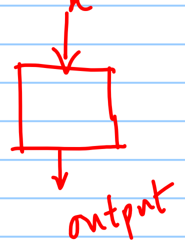
Note Title 2/24/2025

"Review of Calculus"

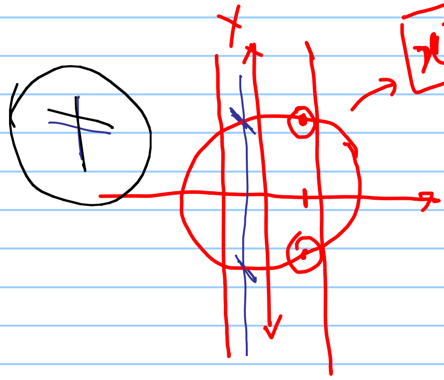
$y = f(x)$ $x \rightarrow$ input

x

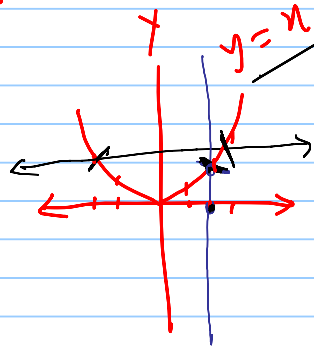


$$\boxed{f(x) = \pm\sqrt{x}}$$

$f(1) = \pm\sqrt{1}$



$\boxed{x^2+y^2=a^2}$



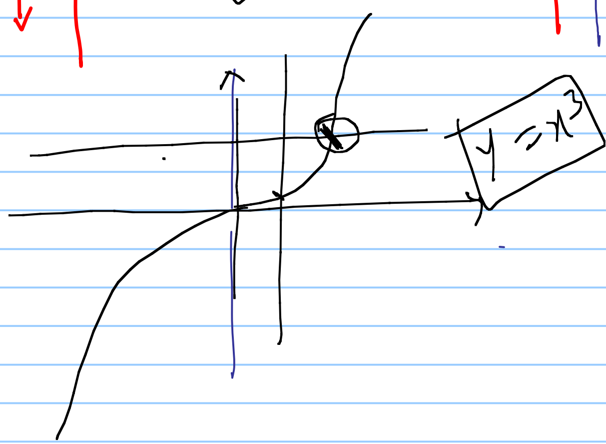
$y = x^2$

$f(x) = x^2$

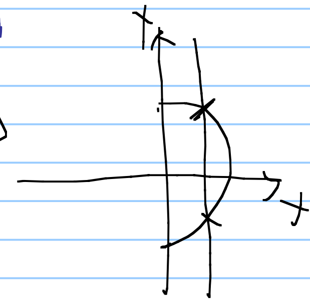
$f(1) = 1$

$f(-1) = 1$

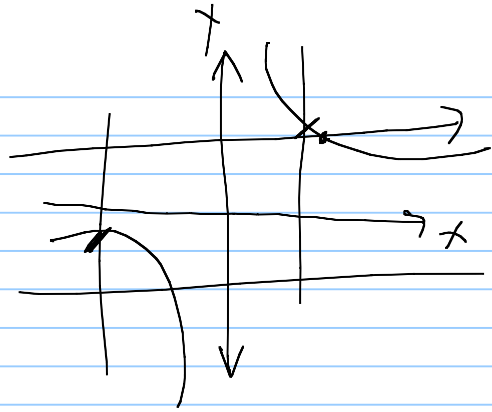
vertical Line Test



$\boxed{y = x^3}$



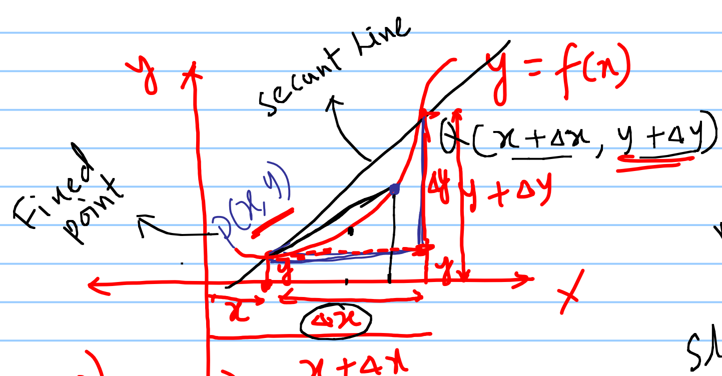
$y = f(x)$



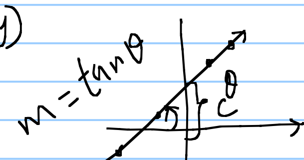
$y = \frac{1}{x}$

(**) $y = f(x)$

Defferentiation:



$y = mx + c$



Slope, $m = \frac{y_2 - y_1}{x_2 - x_1}$

$(x, y) = (x, f(x))$

$(x+\Delta x, y+\Delta y) = (x+\Delta x, f(x+\Delta x))$

$$m_{sec} = \frac{rise}{run} = \frac{y + \Delta y - y}{\cancel{x} + \Delta x - \cancel{x}}$$

$$= \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$m_{sec} = \frac{\Delta f}{\Delta x}$$

if $Q \to P$; then $\Delta x \to 0$,

$$m_{sec} \to m_{tan} = \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x}$$

$$\boxed{\frac{d f(x)}{dx} = \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x}}$$

$$\boxed{\frac{d}{dx} f(x) = \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}}$$

$f(x) = x^2$ $\qquad x = x$

$$f(x+\Delta x) = (x+\Delta x)^2 = x^2 + 2\Delta x\, x + (\Delta x)^2$$

$$\frac{d}{dx} f(x) = \frac{d}{dx}(x^2) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{x^2} + 2\Delta x\, x + (\Delta x)^2 - \cancel{x^2}}{\Delta x}$$

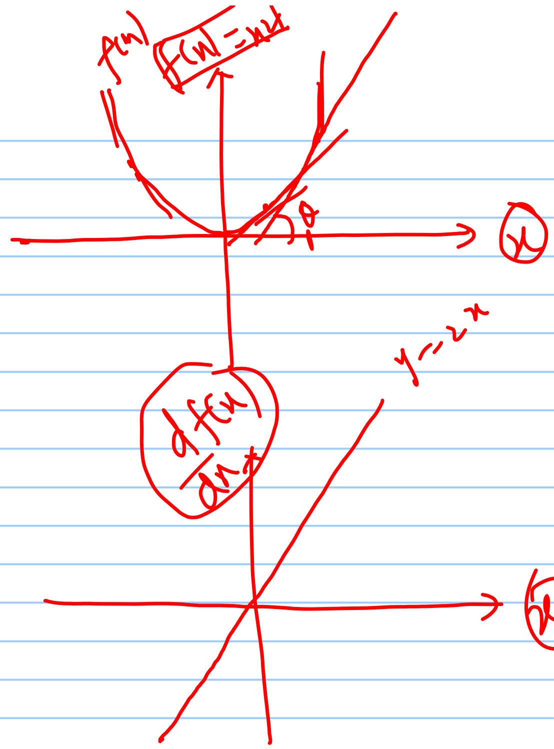
$$\boxed{\frac{d}{dx}(x^n) = n x^{n-1}}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}\,(2x + \Delta x)}{\cancel{\Delta x}}$$

$$= \lim_{\Delta x \to 0} (2x + \Delta x)$$

$$\boxed{\frac{d(x^2)}{dx} = 2x}$$

$x = x$



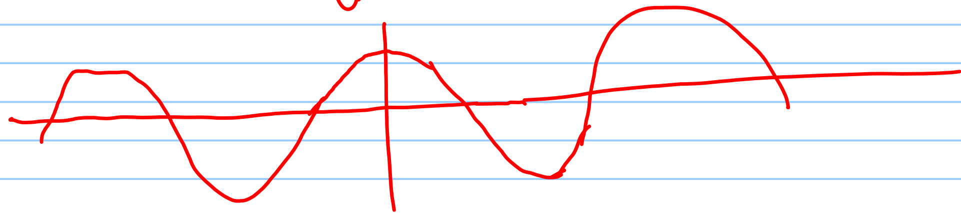
$x = 2$

$$\frac{d}{dx}(x^2)\Big|_{x=2} = 2x\Big|_{x=2}$$

$$= 2 \times 2$$

$y = \sin(x)$

$\cos(x)$



$$\frac{d}{dx}(x^n) = n x^{n-1}$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(\sin 5x) = \cos(5x)\frac{d}{dx}5x$$
$$= 5\cos(5x)$$

$U(V)$

$V(x)$

$$\frac{dU}{dx} = \frac{dU}{dV}\frac{dV}{dx}$$

$$\frac{d}{dx}\left(e^{\ln(5x)}\right) = e^{(\ln 5x)}\frac{d}{dx}\ln(5x)$$
$$= e^{\ln 5x}\frac{1}{5x}\cdot\frac{d}{dx}5x$$
$$= e^{\ln(5x)}\frac{1}{\cancel{5}x}\cdot\cancel{5}$$

* Fundamental Theorem of Calculus:

$$\frac{d}{dx}f(x) = g(x) \longleftrightarrow \int g(x)\,dx = f(x) + c$$

Indefinite Integral

$$\frac{d}{dx}x^2 = 2x \qquad \int 2x\,dx = x^2 + c$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1}\ ; \quad n \neq -1$$

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$$\frac{d}{dx}(\sin x) = \cos x \qquad \int \cos x \, dx = \sin x$$

$$\int 2x\,dx = x^2 + C$$

$$\frac{d}{dx}(x^2+2) = 2x \qquad \int 2x\,dx = x^2 + C$$

$$\frac{d}{dx}(x^2+5) = \underline{2x}$$

<u>Definite Integral:</u>

$$\int_{x=\underline{a}}^{x=b} \underline{f(x)}\,dx = \Big[g(x)\Big]_{x=a}^{x=b} = g(b) - g(a)$$

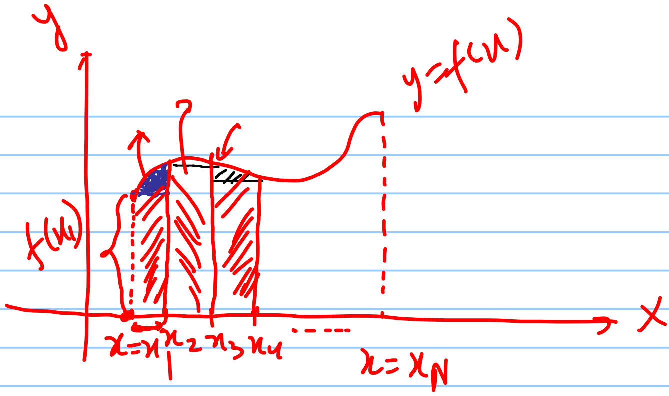
$[a, b]$

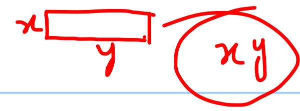
$$|x| = \begin{cases} \underline{-x}; & x < 0 \\ \underline{x}; & x \geq 0 \end{cases}$$

$$\int_{-5}^{+5} |x|\,dx = \int_{-5}^{0} (-x)\,dx + \int_{0}^{5} x\,dx$$

$$= 25$$

$\star$ x





$$\Delta x = x_2 - x_1 = x_3 - x_2$$
$$= x_4 - x_3 = \cdots$$

$$\text{Area} = \underline{f(x_1)\Delta x} + f(x_2)\Delta x + f(x_3)\Delta x + \cdots$$
$$+ f(x_{N-1})\Delta x$$

$$= \underbrace{\sum_{i=1}^{N-1} f(x_i)\Delta x}$$

$N \to \infty$, $\Delta x \to 0$, $\Delta x = dx$

$$= \int_{x=x_1}^{x=x_N} f(x)\,dx$$

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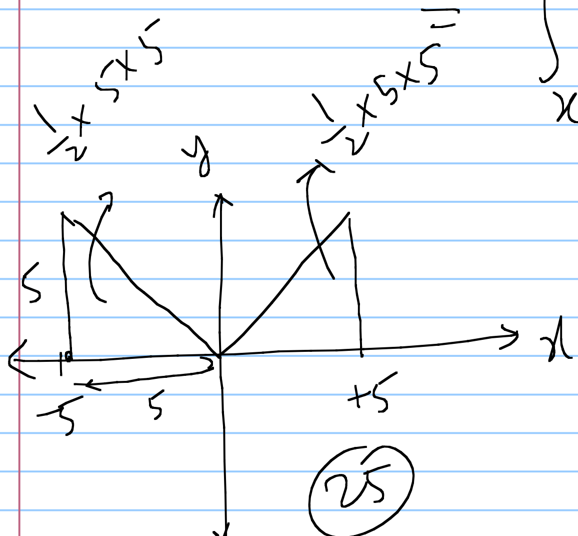

$$\int_{-5}^{+5} |x|\,dx$$