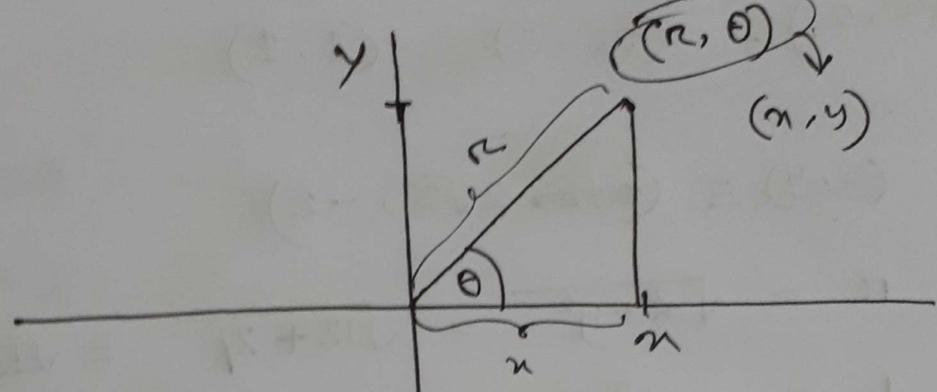
Math 110

Home work sheet #9

Home work Sheet # 9

Poolar cooRadians



$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$r^2 = x^2 + y^2$$

$$\tan\theta = \frac{y}{x}$$

Ans. to the q NO - 01

ⓐ $(2\sqrt{3}, -2)$, ⓑ $(0, -2)$ ⓒ $(1, 1)$

ⓐ $(x, y) \equiv (\cancel{2\sqrt{2}}\ 2\sqrt{3}, -2)$

$r = \sqrt{x^2+y^2} = \sqrt{12+4} = \cancel{\sqrt{16}}\ 4$

$\tan\theta = \dfrac{-2}{2\sqrt{3}} = \dfrac{-1}{\sqrt{3}}$

$\theta = \tan^{-1}\left(\dfrac{-1}{\sqrt{3}}\right) = -30 = \dfrac{-\pi}{6}$

$\therefore (x, y) \equiv (r, \theta) \equiv \cancel{(4, -30)}\ \left(4, \dfrac{-\pi}{6}\right)$

ⓑ $(x, y) \equiv (0, -2)$

$r = \sqrt{0^2+(-2)^2} = \sqrt{4} = 2$

$\tan\theta = \dfrac{-2}{0}$ $\theta = -\dfrac{\pi}{2}$ // $\dfrac{3\pi}{2}$

$\cancel{\cot\theta = \dfrac{0}{-2}}$

$\cancel{\theta = \cot^{-1}(0) =}$

$\therefore (x, y) \equiv \left(2, \dfrac{-\pi}{2}\right)$

or, $\equiv \left(2, \dfrac{3\pi}{2}\right)$

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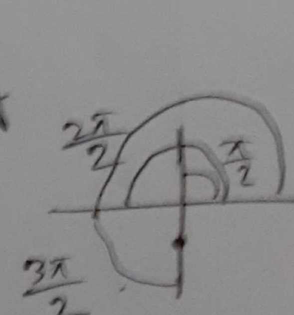


(c) $(x, y) = (1, 1)$

$r = \sqrt{1+1} = \sqrt{2}$

$\tan\theta = \frac{1}{1} = \cancel{\tan}\ 1$

$\theta = \frac{\pi}{4}$

$\therefore (x = y) = \left(\sqrt{2}, \frac{\pi}{4}\right)$

(2) (a) $(7, 2\pi/3)$

$x = r\cos\theta = 7 \cdot \cos\frac{2\pi}{3} = \frac{-7}{2}$

$y = r\sin\theta = 7 \cdot \sin\frac{2\pi}{3} = \frac{7\sqrt{3}}{2}$

Ans: $\left(\frac{-7}{2}, \frac{7\sqrt{3}}{2}\right)$

(b) $(8, 9\pi/4)$

$x = 8 \cdot \cos(9\pi/4) = 4\sqrt{2}$

$y = 8\sin(9\pi/4) = 4\sqrt{2}$

$\therefore$ Ans: $(4\sqrt{2}, 4\sqrt{2})$

(c) $(0, \pi)$

$x = 0$

$y = 0$

Ans: $(0, 0)$

(3)

$$r = a\sin\theta$$

$$\cancel{\sqrt{x^2+y^2} = a\cdot\frac{y}{r}}$$

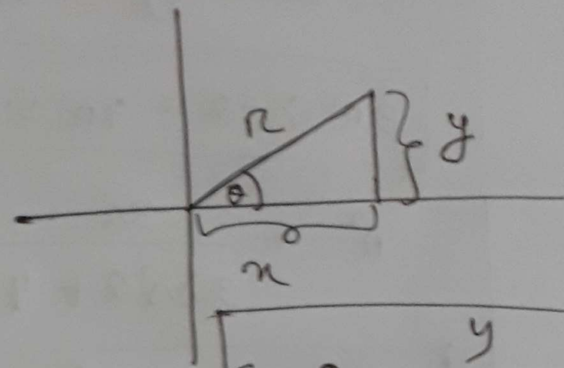
$$\cancel{x^2+y^2 = a^2\cdot\frac{y^2}{r}}$$

$$r = a\cdot\frac{y}{r}$$

$$r^2 = ay$$

$$x^2+y^2 = ay$$

$$x^2+y^2-ay = 0$$



$$\boxed{\sin\theta = \frac{y}{r}}$$

(ii) $\sqrt{r} = \sqrt{a}\cos\left(\frac{\theta}{2}\right)$

$$r = a\cos^2(\theta/2)$$

$$r = a\cdot\frac{1}{2}(1+\cos\theta)$$

$$r = \frac{a}{2}(1+\cos\theta)$$

$$\cancel{\sqrt{x^2+y^2} = \frac{a}{2}(1+}$$

$$r = \frac{a}{2}\left(1+\frac{x}{r}\right)$$

$$r = \frac{a}{2}+\frac{ax}{2r}$$

$$r^2 = \frac{ar}{2}+\frac{ax}{2}$$

$$x^2+y^2 = \frac{a\sqrt{x^2+y^2}}{2}+\frac{ax}{2}$$

$$2x^2+2y^2 = a\sqrt{x^2+y^2}+ax$$

(4)
(i)

$$9x^2 + 4y^2 = 36$$

$$\boxed{9r^2\cos^2\theta + 4r^2\sin^2\theta = 36}$$ Ans.

$$r^2 = \frac{36}{9\cos^2\theta + 4\sin^2\theta}$$

$$\boxed{r = \frac{6}{\sqrt{9\cos^2\theta + 4\sin^2\theta}}} \rightarrow$$

(ii)

$$x^3 = y^2(2a - x)$$

$$r^3\cos^3\theta = r^2\sin^2\theta\,(2a - r\cos\theta)$$

$$r^3\cos^3\theta = 2ar^2\sin^2\theta - r^3\sin^2\theta\cos\theta$$

$$r = \frac{2ar^2\sin^2\theta - r^3\sin^2\theta\cos\theta}{r^2\cos^2\theta}$$

$$\boxed{r = \frac{2a\sin^2\theta}{\cos^2\theta} - \frac{r\sin^2\theta}{\cos\theta}}$$

(5)

$2x^2 + y^2 - 4x + 4y = 0$

New origin : $(1, -2)$

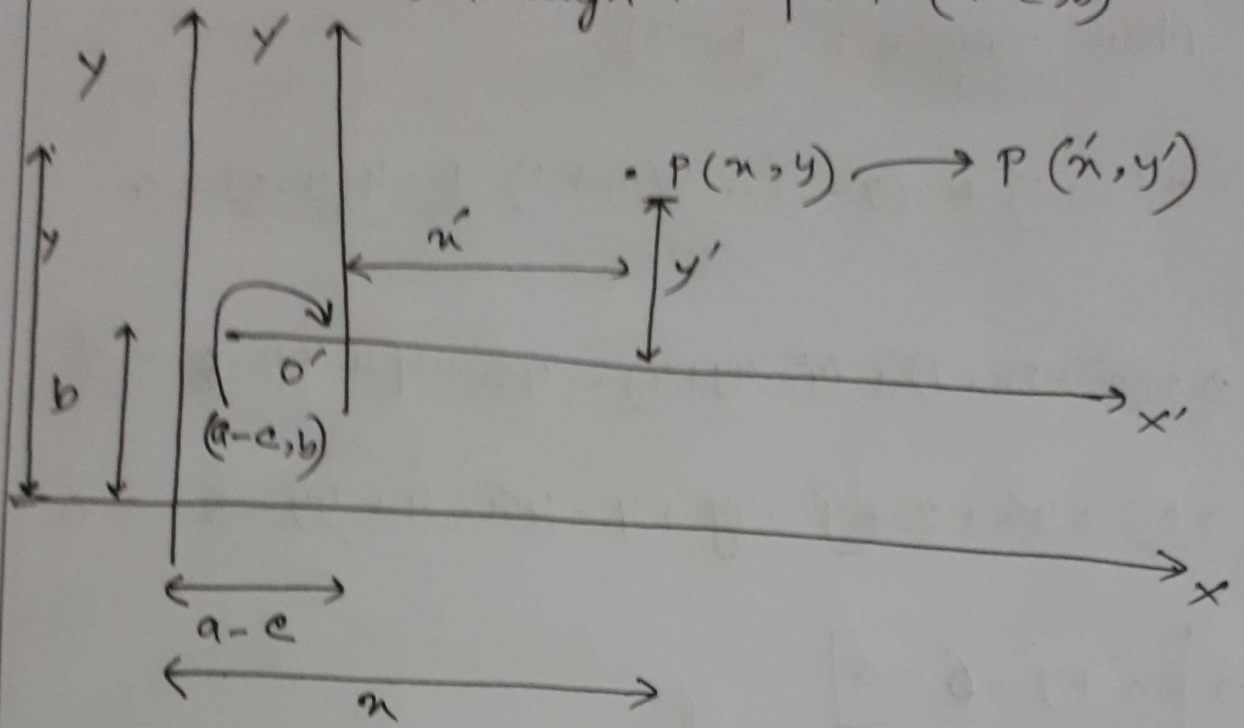
$2(x+1)^2 + (y-2)^2 - 4(x+1) + 4(y-2) = 0$

$\Rightarrow 2(x^2 + 2x + 1) + (y^2 - 4y + 4) - 4x - 4 + 4y - 8 = 0$

$\Rightarrow 2x^2 + \cancel{4x} + 2 + y^2 - \cancel{4y} + 4 - \cancel{4x} - 4 + \cancel{4y} - 8 = 0$

$\Rightarrow \boxed{2x^2 + y^2 - 6 = 0}$

Example: what does the equation $(x-a)^2+(y-b)^2=c^2$ become when it is transformed to parallel access through the point $(a-c, b)$



$\therefore \quad x = a-c+x'$ ——— (i)

$y = b+y'$ ——— (ii)

$\therefore \quad (x-a)^2+(y-b)^2=c^2$

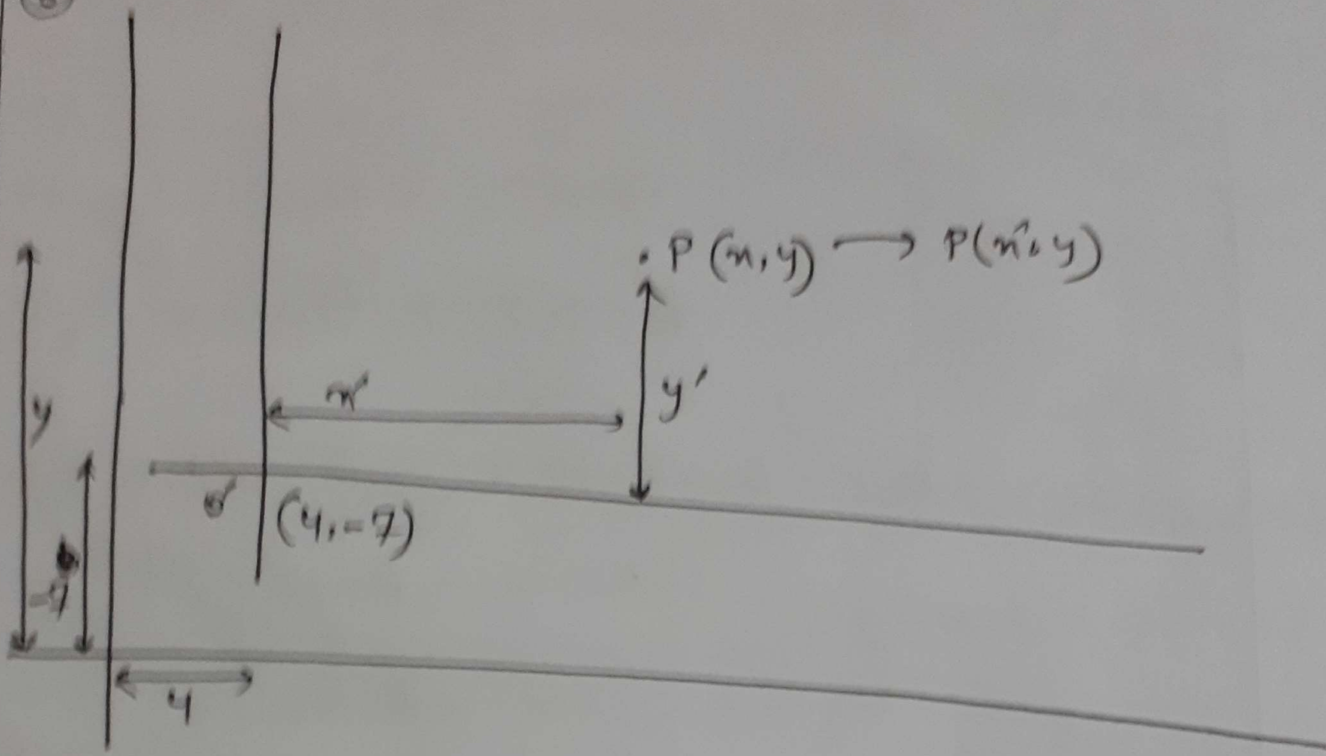
$[(\cancel{a}-c+x')-\cancel{a}]^2+[(\cancel{b}+y')-\cancel{b}]^2=c^2$

$\Rightarrow [x'-c]^2+y'^2=c^2$

$\Rightarrow x'^2+y'^2-2cx'+c^2=c^2$

$\Rightarrow x'^2+y'^2-2cx'=0$

(8)



$x = x' + 4$

$y = y' - 7$

Now,

$x^2 + y^2 - 8x + 14y + 5 = 0$

$\Rightarrow x^2 - 2\cdot x\cdot 4 + 4^2 + y^2 + 2\cdot y\cdot 7 + 7^2 = 16 + 49 - 5$

$\Rightarrow (x-4)^2 + (y+7)^2 = 60$

$\Rightarrow (x' + 4 - 4)^2 + (y + 7 - 7)^2 = 60$

$\Rightarrow x'^2 + y^2 = 60$

General form of a conic section: $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

$$x = x'\cos\theta - y'\sin\theta$$

$$y = x'\sin\theta + y'\cos\theta$$

$$a(x'\cos\theta - y'\sin\theta)^2 + 2h(x'\cos\theta - y'\sin\theta)(y'\cos\theta + x'\sin\theta) + b(y'\cos\theta + x'\sin\theta)^2 + 2g(x'\cos\theta - y'\sin\theta) + 2f(y'\cos\theta + x'\sin\theta) + c = 0$$

$$\Rightarrow (a\cos^2\theta + 2h\cos\theta\sin\theta + b\sin^2\theta)x'^2 + 2(h(\cos^2\theta - \sin^2\theta) - (a-b)(\sin\theta - \cos\theta))x'y' + (a\sin^2\theta - 2h\cos\theta\sin\theta + b\cos^2\theta)y'^2 + 2(g\cos\theta + f\sin\theta)x' + 2(f\cos\theta \mp g\sin\theta)y' + c = 0$$

To find the value of θ that eliminates the $x'y'$ term, we will set the $x'y'$ coefficient to zero.

$$2(h(\cos^2\theta - \sin^2\theta) - (a-b)(\sin\theta\cos\theta) = 0$$

$$2(h\cos 2\theta) - (a-b)(\sin 2\theta) = 0$$

$$\sin 2\theta(a-b) = 2h\cos 2\theta$$

$$\frac{\sin 2\theta}{\cos 2\theta} = \frac{2h}{a-b}$$

$$\boxed{\theta = \frac{1}{2}\tan^{-1}\left(\frac{2h}{a-b}\right)}$$

Rotation of the axes of the general form of a conic section: $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ to ~~elem~~ eliminate the xy coefficient.

If the x and y axes of the cartesian coordinate system are roated conterclockwise through an angle of θ, with the new rotated axes denoted x' and y', then a point whose coordinates are (x, y) in the unrotated system will have coordinates (x', y') in the roated system, where

$$x = x' \cos\theta + y' \sin\theta$$

$$y = y' \cos\theta - x' \sin\theta$$

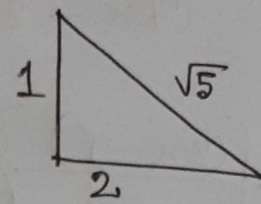
Alternatively, one can view the x and y coordinates as the rotation through an angle of $-\theta$ of the x' and y' coordinates, so

$$x = x' \cos\theta - y' \sin\theta$$

$$y = y' \cos\theta + y' \sin\theta$$

(7)

$$7x^2 - 2xy + y^2 + 1 = 0$$



Here,

$$a = 7$$

$$b = 1$$

$$2h = 2$$

$$x = \sqrt{4+1} = \sqrt{5}$$

$$\tan^{-1}\left(\frac{1}{2}\right) \equiv \sin^{-1}\left(\frac{1}{\sqrt{5}}\right) \equiv \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$$

$$\therefore x = x'\cos\theta - y'\sin\theta$$

$$= x'\cos\cos^{-1}\frac{2}{\sqrt{5}} - y'\sin\sin^{-1}\frac{1}{\sqrt{5}}$$

$$= \frac{2x'}{\sqrt{5}} - \frac{y'}{\sqrt{5}}$$

$$= \frac{2x' - y'}{\sqrt{5}}$$

$$y = x'\sin\theta + y'\cos\theta$$

$$= x'\sin\sin^{-1}\frac{1}{\sqrt{5}} + y'\cos\cos^{-1}\frac{2}{\sqrt{5}}$$

$$= \frac{x'}{\sqrt{5}} + \frac{2y'}{\sqrt{5}}$$

$$= \frac{2y' + x'}{\sqrt{5}}$$

Now,

$$7x^2 - 2xy + y^2 + 1 = 0$$

$$7\cdot\left\{\frac{(2x'-y')}{\sqrt5}\right\}^2 - 2\,\frac{2x'-y'}{\sqrt5}\cdot\frac{x'+2y'}{\sqrt5} + \left(\frac{2y'+x'}{\sqrt5}\right)^2 + 1 = 0$$

$$\frac{7(2x'-y')^2}{5} - \frac{2(2x'-y')(x'+2y')}{5} + \frac{(x'+2y')^2}{5} + 1 = 0$$

$$7(2x'-y')^2 - 2(2x'-y')(x'+2y') + (x'+2y')^2 + 5 = 0$$

$$6(2x'-y')^2 + (2x'-y')^2 - 2(2x'-y')(x'+2y') + (x'+2y')^2 + 5 = 0$$

$$6(4x'^2 - 4x'y' + y'^2) + (2x' - y' - x' - 2y')^2 + 5 = 0$$

$$6(4x'^2 - 4x'y' + y'^2) + (x' - 3y')^2 + 5 = 0$$

$$24x'^2 - 24x'y' + 6y'^2 + x'^2 - 6x'y' + 9y'^2 + 5 = 0$$

$$\cancel{23x'^2 - 18x'y' - 3y'^2 + 5} = 0$$

$$25x'^2 - 30x'y' + 15y'^2 + 5 = 0$$

(8) $11x^2+24xy+4y^2-20x-40y-5=0$

$\Rightarrow 11x^2+22xy+2xy+4y^2-20x-40y-5=0$

$\Rightarrow 11x(x+2y)+2y(x+2y)-20(x+2y)-5=0$

$\Rightarrow (x+2y)(11x+2y)-20(x+2y)-5=0$

$\Rightarrow (x+2y)(11x+2y-20)-5=0$

Now,

$\{(x+2)+2(y-1)\}\{11(x+2)+2(y-1)-20\}-5=0$

$(x+\cancel{2}+2y-\cancel{2})(11x+\cancel{22}+2y-\cancel{2}-\cancel{20})-5=0$

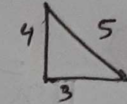
$(x+2y)(11x+2y)-5=0$

$11x^2+2xy+22xy+4y^2-5=0$

$\cancel{11x^2+20xy-4y^2-5=0}$

$11x^2+24xy+4y^2-5=0$

Now,



$$\tan^{-1}\left(\frac{4}{3}\right) \equiv \sin^{-1}\left(\frac{4}{5}\right) \equiv \cos^{-1}\left(\frac{3}{5}\right)$$

$$x = x'\cos\theta - y'\sin\theta$$

$$= x'\cdot\frac{3}{5} + y'\cdot\frac{4}{5} = \frac{3x'}{5} + \frac{4y'}{5}$$

$$= \frac{3x' - 4y'}{5}$$

$$y = y'\cos\theta + x'\sin\theta$$

$$= \frac{3y'}{5} - \frac{4x'}{5} = \frac{3y' + 4x'}{5}$$

Now,

$$11\cdot\frac{(3x'-4y')^2}{25} + 24\cdot\frac{(3x'-4y')(3y'+4x')}{25} - \frac{4(3y'+4x')^2}{25} - 5 = 0$$

$$11(3x'-4y')^2 + 24(3x'-4y')(3y'+4x') + 4(3y'+4x')^2 - 125 = 0$$

$$\cancel{2(3x'-4y')^2 + 9(3x'-4y')^2 + 2\cdot 3(3x'-4y')\cdot 4(3y'+4x')}$$

$$\cancel{11(9x'^2 - 24x'y' + 16y'^2) + 24(9x'y' + 12x'^2 - 12y'^2 - 16x'y')}$$
$$\cancel{-4(9y'^2 + 24x'y' + 16x'^2) - 125} = 0$$

$$99x'^2 - 264x'y' + 176y'^2 + 288x'^2 - 288y'^2 - 168x'y' - 36y'^2 - 96x'y' - 64x'^2 - 125 = 0$$

$$(3x' - 4y')(33x' - 44y' + 72y' + 96x')$$

$$+ 4(9y'^2 + 24x'y' + 16x') - 125 = 0$$

$$\cancel{99x'^2 \quad 132x'y'}$$

$$(3x' - 4y')(129x' + 28y') + 36y'^2 + 96x'y' + 64x' - 125 = 0$$

$$387x'^2 + 84x'y' - 516x'y' - 112y'^2 + 36y'^2 + 96x'y' + 64x'$$

$$- 125 = 0$$

$$451x'^2 - 336x'y' - 76y'^2 - 125 = 0$$

(7)

$f(x,y) = 9x^2 + 15xy + y^2 + 12x - 11y - 5 = 0$

$$\frac{\partial f}{\partial x} = 18x + 15y + 12 = 0$$

$$\frac{\partial f}{\partial y} = 15x + 2y - 11 = 0$$

$(x, y) = (1, -2)$

Suppose,

the equation is:

$9x^2 + 15xy + y^2 + c = 0$

Here, $c = gx_1 + fy_1 + c$

$g = \frac{12}{2} = 6$

$f = \frac{-11}{2} = -\frac{11}{2}$

$c = -5$

$= (6 \times 1) + \left(\frac{-11}{2} \times -2\right) - 5$

$= 6 + 11 - 5 = 12$

~~$9x^2 + 15xy + 12 = 0$~~

$9x^2 + 15xy + y^2 + 12 = 0$

(10) Ans

$11x^2 + 3xy + 7y^2 + 19 = 0$

$a = 11;\ b = 7;\ 2h = 3;\ c = 19$

To remove the xy term, we will rotate the axes through an angle

$$\theta = \frac{1}{2}\tan^{-1}\left(\frac{2h}{a-b}\right)$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{3}{11-7}\right) = \frac{1}{2}\tan^{-1}\frac{3}{4} = 18.43^\circ$$

$$x = x'\cos\theta - y'\sin\theta$$

$$= x'\cos 18.43 - y'\sin 18.43$$

$$x = 0.9x' - 0.3y' = \frac{9x' - 3y'}{10}$$

$$y = x'\sin\theta + y'\cos\theta$$

$$= x'\sin(18.43) + y'\cos(18.43)$$

$$= \frac{3x' + 9y'}{10}$$

$$\therefore 11\cdot\frac{(9x'-3y')^2}{100} + 3\cdot\frac{(9x'-3y')(3x'+9y')}{100} + 7\cdot\frac{(3x'+9y')^2}{100} + 19 = 0$$

$$11(9x'-3y')^2 + 3(9x'-3y')(3x'+9y') + 7(3x'+9y')^2 + 1900 = 0$$

$$(9x'-3y')(99x'-33y'+9x'+27y') + 7(9x'^2+54x'y'+81y'^2) + 1900 = 0$$

$$(9x' - 3y')(108x' - 6y') + 7(9x' + 54x'y' + 81y'^2) + 1900 = 0$$

$$972x'^2 - 54x'y' - 324x'y' + 18y'^2 + 63x' + 378x'y' + 567y'^2 + 1900 = 0$$

$$1035x'^2 + 585y'^2 + 1900 = 0$$

(21)

$$\cancel{2x^2 + 4xy + 5y^2 - 4x - 22y + 7 = 0}$$

$$\cancel{2x^2 + 11xy}$$

$$\cancel{x(2x + 11y)} - x(2x + 11y)$$

(11) $2x^2+4xy+5y^2-4x-22y+7=0$

$$2(x-2)^2+4(x-2)(y+3)+5(y+3)^2-4(x-2)$$
$$-22(y+3)+7=0$$

$$2(x-2)^2-4(x-2)+4(x-2)(y+3)+5(y+3)^2$$
$$-22(y+3)+7=0$$

~~$2(x-2)(1-2x$~~

$$2(x-2)(x-2-2)+(y+3)(4x-8+5y+15-22)+7=0$$

$$2(x-2)(x-6)+(y+3)(4x+5y-15)+7=0$$

$$(2x-4)(x-6)+(y+3)(4x+5y-15)+7=0$$

$$2x^2-\cancel{12x}-4x+24+4xy+5y^2-15y+\cancel{12x}+15y-45+7=0$$

~~$2x^2+5y^2+4xy-22=0$~~

$$2x^2+5y^2-4x-15y-14=0$$
$$+4xy$$

(12)

$$9x^2 + 24xy + 2y^2 + 54 = 0$$

$$\cancel{(3x)^2 + 2.3x.4y + (4y)^2 - 14y^2 + 54 = 0}$$

$$\cancel{(3x + 4y)^2 - 14y^2 + 54 = 0}$$

Here,

If we want to remove the xy term, we have to rotate the equation to an angle $\theta = \frac{1}{2} \tan^{-1}\left(\frac{2h}{a-b}\right)$

$$= \frac{1}{2} \tan^{-1}\left(\frac{24}{9-2}\right)$$

$$= \frac{1}{2} \tan^{-1} \frac{24}{7} = 36.87^\circ$$

$$\therefore x = x' \cos 36.87 - y' \sin 36.87$$

$$= \cancel{x}\, 0.8x' - 0.6y'$$

$$= \frac{8x' - 6y'}{10}$$

$$y = \cancel{x \cos}\; x' \sin 36.87 + y' \cos 36.87$$

$$= \frac{6x' + 8y'}{10}$$

$$9\cdot\frac{(8x'-6y')^2}{100} + 24\cdot\frac{(8x'-6y')(6x'+8y')}{100} + 2\cdot\frac{(6x'+8y')^2}{100} + 54 = 0$$

$$9(8x'-6y')^2 + 24(8x'-6y')(6x'+8y') + 2(6x'+8y')^2 + 5400 = 0$$

$$9(64x'^2 - 96x'y' + 36y'^2) + (12x'+16y')(96x' - 72y' + 6x' + 8y') + 5400 = 0$$

$$\cancel{576x'^2 - 864x'y' + 324y'^2 + 1152x'^2 - 864x'y' + 1536x'y' - 1152y'^2 + 5400 = 0}$$

$$576x'^2 - 864x'y' + 324y'^2 + (12x'+16y')(102x'-64y') + 5400 = 0$$

$$576x'^2 - 864x'y' + 324y'^2 + 1224x'^2 - 768x'y' + 1632y'x' - 1024y'^2 + 5400 = 0$$

$$1800x^2 - 700y^2 + 5400 = 0$$

$$18x^2 - 7y^2 + 54 = 0$$

(13)

$$x^2 + 2\sqrt{3}xy - y^2 = 2a^2$$

$$x = x'\cos\theta - y'\sin\theta$$

$$= x'\cos 30 - y'\sin 30$$

$$= \frac{\sqrt{3}x'}{2} - \frac{y'}{2} = \frac{\sqrt{3}x' - y'}{2}$$

$$y = x'\sin\theta + y'\cos\theta$$

$$= x'\sin 30 + y'\cos 30 = \frac{x'}{2} + \frac{\sqrt{3}y'}{2}$$

$$= \frac{x' + \sqrt{3}y'}{2}$$

$$\therefore x^2 + 2\sqrt{3}xy - y^2 = 2a^2$$

$$\frac{(\sqrt{3}x' - y')^2}{4} + 2\sqrt{3}\cdot\frac{(\sqrt{3}x' - y')(x' + \sqrt{3}y')}{4} - \frac{(x' + \sqrt{3}y')^2}{4} = 2a^2$$

$$(\sqrt{3}x' - y')^2 + 2\sqrt{3}(\sqrt{3}x' - y')(x' + \sqrt{3}y') - (x' + \sqrt{3}y')^2 = 8a^2$$

$$(\sqrt{3}x' - y')(\sqrt{3}x' - y' + 2\sqrt{3}x' + 6y') - x'^2 - 3y'^2 - 2\sqrt{3}x'y' = 8a^2$$

$$(\sqrt{3}x' - y')(3\sqrt{3}x' + 5y') - x'^2 - 3y'^2 - 2\sqrt{3}x'y' = 8a^2$$

$$9x'^2 + 5\sqrt{3}x'y' - 3\sqrt{3}x'y' - 5y'^2 - x'^2 - 3y'^2 - 2\sqrt{3}x'y' = 8a^2$$

$$8x'^2 - 8y'^2 = 8a^2$$

(14)

$E(x,y) = 9x^2 + 24xy + 2y^2 - 6x + 20y + 41 = 0$

$$\frac{\partial E}{\partial x} = 18x + 24y - 6$$

$$\frac{\partial E}{\partial y} = 24x + 4y + 20$$

$(x, y) \equiv (-1, 1)$

Now,

Suppose,

the equation is:

$9x^2 + 24xy + 2y^2 + c = 0$

$K = 41$

$g = -3 \; [-6/2]$

$f = 10 \; [10/2]$

Here,

$c = g x_1 + f y_2 + K$

$\cancel{\{(-1)^2 \times -3\}^2}$

$= \{3 \cdot (-1)\} + \{10 \times (1)^2\} + 41$

$= 3 + 10 + 41 = 54$

$\therefore 9x^2 + 24xy + 2y^2 + 54 = 0$