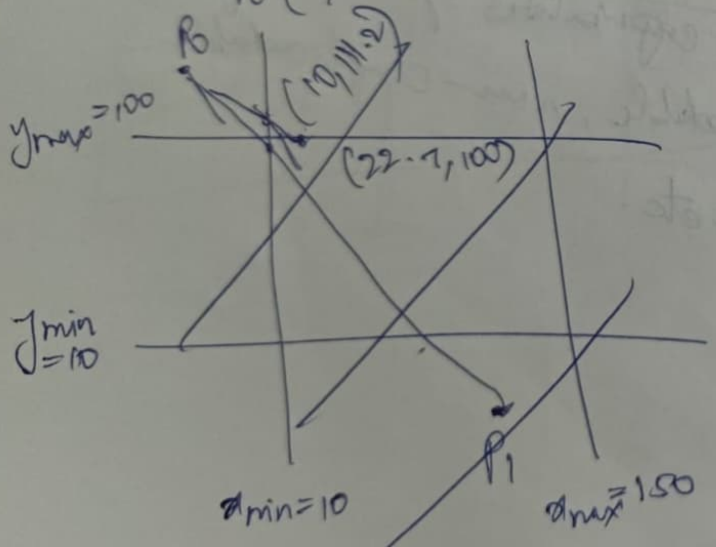
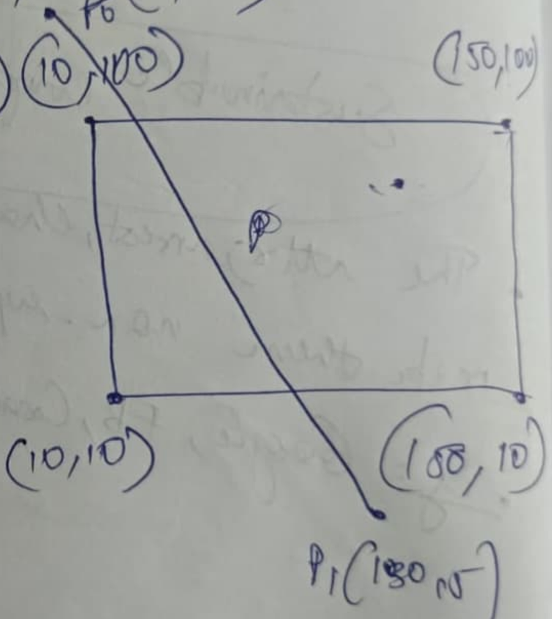
eg:

$(xw_{min}, xw_{max}) = (10, 150)$

$(yw_{min}, yw_{max}) = (10, 100)$ (0,120)

$P_0 = (0, 120)$ $P_1 = (130, 5)$ (10,100)





Top:

$$RC(P_0) = \frac{\text{TBRL}}{1001}$$

$$RC(P_1) = 0100$$

$$TA(OR) = 1101 \neq 0000 \text{ Not TA}$$

$$TR(AND) = 0000$$

Left:

$$\text{slope} = \frac{-115}{130} = -0.88$$

$$x = xw_{min} = 10 \qquad m = \frac{y - y_1}{x - x_1}$$

$$\boxed{y = y1 + m(x - x1)}$$

$$y = \cancel{150}\ 120 + (-0.88)(10 - 0)$$

$$= 120 - 8.8$$

$$= 111.2 \qquad I_1 = \cancel{(111.2)}\ (10, 111.2)$$

$$\Downarrow$$

$$RC(I_1) = 1000$$

$$RC(P_1) = 0100$$

$$TA(OR) = 1100 \neq 0000$$

$$TR(AND) = 0000 \text{ fails TR.}$$

Top:

$$y = y_{max} = 100$$

$$\boxed{x = x1 + (y - y1)/m}$$

$$x = 10 + (100 - 111.2)/(-0.88)$$

$$= 10 + \frac{(-11.2)}{(-0.88)}$$

$$= \cancel{20}\ 22.72$$

$$\Rightarrow RC(I_2) = 0000 \qquad TR(AND) = 0000 \text{ fails TR.}$$

$$RC(P_1) = 0100$$

$$TA(OR) = 0100$$

Bottom:

$y = y_{min} = 10$

$x = x_1 + \left(y - y'\right)/m$

$= 130 + (10 - 5)/(-0.88)$

$= 130 - \frac{5}{0.88}$

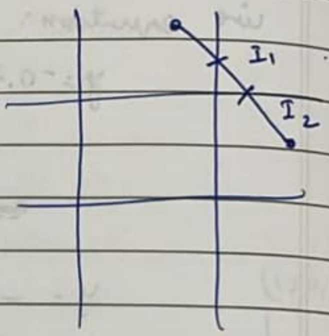
$= 124.31$

$\rightarrow RC(I_2) = 0000$

$RC(I_3) = 0000$

$T.A(OR) = \underline{0000}$ T.A

End points of the clipped line (22.73, 100) and (125, 10)



Cohen Sutherland

disadvantages
- Only rectangular
- extra points calculation

LAMPORT'S DMUTEX ALGORITHM EXAMPLE :

$P_3 \to P_1 \parallel P_3 \to P_2$

Request (Req)
Reply (Rep)
Release (Rel)

P_1 (1, P_3) P_2 (1, P_3) P_3 (1, P_3)

Req(1, P_3, P_1)

Req(1, P_3, {P_1, P_2}) (timestamp, Src, Dest)

Req(1, P_3, P_2)

Rep(3, P_1, P_3)

Rep(3, P_2, P_3)

Rep(3, P_1, P_3), Rep(3, P_2, P_3)

CS

Rel(7, P_3, {P_1, P_2})

Rel(7, P_3, P_2)

Rel(7, P_3, P_1)

Req(10, P_1, {P_2, P_3})

Req(10, P_3, {P_1, P_2})

Req(10, P_3, P_1)

Req(10, P_1, P_2)

Req(10, P_3, P_2)

Req(10, P_1, P_3)

Rep(11, P_2, P_1)

Rep(11, P_2, {P_1, P_3})

Rep(11, P_3, P_2)

Rep(12, P_3, P_1)

Rep(12, P_3, P_1)

CS

Rep(15, P_1, P_3) -

Transform the given point $(3, 2, 1, 1)$ by the following sequence of operations

- Translate by $(-1, -1, -1)$ in (x, y, z)
- rotate by 30° about x-axis and 45° about y axis

$R_y(45^\circ) \cdot R_x(30^\circ) \cdot T(-1, -1, -1) \cdot P$

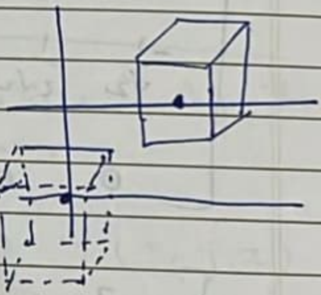
$$\begin{bmatrix} \cos 45^\circ & 0 & \sin 45 & 0 \\ 0 & 1 & 0 & 0 \\ -\sin 45^\circ & 0 & \cos 45^\circ & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 30^\circ & -\sin 30^\circ & 0 \\ 0 & \sin 30^\circ & \cos 30^\circ & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \frac{3}{\sqrt{2}} - \frac{1}{2\sqrt{2}} \\ \frac{\sqrt{3}}{2} \\ -\frac{1}{\sqrt{2}} - \frac{1}{2\sqrt{2}} \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{5}{2\sqrt{2}} \\ \frac{\sqrt{3}}{2} \\ \frac{-3}{2\sqrt{2}} \\ 1 \end{bmatrix} = \begin{bmatrix} 1.766 \\ 0.866 \\ 1.93 \\ 1 \end{bmatrix}$$

-1.061

A cube has vertices $(1,1,1)$, $(2,1,2)$, $(2,2,2)$, $(1,2,2)$, $(1,1,2)$, $(2,1,1)$, $(2,2,1)$, $(1,2,1)$

Rotate the cube by 30° about an axis x' passing through the centroid of the cube and $x' \parallel X$.



$$T(1.5, 1.5, 0) \cdot R_x(30^\circ) \cdot T(-1.5, -1.5, 0) \cdot P$$

$$= \begin{bmatrix} 1 & 0 & 0 & 1.5 \\ 0 & 1 & 0 & 1.5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 30^\circ & -\sin 30^\circ & 0 \\ 0 & \sin 30^\circ & \cos 30^\circ & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & -1.5 \\ 0 & 1 & 0 & -1.5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \{ P$$

$$M' = \begin{bmatrix} 1 & 0 & 0 & 1.5 \\ 0 & 1 & 0 & 1.5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 & 0 \\ 0 & 1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$M = \begin{bmatrix} 1 & 0 & 0 & 1.5 \\ 0 & \sqrt{3}/2 & -1/2 & 1.5 \\ 0 & 1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & -1.5 \\ 0 & 1 & 0 & -1.5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$M'' = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} & \cancel{\frac{\sqrt{3}}{2}1.5}\ +1.5\left(-\frac{\sqrt{3}}{2}+1\right) \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} & -\frac{1.5}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P' = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.866 & -0.5 & 0.2 \\ 0 & 0.5 & 0.866 & -0.75 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 & 1 & 1 & 2 & 2 & 1 \\ 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \\ 1 & 2 & 2 & 2 & 2 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$P' = \begin{bmatrix} 1 & 2 & 2 & 1 & 1 & 2 & 2 & 1 \\ 0.566 & 0.66 & 0.932 & 0.932 & 0.66 & 0.566 & 1.432 & 1.432 \\ 0.616 & 1.482 & 1.982 & 1.982 & 1.482 & 0.616 & 1.116 & 1.116 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Translate a triangle

(1) (10, 25, 5) $t_x = 15$

(5, 10, 5) $t_y = 5$

(20, 10, 10) $t_z = 5$

$$\begin{bmatrix} 1 & 0 & \cancel{0} & \cancel{t_x}\ 15 \\ 0 & 1 & 0 & \cancel{t_y}\ 5 \\ 0 & 0 & 1 & \cancel{t_z}\ 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 & 5 & 20 \\ 25 & 10 & 10 \\ 5 & 5 & 10 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 25 & 20 & 35 \\ 30 & 15 & 15 \\ 10 & 10 & 15 \\ 1 & 1 & 1 \end{bmatrix}$$

(25, 30, 10)

(20, 15, 10)

(35, 15, 15)

(2) Scale the triangle with $S_x = 1.5$, $S_y = 2$, $S_z = 0.5$

$$\begin{bmatrix} 1.5 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 & 5 & 20 \\ 25 & 10 & 10 \\ 5 & 5 & 10 \\ 1 & 1 & 1 \end{bmatrix}$$

~~15 0 0 0~~
~~0 20~~

$$\begin{bmatrix} 15 & 7.5 & 30 \\ 50 & 20 & 20 \\ 2.5 & 2.5 & 5 \\ 1 & 1 & 1 \end{bmatrix}$$

(15, 50, 2.5)

(7.5, 20, 2.5)

(30, 20, 5)

classmate

Date ______
Page ______

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

R_z

$$\begin{matrix} \cos\theta & -\sin\theta & 0 & \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta & 0 & \sin\theta & \cos\theta \\ 0 & 0 & 1 & 0 & 0 \\ \cos\theta & -\sin\theta & 0 & \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta & 0 & \sin\theta & \cos\theta \end{matrix}$$

R_x

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & -\left(\frac{1}{\sqrt{2}} + \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}\right) \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} & -\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right) \\ -\frac{1}{\sqrt{2}} & \frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & -\left(-\frac{1}{\sqrt{2}} + \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}\right) \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \\ 1 \end{bmatrix}$$

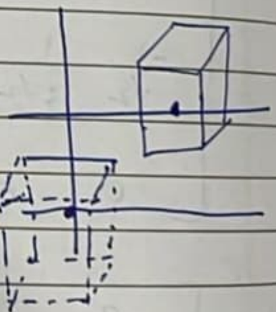
$$\begin{bmatrix} \frac{3}{\sqrt{2}} + \cancel{\frac{1}{\sqrt{2}}} + \cancel{\frac{\sqrt{3}}{2\sqrt{2}}} - \cancel{\frac{1}{\sqrt{2}}} - \frac{1}{2\sqrt{2}} + \cancel{\frac{\sqrt{3}}{2\sqrt{2}}} \\ 0 + \sqrt{3} - \cancel{\frac{1}{2}} - \frac{\sqrt{3}}{2} + \cancel{\frac{1}{2}} \\ -\frac{3}{\sqrt{2}} + \frac{1}{\sqrt{2}} + \cancel{\frac{\sqrt{3}}{2\sqrt{2}}} + \frac{1}{\sqrt{2}} - \frac{1}{2\sqrt{2}} - \cancel{\frac{\sqrt{3}}{2\sqrt{2}}} \\ 0 + 0 + 0 + 1 \end{bmatrix}$$

$$\begin{bmatrix} \frac{3}{\sqrt{2}} - \frac{1}{2\sqrt{2}} \\ \frac{\sqrt{3}}{2} \\ -\frac{1}{\sqrt{2}} - \frac{1}{2\sqrt{2}} \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{5}{2\sqrt{2}} \\ \frac{\sqrt{3}}{2} \\ -\frac{3}{2\sqrt{2}} \\ 1 \end{bmatrix} = \begin{bmatrix} 1.766 \\ 0.866 \\ \cancel{1.95} \ -1.061 \\ 1 \end{bmatrix}$$

A cube has vertices (1, 1, 1), (2, 1, 2), (2, 2, 2), (1, 2, 2), (1, 1, 2), (2, 1, 1), (2, 2, 1), (1, 2, 1)

Rotate the cube by 30° about an axis x' passing through the centroid of the cube and x' ∥ x.

$T(1.5, 1.5, 0) \cdot R_x(30^\circ) \cdot T(-1.5, -1.5, 0) \cdot P$



$$P' = \begin{bmatrix} 1 & 0 & 0 & 1.5 \\ 0 & 1 & 0 & 1.5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 30^\circ & -\sin 30^\circ & 0 \\ 0 & \sin 30^\circ & \cos 30^\circ & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & -1.5 \\ 0 & 1 & 0 & -1.5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \left\{ P \right.$$

4	13010	(13010 − 14009)	14010
5	14010	(14010 − 16009)	16010

$$r_2 = \frac{x_{wmax} - x_1}{\Delta x} = \frac{15}{20} = \frac{3}{4}$$

$$r_3 = \frac{y_1 - y_{min}}{-\Delta y} = \frac{-(3-0)}{6} = -\frac{1}{2} < 0$$

reject

$$r_4 = \frac{y_{max} - y_1}{\Delta y} = \frac{10-(3)}{6} = 7/6 > 1$$

for all $p_k < 0$, for p_1 & p_3

$$u_{min} = \max(0, r_1, r_3) = \max(0, 1/4) = 1/4$$

$$u_{max} = \min(0, r_2, r_4) = \min(0, 3/4) = \frac{3}{4}$$

$u_{min} < u_{max} \Rightarrow$ valid line

$x_1' = x_1 + u_{min} * \Delta x$; $y_1' = y_1 + u_{min} * \Delta y$

$x_2' = x_2 + u_{max} * \Delta x$ $y_2' = y_2 + u_{max} * \Delta y$

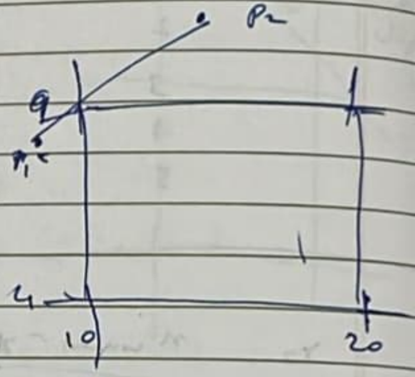
(0, 4.5) (0, 0.75)

Q) Using Liang Barsky :

$x_{min}\ x_{max} = (10, 20)$

$y_{min}\ y_{max} = (4, 9)$

$P_1 = 8, 8$ $P_2\ (16, 14)$



$\Delta x = x_2 - x_1 = 8$

$\Delta y = y_2 - y_1 = 6$

$p_k > 0$

$\Rightarrow r_1 = \frac{x_1 - x_{min}}{-\Delta x} = \frac{8-10}{-8} = \frac{1}{4}$

$r_2 = \frac{x_{max} - x_1}{\Delta x} = \frac{20-8}{8} = \frac{12}{8} = \frac{3}{2}$

$r_3 = \frac{y_1 - y_{min}}{-\Delta y} = \frac{8-4}{-6} = \frac{4}{-6} = -2/3$

$r_4 = \frac{y_{max} - y_1}{\Delta y} = \frac{9-8}{6} = \frac{1}{6}$

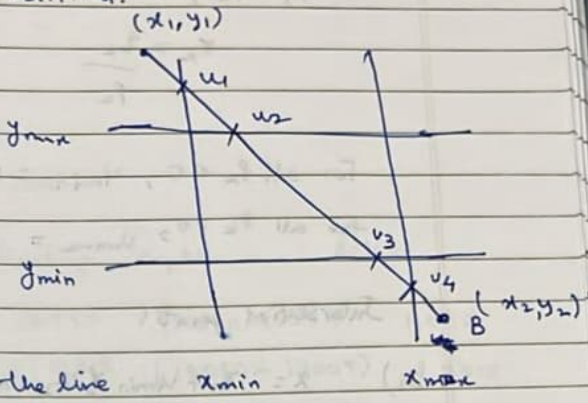
$r_2 > 1,\ r_3 < 0 \Rightarrow$ reject

$u_{min} = \max(0, r_1, \cancel{r_3}) = 1/4$

$u_{max} = \min(1, r_4, \cancel{r_2}) = 1/6$

$u_{min} > u_{max}$ so reject

LIANG BARSKY LINE CLIPPING:



Inside clipping window

$x_{min} \leq x \leq x_{max}$

$y_{min} \leq x \leq y_{max}$

Using parameteric representation of a line

any point (x, y) on the line can be written as

$$x = x_1 + u(x_2 - x_1)$$
$$y = y_1 + u(y_2 - y_1)$$

$$x_{min} \leq x_1 + u\Delta x \leq x_{max}$$
$$y_{min} \leq y_1 + u\Delta y \leq y_{max}$$

$x_{min} \leq x_1 + u\Delta x$ — (1)

$x_1 + u\Delta x \leq x_{max}$ — (2)

$y_{min} \leq y_1 + u\Delta y$ — (3)

$y_1 + u\Delta y \leq y_{max}$ — (4)

(1) $\Rightarrow -u\Delta x \leq x_1 - x_{min}$

(2) $\Rightarrow u\Delta x \leq x_{max} - x_1$

(3) $\Rightarrow -u\Delta y \leq y_1 - y_{min}$

(4) $\Rightarrow u\Delta y \leq y_{max} - y_1$

$$u p_k \leq q_k \Rightarrow u = \frac{q_k}{p_k}$$

$p_k = \pm\Delta x, \pm\Delta y$

$q_k = x_1 - x_{min},\ x_{max} - x_1,\ y_1 - y_{min},\ y_{max} - y_1$

$p_k = 0 \Rightarrow$ line parallel to edge

$\Delta x = 0 \Rightarrow$ parallel to left & Right

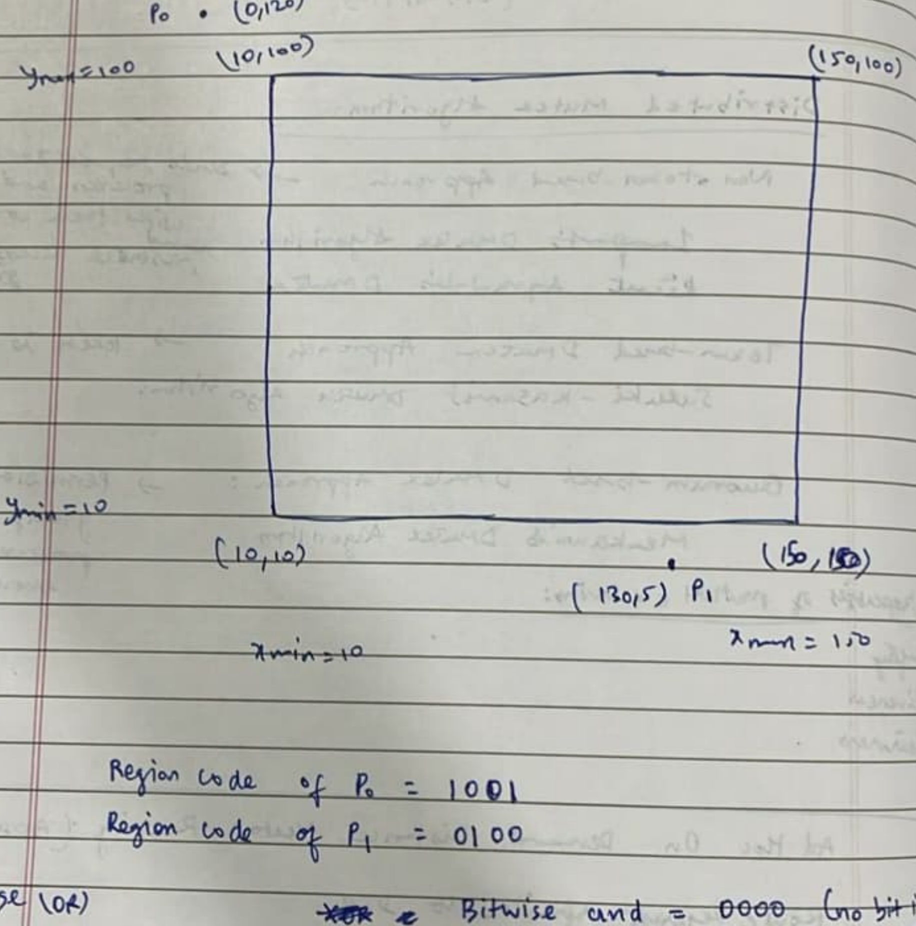
$\Delta y = 0 \Rightarrow$ ||el to top & bottom

$(xw_{min}, \cancel{x}w_{max}) = (10, 150)$

$(yw_{min}, yw_{max}) = (10, 100)$

$P_0 = (0, 120)$

$P_1 = (130, 5)$



Region code of P_0 = 1001

Region code of P_1 = 0100

Bitwise (OR)

1101 ≠ 0000

not Trivially accept

3 intersection points

Bitwise and = 0000 (no bit is set)

So not trivially rejected.

Slope of the line

(0,120), (130,5)

$$\frac{5-120}{130-0} = \frac{-115}{130} > -1$$

$= -0.88 > -1$

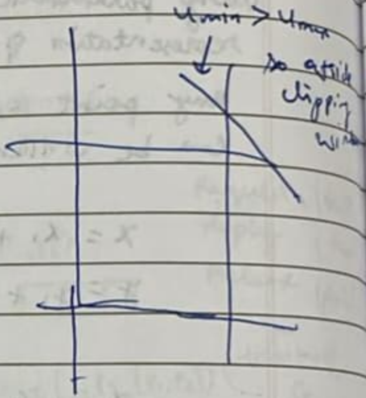
if $p_k \neq 0$

$$r_k = \frac{q_k}{p_k}$$

For all $p_k < 0$, $u_{min} = max(0, r_k)$

for all $p_k > 0$, $u_{max} = min(1, r_k)$

Intersection points:



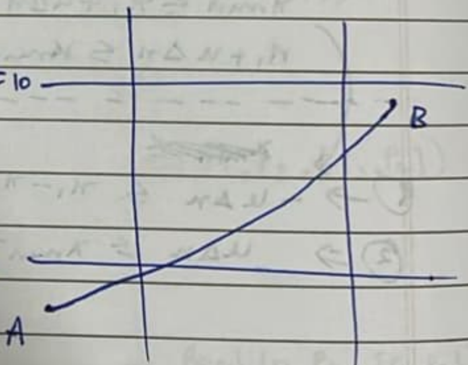
I1) $x = x_1 + u_{min}(x_2 - x_1)$
$y = y_1 + u_{min}(y_2 - y_1)$

I2) $x = x_1 + u_{max}(x_2 - x_1)$
$y = y_1 + u_{max}(y_2 - y_1)$

Window coordinates

(0,0) (10,10)

A (-5,3) B (15,9) $y_{max} = 10$



$\Delta x = 15-(-5) = 20$

$\Delta y = 6$ $y_{min} = 0$

$\Delta x \neq 0$, $\Delta y \neq 0$

$x_{min} = 0$ $x_{max} = 10$

Calculating r values:

$x_1 = -5$

$x_{wmin} = 0$

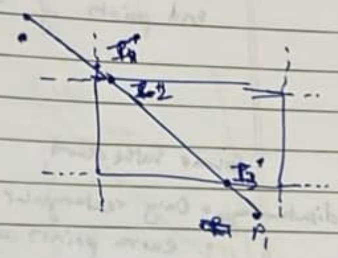
$x_{wmax} = 10$

$x_2 = 15$

$$r_1 = \frac{x_1 - x_{wmin}}{-\Delta x} = \frac{-5-0}{-20} = \frac{1}{4}$$

So intersects with x_{min}

line equation:



$$y = -0.88x + c$$

$$c = 120$$

$$y = -0.88x + 120$$

substitute $x_{min} = 10$

$$y = -8.8 + 120$$

$$y = 111.2 \qquad I_1' = (10, 111.2)$$

Region code (I_1) $= 1000$

Region code (P_1) $= 0100$

or $= 1100$

and $= 0000$

neither TA nor TR

I_2'' Intersects at y_{max}

$$y = -0.88x + 120$$

$$100 = -0.88x + 120$$

$$\frac{100-120}{-0.88} = x$$

$$\frac{20}{0.88} = x$$

$$x = 22.73$$

$$I_2 = (22.73, 100)$$

Region code (I_2) $= 0000$

Region code (P_1) $= 0100$

or $= 0100$ TA ×

and $= 0000$ TR ×

I_3 intersects at $y_{min} = 10$

$$10 = -0.88x + 120$$

$$\frac{-110}{-0.88} = x$$

$$125 = x$$

I_3 (125, 10)

Region code $= 0000$

$I_2 \to 0000$

$\to 0000$

OR $\to 0000$ and 0000

✓TA ×TR