PHY 111: Principles of Physics I Summer 2023

LECTURE 12 — July 13, 2023

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1 The Dynamics of Uniform Circular Motion

Recall that a uniform circular motion occurs when an object moves along a circular path with a constant speed. In this motion, the direction of the object's velocity vector changes continuously, but the magnitude of the velocity vector remains constant. The object is always accelerating because its velocity vector changes and the acceleration is directed toward the circle's center. This acceleration is called *centripetal acceleration*. We dealt with this *kinematics* part of the motion. Now we expand upon its dynamics.

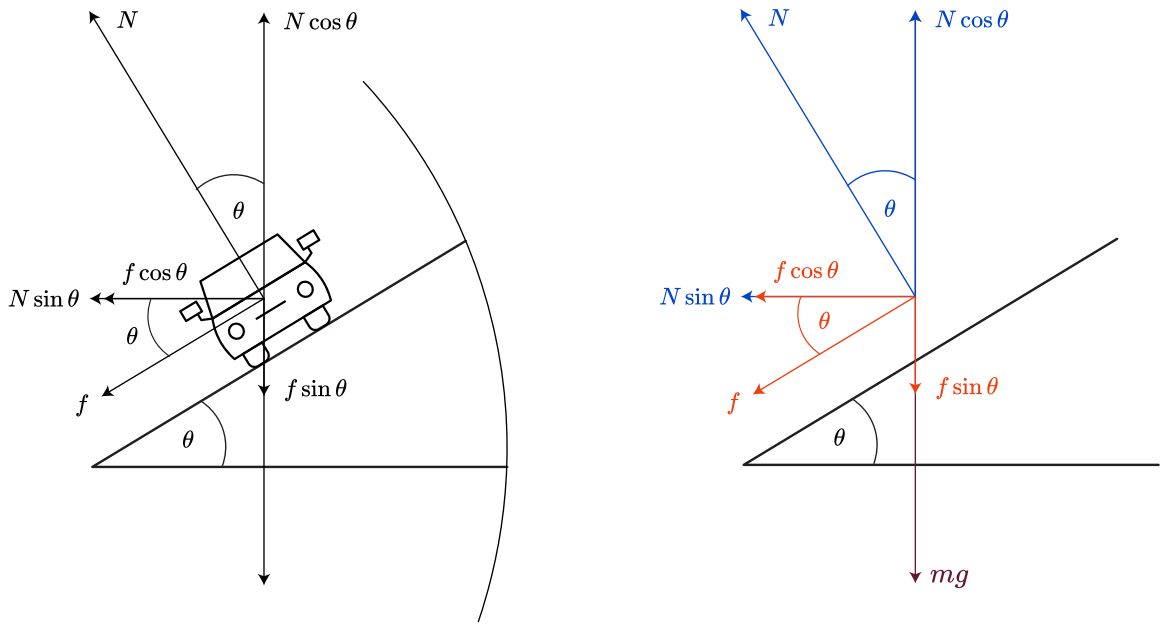
Imagine a car moving around a curve, a satellite orbiting the Earth, or a person on a carnival ride. It has a centripetal force to retain the circular motion. Newton's 2$^{\text{nd}}$ law demands that any acceleration should be accompanied by a change in momentum, which subsequently requires an action of a force. In this case, the object is subject to a centripetal force that keeps it moving in a circle. The magnitude of the centripetal force depends on the object's speed v, the radius of the circle R, and the object's mass m.

Remember, the centripetal acceleration of an object in a circular trajectory is given by: $a_{\perp} = \dfrac{v^2}{R}$. Multiplying this equation with the mass of the object gives us the measurement of the *centripetal force*.

$$F_c = \frac{mv^2}{R} = \frac{4m\pi^2 R}{T^2}. \tag{1}$$

2 Banking of the Circular/Curved Path

The banking of a road refers to tilting the road at a certain angle with respect to the horizontal axis. This angle is called the *bank angle* or the *angle of banking*. When a vehicle moves on a banked road, it experiences a centripetal force that helps it turn around the curve.



Practical scenario for banking

Free-body diagram

FIGURE 1: Force contribution for the banking of a road.

The need for banking on the road arises when we want to ensure the safety of the vehicles and the passengers travelling on them. When a car moves on a curved road, it experiences a pseudo force called *centrifugal force*. This force tries to pull the car outwards from the curve and can lead to a loss of control of the vehicle. The centripetal force is a force that acts on a body moving in a circle and pulls it toward the centre of the circle.

NOTE: **Remember, the centrifugal force is not real. It's a term to describe the sensation of pulling the object feels. There exists no such force in practice.**

On a flat road, the only source of the centripetal force is the friction between the vehicle's tires and the road surface. This friction force works sideways to the tire. This causes the tire to erode faster and increases the chances of accidents. However, on a banked road, the component of the weight of the vehicle perpendicular to the road surface can also provide a part of the required centripetal force, keeping the vehicle stable on the road.

From the figure (1), we find the contributions of the forces and check Newton's 2nd law of motion,

$$N\cos\theta = mg + f\sin\theta \tag{2}$$

$$N\sin\theta + f\cos\theta = \frac{mv^2}{R} \tag{3}$$

The angle θ of the banking of a road is determined based on the speed of the vehicles that are expected to travel on it. When a car moves on a banked road, the net force acting on it is the vector sum of the gravitational force mg and the normal force. The normal force has two

components, one perpendicular to the road surface and the other parallel to it. The component of the normal force perpendicular to the road surface provides the necessary support to the weight of the vehicle, while the component parallel to the road surface provides the required centripetal force for the car to turn around the curve.

$$\theta = \tan^{-1}\left(\frac{a_{\text{rad}}}{g}\right) = \tan^{-1}\left(\frac{v^2}{gR}\right). \tag{4}$$

Intuitively, the banking of the road allows the vehicle's weight to provide a part of the necessary centripetal force, which reduces the load on the friction between the vehicle's tires and the road surface.

This, in turn, reduces the wear and tear of the tires and road surface and the chance of skidding or slipping vehicles. The banked road also helps to increase the speed at which the vehicle can safely travel around the curve. This is because the necessary centripetal force is provided by both the friction and the component of the weight of the vehicle perpendicular to the road surface. Thus, the banking of the road is an important factor in ensuring the safety and efficiency of the vehicles and the passengers traveling on it.