Math 110

Assignment - 1

Set - D

Ans to the Q.N. - 1

$$\lim_{x \to 1} \frac{x^3 - x^2 + 2x - 2}{x^3 + 3x^2 - 4x} = \frac{3}{a}$$

$$\lim_{x \to 1} \frac{x^2(x-1) + 2(x-1)}{x(x^2 + 3x - 4)} = \frac{3}{a}$$

$$\lim_{x \to 1} \frac{(x-1)(x^2+2)}{x(x^2 - x + 4x - 4)} = \frac{3}{a}$$

$$\lim_{x \to 1} \frac{(x-1)(x^2+2)}{x\{x(x-1) + 4(x-1)\}} = \frac{3}{a}$$

$$\lim_{x \to 1} \frac{(x-1)(x^2+2)}{x(x-1)(x+4)} = \frac{3}{a}$$

$$\lim_{x \to 1} \frac{x^2+2}{x(x+4)} = \frac{3}{a}$$

Now,

$$x = 1 \quad ; \quad \frac{1^2+2}{1(1+4)} = \frac{3}{a}$$

$$\Rightarrow \frac{3}{5} = \frac{3}{a}$$

$$\therefore \quad a = \frac{15}{3} = 5 \quad \underline{\text{Ans:}}$$

Ans to the Q NO:- 2

$$\lim_{x \to -1} \frac{x^3 - x^2 - 5x - 3}{x^3 + 6x^2 + 9x + 4} = -\frac{4}{a}$$

$$\lim_{x \to -1} \frac{\frac{x^3}{x^3} - \frac{x^2}{x^3} - \frac{5x}{x^3} - \frac{3}{x^3}}{\frac{x^3}{x^3} + \frac{6x^2}{x^3} + \frac{9x}{x^3} + \frac{4}{x^3}} = -\frac{4}{a}$$

$$\lim_{x \to -1} \frac{1 - \frac{1}{x} - \frac{5}{x^2} - \frac{3}{x^3}}{1 + \frac{6}{x} + \frac{9}{x^2} + \frac{4}{x^3}} = -\frac{4}{a}$$

$$\lim_{x \to -1} \frac{1 - x^{-1} - 5x^{-2} - 3x^{-3}}{1 + 6x^{-1} + 9x^{-2} + 4x^{-3}} = \frac{-4}{a}$$

$$\lim_{x \to -1} \frac{\frac{1}{x^2} + \frac{10}{x^3} + \frac{9}{x^4}}{-\frac{6}{x^2} - \frac{18}{x^3} - \frac{12}{x^4}} = \frac{-4}{a}$$

$$\lim_{x \to -1} \frac{x^{-2} + 10x^{-3} + 9x^{-4}}{-6x^{-2} - 18x^{-3} - 12x^{-4}} = \frac{-4}{a}$$

$$\lim_{x \to -1} \frac{\frac{-2}{x^3} - \frac{30}{x^4} - \frac{36}{x^5}}{\frac{12}{x^3} + \frac{54}{x^4} + \frac{48}{x^5}} = \frac{-4}{a}$$

Now,

$$x \to -1 \; ; \; \frac{\frac{-2}{(-1)^3} - \frac{30}{(-1)^4} - \frac{36}{(-1)^5}}{\frac{12}{(-1)^3} + \frac{54}{(-1)^4} + \frac{48}{(-1)^5}} = \frac{-4}{a}$$

$$\Rightarrow \frac{-2 + 30 - 36}{12 - 54 + 48} = \frac{-4}{a}$$

$$\Rightarrow \frac{-8}{6} = \frac{-4}{a}$$

$$\Rightarrow \frac{-4}{3} = \frac{-4}{a}$$

$$\therefore a = \frac{-12}{-4} = 3 \quad \text{Ans.}$$

Ans to the Q No-3

a) $\lim_{x \to -2} f(x)$

$$\lim_{x \to -2} \frac{1}{x+2} \quad \left[\because x \leq 2\right]$$

Now,

$$x \to -2 \; ; \; \frac{1}{-2+2}$$

$$= \frac{1}{0} \quad \text{Ans.}$$

b) $\lim_{x \to 0} f(x)$

$\lim_{x \to 0} x^2 - 5$ [$\because -2 < x \leq 3$]

Now,

$x \to 0$; $0^2 - 5$

$= -5$ Ans.

c) $\lim_{x \to 3} f(x)$

$\lim_{x \to 3} x^2 - 5$ [$\because -2 < x \leq 3$]

Now,

$x \to 3$; $3^2 - 5$

$= 9 - 5$

$= 4$

Ans.

Ans to the Q No-4

$$\lim_{x \to 3} \frac{x^2 - 5x + 6}{2x^2 - 18} = \frac{1}{p}$$

$$\lim_{x \to 3} \frac{x^2 - 2x - 3x + 6}{2(x^2 - 9)} = \frac{1}{p}$$

$$\lim_{x \to 3} \frac{x(x-2) - 3(x-2)}{2(x^2 - 3^2)} = \frac{1}{p}$$

$$\lim_{x \to 3} \frac{(x-2)(x-3)}{2(x+3)(x-3)} = \frac{1}{p}$$

$$\lim_{x \to 3} \frac{(x-2)}{2(x+3)} = \frac{1}{p}$$

Now,

$\lim_{x \to 3}$; $\frac{(3-2)}{2(3+3)} = \frac{1}{p}$

$$\Rightarrow \frac{1}{12} = \frac{1}{p}$$

$$\therefore p = 12$$ Ans!

Ans to the Q No-5

$$\lim_{x \to 0} \frac{|x|}{x}$$

for, $x < 0$, $\frac{|x|}{x} = \frac{-x}{x} = -1$

for, $x > 0$, $\frac{|x|}{x} = \frac{x}{x} = 1$

Now,

$$\lim_{x \to 0^-} \frac{|x|}{x} = -1$$

$$\lim_{x \to 0^+} \frac{|x|}{x} = 1$$

So, the limit does not exist.