Home work sheet #2

Limit

① $\lim_{x \to 0} \frac{x}{\sqrt{x+1} - 1}$

$= \lim_{x \to 0} \frac{x(\sqrt{x+1} + 1)}{(\sqrt{x+1} - 1)(\sqrt{x+1} + 1)}$

$= \lim_{x \to 0} \frac{x(\sqrt{x+1} + 1)}{x+1 - 1}$

$= \lim_{x \to 0} (\sqrt{x+1} + 1)$

$= \sqrt{0+1} + 1$

$= 2$

② $\lim_{x \to 2} \frac{2x^2 - 5x + 2}{5x^2 - 7x - 6}$

$= \lim_{x \to 2} \frac{(x-2)(2x-1)}{(x-2)(x+3)}$

$= \lim_{x \to 2} \frac{2x-1}{x+3}$

$= \frac{4-1}{2+3} = \frac{3}{5}$ Ans.

③ $f(x) = \begin{cases} x^2+1, & x>0 \\ 1, & x=0 \\ 1+x, & x<0 \end{cases}$

$\lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0^-} 1+x = 1+0 = 1$

Again,

$\lim\limits_{x \to 0^+} f(x) = \lim\limits_{x \to 0^+} x^2+1 = 0+1 = 1$

$\therefore \lim\limits_{x \to 0} f(x) = 1$ Ans.

④ $f(x) = \begin{cases} 3x-1, & x<1 \\ 3-x, & x>1 \end{cases}$

$\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 0^-} (3x-1) = \overset{(3-1) = 2}{\cancel{(1-1) = 1\,0}}$

$\lim\limits_{x \to 1^+} f(x) = \lim\limits_{x \to 1^+} (3-x) = (3-1) = 2$

Ans: 2

(5)

$$\lim_{x \to 0} \frac{x}{|x|}$$

for,

$x < 0$, $\frac{x}{|x|} = \frac{-x}{x} = -1$

$x > 0$, $\frac{x}{|x|} = \frac{x}{x} = 1$

Thus,

$$\lim_{x \to 0^+} \frac{x}{|x|} = 1$$

and,

$$\lim_{x \to 0^-} \frac{x}{|x|} = -1$$

As, $\lim_{x \to 0^+} \frac{x}{|x|} \neq \lim_{x \to 0^-} \frac{x}{|x|}$.

$\therefore$ The limit does not exist.

(6)

$$\lim_{x \to \infty} \frac{3x+5}{6x-8}$$

$$= \lim_{x \to \infty} \frac{3 + \frac{5}{x}}{6 - \frac{8}{x}}$$

$$= \lim_{x \to \infty} \frac{3+0}{6-0} = \frac{1}{2}$$ **Ans.**

(7) $f(x) = \begin{cases} 2-x, & x<1 \\ x^2+1, & x>1 \end{cases}$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} x^2+1 = 1+1 = 2$$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} \cancel{x^2+1}\ 2-x = 2-1 = 1$$

$$\because \lim_{x \to 1^+} f(x) \neq \lim_{x \to 1^-}$$

$\therefore$ The limit does not exist.

⑧ $f(x) = \begin{cases} e^{\frac{-|x|}{2}} & , -1 < x < 0 \\ x^2 & , 0 < x < 2 \end{cases}$

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} x^2 = 0^2 = 0$

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} e^{\frac{-x}{2}} = e^{\frac{-0}{2}} = 1$

$\because \lim_{x \to 0^+} f(x) \neq \lim_{x \to 0^-} f(x)$

$\therefore$ limit does not exist.

⑨ $f(x) = \begin{cases} 1/x + 2 & , x < -2 \\ x^2 - 5 & , -2 < x < 3 \\ \sqrt{x+13} & , x > 3 \end{cases}$

$\lim_{x \to -2^+} f(x) = (-2)^2 - 5 = 4 - 5 = -1$

$\lim_{x \to -2^-} f(x) = \lim_{x \to -2^-} \frac{1}{x} + 2 = \frac{-1}{2} + 2 = -1.5$

$\therefore$ limit does not exist.

again,

$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} \sqrt{x+13} = \sqrt{16} = 4$

$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} x^2 - 5 = 9 - 5 = 4$

$\therefore \lim_{x \to 3} f(x) = 4$ Ans.

(10) $f(x) = \begin{cases} x^2 & , x<1 \\ 2.4 & , x=1 \\ x^2+1 & , x>1 \end{cases}$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} x^2 = 1^2 = 1$$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (x^2+1) = 1+1 = 2$$

$$\therefore \lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x),$$

$\therefore$ limit, does not exist.

(11) $$\lim_{x\to\infty} \sqrt{x^6+5x^3} - x^3)$$

$$= \lim_{x\to\infty} \frac{(\sqrt{x^6+5x^3}-x^3)(\sqrt{x^6+5x^3}+x^3)}{\sqrt{x^6+5x^3}+x^3}$$

$$= \lim_{x\to\infty} \frac{x^6+5x^3-x^6}{\sqrt{x^6(1+\frac{5}{x^2})}+x^3}$$

$$= \lim_{x\to\infty} \frac{5x^3}{x^{6/2}\sqrt{1+\frac{5}{x^2}}+x^3}$$

$$= \lim_{x\to\infty} \frac{5}{\sqrt{1+\frac{5}{x^2}}+1}$$

$$= \lim_{x\to\infty} \frac{5}{\sqrt{1+0}+1} = \frac{5}{1+1} = \frac{5}{2}$$

Ans.

$$y = \lim_{x \to \infty} \left\{1 + \frac{1}{x}\right\}^x \quad \text{——— ①}$$

$$\ln y = \lim_{x \to \infty} \ln \left(1 + \frac{1}{x}\right)^x$$

$$\ln y = \lim_{x \to \infty} x \ln \left(1 + \frac{1}{x}\right)$$

$$= \lim_{x \to \infty} \frac{\ln \left(1 + \frac{1}{x}\right)}{x^{-1}}$$

If I use $x = \infty$, the format will be undefined.

$\therefore$ I must us Li hospital's rule.

$$\ln y = \lim_{x \to \infty} \frac{\frac{1}{1+1/x} \cdot (0 - x^{-2})}{-x^{-2}}$$

$$= \lim_{x \to \infty} \frac{\frac{-x^{-2}}{1 + \frac{1}{x}}}{-x^{-2}}$$

$$= \lim_{x \to \infty} \frac{\cancel{-x^{-2}}}{1 + \frac{1}{x}} \times \frac{1}{\cancel{-x^{-2}}}$$

Hints:
$\ln x = a$
$x = e^a$

$$= \lim_{x \to \infty} \frac{1}{1 + \frac{1}{x}}$$

$$= \frac{1}{1 + \frac{1}{\infty}} = \frac{1}{1+0} = 1$$

$\therefore \ln y = 1 \longrightarrow y = e^1 = e$

$\therefore$ ~~from ① to equation,~~ ~~$\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x$~~ $= e$

from 1 No. equation,

$$\lim_{x \to \infty} \left(1+\frac{1}{x}\right)^{x} = e$$

(13)

$$\lim_{x \to \infty} \sqrt[3]{\frac{3x+5}{6x-8}}$$

$$= \lim_{x \to \infty} \sqrt[3]{\frac{3+\frac{5}{x}}{6-\frac{8}{x}}}$$

$$= \cancel{\lim_{x \to \infty}} \sqrt[3]{\frac{3}{6}}$$

$$= \sqrt[3]{\frac{1}{2}}$$

$$= \frac{1}{\sqrt[3]{2}}$$ Ans.

(14)

$$\lim_{x \to \infty} \frac{4x^2 - x}{2x^3 - 5}$$

$$= \lim_{x \to \infty} \frac{\frac{4}{x} - \frac{1}{x^2}}{2 - \frac{5}{x^3}}$$

$$= \cancel{\lim_{x \to \infty}} \frac{0-0}{2-0}$$

$$= 0$$ Ans

9

(15)

$$f(x) = \begin{cases} 2x+1 & ; x<1 \\ 3-x & ; x>1 \end{cases}$$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (2x+1) = 2+1 = 3$$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (3-x) = 3-1 = 2$$

$$\therefore \lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x)$$

$\therefore$ limit does not exist.