BRAC UNIVERSITY

Inspiring Excellence

Differential Calculus and Co-ordinate Geometry

MATH110 Assignment-04

Ashakuzzaman Odree

ID-20301268

Section-11

Set-D

Ans to the Question NO -1

f(x, y) $= x^2 + xy - 2y - 2x + 1$

$fx = 2x + y - 0 - 2 + 0$

$= 2x + y - 2 = 0 \cdots\cdots\cdots (i)$

$fy = 0 + x - 2 - 0 + 0$

$= x - 2 = 0 \ldots\ldots\ldots (ii)$

$ii) \Rightarrow x - 2 = 0 \qquad x = 2$

$i \Rightarrow 2(2) + y - 2 = 0 \quad [x = 2]$

$4 + y - 2 = 0$

$y = -2$

f$_{xx} = (fx)x = 2$

$f_{yy} = (fy)y = 0$

$fxy = (fx)y = 1$

critical Point $-(2, -2)$

$(fxy)^2 = 1^2 = 1$

$f_{xx} \cdot f_{yy} = 2 \cdot 0 = 0$

$(fxy)^2 > fxx \cdot fyy$

saddle point (2,-2)
Ans:

Ans to the Question NO-2

f(x) =x$^3 + 2x + 1$ $\qquad [x = 3]$
=34
$f'(x) = 3x^2 + 2$
$= 29$
$f''(x) = 6x = 18$
$f'''(x) = 6$
$f^{\text{iv}}(x) = 0$

$P_n(x) = f\,(x_0) + f'_{(x_0)}\,(x - x_0) + \frac{f''(x_0)}{2!}\,(x - x_0)^2 + \frac{f'''(x_0)}{3!}\,(x - x_0)^3 + \ldots \cdots$

$P_n(3) = 34 + 29(x-3) + \frac{18}{2!}(x-3)^2 + \frac{6}{3!}(x-3)^3 + \frac{0}{4!}(x-3)^4 + \cdots\cdots$
$= 34 + 29(x-3) + 9(x-3)^2 + (x-3)^3$

Ans to the Question NO-3

$$
\begin{aligned}
&Given,\\
&f(x,y,z) = 3(2x+5)y^4\left(z^3+1\right)\\
&\qquad\qquad = (6x+15)y^4\left(z^3+1\right)\\
&fx = (6+0)y^4\left(z^3+1\right) = 6y^4\left(z^3+1\right)
\end{aligned}
$$

$$fy = 4(6x+15)\cdot\left(z^3+1\right)y^3$$

f xz=(f x) z=6 y$^4\left(3z^2+0\right)$

$$
\begin{aligned}
&= 6y^4\cdot 3z^2\\
&= 18y^4z^2
\end{aligned}
$$

$Ans:$

$$fyz = (fy)z = 4(6x+15)y^3\cdot 3z^2$$

=$72xz^2y^3 + 180z^2y^3$

$Ans:$

$$
\begin{aligned}
fxyz &= (6+0)4y^3\left(3z^2+6\right) = 6\cdot 4y^3\cdot 3z^2\\
&= 72y^3z^2
\end{aligned}
$$

$Ans:$

Ans to the Question Q NO-4

$$z = ln\left(3x^2 - 2y + 4z^3\right); \quad x = t^{\frac{1}{2}}, y = t^{\frac{2}{3}}, z = t^{-2}$$

$$\frac{\partial z}{\partial x} = \frac{1}{3x^2-2y+4z^3} \cdot 6x^2$$
$$\frac{dx}{dt} = \frac{1}{2}t^{-\frac{1}{2}}$$
$$\frac{\partial z}{\partial y} = \frac{1}{3x^2-2y+4z^3} \cdot -2$$
$$\frac{dy}{dt} = \frac{2}{3}t^{-\frac{1}{3}}$$
$$\frac{\partial z}{\partial z} = \frac{1}{3x^2-2y+4z^3} \cdot 12z^2$$
$$\frac{dz}{dt} = (-2)t^{-3}$$

Chain rule of partial Derivative:

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt} + \frac{\partial z}{\partial z}\frac{dz}{dt}$$

$$= \frac{6x^2}{3x^2-2y+4z^3} \cdot \frac{1}{2}t^{-\frac{1}{2}} + \frac{-2}{3x^2-2y+4z^3} \cdot \frac{2}{3}t^{-\frac{1}{3}} + \frac{12z^2}{3x^2-2y+4z^3}(-2)t^{-3}$$

$$= \frac{1}{3x^2-2y+4z^3}\left(3x^2t^{-\frac{1}{2}} - \frac{4}{3}t^{-\frac{1}{3}} - 24z^2t^{-3}\right)$$

$$= \frac{1}{3t^{\left(\frac{1}{2}\right)^2} - 2t^{\frac{2}{3}} + 4t^{(-2)^3}}\left(3t^{\left(\frac{1}{2}\right)^2} t^{-\frac{1}{2}} - \frac{4}{3}t^{-\frac{1}{3}} - 24t^{(-2)^2}t^{-3}\right)$$

$$= \frac{1}{3t-2t^{\frac{2}{3}}+4t^{-6}}\left(3t - \frac{4}{3}t^{-\frac{1}{3}} - 24t^{-7}\right)$$

Ans to the Question NO-5

a) Given,

$$f(x, y) = x^2 y e^{xy}$$

Now,

$$\frac{\partial f}{\partial x} = 2xye^{xy} + x^2 y e^{xy} \cdot y$$

After putting the value $\cdot \frac{\partial f}{\partial x}(1 \cdot 1)$, we get

$$2 \cdot 1 \cdot 1 \cdot e^{1 \cdot 1} + 1^2 \cdot 1, e^{1 \cdot 1} \cdot 1$$

$$= 3 \text{ e}$$

Again,

$\frac{\partial f}{\partial y} = x^2 e^{xy} + x^2 y e^{xy} \cdot x$

$\frac{\partial f}{\partial y}(1, 1)$, we get

$$1^2 \cdot e^{1 \cdot 1} + 1^2 \cdot 1 \cdot e^{1 \cdot 1} \cdot 1$$

$$= 2 \text{ e}$$

Ans:

b) given,

$$\omega = x^2 \cos xy$$

Now,

$$\frac{\partial \omega}{\partial x} = 2x \cos xy - yx^2 \sin xy$$

After putting the value, $\frac{\partial w}{\partial y}\left(\frac{1}{2}, \pi\right)$ we get

$2 \cdot \frac{1}{2} \cdot \cos \frac{\pi}{2} - \pi \cdot \frac{1}{4} \cdot \sin \frac{\pi}{2}$

$= 0 - \frac{\pi}{4} \cdot 1$

$= -\frac{\pi}{4}$
$Again,$
$\frac{\partial \omega}{\partial y} = -x^2 \sin xy \cdot x$
$= -x^3 \sin xy$

$\frac{\partial w}{\partial y}\left(\frac{1}{2}, \pi\right)$ we get

$$-\frac{1}{8} \cdot \sin \frac{\pi}{2}$$
$$= -\frac{1}{8}$$

Ans:

Ans to the Question NO-6

$f(x) = \frac{1}{1-x}$ at $x = 0$

$f(x) = \frac{1}{1-x} = \frac{1}{1-0} = 1$

$f'(x) = \frac{1}{\left(1-x\right)^2} = \frac{1}{(1-0)^2} = 1$

$f''(x) = \frac{2}{(1-x)^3} = \frac{1}{(1-0)^3} = 2$

$f'''(x) = \frac{6}{(1-x)^4} = \frac{6}{(1-0)^4} = 6$

Toylor series,

$P_n(x) = f\,(x_0) + f'_{(x_0)}\,(x - x_0) + \frac{f''(x_0)}{2!}\,(x - x_0)^2 + \frac{f'''(x_0)}{3!}\,(x - x_0)^3 + \cdots$

$P_n(x) = 1 + (1)x + \frac{2}{2!}x^2 + \frac{6}{3!}\,x^3 + \cdots$
$= 1 + x + x^2 + x^3 + \cdots\cdots$

Ans: