Note Title 5/12/2025

SHM

$$x = \sin(\omega t + \delta)$$

$$\text{or, } x = \cos(\omega t + \delta)$$

Spring-block system:

$$\frac{d^2}{dt^2}x + \frac{k}{m}x = 0$$

$$\boxed{x = \underbrace{x_m}_{\text{amplitude}} \sin\left(\sqrt{\frac{k}{m}}\, t + \delta\right)} \rightarrow \text{periodic}$$

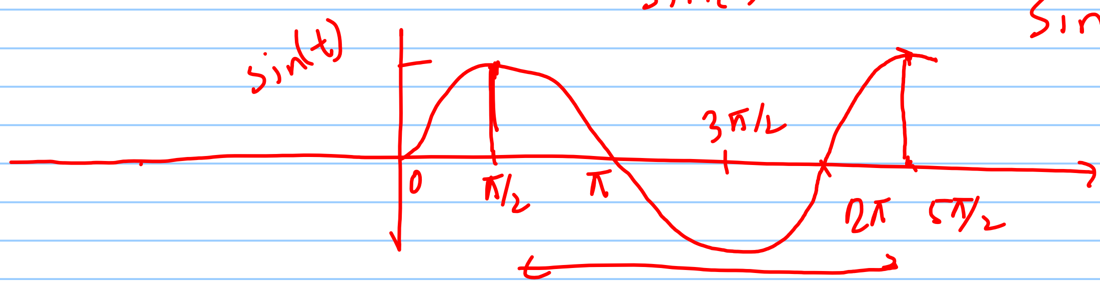
$$\sqrt{\frac{k}{m}} = \omega \leftarrow \text{angular frequency}$$

Periodic function,

$$f(t + T) = f(t)$$

$$T \rightarrow \text{period}$$

$\sin(t)$



$$\sin(t + 2\pi) = \sin t$$

$$T = 2\pi$$

$\sin(2t) \longrightarrow T = \frac{2\pi}{2} = \pi$

$\sin\left(\frac{t}{2}\right) \longrightarrow T = \frac{2\pi}{\frac{1}{2}} = 4\pi$

$\sin(t)$

$T = 2\pi$

$$\boxed{x = x_m \sin\left(\sqrt{\frac{k}{m}}\, t + \delta\right)}$$

$$\boxed{T = \frac{2\pi}{\sqrt{\frac{k}{m}}}}$$

$T = \frac{2\pi}{\omega}$

$$\boxed{\frac{d^2x}{dt^2} + \left(\frac{k}{m}\right)x = 0} \longleftrightarrow \boxed{\omega = \sqrt{\frac{k}{m}}}$$

$[k] = \left[\frac{N}{m}\right]$

$[m] = kg$

$\frac{k}{m} = \frac{N}{m} \times \frac{1}{kg}$

$= \frac{\cancel{kg}\, \cancel{m} s^{-2}}{\cancel{m} \times \cancel{kg}}$

$= s^{-2}$

$\sqrt{\frac{k}{m}} = s^{-1}$

$\omega = 2\pi f$

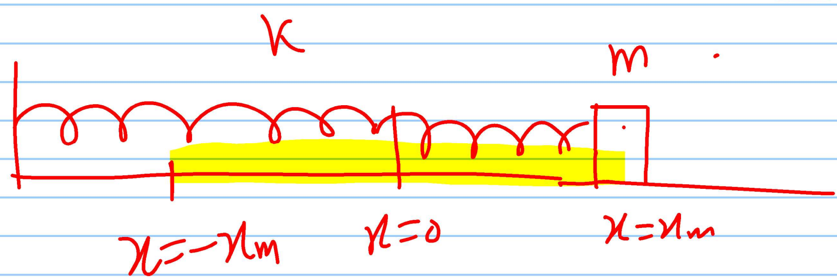
$$\boxed{x = x_m \sin(\omega t + \delta)}$$

$$\boxed{\omega = \sqrt{\frac{k}{m}}}$$

$T = \frac{2\pi}{\omega}$

$$\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$

$$\boxed{f = \frac{1}{T} = \frac{1}{2\pi} \cdot \sqrt{\frac{k}{m}}}$$



$$x = x_m \sin(\omega t + \delta)$$

$$v = \frac{dx}{dt} = \frac{d}{dt}\left[x_m \sin(\omega t + \delta)\right]$$

$$= x_m \frac{d}{dt}\left[\sin(\omega t + \delta)\right]$$

$$\boxed{v(t) = x_m \omega \cos(\omega t + \delta)}$$

$$v = \omega \sqrt{x_m^2 \cos^2(\omega t + \delta)}$$

$$= \omega \sqrt{x_m^2 (1 - \sin^2(\omega t + \delta))}$$

$$= \omega \sqrt{x_m^2 - \underbrace{x_m^2 \sin^2(\omega t + \delta)}}$$

$$\boxed{v = \omega \sqrt{x_m^2 - x^2}}$$

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\underline{x_m w}\ \cos(wt+\delta)\right)$$

$$= x_m w \ \frac{d}{dt}\left[\cos(wt+\delta)\right]$$

$$= - x_m w^2 \sin(wt+\delta)$$

$$\boxed{a = - w^2 \underbrace{x_m \sin(wt+\delta)}}$$

$$\boxed{a = - w^2 x}$$

$$\boxed{a \propto - x}$$

Energy of SHM

$$U(t) = \frac{1}{2} k\, x(t)^2$$

$$= \frac{1}{2} k\, x_m^2 \sin^2(wt+\delta)$$

$$K(t) = \frac{1}{2} m\, V(t)^2$$

$$= \frac{1}{2} m\, x_m^2 w^2 \cos^2(wt+\delta)$$

$$w = \sqrt{k/m}$$

$$\Rightarrow w^2 = k/m$$

$$\Rightarrow \underline{m w^2 = k}$$

$$= \frac{1}{2} m w^2\, x_m^2 \cos^2(wt+\delta)$$

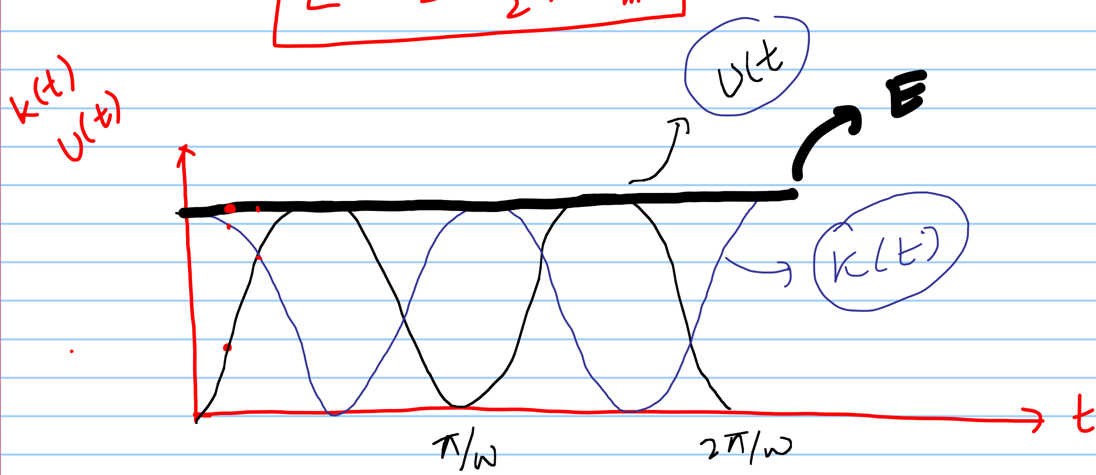
$$= \frac{1}{2} k\, x_m^2 \cos^2(wt+\delta)$$

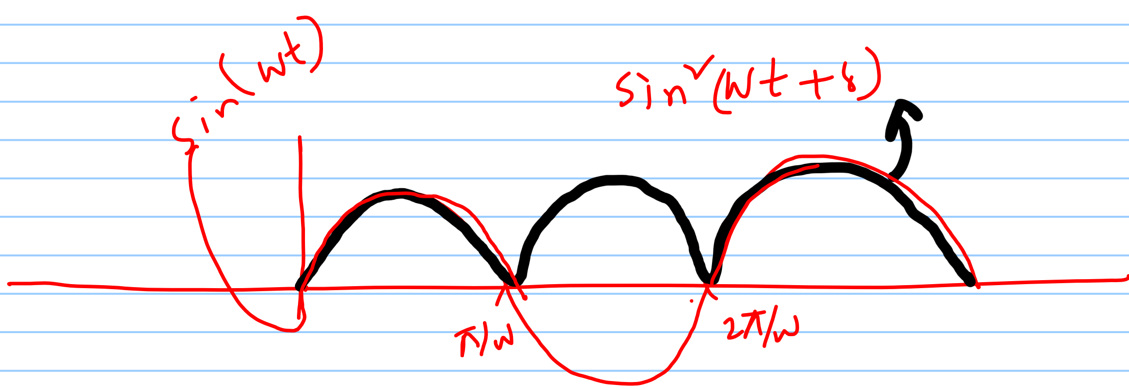
$$E = U(t) + K(t)$$

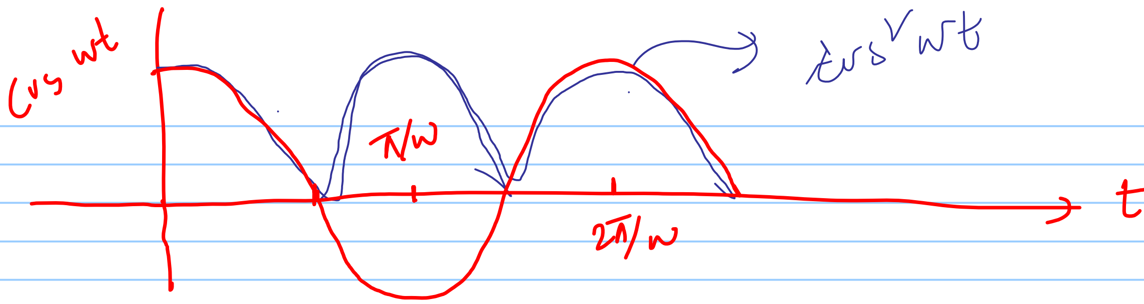
$$= \frac{1}{2} k x_m^2 \sin^2(\omega t + \delta) + \frac{1}{2} k x_m^2 \cos^2(\omega t + \delta)$$

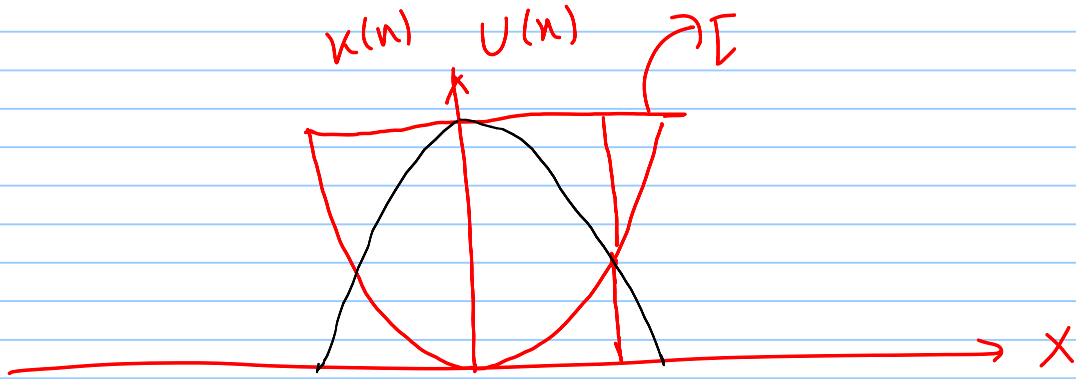
$$= \frac{1}{2} k x_m^2 \left(\underbrace{\sin^2(\omega t + \delta) + \cos^2(\omega t + \delta)}_{1} \right)$$

$$\boxed{E = \frac{1}{2} k x_m^2}$$







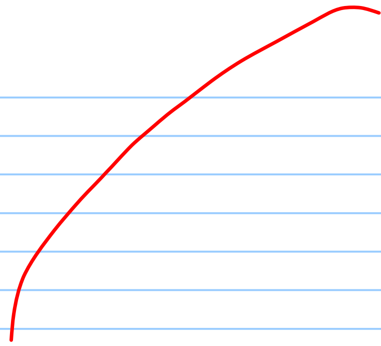


$$U(x) = \frac{1}{2} k x^2$$

$$K(x) = \frac{1}{2} m v^2 = \frac{1}{2} m \omega^2 (x_m^2 - x^2)$$

$$= \frac{1}{2} k (x_m^2 - x^2)$$

$$E = \frac{1}{2} k x_m^2$$

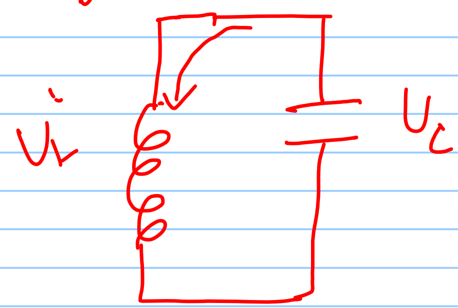


$y = |x-2|$

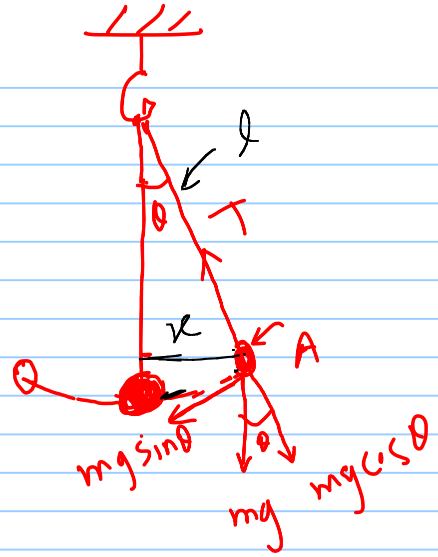
$y-2 = |x-2|$

$y = -|x|$

LC circuit



Simple pendulum



$\sin\theta = \frac{x}{l}$

Restoring Force,

$$F = -mg\sin\theta \quad -(1)$$

$$\sin\theta \simeq \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} + \cdots$$

$0^\circ < \theta < 4^\circ \qquad \sin\theta \simeq \theta$

$$F = -mg\frac{x}{l}$$

Newton's 2nd Law, $\quad F = ma$

$$-mg\frac{x}{l} = m\frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

$$\Rightarrow \boxed{\frac{d^2x}{dt^2} + \left(\frac{g}{l}\right)x = 0}$$

$$\omega = \sqrt{\frac{g}{L}}$$

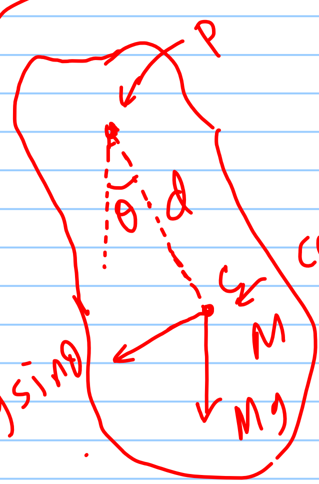
$$\boxed{T = 2\pi\sqrt{\frac{l}{g}}}$$ ✓

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l}{g}}$$

$$f = \frac{1}{T}$$

Any rigid body mounted so that it can swing in a vertical plane about some axis.

Physical pendulum



center of mass

Mg sinθ

$$\tau = -Mgd\sin\theta$$

Newton's 2nd Law For Rotation,

$$\tau = I\alpha$$

$$-Mgd\sin\theta = I\frac{d^2\theta}{dt^2}$$

small oscillation

$\sin\theta \simeq \theta$

$$\Rightarrow -Mgd\,\theta = I\frac{d^2\theta}{dt^2}$$

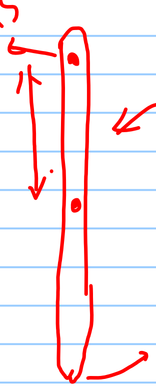
$$\Rightarrow \frac{d^2\theta}{dt^2} + \frac{Mgd}{I}\theta = 0$$

$$\omega^2 = \frac{Mgd}{I}$$

$$\omega = \sqrt{\frac{Mgd}{I}}$$

$$T = 2\pi\sqrt{\frac{I}{Mgd}}$$

axis



$I = \frac{1}{12} ML^2$

$M = 1\,kg$

$L = 1\,m$

$d = \frac{L}{2}$

$$\boxed{T = 2\pi\sqrt{\frac{I}{Mgd}}}$$

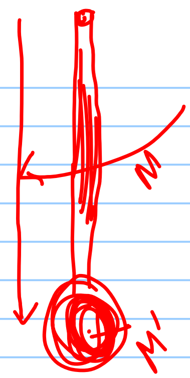
$$\theta = \theta_m \sin(\omega t + \delta)$$

$$\frac{d\theta}{dt} =$$

$$\frac{d^2\theta}{dt^2} =$$

$x\ m^2 d^2$

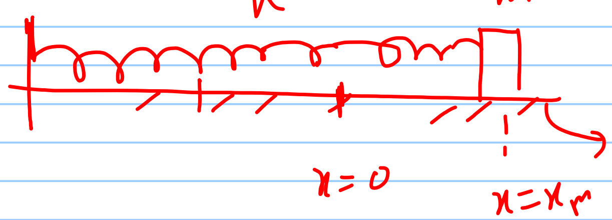
$$I = \frac{1}{12} M L^2 + \frac{2}{5} M' R^2 + M'(L+R)^2$$



$$x_{cm} = \frac{m_1 x_1}{m_1 + m_2}$$

$$T = 2\pi \sqrt{\frac{I}{Mgd}}$$

spring-block



Damping, $F_d = -bv$

b → damping constant

$$F_s = -kx \qquad F_{net} = -kx - kv$$

Newton's 2nd law

$$F_{net} = ma$$

$$\Rightarrow -kx - bv = ma$$

$$\Rightarrow -kx - b\frac{dx}{dt} - m\frac{d^2x}{dt^2} = 0$$

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

$$\Rightarrow \boxed{\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = 0}$$

$$x(t) = e^{\lambda t}$$

$$\frac{d^2}{dt^2}(e^{\lambda t}) + \frac{b}{m}\frac{d}{dt}(e^{\lambda t}) + \frac{k}{m}e^{\lambda t} = 0$$

$$\Rightarrow \lambda^2 e^{\lambda t} + \frac{b}{m}\lambda e^{\lambda t} + \frac{k}{m}e^{\lambda t} = 0$$

$$\Rightarrow e^{\lambda t}\left(\lambda^2 + \frac{b}{m}\lambda + \frac{k}{m}\right) = 0$$

$$e^{\lambda t} \neq 0; \quad \lambda^2 + \frac{b}{m}\lambda + \frac{k}{m} = 0$$

$$\lambda = \frac{-\frac{b}{m} \pm \sqrt{\left(\frac{b}{m}\right)^2 - 4.1.\frac{k}{m}}}{2 \cdot 1}$$

$$\lambda = -\frac{b}{2m} \pm \sqrt{\left(\frac{b}{2m}\right)^2 - \frac{k}{m}}$$

$$x = x_m e^{\left(-\frac{b}{2m} \pm \sqrt{\left(\frac{b}{2m}\right)^2 - \frac{k}{m}}\right)t}$$

$$x = x_m e^{-\frac{b}{2m}t} e^{\pm\sqrt{\left(\frac{b}{2m}\right)^2 - \frac{k}{m}}\, t}$$

$$\left(\frac{b}{2m}\right)^2 < \frac{k}{m}$$ [underdamped condition)

$$x = x_m e^{-\frac{b}{2m}t} e^{\pm i\sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}\, t}$$

$$= x_m e^{-\frac{b}{2m}t}\left[\underline{\cos\sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}\, t} \pm i\,\underline{\sin\sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}\, t}\right]$$

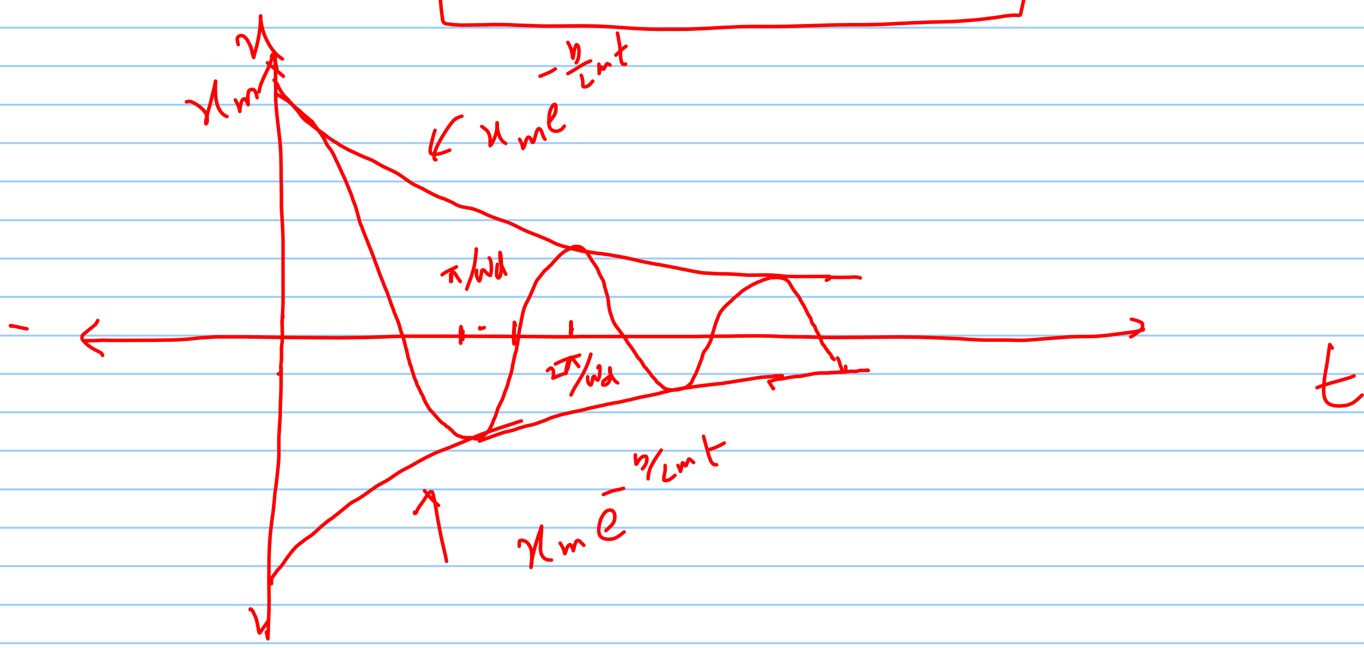
Taking real part,

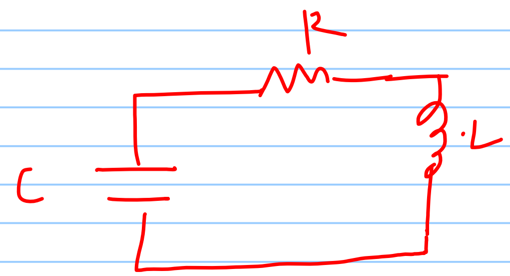
$$\boxed{x = x_m e^{-\frac{b}{2m}t}\cos(\omega_d t + \delta)}$$

$$\omega_0 = \sqrt{\frac{k}{m}}$$

$$\omega_d = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$$

$$\boxed{\omega_d = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2}}$$

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