Assignment - 2

① find the first derivative of $\ln(x^a + x^{-a})$

Let, $y = \ln(x^a + x^{-a})$

$$y' = \frac{1}{x^a + x^{-a}} \cdot (ax^{a-1} - ax^{-a-1})$$

$$y' = \frac{a(x^{a-1} - x^{-a+1})}{x^a + x^{-a}}$$

Ans.

(2) Find the fourth derivative of the function.

$$f(x) = \frac{4x^3+3x}{2x^2}$$

$$f'(x) = \frac{2x^2\frac{d}{dx}(4x^3+3x) - (4x^3+3x)\frac{d}{dx}2x^2}{(2x^2)^2}$$

$$= \frac{2x^2(12x^2+3) - (4x^3+3x)4x}{4x^4}$$

$$= \frac{8x^4-6x^2}{4x^4}$$

$$= \frac{\cancel{4x^2}(2x^2-\frac{6}{4})}{\cancel{4x^2}x^2}$$

$$= \frac{2x^2-\frac{6}{4}}{x^2}$$

$$f''(x) = \frac{2x^2-\frac{6}{4}}{x^2}$$

$$= \frac{x^2\frac{d}{dx}(2x^2-\frac{6}{4}) - (2x^2-\frac{6}{4})\frac{d}{dx}x^2}{(x^2)^2}$$

$$= \frac{x^2(4x) - 4x^3+3x}{x^4}$$

$$= \frac{4x^3-4x^3+3x}{x^4}$$

$$= \frac{3}{x^3}$$

$$f'''(x) = \frac{d}{dx}\frac{3}{x^3} = 3.-3x^{-4} = \frac{-9}{x^4}$$

$$f^4(x) = \frac{-9}{x^4}$$

$$= \frac{d}{dx}\left(\frac{-9}{x^4}\right)$$

$$= \frac{36}{x^5}$$

Ans.

③ find y' if $y = xa^{2x}$

$$y = xa^{2x}$$

$$\ln(y) = \ln(x.a^{2x})$$

$$\ln y = \ln x + \ln a^{2x}$$

$$\ln y = \ln x + 2x.\ln a$$

$$\frac{1}{y}.\frac{dy}{dx} = \frac{d}{dx}(\ln x) + \frac{d}{dx}(2x\ln a)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x} + 2\ln x$$

$$\frac{dy}{dx} = y\left(\frac{1}{x} + 2\ln x\right)$$

$$\frac{dy}{dx} = xa^{2x}\left(\frac{1}{x} + 2\ln x\right)$$

(4) Analyze the differentiability at $x=2$ of the functio

$$f(x) = \begin{cases} x^2 - 4x - 2 & x < 2 \\ -2x^2 + 4x & x \geq 2 \end{cases}$$

$$\frac{d}{dx}(x^2 - 4x - 2) = \frac{d}{dx}(-2x^2 + 4x)$$

$\Rightarrow \quad 2x - 4 = -4x + 4$

$\Rightarrow \quad 2 \times (2) - 4 = -4 \times (2) + 4$ [by putting the value of $x=2$]

$\Rightarrow \quad 0 \neq -4$

The function is not differentiability at $x=2$ point.

Bonus Question.

Quotient rule.

Prove if $h(x) = \frac{f(x)}{g(x)}$ then $h'(x) \; \frac{g(x) f'(x) - f'(x) g(x)}{\{g(x)]^2}$

$$h'(x) = \lim_{h \to 0} \frac{h(x+h) - h(x)}{h}$$

we know that $h(x) = \frac{f(x)}{g(x)}$

$$\therefore h(x+h) = \frac{f(x+h)}{g(x+h)}$$

$$\lim_{h \to 0} \left(\frac{f(x+h)}{g(h+h)} - \frac{f(x)}{g(x)} \right) \frac{1}{h}$$

$$\lim_{h \to 0} \frac{f(x+h) gx - f(x) g(x+h)}{g(x+h) g(x) h}$$

$$\lim_{h \to 0} \frac{f(x+h) g(x) - f(x) g(x) + f(x) g(x) - f(x) g(x+h)}{g(x+h) g(x) h}$$

$$\lim_{h \to 0} \frac{f(x+h) gx - f(x) g(x)}{g(x+h) g(x) h} + \lim_{x \to 0} \frac{f(x) g(x) - f(x) g(x+h)}{g(x+h) g(x) h}$$

$$\lim_{h \to 0} \frac{g(x)}{g(x+h) g(x)} \left(\frac{f(x+h) - f(x)}{h} \right) - \lim_{h \to 0} \frac{f(x) (g(x+h) - g(x))}{g(x+h) g(x) h}$$

$$\lim_{h \to 0} \frac{g(x)}{g(x+h) g(x)} \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} - \lim_{h \to 0} \frac{f(x)}{g(x+h) g(x)} \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$

$$\frac{g(x)}{g(x)g(x)} f'(x) - \frac{f(x)}{g(x)g(x)} g'(x)$$

$$= \frac{f'(x)}{gx} - \frac{f(x)g'(x)}{[g(x)]^2}$$

$$= \frac{f'(x)g(x)}{[g(x)]^2} - \frac{f(x)g'(x)}{[g(x)]^2}$$

$$= \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2}$$

(from first principle.)