1

#10

Pair of straight line

The condition for which the general equation of second degree represents pair of straight lines, angle between the pair of straight lines, point of intersector, equation of the bisector of angle between the pair of straight line :

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

It will be straight line if,

① $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

Angle between straight line

$$\theta = \tan^{-1} \frac{2\sqrt{h^2 - ab}}{a+b}$$

point of intersector

$$f(x, y) = ax^2 + \cancel{2b}\, 2hxy + by^2 + 2gx + 2fy + c = 0$$

$$\frac{\partial f}{\partial y} \;\&\; \frac{\partial f}{\partial x} \longrightarrow (x_1, y_1)$$

Bisector of the angle

$$\frac{(x - x_1)^2 - (y - y_1)^2}{a - b} = \frac{(x - x_1)(y - y_1)}{h}$$

Another method of point of intersection

$$(x_1, y_1) = \left(\frac{hf - bg}{ab - h^2}, \frac{gh - af}{ab - h^2}\right)$$

① $2y^2 - xy - x^2 + y + 2x - 1 = 0$

$-x^2 + 2y^2 - xy + 2x + y - 1 = 0$

$\cancel{\Delta \equiv}$ $a = -1;\ b = 2;\ c = -1;\ f = \frac{1}{2};\ g = 1;\ h = \frac{-1}{2}$

$\therefore \Delta \equiv abc + 2fgh - af^2 - bg^2 - ch^2$

$\equiv (-1 \cdot 2 \cdot -1) + \left(1 \cdot 1 \cdot \frac{-1}{2}\right) - \left(-1 \cdot \frac{1}{4}\right) - (2 \cdot 1) - \left(-1 \cdot \frac{1}{4}\right)$

$= 2 - \frac{1}{2} + \frac{1}{4} - 2 + \frac{1}{4}$

$= \frac{8 - 2 + 1 - 8 + 1}{4} = \frac{0}{4} = 0$

[Proved]

Again, $z(x, y) = 2y^2 - xy - x^2 + y + 2x - 1$

$\frac{\partial z}{\partial x}(x, y) = -y - 2x + 2$

$\frac{\partial z}{\partial y}(x, y) = 4y - x + 1$

Now, $y + 2x - 2 = 0$ ——— (i)

$4y - x + 1 = 0$ ——— (ii)

$(x_1, y_1) \equiv (1, 0)$

$\underline{\text{Angle}}$: $\theta = \tan^{-1} \dfrac{2\sqrt{h^2 - ab}}{a+b}$

$$= \tan^{-1} \frac{2\sqrt{\left(\frac{-1}{2}\right)^2 - (-1 \cdot 2)}}{-1+2}$$

$$= 71.56^\circ$$

(ii) $2x^2 - 2xy + x + 2y - 3 = 0$

$a = 2;\ b = 0;\ c = -3;\ f = 1;\ g = \frac{1}{2};\ h = -1$

$\Delta \equiv abc + 2fgh - af^2 - bg^2 - ch^2$

$\equiv (2 \cdot 0 \cdot -3) + \left\{2 \cdot \frac{1}{2} \cdot (-1)\right\} - (2 \cdot 1) - (0 \cdot \frac{1}{4}) - (-3 \cdot 1)$

$= 0 - 1 - 2 - 0 + 3 = 0$

Again, $z(x, y) = 2x^2 - 2xy + x + 2y - 3$

$$\frac{\partial z}{\partial x} = 4x - 2y + 1$$

$$\frac{\partial z}{\partial y} = -2x + 2$$

$(x_1, y_1) = \left(1, \frac{5}{2}\right)$

$$\theta = \tan^{-1} \frac{2\sqrt{h^2 - ab}}{a+b} = \tan^{-1} \frac{2\sqrt{1 - 0}}{2 + 0}$$

$$= 45^\circ$$

(III)

$$x^2 + 3xy + 2y^2 + \frac{1}{8}x - \frac{1}{32} = 0$$

$$32x^2 + 96xy + 64y^2 + 4x - 1 = 0$$

$a = 32$; $b = 64$; $c = -1$; $f = 0$; $g = 2$; $h = 48$

$$D \equiv abc + 2fgh - af^2 - bg^2 - ch^2$$

$$\equiv (32 \cdot 64 \cdot -1) + (2 \cdot 0 \cdot 2 \cdot 48) - (32 \cdot 0) - (64 \cdot 4) - (-1 \cdot 48^2)$$

$$= -2048 - 256 + 2304 = -2304 + 2304 = 0$$

$$z = \cancel{x^2 + 3xy}\ 32x^2 + 96xy + 64y^2 + 4x - 1$$

$$\frac{\partial z}{\partial x} = 64x + 96y + 4$$

$$\frac{\partial z}{\partial y} = 96x + 128y$$

$$64x + 96y + 4 = 0$$

$$96x + 128y = 0$$

$$(x_1, y_1) \equiv \left(\frac{1}{2}, \frac{3}{8}\right)$$

$$\theta = \tan^{-1} \frac{2\sqrt{h^2 - ab}}{a+b}$$

$$= \tan^{-1} \frac{2\sqrt{48^2 - (32 \times 64)}}{32 + 64}$$

$$= 18.43^\circ$$

(iv)

$$21x^2 + 40xy - 21y^2 + 44x + 122y - 17 = 0$$

$$a = 21 ;\ b = -21 ;\ c = -17 ;\ g = 22 ;\ f = 61 ;\ h = 20$$

$$D \equiv (21 \cdot -21 \cdot -17) + (2 \cdot 61 \cdot 22 \cdot 20) - (21 \cdot 61^2) - (-21 \cdot 22^2)$$
$$- (-17 \cdot 20^2)$$

$$= 7497 + \overset{53680}{\cancel{56320}} - \overset{78141}{\cancel{86016}} + 10164 + 6800$$

$$= 0$$

Now,

$$z = 21x^2 + 40xy - 21y^2 + 44x + 122y - 17$$

$$\frac{\partial z}{\partial x} = 42x + 40y + 44$$

$$\frac{\partial z}{\partial y} = 40x - 42y + 122$$

$$42x + 40y + 44 = 0$$

$$40x - 42y + 122 = 0$$

$$(x_1, y_1) \equiv (-2, 1)$$

$$\cancel{\tan}\ \theta = \tan^{-1}\left(\frac{2\sqrt{h^2 - ab}}{a+b}\right)$$

$$= \tan^{-1}\left(\frac{2\sqrt{400 + 441}}{21 - 21}\right)$$

$$= \tan^{-1}(\infty) = \frac{\pi}{2} = 90^\circ \quad \underline{\underline{\text{Ans.}}}$$

(2)

(1) $2\lambda xy - y^2 + 4x + 2y + 8 = 0$

$a = 0$; $b = -1$; $g = 2$; $f = 1$; $c = 8$; $h = \lambda$

Now,

$$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

$$2 \cdot 1 \cdot 2 \cdot \lambda - (-1 \cdot 4) - (8 \cdot \lambda^2) = 0$$

$$4\lambda + 4 - 8\lambda^2 = 0$$

$$\lambda + 1 - 2\lambda^2 = 0$$

$$\cancel{\lambda - \lambda^2 - \lambda + 1 = 0}$$

$$2\lambda^2 - \lambda - 1 = 0$$

$$2\lambda^2 - 2\lambda + \lambda - 1 = 0$$

$$2\lambda(\lambda - 1) + 1(\lambda - 1) = 0$$

$$(\lambda - 1)(2\lambda + 1) = 0$$

$$\boxed{\lambda = 1 ; \frac{-1}{2}}$$

Ans.

(11)

$$2x^2 + xy - y^2 - 2x - 5y + K = 0$$

$a = 2$; $b = -1$; $f = \frac{-5}{2}$; $g = -1$; $c = K$; $h = \frac{1}{2}$

$$\Delta \equiv abc + 2fgh - af^2 - bg^2 - ch^2$$

$$\equiv 2 \cdot (-1) \cdot K + 2 \cdot \frac{-5}{2} \cdot (-1) \cdot \frac{1}{2} - \left(2 \cdot \frac{25}{4}\right)$$

$$- \left(-1 \cdot \frac{1}{1}\right) - \left(K \cdot \frac{1}{4}\right)$$

$$\equiv -2K + \frac{5}{2} - \frac{25}{2} + 1 - \frac{K}{4}$$

$$\equiv \frac{-9K}{4} - 9$$

$$\therefore \frac{-9K}{4} = 9$$

$$-9K = 9 \times 4$$

$$\boxed{K = -4}$$

Ans.

(iii)

$$x^2 - \lambda xy + 2y^2 + 3x - 5y + 2 = 0$$

$$a = 1;\ b = 2;\ h = \frac{-\lambda}{2};\ g = \frac{3}{2};\ f = \frac{-5}{2};\ c = 2$$

$$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

$$(1\cdot 2\cdot 2) + \left(-5\cdot \frac{3}{2}\cdot \frac{-\lambda}{2}\right) - \left(1\cdot \frac{25}{4}\right) - \left(2\cdot \frac{9}{4}\right) - \left(2\cdot \frac{\lambda^2}{4}\right) = 0$$

$$4 + \frac{15\lambda}{4} - \frac{25}{4} - \frac{9}{2} - \frac{\lambda^2}{2} = 0$$

$$\cancel{\frac{3\lambda}{2} - \frac{\lambda^2}{2} - \frac{27}{4} = 0}$$

$$\cancel{6\lambda - 2\lambda^2 - 27 = 0}$$

$$\frac{15\lambda}{2} - \frac{\lambda^2}{2} - \frac{27}{4} = 0$$

$$30\lambda - 2\lambda^2 - 27 = 0$$

$$2\lambda^2 - 30\lambda + 27 = 0$$

$$16 + 15\lambda - 25 - 18 - 2\lambda^2 = 0$$

$$-27 - 2\lambda^2 + 15\lambda = 0$$

$$\lambda = 3;\ \frac{9}{2}$$ **Ans**

(iv)

$$12x^2 - 10xy + 2y^2 + \underset{g}{\underline{11x}} - \underset{f}{\underline{5y}} + \lambda = 0$$

$$(12 \cdot 2 \cdot \lambda) + \left(-5 \cdot \frac{11}{2} \cdot \cancel{-5}\right) - \left(12 \cdot \frac{25}{4}\right)$$

$$- \left(2 \cdot \frac{121}{4}\right) - (\lambda \cdot 25) = 0$$

$$24\lambda + \frac{275}{2} - 75 - \frac{121}{2} - 25\lambda = 0$$

$$-\lambda + 2 = 0$$

$$\underline{\lambda = 2}$$

3(i)

$$x^2+xy-6y^2-x-8y-2=0$$

$$\frac{\partial z}{\partial y} = x-12y-8$$

$$\frac{\partial z}{\partial x} = 2x+y-1$$

$$\left(\frac{4}{5}, \frac{-3}{5}\right)$$

$$\frac{\left(x-\frac{4}{5}\right)^2-\left(y+\frac{3}{5}\right)^2}{7} = \frac{\left(x-\frac{4}{5}\right)\left(y+\frac{3}{5}\right)}{\frac{1}{2}}$$

$$\frac{x^2-\frac{8x}{5}+\frac{16}{25}-y^2-\frac{6y}{5}-\frac{9}{25}}{7} = 2\left(xy+\frac{3x}{5}-\frac{4y}{5}-\frac{12}{25}\right)$$

$$x^2-y^2-\frac{8x}{5}-\frac{6y}{5}+\frac{7}{25} = 14xy+\frac{42x}{5}-\frac{56y}{5}-\frac{168}{25}$$

$$25x^2-25y^2-40x-30y+7 = 350xy+210x-280y-168$$

$$25x^2-25y^2-250x+250y-350xy+175=0$$

$$\boxed{x^2-y^2-10x+10y-14xy+7=0}$$

Ans.

(11) $8x^2 - 14xy + 6y^2 + 2x - y - 1 = 0$

$$\frac{\partial z}{\partial y} = -14x + 12y - 1$$

$$\frac{\partial z}{\partial x} = 16x - 14y + 2$$

$$(x, y) = \left(\frac{5}{2}, 3\right)$$

$$\frac{\left(x - \frac{5}{2}\right)^2 - (y-3)^2}{8-6} = \frac{\left(x - \frac{5}{2}\right)(y-3)}{-7}$$

$$-7\left(x^2 - 5x + \frac{25}{4} - y^2 + 6y - 9\right) = 2\left(xy - 3x - \frac{5y}{2} + \frac{15}{2}\right)$$

$$-28x^2 + 140x - 175 + 28y^2 - 168y + 252 = 8xy - 24x - 20y + 60$$

$$137 \quad \cancel{135}$$

$$\cancel{28x^2 - 28y^2 - 116x + 188y - 8xy - 137 = 0}$$

$$-28x^2 + 28y^2 + 164x - 148y - 8xy + 17 = 0$$

(iii)

$$2x^2 + xy - y^2 - 3x + 6y - 9 = 0$$

$$\frac{\partial S}{\partial x} = 4x + y - 3$$

$$\frac{\partial S}{\partial y} = x - 2y + 6$$

$$(x_1, y_1) \equiv (0, 3)$$

$$\frac{(x-0)^2 - (y-3)^2}{2+1} = \frac{(x-0)(y-3)}{\frac{1}{2}}$$

$$\frac{x^2 - (y^2 - 6y + 9)}{3} = 2x(y-3)$$

$$x^2 - y^2 + 6y - 9 = 6xy - 18x$$

$$x^2 - y^2 - 6xy + 6y - 9 + 18x = 0$$

$$2x^2+7xy+6y^2+13x+22y+20=0$$

$$\frac{\partial S}{\partial x} = 4x+7y+13$$

$$\frac{\partial S}{\partial y} = 7x+12y+22$$

$$(x,y) \equiv (2,-3)$$

$$\frac{(x-2)^2+(y+3)^2}{-4} = \frac{(x-2)(y+3)}{\frac{7}{2}}$$

$$\frac{7}{2}(x^2-4x+4+y^2+6y+9) = -4(xy+3x-2y-6)$$

$$\frac{7}{2}(x^2+y^2-4x+6y+13) = -4xy-12x+8y+24$$

$$7x^2+7y^2-28x+42y+91 = -14xy-12x+8y+24$$

$$7x^2+7y^2-16x+34y+14xy+67=0$$

$$14(x^2-4x+4+y^2+6y+9) = 16(xy+3x-2y-6)$$

$$7(x^2-4x+y^2+6y+13) = 8(xy+3x-2y-6)$$

$$7x^2-28x+7y^2+42y+91-8xy-24x+16y+48=0$$

$$7x^2+7y^2-52x+58y-8xy+139=0$$

Circle

1 (i) $\boxed{(-2, -1) \ \& \ 4}$

$$(x+2)^2 + (y+1)^2 = 16$$

$$x^2 + 4x + 4 + y^2 + 2y + 1 - 16 = 0$$

$$x^2 + y^2 + 4x + 2y - 11 = 0$$

(ii) $\boxed{(9, 0) \ r = 1}$

$$(x-9)^2 + y^2 - 1 = 0$$

$$x^2 - 18x + 81 + y^2 - 1 = 0$$

$$x^2 + y^2 - 18x + 80 = 0$$

(iii) $x^2 + y^2 - 25 = 0$

2 (i) $5x^2 + 5y^2 - 11x - 9y - 12 = 0$

$$5\left(x^2 + y^2 - \frac{11}{5}x - \frac{9}{5}y\right) - 12 = 0 \qquad \frac{12}{5} + \frac{121}{100} + \frac{81}{100}$$

$$x^2 - 2 \cdot \frac{11}{5 \cdot 2} \cdot x + \frac{121}{100} + y^2 - \frac{9}{2 \cdot 5} \cdot y \cdot 2 + \frac{81}{100} = \frac{12}{5} + \frac{121}{25} + \frac{81}{25}$$

$$\left(x - \frac{11}{10}\right)^2 + \left(y - \frac{9}{10}\right)^2 =$$

② (i) $5x^2+5y^2-11x-9y-12=0$

$$\cancel{g=\frac{-11}{2} \qquad f=\frac{9}{2}}$$

$$\cancel{r=\sqrt{\frac{121}{4}+\frac{81}{4}+12}=r}$$

$$x^2+y^2-\frac{11}{5}x-\frac{9}{5}y-\frac{12}{5}=0$$

$$g=\frac{11}{10} \qquad f=\frac{9}{10}$$

$$r=\sqrt{\frac{121}{100}+\frac{81}{100}+\frac{12}{5}}=\frac{\sqrt{173}}{5}$$

(ii) $x^2+y^2+2x+2y+1=0$

$$(g,f)=(-1,-1) \qquad r=\sqrt{1+1-1}=\sqrt{2}$$

(iii) $x^2+y^2+2x-4y-8=0$

$$(g,f)=(-1,2) \qquad r=\sqrt{1+4+8}=\sqrt{13}$$

③ (i) (1, 3) (2, -1) (-1, 1)

$x^2+y^2+2gx+2fy+c=0$

$\boxed{(1,3)}$

$1+9+2g+6f+c=0$

$\boxed{2g+6f+c+10=0}$

$\boxed{2,-1}$

$4+1+4g-2f+c=0$

$4g-2f+c+5=0$

$\boxed{-1,1}$

$1+1-2g+2f+c=0$

$-2g+2f+c+2=0$

$\cancel{g=-2 \quad f=\frac{-9}{4} \quad c=\frac{-3}{2}}$

$\cancel{x^2+y^2-4x-\frac{9y}{2}-\frac{3}{2}=0}$

$\cancel{2x^2+2y^2-8x-9y-3=0}$

$g=\frac{-11}{10} \quad y=\frac{-9}{10} \quad c=\frac{-12}{5}$

$x^2+y^2-\frac{11x}{5}-\frac{9y}{5}-\frac{12}{5}=0$

$5x^2+5y^2-11x-9y-12=0$

(11) $(-4,-3)$ $(-1,7)$ $(0,0)$

$x^2+y^2+2gx+2fy+c=0$

$\boxed{(-4,-3)}$

$16+9-8g-6f+c=0$

$\boxed{-8g-6f+c+25=0}$

$\boxed{(-1,7)}$

$1+49-2g+14f+c=0$

$\boxed{-2g+14f+c+50=0}$

$\boxed{(0,0)}$

$c=0$; $g=\frac{325}{62}$; $f=\frac{175}{62}$

$\therefore x^2+y^2+\frac{325x}{31}-\frac{175y}{31}=0$

$31x^2+31y^2+325x-175y=0$

3 | m

(3,1) (4,-3) (1,-1)

$x^2+y^2+2gx+2fy+c=0$

$\boxed{3,1}$ $9+1+6g+2f+c=0$

$$\boxed{6g+2f+c+10=0}$$

$\boxed{4,-3}$ $16+9+8g-6f+c=0$

$$\boxed{8g-6f+25+c=0}$$

$\boxed{1,-1}$ $1+1+2g-2f+c=0$

$$\boxed{2g-2f+2+c=0}$$

$g=\frac{-31}{10}$ $f=\frac{11}{10}$; $\frac{32}{5}$

$$x^2+y^2-\frac{31x}{5}+\frac{11y}{5}+\frac{32}{5}=0$$

$$5x^2+5y^2-31x+11y+32=0$$