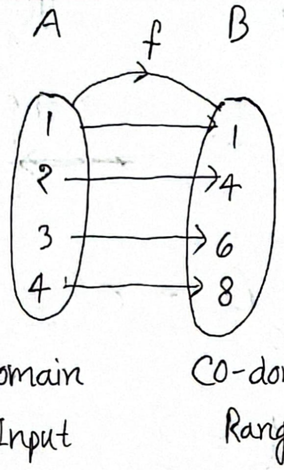
Lec-1

10.2.25

Function: $y = f(x)$



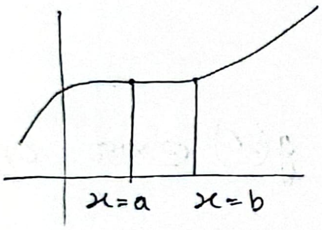
Output

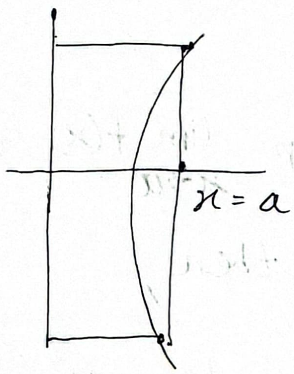
$f(x) = y = x^2$ (Function)

$y^2 = x^2 - 1$

$\Rightarrow y = \pm\sqrt{x^2 - 1}$ (not function)

$\searrow$ 2 output





not Function

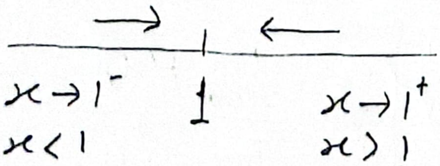
$f(x) = \dfrac{x+2}{x-1}$; $x \neq 1$

$\Rightarrow f(1) = \dfrac{1+2}{1-1}$ = undefined

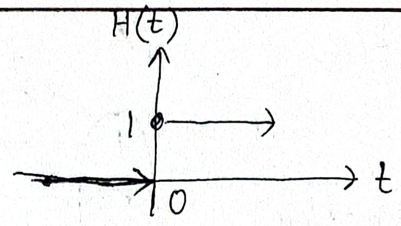
$x \to 1$ $\therefore f(x) = \dfrac{2.9999}{0.9999 - 1} = \dfrac{2.9999}{-0.0001} = 2.99999 \ldots$

$0.9999\ldots$

$$\boxed{\lim_{x \to a} f(x) = L}$$



Heviside Function:



$$H(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$$

$$\lim_{t \to 0^-} H(t) = \lim_{t \to 0} 0 = 0$$

$$\lim_{t \to 0^+} H(t) = t^2 + 1 = 1$$

$$\boxed{0^2+1}$$

$$\lim_{t \to 0^+} H(t) = \lim_{t \to 0} 1 = 1$$

Propertise :-

We will consider $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist and c is any constant then,

i. $\lim_{x \to a} \left[c f(x) \right] = c \lim_{x \to a} f(x)$

ii. $\lim_{x \to a} \left[f(x) \pm g(x) \right] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$

iii. $\lim_{x \to a} \left[f(x) \cdot g(x) \right] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$

iv. $\lim_{x \to a} \left[\frac{f(x)}{g(x)} \right] = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$ where $\lim_{x \to a} g(x) \neq 0$

v. $\lim_{x \to a} \left[f(x) \right]^n = \left[\lim_{x \to a} f(x) \right]^n$ example : $\lim_{x \to a} (x^2+1)^3 = \left[\lim_{x \to a} x^2+1 \right]^3$

→ needed in preswise function

L.H.L $= \lim_{x \to a^-} f(x) = L$

R.H.L $= \lim_{x \to a^+} f(x) = L$

$\Big\}$ same, so limit exist.

$$\lim_{x \to 3} \frac{x^2-9}{x-3} = \lim_{x \to 3} (x+3) = 6$$

$$= \lim_{x \to 3} \frac{(x+3)(x-3)}{(x-3)} \quad ; \; x \neq 3$$ → Important Condition

$$\lim_{x \to -4} \frac{2x+8}{x^2+x-12} = \lim_{x \to -4} \frac{2\cancel{(x+4)}}{\cancel{(x+4)}(x-3)}$$

$$= \lim_{x \to -4} \frac{2}{x-3} \quad ; \; x \neq -4$$

$$= -\frac{2}{7}$$

$$\lim_{x \to 0} \frac{x}{\sqrt{x+1}-1} = \lim_{x \to 0} \frac{x(\sqrt{x+1}+1)}{x+1-1} = \lim_{x \to 0} \sqrt{x+1} + 1 \quad ; \; x \neq 0$$

$$= 2$$

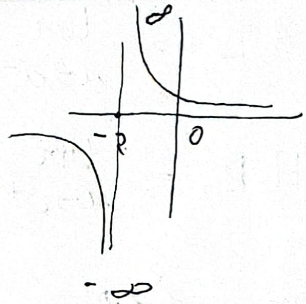
Given that,

$$f(x) = \begin{cases} \dfrac{1}{x+2} & ; \; x < -2 \\ x^2-5 & ; \; -2 \leq x \leq 3 \\ \sqrt{x+13} & ; \; x > 3 \end{cases}$$

↙ Preswise function

Find the $\lim_{x \to -2} f(x)$ if exist

$\Rightarrow$ L.H.L $= \lim\limits_{x \to -2^-} f(x) = \lim\limits_{x \to -2} \frac{1}{x+2}$



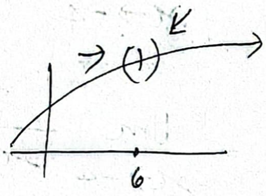
$= -\infty$

R.H.L $= \lim\limits_{x \to -2^+} f(x) = \lim\limits_{x \to -2} x^2 - 5$

$= -1$

$\therefore$ Limit does not exist.

$\lim\limits_{x \to 6} f(x) = \lim\limits_{x \to 6} \sqrt{x+13}$



$\boxed{\text{Lec-2}}$

12.2.25

$|x| = \begin{cases} x & ; x > 0 \\ -x & ; x \le 0 \end{cases}$

$|x-2| = \begin{cases} x-2 & ; x-2 > 0 \Rightarrow x > 2 \\ -(x-2) & ; x-2 \le 0 \Rightarrow x \le 2 \end{cases}$

$\lim\limits_{x \to 0} x \sin\left(\frac{1}{x}\right)$

The squeeze theorem

$g(x) \le f(x) \le h(x)$

if $\lim\limits_{x \to a} g(x) = \lim\limits_{x \to a} h(x) = L$

then $\lim\limits_{x \to a} f(x) = L$

$2 < 3$

$(-1)\, 2 > 3\, (-1)$

$-2 > -3$

$\lim_{x \to 0} x \sin\left(\frac{1}{x}\right)$

$$-1 \le \sin\frac{1}{x} \le 1$$

$$-|x| \le x\sin\frac{1}{x} \le |x|$$

$$-\lim_{x\to 0}|x| \le \lim_{x\to 0} x\sin\frac{1}{x} \le \lim_{x\to 0}|x|$$

Since, $\lim_{x\to 0}|x| = 0$

Applying squeeze theorem we get, $\lim_{x\to 0} |x| \sin\left(\frac{1}{x}\right) = 0$

So, $\lim_{x\to 0} x\sin\frac{1}{x} = 0$ x

Definition of Continuty :-

A function $f(x)$ is continuos of $x=a$ if following condition satisfies :-

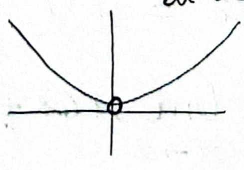
i. $f(a)$ is defined , $f(x) = \frac{x}{x-1}$, $x=1$ $f(1) = \frac{1}{0}$ undefined

ii. $\lim_{x\to a} f(x)$ exist

iii. $\lim_{x\to a} f(x) = f(a)$

$$f(x) = \begin{cases} x^2 & ; x \neq 0 \\ 1 & ; x = 0 \end{cases}$$

at $x=0$



not continuous

$f(0) = 1$

$\lim_{x\to 0} x^2 = 0$

☐ Given that, $f(x) = \begin{cases} x^2+1 & ; \; x>0 \\ 1 & ; \; x=0 \\ x+1 & ; \; x<0 \end{cases}$ is it Continous ?

$\Rightarrow$ $f(0) = 1$. [So the function is defined]

$LHL = \lim_{x \to 0^-} x^2+1 = 1$

$RHL = \lim_{x \to 0^+} x^2+1 = 1$

So limit exists

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = 1 = f(0)$

$\therefore$ The function is continues at $x=0$.

☐ Given that, $f(x) = \begin{cases} x\cos(\frac{1}{x}) & ; \; x \neq 0 \\ 0 & ; \; x=0 \end{cases}$ is it Continous ?

$\Rightarrow$ $-1 \leq \cos\frac{1}{x} \leq 1$

$-|x| \leq x\cos\frac{1}{x} \leq |x|$

$-\lim_{x \to 0} |x| \leq \lim_{x \to 0} x\cos\frac{1}{x} \leq \lim_{x \to 0} |x|$

$\because -\lim_{x \to 0} |x| = \lim_{x \to 0} |x| = 0$ $\therefore \lim_{x \to 0} x\cos\frac{1}{x} = 0$ [Squeeze theorem]

$f(0) = 0$ $\because \lim_{x \to 0} x\cos\frac{1}{x} = f(0)$ $\therefore$ the function is continues at $x=0$

Q-1 Syllabus 2 Questions.

wed

1. Limit
2. Continuoty

Propertise of continuty :-

If the functions of f & g are continuos at $x=a$ then,

1. $f \pm g$ is continuos at $x=a$
2. $f \cdot g$ " " " "
3. f/g " " " " ; $g(a) \neq 0$

* 4. If $\lim_{x \to a} g(x) = L$ and if f is Continuous at ~~x~~ $=a$ then

$$\lim_{x \to a} f(g(x)) = f\left(\lim_{x \to a} g(x)\right)$$

Consider $g(x) = x^2+1$ and $f(x) = \sqrt{x}$ are continuos for $x \geq 0$

If $\lim_{x \to 2} g(x) = 5$ then find $\lim_{x \to 2} f(g(x))$

$$\Rightarrow \lim_{x \to 2} f(g(x)) = f\left(\lim_{x \to 2} g(x)\right) = f(5)$$

$f(x) = \sqrt{x} \quad \therefore f(5) = \sqrt{5}$

Lec-3

$\lim_{x \to \infty} \frac{2-\cos x}{x+3}$ using squeeze theorem :-

$\Rightarrow$

$$-1 \le \cos x < 1$$

$$(-1)(-1) \ge -\cos x \ge (-1)(1)$$

$$1 \ge -\cos x \ge -1$$

$$2+1 \ge 2-\cos x \ge 2-1$$

$$3 \ge 2-\cos x \ge 1$$

$$1 \le 2-\cos x \le 3$$

$$\cancel{\lim_{x \to \infty} 1 \le \lim_{x \to \infty} 2-\cos x \le}$$

$$\frac{1}{x+3} \le \frac{2-\cos x}{x+3} \le \frac{3}{x+3}$$

$$\lim_{x \to \infty} \frac{1}{|x+3|} \le \lim_{x \to \infty} \frac{2-\cos x}{|x+3|} \le \lim_{x \to \infty} \frac{3}{|x+3|}$$

$$0 \le \underset{0}{\downarrow} \le 0$$

$\therefore \lim_{x \to \infty} \frac{2-\cos x}{x+3} = 0$ ✓

$$f(x) = \begin{cases} \dfrac{e^{1/x^2}}{e^{1/x^2} - 1} & ; x \neq 0 \\ 1 & ; x = 0 \end{cases}$$

is the function continuous at $x=0$?

$$\boxed{\frac{0}{0},\ \frac{\infty}{\infty},\ 1^{\infty},\ \infty^{\infty}}$$

$$\Rightarrow \lim_{x\to 0} \frac{e^{1/x^2}}{e^{1/x^2}-1} = \lim_{x\to 0} \frac{e^{1/x^2}\cdot \frac{1}{e^{1/x^2}}}{\frac{1}{e^{1/x^2}}\left(e^{1/x^2}-1\right)} = \lim_{x\to 0} \frac{1}{1-\frac{1}{e^{1/x^2}}}$$

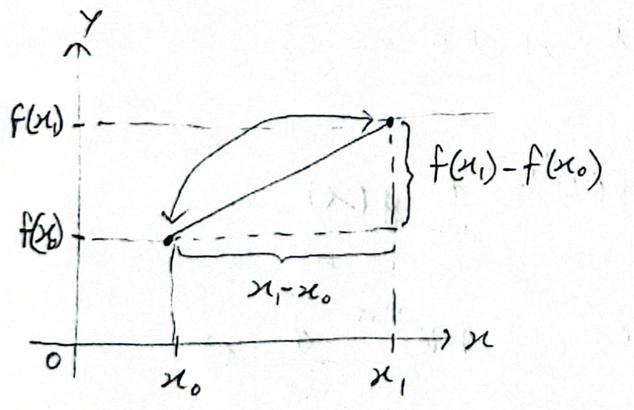
$$= \frac{1}{1-0} = 1$$

Differentiation

Rate of change slope $m = \frac{\Delta y}{\Delta x} = \frac{f(x_1)-f(x_0)}{x_1-x_0} = \frac{\text{rise}}{\text{run}}$

Average rate of change, $r_{avg} = \frac{f(x_1)-f(x_0)}{x_1-x_0}$

Instateneous rate of change at x_0, $r_{in} = \lim_{x_1\to x_0} \frac{f(x_1)-f(x_0)}{x_1-x_0}$



$h = x_1 - x_0$, $x_1 = h + x_0$

$x_1 \to x_0$, $h \to 0$

$$f(x_0) = r_{in} = \lim_{h\to 0} \frac{f(x_0+h)-f(x_0)}{h}$$

is called derivative of $f(x)$ in terds to x at $x = x_0$ for all x

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

Derivative of following function by definition :-

$f(x) = x^2$, $f(x+h) = (x+h)^2$

$$\therefore f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h\to 0} \frac{\cancel{x^2}+2xh+h^2-\cancel{x^2}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}} = 2x + 0 = 2x$$

Notation :- $\frac{df}{dx}$, f' , f_x , f_D , $\dot{f}$

Propentise :-

1. $\frac{d}{dx}\left[c\, f(x)\right] = c \frac{d}{dx} f(x)$

2. $\frac{d}{dx} x^r = r x^{r-1}$, $a^{r\in R}$, $a \in R - \{0\}$

3. $\frac{d}{dx}\left[f(x) \pm g(x)\right] = \frac{d}{dx} f(x) \pm \frac{d}{dx} g(x)$

4. $\frac{d}{dx} e^{mx} = e^{mx} \cdot \frac{d}{dx}(mx) = me^{mx} \cdot \frac{d}{dx}(x)$

Find the value of K for which the given function is continuous at $x = 1$,

$$f(x) = \begin{cases} 7x-2 & ; x \le 1 \\ Kx^2 & ; x > 1 \end{cases}$$

10, 11 (X)

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1} (7x-2) = 5$$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1} Kx^2 = K$$

If $K=5$, then the function is continuous at $x=1$.

$$f(x) = 7x^{-6} \cancel{+} 5\sqrt{x}$$

$$\Rightarrow f'(x) = \frac{d}{dx} 7x^{-6} - \frac{d}{dx} 5\sqrt{x} = 7(-6)x^{-6-1} - \frac{5}{2\sqrt{x}}$$

$$= -42x^{-7} - \frac{5}{2\sqrt{x}}$$

Lec-4

10. 2. 25

Product rule :-

If $f(x)$ & $g(x)$ both are differentiable so, $f \cdot g$ are,

$$\frac{d}{dx}[f(x)\cdot g(x)] = f(x)\frac{d}{dx}g(x) + g(x)\frac{d}{dx}f(x)$$

or

$$(f\cdot g)' = f\cdot g' + g\cdot f'$$

Find $\frac{dy}{dx}$, where $y = (4x^3+1)(7x^2+x)$

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\left[(4x^3+1)(7x^2+x)\right]$

$= (4x^3+1)\frac{d}{dx}(7x^2+x) + (7x^2+x)\frac{d}{dx}(4x^3+1)$

$= (4x^3+1)(14x+1) + (7x^2+x)(12x^2)$

$\left.\frac{dy}{dx}\right|_{x=2} = (32+1)(28+1) + (28+2)\cdot 48$

$= 2397$

Quitient Rule :

If f & g are both differrentiable and $g(x) \neq 0$

so, $\frac{f(x)}{g(x)}$ also differrentiable :-

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}f(x) - f(x)\frac{d}{dx}g(x)}{[g(x)]^2}$$

or

$$\left(\frac{f}{g}\right)' = \frac{g\cdot f' - fg'}{g^2}$$

Trigonometric Function :-

1. $\frac{d}{dx}\sin x = \cos x$

2. $\frac{d}{dx}\cos x = -\sin x$

3. $\frac{d}{dx}\tan x = \sec^2 x$

4. $\frac{d}{dx}\cot x = -\operatorname{cosec}^2 x$

5. $\frac{d}{dx}\sec x = \sec x \tan x$

6. $\frac{d}{dx}\operatorname{cosec} x = -\operatorname{cosec} x \cot x$

$y = \frac{\sin x}{1+\cos x} =$

Chain Rule :-

f & g are both differetiable. say $y = f(g(x))$ & $g(x) = u$

$$y = f(u)$$

$$\frac{dy}{dx} = \frac{df}{du} \cdot \frac{du}{dx} = f'(u) \cdot g'(x) = f'(g(x)) \cdot g'(x)$$

$$\boxed{\frac{dy}{dx} = f'(g(x)) \cdot g'(x)}$$

logarithmus naturalis → ln

next quiz important

$f(x) = \sqrt{x+1}$, $g(x) = x^2 - 2$ Find $F'(x)$ where $F(x) = f(g(x))$

$\Rightarrow F'(x) = (f(g(x)))' = f'(g(x)) \cdot g'(x)$

$f(x) = \sqrt{x+1}$

$y = \sin(2x^2+1)$

$$\frac{dy}{dx} = \cos(2x^2+1) \cdot 4x = 4x\cos(2x^2+1)$$

$$\frac{d}{d\theta}\cos 2\theta = -\sin 2\theta \frac{d}{d\theta}(2\theta) = -2\sin(2\theta)$$

Lec-5

24.2.25

$f(x) = x^2+1$

Implecit Function:

$x^2y + \sin y = 2xy$

$$\frac{d}{dx}[x^2y + \sin y] = \frac{d}{dx}2xy$$

$$\frac{d}{dx}x^2y + \frac{d}{dx}\sin y = 2\frac{d}{dx}xy$$

$$x^2\frac{dy}{dx} + y2x + \cos y\frac{dy}{dx} = 2x\frac{dy}{dx} + 2y$$

$$\frac{dy}{dx}\left[x^2+\cos y+2x\right] = 2y+2xy$$

$$\frac{dy}{dx} = \frac{2y-2xy}{x^2+\cos y-2x}$$

Logarithms & Exponents

i. $\frac{d}{dx}\ln x = \frac{1}{x}$

ii. $\frac{d}{dx}\log_b x = \frac{1}{\ln b}\frac{d}{dx}\ln x = \frac{1}{x\ln b}$

$$\log_b x = \frac{\log_a x}{\log_a b}$$

$$= \frac{\ln x}{\ln b}$$

iii. $\frac{d}{dx}e^{mx} = e^{mx}\frac{d}{dx}mx = me^{mx}$

iv. $\frac{d}{dx}b^x = b^x\ln b$

Say, $y = (x^2+1)^{\sin x}$, $\ln y = \sin \ln(x^2+1)$, $\frac{dy}{dx} = ?$

$$\frac{d}{dx}\left[\ln y\right] = \frac{d}{dx}\left[\sin x\ln(x^2+1)\right] = \frac{\sin x\cdot 2x}{x^2+1} + \cos x\ln(x^2+1)$$

$$\frac{dy}{dx} = y\left\{\frac{2x\sin x}{x^2+1} + \cos x\ln(x^2+1)\right\}$$

$y = (x^2+1)^{\sin x}$

Inverse trigonometric Function

i. $\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$

ii. $\frac{d}{dx} \cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$

iii. $\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$

iv. $\frac{d}{dx} \cot^{-1} x = \frac{-1}{1+x^2}$

v. $\frac{d}{dx} \sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}$

v. $\frac{d}{dx} \csc^{-1} x = \frac{-1}{|x|\sqrt{x^2-1}}$

$y = \sin^{-1} x^3 \Rightarrow \frac{dy}{dx} = \frac{d}{dx} \sin^{-1}(x^3) = \frac{1}{\sqrt{1-x^6}} \cdot 3x^2$

L'Hsosptials Rule:

f & g bot differentiable & $\lim_{x\to a} f(x) = 0$, $\lim_{x\to a} g(x) = 0$

If $\lim_{x\to a} \frac{f(x)}{g(x)}$ exists then

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

$$\lim_{x\to 0} \frac{e^x - 1}{x^3} = \lim_{x \to 0} \frac{e^x}{3x^2} = +\infty$$

$\infty + \infty,\ 0\cdot\infty,\ 0^0,\ 1^\infty,\ \infty^0 \rightarrow \frac{0}{0},\ \frac{\infty}{\infty}$

$$y = \lim_{x\to \frac{\pi}{4}} (1-\tan x)\sec 2x \qquad [0, \infty]$$

$$= \lim_{x\to\frac{\pi}{4}} \frac{(1-\tan x)}{\cos 2x}$$

$$= \lim_{x\to\frac{\pi}{4}} \frac{-\sec^2 x}{-2\sin 2x}$$

$$= \frac{-(\sqrt{2})^2}{-2} = \frac{-2}{-2} = 1$$

Lec-7

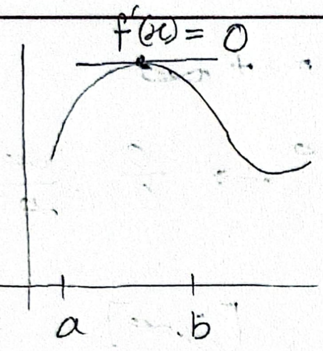
3·3·25

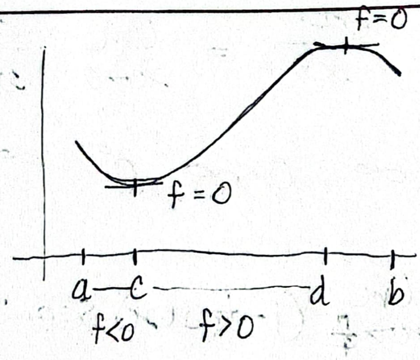
Increasing, Decreasing, Continuity

i. If $f'(x) > 0 \ \forall x \in (a, b)$, then f is increasing on $[a, b]$

ii. If $f'(x) < 0 \ \forall x \in (a, b)$, then f is decreasing on $[a, b]$

iii. If $f'(x) = 0 \ \forall x \in (a, b)$ then f is constant on $[a, b]$





四 Find the intervals on which $f(x) = 3x^4 + 4x^3 - 12x^2 + 2$ is increasing, decreasing, and concavity.

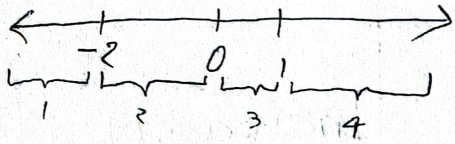
$\Rightarrow F(x) = f'(x) = 12x^3 + 12x^2 - 24x$

For critical point, $f'(x) = 0$.

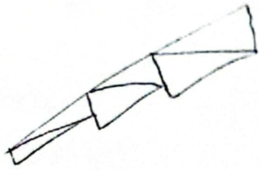
$$12x^3 + 12x^2 - 24x = 0$$

$$12x(x+2)(x-1) = 0$$

$x = 0, -2, 1$



Interval	$12x(x+2)(x-1)$		$f'(x)$	Conclusion
$x < -2$	$(-)(-)(-)$		$-$	f is decreasing on $(-\infty,$
$-2 < x < 0$	$(-)(+)(-)$	$x=-2$ min	$+$	f is increasing on $[-2, 0$
$0 < x < 1$	$(+)(+)(-)$	$x=0$ max	$\cancel{+}$	f 〃 decreasing on $[0, 1]$
$x > 1$	$(+)(+)(+)$	$x=1$ min	$+$	f 〃 〃 on $[1, \infty$



Concavity

Concave up

$f''(x) > 0$

Concave down

$f''(x) < 0$

i. If $f''(x) > 0 \quad \forall x \in (a,b)$ then f is concave up on (a,b)

ii. If $f''(x) < 0$ 〃 〃 〃 f is 〃 down on (a,b)

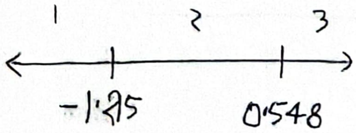
B $F(x) = f'(x)$

$$F'(x) = f''(x) = 36x^2 + 24x - 24 = 0$$

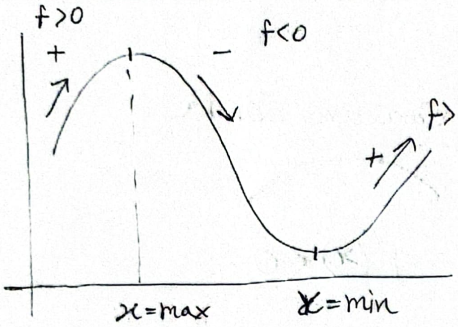
$$36x^2 + 24x - 24 = 0$$

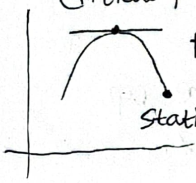
$$x = \frac{-1 \pm \sqrt{7}}{3}$$

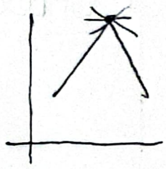
$$x = \begin{cases} \frac{-1-\sqrt{7}}{3} = -1.215 \\ \frac{-1+\sqrt{7}}{3} = 0.548 \end{cases}$$



Interval	$f''(x)$	Conclusion
$x < -1.215$	+	f is concave up on $(-\infty, -1.215)$
$-1.215 < x < 0.548$	-	f is 〃 down 〃 $(-1.215, 0.548)$
$x > 0.548$	+	up $(0.548, +\infty)$



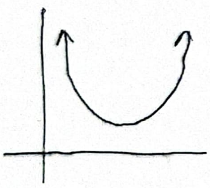




Lec-8

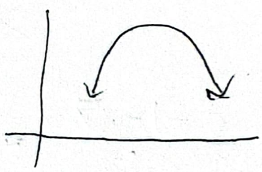
Extrema (max/min value)

- → relative extrema — + to − max
- → Absolute extrema — − to + min



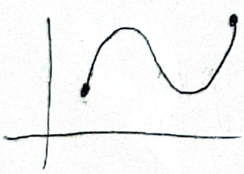
Absolute maxima = X

" Minima = Maybe



A. max = m

A.min = X



has Absolute maxima/Minima

Need Boundary point & critical point for absolute maxima & min

Quiz-2 | Wed

i. Differetiability
ii. Technic of differnention
iii. Increasing, decreasing concavity
iv. Max / Min.

Find the absolute extrema of the function:

$f(x) = 3x^4 + 4x^3 - 12x^2 + 2$ on the interval $[-5, 5]$

$\Rightarrow$ For Critical point, $f'(x) = 0$

$$\Rightarrow 12x^3 + 12x^2 - 24x = 0$$

$$\Rightarrow 12x(x+2)(x-1) = 0$$

$$x = 0, -2, 1$$

For absolute extrema:

$f(0) = 2$

$f(-2) = -30$

$f(1) = -3$

$f(-5) = 1077$

$f(5) = 2077$

Absolute maxima value is 2077 at $x = 5$

And absolute minima value is -30 at -2

* Successive Differentiation

$$y_1 = \frac{dy}{dx}$$

$$y_2 = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{dy_1}{dx}$$

calculus always ln

If $y = \log(x+\sqrt{x^2+1})$, show that $(1+x^2)y_2 + xy_1 = 0$

$\Rightarrow y = \log(x+\sqrt{x^2+1})$

$$y_1 = \frac{1}{x+\sqrt{x^2+1}} \cdot \left[1 + \frac{1}{\cancel{2}\sqrt{x^2+1}} \cdot \cancel{2}x\right]$$

$$y_1 = \cancel{\frac{x}{(x+\sqrt{x^2+1})\sqrt{x^2+1}}} \qquad \frac{1+\frac{x}{\sqrt{x^2+1}}}{x+\sqrt{x^2+1}}$$

$$y_1 = \frac{\frac{\cancel{x+\sqrt{x^2+1}}}{\sqrt{x^2+1}}}{\cancel{x+\sqrt{x^2+1}}}$$

$$y_1 = \frac{1}{\sqrt{x^2+1}}$$

$$y_1\sqrt{x^2+1} = 1$$

$$y_1^2(x^2+1) = 1$$

Differentiate both side in terms of x

uv $\rightarrow (x^2+1)2y_1y_2 + y_1^2(2x+0) = 0$

$$2y_1\{(1+x^2)y_2 + y_1x\} = 0$$

$\therefore (1+x^2)y_2 + y_1x = 0$ (Showed)

Find n^{th} differential Coefficient of $\frac{1}{(1+x)^2}$.

$\Rightarrow y = (1+x)^{-2}$

$y_1 = (-2)(1+x)^{-3}$

$y_2 = (-3)(-2)(1+x)^{-4} = (-1)^2 2 \cdot 3 \cdot (1+x)^{-(2+2)}$

$y_3 = (-2)(-3)(-4)(1+x)^{-5} = (-1)^3 2 \cdot 3 \cdot 4 (1+x)^{-(3+2)}$

$y_2 = (-1)^2 3! (1+x)^{-(2+2)} = (-1)^2 (2+1)! (1+x)^{-(2+2)}$

$y_3 = (-1)^3 4! (1+x)^{-(3+2)} = (-1)^3 (3+1)! (1+x)^{-(3+2)}$

So, $y_n = (-1)^n (n+1)! (1+x)^{-(n+2)}$

Tylors polynominal :

$f(x) \approx f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \ldots\ldots$

Lec-9

10.3.25

$$y_1 = \frac{dy}{dx} \qquad y_2 = \frac{dy_1}{dx} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

If $y = a\cos(\log x) + b\sin(\log x)$, prove that $x^2y_2 + xy_1 + y$.

$$y_1 = -a\sin(\log x)\frac{1}{x} + b\cos(\log x)\frac{1}{x}$$

$$xy_1 = -a\sin\log x + b\cos(\log x)$$

$$xy_2 + y_1' = -a\cos(\log)\frac{1}{x} - b\sin(\log)\frac{1}{x}$$

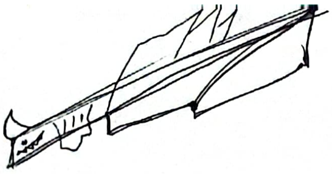
$$x(xy_2 + y_1) = -\underbrace{\left[a\cos(\log x) + b\sin(\log x)\right]}_{y}$$

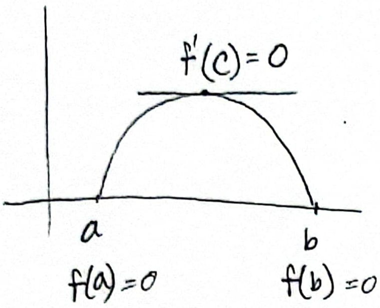
$$x^2y_2 + xy_1 = -y$$

$$\therefore x^2y_2 + xy_1 + y = 0 \quad \text{[showed]}$$

* Rolles Theorem:

Consider f be continuous on closed interval $[a,b]$ and differentiable on (a,b). If $f(a)=0$ and $f(b)=0$, then there is at ~~be~~ least one point C is (a,b) such that $f'(c)=0$.





四 Verify that the hypothesis of Rolles' theorem are satisfy on the given interval and find all values of C in that interval that satisfy the conclusion of the theorem.

$$f(x) = x^2 - 5x + 4 \quad \text{on } [1,4]$$

$\Rightarrow f(1) = 1^2 - 5\cdot 1 + 4 = 0$

$f(4) = 4^2 - 5\cdot 4 + 4 = 0$

$f'(x) = 2x - 5 = 0$

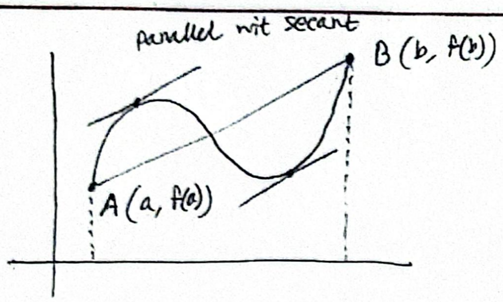
$f'(c) = 2c - 5 = 0 \quad \therefore \quad c = 5/2 \in (1,4)$

$C=1 \notin (1,4)$
$C=4 \notin (1,4)$

* The mean value theorem :-

Consider f be continuous on closed interval $[a,b]$ and differnentiable on (a,b) Then there is at least one point C in (a,b) such that $f'(C) = \dfrac{f(b) - f(a)}{b-a}$ → Average velocity

Instataneous velocity



Determine all the numbers 'C' which satisfy the Conclusion of the Mean value Theorem for the following function. $f(x) = x^3 + 2x^2 - x$ on $[-1, 2]$

* every polynominial function is differrentiable and continuous.

$\Rightarrow$ $f'(x) = 3x^2 + 2x - 1$

$f(-1) = 1^3 + 2 \cdot 1^2 + 1 = 2$

$f(2) = 2^3 + 2 \cdot 2^2 - 2 = 14$

According to MVT,

$f'(c) = 3c^2 + 2c - 1 = \frac{14-2}{2-(-1)} = \frac{12}{3} = 4$

$3c^2 + 4c - 5 = 0$, $c = \frac{-4 \pm \sqrt{76}}{6} = 0.7863 \in (-1, 2)$

$-2.1196 \notin (-1, 2)$

Suppose we know that $f(x)$ is continuous and differentiable on $[6, 15]$ let's also suppose, we know that $f(6) = -2$ & $f'(x) \leq 10$. What is the largest possible value for $f(15)$?

$\Rightarrow$ $f'(c) = \dfrac{f(b) - f(a)}{b-a}$

$f'(c) \leq 10$

By MVT, $f'(c) = \cancel{16} \dfrac{f(15) - f(6)}{15-6} = \dfrac{f(15)+2}{9}$

$9f'(c) = f(15) + 2$

A/Q, $f(x) \leq 10$

So, in particular we know that $f'(c) \leq 10$

Thus $f(15) = 9f'(c) - 2 \leq 9 \cdot 10 - 2 = 88$

Which means the possible largest value of $f(15)$ is 88.