PHY-112 | Principles of Physics-2

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What is our intention with Gauss's Law?

Gauss's Law and Electric fields

How much field through an area?

The total Φ_E passing through a closed surface is proportional to the total electric charge Q_{enc} enclosed within that surface.

$$\oint_S \Phi_E = \oint_S \vec{E} \cdot d\vec{a} = \frac{Q_{\text{enclosed}}}{\epsilon_0}. \qquad \text{(Integral Form)}$$

$$\text{where} \quad \epsilon_0 = 8.854 \times 10^{-12}\,\text{F}\,\text{m}^{-1}$$

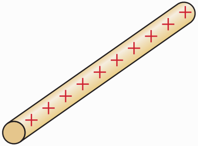
$$\rightarrow \text{Permittivity of free space}$$

Note: Electric Flux Is Independent of Surface Shape and Radius.

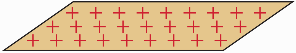
3 Key $\vec{E}$ Field Sources that we will study

Keep the intensity the same all over

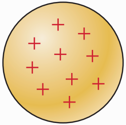
Line Charge



Surface Charge



Volume Charge



The template we shall follow
To simplify our field calculation. For now!!

$$\oint_S \Phi_E = \oint_S \vec{E} \cdot d\vec{a} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

$$\oint E da \cos\phi = \frac{Q_{\text{enc}}}{\epsilon_0}$$

$$\pm E \oint da = \frac{Q_{\text{enc}}}{\epsilon_0}; \ \ \text{provided} \ \ \cos\phi = \pm 1$$

$$EA = \frac{Q_{\text{enc}}}{\epsilon_0}$$

$$E = \frac{1}{A} \times \frac{Q_{\text{enc}}}{\epsilon_0};$$

$$\text{where} \ \ Q_{\text{enc}} = \int dq_{\text{enc}} = \text{charge density} \times \text{distribution element}$$

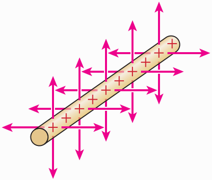
The template we shall follow
To simplify our field calculation. For now!!

TLDR: We want to choose the **Gaussian Surfaces** to be symmetric with the source field. These are the suitable surfaces across where E of the source and are parallel, and the intensity is uniform.

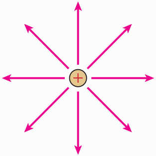
Note: These *symmetric* surfaces are called Equipotential surfaces. We will see what they mean in the potential chapter of the story.

$\vec{E}$ Field for an Infinitely long Line Charge

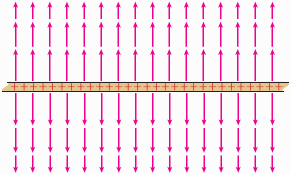
Charge is uniformly distributed across a line



3D view

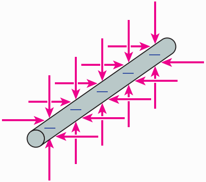


End view

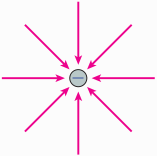


Side view

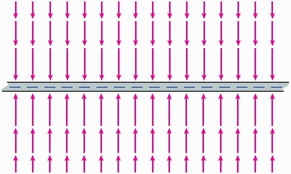
$$\lambda = \frac{Q}{L}$$



3D view



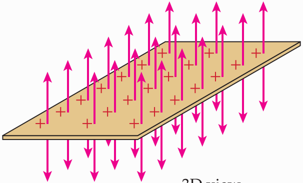
End view



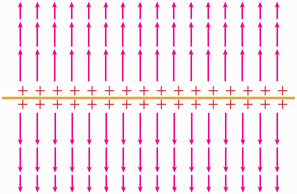
Side view

$\vec{E}$ Field for an Infinitely distributed Surface Charge

Charge is uniformly distributed across a surface

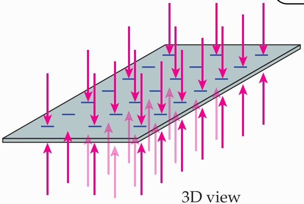


3D view

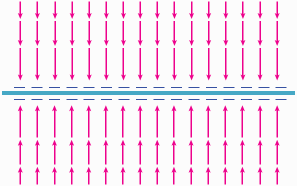


End view = Side view

$$\sigma = \frac{Q}{A}$$

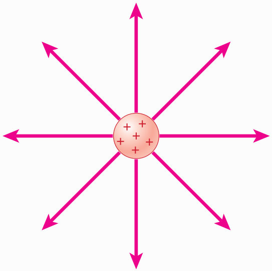


3D view



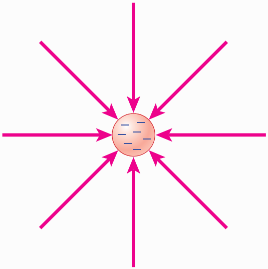
End view = Side view

$\vec{E}$ Field for a Volume Charge
Charge is uniformly distributed across a volume



2D view

$$\boxed{\rho = \frac{Q}{V}}$$



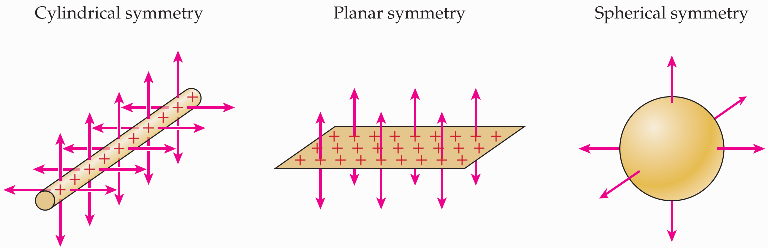
2D view

Field acts radially outward (positive), or inward (negative)

Suitable Gaussian Surfaces

Suitable Gaussian Surfaces for 3 Key $\vec{E}$ Field Sources

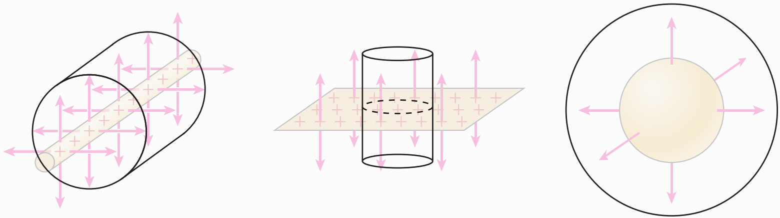
Keep the intensity the same all over



Suitable Gaussian Surfaces for 3 Key $\vec{E}$ Field Sources

Keep the intensity the same all over

Cylindrical symmetry　　Planar symmetry　　Spherical symmetry



Suitable Gaussian Surfaces for 3 Key $\vec{E}$ Field Sources
Glance over at one go

- Line Charge $\longrightarrow$ Current Wire $\longrightarrow$ Long Cylindrical
- Surface Charge $\longrightarrow$ Capacitors $\longrightarrow$ Wide Planar
- Volume Charge $\longrightarrow$ Electrodes/Shell Charges $\longrightarrow$ Spherical

Now shut up! and Calculate

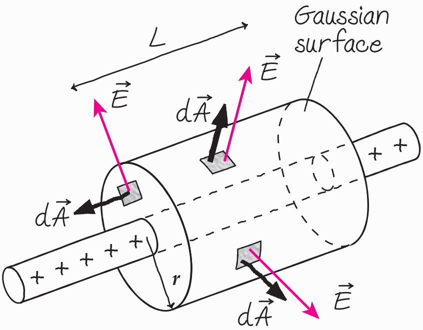
How to apply Gauss's Law to measure $\vec{E}$ fields

One size fits all recipe for easier calculation

1. Check the source and figure out the symmetry of its $\vec{E}$ fields
2. Choose a Gaussian surface matching the field symmetry
3. Calculate the Q_{enc} enclosed by the chosen surface
4. Collect only the area portion of the chosen surface that can record flux, discard the rest
5. Confine everything to one side, keeping E on the left after You have applied Gauss's law. Et voilà!!

$\vec{E}$ Field for an Infinitely long Line Charge: Positive

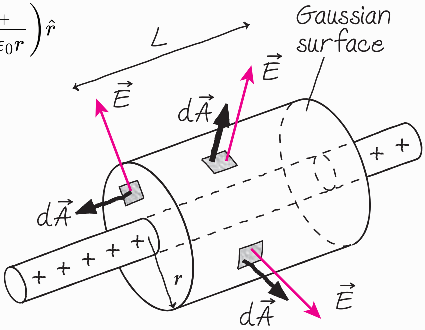
Positive Charge is distributed uniformly over a surface



$\vec{E}$ Field for an Infinitely long Line Charge: Positive

Positive Charge is distributed uniformly over a surface

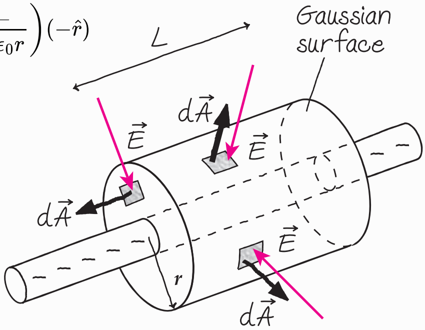
$$\vec{E} = \left(\frac{\lambda_+}{2\pi\epsilon_0 r}\right)\hat{r}$$



$\vec{E}$ Field for an Infinitely long Line Charge: Negative

Negative Charge is distributed uniformly over a surface

$$\vec{E} = \left(\frac{\lambda_-}{2\pi\epsilon_0 r}\right)(-\hat{r})$$



Incepting Ideas (1)

Hint: λ is fixed

Q: A thin, horizontal ($y = 0$), 50 m-long copper wire is charged to $+3.5$ nC. The charge is uniformly distributed on the wire.

- Find $\vec{E}$ at $y = 3$ mm.
- Find $\vec{E}$ at $y = -4$ mm.
- Find $\vec{E}$ at $(x = 3, y = 3)$ mm.
- Find $\vec{E}$ at $(x = 3, y = -6)$ mm.

Alternative parameters to try

Q: A thin, vertical ($x = 0$), 50 m-long copper wire is charged to -3.5 nC. The charge is uniformly distributed on the wire.

Incepting Ideas (2)

Hint: λ is fixed

Q: Two very long uniform lines of charge are vertically parallel and are horizontally separated by 0.0400 mm. Each line of charge has a charge per unit length $+5.20\ \mu\mathrm{C\,m^{-1}}$. What magnitude of force does one line of charge exert on a 0.0400 mm section of the other line of charge? Pick either line to be the source.

Alternative parameters to try the same problem

Q: Two very long uniform lines of charge are vertically parallel and are horizontally separated by 0.0400 mm. Each line of charge has a charge per unit length $\pm 5.20\ \mu\mathrm{C\,m^{-1}}$, respectively, from left to right. What magnitude of force does one line of charge exert on a 0.0400 mm section of the other line of charge? Pick either line to be the source.

Incepting Ideas (3)

Hint: λ is fixed

Homework Practice Problem: Try it Yourself

Q: A very long uniform line of charge has a charge per unit length $4.80\ \mu\text{C}$ and lies along the x-axis. A second long uniform line of charge has charge per unit length $-2.40\ \mu\text{C}$ and is parallel to the x-axis at $y = 0.400$ m. What is the net electric field (magnitude and direction) at the following points on the y-axis: (a) $y = 0.200$ m and (b) $y = 0.800$ m? (Exercise 22.17, p-770 | Young-Freedman)

Q: The electric field 0.335 m from a very long uniform line of charge is $840\ \text{N}\,\text{C}^{-1}$. How much charge is contained in a section of the line of length 2.80 cm? (Exercise 22.18, p-770 | Young-Freedman)

That is it for today!