# MAT 110 : Differential Calculus & Coordinate Geometry

## Lecture 1:

First we want to introduce Function(s). We're going to make sure that you're familiar with functions and function notation.

So, question is " what exactly is a function?"

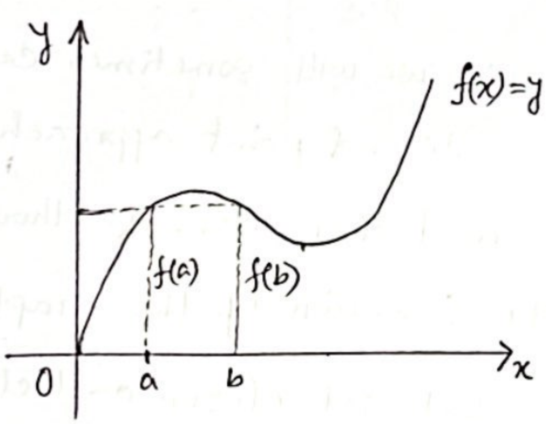
An equation will be a function if, for any $x$ in the domain of the equation, the domain is all the $x$'s that can be plugged into the equation, the equation will yield exactly one value of $y$ when we evaluate the equation at a specific $x$. e.g.,

**Notation**

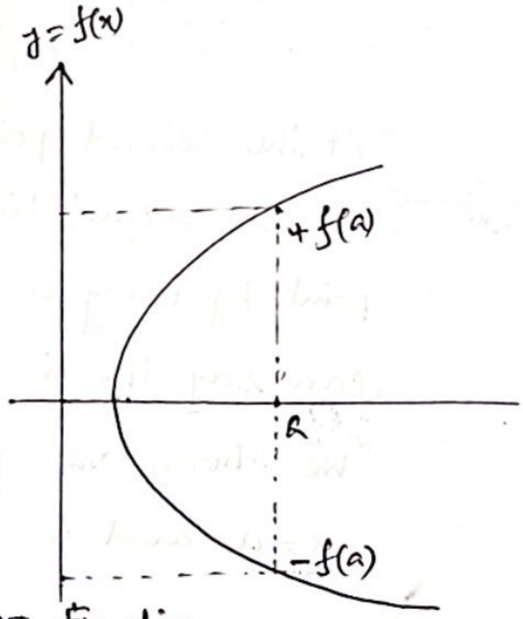
$f(x)$ is a function of $x$ where $x$ is a free or independent variable.

1. $y = x^2+1$ is a function
2. $y^2 = x+1$ is not a function

since $y = \pm\sqrt{x+1}$ , $y$ has TWO values correspond One value of $x$.




Function.




NOT Function

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Assistant Professor
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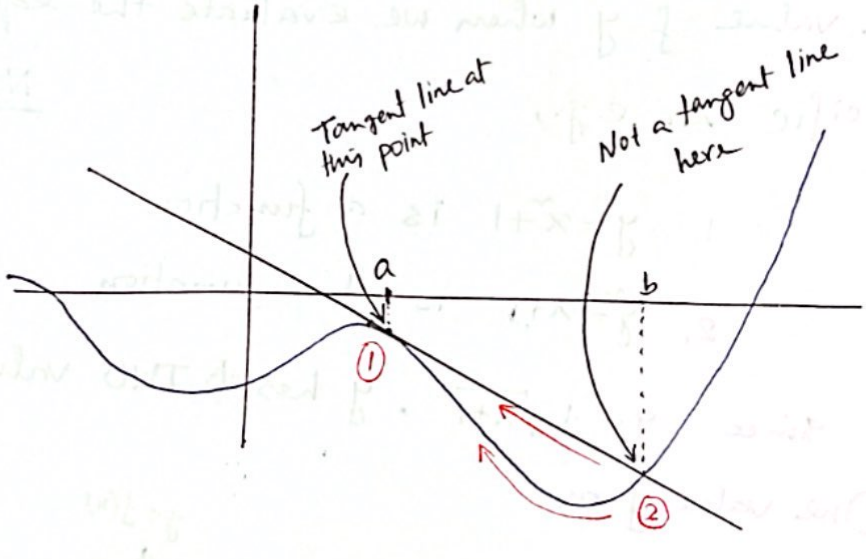
## Limit:

We will discuss just what a limit tells us about a function as well as how they can be used to get the rate of change of a function as well as the slope of the line tangent to the graph of a function.

A tangent line to the function $f(x)$ at the point $x=a$ is a line that just touches the graph of the function at the point in the graph, and is 'parallel' (in some way) to the graph at that point.




At the second point shown we will sometimes call the line a secant line when second point approach the first point by using a process and this process (method) we can say the limit of the function of the graph.

We choose values of $x$ that got closer and closer to $x=a$ and we plugged these into the function.

This process is called 'taking a limit' and we have some notation for this.

$$\lim_{x \to a} f(x) = L.$$

**Definition:**

The limit of $f(x)$ is $L$ as $x$ approaches $a$ and write this

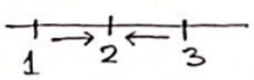
$$\lim_{x \to a} f(x) = L$$

provided we can make $f(x)$ as close to $L$ as we want for all $x$ sufficiently close to $a$, from both sides, without actually letting $x$ be $a$.

Example: Estimate the value of the following limit,

$$\lim_{x \to 2} \frac{x^2+4x-12}{x^2-2x}.$$

| $x$ | $f(x)$ | $x$ | $f(x)$ |
|---|---|---|---|
| 2.5 | 3.4 | 1.9 | 4.157894737 |
| 2.1 | 3.857142857 | 1.99 | 4.015075377 |
| 2.01 | 3.985074627 | 2.999 | 4.001500750 |
| 2.001 | 3.998500750 | 2.9999 | 4.000150008 |
| 2.00001 | 3.999985000 | 2.99999 | 4.000015000 |



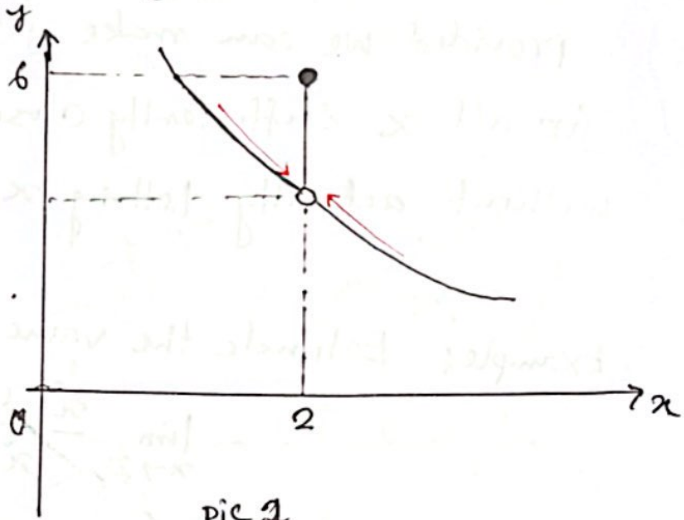
that means the function is going to 4 as $x$ approaches 2, so

$$\lim_{x\to 2} \frac{x^2+4x-12}{x^2-2x} = 4 .$$

Example 2: Estimate the value of the following limit,

$\lim_{x\to 2} g(x)$, where

$$g(x) = \begin{cases} \dfrac{x^2+4x-12}{x^2-2x} & \text{if } x \neq 2 \\ 6 & \text{if } x = 2 . \end{cases}$$



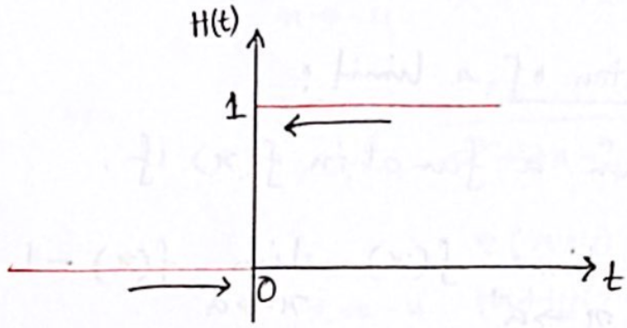

pic 2

From pic 2 (see) [Previous example]

$\lim_{x\to 2} g(x) = 4$ but when $x = 2$, $g(2) = 6$.

Another example called Heaviside or step function.

$$H(t) = \begin{cases} 0 & \text{if } t<0 \\ 1 & \text{if } t>0 \end{cases}.$$




~~Now we details definition of a limit.~~

Now we add Right and Left ~~side~~ hand of a limit.

<u>Right hand limit</u>

We say $\lim\limits_{x \to a^+} f(x) = L$

provided we can make $f(x)$ as close to $L$ as we want for all $x$ sufficiently close to 'a' with $x > a$ without actually letting $x$ be a.

<u>Left hand limit</u>

We say $\lim\limits_{x \to a^-} f(x) = L$

provided we can make $f(x)$ as close to $L$ as we want for all $x$ sufficiently close to 'a' with $x < a$ without actually letting $x$ be a.

so, for the previous example,

$$\lim_{t \to 0^+} H(t) = 1 \quad \text{and} \quad \lim_{t \to 0^-} H(t) = 0$$

Now we are giving a methematical definition of a limit.

<u>Definition of a limit:</u>

Given a function $f(x)$ if,

$$\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x) = L$$

then the normal limit will exist and

$$\lim_{x \to a} f(x) = L.$$

<u>Properties:</u>

We will assume that $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist and $c$ is any constant. Then,

1. $\lim_{x \to a} [c f(x)] = c \lim_{x \to a} f(x)$

2. $\lim_{x \to a} [f(x) \pm g(x)] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$

3. $\lim_{x \to a} [f(x) g(x)] = \lim_{x \to a} f(x) . \lim_{x \to a} g(x)$

4. $\lim_{x \to a} \left[\dfrac{f(x)}{g(x)}\right] = \dfrac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$, and $\lim_{x \to a} g(x) \neq 0$.

5. $\lim_{x \to a} [f(x)]^n = \left[\lim_{x \to a} f(x)\right]^n$ e.g. $\lim_{x \to a} (x^3+1)^3$

$$= \left[\lim_{x \to a} (x^3+1)\right]^3$$

Problem 1: Estimate the limit of the following function :

(i) $\lim_{x \to 3} \frac{x^2-9}{x-3}$ (ii) $\lim_{x \to -4} \frac{2x+8}{x^2+x-12}$.

(i) $\lim_{x \to 3} \frac{x^2-9}{x-3}$

$\lim_{x \to 3} \frac{(x-3)(x+3)}{(x-3)}$

$\lim_{x \to 3} (x+3)$

$= 6$

(ii) $\lim_{x \to -4} \frac{2x+8}{x^2+x-12}$

$\lim_{x \to -4} \frac{2(x+4)}{x^2+4x-3x-12}$

$\lim_{x \to -4} \frac{2(x+4)}{(x+4)(x-3)}$

$\lim_{x \to -4} \frac{2}{x-3} = -\frac{2}{7}$

Book: Calculus: Early Transcendentals (10th Edition) – H. Anton

Page: 84. Example 9:

Page: 87. Exercise 1.2

Problem : 5, 7, 8, 9, 11, 15, 25, 27.