# BRAC UNIVERSITY

## Principles of Physics-II (PHY-112)

Department of Mathematics and Natural Sciences

**Assignment**: 02 — **Section**: 30

Inspiring Excellence

Duration: 7 Days **Summer 2024 (10F-31C)** Marks: 15

**Attempt all questions. Show Your work in detail. Use SI units. 1:1 plagiarism will be strictly penalized.**

1. 100 nC amount of charge is collected and uniformly distributed to a spherical shell-like shape with inner radius $a = 10\,\text{cm}$ and outer radius $b = 15\,\text{cm}$. The charge is distributed only in the $a < r < b$ region. **Note**: You must use Gauss's Law only to solve the following questions.

   (a) Why does $\vec{E}$ inside a volume charge distribution increase with increasing distance from the center? **(1)**

   Inside a volume charge distribution, as you move away from the center, you *enclose* more charge within an observational (Gaussian) sphere of radius $r$. According to Gauss's law, the electric field strength $E$ at a distance $r$ from the center is proportional to the charge enclosed within that radius. Thus, more charge enclosed results in a stronger electric field at greater distances from the center of the distribution.

   (b) What is the volume charge density of the given distribution? **Hint**: You do not have the whole volume available to You. Only parts of it. **(2)**

   To find the volume charge density $\rho$ of the spherical shell:
   Calculate the available volume of the Spherical Shell first where you are to distribute the given charge:

   $$V = \frac{4}{3}\pi(b^3 - a^3)$$

   Given $a = 10\,\text{cm} = 0.1\,\text{m}$ and $b = 15\,\text{cm} = 0.15\,\text{m}$:

   $$V = 9.95 \times 10^{-9}\,\text{m}^3$$

   Next, calculate the Volume Charge Density:

   $$\rho = \frac{Q}{V}$$

   Given $Q = 100\,\text{nC} = 100 \times 10^{-9}\,\text{C}$:

   $$\rho = 10.05 \times 10^{-6}\,\text{C/m}^3$$

   (c) What are the electric field strengths and directions at radial distances 5, 10, 12, 15, and 20 cm from the center of the charged shell? **(4)**

   The electric field $E$ at a radial distance $r$ from the center of the shell is given by:

   $$E = \frac{Q_{\text{enc}}}{4\pi\epsilon_0 r^2}$$

   where $Q_{\text{enc}}$ is the charge enclosed within the spherical shell of radius $r$.
   Calculating $Q_{\text{enc}}$:

   $$Q_{\text{enc}} = \rho \times V = \rho \times \frac{4}{3}\pi\left(r'^3 - a^3\right)$$

   where $r'$ is the radial distance that represents the boundary of the charge enclosing sphere and $r$ is the radial boundary of our observation (Gaussian) sphere. They are the same when measuring the field inside the distribution and unequal when outside the shell.
   Now calculate $E$ at each distance:

1. At $r = 5\,\text{cm} = r'$:

$$Q_{\text{enc}} = 0$$
$$E = \frac{0}{4\pi\epsilon_0(0.05\,\text{m})^2} = 0$$

2. At $r = 10\,\text{cm} = r'$:

$$Q_{\text{enc}} = 0$$
$$E = \frac{0}{4\pi\epsilon_0(0.1\,\text{m})^2} = 0$$

3. At $r = 12\,\text{cm} = r'$:

$$Q_{\text{enc}} = \rho \times V = \rho \times \frac{4}{3}\pi\left(r'^3 - a^3\right) = 3.05 \times 10^{-8}\,\text{C}$$
$$E = \frac{3.05 \times 10^{-8}\,\text{C}}{4\pi\epsilon_0(0.12\,\text{m})^2} = 19.033 \times 10^3\,\text{N}\,\text{C}^{-1}.$$

4. At $r = 15\,\text{cm} = r'$:

$$Q_{\text{enc}} = \rho \times V = \rho \times \frac{4}{3}\pi\left(r'^3 - a^3\right) = 9.95 \times 10^{-8}\,\text{C}$$
$$E = \frac{9.95 \times 10^{-8}\,\text{C}}{4\pi\epsilon_0(0.15\,\text{m})^2} = 39.74 \times 10^3\,\text{N}\,\text{C}^{-1}.$$

5. At $r = 20\,\text{cm}, r' = 15\,\text{cm}$:

$$Q_{\text{enc}} = \rho \times V = \rho \times \frac{4}{3}\pi\left(r'^3 - a^3\right) = 9.95 \times 10^{-8}\,\text{C}$$
$$E = \frac{9.95 \times 10^{-8}\,\text{C}}{4\pi\epsilon_0(0.20\,\text{m})^2} = 22.35 \times 10^3\,\text{N}\,\text{C}^{-1}.$$

Since we have a positive charge distribution, all fields will point radially outward from the center of the shell.

2. The three parallel planes of charge are placed horizontally apart, having surface charge densities $\sigma_1 = -1.5\sigma_2$, $\sigma_2 = +\sigma$, $\sigma_3 = -0.5\sigma_2$. The plates divide the horizontal axis into four regions labeled $A$, $B$, $C$, and $D$. Find the electric fields $\vec{E}_B$ and $\vec{E}_D$ in regions $B$ and $D$. **(5)**

The plates are placed vertically, so our field directions will point horizontally.
Charge density of the first plate: $\sigma_1 = -\frac{\sigma}{2}$.
Charge density of the second plate: $\sigma_2 = +\sigma$.
Charge density of the third plate: $\sigma_3 = +\frac{\sigma}{2}$.
Distance between the rods: $d = 4\,\text{cm} = 0.04\,\text{m}$
The electric field $E$ at any distance from an infinitely extended charged surface with density $\lambda$ is given by:

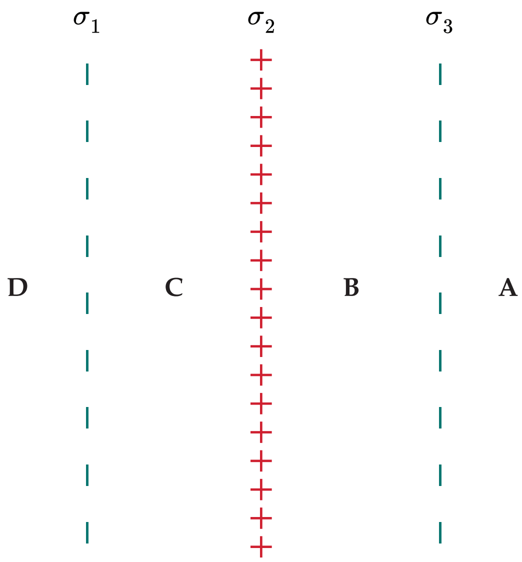
$$E = \frac{\sigma}{2\epsilon_0}$$

. 1. At region $B$:

$$\begin{aligned}
\vec{E}_B &= \vec{E}_1 + \vec{E}_2 + \vec{E}_3 \\
&= -\left(\frac{\sigma_1}{2\epsilon_0}\right)\hat{\imath} + \left(\frac{\sigma_2}{2\epsilon_0}\right)\hat{\imath} + \left(\frac{\sigma_3}{2\epsilon_0}\right)\hat{\imath} \\
&= -\left(\frac{\sigma}{4\epsilon_0}\right)\hat{\imath} + \left(\frac{\sigma}{2\epsilon_0}\right)\hat{\imath} + \left(\frac{\sigma}{4\epsilon_0}\right)\hat{\imath} \\
&= \left(\frac{\sigma}{2\epsilon_0}\right)\hat{\imath}
\end{aligned}$$

2. At region $D$:

$$\begin{aligned}
\vec{E}_D &= \vec{E}_1 + \vec{E}_2 + \vec{E}_3 \\
&= \left(\frac{\sigma_1}{2\epsilon_0}\right)\hat{\imath} - \left(\frac{\sigma_2}{2\epsilon_0}\right)\hat{\imath} + \left(\frac{\sigma_3}{2\epsilon_0}\right)\hat{\imath} \\
&= \left(\frac{\sigma}{4\epsilon_0}\right)\hat{\imath} - \left(\frac{\sigma}{2\epsilon_0}\right)\hat{\imath} + \left(\frac{\sigma}{4\epsilon_0}\right)\hat{\imath} \\
&= 0.
\end{aligned}$$




3. At some instant, the velocity components of an electron moving between two charged parallel plates are $v_x = 1.5 \times 10^5\,\mathrm{m\,s^{-1}}$ and $v_y = 3.0\,\mathrm{km\,s^{-1}}$. Suppose the electric field between the plates is uniform and given by $\vec{E} = -(120\,\mathrm{N\,C^{-1}})\hat{\jmath}$. In unit-vector notation, what are (i) the electron's acceleration in that field and (ii) the electron's velocity when its $x$ coordinate has changed by 2.0 cm? **(3)**

The initial speed of the electron at a given instant

$$\begin{aligned}
\vec{v}_0 &= v_{x0}\hat{\imath} + v_{y0}\hat{\jmath} \\
&= (1.5 \times 10^5\hat{\imath} + 3.0 \times 10^3\hat{\jmath})\,\mathrm{m\,s^{-1}}. \\
|\vec{v}_0| &= \sqrt{v_{x0}^2 + v_{y0}^2}. \\
&= 1.50 \times 10^5\,\mathrm{m\,s^{-1}}.
\end{aligned}$$

(i) Electron's acceleration in the electric field:

$$\begin{aligned}
\vec{a} &= \frac{q_e\vec{E}}{m_e} \\
&= \frac{-1.602 \times 10^{-19}\,\mathrm{C} \times (-120\hat{\jmath})\,\mathrm{N\,m^{-1}}}{9.11 \times 10^{-31}\,\mathrm{kg}}. \\
&= +(2.11 \times 10^{13}\hat{\jmath})\,\mathrm{m\,s^{-2}}.
\end{aligned}$$

(ii) The time elapsed when the electron makes a horizontal displacement of $\Delta x = 2 \times 10^{-2}\,\mathrm{m}$:

$$\begin{aligned}
t &= \frac{\Delta x}{v_x} = \frac{2 \times 10^{-2}\,\mathrm{m}}{1.5 \times 10^5\,\mathrm{m\,s^{-1}}} \\
&= 1.33 \times 10^{-7}\,\mathrm{s}.
\end{aligned}$$

Since the electric field only acts in the $\hat{\jmath}$ direction for this problem, the electron will see no acceleration along the $\hat{\imath}$ direction. Meaning $v_x = v_{x0}$.

Electron's velocity after the horizontal displacement:

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\begin{aligned}
\vec{v} &= v_x\hat{i} + v_y\hat{j} \\
&= v_x\hat{i} + \left(v_{y0} + a_y t\right)\hat{j} \\
&= 1.5 \times 10^5\hat{i} + \left[(3.0 \times 10^3\hat{j} + (2.11 \times 10^{13}\hat{j}) \times (1.33 \times 10^{-7})\right]\ \mathrm{m\,s^{-1}} \\
&= \left(1.5 \times 10^5\hat{i} + 2.80 \times 10^6\hat{j}\right)\ \mathrm{m\,s^{-1}}. \\
|\vec{v}| &= \sqrt{v_x^2 + v_y^2} \\
&= 2.80 \times 10^6\,\mathrm{m\,s^{-1}}.
\end{aligned}
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