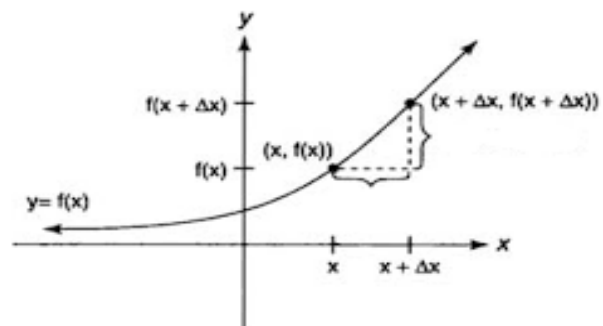


Differentiability

Differentiability: A function is differentiable at 'a' if limit of difference quotient exists,

i. e.,

$$f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}.$$



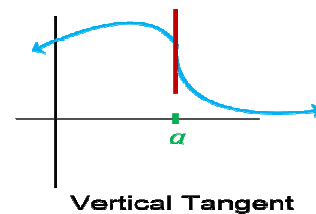
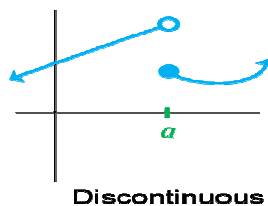
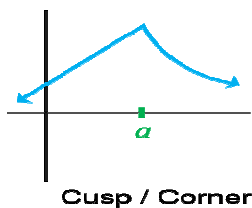
The Derivative of a Function
in terms of Limit

Interpretations: The derivative f' of a function f can be interpreted as follows:

- A function whose value at x is the slope of the graph of $y = f(x)$.
- A function whose value at x is the instantaneous rate of change of y with respect to x at x .
- In particular, when $y = f(t)$ describes the position at time t of an object moving along a straight line, then $f'(t)$ describes the instantaneous velocity of the object at time t .

Non-differentiable points:

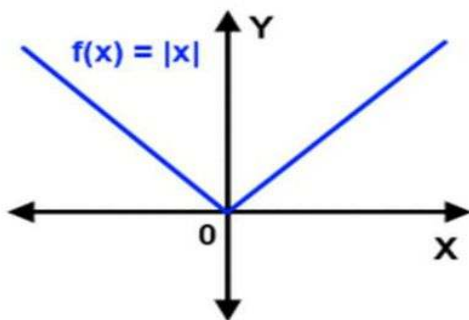
- (i) A corner point
- (ii) A vertical tangent line
- (iii) A discontinuity



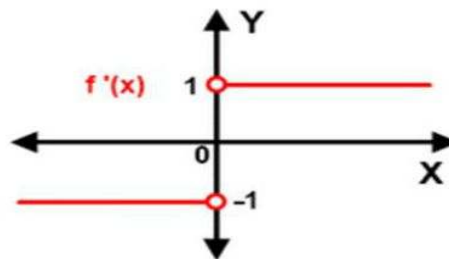
Most of the functions we study in calculus will be differentiable.

If f has a derivative at $x = a$, then f is continuous at $x = a$.

However, if a function is continuous at $x=a$, it is not necessarily differentiable at $x=a$!



Continuous, but NOT
Differentiable at $x=0$!



→