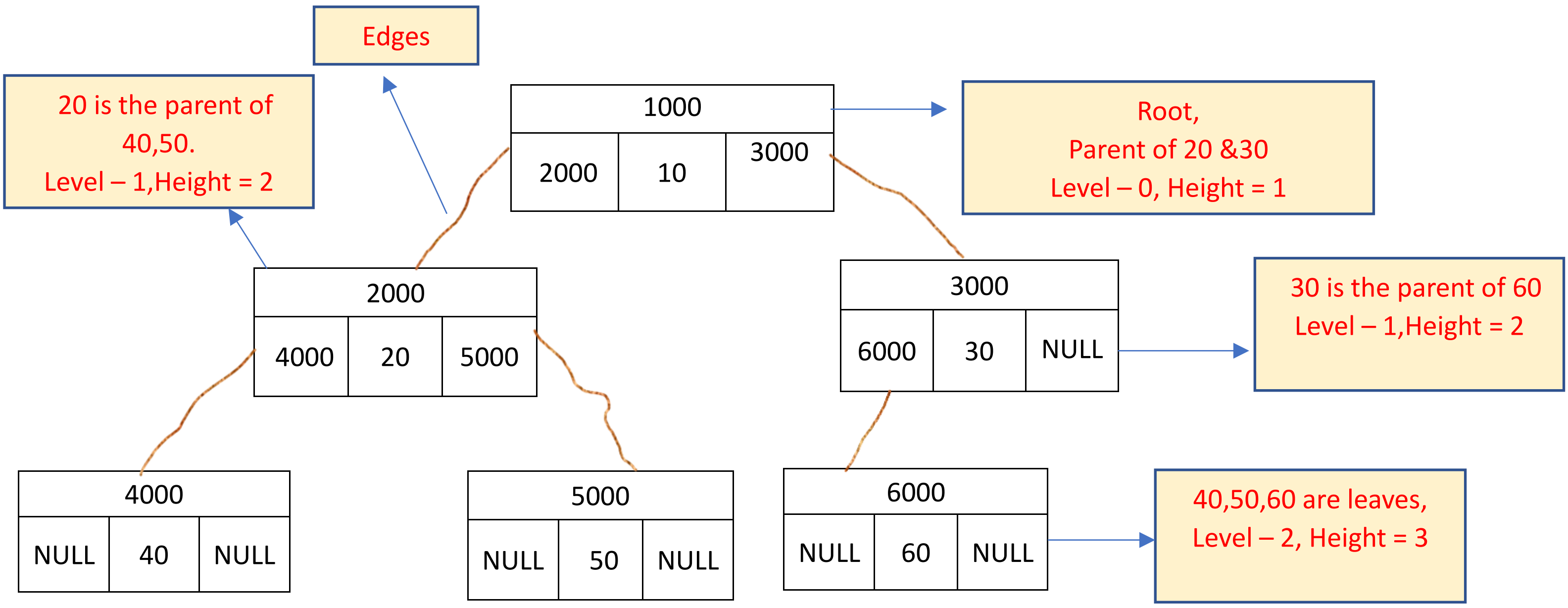
Node structure of binary tree :-

Left	Data	Right

Node structure :-

```
struct Node
{
      struct *left;
      int  data;
      struct *right;
};
```



For the above Binary Tree :-

Nodes = 6 , Edges = 5 [For n nodes, (n-1) edges are there]

Here 40,50,60 are called leaves sinces these nodes have child count as 0.

40,50 are sibiing

Height = level + 1

Maximum nodes at that level 2 power of l (level)   [pow(2,l)]

Height = 3   Total maximum nodes of above tree is $2^3 – 1 = 8 – 1 = 7$

Different ways to traverse through binary tree : -

BFS = Breadth First Search

DFS = Depth First Search

1. **Level Order :-**

   It is same as order of the given tree

   Ex :- 10 20 30 40 50 60

2. **Inorder :-**

   It means Left Subtree - Root - Right Subtree

   Ex :- 40 20 50 10 60 30 70

Left	Root	Right
40	20	50
Left Subtree		

10
Root

Left	Root	Right
60	30	70
Right Subtree		

3. **Post Order :-**

   It means Left Subtree - Right Subtree - Root

   Ex :- 40 50 20 60 70 30 10

Left	Right	Root
40	50	20
Left Subtree		

Left	Right	Root
60	70	30
Right Subtree		

10
Root

4. **Pre Order :-**

   It means Root - Left Subtree - Right Subtree
   Ex :- 10 20 40 50 30 60 70

10
Root

Root	Left	Right
20	40	50
Left Subtree		

Root	Left	Right
30	60	70
Right Subtree		

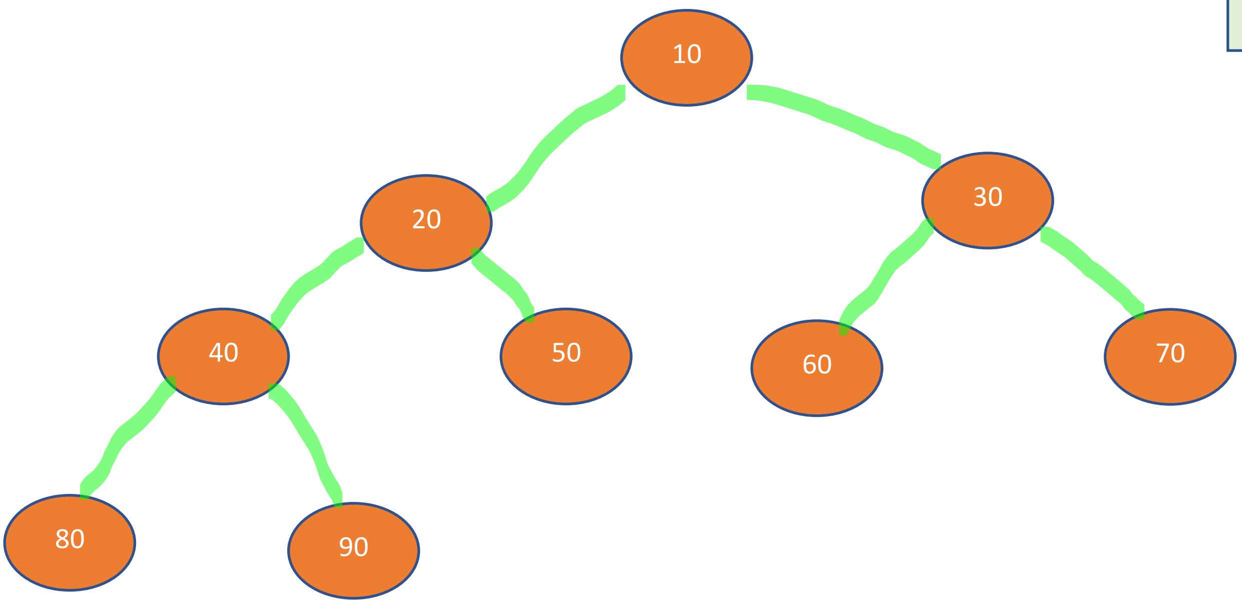
Accessing nodes of binary tree using array :-

0	1	2	3	4	5	6	7	8
10	20	30	40	50	60	70	80	90

Root = Null

Left = 2*i+1

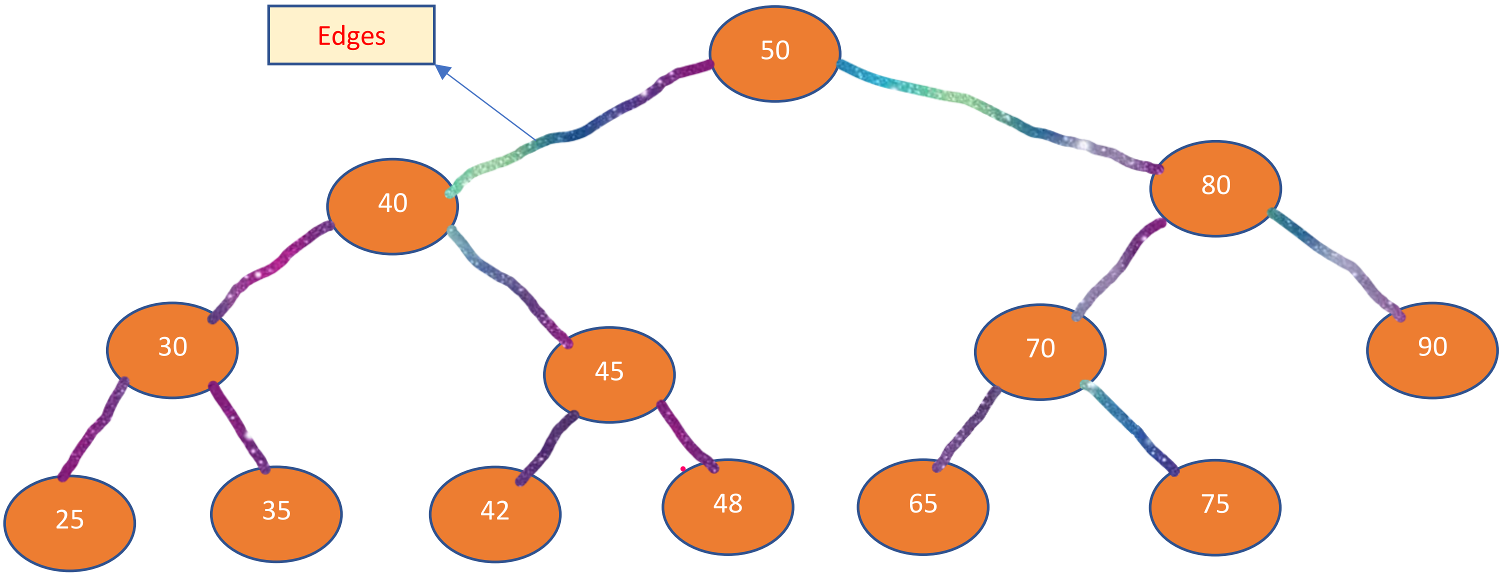
Right = 2*i+2



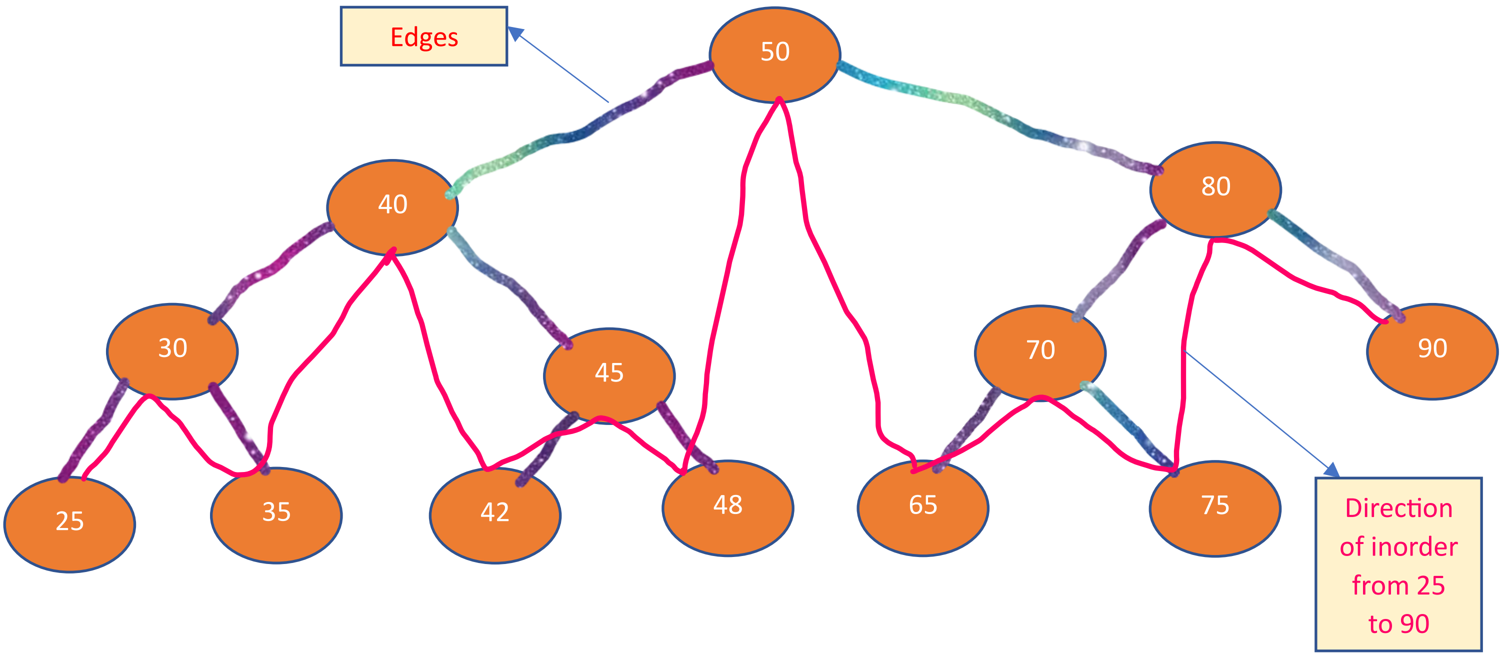
Inorder	Left of 10					ROOT	Right of 10		
	Left of 20				Right of 20		Left of 30		Right of 30
	L (40)		R (40)						
	80	40	90	20	50	10	60	30	70

Postorder	Left of 10					Right of 10			ROOT
	Left of 20			Right of 20		Left of 30	Right of 30		
	L (40)	R (40)							
	80	90	40	50	20	60	70	30	10

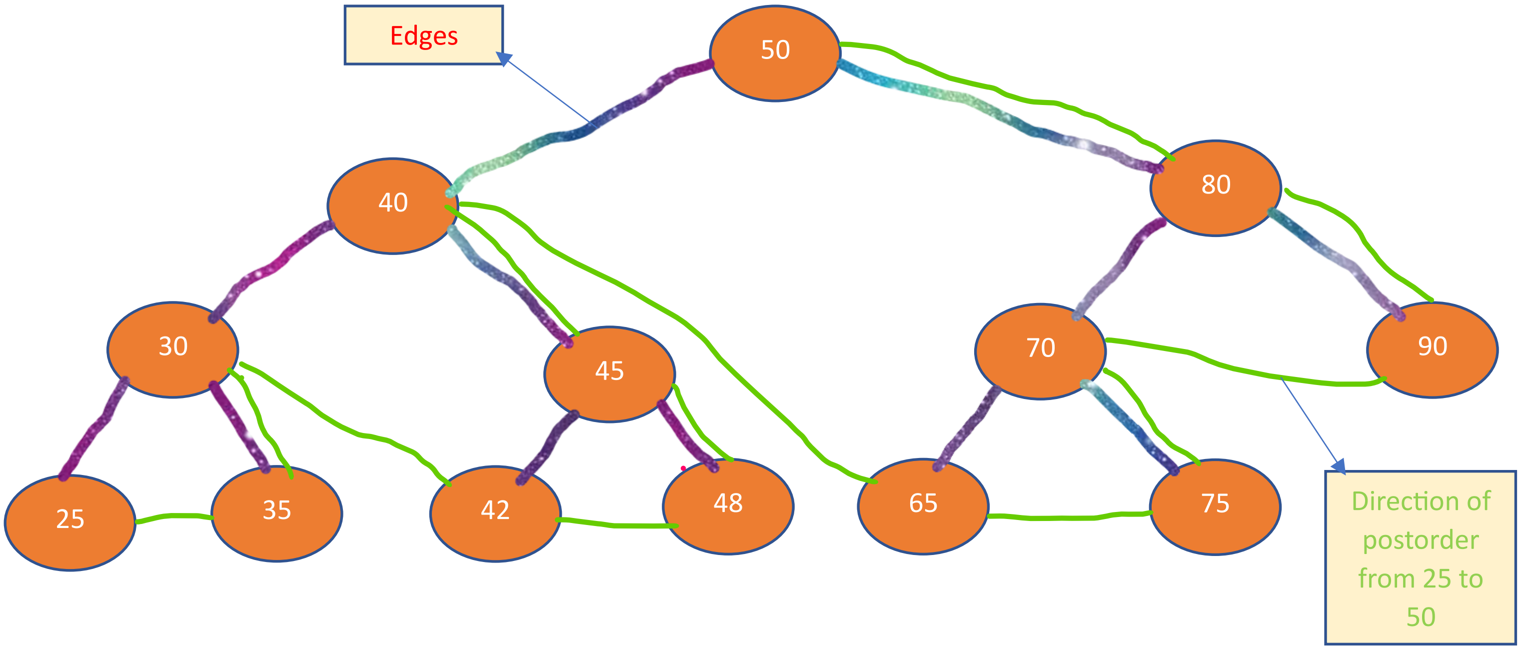
Preorder	ROOT	Left of 10					Right of 10		
			Left of 20			Right of 20		Left of 30	Right of 30
				L (40)	R (40)				
	10	20	40	80	90	50	30	60	70



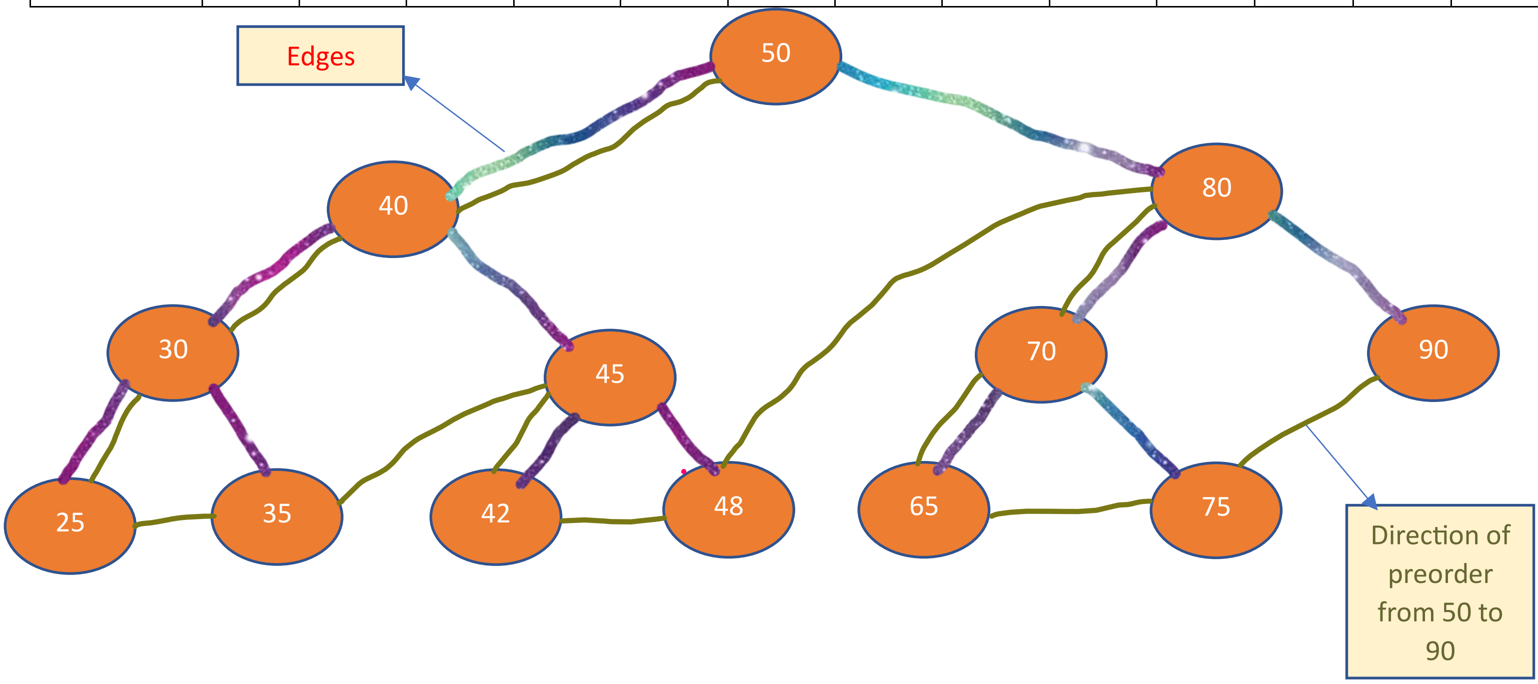
<table>
<tr><td rowspan="4">Inorder</td><td colspan="7">Left of 50</td><td rowspan="3">ROOT</td><td colspan="5">Right of 50</td></tr>
<tr><td colspan="3">Left of 40</td><td rowspan="2"></td><td colspan="3">Right of 40</td><td colspan="3">Left of 80</td><td rowspan="2"></td><td rowspan="2">Right of 80</td></tr>
<tr><td>L (30)</td><td></td><td>R (30)</td><td>L (45)</td><td></td><td>R (45)</td><td>L (70)</td><td></td><td>R (70)</td></tr>
<tr><td>25</td><td>30</td><td>35</td><td>40</td><td>42</td><td>45</td><td>48</td><td>50</td><td>65</td><td>70</td><td>75</td><td>80</td><td>90</td></tr>
</table>



<table>
<tr><td rowspan="4">Postorder</td><td colspan="7">Left of 50</td><td colspan="5">Right of 50</td><td rowspan="3">ROOT</td></tr>
<tr><td colspan="3">Left of 40</td><td colspan="3">Right of 40</td><td rowspan="2"></td><td colspan="3">Left of 80</td><td rowspan="2">Right of 80</td><td rowspan="2"></td></tr>
<tr><td>L (30)</td><td>R (30)</td><td></td><td>L (45)</td><td>R (45)</td><td></td><td>L (70)</td><td>R (70)</td><td></td></tr>
<tr><td>25</td><td>35</td><td>30</td><td>42</td><td>48</td><td>45</td><td>40</td><td>65</td><td>75</td><td>70</td><td>90</td><td>80</td><td>50</td></tr>
</table>



Preorder	ROOT	Left of 50											
			Left of 40			Right of 40				Left of 80			Right of 80
				L (30)	R (30)		L (45)	R (45)			L (70)	R(70)	
	50	40	30	25	35	45	42	48	80	70	65	75	90

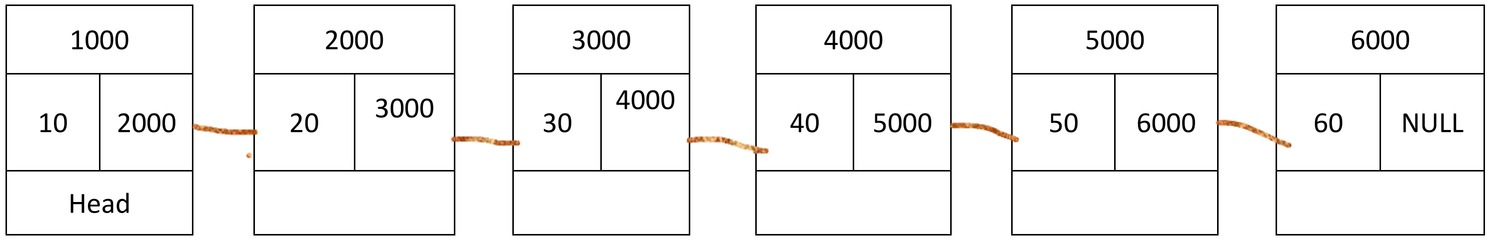


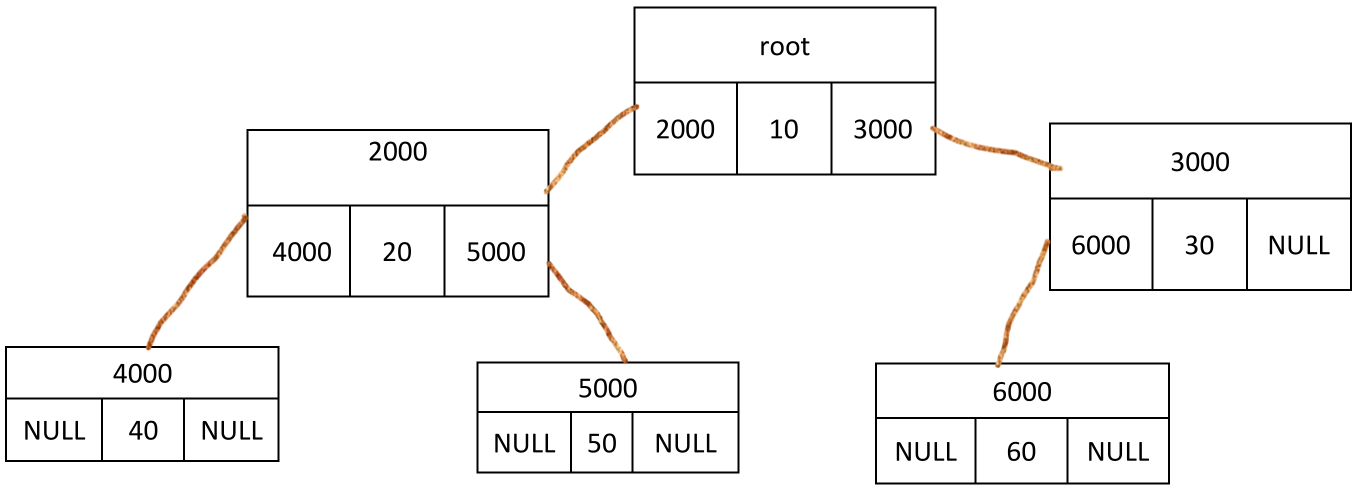
Construction of binary tree using Queue data structure :-

0	1	2	3	4	5	6	7	8
10	20	30	40	50	60	70	80	90

Same Logic
Root = Null
Left = 2*i+1
Right = 2*i+2

Construction of binary tree using Linked Lists :-

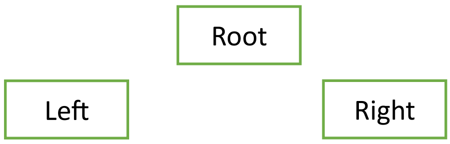




Binary Search Tree :-

Rules

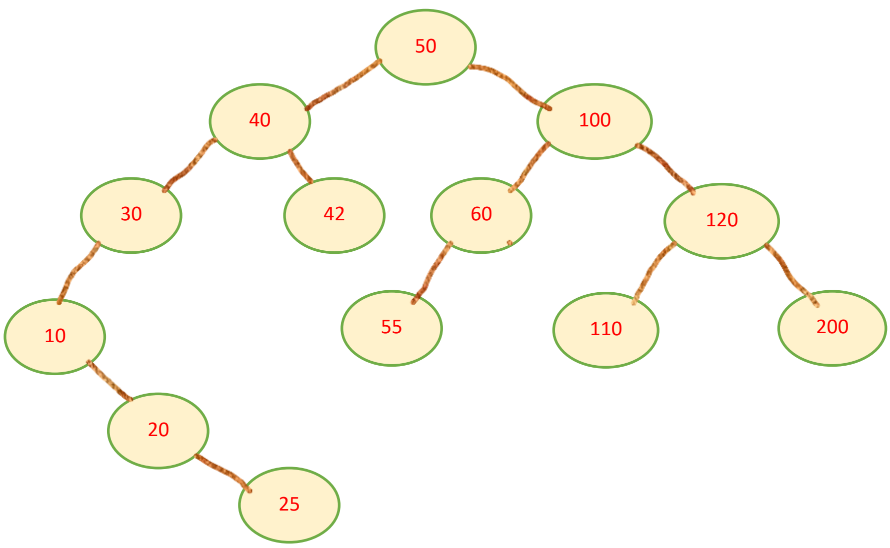
temp = root
temp->val >
temp = temp->left
temp->val <
temp = temp->right



1.Left subtree values are less than root

2. Right subtree values are greater than root

Values :- 50 40 30 42 100 120 60 10 20 55 110 200 25



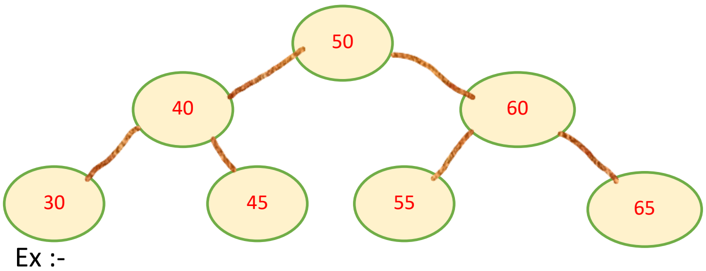
Operaions on Binary Search Tree:-
Insert
Delete
Search
Inorder
Preorder
Postorder
Levelorder

Inorder	10	20	25	30	40	42	50	55	60	100	110	120	200

Postorder	25	20	10	30	42	40	55	60	110	200	120	100	50

Preorder	50	40	30	10	20	25	42	100	60	55	120	110	200

Search operation :-



Ex :-

Search(val)
temp = root
while true
if temp->val == key :-
print element found
if temp->val > key :-
temp = temp->left
if temp->val < key :-
temp = temp->right
if temp == NULL :-
element not found

Search(20)

1. Temp = root = 50

50 == 20　　**50 < 20**　　**50 > 20**

[Since 3rd condition is correct we have to check left of temp i.e; **temp = temp->left**]

2. Temp = 50->left = 40

40 == 20　　**40 < 20**　　**40 > 20**

[Since 3rd condition is correct we have to check left of temp i.e; **temp = temp->left**]

3. Temp = 40->left = 30

30 == 20　　**30 < 20**　　**30 > 20**

[Since 3rd condition is correct we have to check left of temp i.e; **temp = temp->left**]

4. Temp = 30->left = NULL

Since **temp == NULL**, we no need to check any of the three conditions and can say **element is not found** .
So, 20 is not in the above binary tree.

Search(55)

1. Temp = root = 50

50 == 55　　**50 < 55**　　**50 > 55**

[Since 2nd condition is correct we have to check right of temp i.e; **temp = temp->right**]

2. Temp = 50->right = 60

60 == 55　　**60 < 55**　　**60 > 55**

[Since 3rd condition is correct we have to check left of temp i.e; **temp = temp->left**]

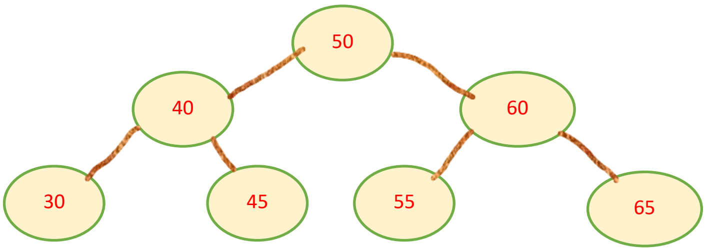
3. Temp = 60->left = 55

55 == 55　　**55 < 55**　　**55 > 55**

Since 1st condition is correct, we no need to check any of the three conditions and can say **element is found** .
So, 55 is found in the above binary tree.

```
Insert(val)
temp = root
while true
if root == NULL :-
root = NN(key)
break
if temp->val == key :-
print element is already in tree
break
if  temp->val > key :-
if temp->left != NULL :-    temp = temp->left
else:-       temp->left = NN(i.e; key)
break
if temp->val < key :-
if temp->right != NULL :-    temp = temp->right
else:-      temp->right = NN(i.e; key)
break
```

Insert Operation :-



Insert(20)

1. Temp = root = 50

50 == 20 **50 < 20** **50 > 20**

[Since 3rd condition is correct we have to check left of temp i.e; **temp = temp->left = 50->left = 40 != NULL**]

2. Temp = 50->left = 40

40 == 20 **40 < 20** **40 > 20**

[Since 3rd condition is correct we have to check left of temp i.e; **temp = temp->left = 40->left = 30 != NULL**]

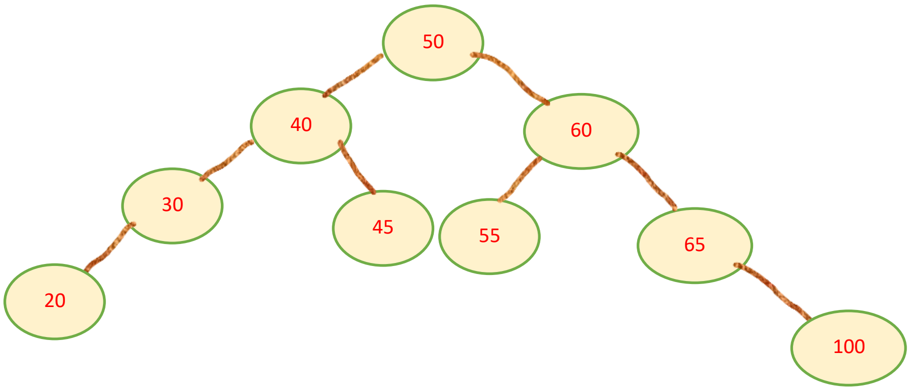
3. Temp = 40->left = 30

30 == 20 **30 < 20** **30 > 20**

[Since 3rd condition is correct we have to check left of temp i.e; **temp = temp->left = 30->left = NULL**]

4. Temp = 30->left = NULL

Since **temp == NULL and 20 < 30** , we no need to check any of the three conditions and can **insert element at left of 30 as new node** .So, 20 is inserted as a left child of 30 in the above binary tree.



Insert(55)

1. Temp = root = 50

50 == 55 **50 < 55** **50 > 55**

[Since 2nd condition is correct we have to check right of temp i.e; **temp = temp->right = 50->right = 60 != NULL**]

2. Temp = 50->right = 60

60 == 55 **60 < 55** **60 > 55**

[Since 3rd condition is correct we have to check left of temp i.e; **temp = temp->left = 60->left = 55 != NULL**]

3. Temp = 60->left = 55

55 == 55 **55 < 55** **55 > 55**

Since 1st condition is correct, we no need to check remaining conditions and can say **element is already present in the binary search tree**.

Insert(100)

1. Temp = root = 50

50 == 100 **50 < 100** **50 > 100**

[Since 2nd condition is correct we have to check right of temp i.e; **temp = temp->right = 50->right = 60 != NULL**]

2. Temp = 50->right = 60

60 == 100 **60 < 100** **60 > 100**

[Since 2nd condition is correct we have to check right of temp i.e; **temp = temp->right = 60->right = 65 != NULL**]

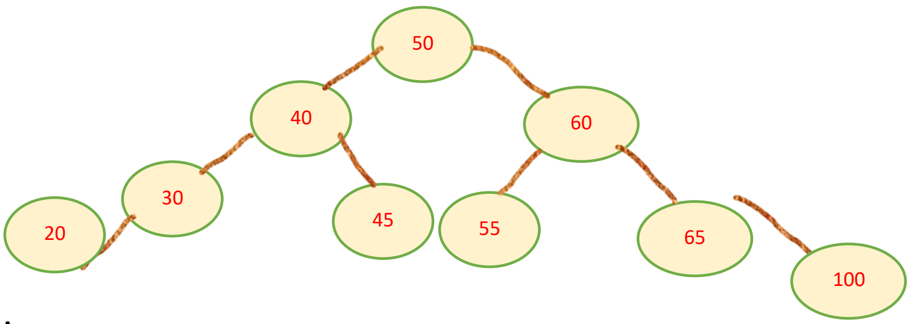
3. Temp = 60->right = 65

65 == 100 **65 < 100** **65 > 100**

[Since 2nd condition is correct we have to check right of temp i.e; **temp = temp->left = 65->right = NULL**]

4. Temp = 65->right = NULL

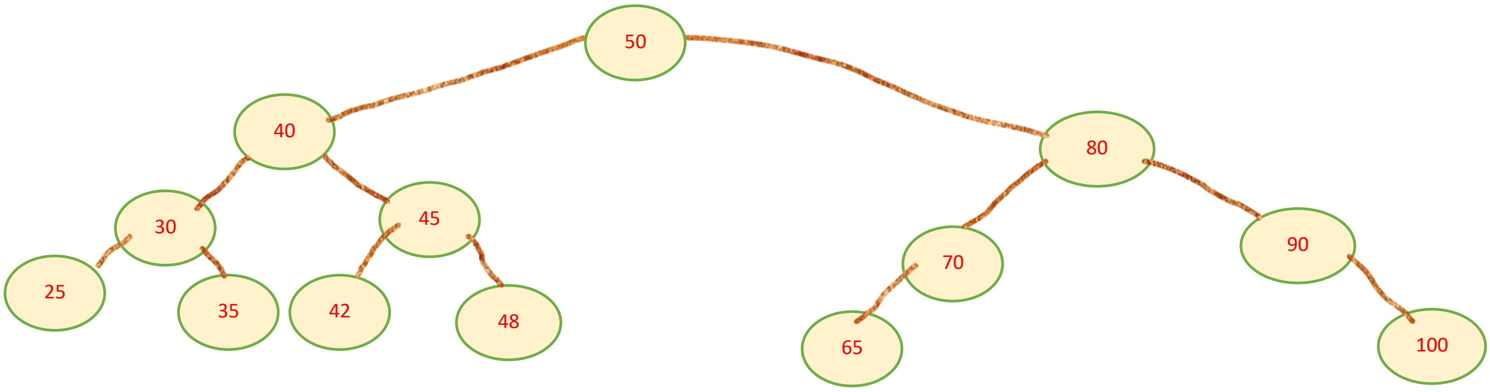
Since **temp ==NULL and 100 > 65**, we no need to check any of the three conditions and can **insert element at right of 65 as new node** .So, 100 is inserted as a left child of 65 in the above binary tree.



<u>Delete Operation :-</u>

Let's work on the following example to understand the delete operation.

50 40 80 30 45 70 90 25 35 42 48 65 100



There are three cases to delete an element

1. Node has zero child → 25, 35, 42, 48, 65, 100
2. Node has one child → 70, 90
3. Node has two child → 30, 45, 40, 80, 50

We want to return the res to main function in any case
if res == NULL :- It prints **element not found**
Else it prints **res free it's memory**

General procedure in deleting any of the node

If root == NULL
we have to return **NULL**
temp = root
parent = NULL
while(temp && temp->data != key)
if temp->data > key :- **parent = temp ;**
temp =temp->left
else :- **parent = temp; temp = temp->right**

delete(25) :-

For deleting Leave nodes
(temp->right == NULL &&
temp->left == NULL)

res = temp
if parent->right != NULL &&
parent ->right->data== key :-
parent->right = NULL
else if parent->left != NULL &&
parent ->left->data== key:-
parent->left == NULL
return res

1. temp = root = 50 and parent = NULL
   50 == 25 **50 < 25** **50 >25**

[Since 3rd condition is correct we have to check left of temp i.e; **parent = temp = 50 and temp = temp->left = 50->left = 40 != NULL**]

2. temp = 40 and parent = 50
   40 == 25 **40 < 25** **40 >25**

[Since 3rd condition is correct we have to check left of temp i.e; **parent = temp = 40 and temp = temp->left = 40->left = 30 != NULL]**

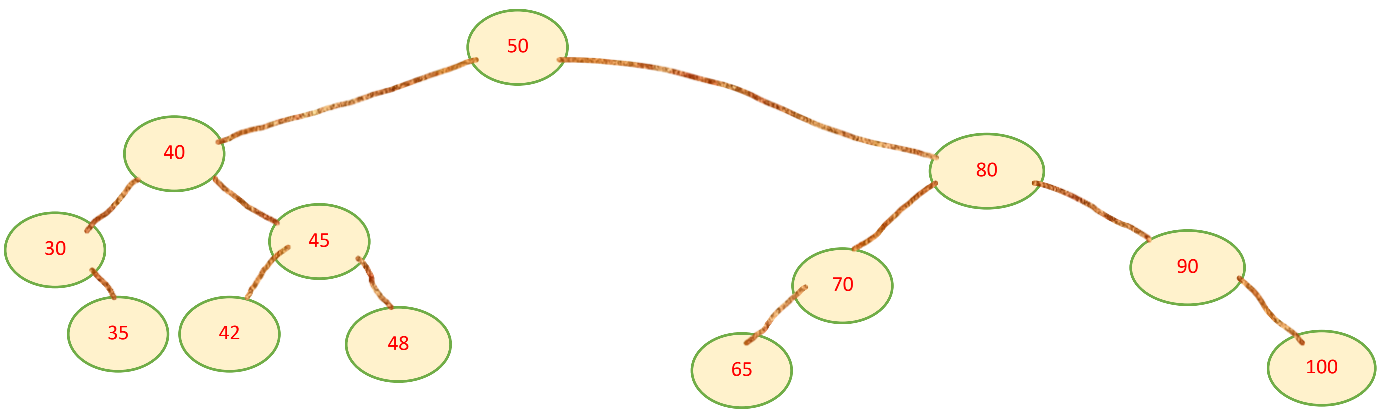
3. temp = 30 and parent = 40
   30 == 25 **30 < 25** **30 >25**

[Since 3rd condition is correct we have to check left of temp i.e; **parent = temp = 30 and temp = temp->left = 30->left = 25 != NULL**]

4. temp = 25 and parent = 30
   25 == 25 **25 < 25** **25 >25**

[Since 3rd condition is correct we have to check left of temp i.e; **parent = temp = 30 and temp = temp->left = 30->left = 25 != NULL**]

Here 1st condition is correct, we no need to check remaining conditions and can copy 25 to res. Since 30->left = **25 != NULL** Now we have to make **30->left as NULL** by returning **res** to main the **element is printed on compiler and it's memory is freed in the binary search tree**.



delete(70) :-

1. temp = root = 50 and parent = NULL

For deleting single child left node (temp->right == NULL)

res = temp
if parent->right != NULL &&
parent ->right->data== key :-
parent->right = temp->left
else if parent->left != NULL &&
parent ->left->data== key:-
parent->left == temp->left
return res

50 == 70 **50 < 70** **50 > 70**

[Since 2nd condition is correct we have to check right of temp i.e; **parent = temp = 50 and temp = temp->right = 50->right = 80 != NULL**]

2. temp = 80 and parent = 50

80 == 70 **80 < 70** **80 > 70**

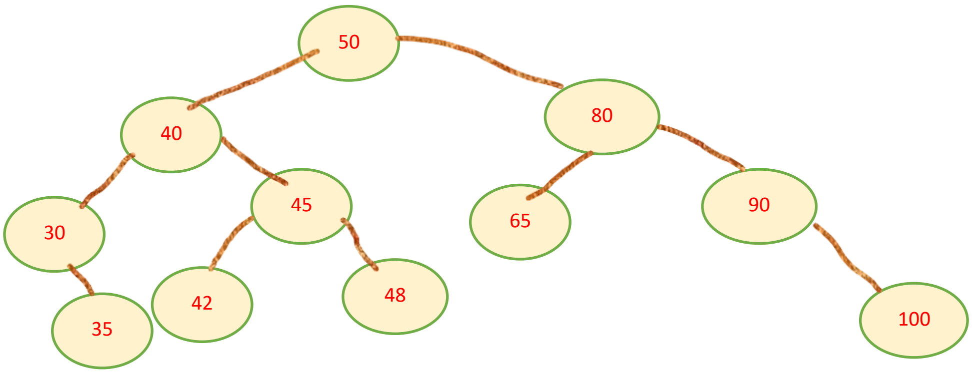
[Since 3rd condition is correct we have to check left of temp i.e; **parent = temp = 80 and temp = temp->left = 80->left = 70 != NULL**]

3. temp = 70 and parent = 80

70 == 70 **70 < 70** **70 > 70**

[Since 1st condition we no need to check remaining conditions and can copy 70 to temp.]

Since 80->left = **70 != NULL and 70 == Key.** Therefore, **80->left = 70->left = 65** by returning **res = 70** to main the **element is printed on compiler and it's memory is freed in the binary search tree**



delete(90) :-

For deleting single child right node (temp->left == NULL)

res = temp
if parent->right != NULL &&
parent ->right->data== key :-
parent->right = temp->right
else if parent->left != NULL &&
parent ->left->data== key:-
parent->left == temp->right
return res

1. temp = root = 50 and parent = NULL

50 == 90 **50 < 90** **50 > 90**

[Since 2nd condition is correct we have to check right of temp i.e; **parent = temp = 50 and temp = temp->right = 50->right = 80 != NULL**]

2. temp = 80 and parent = 50

80 == 90 **80 < 90** **80 > 90**

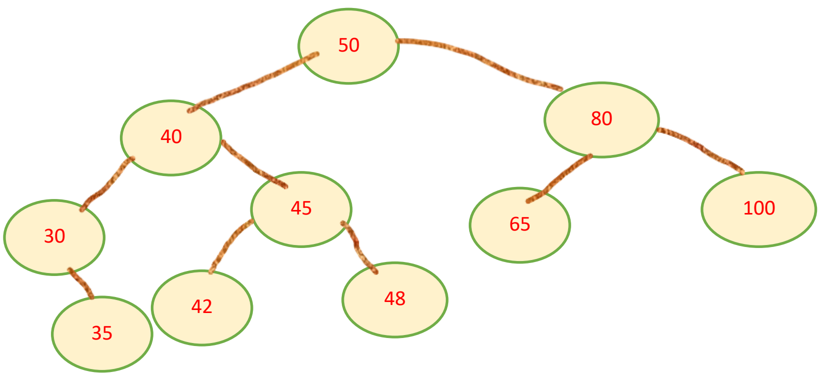
[Since 2nd condition is correct we have to check right of temp i.e; **parent = temp = 80 and temp = temp->right = 80->right = 90 != NULL**]

3. temp = 90 and parent = 80

90 == 90 **90 < 90** **90 > 90**

[Since 1st condition we no need to check remaining conditions and can copy 90 to temp.]

Since 80->right = **90 != NULL and 90 == Key.** Therefore, **80->right = 90->right = 100** by returning **res = 90** to main the **element is printed on compiler and it's memory is freed in the binary search tree.**



Delete (50) :-

1. temp = root = 50 and parent = NULL

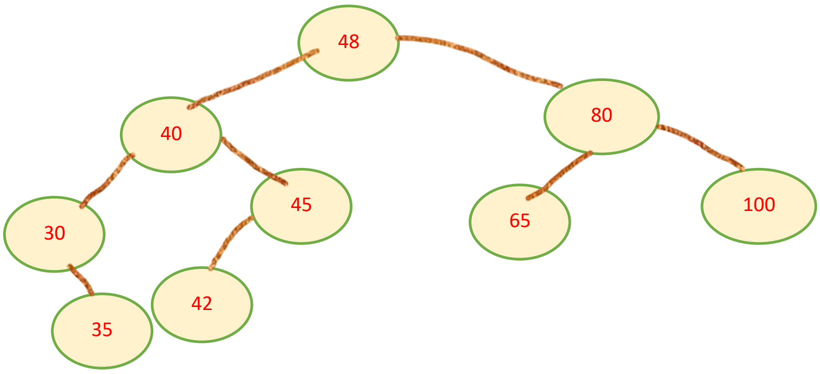
   50 == 50 **50 < 50** **50 > 50**

[Since 1st condition we no need to check remaining conditions and can copy 70 to temp.]

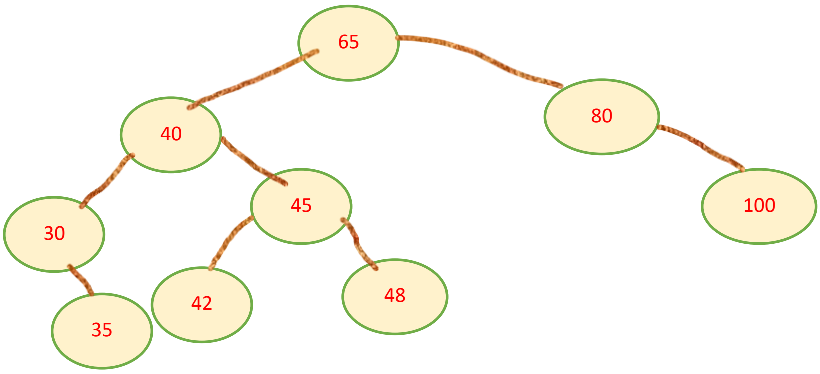
Since 50->right = **80 != NULL** and 50->left = **40 != NULL and 50 == Key.** In this case, **we cannot directly return the res = 50** to main the **element is printed on compiler and it's memory is freed in the binary search tree.**

Before returning we to swap the 50 with it's inorder predecessor or inorder successor

If we swap 50 with it's inorder predecessor the tree will be like this



If we swap 50 with it's inorder successor the tree will be like this



<table>
<tr><th colspan="3">For deleting two child nodes
(temp->right != NULL && temp->left != NULL)</th></tr>
<tr>
<td>In this case, swapping will occur, For this we have to declare two more pointer variables p and t. And val as integer variable instead of key
t = temp->right
p = NULL
while (t->left) :-
p = t
t = t->left</td>
<td>if p != NULL :-
res = t
val = t->data
t->data = temp-> data
temp->data = val
p->left = t->right
return res</td>
<td>else :-
res = t
val = t->data
t->data = temp-> data
temp->data = val
temp->right = t->right
return res</td>
</tr>
</table>

AVL TREE :-

AVL Tree is a Self balancing tree.

Balancing Factor :-

It is the difference between maximum depth of left and maximum right of a node. i.e; (max depth of left – max depth of right)

It must be 0,-1,1

Rotations :-

1. Left Rotation
2. Right Rotation

We use the above rotations to balance the above tree.

4 cases :-

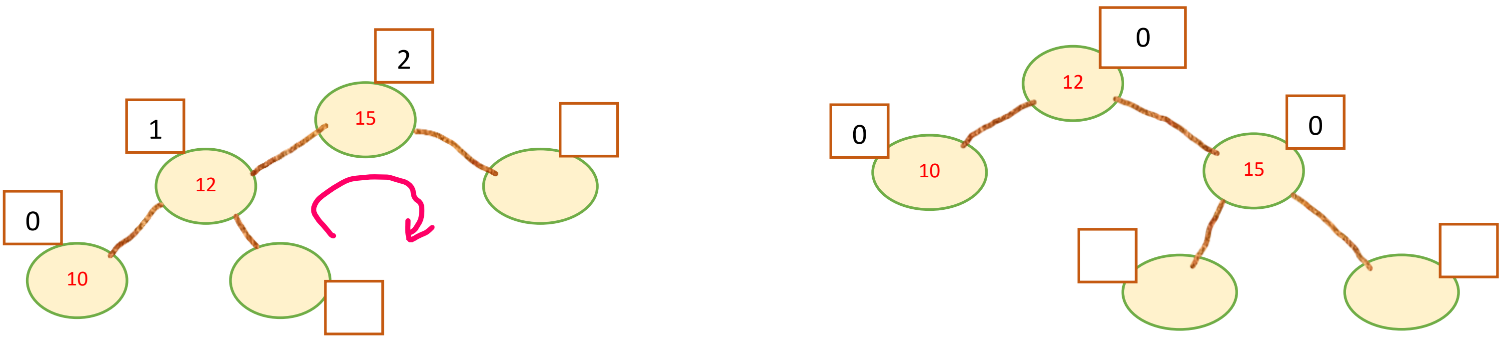
1. Right right case → Left rotation
2. Left left case → Right rotation
3. Right left case → Right rotation and Left rotation
4. Left right case → Left rotation and Right rotation

Left left case → Right rotation

Right rotation :-

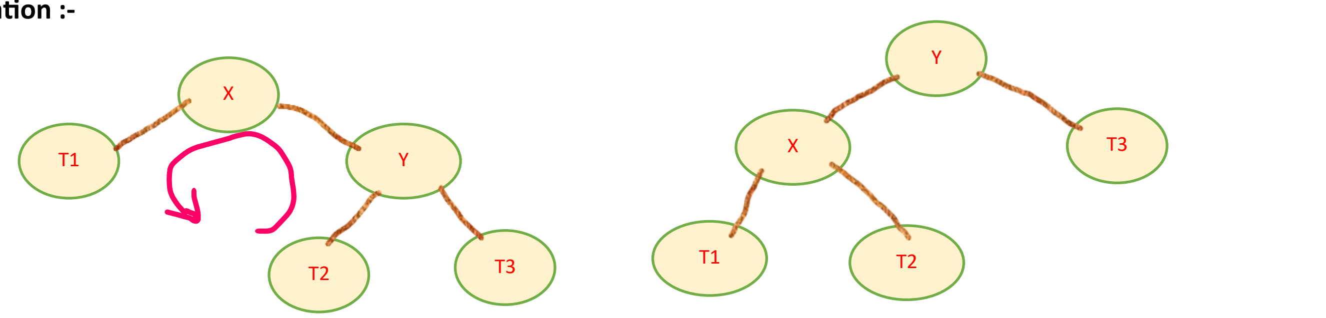
Example :-

Let's consider three numbers 15, 12 , 10 in place of X, Y, T1



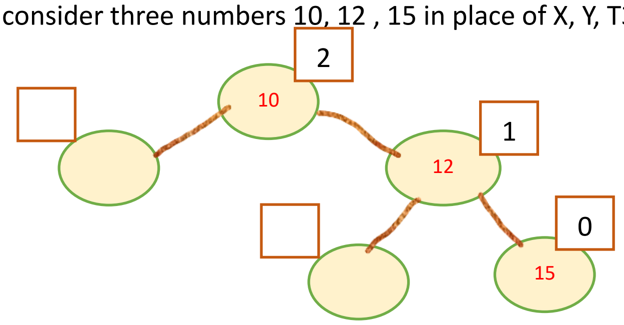
Right right case → Left rotation

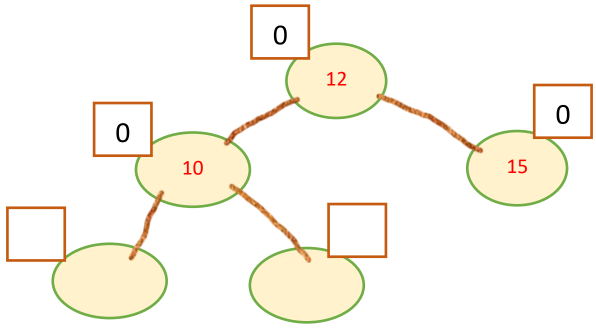
Left rotation :-



Example :-

Let's consider three numbers 10, 12 , 15 in place of X, Y, T3

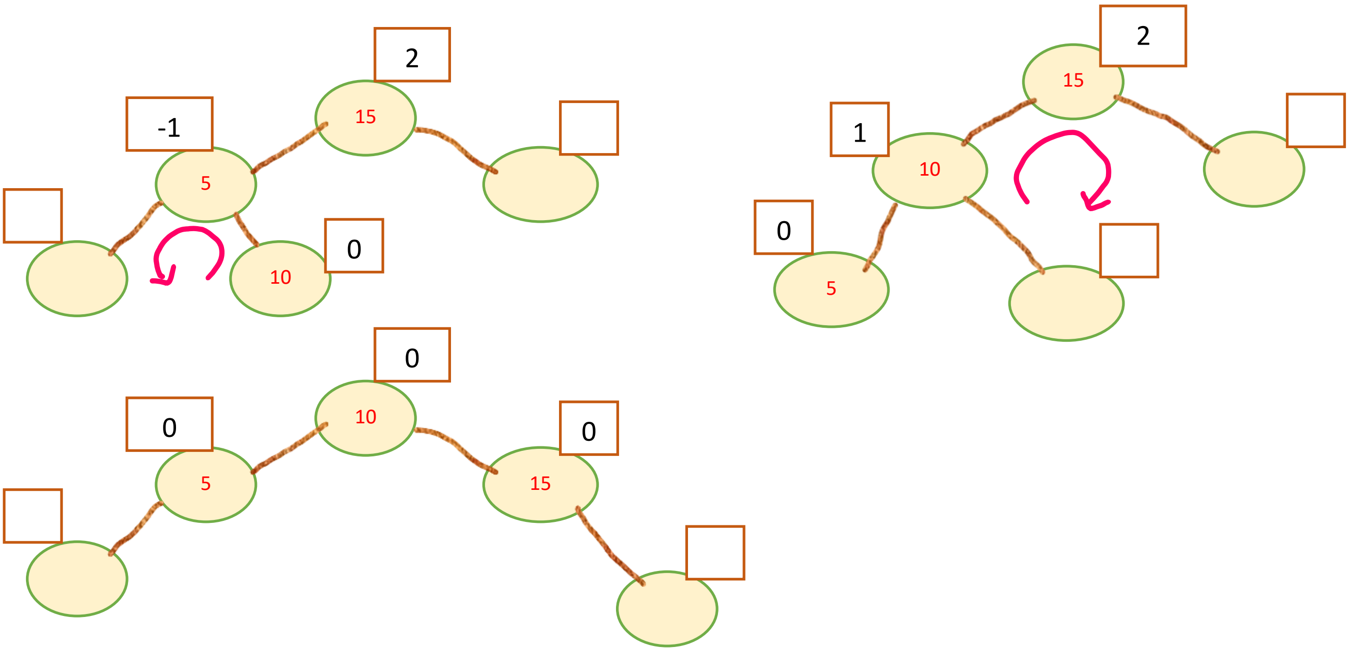




Left right case → Left rotation and Right rotation

Example :-

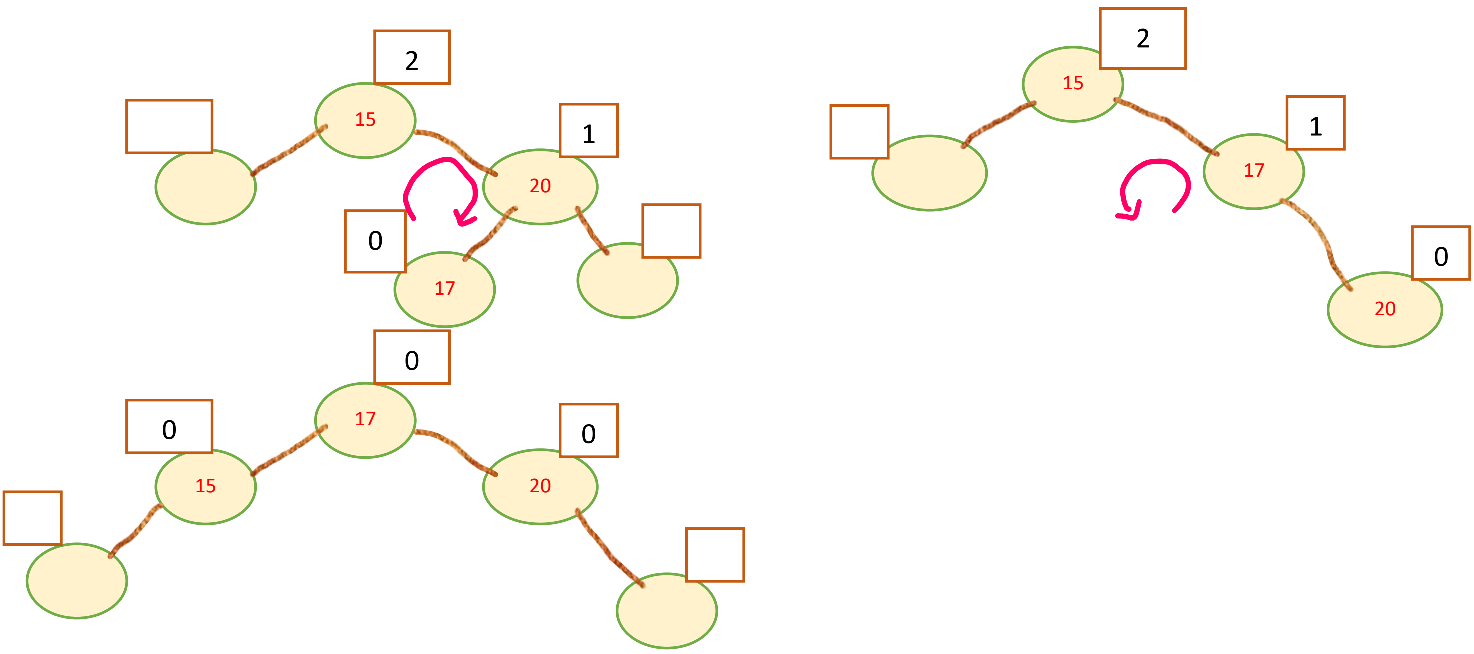
Let's consider three numbers 15, 5 , 10 in place of X, Y, T2



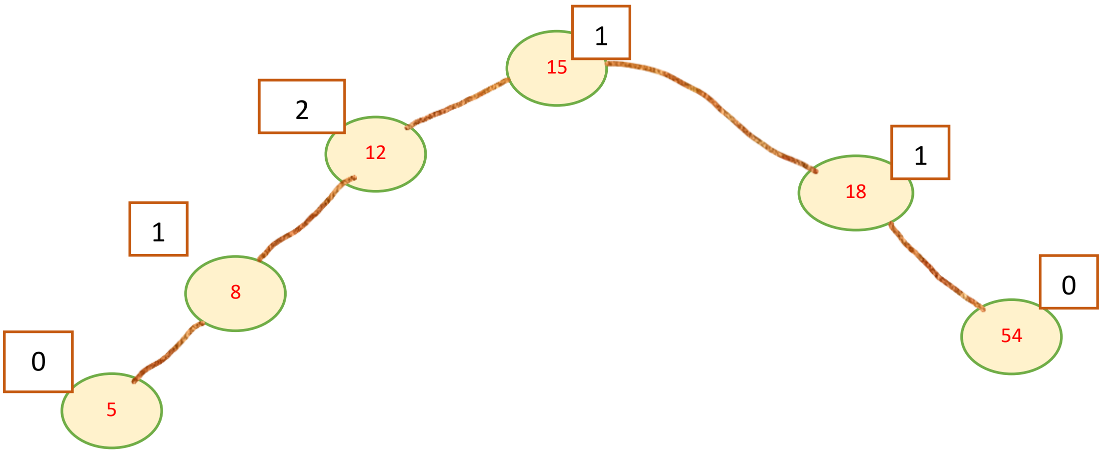
Right left case → Right rotation and Left rotation

Example :-

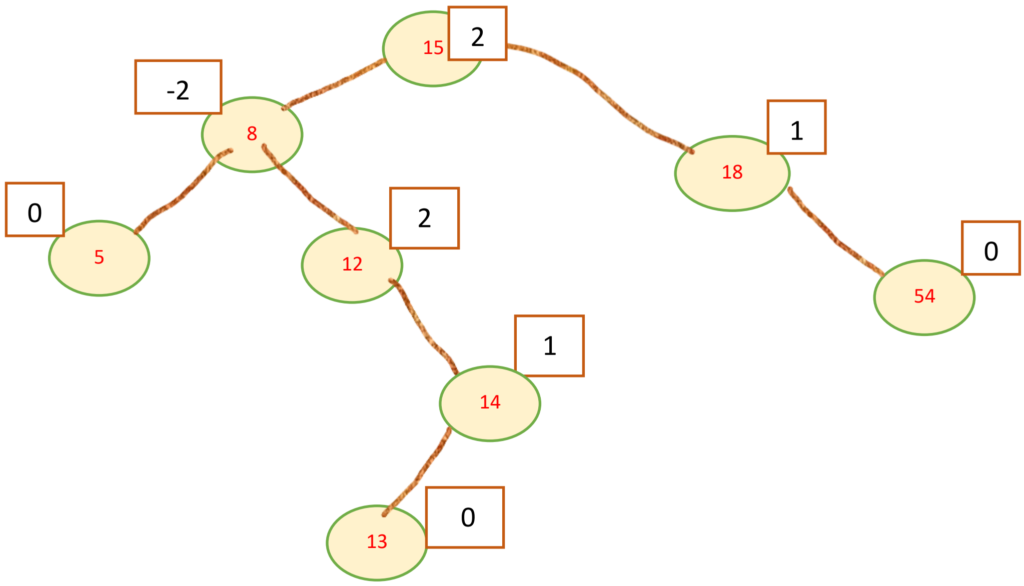
Let's consider three numbers 15, 20 , 17 in place of X, Y, T2



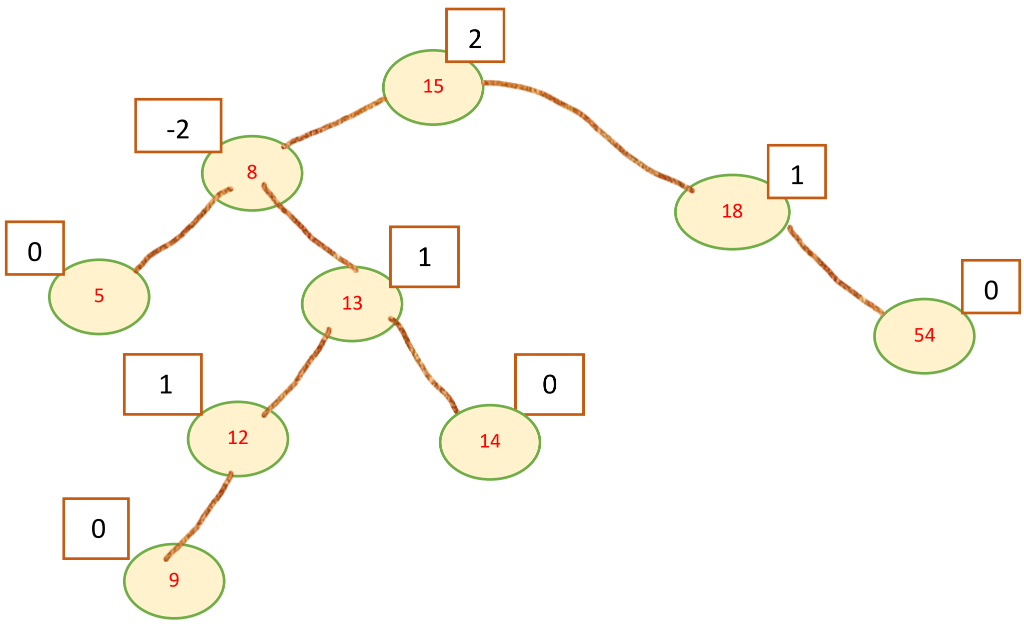
Let's work on this example to clearly understand the above all cases **15, 18, 12, 8, 54, 5, 14, 13, 9, 59, 20, 17, 21**



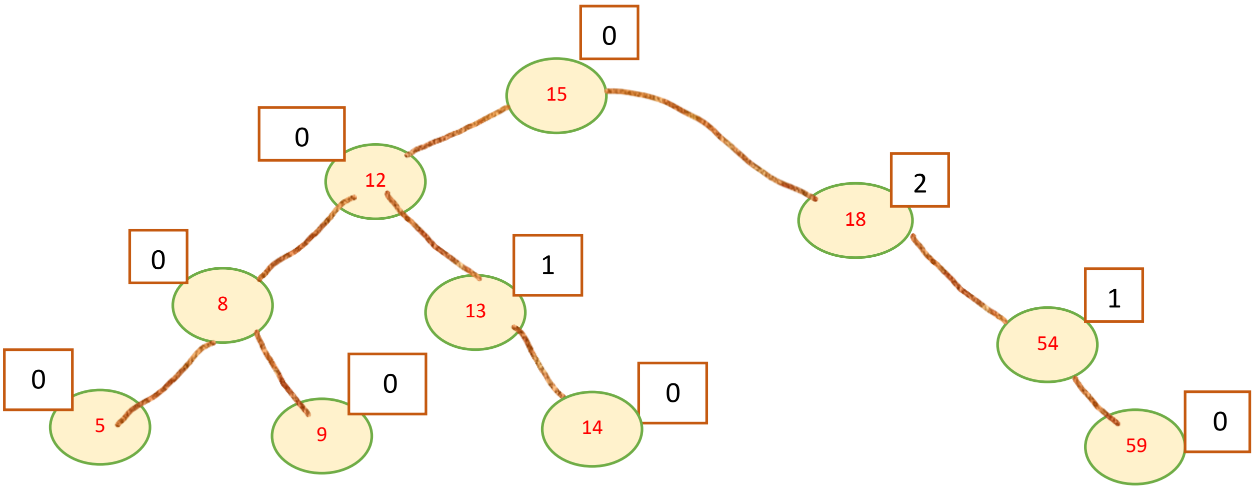
Here balancing factor of 12 is 2 which is unbalanced and this is the left left case.
So we have to apply right rotation on 12,8,5 .



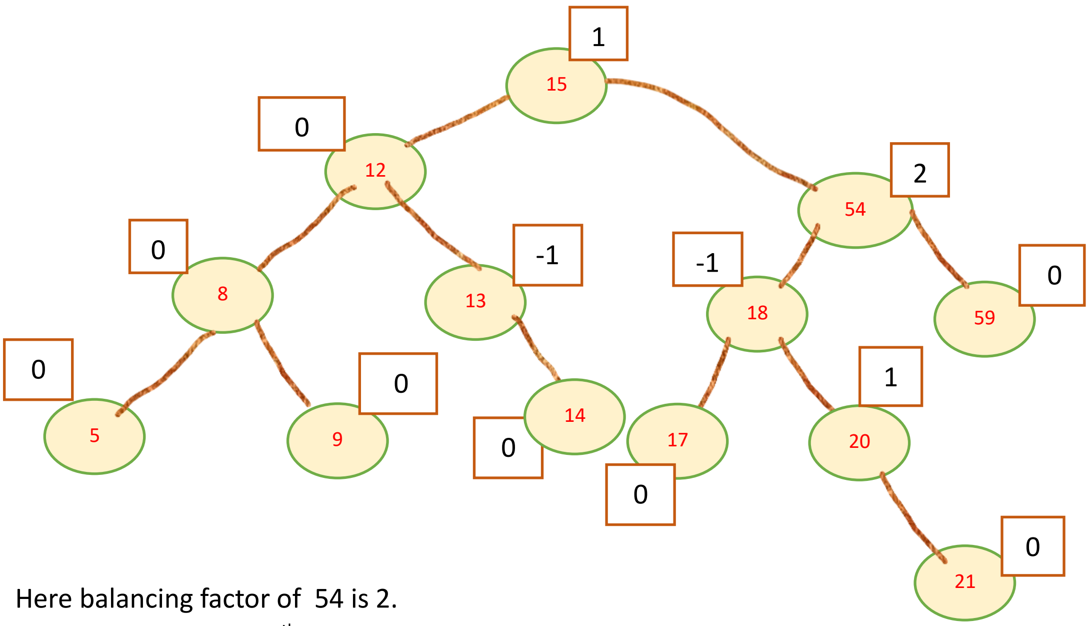
Here balancing factor of 12 is 2 which is unbalanced and this is the right left case. So we have to apply right rotation on 12,14,13 and left rotation on 12,13,14 . Balanced factor of 15 also get balanced when it's left subtree gets balanced.



Here balancing factor of 8 is -2 which is unbalanced and this is right left case. To balance this first we have to apply right rotation on 12, 13, 14. Then, we have to apply left rotation on 8,12,9. Balanced factor of 15 also get balanced when it's left subtree gets balanced.



Here balancing factor of 18 is 2 which is unbalanced and this is right right case.
To balance this we have to apply left rotation on 18, 54, 59.



Here balancing factor of 54 is 2.
we have to balance 54^{th} left which is unbalanced and this is left right case.
To balance this we have to apply left rotation on 18, 20, 21.
Then, we have to apply right rotation on 54, 20,21.