Integration

1. (i) $\int x^n\,dx = \frac{x^{n+1}}{n+1} + c,\ n \neq -1$

   (ii) $\int dx = x + c$

   (iii) $\int k\,dx = kx + c$

2. (i) $\int \frac{1}{x}\,dx = \ln|x| + c$

   (ii) $\int \frac{1}{x^2}\,dx = -\frac{1}{x} + c$

   (iii) $\int \frac{1}{2\sqrt{x}}\,dx = \sqrt{x} + c$

3. $\int e^x\,dx = e^x + c$

4. $\int e^{mx}\,dx = \frac{1}{m}e^{mx} + c$

5. $\int a^x\,dx = \frac{a^x}{\ln a} + c$

6. (i) $\int \sin x\,dx = -\cos x + c$

   (ii) $\int \sin mx\,dx = -\frac{1}{m}\cos mx + c$

7. (i) $\int \cos x\,dx = \sin x + c$

   (ii) $\int \cos mx\,dx = \frac{1}{m}\sin mx + c$

8. $\int \tan x\,dx = \ln|\sec x| + c$

9. $\int \cot x\,dx = \ln|\sin x| + c$

10. $\int \sec x\,dx = \ln|\sec x + \tan x| + c$

    $= \ln\left|\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right| + c$

11. $\int \operatorname{cosec} x\,dx = -\ln|\operatorname{cosec} x + \cot x| + c$

    $= \ln\left|\tan\frac{x}{2}\right| + c$

12. $\int \sec^2 x\, dx = \tan x + c$

13. $\int \operatorname{cosec}^2 x\, dx = -\cot x + c$

14. $\int \sec x \tan x\, dx = \sec x + c$

15. $\int \operatorname{cosec} x \cot x\, dx = -\operatorname{cosec} x + c$

16. (i) $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$

(ii) $\int \frac{dx}{1 + x^2} = \tan^{-1} x + c$

17. (i) $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + c$

(ii) $\int \frac{dx}{\sqrt{1 - x^2}} = \sin^{-1} x + c$

18. $\int \frac{dx}{x\sqrt{x^2 - 1}} = \sec^{-1} x + c$

19. $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left| \frac{x - a}{x + a} \right| + c$

20. $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left| \frac{a + x}{a - x} \right| + c$

21. $\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + c$

22. $\int \frac{f'(x)}{\sqrt{f(x)}} dx = 2\sqrt{f(x)} + c$

23. $\int uv\, dx = u \int v dx - \int \left\{ \frac{d}{dx}(u) \int v\, dx \right\} dx$

24. $\int 0\, dx = c$

25. $\int \sqrt{a^2 - x^2}\, dx = \frac{x\sqrt{a^2 - x^2}}{2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + c$

26. $\int \sqrt{x^2 - a^2}\, dx = \frac{x\sqrt{x^2 - a^2}}{2} - \frac{a^2}{2} \ln \left| x + \sqrt{x^2 - a^2} \right| + c$

27. $\int \sqrt{x^2 + a^2}\, dx = \frac{x\sqrt{x^2 + a^2}}{2} + \frac{a^2}{2} \ln \left| x + \sqrt{x^2 + a^2} \right| + c$

28. $\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln \left| x + \sqrt{x^2 + a^2} \right| + c$

29. $\int \frac{dx}{\sqrt{x^2 - a^2}} = \ln \left| x + \sqrt{x^2 - a^2} \right| + c$

30. The area bounded by the graph of $y = f(x)$, the lines $x = a$, $x = b$ and x-axis is $= \int_a^b y\, dx$

31. The area bounded by the graph of $x = f(y)$, the lines $y = c$, $y = d$ and y-axis is $= \int_c^d x\, dy$

32. The area bounded by the graphs of $y_1 = f_1(x)$, $y_2 = f_2(x)$ and the lines $x = a$, $x = b$ is $= \int_a^b (y_1 - y_2)\, dx$

33. The area bounded by the graphs of $x_1 = f_1(y)$, $x_2 = f_2(y)$ and the lines $y = c$, $y = d$ is $= \int_c^d (x_1 - x_2)\, dy$