**PHY 111: Principles of Physics I** Fall 2023

LECTURE 01 — September 23, 2023

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# 1 Coordinate Axes

The coordinate axes are lines used to define the position of points in space. Typically, a coordinate system is defined by an origin and two or three *mutually perpendicular* (each making 90° with each other) lines called the $x$, $y$, and $z$-axes that intersect at the origin. The point where the axes intersect is called the origin. In physics, we use a coordinate axis to describe the motion of objects in three-dimensional space.

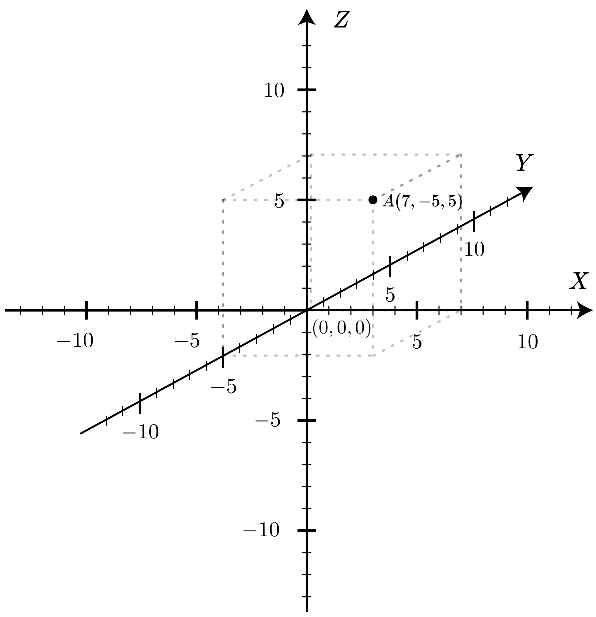



FIGURE 1: Locating a point $A(7, -5, 5)$ in a 3D Cartesian coordinate. Move seven units along the positive $x$-axis, five in the negative $y$-axis, and five along the $z$-axis. The cube is drawn to give a visual location in the grid. Not necessary while locating a coordinate point.

## 1.1 Cartesian Coordinates

Cartesian coordinates $(x, y)$ using a set of two or more number lines, called the $x$-axis and $y$-axis, which are perpendicular to each other and intersect at the origin point. The position of a point in a plane is specified by its distance from the $x$-axis (its $x$-coordinate) and its distance from the $y$-axis (its $y$-coordinate). You can expand it to 3D by adding a $z$-axis perpendicular with both $x$ and $y$-axes.

Cartesian coordinates are helpful when the system in question has planar symmetry.

## 1.2 Polar Coordinates

Polar coordinates $(r, \theta)$ using a single number line, called the **polar axis** (the (**radial coordinate**) $x$-axis), and the distance $r$ (its **radial coordinate**) of a point from the origin (the **pole**) and the angle $\theta$ (its **angular coordinate**) between the polar axis and the line connecting the origin to the point.

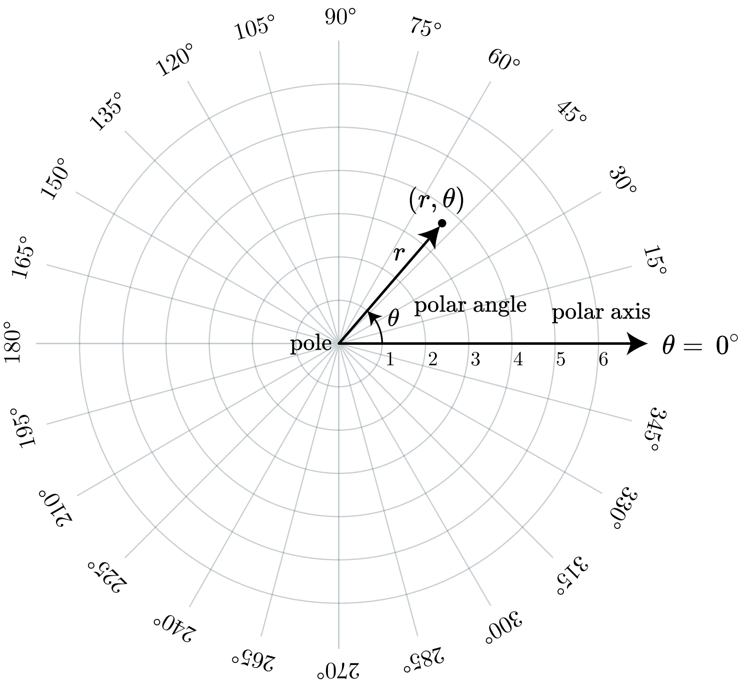



FIGURE 2: Polar coordinate system requires only two coordinates. The radial one $r$ and an angular one $\theta$. In 3D, we will have one more angular parameter to complete the coordinate system, called the *spherical polar coordinate* system.

Polar coordinates are helpful when the system in question has circular symmetry.

## 1.3 Spherical Polar Coordinates

Spherical polar coordinates use a set of three number lines. It describes the position of a point in three-dimensional space using three parameters: radial distance $r$, polar angle $\theta$, and azimuthal angle $\phi$. The radial distance $r$ represents the distance from the origin (typically the center of a

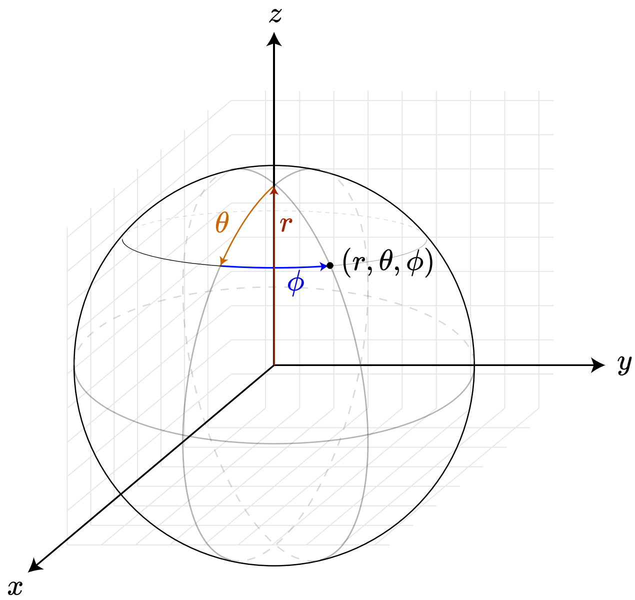



FIGURE 3: Locating a point $(r, \theta, \phi)$ in a Spherical Polar coordinate system. Move $r$ units along the positive $z$-axis, $\theta$ in the $xz$-plane, and $\phi$ along the $xy$-plane. This type of coordinate system is used when the system under scrutiny has a spherical symmetry.

sphere) to the point in question. It is measured along a line segment extending from the origin to the point.

The polar angle $\theta$ is measured from the positive $z$-axis (or the positive $x$-axis in some conventions) to the line segment connecting the origin and the point. It ranges from 0 to $\pi$ (or $0 \leq \theta \leq \pi$), with 0 representing the positive $z$-axis and $\pi$ representing the negative $z$-axis.

The azimuthal angle $\phi$ is measured from the positive $x$-axis to the projection of the line segment connecting the origin and the point onto the $xy$-plane. It ranges from 0 to $2\pi$ (or $0 \leq \phi \leq 2\pi$), completing a full circle.

## 1.4 Cylindrical Coordinates

Cylindrical coordinates are a coordinate system that describes the position of a point in three-dimensional space using three parameters: radial distance $r$, polar angle $\theta$, and height $z$.

The structure of this coordinate system is exactly like the spherical polar one, with one caveat: $\pi$ is replaced with $z$, a length across the $z$-axis.

The cylindrical variables have the following possible ranges:

$$0 \leq r < \infty, \qquad 0 \leq \theta \leq \pi, \qquad -\infty < z < +\infty$$

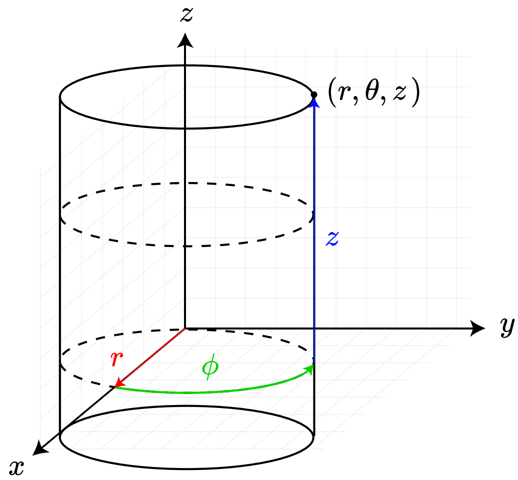



FIGURE 4: Locating a point $(r, \theta, z)$ in a Cylindrical coordinate system. Move $r$ units along the positive $z$-axis, $\theta$ in the $xy$-plane, and $z$ along the $z$-axis. This type of coordinate system is used when the system under scrutiny has a cylindrical symmetry.

## 2 Switching Coordinates

Coordinate transformation allows us to choose a reference system that simplifies our calculation. No one coordinate system shall be used as a rule of thumb. You are free to choose whichever assists you. NOTE: The following convention helps us keep track of the coordinates. You may change the convention at your own risk. It's fine as long as You stick to your pre-defined setup.

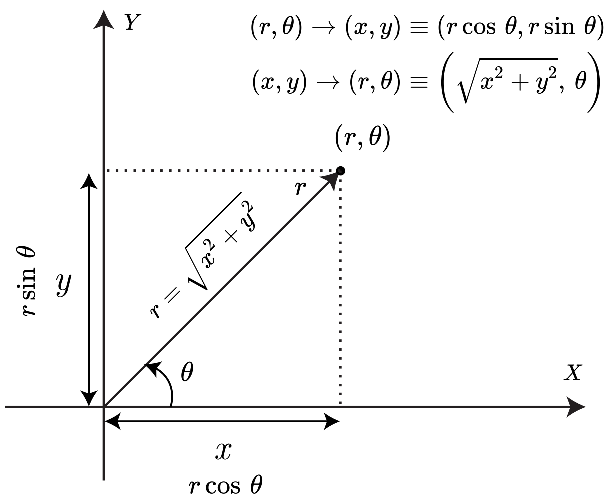



FIGURE 5: Conversion between the Cartesian and Polar coordinate system.

## 2.1 Cartesian to Polar

$$(r,\theta) \rightarrow (x,y) \equiv (r\cos\,\theta, r\sin\,\theta)$$

## 2.2 Polar to Cartesian

$$(x,y) \rightarrow (r,\theta) \equiv \left(\sqrt{x^2+y^2},\, \theta\right)$$

Depending on the sign of the ratio in the tangent, the angular coordinate can have the following formats:

$$\theta = \begin{cases} \tan^{-1}\left|\dfrac{y}{x}\right| & ;\ x>0,\, y>0 \text{ (1}^{\text{st}}\text{ quadrant)} \\ \pi - \tan^{-1}\left|\dfrac{y}{x}\right| & ;\ x<0,\, y>0 \text{ (2}^{\text{nd}}\text{ quadrant)} \\ \pi + \tan^{-1}\left|\dfrac{y}{x}\right| & ;\ x<0,\, y<0 \text{ (3}^{\text{rd}}\text{ quadrant)} \\ 2\pi - \tan^{-1}\left|\dfrac{y}{x}\right| & ;\ x>0,\, y<0 \text{ (4}^{\text{th}}\text{ quadrant)} \end{cases}$$

Other constant angles are given for the following conditions.

$$\theta = \begin{cases} +\dfrac{\pi}{2} & ;\ x=0,\, y>0 \text{ (positive } y\text{-axis)} \\ -\dfrac{\pi}{2} & ;\ x=0,\, y<0 \text{ (negative } y\text{-axis)} \\ 0 & ;\ x>0, y=0 \\ \pi & ;\ x<0, y=0 \\ \text{undefined} & ;\ x=0, y=0 \end{cases}$$

## 2.3 Spherical Polar to Cartesian

$$(r,\theta,\phi) \rightarrow (x,y,z) \equiv (r\sin\theta\cos\phi, r\sin\theta\sin\phi, r\cos\theta)$$

## 2.4 Cylindrical to Cartesian

$$(r,\theta,z) \rightarrow (x,y,z) \equiv (r\cos\theta, r\sin\theta, z)$$

# 3 The Joy of Measuring

## 3.1 Vectors

In physics and mathematics, a vector is a quantity that has **both magnitude and direction**. Examples of vectors include **velocity**, **momentum**, **force**, and **acceleration**. Vectors can be represented graphically as directed line segments and can be added and subtracted using vector algebra. They can also be represented by a set of coordinates in a coordinate system.

### 3.1.1 Magnitude of Vectors

The **magnitude** of a vector is represented by its length, and the angle represents the **direction** it makes with the positive $x$-axis. Vectors can be added and subtracted by adding or subtracting the components in the same coordinate system.

For example, a vector $\vec{A} = \hat{\imath}A_x + \hat{\jmath}A_y + \hat{k}A_z$ has the magnitude given by $|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$.

TAKEAWAY: The magnitude of the vector gets rid of the directional aspect of the vector and only leaves you with a number.

### 3.1.2 Null Vectors

A null vector, also known as a zero vector, is a vector with zero magnitude. In other words, it has no direction or length. Mathematically, a null vector is represented by a vector with all its components equal to zero.

TAKEAWAY: A null vector is an identity element for vector addition. This means that when a null vector is added to any other vector, the result is the original vector.

### 3.1.3 Components of Vectors

The **components** of a vector are the contributions from the vector in a particular direction. For instance, $\vec{A}$ has a component $A_x$ in the $x$-direction and $A_y$ in the $y$-direction.

$$A_x = |\vec{A}| \cos\theta \tag{1}$$
$$A_y = |\vec{A}| \sin\theta \tag{2}$$

TAKEAWAY: *Components are not vectors*. The magnitude of the vector component can only equal the actual vector magnitude when the projection is taken in the same direction as the source vector. Otherwise, the components are always lesser in magnitude.

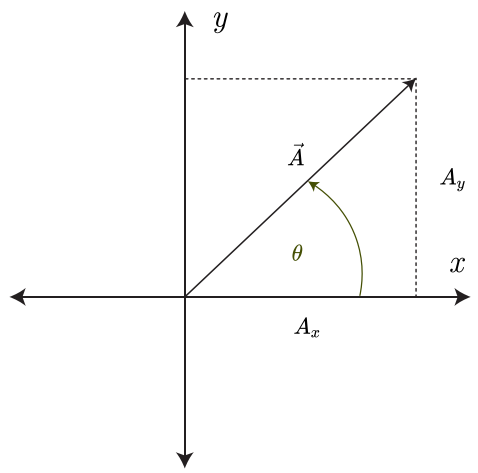



FIGURE 6: The components $A_x$ and $A_y$ are the magnitudes of the main vector in the $x$ and $y$-direction respectively.

### 3.1.4 Unit Vectors

The unit vector is a vector having a magnitude of 1 without a unit.

For example, a vector $\vec{A} = \hat{\imath}A_x + \hat{\jmath}A_y + \hat{k}A_z$ can have a unit vector parallel to its direction in 3D space is given by $\hat{A} = \dfrac{\vec{A}}{|\vec{A}|} = \dfrac{\hat{\imath}A_x + \hat{\jmath}A_y + \hat{k}A_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$.

TAKEAWAY: Unit vectors are usually used to denote where the parent vector is pointing to. The magnitude here is sidelined to have a default unit value. We need not speak of it.

### 3.1.5 Parallel and Anti-parallel Vectors

Parallel vectors (internal angle is 0°) point in the same direction, and the anti-parallel (internal angle is 180°) ones in the opposite direction. Two vectors can only be equal if they share the direction and magnitude. Otherwise, no.

TAKEAWAY: Equal but anti-parallel vectors give you a negative vector corresponding to the other in the pair.

## 3.2 Scalars

Scalars are mathematical quantities with only magnitude and no direction. They are represented by a single number and are often used to represent physical quantities such as temperature, mass, and energy. We don't need any fancy algebra to deal with scalars. Just regular arithmetic would do.

# 4 Vector Algebra

Vector algebra deals with the manipulation and analysis of vectors. It includes the operations of vector **addition**, **subtraction**, scalar **multiplication**, dot (aka scalar) product, and cross (aka vector) product.

## 4.1 Vector Addition and Subtraction

Vector addition is the process of adding two or more vectors together to **form a new vector**. The process of vector addition is similar to the process of adding two or more vectors together. The sum of two or more vectors is known as the **resultant vector**.

Vector **subtraction** is the process of subtracting one vector from another. It is the reverse operation of vector addition.

### 4.1.1 Analytical Approach of Vector Addition

The idea is to resolve all the vectors *to be summed* into their $x$ and $y$-coordinate axes components. Then, manipulate the components to find the resultant vector.

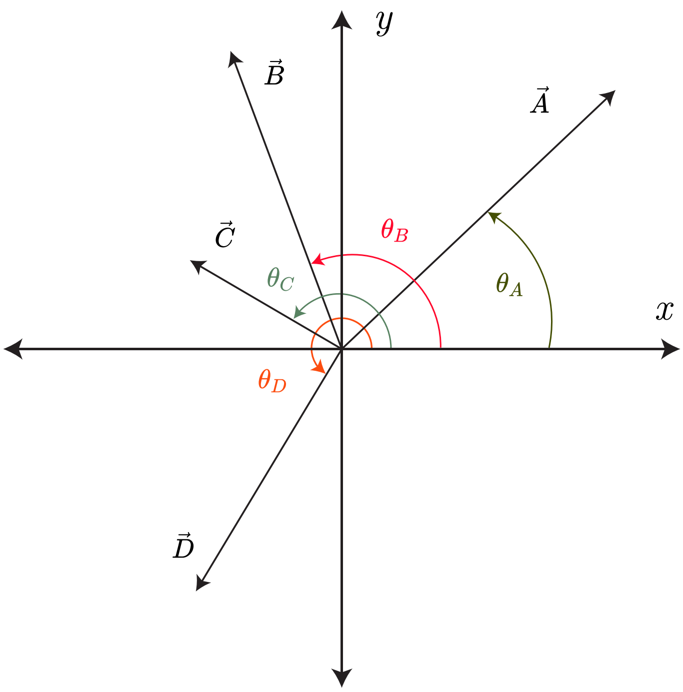



FIGURE 7: A system with four vectors is shown here.

We use the concept of "vector resolution." It means to **resolve** the vector into all possible axis directions it can contribute to. These contributions are the *components* of the main vector.

Assume the following setup. We have four vectors $\vec{A}$, $\vec{B}$, $\vec{C}$, $\vec{D}$ making $\theta_A$, $\theta_B$, $\theta_C$, $\theta_D$ with the $+x$-axis, according to our counter-clockwise convention of angle measurement.

| Vectors | $x$-components | $y$-components |
|---|---|---|
| $\vec{A}$ | $A_x \cos\theta_A$ | $A_y \sin\theta_A$ |
| $\vec{B}$ | $B_x \cos\theta_B$ | $B_y \sin\theta_B$ |
| $\vec{C}$ | $C_x \cos\theta_C$ | $C_y \sin\theta_C$ |
| $\vec{D}$ | $D_x \cos\theta_D$ | $D_y \sin\theta_D$ |

TABLE 1: Resolution of all four vectors into their coordinate components.

The resultant will have its $x$-component as the sum of all vector contributions in the $x$-direction and the $y$-component in the designated $y$-direction.

$$R_x = A_x \cos\theta_A + B_x \cos\theta_B + C_x \cos\theta_C + D_x \cos\theta_D$$
$$R_y = A_y \sin\theta_A + B_y \sin\theta_B + C_y \sin\theta_C + D_y \sin\theta_D$$

The direction of the resultant will be given by

$$\theta = \begin{cases} \tan^{-1}\left|\dfrac{R_y}{R_x}\right| & ;\ R_x > 0,\ R_y > 0 \ (1^{\text{st}} \text{ quadrant}) \\ \pi - \tan^{-1}\left|\dfrac{R_y}{R_x}\right| & ;\ R_x < 0,\ R_y > 0 \ (2^{\text{nd}} \text{ quadrant}) \\ \pi + \tan^{-1}\left|\dfrac{R_y}{R_x}\right| & ;\ R_x < 0,\ R_y < 0 \ (3^{\text{rd}} \text{ quadrant}) \\ 2\pi - \tan^{-1}\left|\dfrac{R_y}{R_x}\right| & ;\ R_x > 0,\ R_y < 0 \ (4^{\text{th}} \text{ quadrant}) \end{cases}$$

Nothing new here. Just the same argument we used for the Polar to Cartesian coordinate transformation.

### 4.1.2 Geometric Approach of Vector Addition

The idea is to draw all the vectors according to the given prompt and geometrically add them using the "**tip-to-tail**" method. The tip of the first vector will be on top of the tail of the second vector, followed by the tip of the second vector on top of the tail of the third vector, and so on and on until all vectors are accounted for.

Then, using the *geometric* vector sum to find the resultant vector.

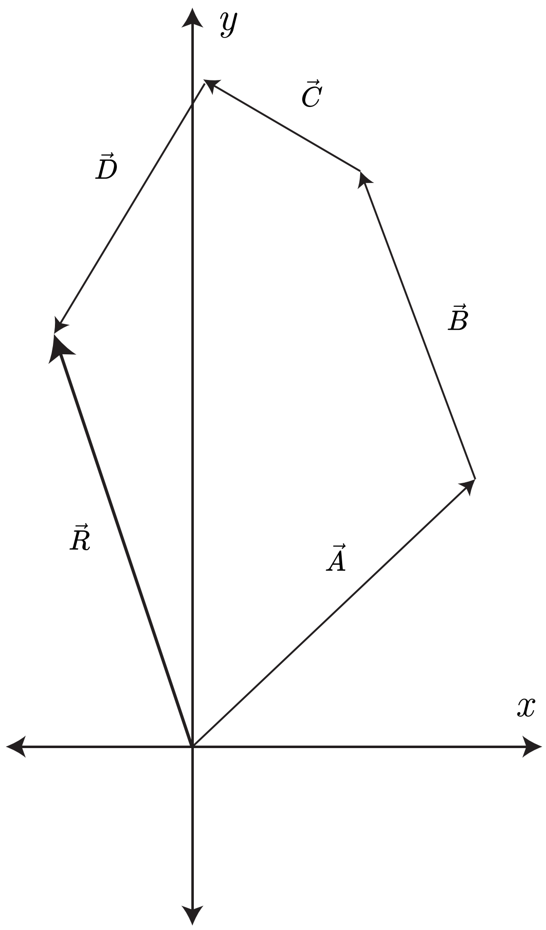



FIGURE 8: A system with four vectors is shown here.

Assume the same setup above. Keep placing the vectors on top of each other using the tip-to-tail method. Finally, add a vector pointing from the tail of the first vector to the tip of the last vector in the series. That is the resultant vector. Although the analytical method of vector addition may not always place the resultant in its actual quadrant, the geometrical method does.

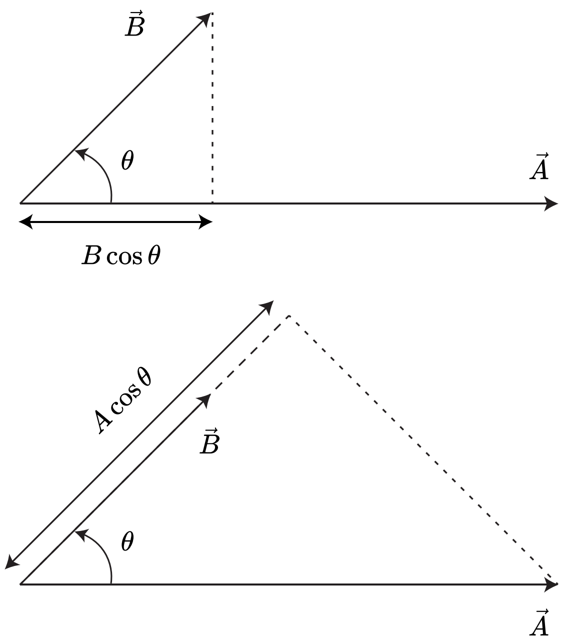
# 5 Vector Multiplication

The last and most crucial part of vector algebra. Unlike scalar multiplication, vectors follow a different concept when it comes to multiplication. We need to understand its physical intuition before we do any math with it.

## 5.1 Scalar (Dot) Multiplication

The dot product is a scalar quantity obtained by multiplying the magnitudes of the two vectors and the cosine of the angle between them. This multiplication takes the **projection** of one vector in a direction **parallel** to another vector.

Take the following vectors $\vec{A}$ and $\vec{B}$, for example. Assume they make an angle $\theta$ with each other. There can be two ways we calculate scalar multiplication.




$$\vec{A} \cdot \vec{B} = AB\cos\theta$$

$$\vec{B} \cdot \vec{A} = BA\cos\theta$$

FIGURE 9: Geometrical definition of scalar multiplication.

### 5.1.1 Analytical

Similar to the vector addition, we first resolve the vectors in the system coordinate axes and then take the products of *like* components and sum all the products.

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z \tag{3}$$

### 5.1.2 Geometrical

Measure the magnitude of the vectors and simply plugin their value in the following formula:

$$\vec{A} \cdot \vec{B} = AB\cos\theta = \vec{B} = BA\cos\theta = \vec{B} \cdot \vec{A} \tag{4}$$

$B\cos\theta$ is the projection of $\vec{B}$ in a direction parallel to $\vec{A}$.

### 5.1.3 Limiting Cases of Scalar Multiplication

1. $\theta = 0°$: The dot product is maximum.
2. $90° > \theta \geq 0°$: The dot product is **positive** in this range.
3. $\theta = 90°$: The dot product is zero.
4. $180° > \theta > 90°$: The dot product is **negative** in this range.
5. $\theta = 180°$: The dot product is maximum in the opposite direction to the $1^{\text{st}}$ case.
6. $270° > \theta > 180°$: The dot product is **negative** in this range.
7. $\theta = 270°$: The dot product is zero.
8. $360° \geq \theta > 270°$: The dot product is **negative** in this range.
9. $\theta = 360°$: The dot product is maximum. Superimposes with the $1^{\text{st}}$ case.

### 5.1.4 What to expect?

1. Two vectors may be given in *vector* notation. Then, usually, you may be asked to calculate the scalar product. You may use the **analytical** method to solve for the answer.
2. Two vectors may be given in *verbal* notation. To calculate the scalar product in this case, you may use the **geometrical** method to solve.
3. Two vectors may be given in either **vector** or **verbal** notation. You may be asked to calculate the angle between them.

$$\theta = \cos^{-1}\left(\frac{\vec{A} \cdot \vec{B}}{AB}\right) = \cos^{-1}\left\{\frac{A_x B_x + A_y B_y + A_z B_z}{\left(\sqrt{A_x^2 + A_y^2 + A_z^2}\right)\left(\sqrt{B_x^2 + B_y^2 + B_z^2}\right)}\right\} \tag{5}$$

TAKEAWAY: Dot multiplication is commutative. That is, $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$.

### 5.2 Vector (Cross) Multiplication

The cross product is a vector quantity that is perpendicular to the plane of the two vectors being multiplied, and its magnitude is equal to the product of the magnitudes of the two vectors and sine of the angle between them. The direction of the cross product is determined by the **right-hand rule**. This multiplication takes the **projection** of one vector in a direction **perpendicular** to another vector.

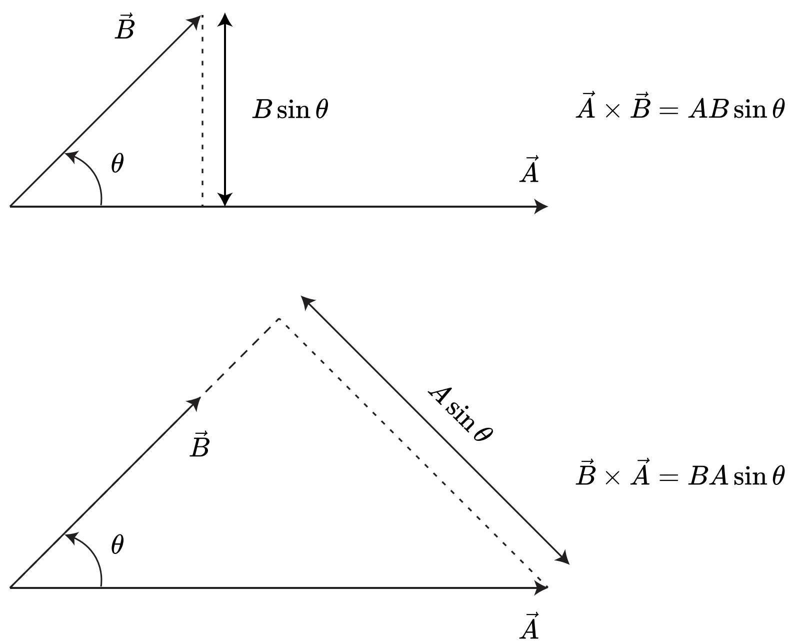



FIGURE 10: Geometrical definition of cross multiplication.

Take the same two vectors $\vec{A}$ and $\vec{B}$ again, still making an angle $\theta$ with each other. There can be two ways we calculate vector multiplication.

#### 5.2.1 Analytical

Similar to the vector addition, we first resolve the vectors in the system coordinate axes and then take the products of all possible combinations of *unlike* components and follow the following format:

$$\vec{A} \times \vec{B} = \left(A_y B_z - A_z B_y\right) \hat{\imath} + \left(A_z B_x - A_x B_z\right) \hat{\jmath} + \left(A_x B_y - A_y B_x\right) \hat{k} \tag{6}$$

$$= \begin{pmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{pmatrix} \tag{7}$$

### 5.2.2 Geometrical

Measure the magnitude of the vectors and simply plugin their value in the following formula:

$$\vec{A} \times \vec{B} = AB \sin\theta \hat{\eta}_{|\vec{A}\times\vec{B}|} \tag{8}$$
$$\vec{B} \times \vec{A} = BA \sin\theta \hat{\eta}_{|\vec{B}\times\vec{A}|} \tag{9}$$
$$\hat{\eta}_{|\vec{A}\times\vec{B}|} = -\hat{\eta}_{|\vec{B}\times\vec{A}|} \tag{10}$$

$B \sin\theta$ is the projection of $\vec{B}$ in a direction perpendicular to $\vec{A}$.

### 5.2.3 Limiting Cases of Vector Multiplication

1. $\theta = 0$: The cross product is zero.
2. $90^\circ \geq \theta > 0^\circ$: The cross product is **positive** in this range.
3. $\theta = 90^\circ$: The cross product is maximum.
4. $180^\circ > \theta > 90^\circ$: The cross product is **positive** in this range.
5. $\theta = 180^\circ$: The cross product is zero and minimum.
6. $270^\circ > \theta > 180^\circ$: The cross product is **negative** in this range.
7. $\theta = 270^\circ$: The cross product is minimum in the opposite direction to the $3^{\text{rd}}$ case.
8. $360^\circ > \theta > 270^\circ$: The cross product is **positive** in this range.
9. $\theta = 360^\circ$: The cross product is zero. Superimposes with the $1^{\text{st}}$ case.

### 5.2.4 What to expect?

1. Two vectors may be given in *vector* notation. Then, usually, you may be asked to calculate the cross-product. You may use the **analytical** method to solve for the answer.
2. Two vectors may be given in *verbal* notation. To calculate the cross product in this case, you may use the **geometrical** method to solve.
3. You can't find the angle between the vectors using vector multiplication. Why not? Let's consider the following, similar to what we did for the scalar product.

$$\theta = \sin^{-1}\left\{\frac{\vec{A}\times\vec{B}}{AB}\right\} \tag{11}$$

   Unlike scalar multiplication, the numerator in the fraction above is a vector that will never return any numerical value for the angle.

We can bypass this just by taking the magnitude of the numerator.

$$\theta = \sin^{-1}\left\{\frac{\left|\vec{A}\times\vec{B}\right|}{AB}\right\} \tag{12}$$

TAKEAWAY: Cross multiplication is anti-commutative. That is, $\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$.

### 5.2.5 Rule of Thumb to Cross Multiplication

Unless mentioned otherwise or explicitly, always **follow the right-handed system** when doing maths with cross products.

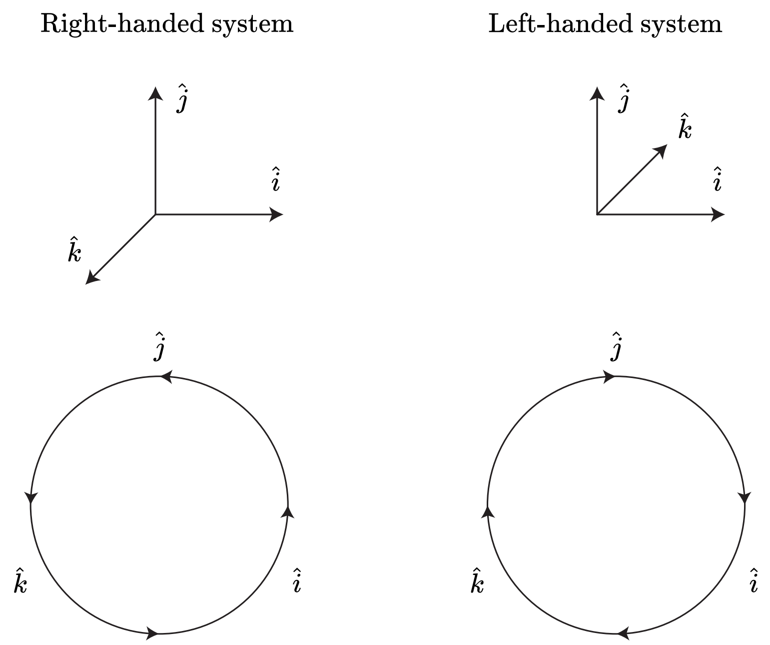



FIGURE 11: The coordinate axes unit vectors follow the cyclic relation in 3D space. The Left-handed system is just the mirror opposite of the Right-handed system. The reflection symmetry lies in the $x$-$y$ plane.

## 6 Vector Division

Vector division is impractical. Because, unlike the mathematical operations of addition and subtraction, which can be applied to vectors with clear geometric interpretations, the operation of division does not have a precise geometric interpretation for vectors.

However, if we divide a vector with a positive scalar, the vector gets down-scaled, more like shrinks, while keeping the same direction. Do the same with a negative scalar; the vector still shrinks but in the opposite direction.