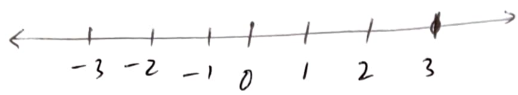
# "Motion Along a Stright line" "Chapter-2"

$x = +3$




position $x = +3$ → distance from ref point "O"; right from ref point "O"

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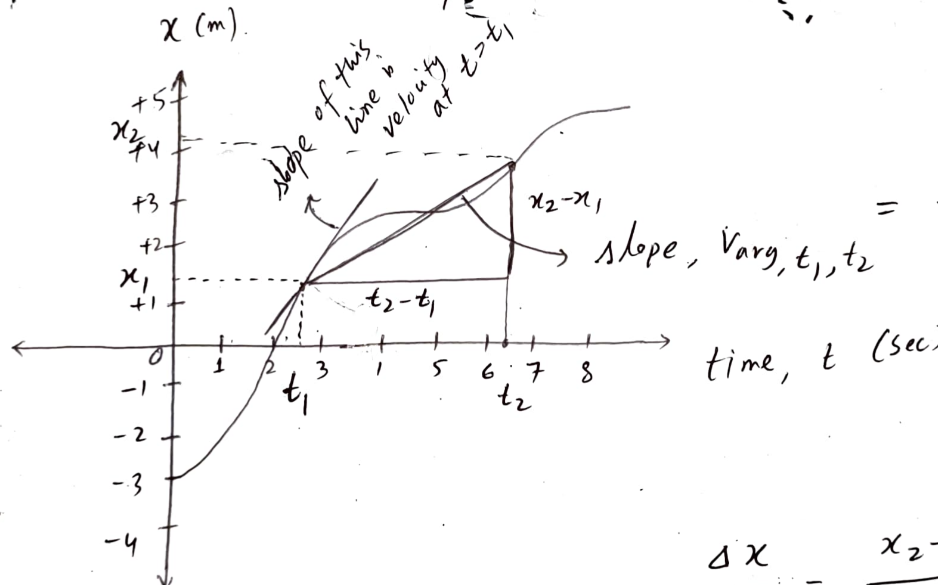
Displacement, $\Delta x = x_2 - x_1$

Final position ($x_2$); Initial position ($x_1$)




$$= \frac{x_2 - x_1}{t_2 - t_1}$$

Average velocity, $V_{avg,t_1,t_2} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{x(t_1+\Delta t) - x(t_1)}{\Delta t}$

Instantaneous velocity / velocity,

$$V(t = t_1) = \lim_{\Delta t \to 0} \frac{x(t_1+\Delta t) - x(t_1)}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}\bigg|_{t=t_1}$$

$$= \frac{dx}{dt}\bigg|_{t=t_1}$$

Average acceleration, $a_{avg,\, t_1, t_2} = \dfrac{V_2 - V_1}{t_2 - t_1} = \dfrac{\Delta V}{\Delta t} = \dfrac{V(t_1+\Delta t) - V(t_1)}{\Delta t}$

Instantaneous acceleration, $a(t = t_1) = \lim_{\Delta t \to 0} \dfrac{V(t_1+\Delta t) - V(t_1)}{\Delta t}$

$= \lim_{\Delta t \to 0} \dfrac{\Delta V}{\Delta t}\Big|_{t=t_1}$

$= \dfrac{dV}{dt}\Big|_{t=t_1}$

$$\boxed{\begin{aligned} v &= \frac{dx}{dt} \\ a &= \frac{dv}{dt} = \frac{d^2x}{dt^2} \end{aligned}}$$

**Sample problem: 2.03**

$x = 4 - 27t + t^3$

(a) $V(t) = \dfrac{dx}{dt} = -27 + 3t^2$

(b) $\therefore V = 0 = -27 + 3t^2 \Rightarrow 3t^2 = 27$

$\Rightarrow t^2 = 9$

$\Rightarrow t = \pm 3\ s$

(c) $x(t) = 4 - 27t + t^3$

$v(t) = -27 + 3t^2$

$a(t) = 6t$

(2)

at, $t = 0$, $x(0) = +4$ m

$v(0) = -27$ m/s

$a(0) = 0$ m/s²

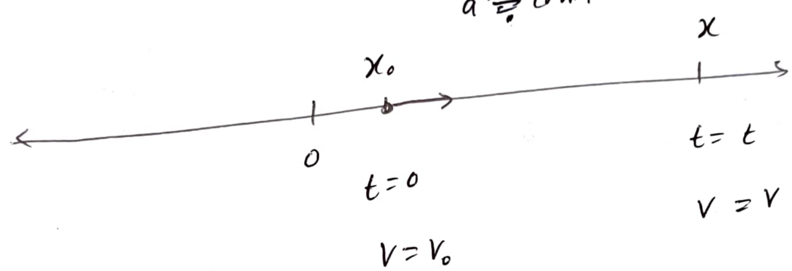
at, $t = 3$, $x(3) = -50$ m

$v(3) = 0$ m/s

$a(3) = 9$ m/s²

Equations for motion with constant acceleration,

$\vec{a} \Rightarrow$ constant




$$a = \frac{dv}{dt}$$

$$\Rightarrow \int_{v_0}^{v} dv = \int_0^t a\,dt$$

| $t$ | 0 | $t$ |
|---|---|---|
| $v$ | $v_0$ | $v$ |

$$\Rightarrow [v]_{v_0}^{v} = a\int_0^t dt$$

$$\Rightarrow v - v_0 = at$$

$$\Rightarrow \boxed{v = v_0 + at}$$

$$a = \frac{v-v_0}{t} \Rightarrow t = \frac{v-v_0}{a}$$

$$\boxed{v^2 = v_0^2 + 2a(\cancel{x}-x_0)}$$

Graph: $v \sim t$

$x \sim t$

| $t$ | 0 | $t$ |
|---|---|---|
| $x$ | $x_0$ | $x$ |

$$v = \frac{dx}{dt}$$

$$\Rightarrow \int_{x_0}^{x} dx = \int_0^t v\,dt$$

$$\Rightarrow [x]_{x_0}^{x} = \int_0^t (v_0 + at)\,dt \Rightarrow x - x_0 = \left[v_0 t + \frac{1}{2}at^2\right]_0^t$$

$$\Rightarrow \boxed{x - x_0 = v_0 t + \frac{1}{2}at^2}$$

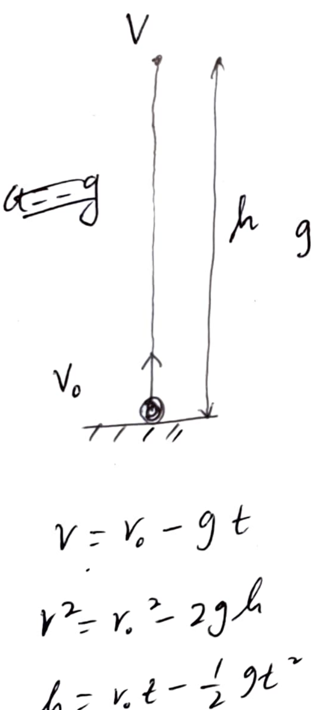
Sample problem 2.03 "Reverse"

$a(t) = 6t \ m/s^2$, $v(t=0) = -27 \ m/s$, $x(t=0) = 4 \ m$

$v(t) = ?$ $x(t) = ?$

## Free Fall Acceleration:

Case-1

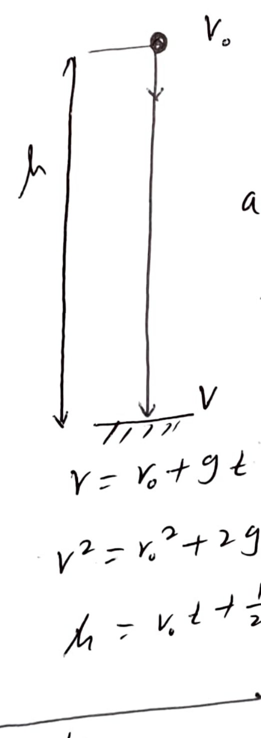



$v = v_0 - gt$

$v^2 = v_0^2 - 2gh$

$h = v_0 t - \frac{1}{2} g t^2$

Case-2

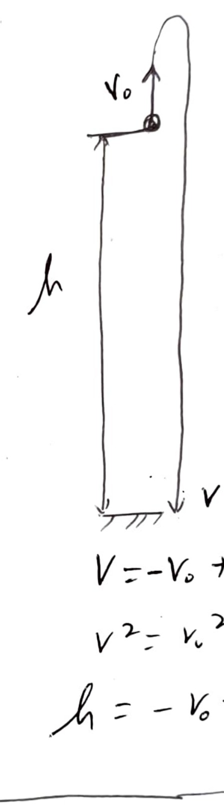



$v = v_0 + gt$

$v^2 = v_0^2 + 2gh$

$h = v_0 t + \frac{1}{2} g t^2$

Case: 3



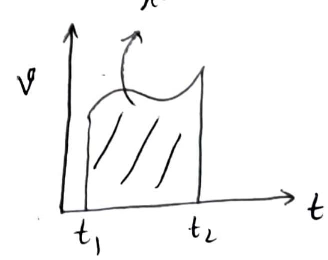

$V = -V_0 + gt$

$v^2 = v_0^2 + 2gh$

$h = -v_0 t + \frac{1}{2} g t^2$

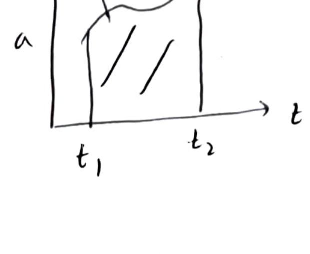
# $v = \frac{dx}{dt} \Rightarrow \int_{x_0}^{x} dx = \int_{t_1}^{t_2} v\,dt \Rightarrow x - x_0 = \int_{t_1}^{t_2} v\,dt$

$x - x_0$ = displacement




$v - v_0 = \int_{t_1}^{t_2} a\,dt$

change of the velocity



$x(t) = 3t - 4t^2 + t^3$

$\boxed{5}$

(a) $t = 1\,s$, $x(1) = (3\times 1 - 4\times 1^2 + 1^3)\,m$
$= \boxed{0\ m}$

(b) $t = 2s$, $x(2) = (3\times 2 - 4\times 2^2 + 2^3)\ m$
$= \boxed{-2m}$

(c) $t = 3\,s$, $x(3) = (3\times 3 - 4\times 3^2 + 3^3)\ m$
$= \boxed{0\ m}$

(d) $t = 4\,s$, $x(4) = (3\times 4 - 4\times 4^2 + 4^3)\ m$
$= \boxed{12\ m}$

(e) Displacement, $\Delta x = x(4) - x(0)$
$= (12\ m - 0\ m) = \boxed{12\ m}$

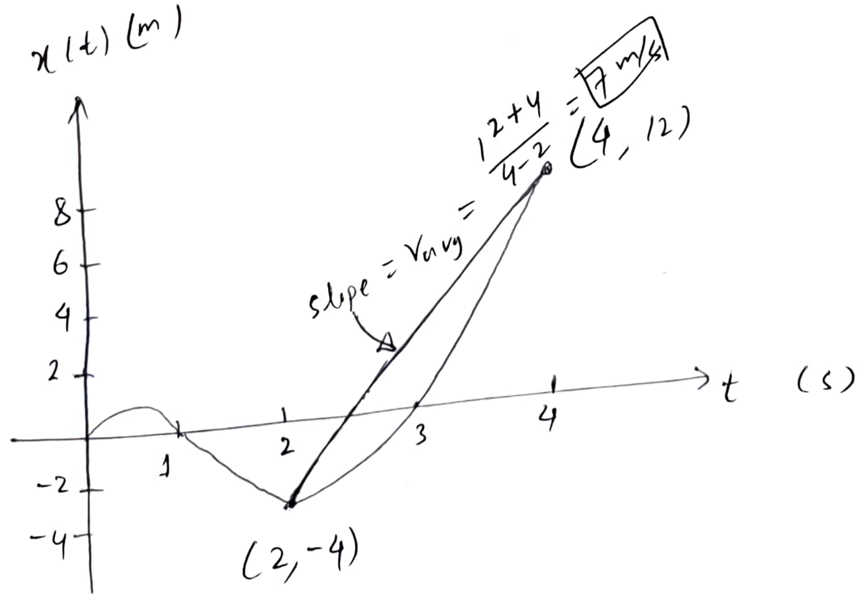
(f) Average velocity, $V_{avg}$, 2s to 4s $= \dfrac{\Delta x}{\Delta t}$

$= \dfrac{x(4) - x(2)}{4-2}$ m/s

$= \dfrac{12 - (-2)}{2}$ m/s

$= \dfrac{14}{2}$ m/s $= \boxed{7\ m/s}$

(g)



15 $x(t) = 4 - 12t + 3t^2$

(a) $v(t) = \frac{dx}{dt} = \frac{d}{dt}(4 - 12t + 3t^2)$

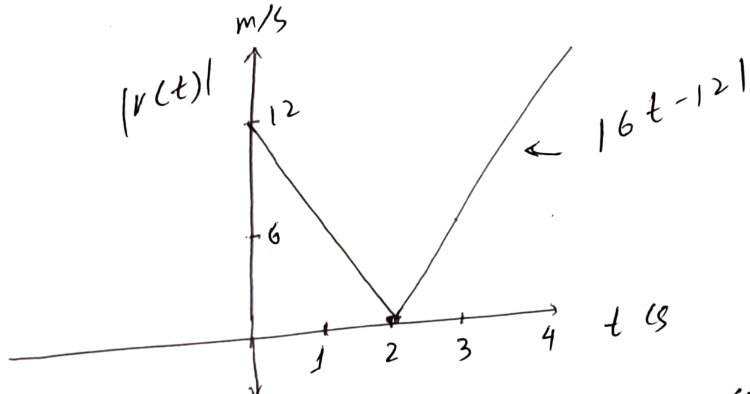
$= -12 + 6t$

$v(1) = (-12 + 6)$ m/s $= \boxed{-6 \text{ m/s}}$

(b) at $t = 1s$, $v = -6$ m/s $< 0$, so the particle is moving from right to left.

(c) speed at $t = 1s$, $|v(1)| = |-6 \text{ m/s}| = 6$ m/s

(d) speed, $|v(t)| = |6t - 12|$




From the graph, $|v(t)|$ vs $t$, we can see $0 < t < 2s$ $|v|$ decreases untill it vanishes.

From, $t > 2s$ $|v|$ increases.

(e) $v(t) = 0$ $-12 + 6t = 0$

$\Rightarrow$ $t = 2s$,

so at, $t = 2s$, $v(t) = 0$.

(f) $v(t) = -12 + 6t$, For $t > 2s$ $v > 0$.

So after $t = 3s$, particle is not moving negative $x$ direction.

$\boxed{18}$ $x(t) = 12t^2 - 2t^3$, $v(t) = \frac{dx(t)}{dt} = 24t - 6t^2$

$a(t) = \frac{dv(t)}{dt} = 24 - 12t$

at, $t = 3s$

(a) Position, $x(3) = [12 \times 3^2 - 2 \times 3^3]$ m

$= \boxed{54 \text{ m}}$

(b) velocity, $v(3) = [24 \times 3 - 6 \times 3^2]$ m/s

$= 18$ m/s

(c) acceleration, $a(3) = [24 - 12 \times 3]$ m/s$^2$

$= -12$ m/s$^2$

(d) For maximum positive co-ordinate,

$$\frac{dx(t)}{dt} = 12 \times 2t - 6t^2 = 0$$

$$\Rightarrow 6t(4 - t) = 0$$

$$\therefore t = 0s, \quad t = 4s$$

So, $x(0) = 12(0)^2 - 2(0^3) = 0$ m

$x(4) = [12 \times 4^2 - 2 \times 4^3]$ m

$= \boxed{64 \text{ m}}$

So, maximum positive co-ordinate of the particle is 64 m at $\boxed{t = 4s}$ (Ans.(e))

(e) Ans : $\boxed{t = 4s}$

(f) For maximum velocity,

$$\frac{dv}{dt} = 0$$

$$24 - 12t = 0$$

$$\Rightarrow \boxed{t = 2s} \leftarrow \text{Ans (g)}$$

$\Rightarrow$ at, $t = s$, $v(t)\big|_{t=2s} = [24 \times 2 - 6 \times 2^2]$ m/s

$$= (48 - 24) \text{ m/s}$$

$$= 24 \text{ m/s}$$

(g) $t = 2s$

(h) From, (d)

For $v = 0$, we have found, $t = 0s$ and $t = 4s$

so, at, $t = 4s$, acceleration, $a(4) = (24 - 12 \times 4)$ m/s$^2$

$$= -24 \text{ m/s}^2$$

(i) $$V_{avg_{t_1, t_2}} = \frac{x(t_2) - x(t_1)}{t_2 - t_1}$$

$$= \frac{x(3) - x(0)}{3 - 0} \text{ m/s}$$

$$= \frac{(12 \times 3^2 - 2 \times 3^3) - (12 \times 0^2 - 2 \times 0^3)}{3 - 0} \text{ m/s}$$

$$= \frac{54}{3} \text{ m/s} = \boxed{18 \text{ m/s}}$$

20

$$x = 20t - 5t^3$$

$$v = \frac{dx}{dt} = 20 - 15t^2$$

$$a = \frac{dv}{dt} = -30t$$

(a) For $v = 0$

$\Rightarrow 20 - 15t^2 = 0$

$\Rightarrow t = \pm\sqrt{\frac{20}{15}}$ s

$= \boxed{1.15\ s}$

(b) For $a = 0$

$\Rightarrow -30t = 0$

$\Rightarrow \boxed{t = 0\ s}$

(c) ~~(d)~~ Here, $a = -30t$

For, $\boxed{t \leq 0\ s}$ acceleration is $\boxed{\text{positive}}$

$\boxed{t > 0\ s}$ " " $\boxed{\text{negative}}$

(e)