9–20 Locate all relative maxima, relative minima, and saddle points, if any. ■

9. $f(x, y) = y^2 + xy + 3y + 2x + 3$

10. $f(x, y) = x^2 + xy - 2y - 2x + 1$

11. $f(x, y) = x^2 + xy + y^2 - 3x$

12. $f(x, y) = xy - x^3 - y^2$

13. $f(x, y) = x^2 + y^2 + \dfrac{2}{xy}$

14. $f(x, y) = xe^y$

15. $f(x, y) = x^2 + y - e^y$

16. $f(x, y) = xy + \dfrac{2}{x} + \dfrac{4}{y}$

17. $f(x, y) = e^x \sin y$

18. $f(x, y) = y \sin x$

19. $f(x, y) = e^{-(x^2+y^2+2x)}$

20. $f(x, y) = xy + \dfrac{a^3}{x} + \dfrac{b^3}{y} \quad (a \neq 0, b \neq 0)$

C **21.** Use a CAS to generate a contour plot of

$$f(x, y) = 2x^2 - 4xy + y^4 + 2$$

for $-2 \le x \le 2$ and $-2 \le y \le 2$, and use the plot to approximate the locations of all relative extrema and saddle points in the region. Check your answer using calculus, and identify the relative extrema as relative maxima or minima.

C **22.** Use a CAS to generate a contour plot of

$$f(x, y) = 2y^2x - yx^2 + 4xy$$

for $-5 \le x \le 5$ and $-5 \le y \le 5$, and use the plot to approximate the locations of all relative extrema and saddle points in the region. Check your answer using calculus, and identify the relative extrema as relative maxima or minima.

23–26 True–False Determine whether the statement is true or false. Explain your answer. In these exercises, assume that $f(x, y)$ has continuous second-order partial derivatives and that

$$D(x, y) = f_{xx}(x, y)f_{yy}(x, y) - f_{xy}^2(x, y) \quad ■$$

23. If the function f is defined on the disk $x^2 + y^2 \le 1$, then f has a critical point somewhere on this disk.

24. If the function f is defined on the disk $x^2 + y^2 \le 1$, and if f is not a constant function, then f has a finite number of critical points on this disk.

25. If $P(x_0, y_0)$ is a critical point of f, and if f is defined on a disk centered at P with $D(x_0, y_0) > 0$, then f has a relative extremum at P.

26. If $P(x_0, y_0)$ is a critical point of f with $f(x_0, y_0) = 0$, and if f is defined on a disk centered at P with $D(x_0, y_0) < 0$, then f has both positive and negative values on this disk.

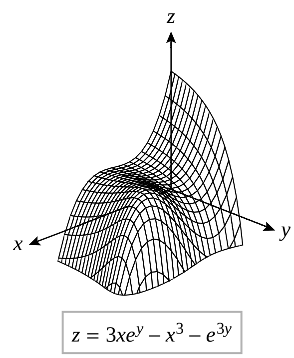
FOCUS ON CONCEPTS

27. (a) Show that the second partials test provides no information about the critical points of the function $f(x, y) = x^4 + y^4$.
(b) Classify all critical points of f as relative maxima, relative minima, or saddle points.

28. (a) Show that the second partials test provides no information about the critical points of the function $f(x, y) = x^4 - y^4$.
(b) Classify all critical points of f as relative maxima, relative minima, or saddle points.

29. Recall from Theorem 4.4.4 that if a continuous function of one variable has exactly one relative extremum on an interval, then that relative extremum is an absolute extremum on the interval. This exercise shows that this result does not extend to functions of two variables.
(a) Show that $f(x, y) = 3xe^y - x^3 - e^{3y}$ has only one critical point and that a relative maximum occurs there. (See the accompanying figure.)
(b) Show that f does not have an absolute maximum.

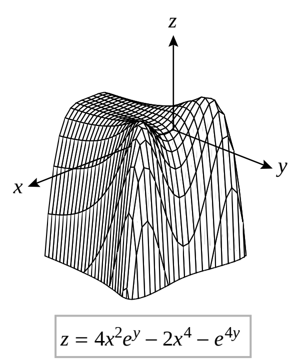
Source: This exercise is based on the article "The Only Critical Point in Town Test" by Ira Rosenholtz and Lowell Smylie, *Mathematics Magazine*, Vol. 58, No. 3, May 1985, pp. 149–150.



◀ **Figure Ex-29**

30. If f is a continuous function of one variable with two relative maxima on an interval, then there must be a relative minimum between the relative maxima. (Convince yourself of this by drawing some pictures.) The purpose of this exercise is to show that this result does not extend to functions of two variables. Show that $f(x, y) = 4x^2e^y - 2x^4 - e^{4y}$ has two relative maxima but no other critical points (see Figure Ex-30).

Source: This exercise is based on the problem "Two Mountains Without a Valley" proposed and solved by Ira Rosenholtz, *Mathematics Magazine*, Vol. 60, No. 1, February 1987, p. 48.



◀ **Figure Ex-30**

31–36 Find the absolute extrema of the given function on the indicated closed and bounded set R. ■

31. $f(x, y) = xy - x - 3y$; R is the triangular region with vertices (0, 0), (0, 4), and (5, 0).