MATH110 Assignment-02

Ashakuzzaman Odree
ID-20301268
Section-11
Set-D

Ans to the question No-1

a) $y = x^2 - \cos x + 1$

$\frac{dy}{dx} = 2x + sinx$

b) $y = \sin^{-1} \left(x^2\right)$

$\frac{dy}{dx} = \frac{1}{\sqrt{1-(x^2)^2}} 2x$

$\frac{dy}{dx} = \frac{2x}{\sqrt{1-x^4}}$

Ans to the question No-2

$f(x) = \frac{5x^3+8}{2x^5}$

$\rightarrow f(x) = \frac{5}{2} + 4x^{-3}$

$\rightarrow f''(x) = 12x^{-4}$

$\rightarrow f'''(x) = 48x^{-5}$

$\rightarrow f'''(x) =$ -240x^{-6}

$\rightarrow f''''(x) = 1440x^{-7}$

$\frac{dy}{dx} = 1440x^{-7}$

Ans to the question No-3

$$f(x) = |x+2|; \begin{cases} -(x+2); x < -2 \\ 0; x = -2 \\ (x+2); \quad x > -2 \end{cases}$$

From the limit def. ,
$for,\ x < -2,$
$L \cdot H \cdot L.$
$= \lim_{h \to 0^-} \quad \frac{f(x+h)-f(x)}{h}$
$= \lim_{h \to 0} \frac{f(x+2+h)-f(x+2)}{h}$
$= \lim \ _{h \to 0} \quad \frac{-(-2+2+h)-(-2+2)}{h}$
$= \lim_{h \to 0} \quad \frac{-h}{h}$

$= -1$
$for,\ x > -2,$
$R \cdot H \cdot L.$
$= \lim_{h \to 0^+} \frac{f(x+h)-f(x)}{h}$
$= \lim_{h \to 0^+} \frac{f(x+2+h)-f(x+2)}{h}$
$= \lim_{h\ \to 0^+} \frac{(-2+2+h)-(-2+2)}{h}$
$= \lim_{h \to 0^+} \frac{h}{h}$
$= 1$
As,
$L \cdot H \cdot L \neq R. \cdot H \cdot L$
So limit does not exits.
$[showed]$

Ans to the question NO-4

$f(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$=\lim_{h \to 0} \frac{cosx.cosh - sinx.sinh - cosx}{h}$

$= lim_{h \to 0} \frac{cosx.cosh - cosx}{h} - \frac{sinx.sinh}{h}$

$= cosx\left(lim_{h \to 0} \frac{cosh-1}{h}\right) - sinx\left(lim_{h \to 0} \frac{sinh}{h}\right)$

$= cosx(0) - sinx(1)$

$= -sinx$

Bonus

Ans to the question No-5

$Quotientrule:$
$\frac{d}{dn} \frac{f(x)}{g(x)} = \lim_{h \to 0} \frac{\frac{f(x+h)}{g(x+h)} - \frac{f(x)}{g(x)}}{h}$
$= \lim_{h \to 0} \frac{\frac{f(x+h)g(x) - f(x)g(x+h)}{g(x+h)g(x)}}{h}$
$= \lim_{h \to 0} \frac{\frac{f(x+h)g(x) - f(x)g(x+h)}{h}}{g(x+h)g(x)}$
$= \lim_{h \to 0} \frac{\frac{f(x+h)-f(x)}{h} g(x) - f(x) \frac{g(x+h)-g(x)}{h}}{g(x+h)g(x)}$
$= \frac{f'(x) \cdot g(x) - g'(x) f(x)}{\{g(x)\}^2}$

$[showed]$

Ans to the question No-6

a) $P(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^m$
$\rightarrow p'(x) = a_1 + 2a_2x + \cdots + ma_nn^{m-1}$

again
$p''(x) = 2a_2 + 6a_3x + \cdots + m(m-1)a_nx^{m-2}$

and
$p'''(x) = 6a_3 + 24a_4x + \ldots + m(m-1)(m-2)a_2n^{m-3}$

b) $P(n) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + \cdots + a_xx^m$
$p'(x) = a_1 + 2a_2x + 3a_3x^2 + 4a_4x^3 + \cdots + ma_nx^{n-1}$
$p''(x) = 2a_2 + 6a_3x + 12a_4x^2 + \ldots + m(m-1)a_nx^{m-2}$
$p'''(x) = 6a_3 + 24a_4x^1 + \cdots + m(m-1)(m-2)a_nx^{m-3}$
$P^m = m!a_n$

c) $P(n) = a_0 + a_1u + a_2n^2 + a_3n^3 + a_nn^n + \cdots + a_nx^m$
$P'(n) = a_1 + 2a_2n + 3a_3n^2 + 4a_nn^3 + \cdots + ma_nn^{m-1}$
$P''(n) = 2a_2 + 6a_3n + 12a_4n^2 + \cdots + m(m-1)a_nn^{m-2}$
$P'''(n) = 6a_3 + 24a_4n + \ldots + m(m-1)(m-2)a_nn^{m-3}$
so, we can say,
$p^m = m!a_n$
Here,
$m!a_n$ is a constant,
so if we differentiate
it again the result will be 0 .
if $n > m$ Then
$p^n = 0$