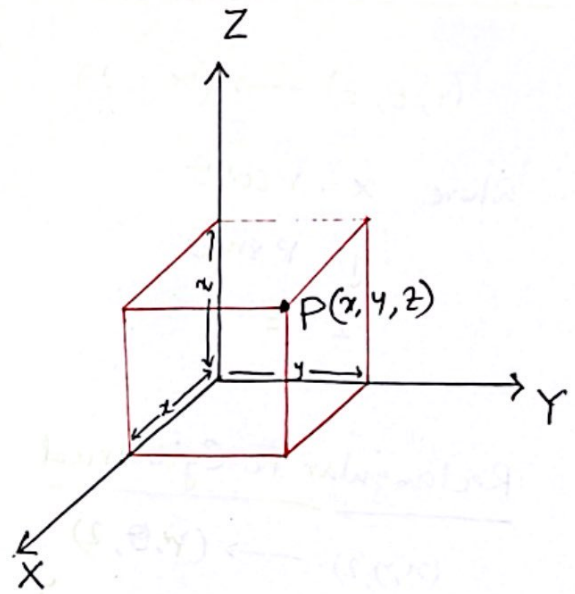
Lecture 16 — Coordinate Systems in 3D

Rectangular Coordinates

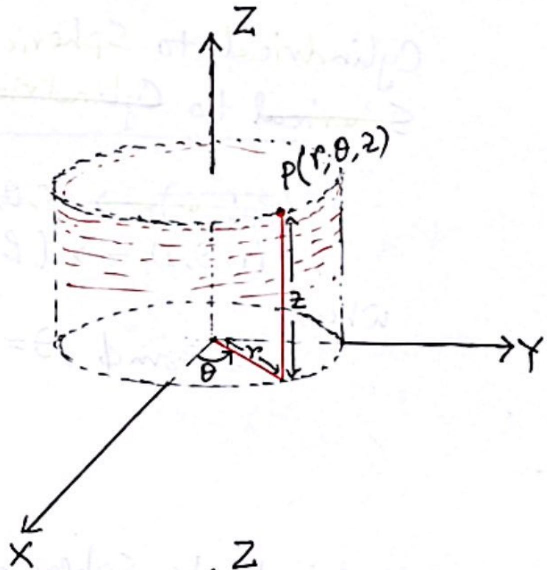
(x, y, z)



Cylindrical Coordinates

(r, θ, z)

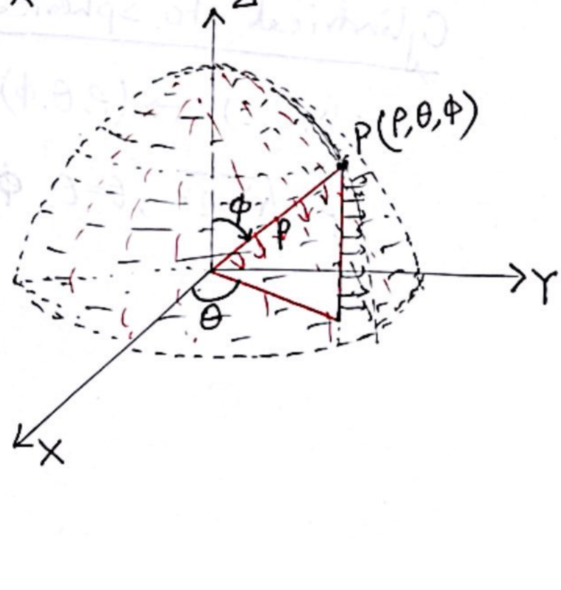
$(r \geq 0,\ 0 \leq \theta \leq 2\pi)$



Spherical coordinates

(ρ, θ, ϕ)

$(\rho \geq 0,\ 0 \leq \theta < 2\pi,\ 0 \leq \phi \leq \pi)$



~~Lecture 17~~

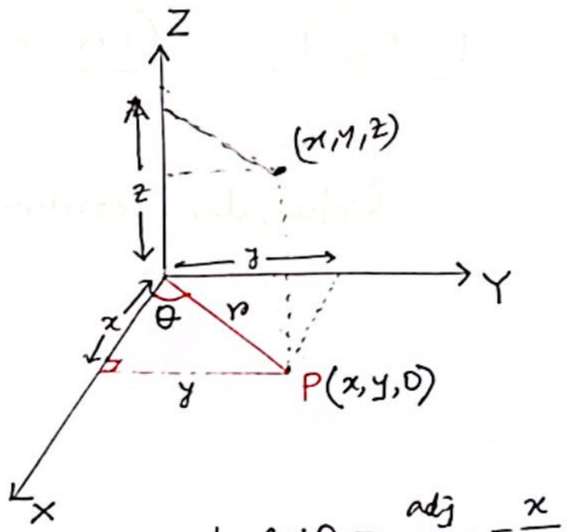
Cylindrical to Rectangular

$$(r, \theta, z) \longrightarrow (x, y, z)$$

where $x = r\cos\theta$

$y = r\sin\theta$

$z = z$



$\cos\theta = \dfrac{adj}{hyp} = \dfrac{x}{r}$

$x = r\cos\theta$

$\sin\theta = \dfrac{y}{r}$

$y = r\sin\theta$

Rectangular to Cylindrical

$$(x, y, z) \longrightarrow (r, \theta, z)$$

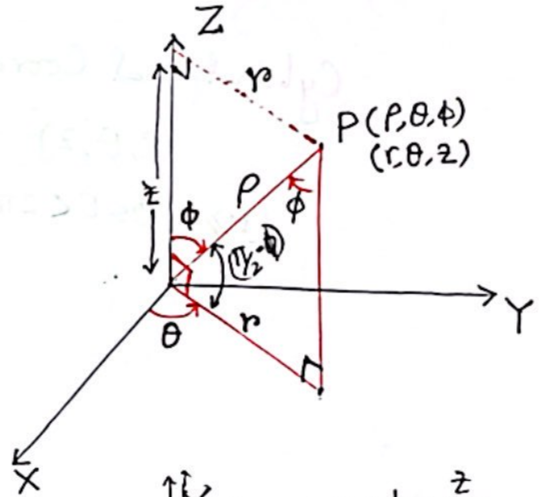
$r = \sqrt{x^2+y^2}$, $\tan\theta = \dfrac{y}{x}$, $z = z$

Spherical to Cylindrical

$$(\rho, \theta, \phi) \rightarrow (r, \theta, z)$$

where

$r = \rho\sin\phi$, $\theta = \theta$, $z = \rho\cos\phi$

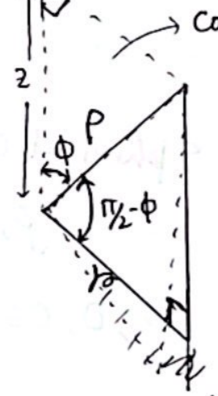


Cylindrical to Spherical

$$(r, \theta, z) \rightarrow (\rho, \theta, \phi)$$

$\rho = \sqrt{r^2+z^2}$, $\theta = \theta$, $\tan\phi = \dfrac{r}{z}$

$\cos\phi = \dfrac{z}{\rho}$



$\cos(\pi/2 - \phi) = \dfrac{r}{\rho}$

$\rho\sin\phi = r$

Spherical to rectangular

$$(\rho, \theta, \phi) \to (x, y, z)$$

where $x = r\cos\theta = \rho\sin\phi\cos\theta$

$y = r\sin\theta = \rho\sin\phi\sin\theta$

$z = \rho\cos\phi$

rectangular to Spherical

$$(x, y, z) \longrightarrow (\rho, \theta, \phi)$$

$\rho = \sqrt{x^2+y^2+z^2}$

$\tan\theta = \dfrac{y}{x}$

$\cos\phi = \dfrac{z}{\rho} = \dfrac{z}{\sqrt{x^2+y^2+z^2}}$

$x^2 = \rho^2\sin^2\phi\cos^2\theta$

$y^2 = \rho^2\sin^2\phi\sin^2\theta$

$z^2 = \rho^2\cos^2\phi$

$x^2+y^2+z^2 = \rho^2\sin^2\phi(\cos^2\theta+\sin^2\theta)+\rho^2\cos^2\phi$

$= \rho^2\sin^2\phi + \rho^2\cos^2\phi$

$= \rho^2(\sin^2\phi+\cos^2\phi)$

$= \rho^2$

Example: Find the rectangular coordinates of the point with cylindrical coordinates $(r, \theta, z) = (4, \pi/3, -3)$.

Soln: $x = r\cos\theta = 4\cos\pi/3 = 2$

$y = r\sin\theta = 4\sin\pi/3 = 2\sqrt{3}$

$z = z = -3$

$\cos\frac{\pi}{3} = \frac{1}{2}$

$\sin\pi/3 = \frac{\sqrt{3}}{2}$

Thus, the rectangular coordinates of the point are

$$(x, y, z) = (2, 2\sqrt{3}, -3)$$

Example: Find the rectangular coordinates of the point with spherical coordinates ~~pf~~

$$(\rho, \theta, \phi) = (4, \pi/3, \pi/4)$$

Sol:

$$x = \rho \sin\phi \cos\theta = 4 \sin\frac{\pi}{4} \cos \pi/3 = 4 \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{2} = \sqrt{2}$$

$$y = \rho \sin\phi \sin\theta = 4 \sin\frac{\pi}{4} \sin \pi/3 = 4 \cdot \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} = \sqrt{6}$$

$$z = \rho \cos\phi = 4 \cos\frac{\pi}{4} = 4 \cdot \frac{1}{\sqrt{2}} = 2\sqrt{2}$$

Thus rectangular coordinates of the point are

$$(x, y, z) = (\sqrt{2}, \sqrt{6}, 2\sqrt{2})$$