BRAC University

Department of Mathematics & Natural Sciences

Principles of Physics-I (PHY 111)

Solution of Suggested Problems (Vector) — Fall, 2024

Formulas

Vector addition:

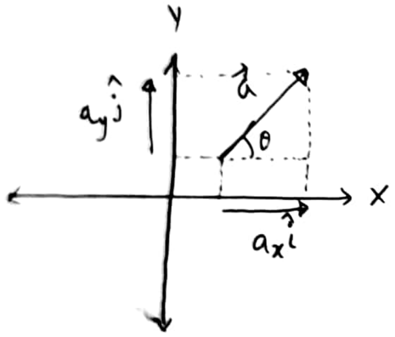
Component

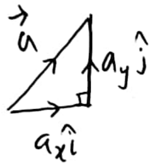
$\vec{c} = \vec{a} + \vec{b}$

Resultant

Unit vector:

$$\hat{a} = \frac{\vec{A}}{|\vec{A}|}$$





$\vec{a} = a_x\hat{i} + a_y\hat{j}$

$a_x = |\vec{a}|\cos\theta$ = x-component of $\vec{a}$

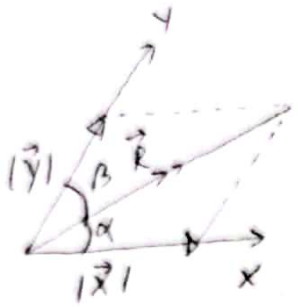
$a_y = |\vec{a}|\sin\theta$ = y- component of $\vec{a}$

$a_x\hat{i}$ = vector component of $\vec{a}$ along the direction of +x-axis

$a_y\hat{j}$ = vector component of $\vec{a}$ along the direction of +y-axis

θ = angle between $\vec{a}$ and +x-axis

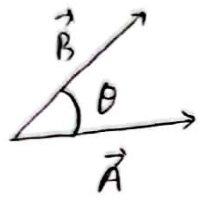
$\tan\theta = \dfrac{a_y}{a_x}$; magnitude of $\vec{a}$, $|\vec{a}| = \sqrt{a_x^2 + a_y^2}$



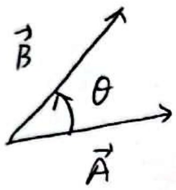
$$|\vec{X}| = \frac{|\vec{R}|\sin\beta}{\sin(\alpha+\beta)}$$

$$|\vec{Y}| = \frac{|\vec{R}|\sin\alpha}{\sin(\alpha+\beta)}$$

<u>Scalar product:</u> $\vec{A}\cdot\vec{B} = |\vec{A}|\,|\vec{B}|\cos\theta$

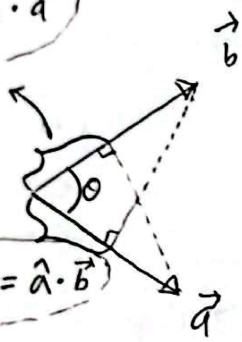


<u>Vector product:</u> $\vec{A}\times\vec{B} = \hat{\eta}\,|\vec{A}|\,|\vec{B}|\sin\theta$



$\hat{\eta} \rightarrow$ unit vector

$|\vec{a}|\cos\theta = \hat{b}\cdot\vec{a}$ $\rightarrow$ component of $\vec{a}$ along the direction of $\vec{b}$

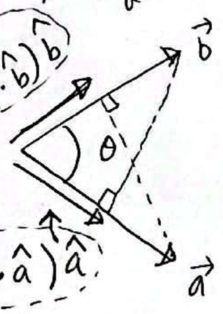


$|\vec{b}|\cos\theta = \hat{a}\cdot\vec{b}$

$\downarrow$

Component of $\vec{b}$ along the direction of $\vec{a}$

$(\vec{a}\cdot\hat{b})\hat{b}$ $\rightarrow$ vector component of $\vec{a}$ along the direction of $\vec{b}$



$(\vec{b}\cdot\hat{a})\hat{a}$

$\downarrow$

vector component of $\vec{b}$ along the direction of $\vec{a}$

$\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$

$\vec{b} = b_x \hat{i} + b_y \hat{j} + b_z \hat{k}$

- $$\vec{a} + \vec{b} = (a_x \hat{i} + a_y \hat{j} + a_z \hat{k}) + (b_x \hat{i} + b_y \hat{j} + b_z \hat{k})$$
$$= (a_x + b_x)\hat{i} + (a_y + b_y)\hat{j} + (a_z + b_z)\hat{k}$$

$$\vec{a} - \vec{b} = (a_x - b_x)\hat{i} + (a_y - b_y)\hat{j} + (a_z - b_z)\hat{k}$$

$$\vec{b} - \vec{a} = (b_x - a_x)\hat{i} + (b_y - a_y)\hat{j} + (b_z - a_z)\hat{k}$$

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

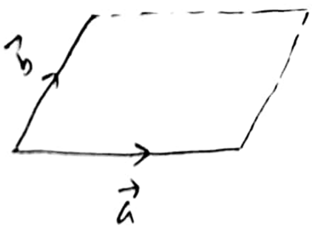
$$\cos(\theta) = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|\,|\vec{b}|}$$

angle between $\vec{a}$ and $\vec{b}$

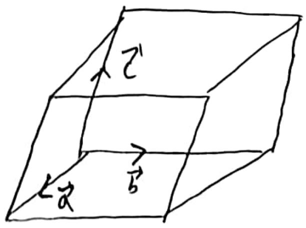
$$|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

$$\left.\begin{aligned} \theta_x &= \cos^{-1}\left(\frac{a_x}{|\vec{a}|}\right) \\ \theta_y &= \cos^{-1}\left(\frac{a_y}{|\vec{a}|}\right) \\ \theta_z &= \cos^{-1}\left(\frac{a_z}{|\vec{a}|}\right) \end{aligned}\right\}$$

$\theta_x, \theta_y, \theta_z \rightarrow$ angle between vector $\vec{a}$ and $+X, +Y, +Z$ axes respectively



$\longrightarrow$ Area of parallelogram $= |\vec{a} \times \vec{b}|$



- Volume of parallelopiped

$$= \vec{a} \cdot \vec{b} \times \vec{c}$$

$$= \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}$$

(Problem No: 17)

1. Three vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ each have a magnitude of 50 m and lie in an xy plane. Their directions relative to the positive direction of the x axis are 30°, 195°, and 315°, respectively. What are (a) the magnitude and (b) the angle of the vector $\vec{a}+\vec{b}+\vec{c}$, and (c)the magnitude and (d) the angle of $\vec{a}-\vec{b}+\vec{c}$? (e)What are the magnitude and (f) angle of a fourth vector $\vec{d}$ such that $(\vec{a}+\vec{b})-(\vec{c}+\vec{d})=0$?

$|\vec{a}| = |\vec{b}| = |\vec{c}| = 50\,m, \quad \theta_1 = 30°, \quad \theta_2 = 195°, \quad \theta_3 = 315°$

$$\vec{a} = a_x \hat{i} + a_y \hat{j} = |\vec{a}| \cos\theta_1 \hat{i} + |\vec{a}| \sin\theta_1 \hat{j}$$

$$= (50)m (\cos 30°) \hat{i} + (50m) \sin(30°) \hat{j}$$

$$= (50m) \frac{\sqrt{3}}{2} \hat{i} + (50m) \frac{1}{2} \hat{j}$$

$$= (25\sqrt{3})\, m\, \hat{i} + (25\, m) \hat{j}$$

$$\vec{b} = b_x \hat{i} + b_y \hat{j} = |\vec{b}| \cos\theta_2 \hat{i} + |\vec{b}| \sin\theta_2 \hat{j}$$

$$= (50m) \cos(195°) \hat{i} + (50m) \sin(195°) \hat{j}$$

$$= (-48.29\, m) \hat{i} + (-12.94\, m) \hat{j}$$

$$\vec{c} = c_x \hat{i} + c_y \hat{j} = |\vec{c}| \cos\theta_3 \hat{i} + |\vec{c}| \sin\theta_3 \hat{j}$$

$$= (50m) \cos(315°) \hat{i} + (50m) \sin(315°) \hat{j}$$

$$= (25\sqrt{2}\, m) \hat{i} + (-25\sqrt{2})\, m\, \hat{j}$$

(a) $$\vec{a}+\vec{b}+\vec{c} = (25\sqrt{3} - 48.29 + 25\sqrt{2})\, m\, \hat{i} + (25 - 12.94 - 25\sqrt{2})\, m\hat{j}$$

$$= \boxed{(30.4\, m)\hat{i} + (-23.3\, m)\hat{j}}$$

$$|\vec{a}+\vec{b}+\vec{c}| = \sqrt{(30.4\, m)^2 + (-23.3\, m)^2} = \boxed{38.3m}$$

(b) $(\vec{a}+\vec{b}+\vec{c})\cdot\hat{i} = |\vec{a}+\vec{b}+\vec{c}|\,|\hat{i}|\cos\theta_x$

$$\Rightarrow \cos\theta_x = \frac{(\vec{a}+\vec{b}+\vec{c})\cdot\hat{i}}{|\vec{a}+\vec{b}+\vec{c}|(1)} = \frac{30.4\text{ m}}{38.3\text{ m}}$$

$$\Rightarrow \theta_x = \cos^{-1}\left(\frac{30.4}{38.3}\right) = 37.5^\circ$$

(c) $\vec{a}-\vec{b}+\vec{c} = (25\sqrt{3}+48.29+25\sqrt{2})\,\text{m}\hat{i} + (25+12.94-\cancel{25.94}\ 25\sqrt{2})\,\text{m}\hat{j}$

$$= (126.95\text{ m})\hat{i} + (2.6\text{ m})\hat{j}$$

$$|\vec{a}-\vec{b}+\vec{c}| = \sqrt{(126.95)^2+(2.6)^2}\ \text{m}$$

$$= \boxed{126.98\ \text{m}}$$

(d) $(\vec{a}-\vec{b}+\vec{c})\cdot\hat{i} = |\vec{a}-\vec{b}+\vec{c}|\,|\hat{i}|\cos\theta_x'$

$$\Rightarrow \cos\theta_x' = \frac{126.95\text{ m}}{126.98\text{ m}}$$

$$\therefore \theta_x' = \cos^{-1}\left(\frac{126.95}{126.98}\right) = \boxed{1.25^\circ}$$

(e) $\vec{a}+\vec{b}-(\vec{c}+\vec{d}) = 0$

$$\Rightarrow \vec{c}+\vec{d} = \vec{a}+\vec{b}$$

$$\Rightarrow \vec{d} = \vec{a}+\vec{b}-\vec{c}$$

$$= (25\sqrt{3}+(-48.29)-25\sqrt{2})\,\text{m}\hat{i} + (25+(-12.94)-(-25\sqrt{2}))\,\text{m}\hat{j}$$

$$= (-40.3)\,\text{m}\hat{i} + (47.4)\,\text{m}\hat{j}$$

$|\vec{d}| = \sqrt{(-40.3\text{ m})^2+(47.4\text{ m})^2} = 62.28$ m; (f) $\theta_x'' = \cos^{-1}\left(-\frac{40.3}{62.28}\right)$

$= \boxed{130}$

(Problem - 35)

2. Two vectors, $\vec{r}$ and $\vec{s}$, lie in the xy plane. Their magnitudes are 4.50 and 7.30 units, respectively, and their directions are 320° and 85.0°, respectively, as measured counterclockwise from the positive x axis. What are the values of (a) $\vec{r}\cdot\vec{s}$ and (b) $\vec{r}\times\vec{s}$?

$|\vec{r}| = 4.5, \quad \theta' = 320°$

$|\vec{s}| = 7.3, \quad \theta'' = 85°$

$$\vec{r} = |\vec{r}|\cos\theta'\,\hat{i} + |\vec{r}|\sin\theta'\,\hat{j}$$

$$= (4.5)\cos(320)\,\hat{i} + (4.5)\sin(320°)\,\hat{j}$$

$$= 3.45\,\hat{i} - 2.89\,\hat{j}$$

$$\vec{s} = |\vec{s}|\cos\theta''\,\hat{i} + |\vec{s}|\sin\theta''$$

$$= (7.3)\cos 85°\,\hat{i} + (7.3)\sin(85°)$$

$$= 0.64\,\hat{i} + 7.27\,\hat{j}$$

(a) $\vec{r}\cdot\vec{s} = (3.45)(0.64) + (-2.89)(7.27)$

$= 2.01 + (-21.01)$

$= \boxed{-19}$

(b)
$$\vec{r}\times\vec{s} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3.45 & -2.89 & 0 \\ 0.64 & 7.27 & 0 \end{vmatrix}$$

$$= \hat{i}\,(-2.89\times 0 - 7.27\times 0)$$
$$- \hat{j}\,(3.45\times 0 - 0.64\times 0)$$
$$+ \hat{k}\,(3.45\times 7.27 + (2.89)\times(.64))$$

$$\vec{r} \times \vec{s} = \hat{k}\ 26.94$$

$$|\vec{r} \times \vec{s}| = |\hat{k}|\ 26.94 = \boxed{26.94}$$

(Problem 37)

3 Three vectors are given by $\vec{a} = 3.0\,\hat{i} + 3.0\,\hat{j} - 2.0\,\hat{k}$, $\vec{b} = -1.0\,\hat{i} - 4.0\,\hat{j} + 2.0\,\hat{k}$, and $\vec{c} = 2.0\,\hat{i} + 2.0\,\hat{j} + 1.0\,\hat{k}$. Find (a) $\vec{a}\cdot(\vec{b}\times\vec{c})$, (b) $\vec{a}\cdot(\vec{b}+\vec{c})$, and (c) $\vec{a}\times(\vec{b}+\vec{c})$.

$$\vec{a} = 3\hat{i} + 3\hat{j} - 2\hat{k}$$
$$\vec{b} = -\hat{i} - 4\hat{j} + 2\hat{k}$$
$$\vec{c} = 2\hat{i} + 2\hat{j} + \hat{k}$$

(a) $$\vec{a}\cdot\vec{b}\times\vec{c} = \begin{vmatrix} 3 & 3 & -2 \\ -1 & -4 & 2 \\ 2 & 2 & 1 \end{vmatrix}$$

$$\boxed{\begin{aligned} &= 3(-4-4) - \cancel{6(\ldots)} - 2(-2+8) \\ &= 24 - \cancel{18} - 12 \end{aligned}}$$

$$= 3(-4-4) - 3(-1-4) - 2(-2+8)$$

$$= -18 + 15 - 12 = \boxed{-21}$$

(b) $$\vec{b}+\vec{c} = (-1+2)\,\hat{i} + (-4+2)\,\hat{j} + (2+1)\,\hat{k}$$
$$= \hat{i} - 2\hat{j} + 3\hat{k}$$

$$\vec{a}\cdot(\vec{b}+\vec{c}) = (3)(1) + (3)(-2) + (-2)(3)$$
$$= 3 - 6 - 6 = \boxed{-9}$$

(c) $$\vec{a}\times(\vec{b}+\vec{c}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 3 & -2 \\ 1 & -2 & 3 \end{vmatrix}$$

$$= \hat{i}(9-4) - \hat{j}(9+2) + \hat{k}(-6-3)$$
$$= \boxed{5\hat{i} - 11\hat{j} - 9\hat{k}}$$

(Problem 63)

$\boxed{4}$. Here are three vectors in meters: $\vec{d_1} = -3.0\hat{i} + 3.0\hat{j} + 2.0\hat{k}$, $\vec{d_2} = -2.0\hat{i} - 4.0\hat{j} + 2.0\hat{k}$, $\vec{d_3} = 2.0\hat{i} + 3.0\hat{j} + 1.0\hat{k}$ What results from: (a) $\vec{d_1} \cdot (\vec{d_2} + \vec{d_3})$ (b) $\vec{d_1} \cdot (\vec{d_2} \times \vec{d_3})$ (c) $\vec{d_1} \times (\vec{d_2} + \vec{d_3})$?

$\vec{d_1} = (-3\hat{i} + 3\hat{j} + 2\hat{k})\,m$

$\vec{d_2} = (-2\hat{i} - 4\hat{j} + 2\hat{k})\,m$

$\vec{d_3} = (2\hat{i} + 3\hat{j} + \hat{k})\,m$

(a) $(\vec{d_2} + \vec{d_3}) = [(-2+2)\hat{i} + (-4+3)\hat{j} + (2+1)\hat{k}]\,m$

$= [-\hat{j} + 3\hat{k}]\,m$

$\vec{d_1} \cdot (\vec{d_2} + \vec{d_3}) = [(-3)(0) + (+3)(-1) + (2)(3)]\,m^2$

$= m^2[-3+6] = \cancel{8} \boxed{3\,m^2}$

(b) $\vec{d_1} \cdot \vec{d_2} \times \vec{d_3} = \begin{vmatrix} -3 & 3 & 2 \\ -2 & -4 & 2 \\ 2 & 3 & 1 \end{vmatrix}$

$= (-3)(-4-6) - 3(-2-4) + 2(-6+8)$

$= (-3)(-10) - 3(-6) + 2(2)$

$= 30 + 18 + 4 = \boxed{52\,m^3}$

(c) $\vec{d_1} \times (\vec{d_1} + \vec{d_2}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 3 & 2 \\ 0 & -1 & 3 \end{vmatrix}$

$$\vec{d}_1 \times (\vec{d}_2 \times \vec{d}_3) = \hat{i}\,(9+2) - \hat{j}\,(-9-0) + \hat{k}\,(3-0)$$

$$= \boxed{(11\,\hat{i} + 9\,\hat{j} + 3\,\hat{k})\ m^2}$$