Set-K

Ans To The Ques No. 01

$$f(x) = \sin(2x-5)$$

$$\Rightarrow \frac{d}{dx}\{f(x)\} = \frac{d}{dx}\{\sin(2x-5)\}$$

$$\Rightarrow \frac{d}{dx}\{f(x)\} = 2\cos(2x-5)$$

$$\Rightarrow f'(x) = 2\cos(2x-5)$$

$$\Rightarrow \frac{d}{dx}\{f'(x)\} = \frac{d}{dx}\{2\cos(2x-5)\}$$

$$\therefore f''(x) = -4\sin(2x-5) \quad \text{(Ans)}$$

Ans To The Ques No. 02

$$f(x) = \cos\left(\ln \frac{2}{x^3}\right)$$

$$\frac{d}{dx} f(x) = \frac{d}{dx} \left\{ \cos\left(\ln \frac{2}{x^3}\right) \right\}$$

$$f'(x) = \frac{d}{dx} \cos\left(\ln \frac{2}{x^3}\right) \times \frac{d}{dx} \left(\ln \frac{2}{x^3}\right)$$

$$= -\sin\left(\ln \frac{2}{x^3}\right) \times \frac{x^3}{2} \times \left(-2 \times \frac{3x^2}{(x^3)^2}\right)$$

$$= -\sin\left(\ln \frac{2}{x^3}\right) \times \left(-\frac{3}{x}\right)$$

$$= \frac{3 \sin\left(\ln \frac{2}{x^3}\right)}{x} \quad \text{(Ans)}$$

Ans To The Ques No. 03

a) $\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$

$= \tan^{-1}\left(\frac{\sin\theta}{\sqrt{1-\sin\theta}}\right)$

$= \tan^{-1}\left(\frac{\sin\theta}{\cos\theta}\right)$

$= \tan^{-1}(\tan\theta)$

$= \theta$

$= \sin^{-1} x$

$\therefore \frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$ (Ans)

let,
$x = \sin\theta$
$\therefore \theta = \sin^{-1} x$

b) $\ln\sqrt{\frac{1-\cos x}{1+\cos x}}$

$= \ln\sqrt{\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}}$

$= \ln\left(\tan\frac{x}{2}\right)$

$= \frac{d}{dx}\left\{\ln\left(\tan\frac{x}{2}\right)\right\}$

$= \frac{1}{\tan\frac{x}{2}} \times \sec^2\frac{x}{2} \times \frac{x}{2}$

$= \frac{1}{\tan\left(\frac{x}{2}\right)} \times \sec\left(\frac{x}{2}\right)^2 \times \frac{x}{2}$

$= \frac{\cos\frac{x}{x}}{\sin\frac{x}{2}} \times \frac{1}{\cos x\left(\frac{x}{2}\right)^2} \times \frac{x}{2}$

$= \frac{1}{\sin x}$

$= \operatorname{cosec} x$ (Ans)

Ans To The Ques No. 04

Ep.

$$y = \ln \frac{1}{x}$$

$$\frac{dy}{dx} = \frac{-1}{\frac{1}{x}} \times \frac{1}{x^2}$$

$$= -\frac{1}{x}$$

$$(y - 0) = \frac{1}{x}(x - 1)$$

$$x + xy = 1 \quad (\text{Ans})$$

$x = -1$

$y = \ln \frac{1}{1}$

$= 0$

Ans To The ques No. 05

$$\frac{d}{dx}\frac{f(x)}{g(x)} = \lim_{h\to 0} \frac{\frac{f(x+h)}{g(x+h)} - \frac{f(x)}{g(x)}}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)\,g(x) - f(x)\,g(x+h)}{h\,g(x)\,g(x+h)}$$

$$= \lim_{h\to 0} \frac{f(x+h)\,g(x) - f(x)\,g(x) + f(x)\,g(x) - f(x)\,g(x+h)}{h\,g(x)\,g(x+h)}$$

$$= \lim_{h\to 0} \frac{g(x)\,\frac{f(x+h)-f(x)}{h} - f(x)\,\frac{g(x+h)-g(x)}{h}}{g(x)\,g(x+h)}$$

$$= \frac{g(x)\,f'(x) - f(x)\,g'(x)}{g(x)^2}$$

(proved)

Ans To The Ques No.06

Find the first and second derivative of following with respect to b:

$$\cos\left(\frac{\pi}{2}\left[\frac{b^4}{4}\left(1-\frac{2\sinh^2(8\pi l_s Q)}{\sinh^2(9\pi l_s Q)}\right)\right]^{1/4}\right)$$

$$= \cos\left((b^4)\frac{\pi}{2}\left[\frac{1}{4}\left(1-\frac{2\sinh^2(8\pi l_s Q)}{\sinh^2(9\pi l_s Q)}\right)\right]^{1/4}\right)$$

$$= \cos\left(b\frac{\pi}{2}\left[\frac{1}{4}\left(1-\frac{2\sinh^2(8\pi l_s Q)}{\sinh^2(9\pi l_s Q)}\right)\right]^{1/4}\right)$$

1st dercivative:

$$= -\sin\left(b\frac{\pi}{2}\left[\frac{1}{4}\left(1-\frac{2\sinh^2(8\pi l_s Q)}{\sinh^2(9\pi l_s Q)}\right)\right]^{1/4}\right)\cdot\left(b^{1-1}\frac{\pi}{2}\left[\frac{1}{4}\left(1-\frac{2\sinh^2(8\pi l_s Q)}{\sinh^2(9\pi l_s Q)}\right)\right]^{-3/4}\right)$$

$$= -\sin\left(b\frac{\pi}{2}\left[\frac{b^4}{4}\left(1-\frac{2\sinh^2(8\pi l_s Q)}{\sinh^2(9\pi l_s Q)}\right)\right]^{1/4}\right)\cdot\left(\frac{\pi}{2}\left[\frac{1}{4}\left(1-\frac{2\sinh^2(8\pi l_s Q)}{2\sinh^2(9\pi l_s Q)}\right)\right]^{1/4}\right)$$

2nd derivative:

$$= -\cos\left(b\frac{\pi}{2}\left[\frac{1}{4}\left(1-\frac{2\sinh^2(8\pi\, ls\, Q)}{\sinh^2(9\pi\, ls\, Q)}\right]^{1/4}\right)\right.$$

$$\left(b^{1-1}\frac{\pi}{2}\left(\frac{1}{4}\left(1-\frac{2\sinh^2(8\pi\, ls\, Q)}{\sinh^2(9\pi\, ls\, Q)}\right]\right]^{1/4}\right)\left(\frac{\pi}{2}\left[\frac{1}{4}\left(1-\frac{2\sinh^2(8\pi\, ls\, Q)}{\sinh^2(9\pi\, ls\, Q)}\right)\right]^{1/4}\right)$$

$$= -\cos\left(b\cdot\frac{\pi}{2}\left[\frac{1}{4}\left(1-\frac{2\sinh^2(8\pi\, ls\, Q)}{\sinh^2(9\pi\, ls\, Q)}\right)\right]^{1/4}\right)\cdot\left(\frac{\pi}{2}\left[\frac{1}{4}\left(1-\frac{2\sinh^2(8\pi\, ls\, Q)}{\sinh^2(9\pi\, ls\, Q)}\right)\right]^{1/4}\right)^2$$

$$\left(\delta_{mn}\right)$$