Reduction Formula (We will discuss about Reduction formula in details later).

$\int u\,dv = uv - \int v\,du$

$$\int \sec^3\theta\, d\theta$$

let $I = \int \sec^3\theta\, d\theta$

let $u = \sec\theta$

$du = \sec\theta\tan\theta\, d\theta$

$dv = \sec^2\theta\, d\theta$

$\Rightarrow \int dv = \int \sec^2\theta\, d\theta$

$\Rightarrow v = \tan\theta + K$

$$= \int \underbrace{\sec\theta}_{u}\, \underbrace{\sec^2\theta\, d\theta}_{dv}$$

$$= \sec\theta\tan\theta - \int \tan\theta \cdot \sec\theta\tan\theta\, d\theta$$

$$= \sec\theta\tan\theta - \int \sec\theta\tan^2\theta\, d\theta$$

$$= \sec\theta\tan\theta - \int \sec\theta(\sec^2\theta - 1)\, d\theta$$

$$= \sec\theta\tan\theta - \int \sec^3\theta\, d\theta + \int \sec\theta\, d\theta$$

$$\Rightarrow I = \sec\theta\tan\theta - I + \ln|\tan\theta + \sec\theta| + c_1$$

$$\Rightarrow 2I = \sec\theta\tan\theta + \ln|\tan\theta + \sec\theta| + c_1$$

$$\Rightarrow I = \frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln|\tan\theta + \sec\theta| + \frac{c_1}{2}$$

$$= \tfrac{1}{2}\sec\theta\tan\theta + \tfrac{1}{2}\ln|\tan\theta + \sec\theta| + c, \quad c = \frac{c_1}{2}$$