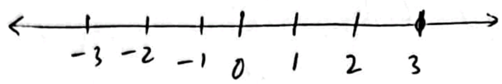
"Motion Along a Stright line"

$x = +3$



distance from ref point "O"

position $x = +3$

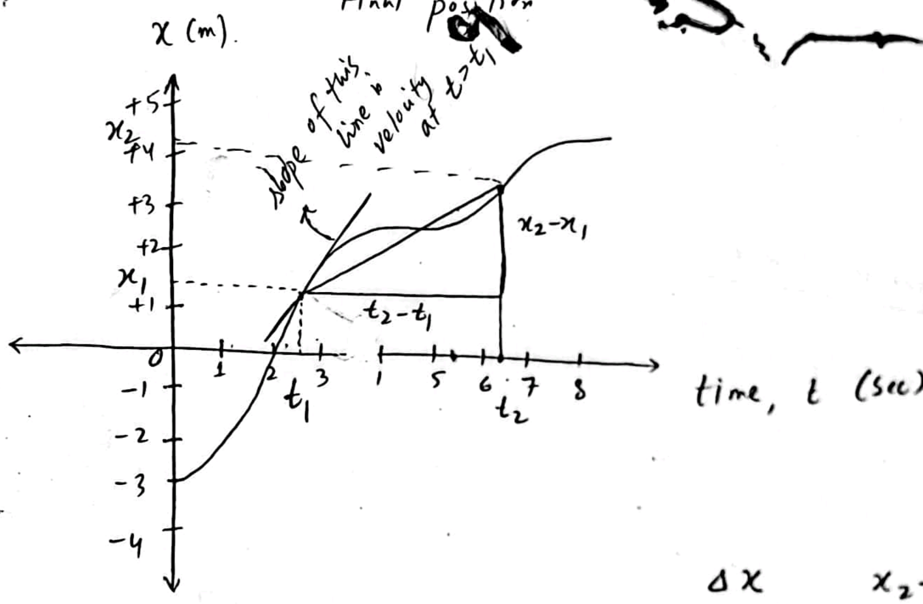
right from ref point "O"

||

Displacement, $\Delta x = x_2 - x_1$

Final position

Initial position



Average velocity, $V_{avg,t_1,t_2} = \dfrac{\Delta x}{\Delta t} = \dfrac{x_2 - x_1}{t_2 - t_1} = \dfrac{x(t_1+\Delta t) - x(t_1)}{\Delta t}$

Instantaneous velocity / velocity,

$$V(t=t_1) = \lim_{\Delta t \to 0} \frac{x(t_1+\Delta t) - x(t_1)}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \left.\frac{\Delta x}{\Delta t}\right|_{t=t_1}$$

$$= \left.\frac{dx}{dt}\right|_{t=t_1}$$

Average acceleration, $a_{avg, t_1, t_2} = \frac{V_2 - V_1}{t_2 - t_1} = \frac{\Delta V}{\Delta t} = \frac{V(t_1+\Delta t) - V(t_1)}{\Delta t} = \frac{\Delta}{\Delta t}$

Instantaneous acceleration, $a(t = t_1) = \lim_{\Delta t \to 0} \frac{V(t_1+\Delta t) - V(t_1)}{\Delta t}$

$$= \lim_{\Delta t \to 0} \left. \frac{\Delta V}{\Delta t} \right|_{t=t_1}$$

$$= \left. \frac{dv}{dt} \right|_{t=t_1}$$

$$v = \frac{dx}{dt}$$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

Sample problem: 2.03

$$x = 4 - 27t + t^3$$

(a) $v(t) = \frac{dx}{dt} = -27 + 3t^2$

(b) $V = 0 = -27 + 3t^2 \Rightarrow 3t^2 = 27$

$\Rightarrow t^2 = 9$

$\Rightarrow t = \pm 3\,s$

(c) $x(t) = 4 - 27t + t^3$

$v(t) = -27 + 3t^2$

$a(t) = 6t$

at, $t=0$, $x(0) = +4$ m

$v(0) = -27$ m/s

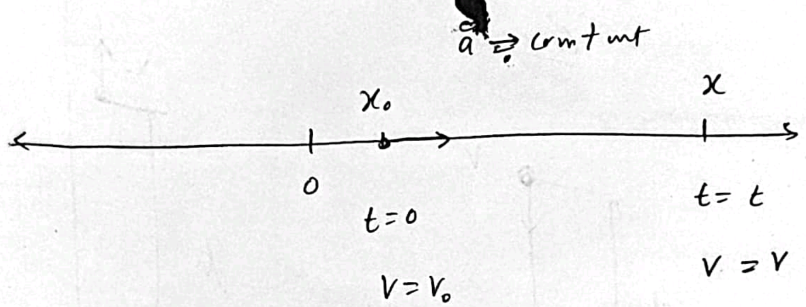
$a(0) = 0$ m/s²

at, $t = 3$, $x(3) = -50$ m

$v(3) = 0$ m/s

$a(3) = 9$ m/s²

Equations for motion with constant acceleration,



$$a = \frac{dv}{dt}$$

$$\Rightarrow \int_{v_0}^{v} dv = \int_0^t a\,dt$$

t	0	t
v	v_0	v

$$\Rightarrow [v]_{v_0}^{v} = a\int_0^t dt$$

$$\Rightarrow v - v_0 = at$$

$$\Rightarrow \boxed{v = v_0 + at}$$

$$a = \frac{v-v_0}{t} \Rightarrow t = \frac{v-v_0}{a}$$

$$\boxed{v^2 = v_0^2 + 2a(x-x_0)}$$

H

Graph: $v \sim t$

$x \sim t$

$$v = \frac{dx}{dt}$$

t	0	t
x	x_0	x

$$\Rightarrow \int_{x_0}^{x} dx = \int_0^t v\,dt$$

$$\Rightarrow [x]_{x_0}^{x} = \int_0^t (v_0 + at)\,dt \Rightarrow x - x_0 = \left[v_0 t + \frac{1}{2}at^2\right]_0^t$$

$$\Rightarrow \boxed{x - x_0 = v_0 t + \frac{1}{2}at^2}$$

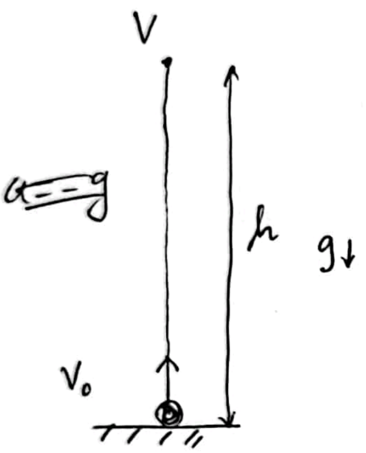
Sample problem 2.03 "Reverse"

$a(t) = 6t \; m/s^2, \quad v(t=0) = -27 \; m/s, \quad x(t=0) = 4 \; m$

$v(t) = ? \qquad x(t) = ?$

Free Fall Acceleration:

Case-1

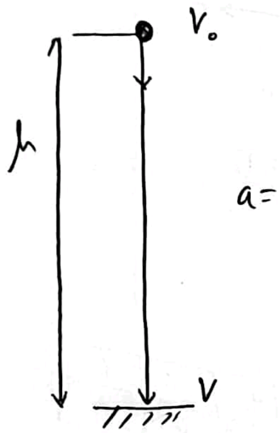


$v = v_0 - gt$

$v^2 = v_0^2 - 2gh$

$h = v_0 t - \frac{1}{2} g t^2$

Case-2



$v = v_0 + gt$

$v^2 = v_0^2 + 2gh$

$h = v_0 t + \frac{1}{2} g t^2$

Case: 3

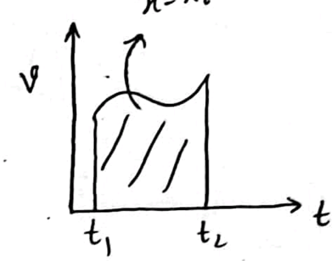
$V = -V_0 + gt$

$v^2 = v_0^2 + 2gh$

$h = -v_0 t + \frac{1}{2} g t^2$

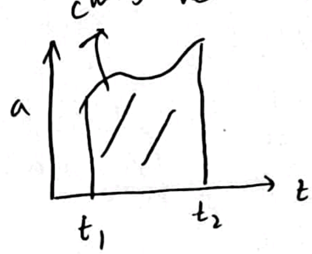
$v = \frac{dx}{dt} \Rightarrow \int_{x_0}^{x} dx = \int_{t_1}^{t_2} v\,dt \Rightarrow x - x_0 = \int_{t_1}^{t_2} v\,dt$

$x - x_0$ = displacement



$v - v_0 = \int_{t_1}^{t_2} a\,dt$

change of the velocity,

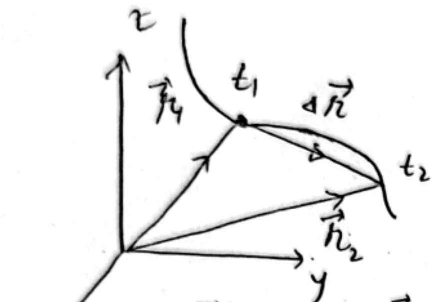


"Motion in '2D' and '3D': "Chapter-4"

Position vector, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

Displacement, $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$

$$= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$



$\vec{r}_1 + \Delta\vec{r} = \vec{r}_2$

$\Rightarrow \Delta\vec{r} = \vec{r}_2 - \vec{r}_1$

Average velocity,

$$\vec{V}_{avg,t_1t_2} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{\Delta\vec{r}}{\Delta t}$$

$$= \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j} + \frac{\Delta z}{\Delta t}\hat{k}$$

$$= V_{avg,x}\hat{i} + V_{avg,y}\hat{j} + V_{avg,z}\hat{k}$$

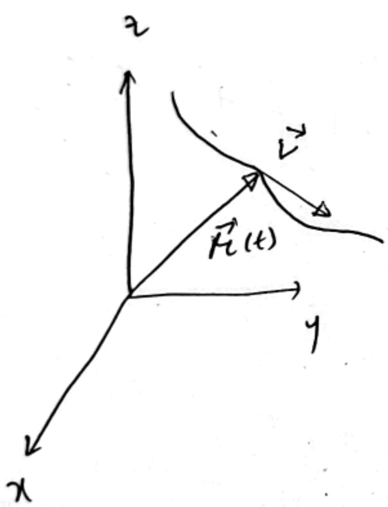
Instantaneous velocity/velocity,

$$\vec{V} = \lim_{\Delta t \to 0}\frac{\Delta\vec{r}}{\Delta t}$$

$$= \frac{d\vec{r}}{dt}$$

$$= \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

$$= v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$$



Average acceleration,

$$\vec{a}_{avg} = \frac{\vec{V}_2 - \vec{V}_1}{t_2 - t_1} = \frac{\Delta\vec{V}}{\Delta t}$$

$$= \frac{\Delta V_x}{\Delta t}\hat{i} + \frac{\Delta V_y}{\Delta t}\hat{j} + \frac{\Delta V_z}{\Delta t}\hat{k}$$

Acceleration, $\vec{a} = \lim_{\Delta t \to 0}\frac{\Delta\vec{V}}{\Delta t} = \frac{d\vec{V}}{dt}$

$$= \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k}$$

$$= a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

See SP: 4.01, 4.02, 4.03

Ch p : 2

SP : 2.03, 2.05

CP : 2, 5

Pr : $\boxed{5}$, $\boxed{15}$, $\boxed{18}$, 20

<u>Chap : 4</u>

SP : 4.01, 4.02, 4.03, 4.04, 4.05

CP : 1, 2, 3, 4, 5

Pr : $\boxed{11}$, 14, $\boxed{16}$, 26, $\boxed{27}$, $\boxed{28}$, 32, 43

Motion in 1D – constant, a

$v = v_0 + at$

$v^2 = v_0^2 + 2a(x - x_0)$

$x - x_0 = v_0 t + \frac{1}{2} a t^2$

<u>Motion in 3D</u> constant $\vec{a}$ (acceleration)

$$\vec{v} = \vec{v}_0 + \vec{a} t$$

$$\vec{v} \cdot \vec{v}^2 = \vec{v}_0 \cdot \vec{v}_0 + 2\vec{a} \cdot (\vec{r} - \vec{r}_0)$$

$$\vec{r} - \vec{r}_0 = \vec{v}_0 t + \frac{1}{2} \vec{a} t^2$$

$$(v_x \hat{i} + v_y \hat{j} + v_z \hat{k}) = (v_{0x} \hat{i} + v_{0y} \hat{j} + v_{0z} \hat{k}) + (a_x t \hat{i} + a_y t \hat{j} + a_z t \hat{k})$$

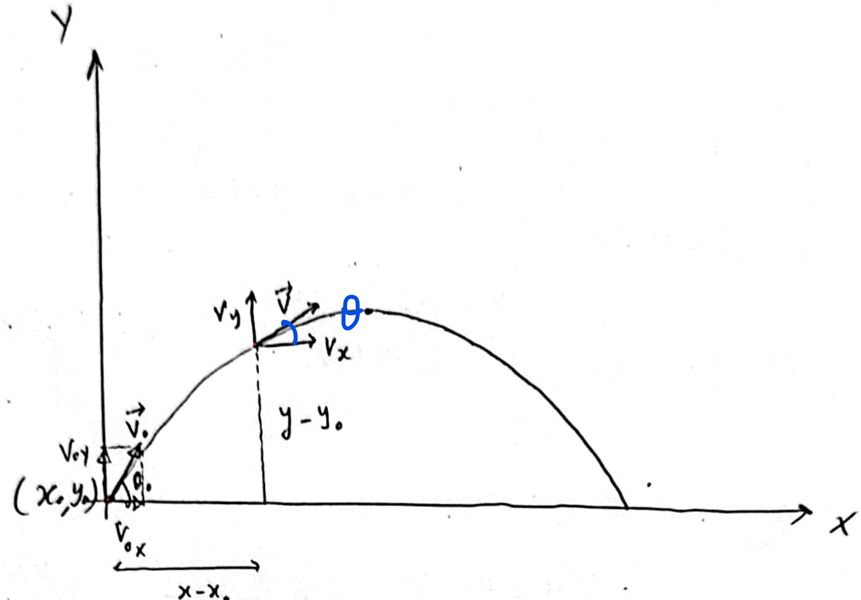
$$\left.\begin{aligned} v_x &= v_{0x} + a_x t \\ v_y &= v_{0y} + a_y t \\ v_z &= v_{0z} + a_z t \end{aligned}\right\}$$

Finally,

$$\left\{\begin{aligned} v_x &= v_{0x} + a_x t \\ v_x^2 &= v_{0x}^2 + 2a_x (x - x_0) \\ x - x_0 &= v_{0x} t + \frac{1}{2} a_x t^2 \end{aligned}\right\}$$

$x \to y \to z$

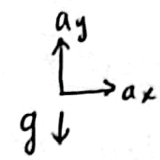
"Projectile Motion" (2D Motion)



$$\left\{\begin{aligned} x - x_0 &= v_{0x} t + \tfrac{1}{2} a_x t^2 \\ v_x &= v_{0x} + a_x t \\ v_x^2 &= v_{0x}^2 + 2a_x(x - x_0) \end{aligned}\right.$$

$x \to y$

$$y - y_0 = v_{0y} t + \tfrac{1}{2} a_y t^2$$



$$\vec{V_0} = \vec{V}_{0x}\hat{i} + \vec{V}_{0y}\hat{j} \quad ---- (1)$$

$$\vec{a} = a_x \hat{i} + a_y \hat{j} \quad .... (2)$$

$$\left.\begin{aligned} a_x &= g\cos 90^\circ = 0 \\ a_y &= g\cos(180^\circ) = -g \end{aligned}\right\} .... (2)a$$

$$\left.\begin{aligned} V_{0x} &= V_0 \cos\theta_0 \\ V_{0y} &= V_0 \sin\theta_0 \end{aligned}\right\} ---- (1)\ a$$

Horizontal Motion:

$$x - x_0 = V_{0x} t + \tfrac{1}{2} a_x t^2$$

$$= V_0 \cos\theta_0\, t + \tfrac{1}{2} \times 0 \times t^2$$

$$\boxed{x - x_0 = V_0 \cos\theta_0\, t} \quad ..... (3)$$

$$V_x = V_{0x} + a_x t$$

$$= V_0 \cos\theta_0 + (0)\, t$$

$$\boxed{V_x = V_0 \cos\theta_0} \quad ---- (4)$$

Vertical Motion:

$$y - y_0 = V_{0y} t + \tfrac{1}{2} a_y t^2$$

$$\boxed{y - y_0 = V_0 \sin\theta_0\, t - \tfrac{1}{2} g t^2} \quad --- (5)$$

$$V_y = V_{0y} + a_y t \Rightarrow \boxed{V_y = V_0 \sin\theta_0 - g t} \quad ---- (6)$$

$$v_y^2 = v_{0y}^2 + 2a_y(y-y_0)$$

$$\boxed{v_y^2 = v_0^2\sin^2\theta_0 - 2g(y-y_0)} \quad \cdots (7)$$

<u>Velocity:</u>

$$\vec{v} = v_x\hat{i} + v_y\hat{j}$$

$$\boxed{\vec{v} = (v_0\cos\theta_0)\hat{i} + (v_0\sin\theta_0 - gt)\hat{j}} \quad \cdots (8)$$

$|\vec{v}| = \sqrt{v_x^2 + v_y^2}$

$$\boxed{\tan\theta = \frac{v_y}{v_x} = \frac{v_0\sin\theta_0 - gt}{v_0\cos\theta_0}} \quad \cdots (9)$$

Equation of Path:

For simplicity, $x_0 = 0$, $y_0 = 0$.

Equation (3), $x = v_0\cos\theta_0 . t \Rightarrow \boxed{t = \frac{x}{v_0\cos\theta_0}}$ ----(10)

From, equation (5) and (10), we get,

$$y = \cancel{v_0}\sin\theta_0 \frac{x}{\cancel{v_0}\cos\theta_0} - \frac{1}{2}g\frac{x^2}{v_0^2\cos^2\theta_0}$$

$$= x(\tan\theta_0) + \left(\frac{-g}{2v_0^2\cos^2\theta_0}\right)x^2$$

$$\boxed{y = ax + bx^2} \quad \cdots\cdots (11)$$

where, $a = \tan\theta_0$

$b = -\frac{g}{2v_0^2\cos^2\theta_0}$

Equation (11) is the equation of Path of projectile, which is the equation of Parabola. So the path is <u>parabolic</u>.

Horizontal Range:

For Horizontal range $x - x_0 = R$, $y - y_0 = 0$,

air time, $t = T$.

From equation, (5);

$$0 = v_0\sin\theta_0 T - \frac{1}{2}gT^2$$

$$\Rightarrow \quad T\left(V_0 \sin\theta_0 - \frac{gT}{2}\right) = 0$$

$$\therefore \; T = 0 \; ; \quad V_0 \sin\theta_0 - \frac{g}{2}T = 0$$

$$\Rightarrow \quad T = \frac{2V_0 \sin\theta_0}{g} \quad \cdots (12) \text{ (air time)}$$

Equation (3), $R = V_0 \cos\theta_0 \dfrac{2V_0 \sin\theta_0}{g}$

Horizontal Range

$$\boxed{R = \frac{V_0^2}{g} \sin(2\theta_0)} \quad \cdots (13)$$

Maximum Height:

At maximum height, $V_y = 0$; $y - y_0 = H$

From equation (6), $0 = V_0 \sin\theta_0 - gt$

$$\Rightarrow \boxed{t = \frac{V_0 \sin\theta_0}{g}} \quad — (14)$$

From equation, (5), $H = (V_0 \sin\theta_0) \dfrac{V_0 \sin\theta_0}{g} - \dfrac{g}{2} \dfrac{V_0^2 \sin^2\theta_0}{g^2}$

$$= \frac{V_0^2 \sin^2\theta_0}{g} - \frac{1}{2}\frac{V_0^2 \sin^2\theta_0}{g}$$

$$\boxed{H = \frac{V_0^2}{2g} \sin^2\theta_0} \quad \cdots (15)$$

See SP: 4.04, 4.05

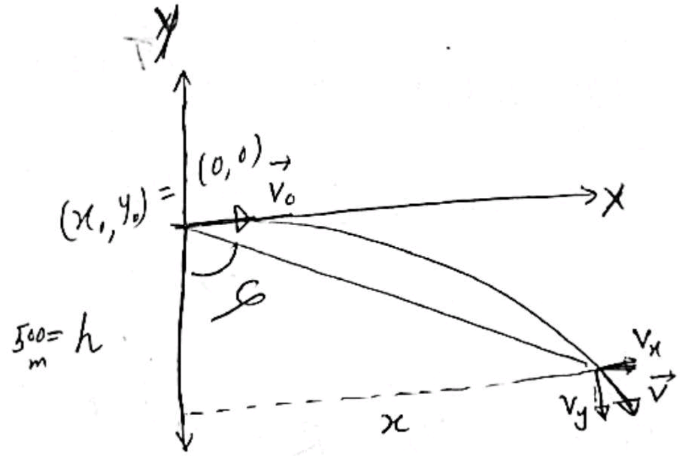
$\swarrow$

x

Sample Problem 4.04

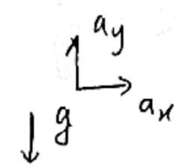
$|\vec{V_0}| = 198$ km/h

$= 55$ m/s



$V_{0x} = V_0 \cos 0^\circ = 55$ m/s

$V_{0y} = V_0 \sin 0^\circ = 0$ m/s



$a_x = g \cos 90^\circ = 0$ m/s²

$a_y = g \cos(180^\circ) = -g$ m/s² $= -9.8$ m/s²

(a) $x - x_0 = V_{0x} t + \frac{1}{2} a_x t^2$

$\Rightarrow x = 55t$ --- (1)

$y - y_0 = V_{0y} t + \frac{1}{2} a_y t^2$

$\Rightarrow -500 = 0 \times t + \frac{1}{2}(-9.8) t^2$

$\Rightarrow t = \sqrt{\frac{2 \times 500}{9.8}}$ s

$\boxed{t = 10.1 \text{ s}}$

From equation, (1). $x = 55 \times 10.1$ m

$= \boxed{555.5 \text{ m}}$

$\tan \varphi = \frac{x}{h} \Rightarrow \varphi = \tan^{-1}\left(\frac{555.5}{500}\right) = 48^\circ$

(b) $\vec{V} = ?$ $\vec{V} = V_x \hat{i} + V_y \hat{j}$

$V_x = V_{0x} + a_x t$

$= (55 + 0 \times 10.1)$ m/s $= 55$ m/s

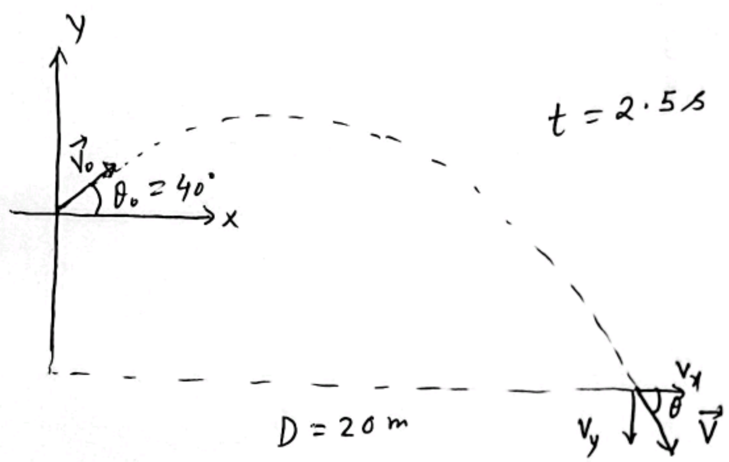
$$v_y = v_{0y} + a_y t$$
$$= (0 + (-9.8) \times 10.1)\ m/s$$
$$= -99\ m/s$$

$$\vec{v} = (55\ m/s)\ \hat{i} + (-99\ m/s)\ \hat{j}$$

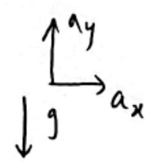
$$|\vec{v}| = \sqrt{(55)^2 + (-99)^2}\ m/s$$
$$= 113.25\ m/s$$

$$\theta = \tan^{-1}\left(\frac{-99}{55}\right)$$
$$= \boxed{-60.9^\circ}$$

Sample Problem 4.05



$$v_{0x} = v_0 \cos\theta_0$$
$$v_{0y} = v_0 \sin\theta_0$$



$V_0 = ?$ $\qquad$ $V = ?$

$$a_x = g\cos(90^\circ) = 0$$
$$a_y = g\cos(180^\circ) = -g = -9.8\ m/s^2$$

$$x - x_0 = v_{0x} t + \tfrac{1}{2} a_x t^2$$
$$\Rightarrow \quad 20 = v_0 \cos\theta_0\, t + \tfrac{1}{2} \times 0 \times t^2$$
$$\Rightarrow \quad v_0 = \frac{20}{(\cos\theta_0)\, t} = \frac{20}{(\cos 40^\circ)\times(2.5)}\ m/s = \boxed{10.4\ m/s}$$

$$V_x = V_{0x} + a_x t$$

$$= V_0 \cos\theta_0 + 0 \times t$$

$$= (10.4) \cos(40^\circ) \text{ m/s}$$

$$= 7.97 \text{ m/s}$$

$$V_y = V_{0y} + a_y t$$

$$= V_0 \sin\theta_0 - 9.8 \times 2.5$$

$$= \left((10.4) \sin(40^\circ) - 9.8 \times 2.5\right) \text{ m/s}$$

$$= -17.81 \text{ m/s}$$

$$V = \sqrt{(7.97)^2 + (-17.81)^2} \text{ m/s}$$

$$= \boxed{19.5 \text{ m/s}}$$

$\boxed{32}$

Here, initial velocity, $V_0 = 25.0 m/s$,

$\theta_0 = 40°$

$x - x_0 = d = 22 m$

(a) $y - y_0 = ?$

$V_{0x} = V_0 \cos\theta_0 = 25 \times \cos 40° \ m/s = 19.15 m/s$

$V_{0y} = V_0 \sin\theta_0 = 25 \times \sin 40° \ m/s = 16.1 m/s$

$y - y_0 = V_{0y} t + \frac{1}{2} a_y t^2$

$= 16.1 \times t - \frac{1}{2} \times 9.8 \times t^2$ —(1)

a_y, a_x, g

$a_x = g \cos 90° = 0 \ m/s^2$

$a_y = g \cos(180°)$

$= -9.8 m/s^2$

$x - x_0 = V_{0x} t + \frac{1}{2} a_x t^2$

$\Rightarrow 22 = 19.15 \times t + \frac{1}{2} \times 0 \times t^2$

$\Rightarrow t = \frac{22}{19.15} s = \boxed{1.15 s}$

From eq. (1), $y - y_0 = [16.1 \times 1.15 - \frac{1}{2} \times 9.8 \times (1.15)^2] m$

$= \boxed{12 (m)}$ $\boxed{Ans}$

(b) $V_x = ?$

(c) $V_y = ?$

$V_x = V_{0x} + a_x t = (19.15 + 0 \times t) m/s$

$= \boxed{19.15 m/s}$ $\boxed{Ans. b}$

$V_y = V_{0y} + a_y t$

$= (16.1 - 9.8 \times 1.15) m/s$

$= \boxed{4.83 m/s}$ $\boxed{Ans. c}$

(d)

$\frac{T}{2} = \frac{2 V_0 \sin\theta_0}{2 \times g} = \frac{2 \times 16.1}{2 \times 9.8} s = \frac{3.29 s}{2} = 1.645 s > t$

So it hasn't passed the heighest point while it hits.