BRAC UNIVERSITY

Principles of Physics-II (PHY-112)

Department of Mathematics and Natural Sciences

Assignment: 03 — **Section**: 30/31/33

Inspiring Excellence

Duration: 6 Days | **Summer 2024 (10F-31C)** | Marks: 15

Attempt all questions. Show Your work in detail. Use SI units. 1:1 plagiarism will be strictly penalized.

1. (a) The two segments of the wire have equal diameters but different conductivities σ_1 and σ_2. Current I passes through this wire. If the conductivities have the ratio $\frac{\sigma_1}{\sigma_2} = 2$, what is the ratio $\frac{E_2}{E_1}$ of the electric field strengths in the two segments of the wire? **(2)**

Given that the conductivities of the two segments of the wire are σ_1 and σ_2 with the ratio:

$$\frac{\sigma_1}{\sigma_2} = 2,$$

and the current I through the wire is constant, we use the relationship between current density J, conductivity σ, and electric field E:

$$J = \sigma E.$$

Since the current is the same in both segments of the wire, the current density in each segment is also the same:

$$\begin{aligned} J_1 &= J_2 \\ \sigma_1 E_1 &= \sigma_2 E_2 \\ \frac{E_2}{E_1} &= \frac{\sigma_1}{\sigma_2} = 2. \end{aligned}$$

(b) The magnitude J of the current density in a certain lab wire with a circular cross-section of radius $R = 2.00\,\text{mm}$ is given by $J = (3.00 \times 10^8)r^2$, with J in A m^{-2} and radial distance r in m. What is the current through the outer section bounded by $r = 0.900R$ and $r = R$? **(3)**

The differential current dI through an infinitesimal ring of radius r and thickness dr is given by:

$$dI = J \cdot dA = J \cdot (2\pi r\, dr).$$

Substitute $J = (3.00 \times 10^8)r^2$:

$$dI = (3.00 \times 10^8)r^2 \cdot (2\pi r\, dr) = (6.00 \times 10^8 \pi) r^3\, dr.$$

Now, integrate from $r = 0.900R$ to $r = R$:

$$I = \int_{0.900R}^{R} (6.00 \times 10^8 \pi) r^3\, dr.$$

Evaluating the integral:

$$I = (6.00 \times 10^8 \pi) \int_{0.900R}^{R} r^3\, dr = (6.00 \times 10^8 \pi) \left[\frac{r^4}{4}\right]_{0.900R}^{R}.$$

Substitute $R = 2.00 \times 10^{-3}\,\text{m}$:

$$\begin{aligned} I &= (6.00 \times 10^8 \pi) \left[\frac{R^4}{4} - \frac{(0.900R)^4}{4}\right] \\ &= (6.00 \times 10^8 \pi) \cdot \frac{R^4}{4} \left(1 - 0.900^4\right) \\ &= (6.00 \times 10^8 \pi) \cdot \frac{(2.00 \times 10^{-3})^4}{4} \cdot (1 - 0.900^4) \approx 2.21\,\text{mA}. \end{aligned}$$

2. (a) You have been assigned to make a $25\,\Omega$ resistor from a poorly conducting material that has conductivity $50\,\Omega^{-1}\,\mathrm{m}^{-1}$. The resistor will be a cylinder with a length of 5 times its diameter. The current will flow lengthwise through the resistor. What should be its length in cm? **(2)**

The resistance R of a cylindrical resistor is given by:

$$R = \frac{L}{\sigma A},$$

Suppose the diameter of the resistor be d, and the length be $L = 5d$. The cross-sectional area A is:

$$A = \frac{\pi d^2}{4}.$$

Substitute into the expression for R:

$$R = \frac{5d}{\sigma \cdot \frac{\pi d^2}{4}} = \frac{20}{\sigma \pi d}$$
$$d = \frac{20}{\sigma \pi R}$$
$$d = \frac{20}{50 \cdot \pi \cdot 25} = \frac{20}{3926.99}\,\mathrm{m} \approx 5.1 \times 10^{-3}\,\mathrm{m}.$$

Now, the length of the resistor is $L = 5d$:

$$L = 5 \times 5.1 \times 10^{-3}\,\mathrm{m} = 2.55 \times 10^{-2}\,\mathrm{m} = 2.55\,\mathrm{cm}.$$

Thus, the length of the resistor is:

$$L \approx 2.55\,\mathrm{cm}.$$

(b) A current-carrying gold wire has diameter 0.87 mm. The electric field in the wire is $0.54\,\mathrm{V\,m^{-1}}$. What are (a) the current carried by the wire; (b) the potential difference between two points in the wire 7.0 m apart; (c) the resistance of a 7.0 m length of this wire? **(3)**

(a) The current I through the wire with uniform current density J and the cross-sectional area A of the wire by:

$$I = J \cdot A.$$

The current density J is related to the electric field E and the conductivity σ_{gold} by:

$$J = \sigma_{\mathrm{gold}} E.$$

The conductivity of gold is $\sigma_{\mathrm{gold}} = 4.1 \times 10^7\,\Omega^{-1}\,\mathrm{m}^{-1}$. Thus:

$$J = (4.1 \times 10^7) \times (0.54) = 2.21 \times 10^7\,\mathrm{A\,m^{-2}}.$$

The cross-sectional area A of the wire is:

$$A = \frac{\pi d^2}{4} = \frac{\pi (0.87 \times 10^{-3})^2}{4} = 5.95 \times 10^{-7}\,\mathrm{m}^2.$$

Now, calculate the current:

$$I = (2.21 \times 10^7) \times (5.95 \times 10^{-7}) = 13.14\,\mathrm{A}.$$

(b) The potential difference V between two points (in a uniform electric field) in the wire is given by:

$$V = EL,$$

where $L = 7.0\,\mathrm{m}$ is the distance between the points. Substituting the given values:

$$V = 0.54\,\mathrm{V\,m^{-1}} \times 7.0\,\mathrm{m} = 3.78\,\mathrm{V}.$$

Thus, the potential difference is:

$$V = 3.78\,\mathrm{V}.$$

(c) The resistance R of the wire is given by:

$$R = \frac{V}{I}.$$

Substitute the values of V and I:

$$R = \frac{3.78\,\mathrm{V}}{13.14\,\mathrm{A}} = 0.288\,\Omega.$$

Thus, the resistance of the wire is:

$$R = 0.288\,\Omega.$$

3. A 0.40 A current runs through a copper wire of cross-sectional area $1.5\,\mathrm{mm}^2$ and through a light bulb. Copper has 8.5×10^{28} free electrons per cubic meter. (i) How many electrons pass through the light bulb each second? (ii) What is the current density of the wire? (iii) At what speed does a typical electron pass by any given point in the wire? (iv) If you were to use wire with a larger cross-sectional area, which of the above answers would change? Would they increase or decrease? **(5)**

(i) The current I is related to the charge ΔQ passing through the wire per second by:

$$I = \frac{\Delta Q}{\Delta t},$$

where $\Delta t = 1\,\mathrm{s}$.
The number of electrons N passing through the wire per second is:

$$N = \frac{\Delta Q}{q_e} = \frac{I \Delta t}{q_e}.$$

Substitute the given values:

$$N = \frac{0.40\,\mathrm{A} \times 1\,\mathrm{s}}{1.6 \times 10^{-19}\,\mathrm{C}} = 2.5 \times 10^{18}\,\mathrm{electrons/s}.$$

Thus, the number of electrons passing through the light bulb per second is:

$$N = 2.5 \times 10^{18}\,\mathrm{electrons/s}.$$

(ii) The current density J is given by:

$$J = \frac{I}{A}.$$

Substitute the given values:

$$J = \frac{0.40\,\mathrm{A}}{1.5 \times 10^{-6}\,\mathrm{m}^2} = 2.67 \times 10^5\,\mathrm{A\,m^{-2}}.$$

Thus, the current density of the wire is:

$$J = 2.67 \times 10^5\,\mathrm{A\,m^{-2}}.$$

(iii) The drift velocity v_d of electrons in the wire is related to the current density by:

$$J = n_e q_e v_d,$$

where n is the number of free electrons per unit volume, and e is the charge of an electron. Solving for v_d:

$$v_d = \frac{J}{n_e q_e}.$$

Substitute the values:

$$v_d = \frac{2.67 \times 10^5\,\mathrm{A\,m^{-2}}}{(8.5 \times 10^{28}\,\mathrm{electrons/m^3})(1.6 \times 10^{-19}\,\mathrm{C})}.$$

Simplifying:

$$v_d = \frac{2.67 \times 10^5}{1.36 \times 10^{10}} = 1.96 \times 10^{-5}\,\mathrm{m\,s^{-1}} \sim 0.0196\mathrm{mm\,s^{-1}}.$$

(iv) Effect of a Larger Cross-Sectional Area If we were to use a wire with a larger cross-sectional area:

- The number of electrons passing through the light bulb per second N would remain the same because the total current I is unchanged.
- The current density J would **decrease**, because $J = \frac{I}{A}$ and increasing the cross-sectional area A reduces J.
- The drift velocity v_d would also **decrease**, because $v_d = \frac{J}{n_e q_e}$ and J decreases with increasing area.