Ans. to the Q. No: 1

$$\lim_{x \to 0} (1 + \sin x)^{\frac{1}{x}} = y$$

$$\Rightarrow \ln y = \lim_{x \to 0} \frac{1}{x} \ln (1 + \sin x)$$

$$\Rightarrow \ln y = \lim_{x \to 0} \frac{\ln (1 + \sin x)}{x}$$

$$\Rightarrow \ln y = \lim_{x \to 0} \frac{\cos x}{1 + \sin x}$$ [L'Hospital rule]

$$\Rightarrow \ln y = \frac{1}{1}$$

$$\Rightarrow \ln y = 1$$

$$\Rightarrow y = e$$

[Proved]

Ans. to the Q. No: 2(a)

$$\lim_{x \to 3^-} \frac{1}{|x-3|}$$

$$= \lim_{x \to 3^-} \frac{1}{-x+3}$$

$$= \frac{1}{-3+3}$$

$$= \infty$$ Ans

Ans. to the Q. No: 2 (b)

$$\lim_{y \to 4} \frac{4-x}{2-\sqrt{y}}$$

$$\Rightarrow \lim_{y \to 4} \frac{4-y}{2-\sqrt{y}} \cdot \frac{2+\sqrt{y}}{2+\sqrt{y}}$$

$$\Rightarrow \lim_{y \to 4} \frac{(4-y)\,(2+\sqrt{y})}{(2-\sqrt{y})\,(2+\sqrt{y})}$$

$$\Rightarrow \lim_{y \to 4} \frac{(4-y)\,(2+\sqrt{y})}{4+\cancel{2\sqrt{y}}-\cancel{2\sqrt{y}}-y}$$

$$\Rightarrow \lim_{y \to 4} \frac{\cancel{(4-y)}\,(2+\sqrt{y})}{\cancel{(4-y)}}$$

$$\Rightarrow \lim_{y \to 4} (2+\sqrt{y})$$

$$\Rightarrow 2+\sqrt{4}$$

$$\Rightarrow 2+2$$

$$\Rightarrow 4 \text{ (Ans)}$$

Ans. to the Q. No: 2(c)

$$\lim_{x \to 4} \frac{3-x}{x^2-2x-8}$$

$$\Rightarrow \lim_{x \to 4} \frac{3-x}{x^2-2x+4x-8}$$

$$\Rightarrow \lim_{x \to 4} \frac{3-x}{x(x-2)+4(x-2)}$$

$$\Rightarrow \lim_{x \to 4} \frac{3-x}{(x+4)(x-2)}$$

$$\Rightarrow \frac{3-4}{(4+4)(4-2)}$$

$$\Rightarrow \frac{-1}{8 \cdot 2}$$

$$\Rightarrow -\frac{1}{16}$$ Ans

Ans. to the Q. No : 3

$$\lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right)$$

$$\Rightarrow -1 \leq \sin\left(\frac{1}{x}\right) \leq 1$$

$$\Rightarrow -x^2 \leq x^2 \sin\left(\frac{1}{x}\right) \leq x^2$$

$$\Rightarrow \lim_{x \to 0} -x^2 \leq \lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right) \leq \lim_{x \to 0} x^2$$

$$\Rightarrow 0 \leq \lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right) \leq 0$$

$$\therefore \lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right) = 0$$

Ans. to the Q. No: 4

$$\lim_{x\to 0} \frac{|x|}{x}$$

$$\Rightarrow \frac{0}{0}$$

$$\therefore \lim_{x\to 0} \frac{|x|}{x}$$

$$L.H.S = \lim_{x\to 0^-} \frac{|x|}{x}$$

$$= \frac{0}{-0}$$

$$= -0$$

$$R.H.S = \lim_{x\to 0^+} \frac{|x|}{x}$$

$$= \frac{0}{0}$$

$$= +0$$

$\therefore L.H.S \neq R.H.S$

$\therefore$ limit dose not exist.

Ans. to the Q. No: 5

$$\lim_{x \to -1} \frac{x^2+6x+5}{x^2-3x-4}$$

$$\Rightarrow \frac{(-1)^2+6(-1)+5}{(-1)^2-3(-1)-4}$$

$$\Rightarrow \frac{1-6+5}{1+3-4}$$

$$\Rightarrow \frac{0}{0}$$

$$\therefore \lim_{x \to -1} \frac{x^2+6x+5}{x^2-3x-4}$$

$$\Rightarrow \lim_{x \to -1} \frac{2x+6}{2x-3}$$ [L' ~~ho~~ hopital's rule]

$$\Rightarrow \frac{2(-1)+6}{2(-1)-3}$$

$$\Rightarrow \frac{-2+6}{-2-3}$$

$$\Rightarrow -\frac{4}{5}$$ Ans