PHY 111: Principles of Physics I Fall 2023

LECTURE 03 — October 30, 2023

SECTION: 35 (UB71001) FACULTY: Akiful Islam (AZW)

BRAC UNIVERSITY

1 Mechanics

More commonly known as *Newtonian Mechanics*. However, Newton comes in the way later. This part deals with the world on a macroscopic level, following Newton's laws of motion and the law of universal gravitation primarily.

1.1 Prerequisite

Before we start our journey through mechanics, we need to satisfy the following:

- The displacement of two position vector or **length** (magnitude of the displacement vector) of all sorts **is absolute**. Regardless of the coordinate system, all observers will measure the same length,
- The **time interval** between two events **is absolute**. The passing of a certain amount of time will be the same for all observers across all coordinate systems.
- The **mass** of the objects under scrutiny **is absolute**. This is a dormant condition for mechanics. Most often, it gets ignored. But still one should keep it in mind.

Mechanics is consisted of **Kinematics** and **Dynamics**.

2 Kinematics

Kinematics is the part of mechanics that studies the motion of a system that does not require the knowledge of force or momentum and how their changes may affect the motion in question. Although force can arise in the system but is absent while determining the trajectory of the system's motion.

2.1 Single-Particle System

For simplicity, we usually consider an object to be a point particle and calculate that point's motion. Usually, when the dimension of the object is very small compared to that of the enclosing system, the particle approximation works charms.

Generalizing the motion for the many-particle system then gives the motion of the entire body. As long as the dimensions of the body are smaller than its surroundings, the particle system gives pretty much the exact measurements a collection of particles would do.

Consider the following situation where a ball is moving in the given trajectory. We may replace the ball with all its mass given to a particle and then measure its motion.

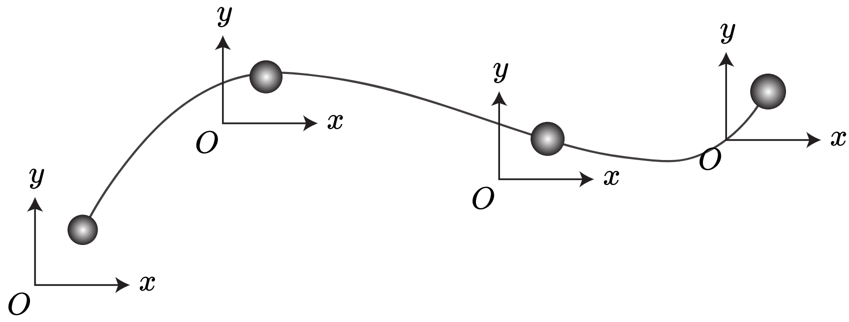

FIGURE 1: Evolution of a system. Only the translational motion is taken here.

For simplicity, we will only look at the system's translational motion and drop the rotational and vibrational motion for now. To account for the translational motion, the particle or the ball will move through space, keeping its original coordinated system intact. If the coordinate system or the object makes a different angle or jittery motion anywhere in the trajectory, we will have to introduce the extra motion accordingly.

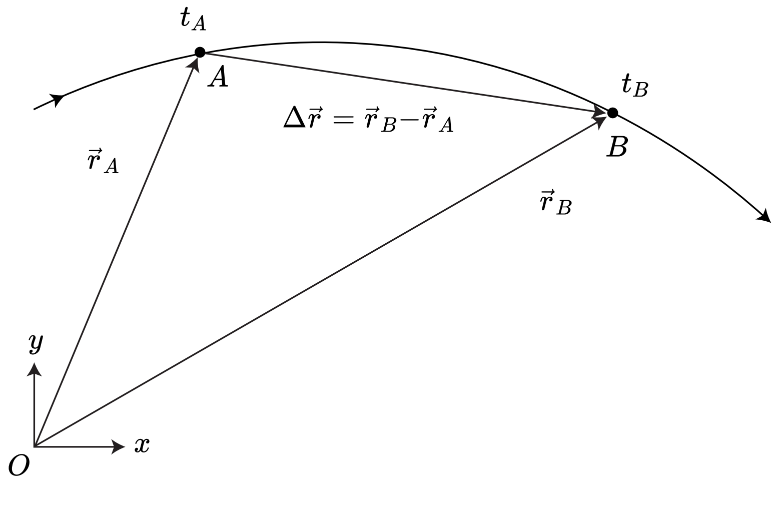

FIGURE 2: Displacement of a point from A to B is defined using the displacement vector. $\vec{r}_A$ and $\vec{r}_B$ here are the position vector of the points A and B at times t_A and t_B respectively. $\Delta\vec{r}$ is the displacement vector.

2.2 Displacement

To define a displacement vector (Figure 2), we need **position vectors** for the initial and final points of a motion. Position vector defines the location of a point in space, generally measured from the origin of a coordinate system.

The vector subtraction of a point's position vector at the initial and final location gives out the net **displacement** of that point.

Consider the following motion: the ball starts at point A and ends at B with position vectors $\vec{r}_A$ and $\vec{r}_B$, respectively. Then the net displacement of the ball is given by

$$\Delta\vec{r} = \vec{r}_B - \vec{r}_A. \tag{1}$$

2.3 Velocity

Velocity is defined by the ***change in displacement*** to the ***change in time***. Consider the same system (Figure 2) as before, where point A was at the time t_A and B at t_B. Then the velocity of the ball is:

$$\vec{v} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_B - \vec{r}_A}{t_B - t_A}. \tag{2}$$

The direction of the velocity is in the same direction as that of the displacements.

$$v\,\hat{v} = \left|\frac{\Delta r}{\Delta t}\right|\hat{r}$$

2.3.1 Average Velocity

Average velocity gives a general sense of velocity for the entire course of a motion. It only depends on the initial and final points. Nothing in between the trajectory interacts with this quantity.

$$\bar{\vec{v}} = \frac{\vec{r}_B - \vec{r}_A}{t_B - t_A}. \tag{3}$$

The instantaneous magnitude of this velocity $|\bar{\vec{v}}|$ is commonly known as *speed*, a scalar.

However, in practice, we would like to know the velocity at each point. Consider the same system (Figure 2) yet again, but this time let's choose an intermediate endpoint $B^{'}$ at time t somewhere in the middle of A and B (Figure 3). This new displacement is closer to a better approximation of our original trajectory. Repeat the process enough time, setting the limit $\Delta t \to 0$, and we see the displacement overlapping with the original trajectory.

2.3.2 Instantaneous Velocity

Instantaneous velocity deals with the velocity **at each instant of the motion**. This is the most practical calculation of velocity in everyday life.

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t} = \frac{d\vec{r}}{dt} \tag{4}$$

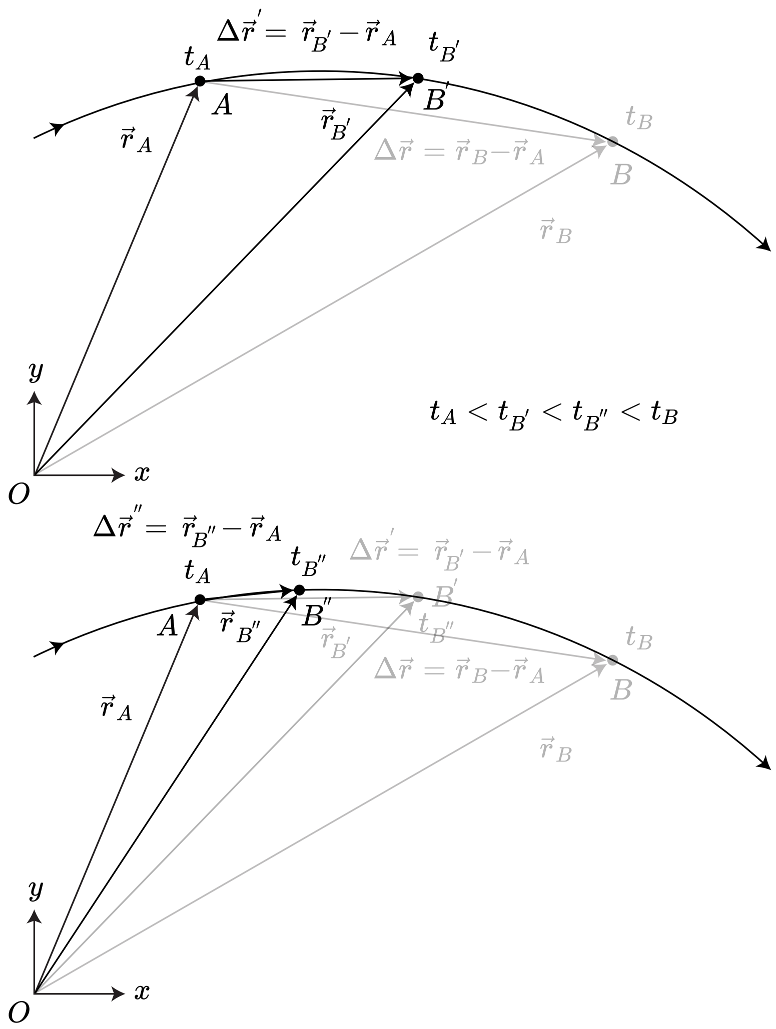

FIGURE 3: Displacement of a point from A to B is defined using the displacement vector. $\vec{r}_A$ and $\vec{r}_B$ here are the position vector of the points A and B at times t_A and t_B respectively. $\Delta\vec{r}$ is the displacement vector.

Geometrically this means the slope of a x-t curve will give us the velocity, and the direction at each point of the trajectory will be given by the **tangent** drawn on that point.

2.4 Constant Velocity

Constant velocity is where the change in velocity is fixed in time. The change in displacement in a certain time interval is the same for all consecutive intervals ahead.

$$\vec{v} = \frac{\vec{r}_B - \vec{r}_A}{t_B - t_A} = \frac{\vec{r}_C - \vec{r}_B}{t_C - t_B} = \frac{\vec{r}_D - \vec{r}_C}{t_D - t_C} = \text{constant.} \tag{5}$$

The slope of such velocity in the x-t curve will be zero.

2.5 Acceleration

Acceleration is defined by the ***change in velocity*** to the ***change in time***. Consider the same system as before where point A was at the time t_A with velocity v_A and B at t_B with velocity v_B. Then the velocity of the ball is:

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_B - \vec{v}_A}{t_B - t_A}. \tag{6}$$

The direction of the acceleration is in the same direction as that of the velocity, which in extension, is in the direction of displacements.

$$a\,\hat{a} = \left|\frac{\Delta v}{\Delta t}\right| \Delta \hat{v} \sim \Delta \hat{r}.$$

2.5.1 Average Acceleration

Like average velocity, average acceleration gives a general sense of acceleration for the entire course of a motion. It only depends on the initial and final points. Nothing in between the trajectory interacts with this quantity.

$$\bar{\vec{a}} = \frac{\vec{v}_B - \vec{v}_A}{t_B - t_A}. \tag{7}$$

2.5.2 Instantaneous Acceleration

Instantaneous velocity deals with the acceleration **at each instant of the motion**. This is the most practical calculation of acceleration in everyday life.

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}. \tag{8}$$

We can use the same argument we used for defining instantaneous velocity here. Just replace the change in displacement with a change in velocity.

Geometrically, this means the slope of a v-t curve will give us the velocity, and the direction at each point of the trajectory will be provided by the **tangent** drawn on that point.

2.5.3 Constant Acceleration

Constant acceleration is when the change in acceleration is fixed in time. The change in velocity in a certain time interval is the same for all consecutive intervals ahead.

$$\vec{a} = \frac{\vec{v}_B - \vec{v}_A}{t_B - t_A} = \frac{\vec{v}_C - \vec{r}_B}{t_C - t_B} = \frac{\vec{v}_D - \vec{v}_C}{t_D - t_C} = \text{constant.} \tag{9}$$

The slope of such acceleration in the v-t curve will be zero.

3 Primitive Equations of Motion

We need only the four laws of motion to track an object's motion across all 3 dimensions in the classical scale. We don't need to introduce any new parameters to derive these laws, but we must remind ourselves of the definitions of the 3 parameters, $r, \vec{v}, \vec{a}$.

$$\text{Displacement } \Delta\vec{r} = \vec{r}_f - \vec{r}_i, \quad \text{Velocity } \vec{v} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}, \quad \text{Acceleration } \vec{a} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}.$$

Equations	$\vec{r}$	$\vec{v}$	$\vec{a}$	t
$\vec{v} = \vec{v}_0 + \vec{a}t$	✗	✓	✓	✓
$\vec{r} - \vec{r}_0 = \vec{r} = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$	✓	✗	✓	✓
$\vec{r} - \vec{r}_0 = \vec{r} = \frac{1}{2}\left(\vec{v} + \vec{v}_0\right)$	✓	✓	✗	✓
$\vec{v}^2 - \vec{v}_0^2 = 2\vec{a}(\vec{r} - \vec{r}_0)$	✓	✓	✓	✗

TABLE 1: All four laws of motion in a nutshell in vector form. For calculation, you'll be given all the ✓ quantities and asked to solve for the ✗ ones.

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LECTURE 04 — October 5, 2023

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4 Application of Laws of Motion

4.1 Horizontal Motion (1D Motion)

Equations	x	v_x	a_x	t
$v_x = v_{x0} + a_x t$	✗	✓	✓	✓
$x - x_0 = x = v_{x0}t + \frac{1}{2}a_x t^2$	✓	✗	✓	✓
$x - x_0 = x = \frac{1}{2}(v_x + v_{x0})$	✓	✓	✗	✓
$v_x^2 - v_{x0}^2 = 2a_x(x - x_0)$	✓	✓	✓	✗

TABLE 2: All four laws of motion in a nutshell for horizontal motion. For calculation, you'll be given all the ✓ quantities and asked to solve for the ✗ ones.

There are no additional constraints on the three parameters in x-dimension.

4.2 Vertical Motion (1D Motion)

Equations	y	v_y	a_y	t
$v_y = v_{y0} - gt$	✗	✓	✓	✓
$y - y_0 = y = v_{y0}t - \frac{1}{2}gt^2$	✓	✗	✓	✓
$y - y_0 = y = \frac{1}{2}(v_y + v_{y0})$	✓	✓	✗	✓
$v_y^2 - v_{y0}^2 = -2g(y - y_0)$	✓	✓	✓	✗

TABLE 3: All four laws of motion in a nutshell for vertical motion. For calculation, you'll be given all the ✓ quantities and asked to solve for the ✗ ones.

We used the constraint $a_y = -g$ here.

Application of Vertical Motion

Time to reach the maximum height

$$t_h = \frac{v_{y0}}{g}. \tag{10}$$

The maximum height

$$h_{\max} = \frac{v_{y0}^2}{2g}. \tag{11}$$

Total time in flight

$$t_{\text{flight}} = 2t_H = \frac{2v_{y0}}{g}. \tag{12}$$

The speed at which the object hits the ground/speed at the moment before hitting the ground

$$v_y = \sqrt{2gh_{\max}}. \tag{13}$$

You only need to check the signs of v_y, v_{y0}, y, y_0. If they are positive, the motion is vertically upward. If negative, they are vertically downward.

Freefall Motion

A freely falling body has no v_{y0}. It simply moves downward due to gravitational acceleration. If You add any initial speed to the body, you will impart an additional acceleration. That would only add to the falling speed.

Equations	y	v_y	a_y	t
$v_y = -gt$	✗	✓	✓	✓
$y - y_0 = y = -\frac{1}{2}gt^2$	✓	✗	✓	✓
$y - y_0 = y = \frac{v_y}{2}t$	✓	✓	✗	✓
$v_y^2 = -2g(y - y_0)$	✓	✓	✓	✗

TABLE 4: All four laws of motion in a nutshell for freefall motion. For calculation, you'll be given all the ✓ quantities and asked to solve for the ✗ ones.

We used the constraint $a_y = -g$ and $v_{y0} = 0$ here.

4.3 Projectile Motion (2D Motion)

A **projectile motion** is where an object is thrown vertically with an initial velocity, making an initial angle with the ground or wherever the base is, and the body follows a **trajectory** that is determined by the gravitational acceleration and air resistance (usually in practice). But we ignore this effect and only consider the effects of g. The object under motion is called a *projectile*.

Projectile motion is vertical motion with an angle.

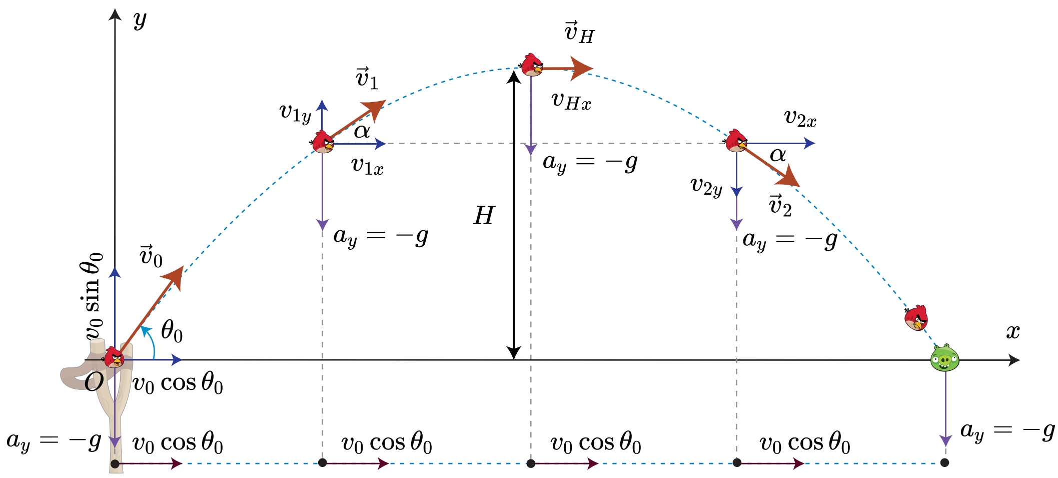

FIGURE 4: Motion of **Red**, the angry bird as a projectile. It has an initial velocity $\vec{v}_0$, an initial angle θ_0. The motion is symmetric with respect to the point where Red reaches a maximum height.

Since the projectile motion requires knowledge of the three primary parameters of kinematics, $\vec{r}$, $\vec{v}$, and $\vec{a}$ in both x and y dimensions, it is a 2D motion. By default, we take the 2D plane to be the x-y plane.

Similar to vertical motion, we only provide acceleration in the y-dimension and none in the x-dimension. Thus, the x-component of $\vec{v}$ stays **constant** throughout the trajectory.

Parameters	x-dimension	y-dimension
$\vec{r}$	$x = (v_0 \cos\theta_0)t$	$y = (v_0 \sin\theta_0)t - \frac{1}{2}gt^2$
$\vec{v}$	$v_x = v_0 \cos\theta_0$	$v_y = v_0 \sin\theta_0 - gt$
$\vec{a}$	$a_x = 0$	$a_y = -g$

TABLE 5: All the essential parameters needed to describe a projectile motion.

The entire parabolic motion may be described using the following expression:

$$y = (\tan\theta_0)x - \frac{1}{2}\left(\frac{g}{v_0^2 \cos^2\theta_0}\right)x^2 \tag{14}$$

This equation proves that a projectile motion is of parabolic fashion.

NOTE: H denotes the maximum achievable height with given $\vec{v}_0$ and θ_0. At the height, H, the y-component of the velocity is zero—only v_x remains. There's no way we can get a null vector at that height. Ideally, the projectile motion is symmetric, with air drag or resistance. Thus, the time of the entire motion t_R is twice that of t_H, the time required to reach H.

Application of Projectile Motion

Time to reach the maximum height

$$t_h = \frac{v_{y0}\sin\alpha_0}{g}. \tag{15}$$

The maximum height

$$h_{\max} = \frac{v_{y0}^2\sin^2\alpha_0}{2g}. \tag{16}$$

Total time in flight

$$t_{\text{flight}} = 2t_H = \frac{2v_{y0}\sin\alpha_0}{g}. \tag{17}$$

The maximum horizontal distance is called **Range** of the projectile only when the initial and final height of the motion is the same.

$$R = \frac{v_{y0}^2\sin 2\alpha_0}{g}. \tag{18}$$

The speed at which the object hits the ground/speed at the moment before hitting the ground

$$v_y = \sqrt{2gh_{\max}}. \tag{19}$$

You only need to check the signs of v_y, v_{y0}, y, y_0. If they are positive, the projectile is moving vertically upward. If negative, it is moving vertically downward. You can always revert back to the equations of vertical motion by substituting $\alpha_0 = 90^\circ$ in the projectile motion equation.