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MAT 110

Assignment - 1

Problem - 1) Find a number 'a' such that the limit

$$\lim_{x \to -2} \frac{3x^2 + ax + a + 3}{x^2 + x - 2}$$

exists. Then find the limit.

Answer: 1)

Given,

$$\lim_{x \to -2} \frac{3x^2 + ax + a + 3}{x^2 + x - 2}$$

$$= \frac{3 \times (-2)^2 + a(-2) + a + 3}{(-2)^2 + (-2) - 2}$$

$$= \frac{(3 \times 4) - 2a + a + 3}{4 - 2 - 2}$$

$$= \frac{12+3-a}{4-4}$$

$$= \frac{15-a}{0}$$

[In order to use L'Hospital's Rule, the given expression should give us $\frac{0}{0}$ indeterminate form when we put $x = -2$]

So, if $15 - a = 0$

or, $a = 15$

We obtain the given expression in the required indeterminate form i.e $\frac{0}{0}$.

Now we can apply L'Hospital's law.

Now, When $a = 15$,

$$\lim_{x \to -2} \frac{3x^2 + ax + a + 3}{x^2 + x - 2}$$

$$= \lim_{x \to -2} \frac{3x^2 + 15x + 15 + 3}{x^2 + x - 2}$$

$$= \lim_{x \to -2} \frac{3x^2 + 15x + 18}{x^2 + x - 2}$$

$$= \lim_{x \to -2} \frac{3(x^2 + 5x + 6)}{x^2 + x - 2}$$

$$= \lim_{x \to -2} \frac{\frac{d}{dx}\{3(x^2+5x+6)\}}{\frac{d}{dx}(x^2+x-2)}$$ [Applying L' Hospital's Rule]

$$= \lim_{x \to -2} \frac{3(2x+5)}{2x+1}$$

$$= \lim_{x \to -2} \frac{6x+15}{2x+1}$$

$$= \frac{6 \times (-2) + 15}{2 \times (-2) + 1}$$

$$= \frac{-12 + 15}{-4 + 1}$$

$$= \frac{3}{-3}$$

$$= -1$$

$\therefore$ The limit of $\lim_{x \to -2} \dfrac{3x^2+ax+a+3}{x^2+x-2}$ is -1 where $a=15$. (Ans).

Problem-2) Find

$$\lim_{x \to 1} \frac{x-1}{\sqrt{x}-1}$$

Answer-2)

Given,

$$\lim_{x \to 1} \frac{x-1}{\sqrt{x}-1}$$

$$= \lim_{x \to 1} \frac{(\sqrt{x})^2-(1)^2}{\sqrt{x}-1}$$

$$= \lim_{x \to 1} \frac{(\sqrt{x}+1)\cancel{(\sqrt{x}-1)}}{\cancel{(\sqrt{x}-1)}}$$

$$= \lim_{x \to 1} (\sqrt{x} + 1)$$

$$= \sqrt{1} + 1$$

$$= 1 + 1$$

$$= 2$$

(Ans).

Problem-3) find the limit

$$\lim_{x \to 4} \frac{x^2 - 16}{x - 4}$$

Answer-3)

Given,

$$\lim_{x \to 4} \frac{x^2 - 16}{x - 4}$$

$$= \lim_{x \to 4} \frac{x^2 - (4)^2}{x - 4}$$

$$= \lim_{x \to 4} \frac{(x+4)\cancel{(x-4)}}{\cancel{(x-4)}}$$

$$= \lim_{x \to 4} (x+4)$$

$$= 4+4$$

$$= 8$$

(Ans).

Problem-4) Evaluate the limit

$$\lim_{x \to 0} \frac{x}{\sqrt{x+1}-1}$$

Answer-4) Given

$$\lim_{x \to 0} \frac{x}{\sqrt{x+1}-1}$$

$$= \lim_{x \to 0} \frac{x(\sqrt{x+1}+1)}{(\sqrt{x+1}-1)(\sqrt{x+1}+1)}$$

$$= \lim_{x \to 0} \frac{x(\sqrt{x+1}+1)}{(\sqrt{x+1})^2 - (1)^2}$$

$$= \lim_{x \to 0} \frac{x(\sqrt{x+1}+1)}{x+1-1}$$

$$= \lim_{x \to 0} \frac{\cancel{x}(\sqrt{x+1}+1)}{\cancel{x}}$$

$$= \lim_{x \to 0} \left(\sqrt{x+1}+1\right)$$

$$= \sqrt{0+1} + 1$$

$$= \sqrt{1} + 1$$

$$= 1 + 1$$

$$= 2$$

(Ans)

Problem-5) First rationalize the numerator and then find the limit:

$$\lim_{x \to 0} \frac{\sqrt{x^2+4} - 2}{x}$$

Answer-5) Given,

$$\lim_{x \to 0} \frac{\sqrt{x^2+4} - 2}{x}$$

$$= \lim_{x \to 0} \frac{(\sqrt{x^2+4} - 2)(\sqrt{x^2+4} + 2)}{x(\sqrt{x^2+4} + 2)}$$

$$= \lim_{x \to 0} \frac{(\sqrt{x^2+4})^2 - (2)^2}{x(\sqrt{x^2+4} + 2)}$$

$$= \lim_{x \to 0} \frac{x^2+4-4}{x(\sqrt{x^2+4} + 2)}$$

$$= \lim_{x \to 0} \frac{\cancel{x^2}\, x}{\cancel{x}\left(\sqrt{x^2+4}+2\right)}$$

$$= \lim_{x \to 0} \frac{x}{\sqrt{x^2+4}+2}$$

$$= \frac{0}{\sqrt{0+4}+2}$$

$$= \frac{0}{\sqrt{4}+2}$$

$$= \frac{0}{2+2}$$

$$= \frac{0}{4}$$

$= 0$ (Ans).

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