# Practice Sheet # 7

## ***Rolle's and Mean Value Theorem***

*a*. Verify the hypothesis of Rolle's Theorem for the following functions:

1. $f(x) = x^2 - 6x + 8; \quad [2,4]$
2. $f(x) = \cos x; \quad [\pi/2, 3\pi/2]$
3. $f(x) = \frac{x}{2} - \sqrt{x}; \quad [0, 4].$

*b*. Verify the hypothesis of Mean Value Theorem for the following functions:

1. $f(x) = x^3 + x - 4; \quad [-1,2]$
2. $f(x) = \sqrt{x+1}; \quad [0,3]$
3. $f(x) = \sqrt{25 - x^2}; \quad [0,5].$

## ***Maclaurin and Taylor Series***

1. Find the Taylor series for the following functions:

   $(i)\ \sin x,$ at $x_0 = \frac{\pi}{2}$. $\qquad (ii)\ \ln x,$ at $x_0 = 2$.

2. Expand $y = \ln x$ in the power of $x-2$ and $y = e^{ax}$ in the power of $x-1$.
3. Find the Maclaurin series for the function $e^{ax}$ and $\cos x$.
4. Find the Maclaurin polynomial $p_0, p_1, p_2, p_3$ for $e^x \cos x$.
5. Expand $y = \ln(x+1)$ and $y = \frac{\sin x}{\cos x}$ in the power of $x$.

## ***Indeterminate Forms***

Find the limit using L' Hospital's rule:

$1. \lim_{x\to 1} \frac{\ln x}{x-1}, \quad 2. \lim_{x\to 3} \frac{x-3}{3x^2 - 13x + 12}, \quad 3. \lim_{x\to 0}\left(\frac{1}{x^2} - \frac{\cos 3x}{x^2}\right), \quad 4. \lim_{x\to \pi} \frac{\sin x}{x-\pi}$

$5. \lim_{x\to 0} \frac{x - \tan^{-1} x}{x^3}, \quad 6. \lim_{x\to +\infty} \frac{e^{3x}}{x^2}, \quad 7. \lim_{x\to 0} \frac{a^x - 1 - x\log a}{x^2}, \quad 8. \lim_{x\to 0} (e^x + x)^{\frac{1}{x}}$

$9. \lim_{x\to 0}\left(\frac{1}{x} - \frac{1}{\sin x}\right), \quad 10. \lim_{x\to \pi} (x-\pi)\cot x, \quad 11. \lim_{x\to 0} \frac{\ln(\sin x)}{\ln(\tan x)}, \quad 12. \lim_{x\to \infty} xe^{-x}$

$13. \lim_{x\to 0} \frac{\sin 2x}{x}, \quad 14. \lim_{x\to 0} \frac{\sin x}{x^2}, \quad 15. \lim_{x\to \infty} \frac{x}{e^x}, \quad 16. \lim_{x\to 0}\left(\frac{1}{x} - \frac{1}{xe^x}\right), \quad 17. \lim_{x\to 0} \frac{\sin 2x}{\sin 5x}.$

# ***Tangent and normal***

1. Find the equations of the tangent and normal at the point (2, 4) of the curve $y = x^3 - 3x + 2$.
2. Find the lengths of the subtangent, subnormal, tangent and normal of $y = 4x^2$ at the point $(1, 4)$.
3. Find the equations of the tangent and normal at the point (2, 2) of the curve $xy^2 = 4(4 - x)$.
4. Find the lengths of the subtangent, subnormal, tangent and normal of $xy + 2x - y = 5$ at the point (2,1).