(9)

$$x = 6\cos(3\pi t + \frac{\pi}{3})$$

at, $t = 2s$,

(a) displacement,

$$x(t=2) = 6\cos(3\pi \times 2 + \frac{\pi}{3}) \text{ m}$$

$$= \boxed{3\text{ m}}$$

(b) velocity, $v = \frac{dx}{dt} = \frac{d}{dt}\left[6\cos(3\pi t + \frac{\pi}{3})\right]$

$$= -6 \times 3\pi \sin(3\pi t + \frac{\pi}{3}) \text{ m/s}$$

$$v(t=2) = -6 \times 3\pi \sin(3\pi \times 2 + \frac{\pi}{3}) \text{ m/s}$$

$$= \boxed{-49\text{ m/s}}$$

(c) acceleration, $a = \frac{dv}{dt}$

$$= \frac{d}{dt}\left(-6 \times 3\pi \sin(3\pi t + \frac{\pi}{3})\right)$$

$$= -6 \times (3\pi)^2 \cos(3\pi t + \frac{\pi}{3}) \text{ m/s}^2$$

$$a(t=2) = -6 \times (3\pi)^2 \cos(3\pi \times 2 + \frac{\pi}{3}) \text{ m/s}^2$$

$$= -2.7 \times 10^2 \text{ m/s}^2$$

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(d) at $t=2$, $\phi = (\omega t + \frac{\pi}{3})$

$= (3\pi \times 2 + \frac{\pi}{3})$ rad

$= 20$ rad.

(e) $\omega = 3\pi$ rad/s

$f = \frac{\omega}{2\pi}$

$\left\{ \begin{array}{l} T = \frac{2\pi}{\omega} \\ f = \frac{\omega}{2\pi} \end{array} \right.$

$= \frac{3\pi}{2\pi}$ s-1

$= \boxed{1.5 \; s^{-1}}$

(f) $T = \frac{1}{f} = \frac{1}{1.5 s^{-1}} \simeq \boxed{0.67 \; s}$

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$m = 0.5$ kg

$x_m = 35$ cm $= 0.35$ m

$T = 0.5$ s

(a) $\boxed{T = 0.5 \; s}$

(b) $f = \frac{1}{T} = \frac{1}{0.5 s} = \boxed{2 s^{-1}}$

(c)

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.5} \text{ rad/s}$$
$$= 12.6 \text{ rad/s}$$

(d)

$$\omega = \sqrt{\frac{k}{m}}$$

$$\Rightarrow \quad k = \omega^2 m = (12.6)^2 \times 0.5 \quad \text{N/m}$$
$$= \boxed{79 \text{ N/m}}$$

(e)

$$V_{max} = \omega x_m = (12.6) \times 0.35) \text{ m/s}$$
$$= \boxed{4.4 \text{ m/s}}$$

(f)

$$F_{max} = k x_m = (79 \text{ N/m}) \times (0.35) \text{ m} = \boxed{27.6 \text{N}}$$

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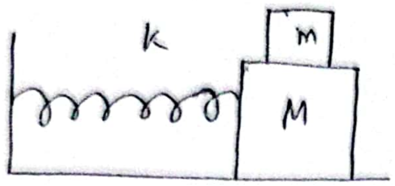
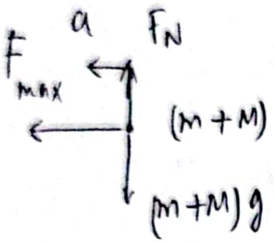



$k = 200\,N/m$

$\mu_s = 0.4$

$m = 1.8\,kg$

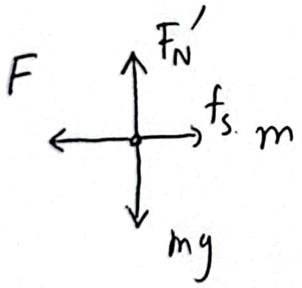
$M = 10\,kg$




$$F_{max} = (m+M)\,a$$

$$\Rightarrow \; kx_m = (m+M)a$$

$$\Rightarrow \; a = \frac{kx_m}{(m+M)}$$




$$f_{s,max} = \mu_s F_N'$$

$$= \mu_s mg$$

$$F = f_{s,max}$$

$$\Rightarrow \; ma = \mu_s mg$$

$$\Rightarrow \; \cancel{m}\frac{kx_m}{(m+M)} = \mu_s \cancel{m}g$$

$$x_m = \frac{\mu_s (m+M)}{k}g$$

$$= \frac{0.4(1.8+10)}{200} \times 9.8\ m$$

$$= \boxed{0.23\ m}$$

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$$M = 250\,g = 250 \times 10^{-3}\,kg$$

$$k = 85\,N/m$$

$$b = 70\,g/s = 70 \times 10^{-3}\,kg/s$$

$$T = \frac{2\pi}{\omega_d} = \frac{2\pi}{\sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}}$$

$$= \frac{2\pi}{\sqrt{\frac{85}{250 \times 10^{-3}} - \left(\frac{70 \times 10^{-3}}{2 \times 250 \times 10^{-3}}\right)^2}}\ s = \boxed{0.34\,s}$$

Now, $t = 20T$,

So, $e^{-\frac{b}{2m}t} = e^{-\left(\frac{70 \times 10^{-3}}{2 \times 250 \times 10^{-3}}\right) \times 20 \times 0.34}$

$$= \boxed{0.39}$$

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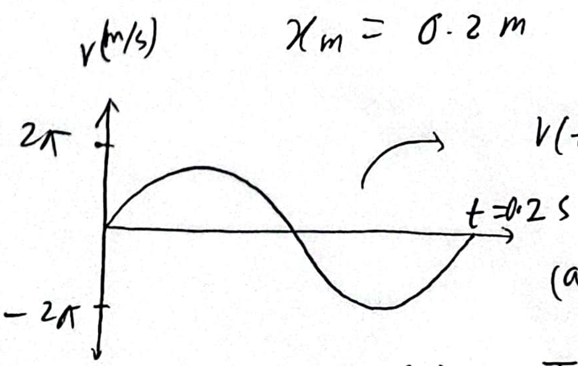
$k = 200\,N/m$

$x_m = 0.2\,m$

$$\omega = \sqrt{\frac{k}{m}}$$




$$v(t) = 2\pi \sin(\omega t) \quad -(1)$$

(a) $T = \boxed{0.2\,s}$

(b) $T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{k}{m}}}$

$$\Rightarrow T^2 = \frac{4\pi^2}{\frac{k}{m}}$$

$$\Rightarrow \frac{k}{m} = \frac{4\pi^2}{T^2}$$

$$\Rightarrow \frac{m}{k} = \frac{T^2}{4\pi^2}$$

$$\Rightarrow m = k\,\frac{T^2}{4\pi^2} = \boxed{0.2\,kg}$$

(c) at, $t=0$, $V=0$ m/s → (From graph)

We know that $V = \omega\sqrt{x_m^2 - x^2}$

$\Rightarrow 0 = \omega\sqrt{x_m^2 - x^2}$

$\Rightarrow \omega^2 x^2 = \omega^2 x_m^2$

$\Rightarrow x = \pm\sqrt{\frac{\omega^2 x_m}{\omega^2}}$

$\Rightarrow x = \pm x_m$

$= \pm 0.20$ m

From graph slope of the velocity $= a > 0$

and $ma = -kx$

$x = -\frac{ma}{k}$

So, $x$ should be negative,

$\therefore x = \boxed{-0.2\text{ m}}$.

(d) $a = \pm\omega^2 x$

$= -\left(\frac{200}{0.2}\right)(0.2)$ m/s²

$= \boxed{-200\text{ m/s}^2}$

Here, at $t=0$, $x = -x_m$

$t = 0.1s = \frac{T}{2}s$

So, $x = x_m$

$ma = -kx$

$\Rightarrow a = -\frac{k}{m}x$

$x = x_m$ (positive) $\therefore a$ (negative)

(e) $K_{max} = \frac{1}{2}kx_m^2 = \boxed{4\text{ J}}$