12/8/21

Bresenhams line drawing algorithm: Exercise:

Draw line b/w (2, 1) & (10, 12)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{12-1}{10-2} = \frac{11}{8} = 1.375 > 1$$

$P_0 = 2\Delta x - \Delta y$

$P_0 = 2(8) - 11 = 16 - 11 = \underline{5}$

(x_{k+1}, y_{k+1})

$P_k + ve$, $P_{k+1} = P_k + 2\Delta x - 2\Delta y$

$P_k - ve$, $P_{k+1} = P_k + 2\Delta x$

Sample along y & calc x.

k	P_k	x_{k+1}, y_{k+1}	P_{k+1}	(x_k, y_{k+1})
0	5	(3, 2)	5 + 2(8) - 2(11)	
1	-1	(3, 3)	-1 + 2(8)	
2	15	(4, 4)	15 + 2(8) - 2(11)	
3	9	(5, 5)	9 + 2(8) - 2(11)	
4	3	(6, 6)	3 + 2(8) - 2(11)	
5	-3	(6, 7)	-3 + 2(8)	
6	13	(7, 8)	13 + 2(8) - 2(11)	
7	7	(8, 9)	7 + 2(8) - 2(11)	
8	1	(9, 10)	1 + 2(8) - 2(11)	
9	-5	(9, 11)	-5 + 2(8)	
10	11	(10, 12)	-	

Rotation:

Rotate a Δ^{le} (4,6) (2,2) (6,2) about vertex (4,6) by 180° counterclockwise & find new vertex.

Rot. w.r.t. fixed pt =

$$\begin{bmatrix} \cos\theta & -\sin\theta & x_r(1-\cos\theta)+y_r\sin\theta \\ \sin\theta & \cos\theta & y_r(1-\cos\theta)-x_r\sin\theta \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos 180^\circ & -\sin 180^\circ & 4(1-\cos 180^\circ)+6\cdot\sin 180^\circ \\ \sin 180^\circ & \cos 180^\circ & 6(1-\cos 180^\circ)+4\cdot\sin 180^\circ \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 & 4(-2)+6(0) \\ 0 & -1 & 6(2)+4(0) \\ 0 & 0 & 1 \end{bmatrix}$$

$$R. = \begin{bmatrix} -1 & 0 & 8 \\ 0 & -1 & 12 \\ 0 & 0 & 1 \end{bmatrix}$$

$P' = R \cdot P$

For vertex (2,2)

$$P' = \begin{bmatrix} -1 & 0 & 8 \\ 0 & -1 & 12 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ 1 \end{bmatrix}$$

For vertex (6,2)

$$P' = \begin{bmatrix} -1 & 0 & 8 \\ 0 & -1 & 12 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 10 \\ 1 \end{bmatrix}$$

Vertex (4,6) remains fixed.

New triangle is drawn at vertices (4,6) (6,10) & (2,10)

Reflection about the line $y=-x$ is equivalent to a reflection relative to the y-axis followed by a counterclockwise rotation of θ degrees. Find the value of θ.

Solution:

Given, $\text{Ref}_{y=-x} = \text{Rot.}(\theta) \cdot \text{Ref}_{y\text{-axis}}$

i.e. $\begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow -\cos\theta = 0$

$-\sin\theta = -1$

$\therefore \theta = +90^\circ$