LIMIT & CONTINUITY

PROBLEM :

Find the limit of the following functions:

1. $\lim_{x\to 0}\dfrac{x}{\sqrt{x+1}-1}$
2. $\lim_{x\to 2}\dfrac{2x^2-5x+2}{5x^2-7x-6}$
3. $\lim_{x\to 0}\dfrac{x}{|x|}$
4. $f(x)=\begin{cases}2-x & ,x<1\\ x^2+1 & ,x>1\end{cases}$ find $\lim_{x\to 1} f(x)$
5. $f(x)=\begin{cases}3x-1 & ,x<1\\ 3-x & ,x>1\end{cases}$ find $\lim_{x\to 1} f(x)$
6. $f(x)=\begin{cases}\dfrac{1}{x+2} & ,x<-2\\ x^2-5 & ,-2<x<3\\ \sqrt{x+13} & ,x>3\end{cases}$ find $\lim_{x\to -2} f(x)$ and $\lim_{x\to 3} f(x)$
7. $\lim_{x\to\infty}\dfrac{3x+5}{6x-8}$
8. $\lim_{x\to\infty}\sqrt[3]{\dfrac{3x+5}{6x-8}}$
9. $\lim_{x\to-\infty}\dfrac{4x^2-x}{2x^3-5}$
10. $\lim_{x\to\infty}\left(\sqrt{x^6+5}-x^3\right)$
11. $f(x)=\begin{cases}x^2+1 & ,x>0\\ 1 & ,x=0\\ 1+x & ,x<0\end{cases}$ find $\lim_{x\to 0} f(x)$
12. $\lim_{x\to\infty}\left(\sqrt{x^6+5x^3}-x^3\right)$
13. $f(x)=\begin{cases}e^{\frac{-|x|}{2}} & ,-1<x<0\\ x^2 & ,0<x<2\end{cases}$

    Find $\lim_{x\to 0} f(x)$
14. $f(x)=\begin{cases}x^2 & ,x<1\\ 2.4 & ,x=1\\ x^2+1 & ,x>1\end{cases}$

    Does $\lim_{x\to 1} f(x)$ exist ?
15. $f(x)=\begin{cases}2x+1 & ,x<1\\ 3-x & ,x>1\end{cases}$

    find $\lim_{x\to 1} f(x)$
16. Prove that $\lim_{x\to\infty}\left(1+\dfrac{1}{x}\right)^x=e$

Test the continuity of the following functions:

1. $f(x)=\begin{cases} Cosx & ,x\geq 0 \\ -Cosx & ,x<0 \end{cases}$ at x = 0

2. $f(x)=\begin{cases} xCos(1/x) & ,x\neq 0 \\ 0 & ,x=0 \end{cases}$ at x = 0

3. $f(x)=\begin{cases} e^{1/x} & ,x\neq 0 \\ 1 & ,x=0 \end{cases}$ at x = 0

4. $f(x)=\begin{cases} \sqrt{|x|} & ,x\geq 0 \\ -\sqrt{|x|} & ,x<0 \end{cases}$ at x = 0

5. $f(x)=\begin{cases} e^{\frac{-|x|}{2}} & ,-1<x<0 \\ x^2 & ,0\leq x<2 \end{cases}$ at x = 0

6. $f(X)=\begin{cases} (x-a)Sin\dfrac{1}{x-a} & x\neq a \\ 0 & x=a \end{cases}$ at x = a

7. $f(x)=\begin{cases} xSin(1/x) & ,x\neq 0 \\ 0 & x=0 \end{cases}$ at x = 0

8. $f(x)=\begin{cases} 1 & ,x<0 \\ 1+Sinx & ,0\leq x<\pi/2 \\ 2+(x-\pi/2)^2 & ,x\geq \pi/2 \end{cases}$ at x = 0 and $x=\pi/2$

9. $f(x)=|x|+|x-1|$ at x = 0 and x = 1

10. $f(x)=\begin{cases} \dfrac{|x-3|}{x-3} & ,x\neq 3 \\ 0 & ,x=3 \end{cases}$ at x = 3

11. $f(x)=\begin{cases} (1+x)^{1/x} & ,x\neq 0 \\ 1 & ,x=0 \end{cases}$ at x = 0

12. $f(x)=\begin{cases} \dfrac{e^{1/x^2}}{e^{1/x^2}-1} & ,x\neq 0 \\ 1 & ,x=0 \end{cases}$ at x = 0