Optimal Control

Course Project #2

Optimal Control of an Actuated Flexible Surface

November 26, 2024

In this project, you are required to design an optimal trajectory for an under-actuated flexible surface. A section of the system will be modeled. The model is represented in Figure 1.

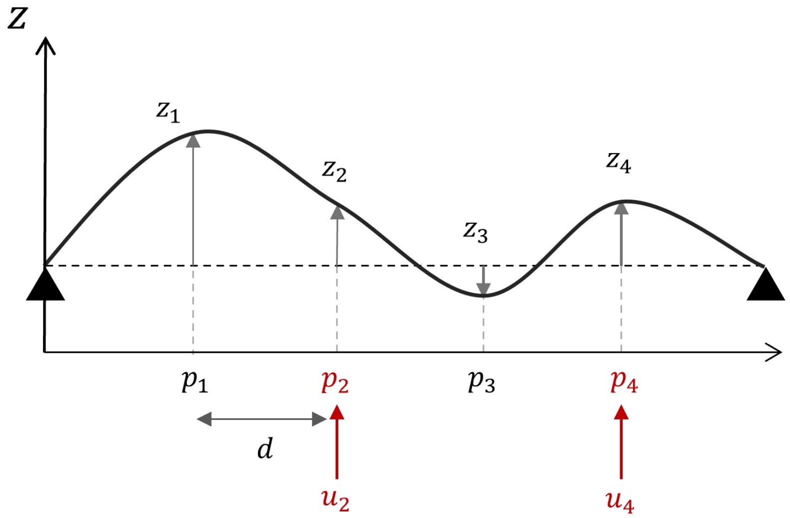

Figure 1: Model of a section of a flexible surface

The state space consist in $x = [z_1, z_2, z_3, z_4, \dot{z}_1, \dot{z}_2, \dot{z}_3, \dot{z}_4]^\top$, where each z_i represents the vertical displacement of one of the points of the surface. Assume that there are no on-plain forces, and stiffness interaction between the points of the surface only produces vertical forces.

Only points p_2 and p_4 are actuated. Dynamics of the system is described by the following equation:

$$\ddot{z}_i = \frac{1}{m_i}\left[F_i - \alpha \sum_{j\in\mathcal{A}} \frac{z_i - z_j}{L_{ij}\left(L_{ij}^2 - (z_i - z_j)^2\right)} - c\dot{z}_i\right]$$

Where $F_i = u_i$ only for the actuated points, and $F_i = 0$ for all the others, $\mathcal{A}$ is the set of all the points indexes, L_{ij} is the distance between points i and j, and α is a mechanical coupling coefficient. All the points are equally spaced, and the distance between direct neighbors is d. All the parameters of the model are available in table 1.

Task 0 – Problem setup

Discretize the dynamics, write the discrete-time state-space equations and code the `dynamics` function.

Parameters: Set 1	
m	0.1
m_{act}	0.2
d	0.2
α	$128 \cdot 0.25$
c	0.1

Parameters: Set 2	
m	0.2
m_{act}	0.3
d	0.25
α	$128 \cdot 0.2$
c	0.1

Parameters: Set 3	
m	0.1
m_{act}	0.4
d	0.30
α	$128 \cdot 0.15$
c	0.1

Table 1: Model parameters with variations.

Task 1 – Trajectory generation (I)

Compute two equilibria for your system and define a reference curve between the two. Compute the optimal transition to move from one equilibrium to another exploiting the Newton's-like algorithm (in closed-loop version) for optimal control.
Hint: you can exploit any numerical root-finding routine to compute the equilibria.
Hint: define two long constant parts between the two equilibria with a transition in between. Try to keep everything as symmetric as possible, see, e.g., Figure 2.

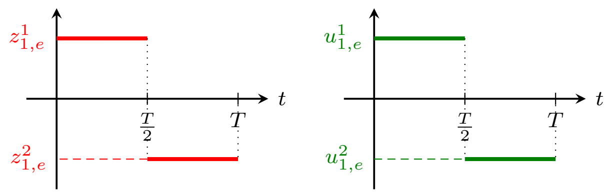

Figure 2: Example of a possible desired transition for one of the actuated points of the surface.

Task 2 – Trajectory generation (II)

Generate a desired (smooth) state-input curve and perform the trajectory generation task (Task 1) on this new desired curve.
Hint: as initial guess you may need to compute a quasi-static trajectory, i.e., a collection of equilibria, and generate the first trajectory by tracking this quasi-static trajectory via the feedback matrix solution of an LQR problem computed on the linearization of the system about the quasi-static trajectory with a user-defined cost.

Task 3 – Trajectory tracking via LQR

Linearizing the dynamics about the generated trajectory $(\mathbf{x}^{\text{traj}}, \mathbf{u}^{\text{traj}})$ computed in Task 2, exploit the LQR algorithm to define the optimal feedback controller to track this reference trajectory. In particular, you need to solve the LQ Problem

$$\begin{aligned}
\min_{\substack{\Delta x_1,\ldots,\Delta x_T \\ \Delta u_0,\ldots,\Delta u_{T-1}}} \; & \sum_{t=0}^{T-1} \Delta x_t^\top Q^{\text{reg}} \Delta x_t + \Delta u_t^\top R^{\text{reg}} \Delta u_t + \Delta x_T^\top Q_T^{\text{reg}} \Delta x_T \\
\text{subj.to } & \Delta x_{t+1} = A_t^{\text{traj}} \Delta x_t + B_t^{\text{traj}} \Delta u_t \qquad t = 0, \ldots, T-1 \\
& x_0 = 0
\end{aligned}$$

where A_t^{traj}, B_t^{traj} represent the linearization of the (nonlinear) system about the generated trajectory. The cost matrices of the regulator are a degree-of-freedom you have.

Hint: to showcase the tracking performances, consider a perturbed initial condition, i.e., different than x_0^{traj}.

Task 4 – Trajectory tracking via MPC

Linearizing the dynamics about the trajectory $(\mathbf{x}^{\text{traj}}, \mathbf{u}^{\text{traj}})$ computed in Task 2, exploit an MPC algorithm to track this reference trajectory.
Hint: to showcase the tracking performances, consider a perturbed initial condition, i.e., different than x_0^{traj}.

Task 5 – Animation

Produce a simple animation of the surface executing Task 3. You can use PYTHON or any other visualization tool.

Required plots

For Tasks 1-2, you are required to attach to the report the following plots

- Optimal trajectory and desired curve.
- Optimal trajectory, desired curve and few intermediate trajectories.
- Armijo descent direction plot (at least of few initial and final iterations).
- Norm of the descent direction along iterations (semi-logarithmic scale).
- Cost along iterations (semi-logarithmic scale).

For the other tasks, you are required to attach to the report the following plots

- System trajectory and desired (optimal) trajectory.
- Tracking error for different initial conditions.

Guidelines and Hints

- As optimization algorithm, you can use the (regularized) closed-loop version of the Newton's-like method for optimal control introduced during the lectures based on the Hessians of the cost only.
- In the definition of the desired curve, you may try to calculate the desired trajectories using a simplified model, e.g., a simplified kinematic model.

Notes

1. Each group must be composed of 3 students (except for exceptional cases to be discussed with the instructor).
2. Any other information and material necessary for the project development will be given during project meetings.
3. The project report must be written in LaTeX and follow the main structure of the attached template.

4. Any email for project support must have the subject:

   "[OPTCON2024]-Group X: rest of the subject".

5. **All** the emails exchanged **must be cc-ed** to professor Notarstefano, dr. Falotico and the other group members.

IMPORTANT: Instructions for the Final Submission

1. The final submission **deadline** is **one** week before the exam date.

2. One member of each group must send an email with subject "[OPTCON2024]-Group X: Submission", with attached a link to a OneDrive folder shared with professor Notarstefano, dr. Falotico and the other group members.

3. The final submission folder must contain:

   - `report_group_XX.pdf`
   - `report` – a folder containing the LaTeX code and `figs` folder (if any)
   - `code` – a folder containing the code, including `README.txt`