LECTURE 05 — October 12, 2023

SECTION: 35 (UB71001) FACULTY: Akiful Islam (AZW)

BRAC UNIVERSITY

1 Motion in a Plane

By plane, we mean a 2D surface or area where the object is allowed to move. We will be using the same four equations yet again. This time, we need both the x and y variations of these laws. Lastly, we may add them vectorially to reconstruct the 2D vectors.

We must not forget the 4 primary formulae. They are the bread and butter of projectile motion calculation.

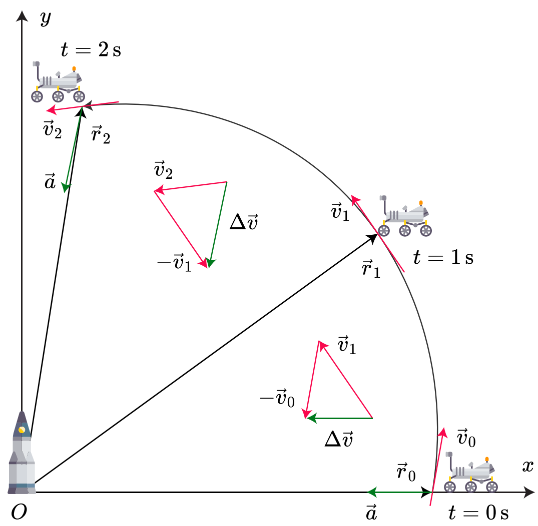

FIGURE 1: Mars rover, Perseverance makes a 2D motion around the lander from $t = 0$s to $t = 2$s. The acceleration here is in the direction of the change in velocity $\Delta\vec{v}$.

1.1 Acceleration in a Curved Path

Unlike linear motion, we must be careful about the notion of vector direction. More precisely, the directions of the displacement, velocity, and acceleration vectors. We must remember the followings:

- Displacement vector is the change in **position vector** with respect to time. Its direction lies in the direction of the difference vectors,
- Velocity vector is the change in **displacement vector** with respect to time. Its direction lies in the direction of the displacement vector,
- Acceleration vector is the change in **velocity vector** with respect to time. Its direction lies in the direction of the difference of velocity vectors.

For example, in linear motion, we found that the direction of $\Delta\vec{r}$, $\Delta\vec{v}$, and $\vec{a}$ all lie in the same direction, but in 2D, they are most likely to change.

1.2 Components of Acceleration

Almost similar to the coordinate components of acceleration vector, $\vec{a}$, we may define a different set of components of the same $\vec{a}$. In this case, $\vec{a}$ will be measured with respect to the direction of $\Delta\vec{v}$. These accelerations will provide us with information about the changes in the velocity.

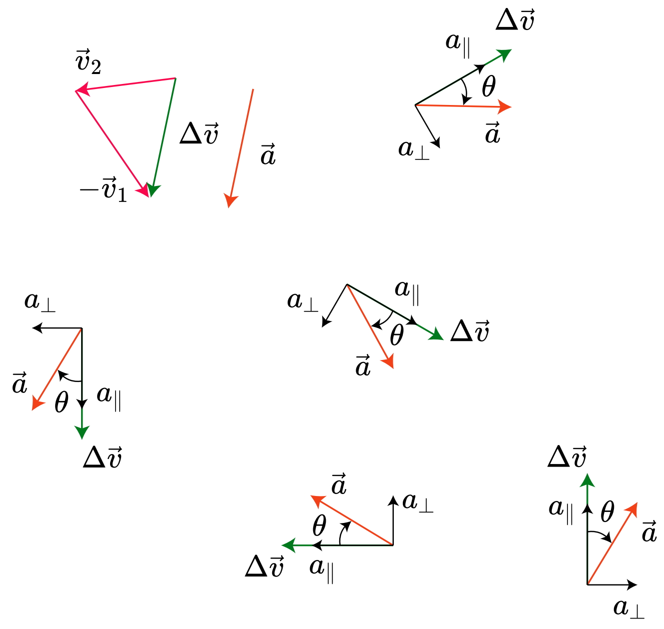

FIGURE 2: We are defining two new components of $\vec{a}$ in the direction of $\Delta\vec{v}$. Regardless of $\Delta\vec{v}$'s orientation in space, $a_{\parallel}$ is across and $a_{\perp}$ is perpendicular to the direction of $\Delta\vec{v}$.

1.2.1 Tangential (or Parallel) Acceleration

This component of acceleration $\vec{a}$ points in the direction parallel to the direction of $\Delta\vec{v}$, labeled $a_{\parallel}$. This component tells us about the changes in an object's **speed**.

$$a_{\parallel} = \frac{dv}{dt}. \tag{1}$$

1.2.2 Radial (or Perpendicular) Acceleration

This component of acceleration $\vec{a}$ points in the direction perpendicular to the direction of $\Delta\vec{v}$, labeled $a_{\perp}$. This component tells us about the changes in the object's **direction of motion**.

$$a_{\perp} = \frac{v^2}{r}. \tag{2}$$

Basically, we use these components to keep track of the velocity changes in the motion. Consider the following case of a motion in a random direction. In the diagram, we only draw the direction of $\Delta\vec{v}$ and $\vec{a}$. Assuming the acceleration makes an angle θ with the direction of $\Delta\vec{v}$. Then we may calculate the components as follows:

$$a_{\parallel} = \frac{dv}{dt} = a\cos\theta \tag{3}$$
$$a_{\perp} = \frac{v^2}{r} = a\sin\theta \tag{4}$$

2 Uniform Circular Motion

Uniform circular motion is a type of motion in which an object moves in a circular path at a **constant speed**. In other words, the speed of the object is *constant*, but its velocity changes as it moves around the circle. This change in velocity is due to the fact that the direction of motion, $\Delta\vec{v}$, is constantly changing as the object moves in a circle.

TAKEAWAY: An object in a circular and/or nearly circular motion must accelerate at all times.

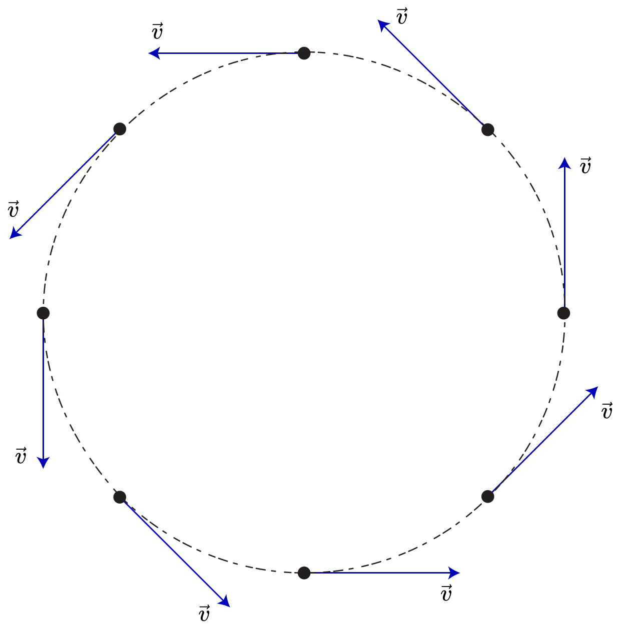

FIGURE 3: Take a ball and let it move in a circle, not rotate on its own axis. At each point in time, the ball has a velocity tangent to the circle's circumference.

2.1 Centripetal Acceleration

Consider the situation shown in the diagram. An object is rotating counterclockwise in a circular fashion. At each instant, the object has an instantaneous velocity denoted by the tangent drawn on that point. Remember the magnitude of this instantaneous velocity. i.e., is constant, but its direction changes at each instant. Because of that, there should be a net change in the velocity vector. We measure that by $\Delta\vec{v}$.

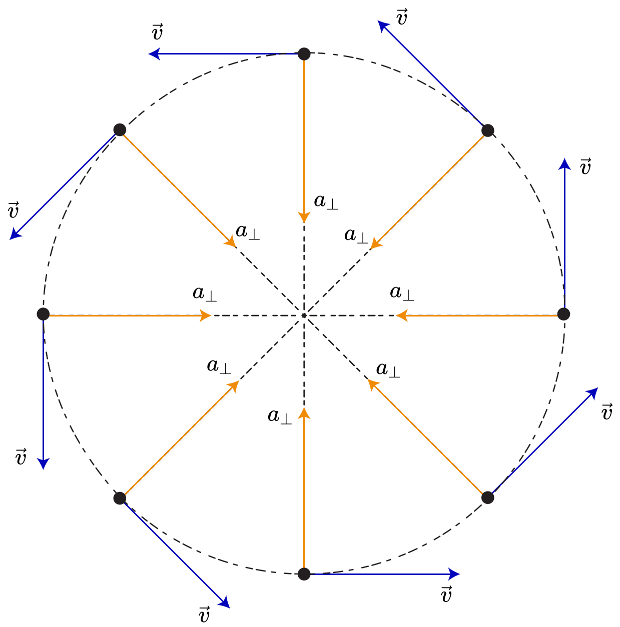

FIGURE 4: A uniform circular motion must be pulled inward with a radial acceleration to maintain a stable orbit.

We shall use the radial and tangential acceleration to keep track of this change in $\Delta\vec{v}$. Remember, $a_\parallel$ points in the direction of $\Delta\vec{v}$, and $a_\perp$, in the direction perpendicular to $\Delta\vec{v}$. Since there is no change in speed $a_\parallel = 0$. But, we should have a net $a_\perp$ since the direction is constantly changing. This acceleration is known as the **centripetal acceleration**.

This acceleration may be evaluated by checking the change in velocity on the circular motion. The expression for this radial acceleration or centripetal acceleration is given by

$$a_\perp = \frac{v^2}{R} = \frac{4\pi^2 R}{T^2}. \tag{5}$$

Remember, this component of the acceleration points radially inward to the center of the circular motion. Hence, the name *radial* acceleration.

2.2 Changing Orbits

If we were to increase the speed, we would have a non-zero $a_\parallel$ in addition to $a_\perp$. This additional $a_\parallel$ will take the object to a wider orbit with a larger radius. Decrease the speed, and the orbit or the radius will shrink.

PHY 111: Principles of Physics I Fall 2023

LECTURE 06 — October 14, 2023

SECTION: 35 (UB71001) FACULTY: Akiful Islam (AZW)

BRAC UNIVERSITY

3 Relative motion

Relative motion refers to the motion of one object with respect to another. In other words, relative motion is the comparison of the motion of one object to that of another.

For example, consider a car moving down the highway. If you are inside the car, you may feel as though you are stationary and the world is moving past you. But if you step outside the car and observe it from a fixed point on the side of the road, you will see the car moving **relative** to you. The same concept applies to two boats moving relative to each other on a lake or two airplanes flying in formation.

Similar to Kinematics, a relative motion has relative displacements, relative velocities, and relative accelerations.

3.1 Relative Displacement

Relative displacement is the change in one object's position relative to another. In other words, it is the distance between the initial and final positions of one object as measured from the position of another object.

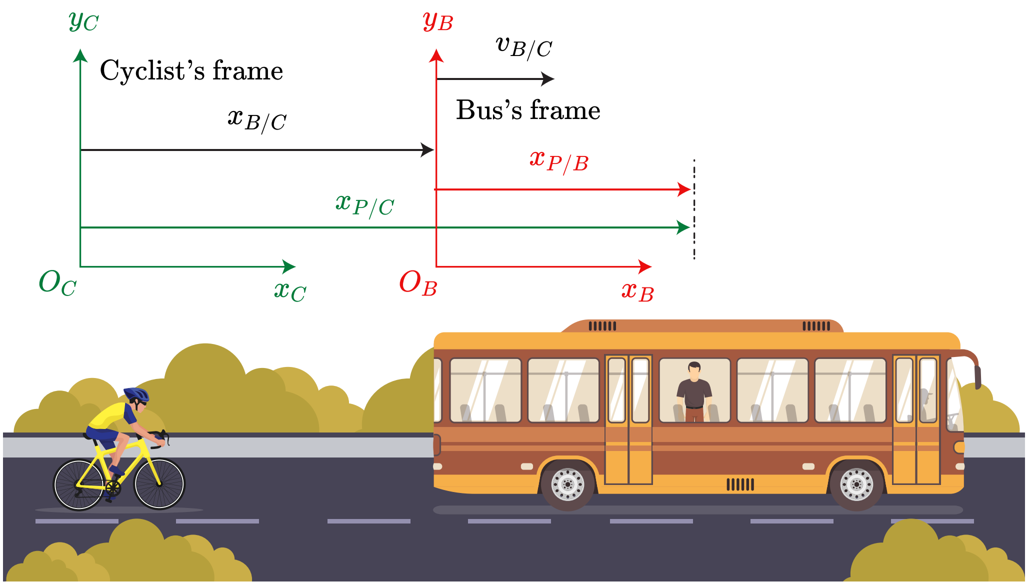

FIGURE 5: Relative motion between an observer and observee. The system is in 1D.

It is a vector quantity, meaning it has both magnitude and direction. The magnitude of the relative displacement is the distance between the initial and final positions of one object as measured from the position of another, and the direction of the relative displacement is the direction of the shortest path between the two positions.

Consider the following system where a cyclist observes a passenger moving onboard a bus. Assuming the bus is moving with a **constant** velocity $v_{B/C}$ away from the cyclist. Thus, the cyclist will experience a relative motion between himself and the passenger onboard the bus.

The position of the passenger relative to the cyclist $x_{P/C}$ will be the sum of the position of the passenger relative to the bus $x_{P/B}$, and the velocity of the bus frame relative to the cyclist $x_{B/C}$.

$$x_{P/C} = x_{P/B} + x_{B/C} \tag{6}$$

This apparent sense of displacement causes relative motion.

3.2 Relative Velocity

Relative velocity is the velocity of an object relative to another object. In other words, it is the velocity of one object with respect to another. The concept of relative velocity is essential in kinematics, where the motion of an object can be affected by the motion of other nearby objects.

We consider the same system mentioned above and differentiate (6) with respect to time, giving us the velocity equation for the system.

$$\frac{dx_{P/C}}{dt} = \frac{dx_{P/B}}{dt} + \frac{dx_{B/C}}{dt}$$

$$\therefore v_{P/C} = v_{P/B} + v_{B/C} \tag{7}$$

Relative velocity is a vector quantity that has both magnitude and direction. The magnitude of the relative velocity is equal to the difference in the magnitudes of the velocities of the two objects. In contrast, the direction of the relative velocity is the direction in which the second object moves relative to the first object.

3.3 Relative Acceleration

For the same system, we get the following acceleration equation by differentiating (7) with respect to time:

$$\frac{dv_{P/C}}{dt} = \frac{dv_{P/B}}{dt} + \frac{dv_{B/C}}{dt}$$

$$\therefore a_{P/C} = a_{B/C} \tag{8}$$

Since the bus is moving away from the cyclist with a *constant* velocity, the acceleration between the two systems is zero. Thus, the cyclist will measure the same acceleration in his frame as the one in the bus's frame. This is a case of constant relative acceleration.

TAKEAWAY: Relative acceleration is not always constant. The relative acceleration between two objects depends on the forces acting on them and can change over time.ij