BRAC UNIVERSITY

Inspiring Excellence

Department of Mathematics and Natural Sciences

MAT 110

Final Examination

SET: 59

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*Please write your name and ID on the exam script. The duration of the exam is **6 hours**. Solve all the problems. The number of marks per question is written on the right margin.*

Any information you need to solve this exam is given in the question.

Be creative, use your intuition. Answer the questions by yourself. Cheating and Copying will lead to an F grade in the course. ***Total marks is 250.****It will be converted to 25.*

P.S: Name your PDF according to the format Set_ Member1ID_ Member2ID_ $\cdots$

1. Solve the following Taylor Expansion Problems

   (a) Find the Taylor Series for $f(x) = xcos\,(x)$ about $x = \pi$ [15]

   (b) Find the Taylor Series for $f(x) = x^7 e^x$ about $x = 0$ [15]

2. (a) Find f_{xxy} and f_{xyz} of the following function: $f(x, y, z) = y^2 \cos(6zx) + x^3 \sin(2y - 5z)$ [15]

   (b) Determine the critical points and locate any relative minima, maxima and saddle points of function f defined by $f(x, y) = 4x^2 + 4xy + 4y^2 - 12x$ [15]

3. Calculate the second-degree Taylor polynomial of:

   (a) $f(x, y) = f(x, y) = e^{3x} \cos 2y$ at the point (0,0) [15]

   (b) $f(x, y) = \cos x \cos y$ at the point (0,0). [15]

4. Compute the Divergence and Curl of the following vector $\vec{F}$:

   (a) $\vec{F} = (4z - \cos(2x))\,\vec{i} - z^3\mathbf{e}^{5x}\vec{j} + (y^3 + 8z^2)\,\vec{k}$ [15]

   (b) $\vec{F} = -(4y + z)\,\vec{i} + y^2 \sin x\vec{j} + (3x + 3y)\,\vec{k}$ [15]

5. (a) Write the equation into the standard form of the equation of the ellipse: $2x^2 - 4x + 2y^2 + 6y - 10 = 0$ [15]

   (b) Write the equation into the standard form of the equation of the hyperbola: $32x^2 - 2y^2 - 64x - 12y = 114$ [15]