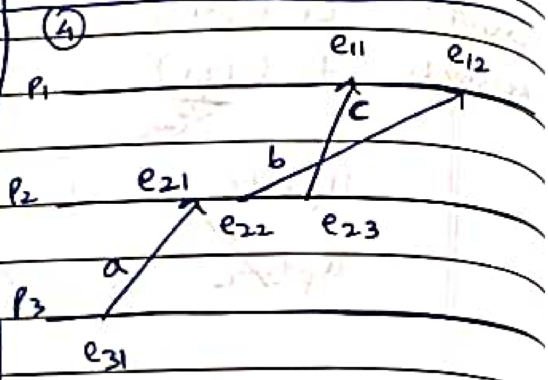
3



2



4



Design challenges of distributed systems
algorithmic challenges
System challenges
} from TB

Homogeneous coordinates

Translation
$$T(t_x, t_y) = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \qquad P' = T \cdot P$$

Rotation wrt origin
$$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad P' = R \cdot P$$

Scaling
$$S(S_x, S_y) = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad P' = S \cdot P$$

$$(x, y) = (x_h, y_h, h)$$

$$x = \frac{x_h}{h} \qquad y = \frac{y_h}{h}$$

$$\begin{array}{ccccc} P' & & T & \cdot & P \\ h=1 \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} & = & \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} & & \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \end{array}$$

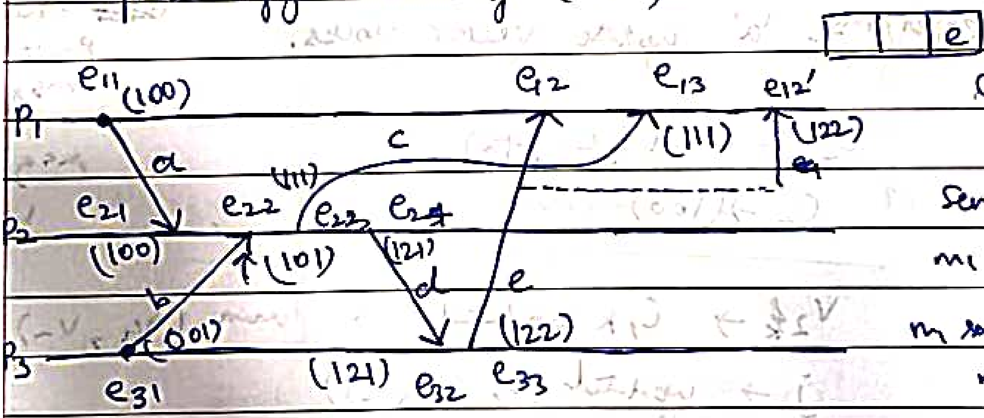
$$x' = x + t_x$$
$$y' = y + t_y$$

~~so~~ Multiplication alone is used to be able to generalise.

$$P' = \underbrace{M_2 M_1}_{\text{Not commutative.}} \cdot P = M \cdot P$$

Causal ordering of unicast and multicast protocol:

Schiphen-Eggi-Sandoz (SES) Protocol:



Causal ordering

P_1	P_2
Sender	Recei
$m_1 \to m_2$	$m_1 \to$
m_1 sent before m_2	m_1 rec before m

time →

Transitive depen

~~exist~~

$P_1(a) \to P_2(b)$

$\to$

$P_1(c) \to P_3$

Causal order

Process → clock, Vector
c, $v \Rightarrow [V]$

message → t_m, $V_m \Rightarrow [V]$
vector of vectors

Pixel addressing :— lower left corner or centre of the pixel

~~Translation:~~

$$X = \begin{bmatrix} 10 \\ 2 \end{bmatrix} \quad Y = \begin{bmatrix} 20 \\ 2 \end{bmatrix} \quad Z = \begin{bmatrix} 15 \\ 5 \end{bmatrix}$$

$$T = \begin{bmatrix} -5.5 \\ 3.75 \end{bmatrix}$$

$X' = X+T \quad Y' = Y+T \quad Z' = Z+T$

$$X' = \begin{bmatrix} 4.5 \\ 5.75 \end{bmatrix} \quad Y' = \begin{bmatrix} 14.5 \\ 5.75 \end{bmatrix} \quad Z' = \begin{bmatrix} 9.5 \\ 8.75 \end{bmatrix}$$

Scaling:

$S_x = 0.5$ $(2,2)$

$S_y = 2$ $(4,2)$

P_1

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$: P_2$

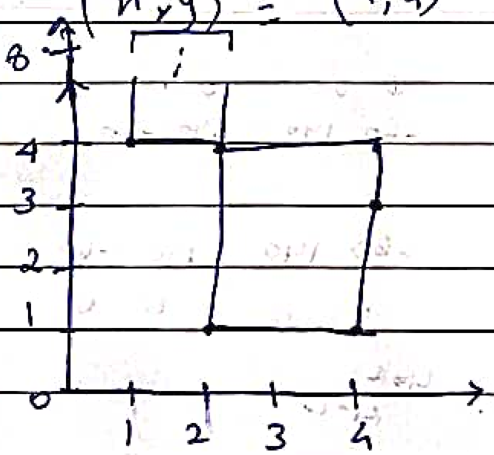
$$= \begin{bmatrix} 0.5 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \end{bmatrix} \qquad \begin{bmatrix} 0.5 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

$(2,4)$

$(x', y') = (1, 4)$

$2,4 \to 1,8$

$4,4 \to 2,8$



$(x_f, y_f) \to$ scaling is done wrt the fixed point

$$x' = x \cdot S_x + x_f (1 - S_x)$$
$$y' = y \cdot S_y + y_f (1 - S_y)$$

$P_{k+1} = P_k + 2(x_k+1) + 1 + (y_{k+1} - \frac{1}{2})^2 - (y_k - \frac{1}{2})^2$

$P_k < 0 \Rightarrow y_{k+1} = y_k$

$$\boxed{P_{k+1} = P_k + 2(x_k+1) + 1}$$

$P_k \geq 0 \Rightarrow y_{k+1} = y_k - 1$

$P_{k+1} = P_k + 2(x_k+1) + 1 + (y_k - \frac{3}{2})^2 - (y_k - \frac{1}{2})^2$

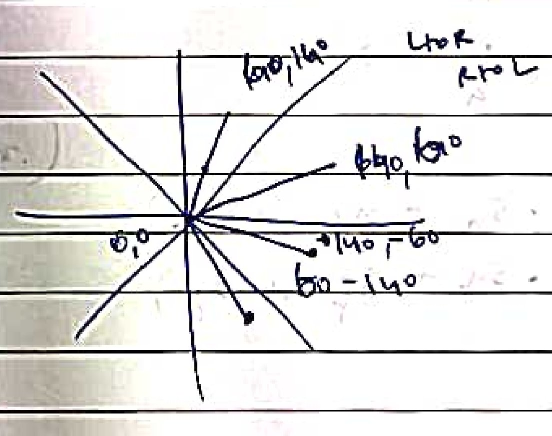
$= P_k + 2(x_k+1) + 1 + \cancel{y_k^2} - 3y_k + \frac{9}{4} - \cancel{y_k^2} - \frac{1}{4} + y_k$

$P_{k+1} = P_k + 2(x_k+1) + \cancel{1} - 2y_k + 2$

$= P_k + 2(x_k+1) - 2(y_k-1) + 1$

$$\boxed{P_{k+1} = P_k + 2x_{k+1} - 2y_{k+1} + 1}$$

0 0	~~140 60~~	0 0	0 0
60 140	0 0	-60 140	140 -60
0 0	60 140	-60 140	140 -60
140 60	0 0	0 0	0 0



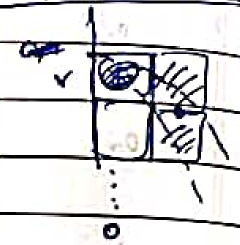
$P_k = (x_k+1)^2 + y_k^2 - r^2$

$P_{k+1} = (x_{k+1}+1)^2 + (y_{k+1})^2 - r^2$

$P_{k+1} - P_k = (x_k + 1 + 1)^2 - x_k^2 + (y_{k+1})^2 - y_k^2 \cancel{-r^2} \cancel{+r^2}$

$= \cancel{x_k^2} + 2x_k + 1 - \cancel{x_k^2} + (y_{k+1})^2 - (y_k)^2$

$P_{k+1} = P_k + (y_{k+1})^2 - (y_k)^2 + 2x_k + 1$



If $P_k < 0 \Rightarrow y_{k+1} = y_k$

$\boxed{P_{k+1} = P_k + 2x_k + 1}$

If $P_k < 0$, point b/w circle

$x_k^2 + y_k^2 < r^2$

$\Rightarrow y_{k+1} = y_k$

If $P_k \leq 0 \Rightarrow y_{k+1} = y_k - 1$

$P_{k+1} = P_k + (y_k - 1)^2 - (y_k)^2 + 2x_k + 1$

$= P_k + \cancel{y_k^2} + 1 - 2y_k - \cancel{y_k^2} + 2x_k + 1$

$= P_k + 2x_k - 2(y_k - 1)$

$= P_k + 2x_k - 2y_{k+1}$

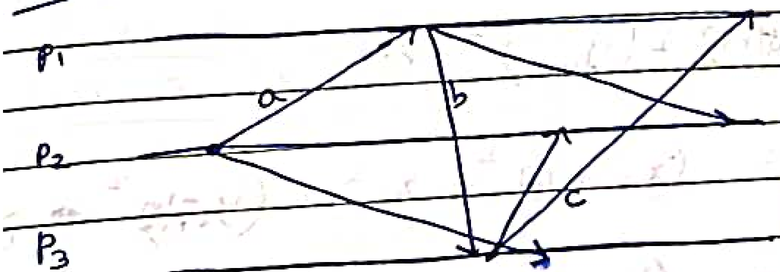
$P_k = (x_k+1)^2 + \left(y_k - \frac{1}{2}\right)^2 - r^2$

$P_{k+1} = ((x_k+1)+1)^2 + \left(y_{k+1} - \frac{1}{2}\right)^2 - r^2$

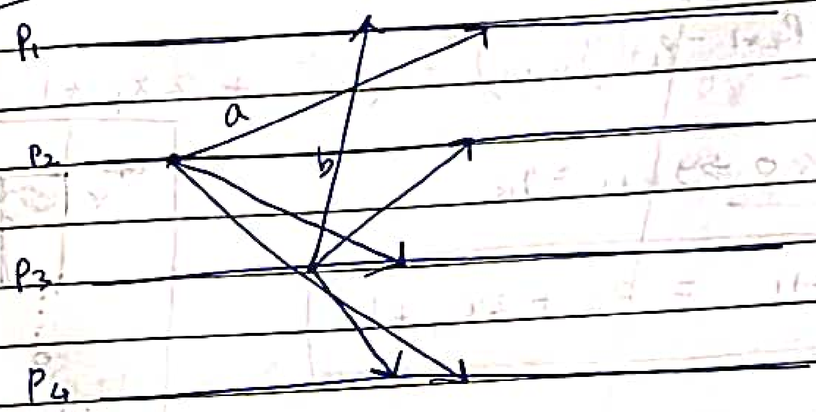
$P_{k+1} - P_k = \cancel{(x_k+1)^2} + 1 + 2(x_k+1) - \cancel{(x_k+1)^2}$

$+ \left(y_{k+1} - \frac{1}{2}\right)^2 - \left(y_k - \frac{1}{2}\right)^2 - \cancel{r^2} + \cancel{r^2}$

Case 3:

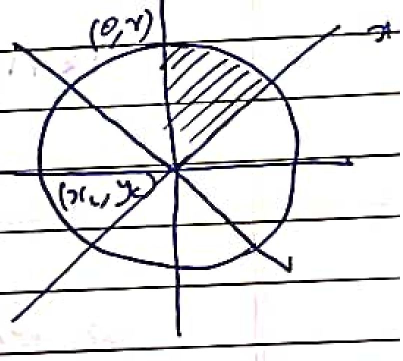


Case 4:



No ~~transitive~~ dependency

Circle drawing Algorithm - Mid Point Algorithm :



$(x - x_c)^2 + (y - y_c)^2 = r^2$

$(x_c, y_c) = (0,0) \Rightarrow x^2 + y^2 - r^2 = 0$

$P_k = f(x, y) = x^2 + y^2 - r^2$

Sample along x

$P_k = f(x_k + 1, y_k - \frac{1}{2}) \Rightarrow P_0 = f(1, r - 1/2)$

$[(x_0, y_0) = (0, r)]$

$P_0 = 1 + (r - \frac{1}{2})^2 - r^2$

$= 1 + \cancel{r^2} - r + \frac{1}{4} - \cancel{r^2}$

$P_0 = \frac{5}{4} - r \approx 1 - r$

	k	p_k	x_{k+1}, y_{k+1}	$2r_y^2 x_{k+1}$	$2r_x^2 y_{k+1}$
	6.	$P_6 = 288 + 432 + 36 - 512 = 244$	$(7,3)$ (x_0, y_0)	$2 \times 36 \times 7 = 504$	$2 \times 64 \times 3 = 384$
Region 2	0.	$P_0 = \frac{225}{4} \times 36 + 4 \times 64 - 36 \times 64 = -23$	$x_{k+1} = x_k + 1 = 8$; $y_{k+1} = y_k - 1 = 2$; $(8,2)$	$2 \times 36 \times 8 = 576$	$2 \times 64 \times 2 = 256$
	1	$P_1 = -23 + 576 + 64 - 256 = 361$	$x_{k+1} = x_k = 8$; $y_{k+1} = y_k - 1 = 1$; $(8,1)$	576	128
	2	$P_2 = 361 - 128 + 64 = 297$	$x_{k+1} = x_k = 8$; $y_{k+1} = y_k - 1 = 0$; $(8,0)$	576	0

Draw ellipse with $r_x = 8$, $r_y = 6$, centre $= (0,0)$

First point $= (0, r_y) = (0, 6)$

Region	k	P_k	(x_{k+1}, y_{k+1})	$2r_y^2 x_{k+1}$	$2r_x^2 y_{k+1}$
	0	$P_0 = r_y^2 + r_x^2\left(\frac{1}{4} - r_y\right)$ $= 36 + 64\left(\frac{1}{4} - 6\right)$ $= 36 + 64 \times \frac{-23}{4}$ $= 36 + 16 \times -23$ $= -332$	$x_{k+1} = 1$ $y_{k+1} = 6$ $(1, 6)$	2×36 $= 72$	$2 \times 64 \times 6$ $= \cancel{2 \times 384}$ $= 768$
	1	$P_1 = -332 + 72 + 36$ $= -224$	$(2, 6)$	$2 \times 36 \times 2$ $= 144$	$2 \times 64 \times 6$ $= 384 \times 2$ $= 768$
	2	$P_2 = -224 + \cancel{72}\ 144 + 36$ $= -224 + 108$ $= -44$	$(3, 6)$	216	768
	3	$P_3 = -\cancel{116}\ 44 + \cancel{72}\ 216 + 36$ $= 208$	$(4, 5)$	288	$\cancel{768}$ 640
	4	$P_4 = 208 + \cancel{72}\ 288 + 36 - 640$ $= -108$	$(5, 5)$	360	640
	5	$P_5 = -108 + \cancel{72}\ 360 + 36$ $= 288$	$(6, 4)$	432	512

$P2_k \le 0$, $x_{k+1} = x_k + 1$

$$P2_{k+1} = P2_k - 2y_{k+1} r_x^2 + r_x^2 + \left[\left(x_k + 1 + \frac{1}{2}\right)^2 - \left(x_k + \frac{1}{2}\right)^2\right] r_y^2$$

$$= P2_k - 2y_{k+1} r_x^2 + r_x^2 + 2\left(x_k + \frac{1}{2}\right) r_y^2 + \frac{1}{4} r_y^2$$

...

$$\left[\cancel{\left(x_k + \frac{1}{2}\right)^2} + 1 + 2\left(x_k + \frac{1}{2}\right) - \cancel{\left(x_k + \frac{1}{2}\right)^2}\right] r_y^2$$

$$(2x_k + 2) r_y^2$$

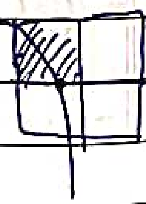
$$= 2(x_k + 1) r_y^2$$

$$= 2x_{k+1} r_y^2$$

$$\boxed{P2_{k+1} = P2_k - 2y_{k+1} r_x^2 + r_x^2 + 2x_{k+1} r_y^2}$$

$$P2_0 = f\left(x_k + \frac{1}{2}, y_k - 1\right)$$

$$= f\left(x_0 + \frac{1}{2}, y_0 - 1\right)$$



$\cancel{P2_0 = x_0 r_y^2 + r_x^2}$

$$P2_0 = \left(x_0 + \frac{1}{2}\right)^2 r_y^2 + (y_0 - 1)^2 r_x^2 - r_x^2 r_y^2$$

(x_0, y_0) point where region 1 ends

Region 2 :

$$p2_k = f\Big(\underbrace{x_k + \tfrac{1}{2},\ y_k - 1}_{\text{midpoint}}\Big)$$

$$p2_k = \left(x_k + \tfrac{1}{2}\right)^2 r_y^2 + (y_k - 1)^2 r_x^2 - r_x^2 r_y^2 \quad \text{— (1)}$$

$$p2_{k+1} = \left(x_{k+1} + \tfrac{1}{2}\right)^2 r_y^2 + \left[(y_k - 1) - 1\right]^2 r_x^2 - r_x^2 r_y^2 \quad \text{— (2)}$$

(2) − (1)

$$p2_{k+1} - p2_k = \left[\left(x_{k+1} + \tfrac{1}{2}\right)^2 - \left(x_k + \tfrac{1}{2}\right)^2\right] r_y^2 + \left[-(y_k - 1)^2 + \left((y_k - 1) - 1\right)^2\right] r_x^2$$

$$= \left[\left(x_{k+1} + \tfrac{1}{2}\right)^2 - \left(x_k + \tfrac{1}{2}\right)^2\right] r_y^2 - 2(y_k - 1) r_x^2 + r_x^2$$

$$p2_{k+1} = p2_k + \left[\left(x_{k+1} + \tfrac{1}{2}\right)^2 - \left(x_k + \tfrac{1}{2}\right)^2\right] r_y^2 - 2(y_k - 1) r_x^2 + r_x^2$$

$p2_k > 0$, $x_{k+1} = x_k$

$$p2_{k+1} = p2_k - 2(y_k - 1) r_x^2 + r_x^2$$

$$\boxed{p2_{k+1} = p2_k - 2y_{k+1} r_x^2 + r_x^2}$$

If $p_k < 0$, $y_{k+1} = y_k$

$$\boxed{p_{1k+1} = p_{1k} + 2r_y^2 x_{k+1} + r_y^2}$$

else if $p_k \geq 0$, $y_{k+1} = y_k - 1$

$$p_{1k+1} = p_{1k} + 2r_y^2(x_k+1) + r_y^2 + r_x^2\left[\left(y_k - 1 - \frac{1}{2}\right)^2 - \left(y_k - \frac{1}{2}\right)^2\right]$$

$$p_{1k+1} = \ldots + r_x^2\left[\left(y_k - \frac{3}{2}\right)^2 - \left(y_k - \frac{1}{2}\right)^2\right]$$

$$\Downarrow$$

$$= \ldots + r_x^2(2y_k - 2)(-1)$$

$$\Downarrow$$

$$= \ldots + 2r_x^2(1 - y_k)$$

$$p_{1k+1} = p_{1k} + 2r_y^2(x_k+1) + r_y^2 + 2r_x^2(1-y_k)$$

$$\boxed{p_{1k+1} = p_{1k} + 2r_y^2 x_{k+1} + r_y^2 - 2r_x^2 y_{k+1}}$$

$$f\left(x_0+1, y_0 - \frac{1}{2}\right) = f\left(0+1, r_y - \frac{1}{2}\right)$$

$$= f\left(1, r_y - \frac{1}{2}\right) = r_y^2 + r_x^2\left(r_y - \frac{1}{2}\right)^2 - r_x^2 r_y^2$$

$$= r_y^2 + r_x^2\left[r_y^2 + \frac{1}{4} - r_y\right] - r_x^2 r_y^2$$

$$= r_y^2 + \frac{r_x^2}{4} - r_y r_x^2$$

$$\boxed{p_{10} = f\left(1, r_y - \frac{1}{2}\right) = r_y^2 + r_x^2\left(\frac{1}{4} - r_y\right)}$$

REGION 1

$$P_{1k} = f(x_k+1,\ y_k - \tfrac{1}{2})$$

$$P_{1k} = r_y^2(x_k+1)^2 + (y_k - \tfrac{1}{2})^2 r_x^2 - r_x^2 r_y^2 \quad \text{— (1)}$$

$$P_{1k+1} = r_y^2\left[(x_k+1)+1\right]^2 + (y_{k+1} - \tfrac{1}{2})^2 r_x^2 - r_x^2 r_y^2 \quad \text{— (2)}$$

(2) − (1)

$$P_{1k+1} - P_{1k} = r_y^2\left[\left[(x_k+1)+1\right]^2 - (x_k+1)^2\right] + \left[(y_{k+1} - \tfrac{1}{2})^2 - (y_k - \tfrac{1}{2})^2\right] r_x^2$$

$$= r_y^2\left[\cancel{x_k^2} + 4 + 4x_k - \cancel{x_k^2} - 1 - 2x_k\right]$$

$$r_y^2(3 + 2x_k) +$$

$$y_k \quad r_y^2\left[\cancel{(x_k+1)^2} + 1\right] + \cancel{2(x_k+1)}$$

$$= r_y^2\left[\left((x_k+2) + (x_k+1)\right)(x_k+2-x_k-1)\right]$$

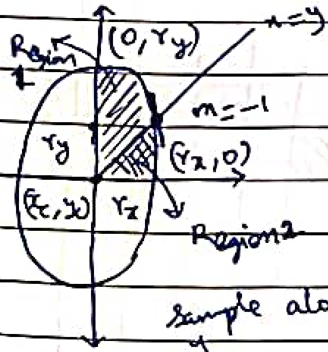
$$= r_y^2(2x_k+3) + r_x^2\left((y_{k+1} - \tfrac{1}{2})^2 - (y_k - \tfrac{1}{2})^2\right)$$

$$P_{1k+1} = P_{1k} + 2(x_k+1)r_y^2 + r_y^2 + r_x^2\left[(y_{k+1} - \tfrac{1}{2})^2 - (y_k - \tfrac{1}{2})^2\right]$$

classmate

Date ______
Page ______

ELLIPSE DRAWING ALGORITHM:



$$\left(\frac{x-x_c}{r_x}\right)^2 + \left(\frac{y-y_c}{r_y}\right)^2 = 1$$

If $(x_c, y_c) = (0, 0)$

sample along y, find x

$$\frac{x^2}{r_x^2} + \frac{y^2}{r_y^2} = 1$$

$$f(x,y) \Rightarrow x^2 r_y^2 + y^2 r_x^2 - r_x^2 r_y^2 = 0$$

Differentiate

$$2x r_y^2 + 2y r_x^2 \frac{dy}{dx} = 0$$

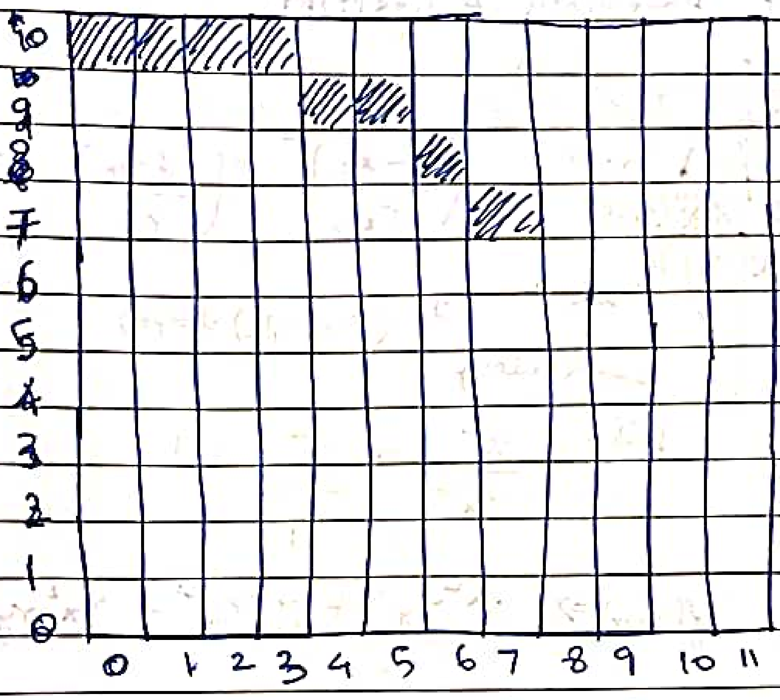
$$\frac{dy}{dx} = -\frac{2x r_y^2}{2y r_x^2} = \frac{-x r_y^2}{y r_x^2}$$

When $\frac{dy}{dx} = -1$, $\Rightarrow \frac{-x r_y^2}{y r_x^2} = -1$

$$\frac{x r_y^2}{y r_x^2} = 1$$

$$x r_y^2 = y r_x^2$$

Change when $x r_y^2 \geq y r_x^2$



Points corresponding to (1,10) in other octets are
~~(10,1)~~, (-1,10), (-1,-10), (1,-10), (10,1) (-10,1), (-10,-1),
(10,-1)

Bresenham algorithm:

end points (7,8), (12,19)

$m = \frac{19-8}{12-7} = \frac{9}{5}$ $\quad |m| > 1$

$y_{k+1} = y_k + 1$

$x_{k+1} = x_k$ if $P_k < 0$

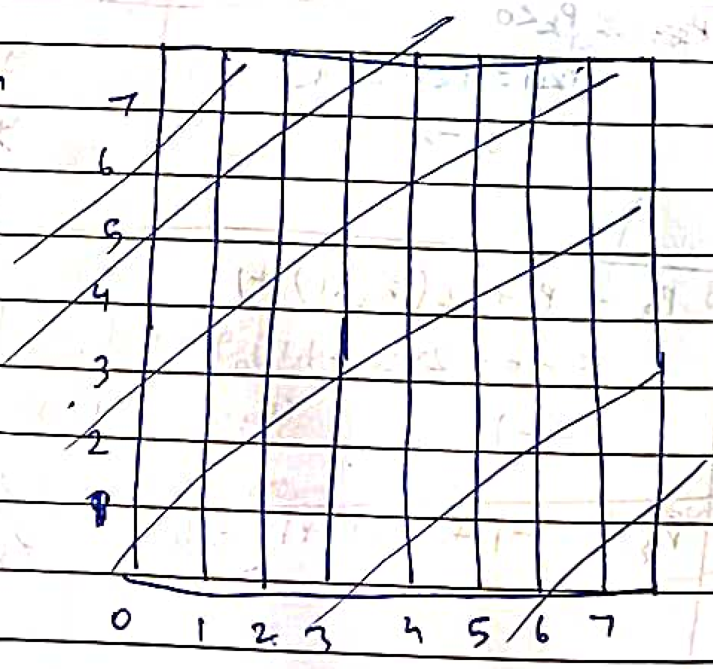
$x_{k+1} = x_{k+1}$ if $P_k \geq 0$

$P_{k+1} = P_k + 2\Delta x$ $\quad P_k < 0$

$P_{k+1} = k_k + 2\Delta x - 2\Delta y$ $\quad P_k \geq 0$

4	$P_k > 0$ $P_{k+1} = P_k + 2x_{k+1} - 2y_{k+1} + 1$ $P_4 = 6 + 2 \times 4 - 2 \times 9 + 1$ $= -3 < 0$	$y_{k+1} = y_k = 9$ $x_{k+1} = 5$ $(5, 9)$
5	$P_5 = P_4 + 2 \times x_4 + 1$ $= -3 + 2 \times 5 + 1$ $= 8 > 0$	$y_{k+1} = y_k - 1$ $= 8$ $x_{k+1} = 6$ $(6, 8)$
6	$P_6 = P_5 + 2 \times x_5 - 2y_5 + 1$ $= 8 + 2 \times 6 - 2 \times 8 + 1$ $= 5 > 0$	$y_{k+1} = y_k - 1$ $= 7$ $x_{k+1} = x_k + 1$ $= 7$ $(7, 7)$

Sim



$$P_{k+1} = 2\Delta y\, x_{k+1} - 2\Delta x\, y_{k+1} + c \quad - (6)$$

(5) − (6)

$$\Rightarrow P_{k+1} - P_k = 2\Delta y x_{k+1} - 2\Delta y x_k - 2\Delta x y_{k+1} + 2\Delta x y_k$$

$$= 2\Delta y (x_{k+1} - x_k) - 2\Delta x (y_{k+1} - y_k)$$

$$P_{k+1} = P_k + 2\Delta y \underbrace{(x_{k+1} - x_k)} - 2\Delta x (y_{k+1} - y_k)$$

sampling along $x \Rightarrow$

$$x_{k+1} - x_k = 1$$

$$P_{k+1} = P_k + 2\Delta y - 2\Delta x (y_{k+1} - y_k)$$

If $P_k < 0$

$$P_{k+1} = P_k + 2\Delta y - 2\Delta x (y_k - y_k) = P_k + 2\Delta y$$

If $P_k \geq 0$

$$P_{k+1} = P_k + 2\Delta y - 2\Delta x (y_k + 1 - y_k)$$

$$= P_k + 2\Delta y - 2\Delta x$$

$$P_0 = 2\Delta y\, x_0 - 2\Delta x\, y_0 + c$$

$$= 2\Delta y\, x_0 - 2\Delta x\, y_0 + 2\Delta y + \Delta x (2y - mx) - 1)$$

$$= 2\Delta y x_0 - 2\Delta x y_0 + 2\Delta y + 2\Delta x y - 2m\Delta x x - \Delta x$$

$$= 2\Delta y x_0 - 2\Delta x y_0 + 2\Delta y + 2\Delta x y_0 - \frac{2\Delta y \times \cancel{\Delta x}\, x_0}{\cancel{\Delta x}} - \Delta x$$

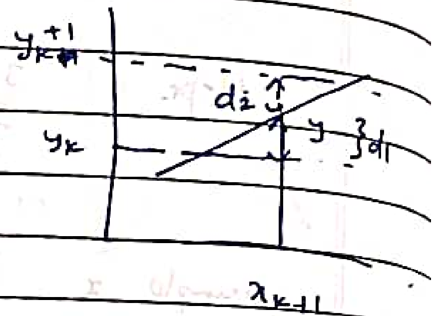
$$= 2\Delta y x_0 - \cancel{2\Delta x y_0} + 2\Delta y + \cancel{2\Delta x y_0} - 2\Delta y x_0 - \Delta x$$

$$P_0 = 2\Delta y - \Delta x$$

Bresenham's line drawing algorithm:

+ve L to R $|m| \le 1$

At $x_k + 1$, $y_{k+1} < \begin{matrix} y_k \\ y_k + 1 \end{matrix}$



$d_1 - d_2 < 0 \Rightarrow y_k$ closer

At $x_k + 1$,

$$y = m(x_k + 1) + b \quad - ①$$

$$d_1 = y - y_k = m(x_k + 1) + b - y_k \quad - ②$$

$$d_2 = y_k + 1 - y = y_k + 1 - m(x_k + 1) - b \quad - ③$$

$$d_1 - d_2 = m(x_k + 1) + b - y_k - y_k + 1 + m(x_k + 1) + b$$

$$= 2(m(x_k + 1) + b) - 2y_k - 1$$

$$= 2m(x_k + 1) + 2b - 2y_k - 1 \quad - ④$$

$P_k \propto d_1 - d_2 \Rightarrow P_k = \Delta x(d_1 - d_2)$; $m = \frac{\Delta y}{\Delta x}$

$$\Rightarrow P_k = \Delta x\left(\frac{2\Delta y}{\Delta x}(x_k + 1) + 2b - 2y_k - 1\right)$$

$$= 2\Delta y(x_k + 1) + 2b\Delta x - 2y_k\Delta x - \Delta x$$

$$= 2\Delta y x_k + 2\Delta y + 2b\Delta x - 2y_k\Delta x - \Delta x$$

$$P_k = 2\Delta y x_k - 2\Delta x y_k + c \quad - ①$$

$c = 2\Delta y + \Delta x(2b - 1)$ [no x_k or y_k term]

If $P_k < 0 \Rightarrow y_{k+1} = y_k$ else

$y_{k+1} = y_k + 1$

DDA:

A(0,0) B(7,3)

Slope $= \frac{3-0}{7-0} = \frac{3}{7}$ (+ve) $= 0.428$

$|m| \le 1$

Sample x and calculate y

$x_{k+1} = x_k + 1$

$y_{k+1} = y_k + m$

$= y_k + \frac{3}{7}$

$x_1 = 0 + 1 = 1$

$y_1 = 0 + 0.428 = 0.428 \approx 0$

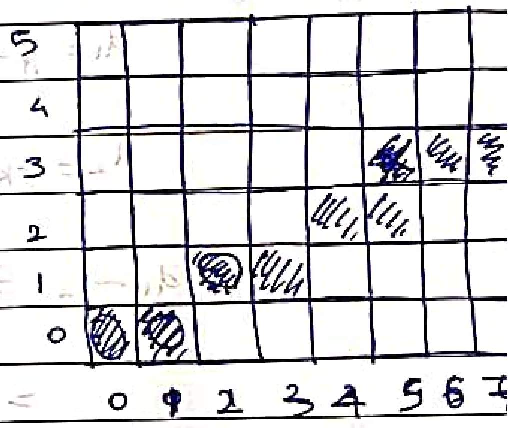
(1,0)

$x_2 = 2$

$y_2 = 0.428 + 0.428 = 0.856$

≈ 1

(2,1)

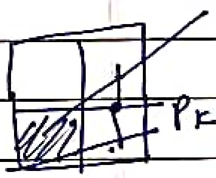


$x_3 = 3$

$y_3 = 0.856 + 0.428 = 1.284$

≈ 1

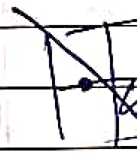
(3,1)



$x_4 = 4$

$y_4 = 1.284 + 0.428 = 1.712 \approx 2$

(4,2)



$x_5 = 5$

$y_5 = 1.712 + 0.428 = 2.140 \approx 2$ (5,2)

$x_6 = 6$

$y_6 = 2.140 + 0.428 = 2.568 = 3$ (6,3)

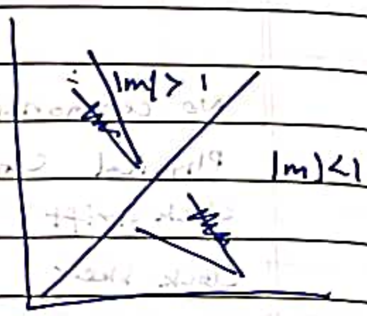
$x_7 = 7$

$y_7 = 2.568 + 0.428 = 2.996 \approx 3$ (7,3)

-ve slope R to L

$|m| \le 1$ $\quad$ $x_{k+1} = x_k - 1$ $\quad$ $y_{k+1} = y_k + m$

$|m| > 1$ $\quad$ $y_{k+1} = y_k + 1$ $\quad$ $x_{k+1} = x_k + \frac{1}{m}$



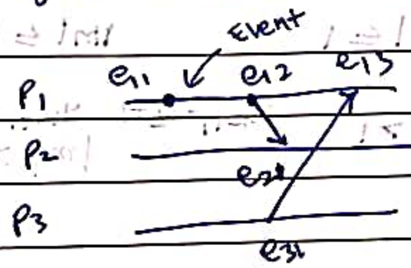
SNA

State Free characteristics

Power law

affiliation network

Logical Clock / Scalar clock



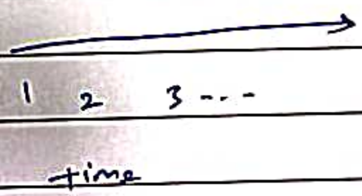
tm → time stamp

Internal event } → $c_{i+1} = c_i + 1$

Send event } $\quad$ $tm = c_i + 1$

Receive event

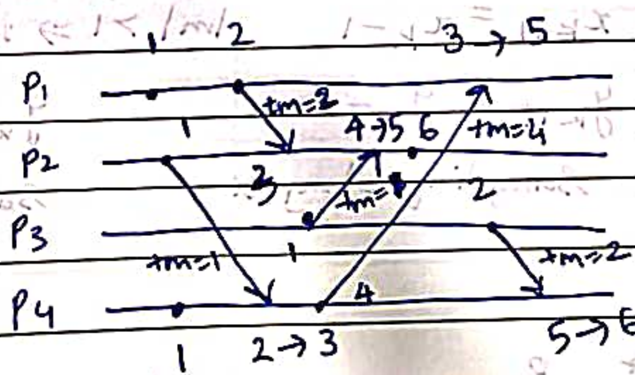
$c_j = \max(tm, c_j) + 1$



i → sender

j → receiver

For every event, increment and check



Drawbacks

Two different processes

$e_1 \to e_2$ $\quad$ e_1 before

$c(e_1) < c(e_2)$

If $c(e_1) < c(e_2)$

$\not\Rightarrow e_1 \to e_2$

$x_1' = x_1 + u_{min} * \Delta x$; $y_1' = y_1 + u_{min} * \Delta y$

$x_2' = x_2 + u_{max} * \Delta x$ $y_2' = y_2 + u_{max} * \Delta y$

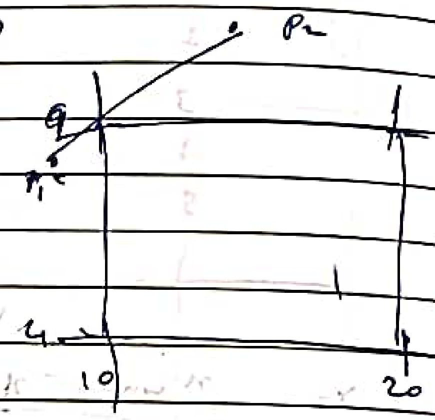
$(0, 4.5)$ $(0, 0.75)$

Q) Using ~~Liang Barsky~~ :

$x_{min}\ x_{max} = (10, 20)$

$y_{min}\ y_{max} = (4, 9)$

$P_1 = 8, 8$ $P_2\ (16, 14)$



$\Delta x = 8$ $x_2 - x_1 = 8$

$\Delta y = 6$ $y_2 - y_1 = 6$

$p_k > 0$

$\Rightarrow r_1 = \dfrac{x_1 - x_{min}}{-\Delta x} = \dfrac{8 - 10}{-8} = \dfrac{1}{4}$

$r_2 = \dfrac{x_{max} - x_1}{\Delta x} = \dfrac{20 - 8}{8} = \dfrac{12}{8} = \dfrac{3}{2}$

$r_3 = \dfrac{y_1 - y_{min}}{-\Delta y} = \dfrac{8 - 4}{-6} = \dfrac{4}{-6} = -2/3$

$r_4 = \dfrac{y_{max} - y_1}{\Delta y} = \dfrac{9 - 8}{6} = \dfrac{1}{6}$

$r_2 > 1,\ r_3 < 0 \Rightarrow$ reject

$u_{min} = \max(0, r_1) = 1/4$

$u_{max} = \min(1, r_4) = 1/6$

$u_{min} > u_{max}$ so reject

Transform the given point (3, 2, 1, 1) by the following sequence of operations

- Translate by (−1, −1, −1) in (x, y, z), rotate by 30° about x-axis and 45° about y axis

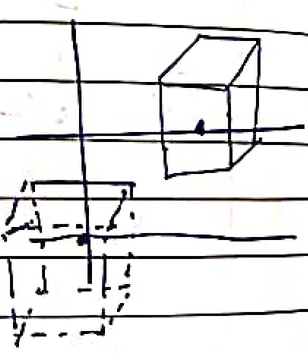
$$R_y(45^\circ) \cdot R_x(30^\circ) \cdot T(-1,-1,-1) \cdot P$$

$$\begin{bmatrix} \cos 45^\circ & 0 & \sin 45 & 0 \\ 0 & 1 & 0 & 0 \\ -\sin 45 & 0 & \cos 45^\circ & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 30^\circ & -\sin 30 & 0 \\ 0 & \sin 30^\circ & \cos 30 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \frac{3}{\sqrt{2}} - \frac{1}{2\sqrt{2}} \\ \frac{\sqrt{3}}{2} \\ -\frac{1}{\sqrt{2}} - \frac{1}{2\sqrt{2}} \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{5}{2\sqrt{2}} \\ \frac{\sqrt{3}}{2} \\ -\frac{3}{2\sqrt{2}} \\ 1 \end{bmatrix} = \begin{bmatrix} 1.766 \\ 0.866 \\ -1.061 \\ 1 \end{bmatrix}$$

A cube has vertices (1, 1, 1), (2, 1, 2), (2, 2, 2), (1, 2, 2), (1, 1, 2), (2, 1, 1), (2, 2, 1), (1, 2, 1)

Rotate the cube by 30° about an axis x' passing through the centroid of the cube and x' || x.



$T(1.5, 1.5, 0) \cdot R_x(30°) \cdot T(-1.5, -1.5, 0) \cdot P$

$$= \begin{bmatrix} 1 & 0 & 0 & 1.5 \\ 0 & 1 & 0 & 1.5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 30° & -\sin 30° & 0 \\ 0 & \sin 30° & \cos 30° & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & -1.5 \\ 0 & 1 & 0 & -1.5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \{ P \}$$

$$M' = \begin{bmatrix} 1 & 0 & 0 & 1.5 \\ 0 & 1 & 0 & 1.5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 & 0 \\ 0 & 1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$m = \begin{bmatrix} 1 & 0 & 0 & 1.5 \\ 0 & \sqrt{3}/2 & -1/2 & 1.5 \\ 0 & 1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & -1.5 \\ 0 & 1 & 0 & -1.5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} & 1.5\left(\frac{-\sqrt{3}}{2}+1\right) \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} & -\frac{1.5}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P' = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.866 & -0.5 & 0.2 \\ 0 & 0.5 & 0.866 & -0.75 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 & 1 & 1 & 2 & 2 & 1 \\ 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \\ 1 & 2 & 2 & 2 & 2 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

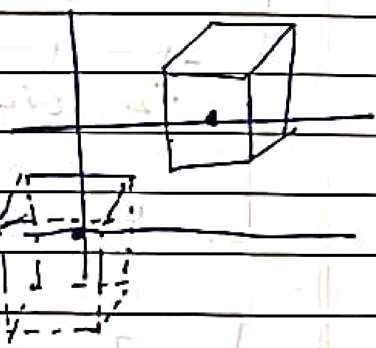
$$P' = \begin{bmatrix} 1 & 2 & 2 & 1 & 1 & 2 & 2 & 1 \\ 0.566 & 0.66 & 0.932 & 0.932 & 0.66 & 0.566 & 1.432 & 1.432 \\ 0.616 & 1.482 & 1.982 & 1.982 & 1.482 & 0.616 & 1.116 & 1.116 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \frac{3}{\sqrt{2}} - \frac{1}{2\sqrt{2}} \\ \frac{\sqrt{3}}{2} \\ -\frac{1}{\sqrt{2}} - \frac{1}{2\sqrt{2}} \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{5}{2\sqrt{2}} \\ \frac{\sqrt{3}}{2} \\ -\frac{3}{2\sqrt{2}} \\ 1 \end{bmatrix} = \begin{bmatrix} 1.766 \\ 0.866 \\ \cancel{1.93} \\ 1 \end{bmatrix}$$

-1.061

A cube has vertices (1, 1, 1), (2, 1, 2), (2, 2, 2), (1, 2, 2), (1, 1, 2), (2, 1, 1), (2, 2, 1), (1, 2, 1)

Rotate the cube by 30° about an axis x' passing through the centroid of the cube and x' ∥ X.



$$T(1.5, 1.5, 0) \cdot R_x(30^\circ) \cdot T(-1.5, -1.5, 0) \cdot P$$

$$= \begin{bmatrix} 1 & 0 & 0 & 1.5 \\ 0 & 1 & 0 & 1.5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 30^\circ & -\sin 30^\circ & 0 \\ 0 & \sin 30^\circ & \cos 30^\circ & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & -1.5 \\ 0 & 1 & 0 & -1.5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \left\{ P \right\}$$

$$\begin{bmatrix} \frac{1}{\sqrt2} & 0 & \frac{1}{\sqrt2} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{-1}{\sqrt2} & 0 & \frac{1}{\sqrt2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sqrt3}{2} & -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} & \frac{\sqrt3}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{matrix} \cos\theta & -\sin\theta & 0 & \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta & 0 & \sin\theta & \cos\theta \\ 0 & 0 & 1 & 0 & 0 \\ \cos\theta & -\sin\theta & 0 & \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta & 0 & \sin\theta & \cos\theta \end{matrix}$$

R_z R_x

$$\begin{bmatrix} \frac{1}{\sqrt2} & \frac{1}{2\sqrt2} & \frac{\sqrt3}{2\sqrt2} & 0 \\ 0 & \frac{\sqrt3}{2} & \frac{-1}{2} & 0 \\ \frac{-1}{\sqrt2} & \frac{1}{2\sqrt2} & \frac{\sqrt3}{2\sqrt2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{\sqrt2} & \frac{1}{2\sqrt2} & \frac{\sqrt3}{2\sqrt2} & -\left(\frac{1}{\sqrt2} + \frac{1}{2\sqrt2} + \frac{\sqrt3}{2\sqrt2}\right) \\ 0 & \frac{\sqrt3}{2} & \frac{-1}{2} & -\left(\frac{\sqrt3}{2} - \frac{1}{2}\right) \\ \frac{-1}{\sqrt2} & \frac{1}{2\sqrt2} & \frac{\sqrt3}{2\sqrt2} & -\left(\frac{-1}{\sqrt2} + \frac{1}{2\sqrt2} + \frac{\sqrt3}{2\sqrt2}\right) \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \frac{3}{\sqrt2} + \cancel{\frac{1}{\sqrt2}} + \cancel{\frac{\sqrt3}{2\sqrt2}} - \cancel{\frac{1}{\sqrt2}} - \frac{1}{2\sqrt2} - \cancel{\frac{\sqrt3}{2\sqrt2}} \\ 0 + \sqrt3 - \cancel{\frac{1}{2}} - \frac{\sqrt3}{2} + \cancel{\frac{1}{2}} \\ -\frac{3}{\sqrt2} + \frac{1}{\sqrt2} + \cancel{\frac{\sqrt3}{2\sqrt2}} + \frac{1}{\sqrt2} - \frac{1}{2\sqrt2} - \cancel{\frac{\sqrt3}{2\sqrt2}} \\ 0 + 0 + 0 + 1 \end{bmatrix}$$

classmate
Date
Page

Translate a triangle

(1) (10, 25, 5) $t_x = 15$
(5, 10, 5) $t_y = 5$
(20, 10, 10) $t_z = 5$

$$\begin{bmatrix} 1 & 0 & 0 & \cancel{t_x}\ 15 \\ 0 & 1 & 0 & \cancel{t_y}\ 5 \\ 0 & 0 & 1 & \cancel{t_z}\ 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 & 5 & 20 \\ 25 & 10 & 10 \\ 5 & 5 & 10 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 25 & 20 & 35 \\ 30 & 15 & 15 \\ 10 & 10 & 15 \\ 1 & 1 & 1 \end{bmatrix}$$

(25, 30, 10)
(20, 15, 10)
(35, 15, 15)

(2) Scale the triangle with $S_x = 1.5$, $S_y = 2$, $S_z = 0.5$

$$\begin{bmatrix} 1.5 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 & 5 & 20 \\ 25 & 10 & 10 \\ 5 & 5 & 10 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 15 & 7.5 & 30 \\ 50 & 20 & 20 \\ 2.5 & 2.5 & 5 \\ 1 & 1 & 1 \end{bmatrix}$$

(15, 50, 2.5)
(7.5, 20, 2.5)
(30, 20, 5)

if $p_k \neq 0$

$$r_k = \frac{q_k}{p_k}$$

~~For all $p_k < 0$, $u_{min} = max(0, r_k)$~~

for all $p_k > 0$, $u_{max} = min(1, r_k)$

Intersection points:

I_1) $x = x_1 + u_{min}(x_2 - x_1)$
$y = y_1 + u_{min}(y_2 - y_1)$

I_2) $x = x_1 + u_{max}(x_2 - x_1)$
$y = y_1 + u_{max}(y_2 - y_1)$

Window coordinates

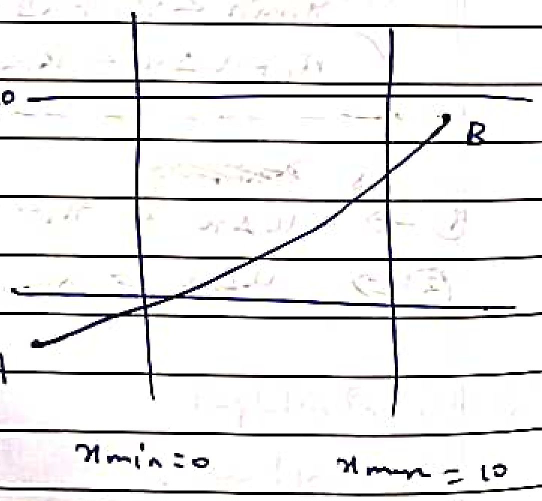
(0,0) (10,10)

(-5,3) (15,9)

$\Delta x = 15 - (-5) = 20$

$\Delta y = 6$

$\Delta x \neq 0$, $\Delta y \neq 0$

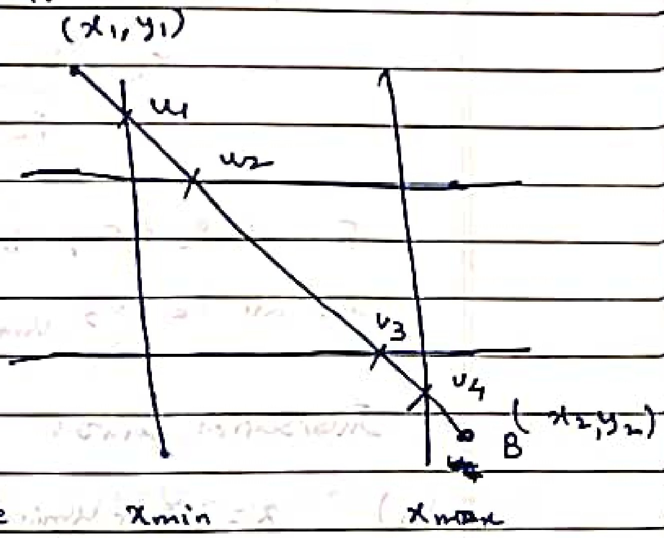


Calculating r values:

$x_1 = -5$
$x_{wmin} = 0$
$x_{wmax} = 10$
$x_2 = 15$

$$r_1 = \frac{x_1 - x_{wmin}}{-\Delta x} = \frac{-5 - 0}{-20} = \frac{1}{4}$$

LIANG BARSKY LINE CLIPPING:



Inside clipping window
$x_{min} \le x \le x_{max}$
~~Out~~ $y_{min} \le x \le y_{max}$

Using parametric representation of a line

any point (x,y) on the line can be written as

$$x = x_1 + u(x_2 - x_1)$$
$$y = y_1 + u(y_2 - y_1)$$

$$x_{min} \le x_1 + u\Delta x \le x_{max}$$
$$y_{min} \le y_1 + u\Delta y \le y_{max}$$

$x_{min} \le x_1 + u\Delta x$ — ①
$x_1 + u\Delta x \le x_{max}$ — ②

$y_{min} \le y_1 + u\Delta y$ — ③
$y_1 + u\Delta y \le y_{max}$ — ④

① ⇒ $-u\Delta x \le x_1 - x_{min}$
② ⇒ $u\Delta x \le x_{max} - x_1$

③ ⇒ $-u\Delta y \le y_1 - y_{min}$
④ ⇒ $u\Delta y \le y_{max} - y_1$

$$u p_k \le q_k \Rightarrow u = \frac{q_k}{p_k}$$

$p_k = \pm\Delta x, \pm\Delta y$

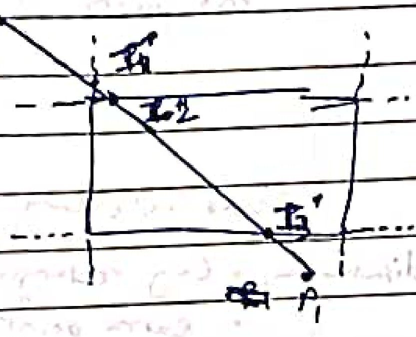
$q_k = x_1 - x_{min}, \; x_{max} - x_1, \; y_1 - y_{min}, \; y_{max} - y_1$

$p_k = 0 \Rightarrow$ line parallel to edge

$\Delta x = 0 \Rightarrow$ parallel to left & Right ~~edge~~

$\Delta y = 0 \Rightarrow$ ∥el to top & bottom

So intersects with x_{min}



line equation:

$y = -0.88x + c$

$c = 120$

$y = \underbrace{-0.88x}_{} + 120$

~~substitute $x_{min} = 10$~~

$y = -8.8 + 120$

$y = 111.2$ $\qquad I_1' = (10, 111.2)$

Region code $(I_1) = 1000$

Region code $(P_1) = 0100$

or $= 1100$

and $= 0000$

neither TA nor TR

I_2'' Intersects at y_{max}

$y = -0.88x + 120$

$100 = -0.88x + 120$

$\frac{100-120}{-0.88} = x$

$\frac{20}{0.88} = x$

$x = 22.73$

$I_2 = (22.73, 100)$

Region code $(I_2) = 0000$

Region code $(P_1) = 0100$

or $= 0100$ TA ✗

and $= 0000$ TR ✗

I_3 intersects at $y_{min} = 10$

$10 = -0.88x + 120$

$\frac{-110}{-0.88} = x$

$125 = x$

$I_3\ (125, 10)$

Region code $= 0000$

$I_2 \rightarrow 0000$

OR $\rightarrow 0000$ ✓TA

and $\rightarrow 0000$ ✗TR