**PHY 111: Principles of Physics I** Summer 2023

# LECTURE 14 — August 5, 2023

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## 1 Center of Mass

The center of mass of an object is a point where the entire mass of the object can be considered to be concentrated. It is the average position of all the mass of an object. The center of mass of an object is a handy concept in physics because it simplifies the analysis of the motion of an object. The motion of an object can be treated as if it were a point mass located at its center of mass.

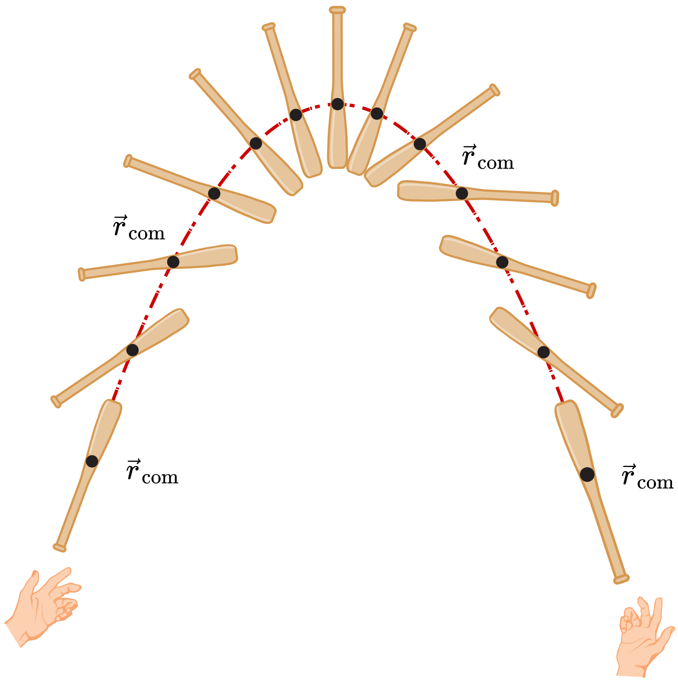
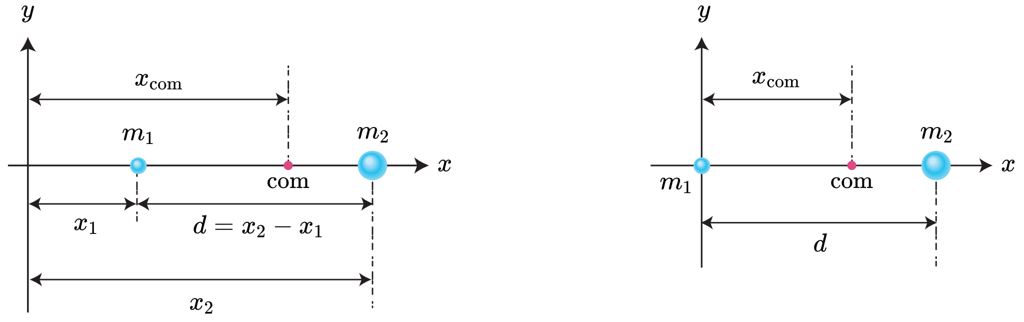
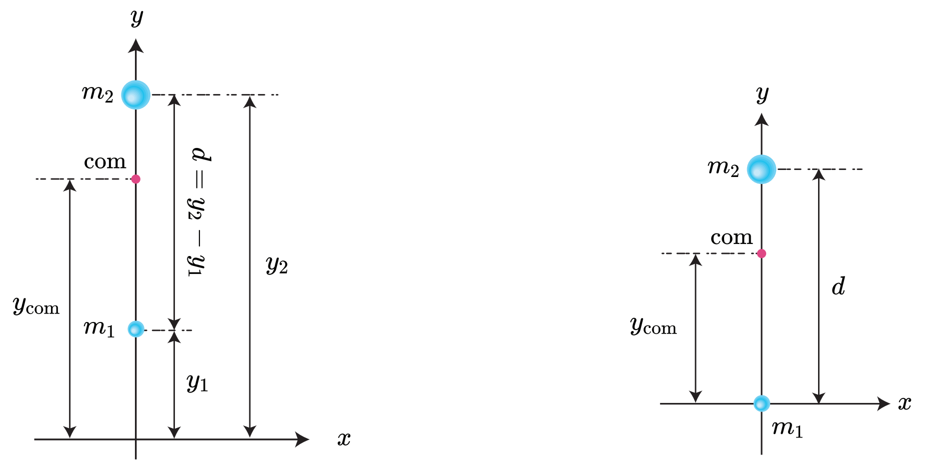



FIGURE 1: A juggler throws a juggling pin into the air and follows a parabolic path. The center of mass (**COM**) (black dot) of the pin flipped into the air follows a parabolic path, but all other points of the bat follow more complicated curved paths.

Intuitively, the center of mass is like the balancing point of an object. If you try to balance a stick on your hand, the point where it balances is the center of mass.




(a)




(b)

FIGURE 2: (a) Two particles of masses $m_1$ and $m_2$ are separated by distance $d = x_2 - x_1$. The dot labeled com shows the position of the center of mass from the origin. (a) The same as (a) except that $m_1$ is located at the origin. We use this scenario to find the $x_{\rm com}$ and $y_{\rm com}$.

The center of mass of a two-body system is defined as

$$x_{\rm com} = \left(\frac{m_2}{m_1 + m_2}\right) d. \tag{1}$$

Notice one of the bodies is placed at the origin of the reference frame. If it were not, then the center of mass would be at

$$x_{\rm com} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}. \tag{2}$$

For a many-particle system, this takes the following form:

$$x_{\rm com} = \frac{m_1 x_1 + m_2 x_2 + \ldots \ldots + m_n x_n}{M} = \frac{1}{M} \sum_{i=1}^{n} m_i x_i. \tag{3}$$

where $M = \sum_{i=1}^{n} m_i$. Extending this to 3D, we get the center of mass of a many-particle system.

$$\vec{r}_{\text{com}} = x_{\text{com}}\hat{i} + y_{\text{com}}\hat{j} + z_{\text{com}}\hat{k} \tag{4}$$

$$= \hat{i}\frac{1}{M}\sum_{i=1}^{n} m_i x_i + \hat{j}\frac{1}{M}\sum_{i=1}^{n} m_i y_i + \hat{k}\frac{1}{M}\sum_{i=1}^{n} m_i z_i. \tag{5}$$

## 2 Rigid Body

A rigid body is an idealized object that is assumed to have a fixed shape and size and to maintain this shape and size under any external loads or forces. In other words, a rigid body does not deform or change shape, even when subjected to external forces.

For a rigid body, the motion of the body can be analyzed as if it were a single-point mass located at its center of mass.

The center of mass for a rigid body is defined as:

$$\vec{r}_{\text{com}} = \frac{1}{M}\left(\hat{i}\int xdm + \hat{j}\int ydm + \hat{k}\int zdm\right). \tag{6}$$

The integrals effectively allow us to use the above formula for many particles, an effort that otherwise would take many years.

## 3 Linear Momentum

Linear momentum is a measure of the motion of an object. It is the product of the mass and velocity of an object. In other words, the linear momentum of an object is the quantity of motion it possesses. The linear momentum of an object is conserved in the absence of external forces. This means that the total linear momentum of a system of objects remains constant, provided no external forces act on the system.

The linear momentum of a system of particles can be defined by differentiating Eq. (5) once.

$$M\frac{d}{dt}\vec{r}_{\text{com}} = \frac{d}{dt}\left(x_{\text{com}}\hat{i} + y_{\text{com}}\hat{j} + z_{\text{com}}\hat{k}\right)$$

$$\Rightarrow M\vec{v}_{\text{com}} = \frac{d}{dt}\left(\hat{i}\sum_{i=1}^{n} m_i x_i + \hat{j}\frac{1}{M}\sum_{i=1}^{n} m_i y_i + \hat{k}\frac{1}{M}\sum_{i=1}^{n} m_i z_i\right)$$

$$\therefore \vec{p}_{\text{com}} = \hat{i}\sum_{i=1}^{n} m_i v_{xi}^{\text{com}} + \hat{j}\sum_{i=1}^{n} m_i v_{yi}^{\text{com}} + \hat{k}\sum_{i=1}^{n} m_i v_{zi}^{\text{com}}$$

$$= \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \ldots\ldots + \vec{p}_n. \tag{7}$$

Time differentiating Eq. (7) once we revive Newton's 2nd law of motion for a system of particles.

$$\frac{d\vec{p}_{\text{com}}}{dt} = \vec{F}_{\text{com}} = M\vec{a}_{\text{com}} = M\left(\vec{a}_1 + \vec{a}_2 + \ldots\ldots + \vec{a}_n\right) = \vec{F}_1 + \vec{F}_2 + \ldots\ldots + \vec{F}_n = \vec{F}_{\text{net}}, \quad (8)$$

where $\vec{F}_{\text{com}}$ is the net force applied on the rigid body's center of mass. It is also the net force applied to the body.

### 3.1 Impulse

Intuitively, the impulse of a force is like the ***kick*** that a force gives to an object. Imagine you're playing baseball and want to give the ball more speed. You could just swing the bat harder, but another way to do it is to apply force to the ball for a longer period of time. By doing this, you increase the impulse of the force on the ball, increasing the ball's speed.

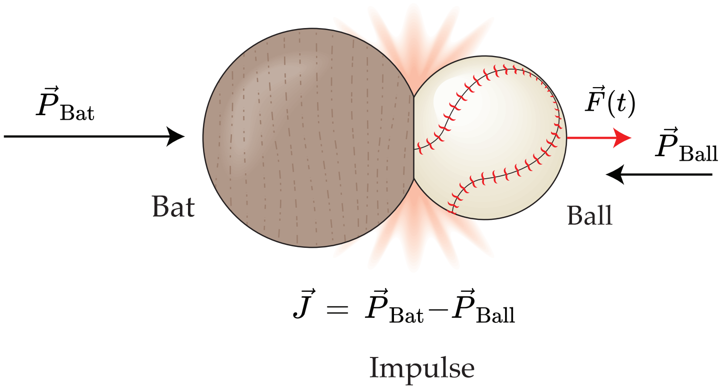



FIGURE 3: The impulse $\vec{J}$ of the net force $\vec{F}(t)$ applied by the bat on the ball equals the change in momentum $\Delta\vec{P} = \vec{P}_{\text{Bat}} - \vec{P}_{\text{Ball}}$, within the system.

In practical terms, the impulse of a force is defined as the product of the force applied to an object and the duration of time for which the force is applied. Mathematically, it can be expressed as:

$$J = F_{\text{avg}}\Delta t. \quad (9)$$

The physical significance of impulse is that it represents the change in momentum of an object that results from applying a force. In other words, the impulse of a force is equal to the change in momentum of an object that results from that force. This is expressed mathematically as:

$$\vec{J} = \vec{p}_f - \vec{p} = \Delta\vec{p}. \quad (10)$$

An impulse is a measure of the change in momentum of an object. It is the product of the force and the time for which the force acts on the object. In other words, the impulse is the quantity of motion imparted to an object by force. The **impulse-momentum theorem**, stated by Eq. (10), says that the impulse on an object is equal to the object's momentum change. This means that the impulse on an object can cause a change in the momentum of the object.

For a single collision, the impulse becomes,

$$\vec{J} = \int_{p_i}^{p_f} d\vec{p} = \int_{t_f}^{t_i} \vec{F}(t)dt = \hat{\imath}\int_{t_f}^{t_i} F_x dt + \hat{\jmath}\int_{t_f}^{t_i} F_y dt + \hat{k}\int_{t_f}^{t_i} F_z dt. \tag{11}$$

### 3.2 Conservation of Linear Momentum

The conservation of linear momentum is a fundamental principle in physics that states that ***the total momentum of a closed system remains constant, provided no external forces act on the system***. Intuitively, this means that a system's total *quantity of motion* remains constant without external forces. This principle applies to all types of motion, including the motion of objects in space, the motion of subatomic particles, and the motion of everyday objects like cars and balls.

$$\Delta\vec{p} = 0; \qquad \text{if } \sum \vec{F}_{\text{ext}} = 0 \text{ (Closed, Isolated system)} \tag{12}$$

To understand the physical significance of the conservation of linear momentum, consider the following example. Imagine two billiard balls of equal mass moving toward each other at equal speed. When the balls collide, they rebound in opposite directions with equal speed. According to the principle of conservation of linear momentum, the system's total momentum of two balls must be conserved. This means that the momentum of one ball is transferred to the other during the collision, and the system's total momentum remains constant.

$$\begin{aligned} \Delta\vec{p} &= \vec{p}_f - \vec{p}_i = 0 \\ \therefore \vec{p}_f &= \vec{p}_i \end{aligned} \tag{13}$$

$$\begin{pmatrix} \text{total linear momentum} \\ \text{at some initial time } t_i \end{pmatrix} = \begin{pmatrix} \text{total linear momentum} \\ \text{at some final time } t_f \end{pmatrix} \tag{14}$$

CAUTION: Momentum should not be confused with energy. In the sample problems of this module, momentum is conserved, but energy is definitely not.

Another example of the conservation of linear momentum is the recoil of a gun. When a bullet is fired from a gun, the gun recoils in the opposite direction. This is because the bullet and gun have equal and opposite momenta, and the system's total momentum must remain constant. This principle is also used in rocket propulsion, where fuel is expelled in one direction to generate thrust in the opposite direction, propelling the rocket forward.

# 4 Collision

A collision in physics occurs when two or more objects come into contact with each other and exchange energy and/or momentum. Several types of collisions can occur, each of which is characterized by how the objects involved interact with each other.

$$\begin{aligned} \vec{p}_{1i} + \vec{p}_{2i} &= \vec{p}_{1f} + \vec{p}_{2f} \\ \Rightarrow m_1\vec{v}_{1i} + m_2\vec{v}_{2i} &= m_1\vec{v}_{1f} + m_2\vec{v}_{2f} \qquad \text{(Conservation of Linear Momentum)} \qquad (15) \\ K_{1i} + K_{2i} &= K_{1f} + K_{2f} \\ \Rightarrow \frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 &= \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2 \qquad \text{(Conservation of Energy)} \qquad (16) \end{aligned}$$

## 4.1 Elastic Collision

In an elastic collision, the total kinetic energy of the objects involved is conserved. This means that the objects rebound off each other with no loss of energy, and their velocities are exchanged without any permanent deformation. An everyday example of an elastic collision is when two pool balls collide on a pool table. If the collision is perfectly elastic, the balls will bounce off each other with no energy loss, and their velocities will be exchanged without any deformation of the balls. In one dimension, when the second body is the target, and the first body is a projectile, then the conservation of linear momentum and energy leads up to:

$$\vec{v}_{1f} = \frac{m_1 - m_2}{m_1 + m_2}\vec{v}_{1i} + \frac{2m_1}{m_1 + m_2}\vec{v}_{2i}, \qquad (17)$$

$$\text{and} \quad \vec{v}_{2f} = \frac{2m_1}{m_1 + m_2}\vec{v}_{1i} + \frac{m_2 - m_1}{m_1 + m_2}\vec{v}_{2i}. \qquad (18)$$

LIMITING CASES: $m_1 = m_2$:

$$\vec{v}_{1f} = \vec{v}_{2i}, \quad \text{and} \quad \vec{v}_{2f} = \vec{v}_{1i} \qquad (19)$$

Eq. (19) means the two bodies will only exchange velocities. $\vec{v}_{2i} = 0$:

$$\vec{v}_{1f} = \frac{m_1 - m_2}{m_1 + m_2}\vec{v}_{1i}, \quad \text{and} \quad \vec{v}_{2f} = \frac{2m_1}{m_1 + m_2}\vec{v}_{1i} \qquad (20)$$

Eq. (20) means the two bodies will only exchange velocities twice the amount.
$m_2 \gg m_1$:

$$\vec{v}_{1f} \simeq -\vec{v}_{2i}, \quad \text{and} \quad \vec{v}_{2f} \simeq 2\vec{v}_{1i} \qquad (21)$$

Eq. (21) means the first body will recoil toward the incident direction with the second body's velocity, and the second body will take off with twice the velocity of the first body.

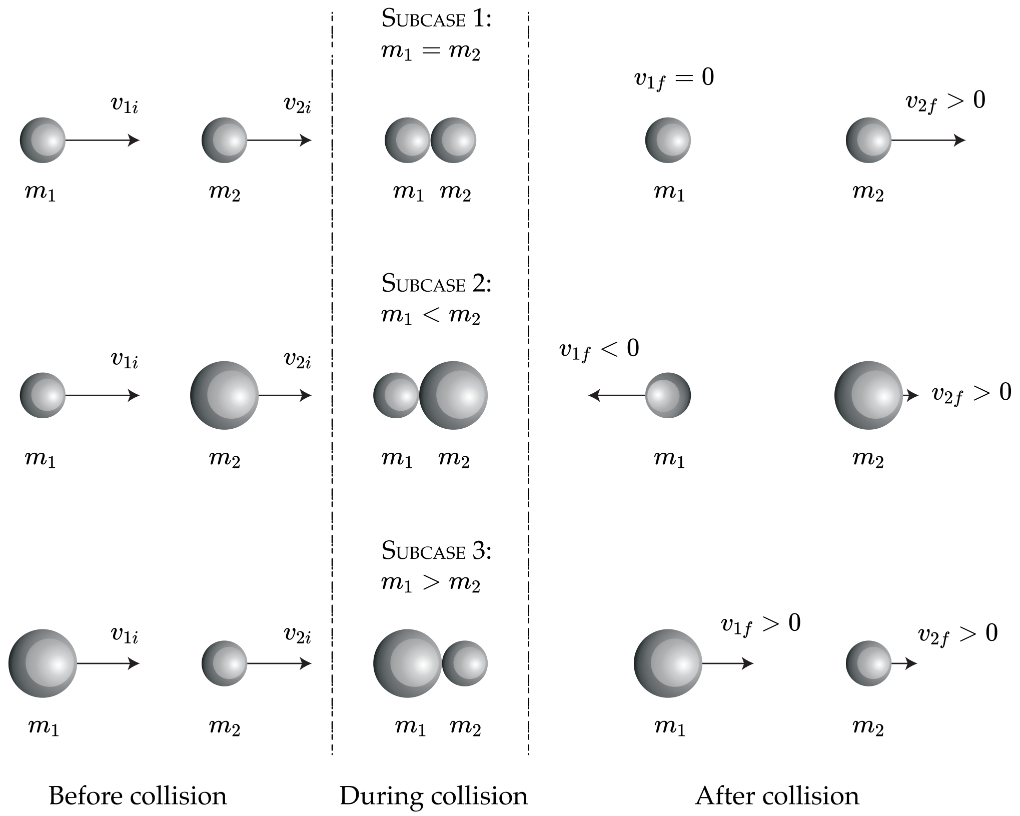



FIGURE 4: After a perfectly elastic collision where both bodies are under relative motion initially, they will have individual initial velocities of their own. After the collision, both moves with their own final velocities given by Eq.(17-18).

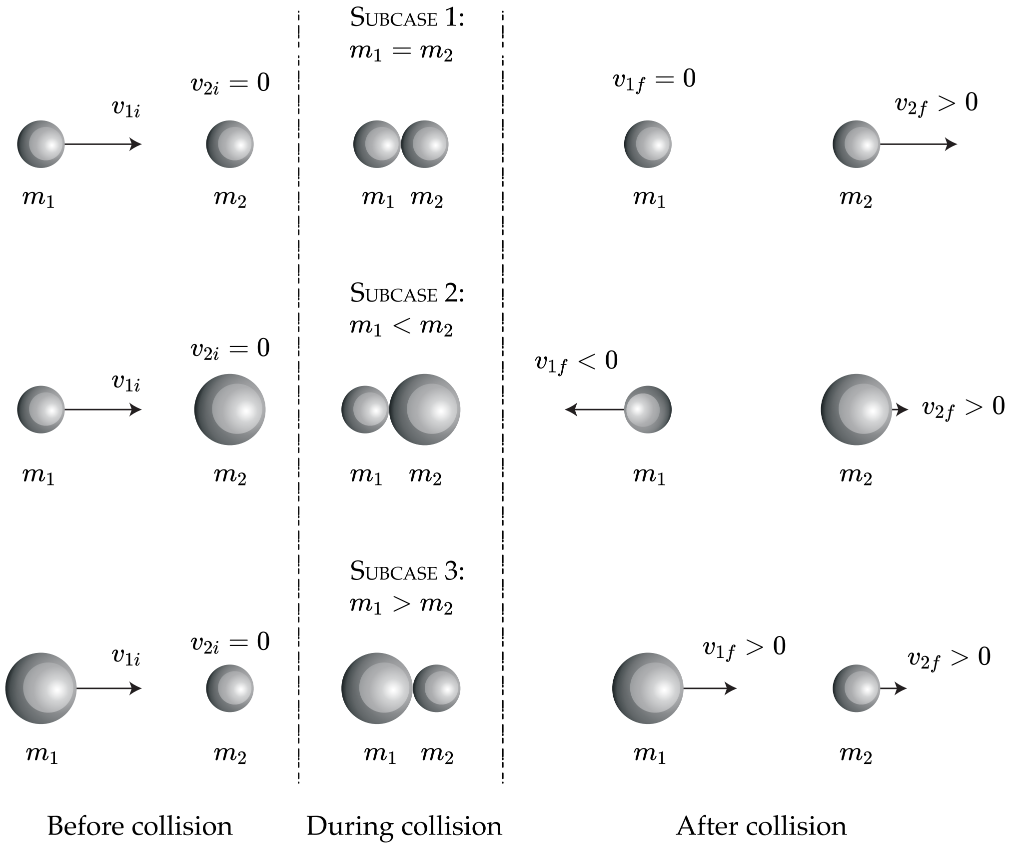



FIGURE 5: For a special case of elastic collision where the second body stays at rest initially, the first body behaves as a projectile. After the collision, both moves with their own final velocities given by Eq.(20-22).

$m_2 \ll m_1$:

$$\vec{v}_{1f} \simeq \vec{v}_{1i}, \quad \text{and} \quad \vec{v}_{2f} = -\vec{v}_{2i} \tag{22}$$

Eq. (22) means the first body will recoil forward with a velocity twice that of the second body added to its own, and the second body will recoil backward with a velocity twice that of the first body subtracted from its own.

### 4.2 Inelastic Collision

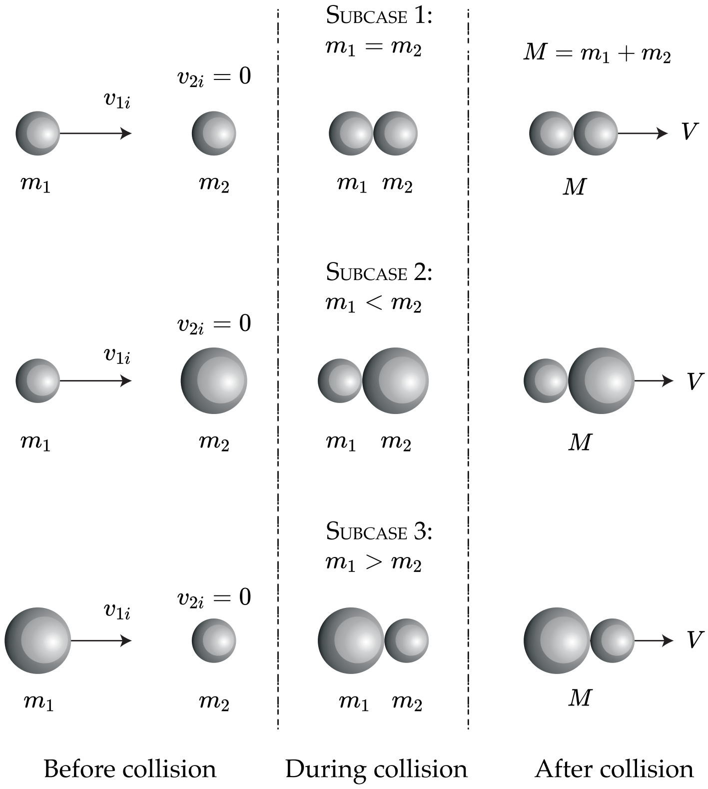



FIGURE 6: After an inelastic collision, the body merges into one with mass $M = m_1 + m_2$ and moves with a common velocity $V$.

In an inelastic collision, the total kinetic energy of the objects is not conserved. Some of the energy is lost in the form of heat or deformation, and the objects may stick together or deform permanently. An everyday example of an inelastic collision is when a car collides with a wall. The car's kinetic energy is converted into other forms of energy, such as heat and deformation, and the car may be damaged or crushed due to the collision.

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = (m_1 + m_2)\,\vec{v} \qquad \text{(Conservation of Linear Momentum)} \tag{23}$$

### 4.2.1 Perfectly Inelastic Collision

In a perfectly inelastic collision, the objects involved stick together after the collision, and their combined kinetic energy is not conserved. An everyday example of a perfectly inelastic collision is when two clay pieces are thrown at each other and stick together upon impact.

### 4.2.2 Partial Inelastic Collision

Finally, there are also partially inelastic collisions, where the objects involved stick together to some degree but not completely.

## 4.3 Real Life Examples of Collisions

Newton's cradle is a classic demonstration of a perfectly elastic collision. It consists of a series of metal balls of equal mass suspended on strings or wires arranged in a straight line.

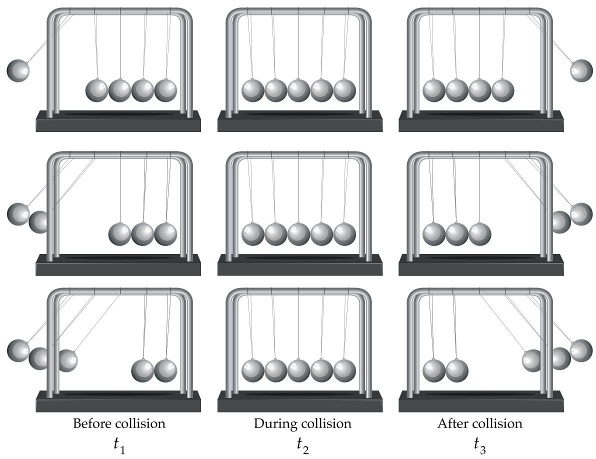



FIGURE 7: In Newton's cradle, the number of balls displaced and swung from one end, the same amount of balls will be displaced from the other end with the same momentum and will use up the same energy.

When one (or few) of the balls on one end is lifted and released, it swings down and strikes the next ball, causing it to move upward while the original ball (or balls) stops. This momentum and energy transfer is then repeated through the rest of the balls in the line, with the balls on the opposite end of the cradle swinging up and back in the opposite direction.

Fatal automobile accidents are examples of inelastic collisions. In this scenario, after the clash, both cars' initial kinetic energy may be transformed into other forms of energy, such as heat and sound. Thus, the mechanical energy within the collision will not be conserved. Heat energy produced during the collision may often be enough to melt parts of the cars and fuse those cars together. If so, the collision will be deemed a *perfectly inelastic*.

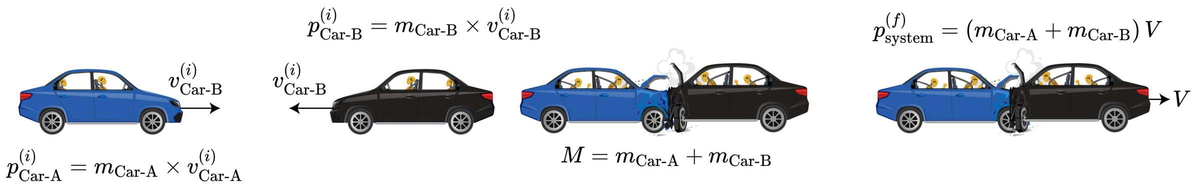



FIGURE 8: After a fatal (perfectly) inelastic collision between two cars with different masses and velocities, the cars merge into one and recoil with a common velocity $V$.

However, for regular fender-bender automobile collisions, the heat energy produced is insufficient to fuse the cars. In that case, the collision is a *partially inelastic* one.