**PHY 111: Principles of Physics I** Summer 2023

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# 1 Work

Work measures the amount of energy transferred from one object to another or the environment. Work is done when a force is applied to an object and causes it to move through a distance. The work done is equal to the force applied times the distance it is applied. Mathematically, work can be expressed as:

$$W = \vec{F} \cdot \vec{s} = Fs\cos\theta. \tag{1}$$

where $W$ is the work done, $\vec{F}$ is the force applied, $s$ is the distance over which the force is applied, and $\theta$ is the angle between the direction of the force and the direction of motion.

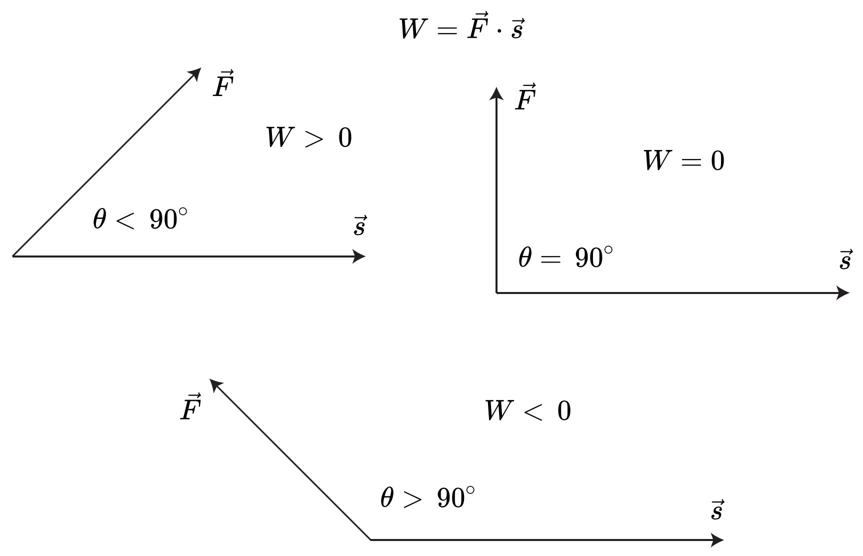


FIGURE 1: Measurement of work and its dependency on the angle between the force applied and the displacement made.

This only applies to work done by a variable force. In practice, we would have forces that are variable. In that case, work done is given by

$$W = \int_{x_i}^{x_f} F dx. \tag{2}$$

In 3D, this becomes a line integral of the following form

$$W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot d\vec{r} = \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy + \int_{z_i}^{z_f} F_z dz. \tag{3}$$

# 2 Energy

Energy is a property of an object or system property that allows it to do work. There are many different forms of energy, including kinetic, potential, thermal, and electrical energy.

## 2.1 Kinetic Energy

Kinetic energy is the energy an object possesses due to its motion. The more an object is moving, the more kinetic energy it has. Intuitively, you can think of kinetic energy as the energy an object has because of how fast it is moving.

$$K = \frac{1}{2}mv^2. \tag{4}$$

## 2.2 Potential Energy

Potential energy is the energy an object possesses because of its position or state. For example, an object lifted above the ground has potential energy because it has the potential to fall and do work as it falls. The more a thing is raised, the more potential energy it has.

$$U = -\int F dr. \tag{5}$$

Gravitational potential energy $U_{\text{grav}}$ is given by a constant force, that is the gravitational force, and Elastic potential energy $U_{\text{elas}}$ is given by a variable force.

$$U_{\text{grav}} = mg\Delta y. \tag{6}$$

$$U_{\text{elas}} = \frac{1}{2}kr^2. \tag{7}$$

## 2.3 Conservation of Energy

The total energy of a system is conserved, meaning that the total energy of a system remains constant, even as the energy is transformed from one form to another. For example, as an object falls, its potential energy decreases, and its kinetic energy increases. However, the sum of its potential and kinetic energies remains constant.

$$K + U = E \tag{8}$$

$$\text{and} \quad \Delta K + \Delta U = 0. \tag{9}$$

Intuitively, you can think of energy conservation as a "balancing act." Energy can be transformed from one form to another, but the total amount of energy in a system remains constant. This principle is known as the law of conservation of energy.

$$\Delta K = -\Delta U. \tag{10}$$

## 3 The Work-Energy Theorem

The work-energy theorem states that the work done on an object equals the change in its kinetic energy. In other words, when a force is applied to an object, work is done on the object, resulting in a change in the object's kinetic energy.

Intuitively, this means that you need to apply a force to change an object's motion. The work done by this force will cause the object to accelerate, and as the object accelerates, it gains kinetic energy. The more work done on the object, the more kinetic energy it achieves and the faster it will move.

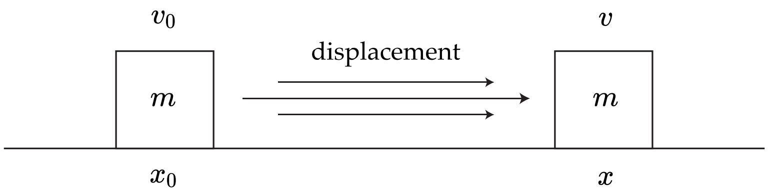


FIGURE 2: The work done to displace a body of mass $m$ to an amount $x - x_0$ with velocity change $v - v_0$, will be the change in kinetic energy.

Conversely, you must apply a force in the opposite direction to slow down or stop an object. The work done by this force will decrease the object's kinetic energy, eventually bringing it to a stop.

$$\Delta W = W_f - W_i = \frac{1}{2}m(v^2 - v_0^2) = \Delta K \tag{11}$$

$$\Rightarrow \Delta W = -\Delta U$$

$$\Rightarrow F dx = -dU$$

$$\Rightarrow F = -\frac{dU}{dx} \tag{12}$$

So the work-energy theorem tells us that any time work is done on an object, there will be a corresponding change in the object's kinetic energy. And this is true whether a force, heat, or other means do the work.

# 4 Power

Power measures the rate at which work is done or energy is transferred. Power equals the work done divided by the time it takes to do the work. Mathematically, power can be expressed as:

$$\bar{P} = \frac{\Delta W}{\Delta t} = F\frac{\Delta x}{\Delta t} = Fv. \tag{13}$$

where $P$ is the average power, $W$ is the work done, and $t$ is the time it takes to do the work.

Intuitively, work can be considered the amount of effort required to move an object, and power can be viewed as the speed at which work is done. For example, lifting a heavy object slowly requires more work than lifting a lighter object quickly, but the power used in the two cases would be the same if the same amount of work is done in the same amount of time. Power is often used to describe the energy consumption rate, such as in the case of an electrical appliance, where the power rating tells you how much energy it operates in a given amount of time.

To find the instantaneous power, we simply set the limit of $\Delta t \to 0$.

$$P = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dW}{dt}. \tag{14}$$