CURVE FITTING

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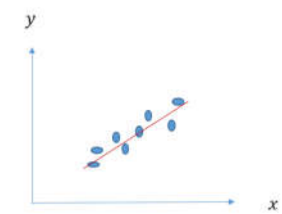
Curve Fitting

- ✓ Curve fitting is the process of finding the 'best-fit' curve for a given set of data. It is the representation of the relationship between two variables by means of an algebraic equation.



Fitting the straight line

$$y = ax + b$$

Working Procedure

To fit the straight line $y = ax + b$

- ✓ Form the normal equations

$$\sum y = a \sum x + nb$$

$$\sum xy = a \sum x^2 + b \sum x$$

- ✓ Solve these normal equations as simultaneous equation for a & b.
- ✓ Substitute the values of a & b in $y = ax + b$.Which is the required line of best fit.

Working Procedure

To fit the straight line $y = a + bx$

✓ Form the normal equations

$$\sum y = na + b\sum x$$

$$\sum xy = a\sum x + b\sum x^2$$

✓ Solve these normal equations as simultaneous equation for a & b.

✓ Substitute the values of a & b in $y = a + bx$.Which is the required line of best fit.

Example 1:

Find the equation $y = ax + b$ of the best fitting straight line for the following data

x	-1	0	1	2
y	1	0	1	4

Solution:

Let the straight line to be fitted to the data be $\boldsymbol{y = ax + b}$

The normal equations are

$$\sum y = a\sum x + nb \qquad (1)$$

$$\sum xy = a\sum x^2 + b\sum x \qquad (2)$$

Example 1:

x	y	x^2	xy
-1	1	1	-1
0	0	0	0
1	1	1	1
2	4	4	8
$\sum x = 2$	$\sum y = 6$	$\sum x^2 = 6$	$\sum xy = 8$

Example 1:

Here, Total no. of data points $\boxed{n = 4}$

Substituting these values in equation (1) and (2)

$$\sum y = a\sum x + nb \Rightarrow \boxed{6} = \boxed{2}a + \boxed{4}b \qquad \ldots\ldots\ldots(3)$$

$$\sum xy = a\sum x^2 + b\sum x \Rightarrow \boxed{8} = \boxed{6}a + \boxed{2}b \qquad \ldots\ldots\ldots(4)$$

Solving equation (3) and (4)

$i.e., Eq^n(3) - 2 \text{ X } Eq^n(4)$

$2a + 4b = 6$

$12a + 4b = 16$

$- \quad - \quad -$

$-10a = -10$

$\Rightarrow a = 1$

x	y	x^2	xy
-1	1	1	-1
0	0	0	0
1	1	1	1
2	4	4	8
$\boxed{\sum x = 2}$	$\boxed{\sum y = 6}$	$\boxed{\sum x^2 = 6}$	$\boxed{\sum xy = 8}$

Example 1:

Using $\text{Eq}^n(3)$

We get,

$6 = 2a + 4b$

$\Rightarrow 6 = 2(1) + 4b \qquad (\because a = 1)$

$\Rightarrow 4b = 4$

$\Rightarrow b = 1$

So, $a = 1$ and $b = 1$

Hence, the required Eq^n of straight line is,

$y = ax + b \Rightarrow y = x + 1$

Example 2:

Find the equation $y = ax + b$ of the best fitting straight line for the following data

X	-5	-3	-1	0	1	2	4
Y	0.4	-0.1	-0.2	-0.3	-0.3	0.1	0.4

Solution:

Let the straight line to be fitted to the data be $\boldsymbol{y = ax + b}$

The normal equations are

$$\sum y = a\sum x + nb \qquad (1)$$

$$\sum xy = a\sum x^2 + b\sum x \qquad (2)$$

Example 2:

x	y	x^2	xy
-5	0.4	25	-2
-3	-0.1	9	0.3
-1	-0.2	1	0.2
0	-0.3	0	0
1	-0.3	1	-0.3
2	0.1	4	0.2
4	0.4	16	1.6
$\sum x = -2$	$\sum y = 0$	$\sum x^2 = 56$	$\sum xy = 0$

Example 2:

Total no. of data points $\boxed{n = 7}$

Substituting these values in equation (1) and (2)

$$\sum y = a\sum x + nb \Rightarrow \boxed{0} = \boxed{-2}a + \boxed{7}b \qquad \ldots\ldots\ldots(3)$$

$$\sum xy = a\sum x^2 + b\sum x \Rightarrow \boxed{0} = \boxed{56}a \boxed{-2}b \qquad \ldots\ldots\ldots(4)$$

Solving equation (3) and (4)

$i.e., 2 \text{ X } Eq^n(3) + 7 \text{ X } Eq^n(4)$

$-4a + 14b = 0$

$392a - 14b = 0$

$388a = 0$

$\Rightarrow a = 0$

x	y	x^2	xy
-5	0.4	25	-2
-3	-0.1	9	0.3
-1	-0.2	1	0.2
0	-0.3	0	0
1	-0.3	1	-0.3
2	0.1	4	0.2
4	0.4	16	1.6
$\sum x = \boxed{-2}$	$\sum \boxed{y = 0}$	$\sum \boxed{x^2 = 56}$	$\sum \boxed{xy = 0}$

Example 2:

Using $\text{Eq}^n(3)$

We get,

$-2a + 7b = 0$

$\Rightarrow -2(0) + 7b = 0$ $\quad (\because a = 0)$

$\Rightarrow 7b = 0$

$\Rightarrow b = 0$

So, $a = 0$ and $b = 0$

Hence, the required Eq^n of straight line is,

$y = ax + b \Rightarrow y = (0)x + 0 = 0$

Example 3:

Find the equation $y = ax + b$ of the best fitting straight line for the following data

x	-1	0	1	2
Y	1	1	1	-5

Solution:

Let the straight line to be fitted to the data be $\boldsymbol{y = ax + b}$

The normal equations are

$$\sum y = a \sum x + nb \qquad (1)$$

$$\sum xy = a \sum x^2 + b \sum x \qquad (2)$$

Example 3:

x	y	x^2	xy
-1	1	1	-1
0	1	0	0
1	1	1	1
2	-5	4	-10
$\sum x = 2$	$\sum y = -2$	$\sum x^2 = 6$	$\sum xy = -10$

Example 3:

Here, Total no. of data points n=4

Substituting these values in equation (1) and (2)

$$\sum y = a \sum x + nb \Rightarrow -2 = 2a + 4b \qquad (3)$$

$$\sum xy = a \sum x^2 + b \sum x \Rightarrow -10 = 6a + 2b \qquad (4)$$

Solving equation (3) and (4)

$i.e., Eq^n(3) + 2\,X\,Eq^n(4)$

$2a + 4b \;\; = -2$
$12a + 4b = -20$
$- \quad - \quad -$
$-10a = 18$
$\Rightarrow a = -1.8$

x	y	x^2	xy
-1	1	1	-1
0	1	0	0
1	1	1	1
2	-5	4	-10
$\sum x = 2$	$\sum y = -2$	$\sum x^2 = 6$	$\sum xy = -10$

Example 3:

Using $\text{Eq}^n(3)$

We get,

$2a + 4b = -2$
$\Rightarrow 2(-1.8) + 4b = -2 \qquad (\because a = -1.8)$
$\Rightarrow 4b = -2(-1.8) - 2$
$\Rightarrow b = 0.4$

So, $a = -1.8$ and $b = 0.4$

Hence, the required Eq^n of straight line is,

$y = ax + b \Rightarrow y = -1.8x + 0.4$

Exercise:

1. **Fit a straight line to the given points (x, y) by method of least squares.**

x	0	2	3	5
$f(x)$	3	1	-1	-2

Ans: $y = 2.845 - 1.038x$

2. By method of least squares, fit a straight line to the following data.

x	-1.3	-0.1	0.2	1.3
$f(x)$	0.103	1.099	0.808	1.897

Ans: $y = 0.9601 + 0.6670x$

Fitting the Parabola

$$y = ax^2 + bx + c$$

Working Procedure

To fit the parabola $y = ax^2 + bx + c$

✓ Form the normal equations

$$\sum y = a\sum x^2 + b\sum x + nc$$

$$\sum xy = a\sum x^3 + b\sum x^2 + c\sum x$$

$$\sum x^2y = a\sum x^4 + b\sum x^3 + c\sum x^2$$

✓ Solve these equations for $a, b, \& c$.
✓ Substitute the values of a, b, c in $y = ax^2 + bx + c$

Working Procedure

To fit the parabola $y = a + bx + cx^2$

✓ Form the normal equations

$$\sum y = na + b\sum x + c\sum x^2$$

$$\sum xy = a\sum x + b\sum x^2 + c\sum x^3$$

$$\sum x^2y = a\sum x^2 + b\sum x^3 + c\sum x^4$$

✓ Solve these equations for $a, b, \& \ c$.
✓ Substitute the values of a, b, c in $y = a + bx + cx^2$

Example 1:

Fit a parabola of second degree $y = a + bx + cx^2$.

x	-1	0	1	2
y	-2	1	2	4

Solution:
Let the equation of the parabola be $y = a + bx + cx^2$.
The normal equations are

$$\sum y = na + b\sum x + c\sum x^2 \quad \ldots.(1)$$

$$\sum xy = a\sum x + b\sum x^2 + c\sum x^3 \quad \ldots(2)$$

$$\sum x^2y = a\sum x^2 + b\sum x^3 + c\sum x^4 \quad \ldots.(3)$$

x	y	x^2	x^3	x^4	xy	x^2y
-1	-2	1	-1	1	2	-2
0	1	0	0	0	0	0
1	2	1	1	1	2	2
2	4	4	8	16	8	16
$\sum x = 2$	$\sum y = 5$	$\sum x^2 = 6$	$\sum x^3 = 8$	$\sum x^4 = 18$	$\sum xy = 12$	$\sum x^2y = 16$

Here $\boxed{n = 4}$
Substituting these values in equations (1), (2), & (3). We get,

$$\sum y = na + b\sum x + c\sum x^2 \Rightarrow \boxed{5} = \boxed{4}a + \boxed{2}b + \boxed{6}c \quad ----(4)$$

$$\sum xy = a\sum x + b\sum x^2 + c\sum x^3 \Rightarrow \boxed{12} = \boxed{2}a + \boxed{6}b + \boxed{8}c \quad ----(5)$$

$$\sum x^2y = a\sum x^2 + b\sum x^3 + c\sum x^4 \Rightarrow \boxed{16} = \boxed{6}a + \boxed{8}b + \boxed{18}c \quad ----(6)$$

So, we get
$5 = 4\,a + 2\,b + 6\,c \quad \ldots\ldots(4)$
$12 = 2\,a + 6\,b + 8\,c \quad \ldots\ldots(5)$
$16 = 6\,a + 8\,b + 18\,c \quad \ldots\ldots(6)$

Solving equation (4) & (5)
i.e., $Eq^n.\ (4) + 2 * Eq^n(5)$
$5 = 4a + 2b + 6c$
$24 = 4a + 12b + 16c$
$\overline{19 = \quad 10b + 10c} \quad \ldots\ldots(7)$

Solving equation (5) & (6)
i.e., $3 * Eq^n.\ (5) + Eq^n(6)$
$36 = 6a + 18b + 24c$
$16 = 6a + 8b + 18c$
$\overline{20 = \quad 10b + 6c} \quad \ldots\ldots(8)$

Solving equation (7) & (8)
$19 = 10b + 10c$
$20 = 10b + \ 6c$
$\overline{-1 = 4c} \quad \Rightarrow c = -\frac{1}{4} \Rightarrow c = -0.25$

Using eq. (8)

$20 = 10b + 6c$

$\Rightarrow 20 = 10b + 6(-0.25)$

$\Rightarrow 20 = 10b - 1.5$

$\Rightarrow b = 2.15$

Using eq. (4)

$5 = 4a + 2b + 6c$

$\Rightarrow 5 = 4a + 2(2.15) + 6(-0.25)$

$\Rightarrow a = 0.55$

Hence, the required Eq^n of Parabola is,

$y = a + bx + cx^2 \Rightarrow y = 0.55 + 2.15x - 0.25x^2$

Example 2:

Fit a parabola of second degree $y = a + bx + cx^2$.

x	-1	0	1	2
y	2	1	0	-2

Solution:

Let the equation of the parabola be $y = a + bx + cx^2$.

The normal equations are

$$\sum y = na + b\sum x + c\sum x^2 \quad \ldots(1)$$

$$\sum xy = a\sum x + b\sum x^2 + c\sum x^3 \quad \ldots(2)$$

$$\sum x^2y = a\sum x^2 + b\sum x^3 + c\sum x^4 \quad \ldots(3)$$

x	y	x^2	x^3	x^4	xy	x^2y
-1	2	1	-1	1	-2	2
0	1	0	0	0	0	0
1	0	1	1	1	0	0
2	-2	4	8	16	-4	-8
$\sum x = 2$	$\sum y = 1$	$\sum x^2 = 6$	$\sum x^3 = 8$	$\sum x^4 = 18$	$\sum xy = -6$	$\sum x^2y = -6$

Here $n = 4$. Substituting these values in equations 1, 2, & 3. We get,

$$1 = 4a + 2b + 6c \quad ---(4)$$
$$-6 = 2a + 6b + 8c \quad ---(5)$$
$$-6 = 6a + 8b + 18c \quad ---(6)$$

By 4, & 5

$$\begin{aligned} 1 &= 4a + 2b + 6c \\ -12 &= 4a + 12b + 16c \\ \hline 13 &= -10b - 10c \quad ---(7) \end{aligned}$$

By 5, & 6

$$\begin{aligned} -18 &= 6a + 18b + 24c \\ -6 &= 6a + 8b + 18c \\ \hline -12 &= 10b + 6c \quad ---(8) \end{aligned}$$

By using eq. 7, &8

$$\begin{aligned} 13 &= -10b - 10c \\ -12 &= 10b + 6c \\ \hline 1 &= -4c \end{aligned}$$

$\Rightarrow c = -\dfrac{1}{4}$

$\Rightarrow c = -0.25$

Using eq. 7

$13 = -10b - 10c$

$\Rightarrow 13 = -10b - 10(-0.25)$

$\Rightarrow 10b = -13 + 2.5$

$\Rightarrow b = -1.05$

Using eq. 4

$1 = 4a + 2(-1.05) + 6(-0.25)$

$\Rightarrow a = 1.15$

$y = a + bx + cx^2 \Rightarrow y = 1.15 - 1.05x - 0.25x^2$

Exercise:

1. **Fit a parabola to the given points (x, y) to the following data.**

x	1	2	3	4	5	6	7	8	9
y	2	6	7	8	10	11	11	10	9

Ans: $y = -0.8995 + 3.523x - 0.2673x^2$

2. **By method of least squares, fit a parabola to the following data.**

x	-1	0	1	2
y	-2	1	2	4

Ans: $y = 0.55 + 2.15x - 0.25x^2$