① $f(x) = x^7 - x^8$

$f'(x) = 7x^6 - 8x^7$

$f''(x) = 42x^5 - 56x^6$

$f'''(x) = 210x^4 - 336x^5$

$f^4(x) = 840x^3 - 1680x^4$

$f^5(x) = 2520x^2 - 6720x^3$

② (a) $f(x) = \frac{2}{x+1}$

$\frac{d}{dx}\left(\frac{2}{x+1}\right)$

$= 2.\left\{-\frac{1}{(x+1)^2}\right\}$

$= -\frac{1}{2.(x+1)^2}$

(b) $f(x) = x^2 \sin(\ln x)$

$\frac{d}{dx}\left(x^2 \sin(\ln x)\right)$

$= x^2.\cos(\ln x).\frac{1}{x^2} + \sin(\ln x).2x$

ans

③ $$f(x) = \begin{cases} x^2 - 4x - 2 & , x < 2 \\ -2x^2 + 4x & , x > 2 \end{cases}$$ at $x = 2$

if, $f(x) = x^2 - 4x - 2$

$$f(x+h) = (x+h)^2 - 4(x+h) - 2$$

$$= x^2 + 2hx + h^2 - 4x - 4h - 2$$

$f(x) = -2x^2 + 4x$

$$f(x+h) = -2(x+h)^2 + 4(x+h)$$

$$= -2(x^2 + 2hx + h^2) + 4x + 4h$$

L.S: $$\lim_{h \to 0^-} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0^-} \frac{x^2 + 2hx + h^2 - x^2 + 4x + 2}{h} \quad \begin{matrix} -4x - 4h - 2 \end{matrix}$$

$$= \lim_{h \to 0^-} \frac{h^2 + 2hx - 4h}{h}$$

$$= 2x - 4$$

L.S: $= 2 \cdot 2 - 4$

$= 0$

R.S: $$\lim_{h \to 0^+} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0^+} \frac{-2x^2 - 4hx - 2h^2 + 4x + 4h + 2x^2 - 4x}{h}$$

$$= \lim_{h \to 0^+} \frac{h(4 - 2h - 4x)}{h}$$

$= 4 - 4x$

R.S $= 4 - 4 \cdot 2 = -4$

$\therefore$ L.S $\neq$ R.S

(a) Given, $f(x) = \sqrt{x}\, g(x)$

$g(4) = 2$

$g'(4) = 3$

$f'(4) = ?$

$f'(x) = \sqrt{x} \cdot g'(x) + g(x) \cdot \frac{1}{2\sqrt{x}}$

$f'(4) = \sqrt{4} \cdot g'(4) + g(4) \cdot \frac{1}{2\sqrt{4}}$

$= (2 \times 3) + \left(\frac{2}{4}\right)$

$= 27/4$ (Ans)