PHY 111: Principles of Physics I Fall 2023

LECTURE 07 — October 19, 2023

SECTION: 35 (UB71001) FACULTY: Akiful Islam (AZW)

BRAC UNIVERSITY

1 Dynamics

Dynamics is part of mechanics that studies the motion of a system that comes into play because of the application of external forces. Dynamics deals with all the motions that did not have a cause previously. Force and momentum are the cause.

2 Prerequisites for the Force Laws

2.1 What is Force?

In an introductory course on Physics like PHY111, Force can be considered a push or a pull applied to an object. It is a vector quantity, meaning it has both magnitude (how strong the force is) and direction. Forces can cause an object to start moving, change its velocity, or alter its direction of motion.

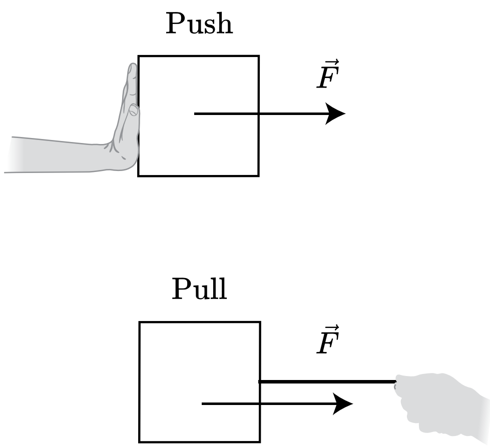

FIGURE 1: Force applied from two directions on the same body.

In nature, we only have four types of fundamental forces. These bind our universe and existence. The force we will be dealing with in this course is localized **push-and-pull** and a bit of gravitational force. We will see forces like the tension on a rope, the weight of an object, the friction of an object, and normal force applied on an object by the plane it is sitting on/sliding off of.

The fundamental forces in physics are the four basic forces that govern the behavior of all matter and energy in the universe. These forces are:

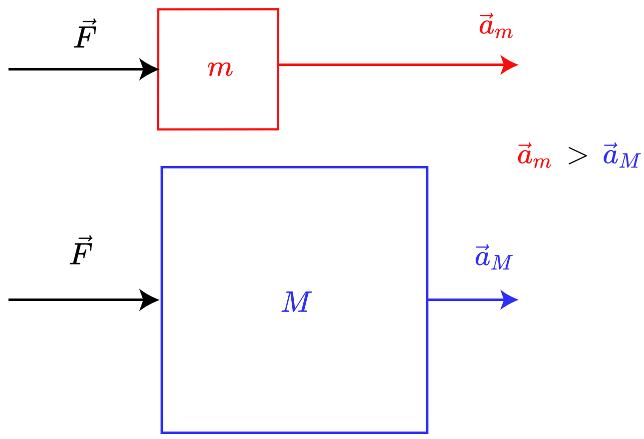

FIGURE 2: The same force applied on two different masses will cause different amounts of acceleration.

1. **Gravitational Force**: The force of attraction between objects with mass. It is why the Earth and the moon remain in orbit around the sun and why objects fall to the ground when dropped.
2. **Electromagnetic Force**: The force that arises between electrically charged objects. It is responsible for many everyday phenomena, such as electric currents, light, and magnetism.
3. **Weak Nuclear Force**: The force responsible for certain types of radioactive decay and the fusion of elements in stars. It is one of the least understood of the four fundamental forces.
4. **Strong Nuclear Force**: The force that holds the nucleus of an atom together. It is the strongest of the four fundamental forces and overcomes the electromagnetic repulsion between the positively charged protons in the nucleus.

Each of these fundamental forces operates over different distances and with different strengths. The electromagnetic force and the weak nuclear force operate on the smallest of scales and are responsible for the behavior of subatomic particles. The strong nuclear force acts on the scale of atomic nuclei. In contrast, gravitational force describes the relationship between two or multiple objects' motion at the scale of entire galaxies, even cosmological. We won't touch Electromagnetic, Strong, or Weak nuclear forces. You'll see Electromagnetic force at play in the PHY112 course. You must study Quantum Field Theory and Particle Physics for the Strong and Weak forces.

2.2 What is Mass?

Mass is a scalar quantity describing the amount of matter in an object. It measures an object's resistance to a change in its velocity and is often considered a measure of its **inertia**. In physics, mass is defined as the amount of matter in an object and is distinct from weight, which is the force exerted on an object by gravity. An object with a large mass has a greater amount of matter and is more difficult to move or change its velocity, while an object with a smaller mass is easier to move or change its velocity.

2.3 What is Momentum?

momentum measures an object's **motion mass**. It is the product of an object's mass and its velocity, and it describes the amount of motion that an object has. Like force, momentum is also a vector quantity, meaning it has both magnitude and direction.

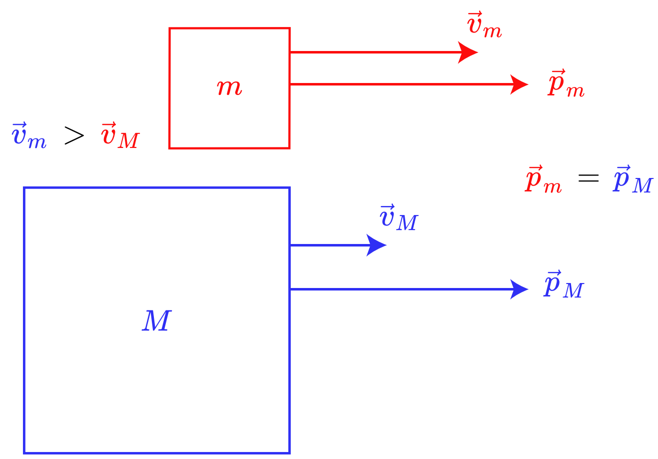

FIGURE 3: Momentum of two different masses can be equal if their velocity retains the product of m and $\vec{v}$ constant.

Intuitively, you can think of force as something that makes an object change its state of motion and momentum as a measure of an object's existing state of motion. For example, if you push a heavy object, you must apply a large force to get it moving. Once the object moves, it has a certain amount of momentum, which determines how difficult it is to stop or change its direction.

2.4 What is Acceleration?

You already know what it is. If not, follow through with Lecture 4.

3 Newton's Laws of Motion

Newton's laws of motion are three fundamental laws describing the relationship between an object's motion and the forces acting on it. These laws were first formulated by Sir Isaac Newton in the 17^{th} century and have since become a cornerstone of classical mechanics, providing a basis for understanding the motion of objects in the physical world. We use them to understand how objects move and how forces affect their motion.

3.1 Newton's First Law of Motion

Newton's first law, also known as the **law of inertia**, states that a system will retain its state of motion, whether at rest or in motion, unless some external intervention, i.e., force, tempers it. This notion of *persisting* to maintain the same state by the system is known as **inertia**.

There is no rigorous mathematical formula to describe the first law. But we can set one for understanding's purpose. According to this law, if we keep $\sum \vec{F}_{\text{ext}} = 0$ on a system, the system will have no new acceleration. Whatever its acceleration was before the application of the force, it will remain the same. That is, $\Delta\vec{a} = 0$.

Intuitively, this law can be understood as **a body in motion tends to stay in motion, and a body at rest tends**

to stay at rest. This means that if no forces are acting on an object, it will continue to move at a constant speed in a straight line. On the other hand, if an object is at rest, it will remain at rest unless a force is applied to it.

3.2 Newton's Second Law of Motion

Newton's second law states that if the momentum of a system changes in time, we get a net force, which subsequently causes the system to accelerate. This acceleration of an object is directly proportional to the magnitude of the net force acting on the object and inversely proportional to its mass.

This means that the more force applied to an object, the greater its acceleration; the greater the object's mass, the smaller its acceleration will be for a given force.

$$\vec{F} \propto \frac{d\vec{p}}{dt} \tag{1}$$
$$= Km\frac{d\vec{v}}{dt}$$
$$= Km\vec{a} \tag{2}$$

K is a force constant that relieves the proportionality of (1). Conventionally, we define $K = 1\,\mathrm{N\,m^{-2}\,s^{2}}$. This K sets the definition of 1N. To sum up, 1N is the amount of force needed to give an object of mass 1kg, an acceleration of $1\,\mathrm{m\,s^{-2}}$. (2) then turns into the famous force equation that we all know and love.

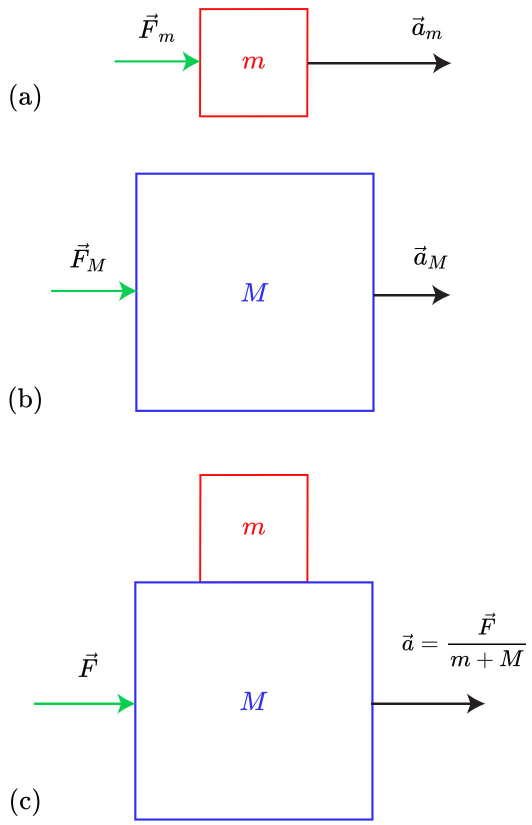

FIGURE 4: (a) $\vec{F}_m$ applied on m mass produces acceleration $\vec{a}_m$, (b) $\vec{F}_M$ applied on M mass produces acceleration $\vec{a}_M$, (c) $\vec{F}$ applied on $m + M$ mass system produces acceleration $\vec{a} = \dfrac{\vec{F}}{m+M}$.

$$\vec{F} = m\vec{a} \tag{3}$$

Intuitively, this law can be understood as **the force acting on an object is equal to its mass multiplied by its acceleration**. If you want to accelerate a heavy object, you will need to apply more force than you would need to accelerate a lighter object.

3.3 Newton's Third Law of Motion

Newton's third law states that for every action, there is an equal and opposite reaction. This means that if object A exerts a force on object B, object B will exert an equal and opposite force on object A.

$$\vec{F}_{A\to B} = -\vec{F}_{B\to A} \tag{4}$$

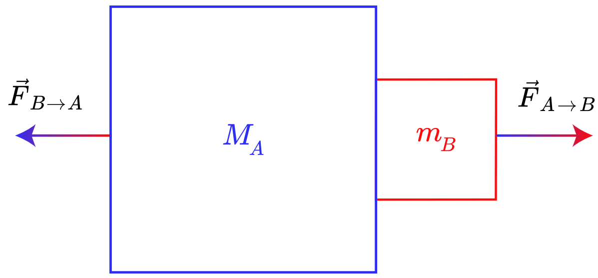

FIGURE 5: Reaction forces always come in pairs. It's the same in magnitude but opposite in direction.

Intuitively, this law can be understood as **for every push, there is an equal and opposite pushback**. For example, if you push on a wall, the wall will push back on you with an equal and opposite force. These two forces always come in pairs. You will never get a system where they are isolated. That would be a violation of the third law.

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4 Friction Force

Friction is a force that resists motion between two surfaces in contact with each other. When you try to slide a book across a table, you'll notice that it takes more force to get it moving than it does to keep it moving. This is due to the force of friction between the book and the table. Friction can be helpful in some cases, such as when you're trying to walk or drive a car, but it can also be a hindrance when trying to slide something across a surface.

We will be looking at the following three friction forces:

1. **Static friction** - this is the force that resists the initiation of motion between two surfaces in contact. For example, if you push a heavy box across the floor, the static friction force will make it difficult to move the box.
2. **Kinetic friction** - this is the force that resists the motion of two surfaces that are already in motion relative to each other. Once you've gotten the box moving, the kinetic friction force will make it harder to stop the box or change its direction.
3. **Rolling friction** - this is the force that resists an object that is rolling over a surface. When an object rolls over a surface, the deformation of the object and the surface it rolls over creates a resistance to motion. Rolling friction is usually less than sliding friction and is one of the reasons why rolling is often preferred over sliding motion.

4.1 Static Friction

The force of static friction is directly proportional to the force pushing the two surfaces together. It is proportional to the coefficient of static friction, which is a property of the two surfaces in contact. When an external force is applied to an object at rest, static friction will act to prevent the object from moving. The maximum amount of static friction that can be exerted is equal to the force that is pushing the two surfaces together, multiplied by the coefficient of static friction. The object will remain at rest if the external force is not large enough to overcome the maximum amount of static friction.

The coefficient of static friction can be determined experimentally by measuring the maximum external force that can be applied to an object at rest before it starts to move. This is often done using a device called a force gauge or a spring scale.

The equation for static friction is:

$$f_s \leq \mu_s N, \tag{5}$$

where f_s is the force of static friction, μ_s is the coefficient of static friction, and N is the normal force, which is the force perpendicular to the surface that the object is resting on. The equality holds only when $f_s = f_s^{\max}$.

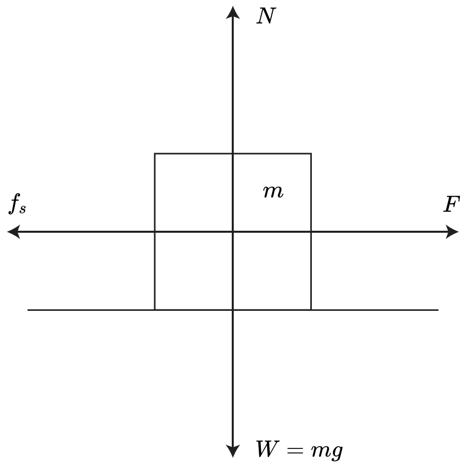


FIGURE 6: Static friction resists the applied force F until the object starts moving. This friction has a maximum limit $f_s^{\max}$, which must be met to move the object.

The physical significance of each term in the equation is as follows:

1. f_s is the force of static friction, which is the force that opposes the tendency of an object to move when a force is applied to it.
2. μ_s is the coefficient of static friction, which is a property of the two surfaces in contact and represents the amount of friction that can be exerted before the object starts to move.
3. N is the normal force, which is the force perpendicular to the surface that the object is resting on and is equal to the weight of the object in most cases.

4.2 Kinetic Friction

When two surfaces are in contact, the irregularities in their surfaces cause them to interlock with each other. As a result, when an object is pushed or pulled across a surface, the interlocking points must break and re-form continuously. This breaking and re-forming of interlocking points require energy and create resistance or friction. This takes over the system once $f_s^{\max}$ is met, and the object moves. The magnitude of the kinetic frictional force is proportional to the normal force, which is the force exerted by the surface perpendicular to the object's motion. The direction of the kinetic friction force is opposite to the direction of the object's motion.

The coefficient of kinetic friction, denoted by μ_k, is a dimensionless quantity that represents the ratio of the kinetic frictional force to the normal force. It depends on the nature of the surfaces in contact and their smoothness. The value of μ_k ranges between 0 and 1, and it is always less than the coefficient of static friction for the same two surfaces.

The equation for kinetic friction is:

$$f_k = \mu_k N, \tag{6}$$

where f_k is the force of kinetic friction, μ_k is the coefficient of kinetic friction, and N is the normal force, which is the force perpendicular to the surface that the object is resting on.

The physical significance of each term in the equation is as follows:

1. f_k is the force acting on the object due to the sliding motion across the surface.

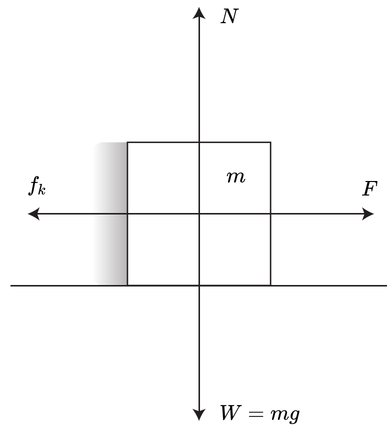


FIGURE 7: Kinetic friction resists the applied force F when the object is moving. This friction has no maximum or minimum limit. As long as the system moves with a constant velocity, f_k pretty much stays the same with a minuscule variation.

2. μ_k is a measure of the surface's roughness and determines the strength of the frictional force.
3. N is the force perpendicular to the surface exerted by the surface on the object. It determines the magnitude of the frictional force.

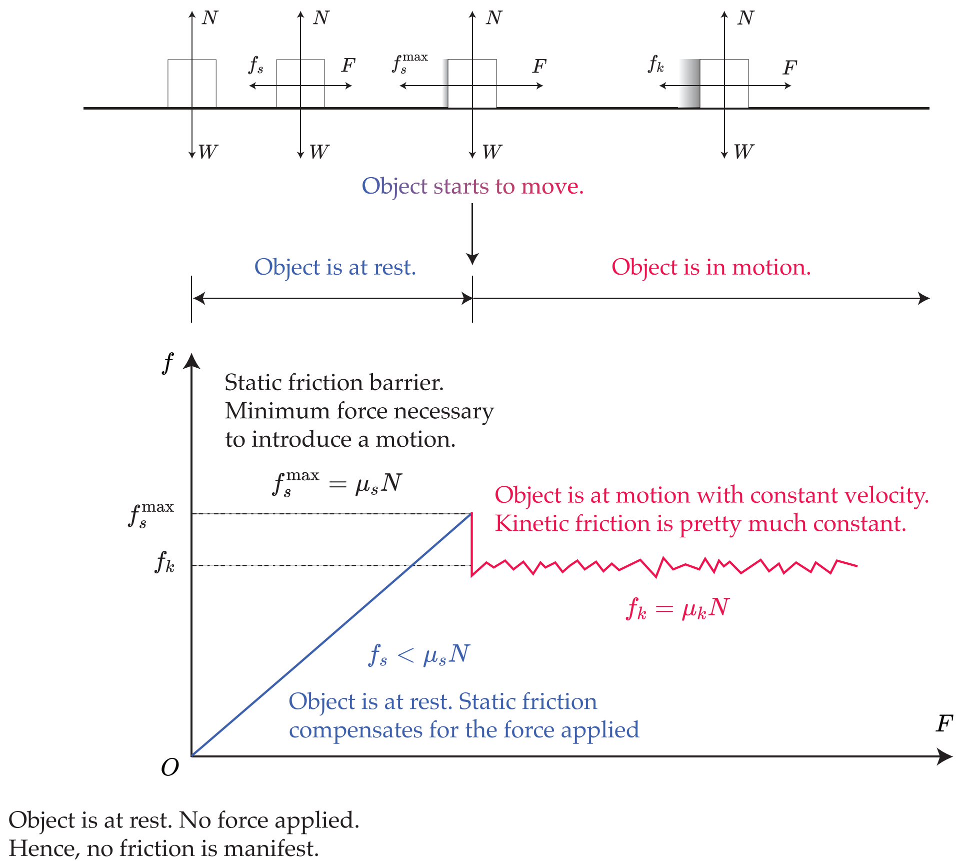


FIGURE 8: A graph representing the platform's friction force f response depending on the object's state of motion with increasing applied force F.

4.3 Rolling Friction

The magnitude of rolling friction depends on several factors, such as the materials of the object and the surface it rolls over, the object's shape, the surface area in contact with the surface, and the object's speed. In general, the rougher the surface or the softer the material of the object, the greater the rolling friction. Rolling friction also depends on the normal force exerted on the object, which is the force exerted by the surface perpendicular to the object.

Rolling friction can be measured by placing an object on a flat surface and rolling it at a constant speed. The force required to maintain the constant speed is equal and opposite to the rolling friction. Alternatively, the coefficient of rolling friction can be measured by applying a known force to the object and measuring the resulting acceleration. The coefficient of rolling friction is the ratio of the force of rolling friction to the normal force exerted by the surface on the object. The equation for static friction is:

$$f_r = \mu_r N \tag{7}$$

where f_r is the force of rolling friction, μ_r is the coefficient of rolling friction, and N is the normal force exerted by the surface on the object.

The physical significance of each term in the equation is as follows:

1. f_r is the force that opposes the rolling motion of the object.
2. μ_r is a dimensionless quantity that measures the frictional properties of the materials in contact.
3. N is the force exerted perpendicular to the surface. This force is responsible for the deformation of the object and the surface it rolls over, which creates the resistance to motion known as rolling friction.

5 The Dynamics of Uniform Circular Motion

Recall that a uniform circular motion occurs when an object moves along a circular path with a constant speed. In this motion, the direction of the object's velocity vector changes continuously, but the magnitude of the velocity vector remains constant. The object is always accelerating because its velocity vector changes and the acceleration is directed toward the circle's center. This acceleration is called *centripetal acceleration*. We dealt with this *kinematics* part of the motion. Now we expand upon its dynamics.

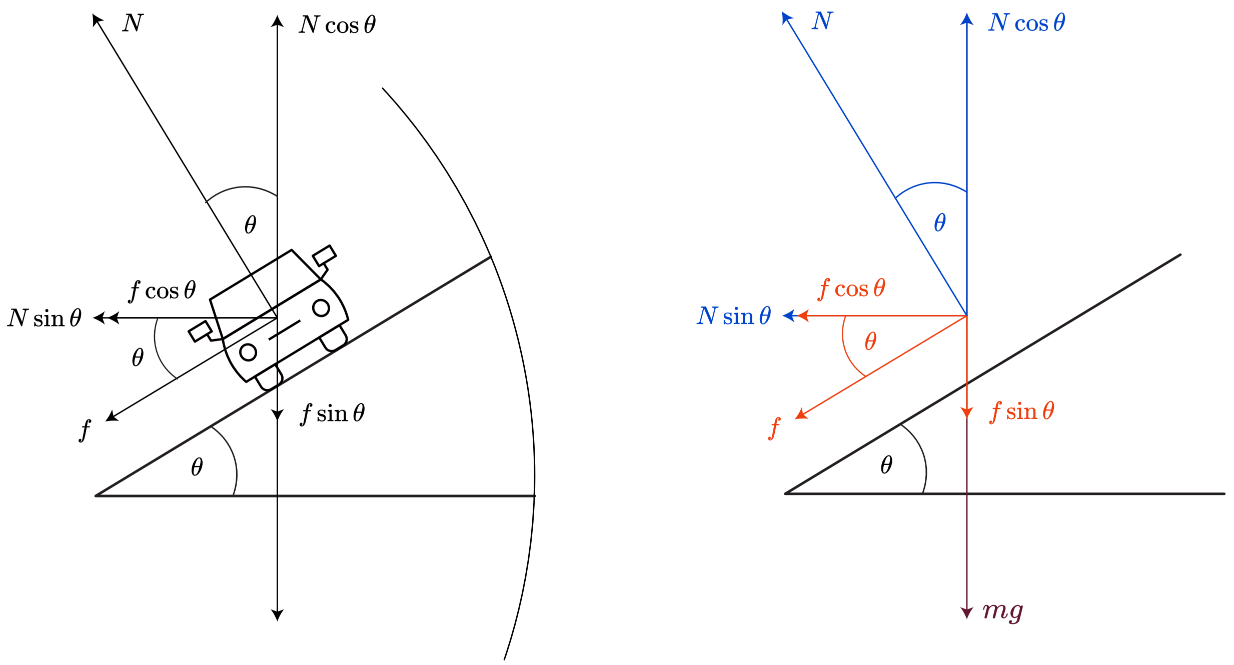
Imagine a car moving around a curve, a satellite orbiting the Earth, or a person on a carnival ride. It has a centripetal force to retain the circular motion. Newton's 2$^{\text{nd}}$ law demands that any acceleration should be accompanied by a change in momentum, which subsequently requires an action of a force. In this case, the object is subject to a centripetal force that keeps it moving in a circle. The magnitude of the centripetal force depends on the object's speed v, the radius of the circle R, and the object's mass m.

Remember, the centripetal acceleration of an object in a circular trajectory is given by: $a_\perp = \dfrac{v^2}{R}$. Multiplying this equation with the mass of the object gives us the measurement of the *centripetal force*.

$$F_c = \frac{mv^2}{R} = \frac{4m\pi^2 R}{T^2}. \tag{8}$$

6 Banking of the Circular/Curved Path

The banking of a road refers to tilting the road at a certain angle with respect to the horizontal axis. This angle is called the *bank angle* or the *angle of banking*. When a vehicle moves on a banked road, it experiences a centripetal force that helps it turn around the curve.



Practical scenario for banking

Free-body diagram

FIGURE 9: Force contribution for the banking of a road.

The need for banking on the road arises when we want to ensure the safety of the vehicles and the passengers traveling on them. When a car moves on a curved road, it experiences a pseudo force called *centrifugal force*. This force tries to pull the car outwards from the curve and can lead to a loss of control of the vehicle. The centripetal force is a force that acts on a body moving in a circle and pulls it toward the center of the circle.

NOTE: **Remember, the centrifugal force is not real. It's a term to describe the sensation of pulling the object feels in a non-inertial frame. There exists no such force in practice.**

On a flat road, the only source of the centripetal force is the friction between the vehicle's tires and the road surface. This friction force works sideways to the tire. This causes the tire to erode faster and increases the chances of accidents. However, on a banked road, the component of the weight of the vehicle perpendicular to the road surface can also provide a part of the required centripetal force, keeping the vehicle stable on the road.

From the figure (9), we find the contributions of the forces and check Newton's 2$^{\text{nd}}$ law of motion,

$$N\cos\theta = mg + f\sin\theta \tag{9}$$

$$N\sin\theta + f\cos\theta = \frac{mv^2}{R} \tag{10}$$

The angle θ of the banking of a road is determined based on the speed of the vehicles that are expected to travel on it. When a car moves on a banked road, the net force acting on it is the vector sum of the gravitational force mg and the normal force. The normal force has two components, one perpendicular to the road surface and the other parallel to it. The component of the normal force perpendicular to the road surface provides the necessary support to the weight of the vehicle, while the component parallel to the road surface provides the required centripetal force for the car to turn around the curve.

We start by dividing Eq.(9) by Eq. (10) after a simple rearrangement of the equations.

$$\begin{aligned}
\frac{\left(\frac{mv^2}{R}\right)}{mg} &= \frac{N\sin\theta + f\cos\theta}{N\cos\theta - f\sin\theta} \\
\Rightarrow \frac{v^2}{gR} &= \frac{N\sin\theta + \mu_s N\cos\theta}{N\cos\theta - \mu_s N\sin\theta} \\
\Rightarrow \frac{v^2}{gR} &= \frac{\left(\frac{N\sin\theta + \mu_s N\cos\theta}{N\cos\theta}\right)}{\left(\frac{N\cos\theta - \mu_s N\sin\theta}{N\cos\theta}\right)} \qquad (11) \\
\Rightarrow \frac{v^2}{gR} &= \frac{\tan\theta + \mu_s}{1 - \mu_s\tan\theta} \\
\therefore v &= \sqrt{gR} \times \sqrt{\frac{\tan\theta + \mu_s}{1 - \mu_s\tan\theta}}. \qquad (12)
\end{aligned}$$

We divided the Numerator and Denominator on the right-hand side of the Eq. (11) by $N\cos\theta$. Why? To simplify the formula. No other physical significance to it. It's just another mathematical manipulation.

The formula for calculating θ from Eq.(12) would be:

$$\theta = \tan^{-1}\left(\frac{v^2 - \mu_s gR}{gR + \mu_s v}\right). \qquad (13)$$

For a frictionless road, set, $\mu_s = 0$, which is the case for $f = 0$. It'll take You back to the familiar formats.

$$v = \sqrt{gR\tan\theta}. \qquad (14)$$

$$\theta = \tan^{-1}\left(\frac{v^2}{gR}\right) = \tan^{-1}\left(\frac{a_{\text{rad}}}{g}\right). \qquad (15)$$

Intuitively, the banking of the road allows the vehicle's weight to provide a part of the necessary centripetal force, which reduces the load on the friction between the vehicle's tires and the road surface.

This, in turn, reduces the wear and tear of the tires and road surface and the chance of skidding or slipping vehicles. The banked road also helps to increase the speed at which the vehicle can safely travel around the curve. This is because the necessary centripetal force is provided by both the friction and the component of the weight of the vehicle perpendicular to the road surface. Thus, the banking of the road is an important factor in ensuring the safety and efficiency of the vehicles and the passengers traveling on it.