



Differential Calculus and Co-ordinate Geometry

MATH110 Assignment-04

Ashakuzzaman Odree

ID-20301268

Section-11

Set-D

Ans to the Question NO -1

$$\begin{aligned}f(x, y) &= x^2 + xy - 2y - 2x + 1 \\f_x &= 2x + y - 0 - 2 + 0 \\&= 2x + y - 2 = 0 \dots\dots\dots (i)\end{aligned}$$

$$\begin{aligned}f_y &= 0 + x - 2 - 0 + 0 \\&= x - 2 = 0 \dots\dots\dots(ii)\end{aligned}$$

$$ii) \Rightarrow x - 2 = 0 \quad x = 2$$

$$\begin{aligned}i \Rightarrow 2(2) + y - 2 &= 0 \quad [x = 2] \\4 + y - 2 &= 0 \\y &= -2\end{aligned}$$

$$f_{xx} = (f_x)_x = 2$$

$$f_{yy} = (f_y)_y = 0$$

$$f_{xy} = (f_x)_y = 1$$

critical Point $-(2, -2)$

$$(f_{xy})^2 = 1^2 = 1$$

$$f_{xx} \cdot f_{yy} = 2 \cdot 0 = 0$$

$$(f_{xy})^2 > f_{xx} \cdot f_{yy}$$

saddle point (2,-2)

Ans:

Ans to the Question NO-2

$$f(x) = x^3 + 2x + 1 \quad [x = 3]$$

$$= 34$$

$$f'(x) = 3x^2 + 2$$

$$= 29$$

$$f''(x) = 6x = 18$$

$$f'''(x) = 6$$

$$f^{iv}(x) = 0$$

$$P_n(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 + \frac{f'''(x_0)}{3!}(x - x_0)^3 + \dots\dots$$

$$P_n(3) = 34 + 29(x - 3) + \frac{18}{2!}(x - 3)^2 + \frac{6}{3!}(x - 3)^3 + \frac{0}{4!}(x - 3)^4 + \dots\dots$$

$$= 34 + 29(x - 3) + 9(x - 3)^2 + (x - 3)^3$$

Ans to the Question NO-3

Given,

$$\begin{aligned}f(x, y, z) &= 3(2x + 5)y^4(z^3 + 1) \\ &= (6x + 15)y^4(z^3 + 1)\end{aligned}$$

$$fx = (6 + 0)y^4(z^3 + 1) = 6y^4(z^3 + 1)$$

$$fy = 4(6x + 15) \cdot (z^3 + 1) y^3$$

$$\begin{aligned}f_{xz} &= (fx)_z = 6y^4(3z^2 + 0) \\ &= 6y^4 \cdot 3z^2 \\ &= 18y^4z^2\end{aligned}$$

Ans :

$$\begin{aligned}fyz &= (fy)z = 4(6x + 15)y^3 \cdot 3z^2 \\ &= 72xz^2y^3 + 180z^2y^3\end{aligned}$$

Ans :

$$\begin{aligned}fxyz &= (6 + 0)4y^3(3z^2 + 6) = 6 \cdot 4y^3 \cdot 3z^2 \\ &= 72y^3z^2\end{aligned}$$

Ans :

Ans to the Question Q NO-4

$$z = \ln(3x^2 - 2y + 4z^3); \quad x = t^{\frac{1}{2}}, y = t^{\frac{2}{3}}, z = t^{-2}$$

$$\begin{aligned}\frac{\partial z}{\partial x} &= \frac{1}{3x^2 - 2y + 4z^3} \cdot 6x^2 \\ \frac{dx}{dt} &= \frac{1}{2}t^{-\frac{1}{2}} \\ \frac{\partial z}{\partial y} &= \frac{1}{3x^2 - 2y + 4z^3} \cdot -2 \\ \frac{dy}{dt} &= \frac{2}{3}t^{-\frac{1}{3}} \\ \frac{\partial z}{\partial z} &= \frac{1}{3x^2 - 2y + 4z^3} \cdot 12z^2 \\ \frac{dz}{dt} &= (-2)t^{-3}\end{aligned}$$

Chain rule of partial Derivative:

$$\begin{aligned}\frac{dz}{dt} &= \frac{\partial z}{\partial x} \frac{dx}{dt} + \frac{\partial z}{\partial y} \frac{dy}{dt} + \frac{\partial z}{\partial z} \frac{dz}{dt} \\ &= \frac{6x^2}{3x^2 - 2y + 4z^3} \cdot \frac{1}{2}t^{-\frac{1}{2}} + \frac{-2}{3x^2 - 2y + 4z^3} \cdot \frac{2}{3}t^{-\frac{1}{3}} + \frac{12z^2}{3x^2 - 2y + 4z^3} (-2)t^{-3} \\ &= \frac{1}{3x^2 - 2y + 4z^3} \left(3x^2 t^{-\frac{1}{2}} - \frac{4}{3}t^{-\frac{1}{3}} - 24z^2 t^{-3} \right) \\ &= \frac{1}{3t^{(\frac{1}{2})^2} - 2t^{\frac{2}{3}} + 4t^{(-2)^3}} \left(3t^{(\frac{1}{2})^2} t^{-\frac{1}{2}} - \frac{4}{3}t^{-\frac{1}{3}} - 24t^{(-2)^2} t^{-3} \right) \\ &= \frac{1}{3t - 2t^{\frac{2}{3}} + 4t^{-6}} \left(3t - \frac{4}{3}t^{-\frac{1}{3}} - 24t^{-7} \right)\end{aligned}$$

Ans to the Question NO-5

a) Given,

$$f(x, y) = x^2 y e^{xy}$$

Now,

$$\frac{\partial f}{\partial x} = 2xy e^{xy} + x^2 y e^{xy} \cdot y$$

After putting the value $\cdot \frac{\partial f}{\partial x}(1 \cdot 1)$, we get

$$\begin{aligned} & 2 \cdot 1 \cdot 1 \cdot e^{1 \cdot 1} + 1^2 \cdot 1 \cdot e^{1 \cdot 1} \cdot 1 \\ & = 3 e \end{aligned}$$

Again,

$$\frac{\partial f}{\partial y} = x^2 e^{xy} + x^2 y e^{xy} \cdot x$$

$\frac{\partial f}{\partial y}(1, 1)$, we get

$$\begin{aligned} & 1^2 \cdot e^{1 \cdot 1} + 1^2 \cdot 1 \cdot e^{1 \cdot 1} \cdot 1 \\ & = 2 e \end{aligned}$$

Ans:

b) given,

$$\omega = x^2 \cos xy$$

Now,

$$\frac{\partial \omega}{\partial x} = 2x \cos xy - yx^2 \sin xy$$

After putting the value, $\frac{\partial w}{\partial y} \left(\frac{1}{2}, \pi \right)$ we get

$$2 \cdot \frac{1}{2} \cdot \cos \frac{\pi}{2} - \pi \cdot \frac{1}{4} \cdot \sin \frac{\pi}{2}$$

$$= 0 - \frac{\pi}{4} \cdot 1$$

$$= -\frac{\pi}{4}$$

Again,

$$\begin{aligned} \frac{\partial \omega}{\partial y} &= -x^2 \sin xy \cdot x \\ &= -x^3 \sin xy \end{aligned}$$

$\frac{\partial w}{\partial y} \left(\frac{1}{2}, \pi \right)$ we get

$$\begin{aligned} & -\frac{1}{8} \cdot \sin \frac{\pi}{2} \\ &= -\frac{1}{8} \end{aligned}$$

Ans:

Ans to the Question NO-6

$$f(x) = \frac{1}{1-x} \text{ at } x = 0$$

$$f(x) = \frac{1}{1-x} = \frac{1}{1-0} = 1$$

$$f'(x) = \frac{1}{(1-x)^2} = \frac{1}{(1-0)^2} = 1$$

$$f''(x) = \frac{2}{(1-x)^3} = \frac{1}{(1-0)^3} = 2$$

$$f'''(x) = \frac{6}{(1-x)^4} = \frac{6}{(1-0)^4} = 6$$

Taylor series,

$$P_n(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 + \frac{f'''(x_0)}{3!}(x - x_0)^3 + \dots$$

$$\begin{aligned} P_n(x) &= 1 + (-1)x + \frac{2}{2!}x^2 + \frac{-6}{3!}x^3 + \dots \\ &= 1 - x + x^2 - x^3 + \dots \end{aligned}$$

Ans: