BRAC UNIVERSITY

Principles of Physics-I (PHY-111)

Department of Mathematics and Natural Sciences

Assignment: 04 — **Section**: 36

Duration: 7 Days | **Summer 2023 (UB41403)** | Marks: 25

Attempt all three questions. Show Your work in detail. 1:1 plagiarism will be penalized.

1. The flywheel of a steam engine runs with a constant angular velocity of 160 rpm. When steam is shut off, the friction of the bearings and of the air stops the wheel in 2.2 h.

   (a) What is the constant angular acceleration, in rpm, of the wheel during the slowdown? **(1)**

$$\begin{aligned}\omega &= \omega_0 + \alpha t\\ 0 &= \omega_0 - \alpha t\\ \alpha &= \frac{\omega_0}{t} = 0.0021\,\mathrm{rad\,s^{-2}}\\ &\sim 1.21\,\mathrm{revmin^{-2}}.\end{aligned}$$

   (b) At the instant the flywheel is turning at 75 rpm, what is the tangential component of the linear acceleration of a flywheel particle that is 50 cm from the axis of rotation? Find the torque if a 5 N force is applied tangentially to the wheel at that point. **(2)**

$$\begin{aligned}\omega &= 75\,\mathrm{rpm} = 7.33\,\mathrm{rad\,s^{-1}}\\ a &= \alpha R = -10.5\times10^{-5}\,\mathrm{m\,s^{-2}}\\ \tau &= F_\perp R = 2.5\,\mathrm{N\,m}\end{aligned}$$

   (c) How many revolutions does the wheel make before stopping? **(2)**

$$\begin{aligned}2\alpha\Delta\theta &= \omega^2 - \omega_0^2.\\ \Delta\theta &= \frac{-\omega_0^2}{2\alpha} = 66368.963\,\mathrm{rad}\\ N &= \frac{\Delta\theta}{2\pi} = 10637.99 \sim 10638.\end{aligned}$$

2. A large yo-yo is released from a crane. The 115 kg yo-yo consisted of a solid cylinder of radius 32 cm a massless string wrapped around. The string unwinds at the height of 25 m but does not slip or stretch as the cylinder descends and rotates.

   (a) Can a single force applied to an object change both its translational and rotational motions? Explain. **(1)**

   Yes, a single force applied to an object can change both its translational and rotational motions. The effect of the force on the translational motion of the object depends on the point of application of the force and the distribution of mass in the object. If the force is applied at the center of mass of the object, it will only cause translational motion without any rotation. However, if the force is applied off-center, it will create both translational and rotational motion.

(b) Find the speed of the cylinder's center of mass after it has descended completely. **(2)**

$$\begin{aligned} K_1 + U_1 &= K_2 + U_2 \\ 0 + MgH &= \frac{1}{2}Mv_{\text{cm}}^2 + \frac{1}{2}I\omega^2 + 0 \\ MgH &= \frac{1}{2}Mv_{\text{com}}^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v_{\text{com}}}{R}\right)^2 \\ MgH &= \frac{3}{4}Mv_{\text{com}}^2 \\ v_{\text{com}} &= \sqrt{\frac{4}{3}gH}. \\ \therefore v_{\text{com}} &= 18.80\,\text{m}\,\text{s}^{-1}. \end{aligned}$$

(c) Find the downward acceleration of the cylinder and the tension in the string. **(3)**
Draw the free-body diagram and apply Newton's 2[nd] law, and the torque definition on the yo-yo:

$$\begin{aligned} \sum F_y = T - Mg &= -Ma_{\text{com}} \\ T &= M(g - a_{\text{com}}) \end{aligned}$$

$$\begin{aligned} \sum \tau = TR &= I_{\text{com}}\alpha = \frac{1}{2}MR^2 = \frac{1}{2}M(R\alpha)R \\ TR &= \frac{1}{2}Ma_{\text{com}}R \\ T &= \frac{1}{2}Ma_{\text{com}} \end{aligned}$$

$$\begin{aligned} M(g - a_{\text{com}}) &= \frac{1}{2}Ma_{\text{com}} \\ a_{\text{com}} &= \frac{2}{3}g = 6.54\,\text{m}\,\text{s}^{-2}. \\ T &= \frac{1}{2}Ma_{\text{com}} = 376.05\,\text{N}. \end{aligned}$$

(d) Find the total kinetic energies of the yo-yo at a distance (i) one-third, (ii) half, (iii) three-fourths from the top, and (iv) at the bottom, **(4)**
Import v_{com} from (b),

$$K = \frac{1}{2}Mv_{\text{com}}^2 = \frac{1}{2}M\left(\frac{4}{3}gH\right) = \frac{2}{3}MgH$$

$$K = \begin{cases} 6267.5\,\text{J} & H = \frac{1}{3}h \\ 9401.25\,\text{J} & H = \frac{1}{2}h \\ 14101.875\,\text{J} & H = \frac{3}{4}h \\ 18802.5\,\text{J} & H = h \end{cases}$$

3. A 10.0 g marble slides to the left at a speed of $0.400\,\text{m}\,\text{s}^{-1}$ on the frictionless, horizontal surface and has a head-on collision with a larger 30.0 g marble sliding to the right at a speed of $0.200\,\text{m}\,\text{s}^{-1}$.

(a) The work done by a force is the product of force and distance. The torque due to a force is the product of force and distance. Does this mean that torque and work are equivalent? Explain. **(1)**

No, torque and work are not equivalent, even though both involve the product of force and distance. The key difference between torque and work lies in the nature of the motion they describe. Torque is associated with rotational motion, while work is associated with translational motion.

In the case of torque, the force applied causes an object to rotate around an axis. The distance involved in torque calculation is the perpendicular distance from the axis of rotation to the line of action of the force. Torque does not involve the displacement of the object itself.

In the case of work, the force applied causes a displacement of the object in the direction of the force. The distance involved in work calculation is the actual displacement of the object, not the distance from an axis of rotation.

(b) Find the velocity of each marble (magnitude and direction) after the collision. (Since the collision is head-on, all motion is along a line.) **(2)**

Since the incident is occurring in a frictionless plane, energy won't be lost during the collision, making it an elastic collision. We can use the specialized elastic collision equations.

$$\vec{v}_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1ix} + \left(\frac{2m_2}{m_1 + m_2}\right) v_{2ix} = +(0.5\,\mathrm{m\,s^{-1}})\hat{\imath}$$
$$\vec{v}_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1ix} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right) v_{2ix} = -(0.10\,\mathrm{m\,s^{-1}})\hat{\imath}.$$

(c) Find the impulse imparted to the smaller marble during the collision. Find the average force that caused this impulse if the collision only took place for 1 ms. **(3)**

$$\vec{J} = \Delta\vec{p} = m_1\vec{v}_{1f} - m_1\vec{v}_{1i} = m_1(\vec{v}_{1f} + \vec{v}_{1i}) = (9 \times 10^{-3}\,\mathrm{N\,s})\hat{\imath}.$$
$$F_{\text{avg}} = \frac{|\vec{J}|}{t} = 9\,\mathrm{N}.$$

(d) Calculate the change in kinetic energy for each marble. Compare your values for each marble. Determine the type of collision. Elastic or Inelastic? **(4)**

$$\Delta K_1 = \frac{1}{2}m_1(v_{1f}^2 - v_{1i}^2) = 4.5 \times 10^{-4}\,\mathrm{J}$$
$$\Delta K_2 = \frac{1}{2}m_2(v_{2f}^2 - v_{2i}^2) = -6 \times 10^{-4}\,\mathrm{J}.$$

This implies m_1 gained some energy while m_2 lost some.

Now, calculate the kinetic energy of the system before and after the collision.

$$K_i = \frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_1v_{2i}^2 = 0.002\,\mathrm{J}$$
$$K_f = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_1v_{2f}^2 = 0.002\,\mathrm{J}$$

Clearly, $K_i = K_f$. Hence, the collision is elastic.