ASSIGNMENT 1

(Set N)

MATH110

Differential Calculus and Co-ordinate Geometry

Submitted

By

Jennifer Abedin

Student ID : 20301219

Section : 15

1. Evaluate the Limit

$$lim_{y\to4}\frac{4-y}{2-\sqrt{y}}$$

Answer

$lim_{y\to4}\frac{4-y}{2-\sqrt{y}}$

$= lim_{y\to4}\frac{2^2-(\sqrt{y})^2}{2-\sqrt{y}}$

$= lim_{y\to4}\frac{(2+\sqrt{y})(2-\sqrt{y})}{(2-\sqrt{y})}$

$= lim_{y\to4}(2+\sqrt{y})$

$= (2+2)$

$= 4$

2. Evaluate the Limit using algebraic manipulation

$$a. \qquad lim_{x\to1}\frac{x^2+6x-7}{x^2-1}$$

Answer

$lim_{x\to1}\frac{x^2+6x-7}{x^2-1}$

$= lim_{x\to1}\frac{x^2+7x-x-7}{x^2-1}$

$= lim_{x\to1}\frac{(x-1)(x+7)}{(x-1)(x+1)}$

$= lim_{x\to1}\frac{(x+7)}{(x+1)}$

$= \frac{(1+7)}{(1+1)}$

$= 4$

$$b. \qquad lim_{x\to 0}\frac{\sqrt{1-2x^2}-\sqrt{1+2x^2}}{x^2}$$

Answer

$lim_{x\to 0}\frac{\sqrt{1-2x^2}-\sqrt{1+2x^2}}{x^2}$

$lim_{x\to 0}\frac{(\sqrt{1-2x^2}-\sqrt{1+2x^2})(\sqrt{1-2x^2}+\sqrt{1+2x^2})}{x^2(\sqrt{1-2x^2}+\sqrt{1+2x^2})}$

$= lim_{x\to 0}\frac{(\sqrt{1-2x^2})^2-(\sqrt{1+2x^2})^2}{x^2(\sqrt{1-2x^2}+\sqrt{1+2x^2})}$

$= lim_{x\to 0}\frac{1-2x^2-1-2x^2}{x^2(\sqrt{1-2x^2}+\sqrt{1+2x^2})}$

$= lim_{x\to 0}\frac{-4x^2}{x^2(\sqrt{1-2x^2}+\sqrt{1+2x^2})}$

$= \frac{-4}{1+1}$

$= \frac{-4}{2}$

$= -2$

$$c. \qquad lim_{x\to\infty}\frac{x^4+2x^2+1}{2x^4-3x^3+x}$$

Answer

$lim_{x\to\infty}\frac{x^4+2x^2+1}{2x^4-3x^3+x}$

$= lim_{x\to\infty}\frac{1+\frac{2}{x^2}+\frac{1}{x^4}}{2-\frac{3}{x}+\frac{1}{x^3}}$

$= lim_{x\to\infty}\frac{1+\frac{2}{\infty}+\frac{1}{\infty}}{2-\frac{3}{\infty}+\frac{1}{\infty}}$

$= \frac{1}{2}$

3. Evaluate

$$lim_{x\to 2}\frac{x^2+4x-12}{x^2-4}$$

Answer

$lim_{x\to 2}\frac{x^2+4x-12}{x^2-4}$

$= lim_{x\to 2}\frac{x^2+6x-2x-12}{x^2-4}$

$= lim_{x\to 2}\frac{x(x+6)-2(x+6)}{(x+2)(x-2)}$

$= lim_{x\to 2}\frac{x(x+6)-2(x+6)}{(x+2)(x-2)}$

$= lim_{x\to 2}\frac{(x-2)(x+6)}{(x+2)(x-2)}$

$= lim_{x\to 2}\frac{(x+6)}{(x+2)}$

$= \frac{8}{4}$

$= 2$

4.

$$\text{If } f(x) = \begin{cases} x-1, & x \leq 3 \\ 3x-7, & x > 3. \end{cases}$$

$$a) find \quad lim_{x\to 3^-} f(x)$$

$$b) find \quad lim_{x\to 3^+} f(x)$$

$$c) find \quad lim_{x\to 3} f(x)$$

Answer

a) Left handside, $lim_{x\to 3^-} f(x)$

$= lim_{x\to 3^-}(x-1)$

$= (3-1)$

$= 2$

b) Right handside, $lim_{x \to 3^+} f(x)$

$= lim_{x \to 3^+}(3x - 7)$

$= (9 - 7)$

$= 2$

c) $lim_{x \to 3} f(x)$

$= lim_{x \to 3}(x - 1)$

$= (3 - 1)$

$= 2$

5.

If $lim_{x \to 1} \frac{x^3 - x^2 + 2x - 2}{x^3 + 3x^2 - 4x} = \frac{3}{a}$
then a = ?

Answer

$lim_{x \to 1} \frac{x^3 - x^2 + 2x - 2}{x^3 + 3x^2 - 4x} = \frac{3}{a}$

$=> lim_{x \to 1} \frac{x^3 - x^2 + 2x - 2}{x^3 + 3x^2 - 4x} = \frac{3}{a}$

$=> lim_{x \to 1} \frac{x^2(x-1) + 2(x-1)}{x(x^2 + 3x - 4)} = \frac{3}{a}$

$=> lim_{x \to 1} \frac{(x^2+2)(x-1)}{x(x+4)(x-1)} = \frac{3}{a}$

$=> lim_{x \to 1} \frac{(x^2+2)}{x(x+4)} = \frac{3}{a}$

$=> \frac{(1+2)}{(1+4)} = \frac{3}{a}$

$=> \frac{3}{5} = \frac{3}{a}$

$=> a = 5$

THE END