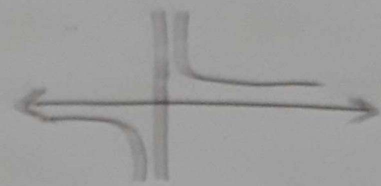
⊛ $f(x) = \frac{1}{x}$

<u>Graph</u> :



<u>Domain</u> : $\mathbb{R} - \{0\}$

<u>Range</u> : $y = \frac{1}{x}$

$x = \frac{1}{y}$

$f^{-1}(x) = \frac{1}{x}$

$\therefore$ Range : $\mathbb{R} - \{0\}$

⊛ $f(x) = \sqrt{x}$

<u>Graph</u> :

<u>Domain</u> : ~~$\mathbb{R}$~~ $- [0, +\infty)$

Again,

$f(x) = \sqrt{x}$

$y = \sqrt{x}$

$x = y^2$

$f^{-1}(x) = x^2$

$\therefore$ <u>Range</u> : $[0, +\infty)$

⊛ $f(x) = |x|$

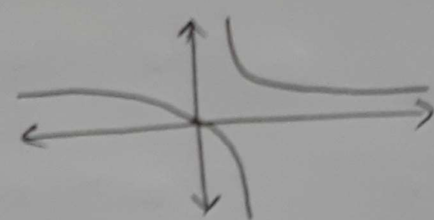
Graph :

Domain : $(-\infty, +\infty)$

Range : $[0, +\infty)$

⊛ $f(x) = \frac{2x}{x-4}$

Graph :



Domain : $\mathbb{R} - \{4\}$

Again :

$$y = \frac{2x}{x-4}$$

$$xy - 4y = 2x$$

$$xy - 2x = 4y$$

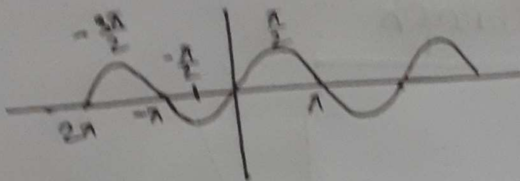
$$x = \frac{4y}{y-2}$$

$$f^{-1}(x) = \frac{4x}{x-2}$$

Domain $f^{-1}(x)$ = Range of $f(x) = \mathbb{R} - 2$

⊛ $f(x) = \sin x$

Graph of $\sin x$

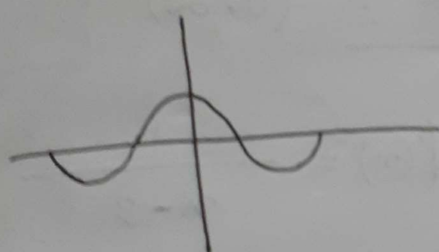


Domain : $(-\infty, +\infty)$

Range : $[-1, 1]$

⊛ $f(x) = \cos x$

Graph of $\cos x$

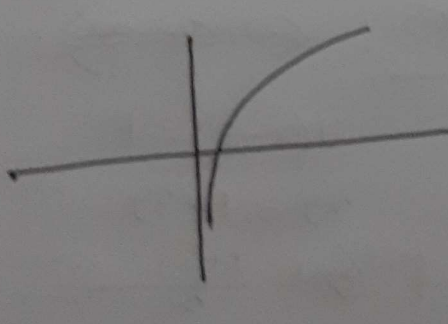


Domain : $(-\infty, \infty)$

Range : $[-1, 1]$

⊛ $f(x) = \ln x$

Graph of $f(x)$

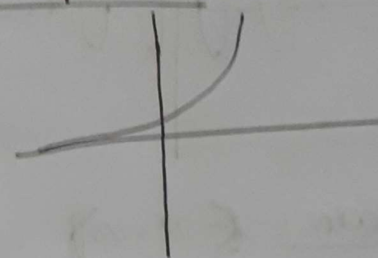


Domain : $(0, +\infty)$

Range : $(-\infty, +\infty)$

⊛ $f(x) = e^x$

Graph



Domain : $(-\infty, +\infty)$

Range : $(0, \infty)$

⊛ $f(x) = \sin 2x$

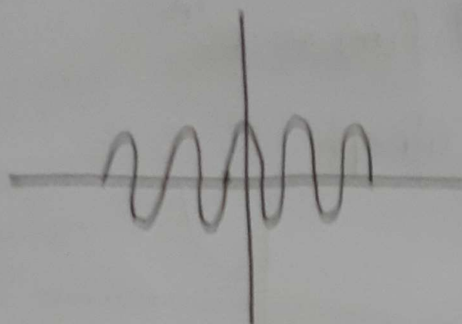
Graph

Domain : $(-\infty, \infty)$

Range : $[-1, 1]$

$f(x) = \cos 2x$

Graph:



Domain: $[-\infty, \infty]$

Range: $[-1, 1]$

$f(x) = e^{x-1}$

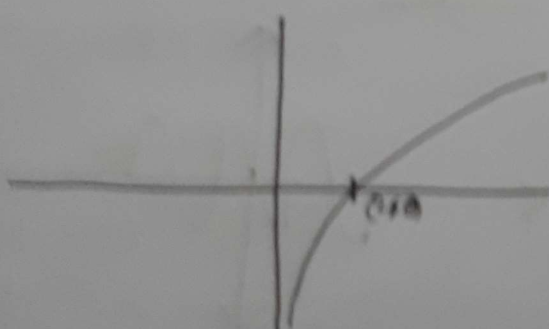
Graph:

Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

$f(x) = \ln(x-1)$

Graph:

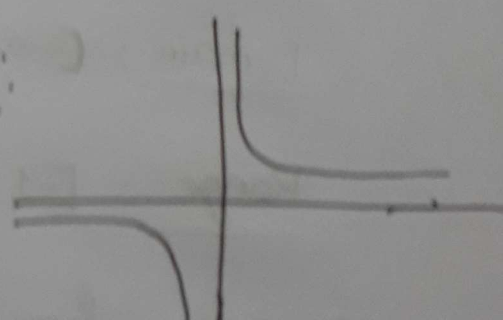


Domain: ~~( )~~ $[1, \infty$

Range: $\mathbb{R} - \{1\}$

$f(x) = \dfrac{1}{x-3}$

Graph:



Domain: $\mathbb{R} - \{3\}$

Range: ~~$(-\infty, \infty)$~~

$y(x-3) = 1$

$xy = 1 + 3y$

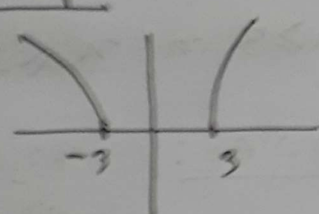
$f^{-1}(x) = \dfrac{1+3y}{x}$

∴ Domain of $f^{-1}(x) = \mathbb{R}-\{0\}$

∴ Range of $f(x) = \mathbb{R}-\{0\}$

$f(x) = \sqrt{x^2-9}$

Graph:



Domain:

$x^2-9=0$

$x=\pm 3$

∴ Domain:

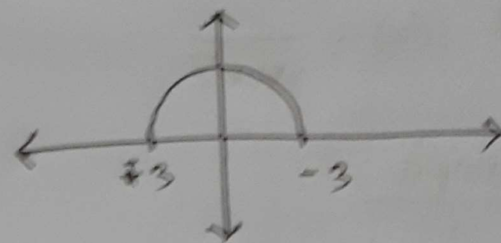
$(-\infty,-3] \cup [3,+\infty)$

Range:

$[0,+\infty)$

$f(x) = \sqrt{9-x^2}$

Graph:

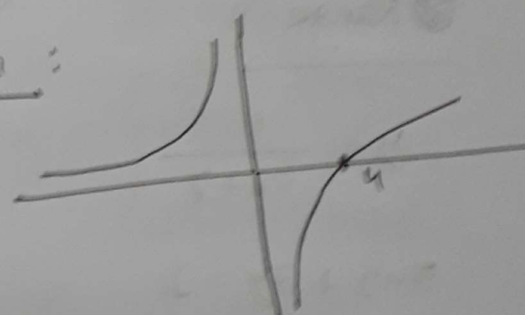


Domain: $[-3,+3]$

Range: $[0,3]$

$f(x) = \dfrac{x-4}{x}$

Graph:



Domain: $\mathbb{R}-\{0\}$

Range: $y = \dfrac{x-4}{x}$

$x-4 = xy$

$x-xy = 4$

$x = \dfrac{4}{1-y}$

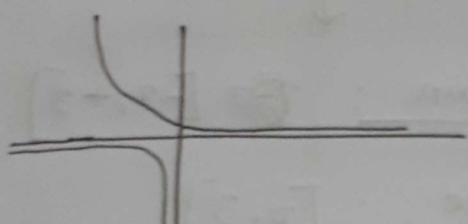
∴ $f^{-1}(x) = \dfrac{4}{1-x}$

∴ Range: $\mathbb{R}-\{1\}$

$\therefore$ ~~Range = $\mathbb{R} - \{2\}$~~

⊛ $f(x) = \dfrac{1}{5x+7}$

Graph :



Domain : $\mathbb{R} - \left\{-\dfrac{7}{5}\right\}$

⊛ Range :

$y = \dfrac{1}{5x+7}$

$5xy + 7y = 1$

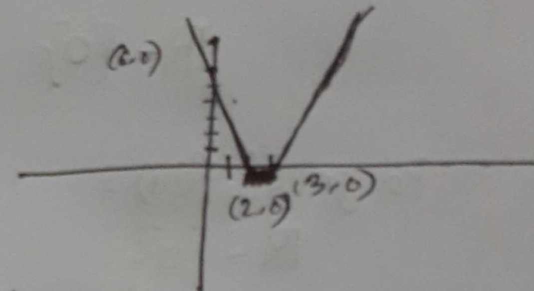
$5xy = 1 - 7y$

$x = \dfrac{1-7y}{5y}$

$f^{-1}(x) = \dfrac{1-7x}{5x}$

$\therefore$ Rang: $\mathbb{R} - \{0\}$

⊛ $f(x) = \sqrt{x^2 - 5x + 6}$



Here,

$x^2 - 5x + 6 \geq 0$

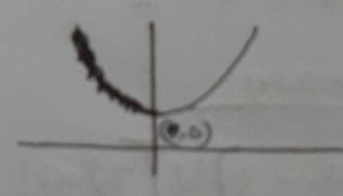
$(x-3)(x-2) \geq 2$

$\therefore x \geq 3$ or $x \leq 2$

$\therefore$ Domain : $(-\infty, 2] \cup [3, \infty)$

Range : $[0, +\infty)$

⊛ $f(x) = \sqrt{9 + x^2}$



Domain : $(-\infty, \infty)$

Range : $[3, +\infty)$

$f(x) = \frac{x}{x}$

Domain: $\mathbb{R} - \{0\}$

Range: 1

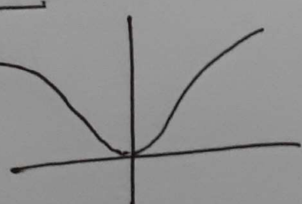
$f(x) = \frac{|x|}{x} \begin{cases} 1, & x > 0 \\ -1, & x < 0 \end{cases}$

Domain: $(-\infty, 0) \cup (0, \infty)$

Range: $\{1\}, \{-1\}$

$f(x) = \ln(x^2 + 1)$

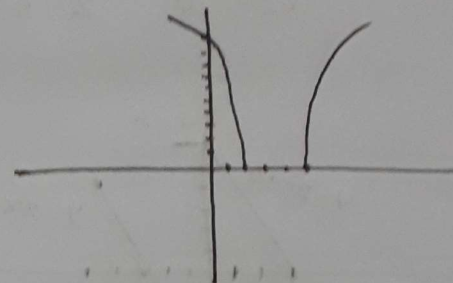
Graph:



Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

$f(x) = \sqrt{x^2 - 7x + 10}$



Domain: $x^2 - 7x + 10 \geq 0$

$(x-2)(x-5) \geq 0$

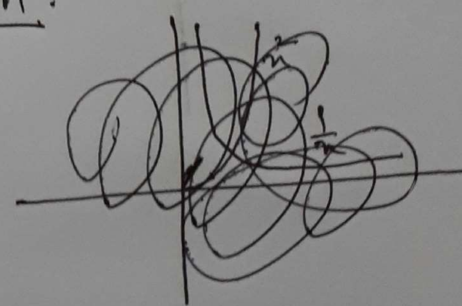
$\therefore x \geq 5$ or $x \leq 2$

$(-\infty, 2] \cup [5, +\infty)$

Range: $[0, +\infty)$

$f(x) = \begin{cases} x^2 & x < 0 \\ x & 0 \leq x \leq 1 \\ \frac{1}{x} & x > 1 \end{cases}$

Graph:



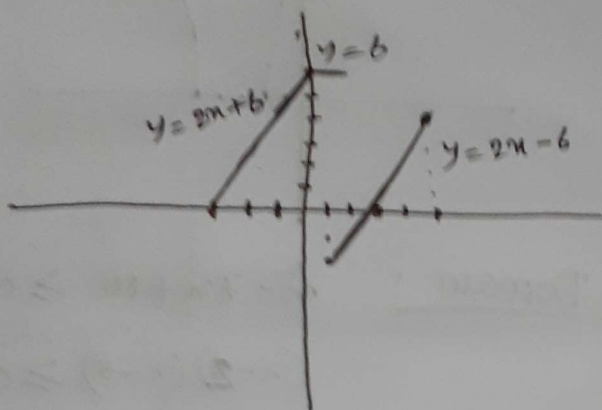
Domain: $(-\infty, 0) \cup [0, 1] \cup (1, +\infty)$

$\Rightarrow (-\infty, +\infty)$

Range: $[0, +\infty) \cup [0, 1] \cup (1, +\infty)$

$[0, +\infty)$

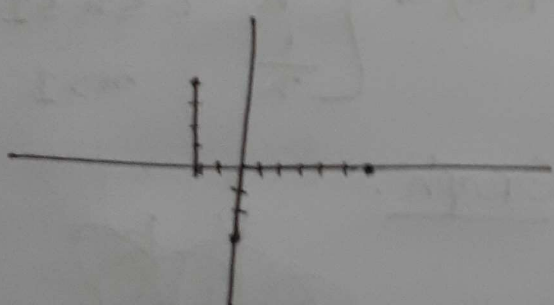
$$f(x) = \begin{cases} 2x+6 & -3 \le x \le 0 \\ 6 & 0 \le x \le 2 \\ 2x-6 & 2 \le x \le 5 \end{cases}$$



Domain: $[-3, 0] \cup [0, 2] \cup [2, 5]$

Range: $[0, 6] \cup \{6\} \cup [-2, 4]$

Graph of Range



$$f(x) = \begin{cases} \dfrac{x^2-1}{x-1} & x \ne 1 \\ 2 & x \ne 1 \end{cases}$$

Domain: $\mathbb{R} - \{1\}$

Range: $y = x+1$

$x = y-1$

$f^{-1}(x) = x-1$

$\therefore$ Range: $\mathbb{R}$

⊛ $f(x) = \sqrt{x^3 - 64x}$

Here,

$x^3 - 64x \geq 0$

$x(x^2 - 64) \geq 0$

$x(x+8)(x-8) \geq 0$

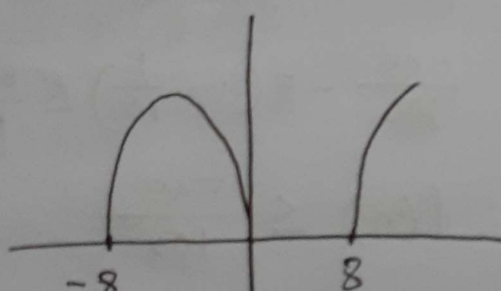
$\therefore x \geq 8$ or $-8 \leq x \leq 0$

Domain:

$[-8, 0] \cup [8, +\infty)$

Range: $[0, +\infty)$

Graph:



Explanation

$x^3 - 64x \geq 0$

$x(x+8)(x-8) \geq 0$

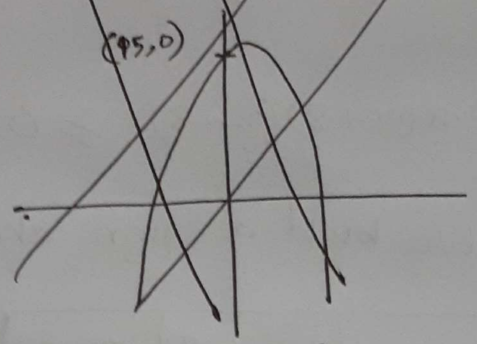
We build a sign chart

x	$-\infty \to -8$	$-8 \to 0$	$0 \to +8$	$+8 \to \infty$
$x+8$	$-$	$+$	$+$	$+$
x	$-$	$-$	$+$	$+$
$x-8$	$-$	$-$	$-$	$+$
$g(x)$	$-$	$+$	$-$	$+$

$\therefore$ **Domain:** $[-8, 0) \cup [8, +\infty)$

$\boxed{\rightarrow}$

$f(x) = -5x^2 + x + 15$



~~Domain : $(-\infty, +\infty)$~~

~~Range : $(-\infty, 15]$~~

$\boxed{\rightarrow}$

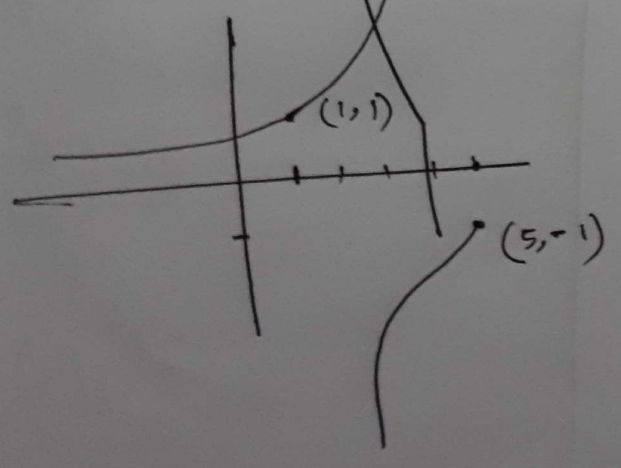
$$\boxed{f(x) = \frac{1}{\sqrt{5-x} - 1}}$$

Domain : ~~$(-\infty, 4]$~~

$(-\infty, 4)$

Range : $(-\infty, -1] \cup [1, +\infty)$

$y = \sqrt{5-x} - 1$



$f(x) = -5x^2 + x + 15$

$f(x) = -5\left(x^2 - \frac{x}{5} - 1\right)$

Completing the square,

$$f(x) = -5\left\{\left(x^2 - \frac{1}{5}x + \frac{1}{100}\right) - \frac{101}{100}\right\}$$

$$= -5\left\{\left(x - \frac{1}{10}\right)^2 - \frac{101}{100}\right\}$$

$$\therefore f(x) = -5\left(x - \frac{1}{10}\right)^2 + \frac{505}{100}$$

$$\because \left(x - \frac{1}{10}\right)^2 \geq 0$$

$$\therefore -5\left(x - \frac{1}{10}\right)^2 \leq 0$$

$$\therefore \frac{505}{100} - 5\left(x - \frac{1}{10}\right) \leq \frac{505}{100}$$

$$\therefore f(x) \leq \frac{505}{100}$$

$\therefore$ Range : $(-\infty, 5.05]$

$f(x) = \sqrt{x - 3x^2}$

$\Rightarrow$ To figure out the domain and range of the function we need to figure out when it is defined or there is a y value associated with the x value. So, when the part inside the square root ≥ 0.

Thus,

$\Rightarrow$ $x - 3x^2 \geq 0$. By solving this ~~equation~~ we can figure out that the function intercepts the x axis at,

$$x - 3x^2 = 0$$

$$x(\cancel{1} - 3x) = 0$$

$$x = 0 \text{ or } \frac{1}{3}$$

$\therefore$ the domain is $[0, \frac{1}{3}]$

For the range,

we need to find when $\sqrt{x - 3x^2}$ is the largest. That is largest when ~~$\sqrt{x - 3x^2}$~~ $x - 3x^2$ is the largest. This is possible by using derivatives since at the highest point the slope will be 0.

$f(x) = x - 3x^2$

$\therefore f'(x) = 1 - 6x$

and when the slope is zero,

$0 = 1 - 6x$

$x = \frac{1}{6}$

If we substitute that into the equation we get that,

$f(x) = \sqrt{x - 3x^2}$

$= \sqrt{1/6 - 3\times(1/6)^2}$

$= \sqrt{\frac{1}{12}}$

$\therefore \sqrt{\frac{1}{12}}$ is the highest point.

$\therefore$ Range : $[0, \sqrt{\frac{1}{12}}]$