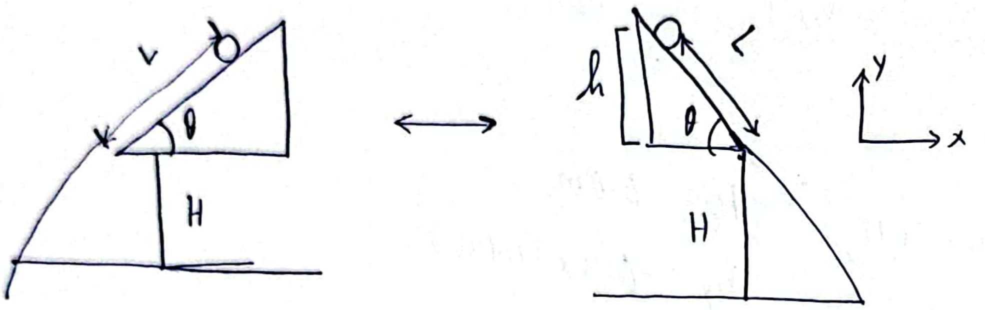
Chapter - 11 (Problems)

7

$$h = L \sin\theta$$
$$= 6 \sin(30^\circ) = 3\ m$$



(a)

$$mgh = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$$

$$\Rightarrow mgh = \frac{1}{2} m \omega^2 R^2 + \frac{1}{2} \times \frac{1}{2} m R^2 \omega^2$$

$$\Rightarrow \cancel{m} g h = \frac{3}{4} \cancel{m} R^2 \omega^2$$

$$\Rightarrow \omega^2 = \frac{4}{3} \frac{1}{R^2} g h$$

$R = 0.1\ m$

$h = 3\ m$

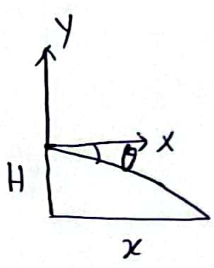
$$\Rightarrow \omega = \frac{1}{R} \sqrt{\frac{4}{3} g h}$$

$$= \frac{1}{0.1} \sqrt{\frac{4}{3} \times 9.8 \times 3}\ rad/s$$

$$= \boxed{63\ rad/s}$$

(b)

$$V_0 = \omega R$$
$$= (63 \times 0.1)\ m/s$$
$$= 6.3\ m/s$$



$a_x = 0\ m/s^2$

$a_y = -9.8\ m/s^2$

$$V_x = V_0 \cos(-\theta) = V_0 \cos\theta$$
$$V_y = V_0 \sin(-\theta) = -V_0 \sin\theta$$

$$Y - Y_0 = V_{0y} t + \frac{1}{2} a_y t^2$$
$$\Rightarrow -H = -V_0 \sin\theta\, t - \frac{1}{2} g t^2 \quad (1)$$

$$x - x_0 = v_{0x} t + \frac{1}{2} a_x t^2$$

$$\Rightarrow x - x_0 = v_{0x} t \quad — (2)$$

From (1), $H = 5.0\,m$

$v_{0y} = -6.3 \times \sin(30^\circ)$

$= -3.15\,m/s$

$$-5 = -3.15\,t - 4.9t^2$$

$$\Rightarrow 4.9t^2 + 3.15\,t - 5 = 0$$

$$\Rightarrow t = \boxed{0.738\,s}$$

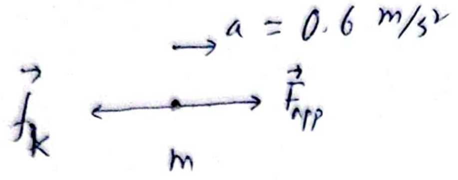
From equation, (2), $x - x_0 = v_0 \cos\theta\; t$

$= (6.3 \cos(30^\circ) \times 0.74)\ m$

$= \boxed{4\,m}$ (Am)

11

(a)



Applying Newton's 2nd law,

$$F_{app} - f_k = ma$$

$$\Rightarrow f_k = F_{app} - ma$$

$$= (10 - 10 \times 0.6)\ N$$

$$= \boxed{4N} \quad (a)$$

in unit vector notation $\vec{f}_k = (4N)\hat{i}$

(b) Newton's 2nd law for rotation

$$\tau = I\alpha$$

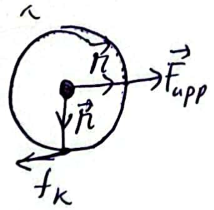
$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\Rightarrow R f_k = I \frac{a}{R}$$

$$\Rightarrow I = \frac{R^2 f_k}{a}$$

$$= \frac{(0.3)^2 \times 4}{0.6}\ kg\,m^2$$

$$= \boxed{0.6\ kg\,m^2}$$



For, $\vec{F}_{app}$,

$$\vec{\tau}_{app} = \vec{r} \times \vec{F}_{app}$$

$$\vec{r} \parallel \vec{F}_{app}$$

$$\therefore \tau_{app} = 0$$

For, $\vec{f}_k$

$$\vec{\tau}_k = \vec{r} \times \vec{f}_k$$

$$\vec{r} \perp \vec{f}_k$$

$$\tau_k = \tau = R f_k$$

$\boxed{25}$ $\vec{r} = 3m\,\hat{i} + 4m\,\hat{j}$

$\vec{F} = (-8N\hat{i}) + (6N\hat{j})$

(a) $\vec{\tau} = \vec{r} \times \vec{F}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3m & 4m & 0m \\ -8N & 6N & 0N \end{vmatrix}$$

$$= \hat{i}\,(4\times 0 - 6\times 0)\,Nm - \hat{j}\,(3\times 0 + 8\times 0)\,Nm + \hat{k}\,(18 + 24)\,Nm$$

$$= (50\,\hat{k})\,Nm$$

(b) $$\cos\theta = \frac{\vec{r}\cdot\vec{F}}{|\vec{r}|\,|\vec{F}|}$$

$$= \frac{3\times(-8) + 4\times 6}{\sqrt{3^2+4^2}\,\sqrt{(-8)^2+(6)^2}}$$

$$\theta = \cos^{-1}\left(\frac{-24+24}{\sqrt{3^2+4^2}\,\sqrt{(-8)^2+(6)^2}}\right)$$

$$= \cos^{-1}(0)$$

$$\boxed{\theta = 90^\circ}$$

$\boxed{37}$ ***

$\boxed{5}$

(a)

$$I = \sum_i m_i r_i^2$$

$$= md^2 + m(2d)^2 + m(3d)^2$$

$$= md^2 + 4md^2 + 9md^2$$

$m = 23 \times 10^{-3}$ kg

$d = 12 \times 10^{-2}$ m

$$= 14md^2$$

$$= 14 \times 23 \times 10^{-3} \times (12 \times 10^{-2})^2 \text{ kgm}^2$$

$$= \boxed{4.6 \times 10^{-3} \text{ kgm}^2}$$

(b)

$$L = I'w$$

$$= [4 \times 23 \times 10^{-3} \times (12 \times 10^{-2})^2 \times 0.85] \text{ kg m}^2/\text{s}$$

$$= \boxed{1.1 \times 10^{-3} \text{ kg.m}^2/\text{s}}$$

$$I' = m(2d)^2$$

$$= 4md^2$$

$$= 4 \times 23 \times 10^{-3} \times (12 \times 10^{-2})^2 \text{ kg m}^2$$

(c)

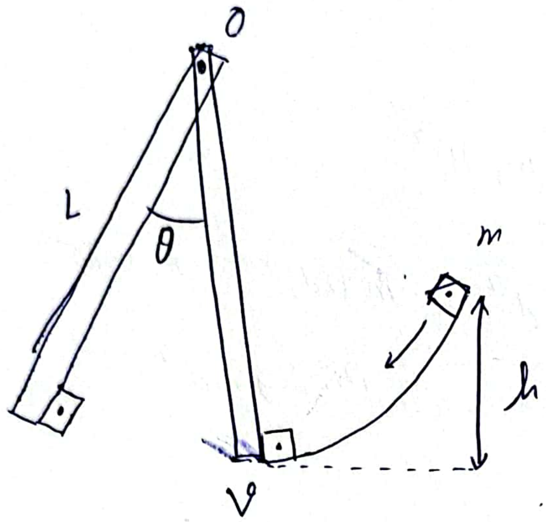
$$L = Iw$$

$$= (4.6 \times 10^{-3}) \times (0.85) \text{ kg m}^2/\text{s}$$

$$= \boxed{3.9 \times 10^{-3} \text{ kg m}^2/\text{s}}$$

CamScanner

66



$$\cancel{m} g h = \frac{1}{2} \cancel{m} v^2$$

$$\Rightarrow \quad v = \sqrt{2gh}$$

Conservation of angular momentum

$$L_i = L_f$$

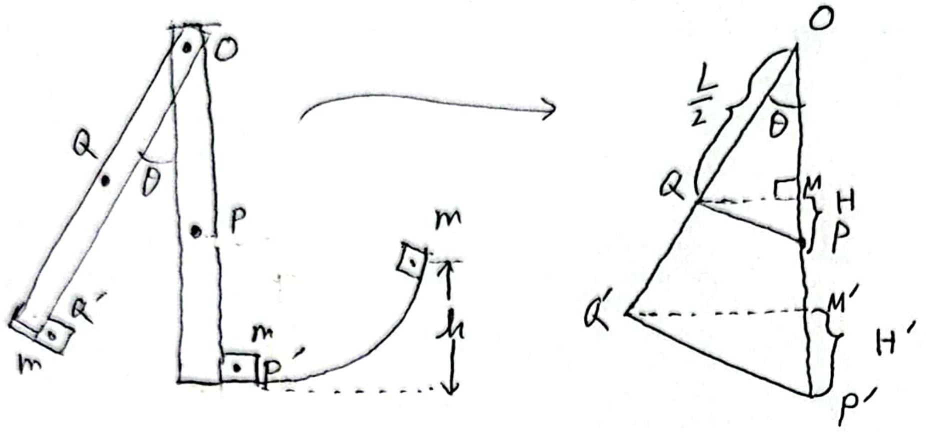
$$\Rightarrow \quad L_m + L_{rod} = L_m' + L_{rod}'$$

$$\Rightarrow \quad m v L + 0 = m v' L + I_{rod}\, \omega$$

$$\Rightarrow \quad m v L = m \omega L^2 + \frac{1}{3} M L^2 \omega$$

$$\Rightarrow \quad m v L = \left(m + \frac{M}{3} \right) \omega L^2$$

$$\Rightarrow \quad \omega = \frac{m v}{\left(m + \frac{M}{3} \right) L} = \frac{m \sqrt{2gh}}{\left(\frac{M}{3} + m \right) L}$$



ΔOQM, $\cos\theta = \dfrac{OM}{OQ}$

$$\Rightarrow \quad OM = \frac{L}{2}\cos\theta$$

$$MP = OP - OM$$
$$= \frac{L}{2} - \frac{L}{2}\cos\theta$$

$$\boxed{H = \frac{L}{2}(1-\cos\theta)}$$

Similarly $H' = L(1-\cos\theta)$

According to Conservation of Energy,

$$\frac{1}{2}(I_{rod} + mL^2)\,\omega^2 = mgH' + MgH$$

$$\Rightarrow \frac{1}{2}\left(\frac{1}{3}ML^2 + mL^2\right)\left(\frac{m^2 \times 2gh}{\left(\frac{M}{3}+m\right)L}\right) = mgL(1-\cos\theta) + Mg\frac{L}{2}(1-\cos\theta)$$

$$\Rightarrow \frac{1}{2}\left(\frac{1}{3}ML^2 + mL^2\right)\left(\frac{2m^2gh}{\left(\frac{M}{3}+m\right)L}\right) = mgL + Mg\frac{L}{2} - \left(mgL + Mg\frac{L}{2}\right)\cos\theta$$

$$\Rightarrow \left(mgL + Mg\frac{L}{2}\right)\cos\theta = \left(mgL + Mg\frac{L}{2}\right) - \frac{1}{2}\left(\frac{1}{3}ML^2 + mL^2\right)\left(\frac{2m^2gh}{\left(\frac{M}{3}+m\right)L}\right)$$

$$\Rightarrow \quad \cos\theta = \frac{\left(mgL + Mg\frac{L}{2}\right) - \frac{1}{2}\left(\frac{1}{3}ML^2 + mL^2\right)\left(\frac{2m^2gh}{\left(\frac{M}{3}+m\right)L}\right)}{\left(mgL + Mg\frac{L}{2}\right)}$$

$\theta = \cos^{-1}(0.85) \simeq \boxed{32^{0}}$