Force and Motion-I
(Chapter 5)

Dynamics

Chapter 5

SP: 5.01, 5.02, 5.03, 5.04, 5.05, 5.06, 5.07

CP: 1 - 5

Pr: 9*, 13, 17, 34*, 50, 53*, 55, 57*, 67

Newton's Law:

Newton's 1st Law: If no net force acts on a body, the body's velocity can't change; that is the body can't accelerate.

Newton's 2nd Law: The net force on a body is equal to the product of the body's mass and it's acceleration.

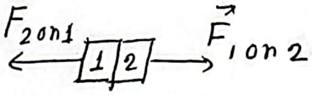
$$\vec{F}_{net} = m\vec{a}$$

unit of Force → N (SI) $= kg\,m/s^2$

CGS → dyne = $g\,cm/s^2$

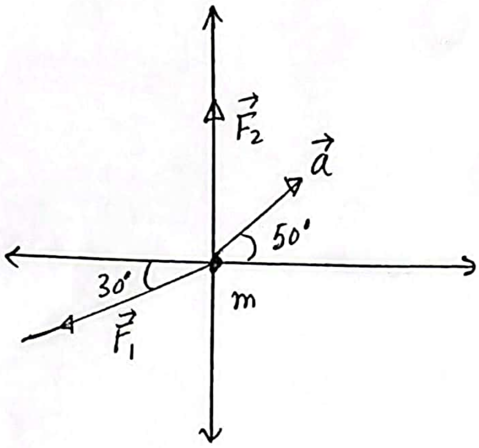
British → pound = slung ft/s^2

Newton's 3rd Law: When two bodies interact, the forces on the bodies from each other are always equal in magnitude and opposite in direction.



$$\vec{F}_{1on2} = -\vec{F}_{2on1}$$

'Sample Pr: 5.02"



$|\vec{F_1}| = 10N$

$|\vec{F_2}| = 20N$

$|\vec{a}| = 3\ m/s$

$m = 2kg$

$\vec{F_3} = ?\ ,\quad |\vec{F_3}| = ?\quad \theta = ?$

$$\vec{F}_{net} = m\vec{a}$$

$$\Rightarrow \vec{F_1} + \vec{F_2} + \vec{F_3} = m\vec{a}$$

$$\Rightarrow \vec{F_3} = m\vec{a} - \vec{F_1} - \vec{F_2}$$

Now,

$$m\vec{a} = m a_x \hat{i} + m a_y \hat{j}$$

$$= 2\,|\vec{a}|\cos(50°)\,\hat{i} + 2\,|\vec{a}|\sin(50°)\,\hat{j}$$

$$= [2 \times 3 \times \cos(50°)\,\hat{i} + 2 \times 3 \times \sin(50°)\,\hat{j}]\ N$$

$$= [3.86\,\hat{i} + 4.60\,\hat{j}]\,N$$

$$\vec{F_1} = |\vec{F_1}|\cos(180° + 30°)\,\hat{i} + |\vec{F_1}|\sin(180° + 30°)\,\hat{j}$$

$$= [-5\sqrt{3}\,\hat{i} + (-5)\,\hat{j}]\,N$$

$$\vec{F_2} = [0\,\hat{i} + 20\,\hat{j}]\,N$$

$$\therefore \vec{F_3} = [(3.86\,\hat{i} + 4.60\,\hat{j}) - (-5\sqrt{3}\,\hat{i} - 5\,\hat{j}) - (0\,\hat{i} + 20\,\hat{j})]\,N$$

$$= [(3.86 + 5\sqrt{3} - 0)\,\hat{i} + (4.60 + 5 - 20)\,\hat{j}\,]\,N$$

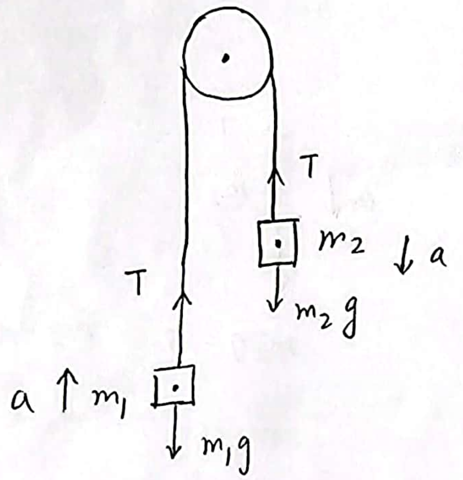
$$= 12.5\,\hat{i} - 10.4\,\hat{j}$$

$$|\vec{F_3}| = \sqrt{(12.5)^2 + (-10.4)^2} = \boxed{16.26\ N}$$

$$\theta = \tan^{-1}\left(\frac{F_{3,y}}{F_{3,x}}\right)$$

$$= \tan^{-1}\left(\frac{-10.4}{12.5}\right) = \boxed{-39.76^\circ}$$

$\boxed{8\ ✱}$

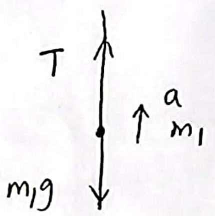


$m_1 = 1.5\ kg$

$m_2 = 3.1\ kg$

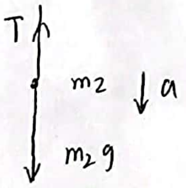
$a = ?$

$T = ?$



$$T - m_1 g = m_1 a$$

$$\Rightarrow T = m_1 a + m_1 g \quad \text{—(1)}$$



$$m_2 g - T = m_2 a$$

$$\Rightarrow m_2 g - m_2 a = T$$

$$\Rightarrow m_2 g - m_2 a = m_1 a + m_1 g$$

$$\Rightarrow m_2 g - m_1 g = m_1 a + m_2 a$$

$$\Rightarrow a = \frac{m_2 g - m_1 g}{m_1 + m_2}$$

$$\therefore a = \left(\frac{m_2 - m_1}{m_1 + m_2}\right) g$$

$$= \left(\frac{3.1 - 1.5}{3.1 + 1.5}\right) \times 9.8\ ms^{-2}$$

$$= \boxed{3.40\ ms^{-2}}$$

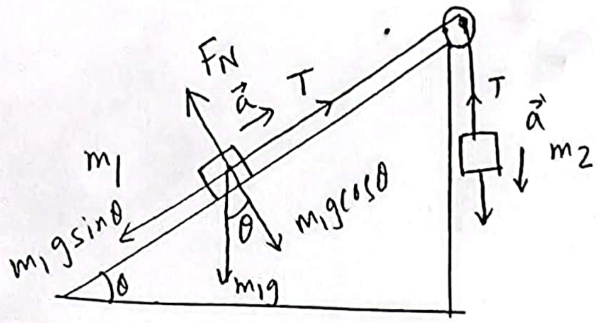
In equation (1),

$$T = (3.1 \times 9.8 + 3.1 \cdot [illegible] \times 3.4)\ ms^{-2}$$

$$= \boxed{19.8\ N}$$

See S.P: 5.06, 5.07

$\boxed{57}$

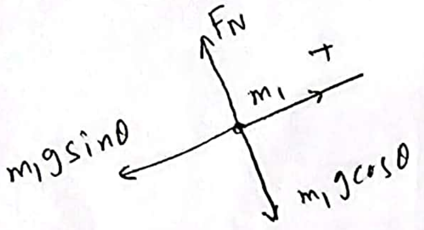


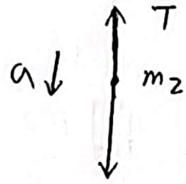
$m_1 = 3.7\ kg$

$m_2 = 2.3\ kg$

$a = ?$

$\theta = 30°$





$$m_2 g - T = m_2 a$$

$$T - m_1 g \sin\theta = m_1 a$$

$$m_2 g - T - m_2 a = 0 \quad \text{—(2)}$$

$$\Rightarrow T - m_1 g \sin\theta - m_1 a = 0 \quad \text{——(1)}$$

(1) + (2) $\Rightarrow$

$$- m_1 g \sin\theta - m_1 a + m_2 g - m_2 a = 0$$

$$\Rightarrow m_2 g - m_1 g \sin\theta = m_1 a + m_2 a$$

$$\Rightarrow a = \frac{(m_2 - m_1 \sin\theta)\, g}{(m_1 + m_2)}$$

$$= \frac{(2.3 - 3.7 \times \sin 30°) \times 9.8}{2.3 + 3.7}\ m/s^2$$

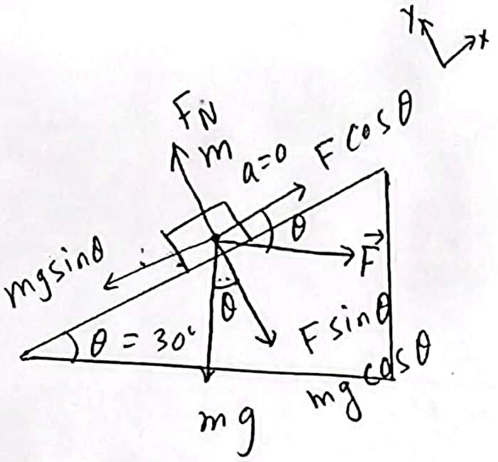
$$= \boxed{0.735\ m/s^2}$$

From equation (2), $T = m_2(g - a)$

$$= 2.3 \times (9.8 - 0.735)\ N$$

$$= \boxed{20.85\ N}$$

34



$m = 100\ kg$

$\theta = 30°$

Applying Newton's 2nd along x-axis,

(a) $F\cos\theta - mg\sin\theta = 0$

$\Rightarrow \quad F\cos\theta = mg\sin\theta$

$\Rightarrow \quad F = mg\tan\theta$

$= 100 \times 9.8 \times \tan 30°\ N$

$= \boxed{565.8\ N}$

Applying Newton's 2nd law along y-axis,

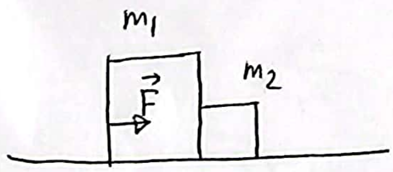
(b) $F_N - F\sin\theta - mg\cos\theta = 0$

$\Rightarrow \quad F_N = F\sin\theta + mg\cos\theta$

$= [565.8 \times \sin(30°) + 100 \times 9.8 \times \cos(30°)]\ N$

$= \boxed{1131.6\ N}$

55

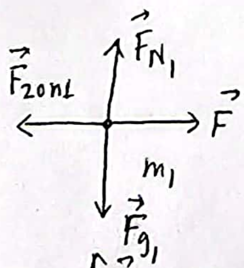


$m_1 = 2.3\ kg$

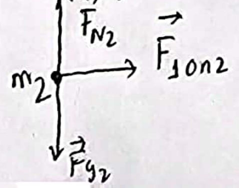
$m_2 = 1.2\ kg$

$F = 3.2\ N$

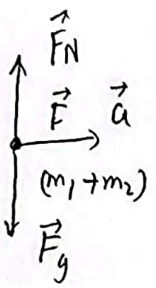
(a)



$F - F_{2on1} = m_1 a \quad \text{--- (1)}$



$F_{2on1} = m_2 a \quad \text{--- (2)}$



$$F = (m_1+m_2)a$$

$$\Rightarrow a = \frac{F}{m_1+m_2} \quad \text{——(3)}$$

Putting equation (3) into (1),

$$F - F_{2on1} = m_1 \frac{F}{m_1+m_2}$$

$$\Rightarrow F - \frac{m_1}{m_1+m_2}F = F_{2on1}$$

$$\Rightarrow \left(1 - \frac{m_1}{m_1+m_2}\right)F = F_{2on1}$$

$$\Rightarrow \frac{m_2}{m_1+m_2}F = F_{2ON1}$$

$$\therefore F_{2on1} = \frac{1.2}{2.3+1.2} \times 3.2\ N$$

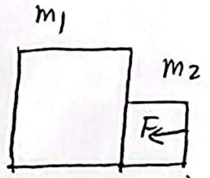
$$= \boxed{1.1\ N}$$

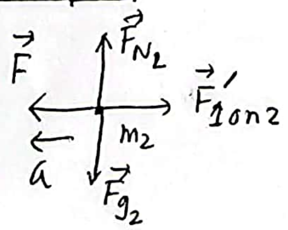
Putting equation (3) into (2),

$$F_{1on2} = \frac{m_2}{m_1+m_2}F$$

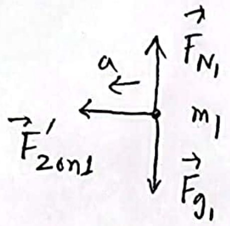
$$= \boxed{1.1\ N}$$

(b)

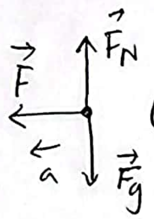




$$F - F'_{1on2} = m_2 a \quad \text{——(1)}$$



$$F'_{2on1} = m_1 a \quad -(2)$$



$$F = (m_1+m_2)\,a$$

$$a = \frac{F}{m_1+m_2} \quad -(3)$$

Putting equation (3) into (1),

$$F - F'_{1on2} = m_2 \frac{F}{m_1+m_2}$$

$$\Rightarrow F - \frac{m_2}{m_1+m_2} F = F'_{1on2}$$

$$\Rightarrow F'_{1on2} = \frac{m_1}{m_1+m_2} F$$

$$= \frac{2.3}{2.3+1.2} \times 3.2N$$

$$= \boxed{2.1\,N}$$

Putting equation (3) into (2),

$$F'_{2on1} = \frac{m_1}{m_1+m_2} F$$

$$= \boxed{2.1\,N}$$

(c) $F'_{1on2} = F'_{2on1} \sim m_1$ and $F_{1on2} = F_{2on1} \sim m_2$

since, $m_1 > m_2$, and $F'_{2on1} > F_{2on1}$

$\therefore F'_{1on2} > F_{1on2}$

9

Here, mass of the particle $m = 0.340\,kg$

Position vector of the particle, $\vec{r} = (-15 + 2t - 4t^3)\hat{i} + (25 + 7t - 9t^2)$

at, $t = 0.700\,s$, $|\vec{F}| = ?$, $\theta = ?$

According to Newton's 2nd law,

$$\vec{F} = m \frac{d^2\vec{r}}{dt^2}$$

$$= 0.340 \times \frac{d}{dt}\left(\frac{d}{dt}(-15 + 2t - 4t^3)\hat{i} + \frac{d}{dt}(25 + 7t - 9t^2)\hat{j}\right)$$

$$= 0.340 \times \frac{d}{dt}\left((2 - 12t^2)\hat{i} + (7 - 18t)\hat{j}\right)$$

$$= 0.340 \times (-24t\,\hat{i} - 18\hat{j})$$

$$\vec{F}\Big|_{t=0.700\,s} = 0.340 \times (-24 \times 0.700\,\hat{i} - 18\hat{j})$$

$$= -0.340 \times 24 \times 0.700\,\hat{i} - 0.340 \times 18\,\hat{j}$$

$$= -5.71\,\hat{i} - 6.12\,\hat{j}$$

(a) $\therefore |\vec{F}| = \sqrt{(-5.71)^2 + (-6.12)^2} = \boxed{8.37\,N}$

(b) $\theta = \tan^{-1}\left(\frac{-6.12}{-5.71}\right) = \pi + \tan^{-1}\left(\frac{6.12}{5.71}\right)$

$= 180° + 47° = \boxed{227°}$

(c) Particle's direction of travel is the direction of the particle's velocity, so,

$$\vec{v} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$$

$$= \frac{d}{dt}(-15 + 2t - 4t^3)\hat{i} + \frac{d}{dt}(25 + 7t -$$

$\vec{v}(t) = +(2 - 12t^2)\hat{i} + (.7 - 18t)\hat{j}$

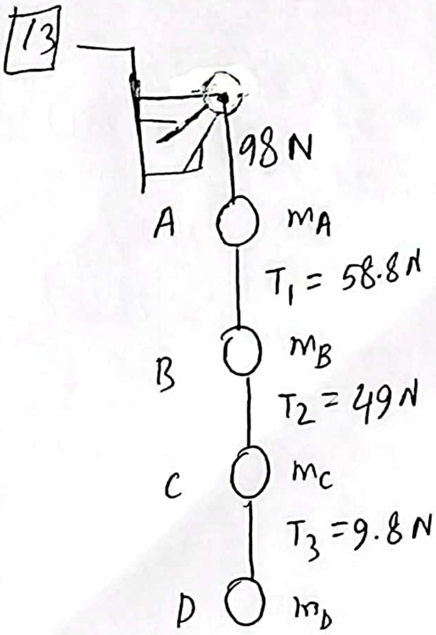
$\therefore \vec{v}(0.7) = [(2 - 12 \times 0.7^2)\hat{i} + (7 - 18 \times 0.7)\hat{j}]\ m/s = [-3.88\hat{i} - 5.60\hat{j}]$

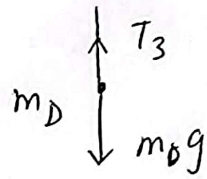
$\theta' = \tan^{-1}\left(\frac{-5.60}{-3.88}\right)$

$= 180° + \tan^{-1}\left(\frac{5.60}{3.88}\right)$

$= 180° + 55.28° = \boxed{235.28°}$

$\boxed{13}$



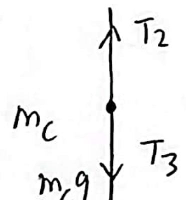


Applying Newton's 2nd Law on D.

$T_3 - m_D g = 0$

$\Rightarrow m_D = \frac{T_3}{g} = \frac{9.8}{9.8}\ kg$

$= \boxed{1\ kg}$



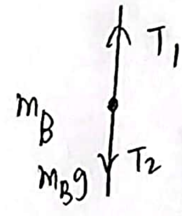
Apply Newton's 2nd Law on C,

$T_2 - T_3 - m_C g = 0$

$\Rightarrow m_C = \frac{T_2 - T_3}{g}$

$= \frac{(49 - 9.8)\ N}{9.8\ m/s^2}$

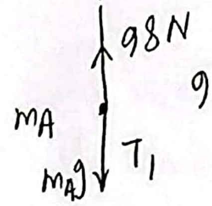
$= \boxed{4\ kg}$



Applying Newton's 2nd Law on B,

$T_1 - T_2 - m_B g = 0$

$\Rightarrow m_B = \frac{T_1 - T_2}{g} = \frac{58.8 - 49}{9.8}\ kg = \boxed{1\ kg}$

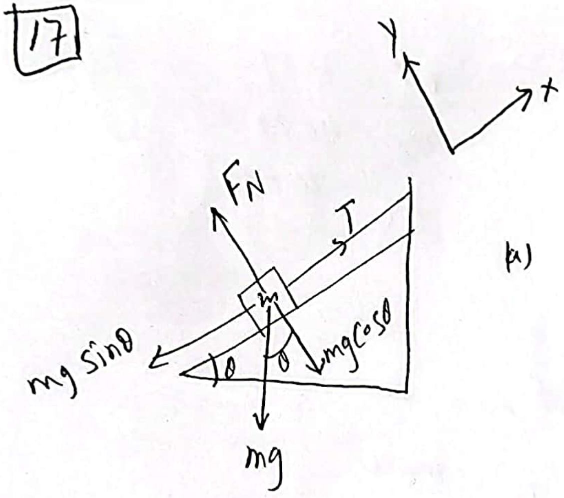


$98 - T_1 - m_A g = 0$

$\Rightarrow m_A = \frac{98 - T_1}{g} = \frac{98 - 58.8}{9.8}\ kg$

$= \boxed{4\ kg}$

17



(a) Applying Newton's 2^{nd} Law into x-axis

$$T - mg\sin\theta = 0$$

$$\Rightarrow T = mg\sin\theta$$

$$= 8.5 \times 9.8 \times \sin(30^\circ)\ N = \boxed{41.65\ N}$$

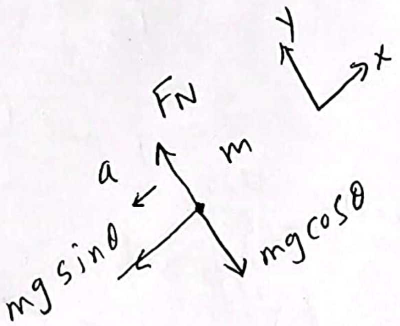
(b) Applying Newton's 2^{nd} Law into y-axis

$$F_N - mg\cos\theta = 0$$

$$\Rightarrow F_N = mg\cos\theta$$

$$= 8.5 \times 9.8 \times \cos(30^\circ)\ N = \boxed{72.14\ N}$$

(c)



When we cut the cord there is no more tension force of the cord. Appyling Newton's 2^{nd} Law along x-axis,

$$-mg\sin\theta = -ma$$

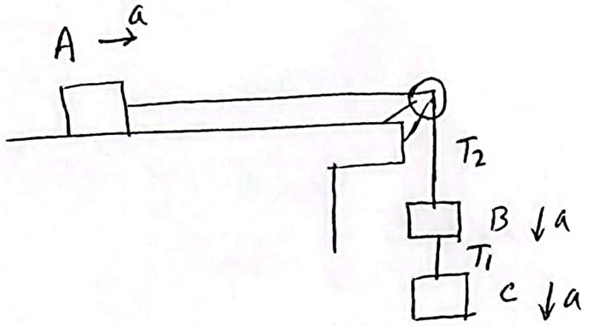
$$\Rightarrow a = g\sin\theta$$

$$= 9.8 \times \sin(30^\circ)$$

$$= \boxed{4.9\ m/s^2}$$

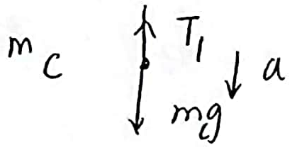
CamScanner

$\boxed{50}$



$m_A = 30\,kg$
$m_B = 40\,kg$
$m_C = 10\,kg$

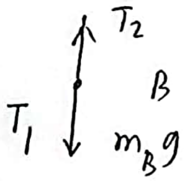
Applying Newton's 2nd Law on C,



$$m_C g - T_1 = m_C a$$

$$T_1 = m_C g - m_C a \quad \text{—(1)}$$

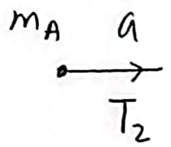
Applying Newton's 2nd Law on B,



$$(-T_2 + T_1 + m_B g) = m_B a$$

$$T_2 = -m_B a + T_1 + m_B g \quad \text{—(2)}$$

Applying Newton's 2nd Law on C,



$$T_2 = m_A a \quad \text{—(3)}$$

$\boxed{\text{Combining equation (1), (2) and (3)}}$

Putting (3) into (2),

$$m_A a = -m_B a + T_1 + m_B g$$

$$\Rightarrow \quad m_A a + m_B a - m_B g = T_1 \quad \text{—(4)}$$

From (1) and (4), $m_C g - m_C a = m_A a + m_B a - m_B g$

$$\Rightarrow \quad m_C g + m_B g = m_A a + m_B a \; m_C a$$

$$\Rightarrow \quad a = \frac{m_C g + m_B g}{m_A + m_B + m_C}$$

$$a = \frac{10\times9.8 + 40\times9.8}{30+40+10} \; m/s^2$$

$$= 6.125 \; m/s$$

Putting a into equation (1), $T_1 = (10 \times 9.8 - 10 \times 6.125)N$

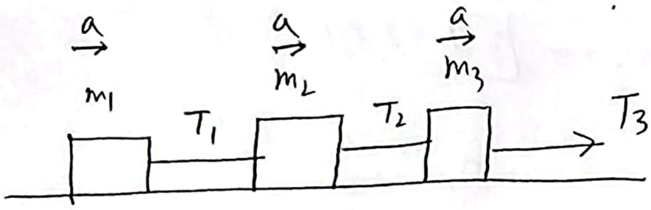
$$= \boxed{36.75 N} \quad (\text{Ans a})$$

Distance travel A, $h = \frac{1}{2} a t^2$

$$= \frac{1}{2} \times 6.125 \times (0.250)^2 \; m$$

$$= \boxed{0.191 \; m} \quad (\text{Ans b})$$

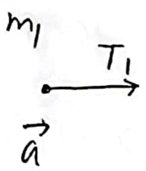
$\boxed{53}$



$T_1 = ?$ $\quad m_1 = 12 kg$

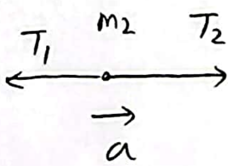
$T_2 = ?$ $\quad m_2 = 24 kg$

$T_3 = 65 N$ $\quad m_3 = 31 kg$



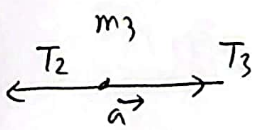
Applying Newton's 2nd Law on m_1

$$T_1 = m_1 a \quad -(1)$$



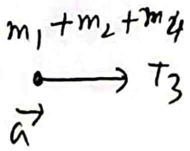
Appyling Newton's 2nd Law on m_2

$$T_2 - T_1 = m_2 a \quad -(3)$$



Appyling Newton's 2nd Law on m_3

$$T_3 - T_2 = m_3 a \quad -(3)$$



Appyling Newton's 2nd Law on total system

$$T_3 = (m_1 + m_2 + m_3) a \quad -(4)$$

$$\Rightarrow \quad a = \frac{T_3}{m_1+m_2+m_3} = \frac{65}{12+24+31} \; m/s^2$$

$\boxed{a = 0.97\ m/s^2}$ —(5) (Ans a)

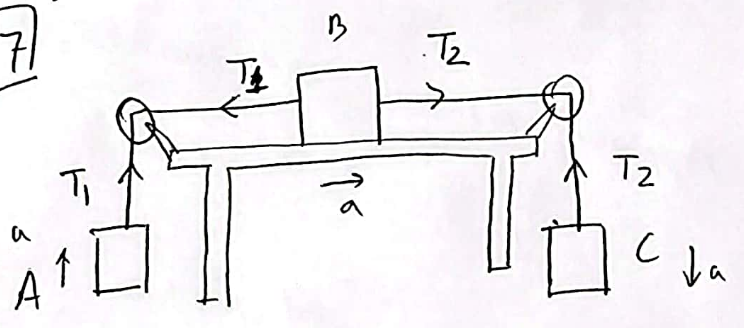
Putting the value of a into (1)

$$T_1 = m_1 a$$
$$= (12 \times .97)\ N$$
$$= \boxed{11.64 N} \quad \text{(Ans b)}$$

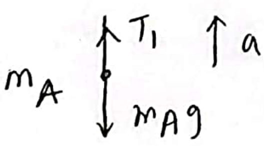
Putting the value of a, T_1 into (2),

$$T_2 = T_1 + m_2 a$$
$$= (11.64 + 24 \times 0.97)\ N$$
$$= \boxed{34.92 N} \quad \text{(Ans c)}$$

$\boxed{67}$



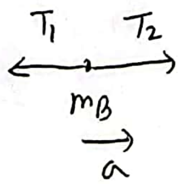
$m_A = 6\ kg$
$m_B = 8\ kg$
$m_C = 10\ kg$
$T_2 = ?$



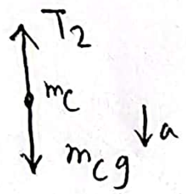
Applying Newton's 2nd law on A,

$$T_1 - m_A g = m_A a \quad \text{—(1)}$$
$$\Rightarrow T_1 = m_A a + m_A g$$

Applying Newton's 2nd law on B,



$$T_2 - T_1 = m_B a \quad \text{—(2)}$$
$$\Rightarrow T_2 = T_1 + m_B a$$
$$= m_A a + m_A g + m_B a \quad \text{—(3)}$$



Applying Newton's 2nd Law on C,

$$m_C g - T_2 = m_C a$$

$$\Rightarrow \quad T_2 = m_C g - m_C a \quad \text{—(4)}$$

From equation (4) and (3),

$$m_C g - m_C a = m_A a + m_A g + m_B a$$

$$\Rightarrow \quad m_C g - m_A g = m_A a + m_B a + m_C a$$

$$\Rightarrow \quad a = \frac{m_C g - m_A g}{m_A + m_B + m_C}$$

$$= \frac{10 \times 9.8 - 6 \times 9.8}{6 + 8 + 10} \ m/s^2$$

$$= 1.63 \ m/s^2$$

From equation (4), $T_2 = (10 \times 9.8 - 10 \times 1.63)N$

$$= \boxed{81.7N} \ \boxed{Ans}$$