Assignment 2

MAT 110 : Differential Calculus and Co-ordinate Geometry.

SET : I

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ANSWER TO THE QUESTION NO: 1

Given,

$$f(x) = \begin{cases} x^2 - 4x - 2, & x < 2 \\ -2x^4 + 4x, & x > 2 \end{cases}$$

$$f'(x) = \begin{cases} 2x - 4, & x < 2 \\ -4x + 4, & x > 2 \end{cases}$$

$\Rightarrow f'(x) = 2 \cdot 2 - 4 = 0$

$\Rightarrow \lim_{x \to 2^-} f(x) = -2 \cdot 2 + 4 = -4$

$Since, \quad f'(x) \neq \lim_{x \to 2^-} f'(x)$so, the function is not differentiable.

ANSWER TO THE QUESTION NO: 2

$$\begin{aligned} f(x) &= \frac{5x^3 - 8}{2x^3} \\ &= \frac{5x^3}{2x^3} + \frac{8}{2x^3} \\ &= \frac{5}{2} + 4x^{-3} \\ f'(x) &= \frac{d}{dx}\left(\frac{5}{2}\right) + \frac{d}{dx}\left(4x^{-3}\right) \\ &= -12x^{-4} \\ f''(x) &= \frac{d}{dx}\left(-12x^{-4}\right) \\ &= 48x^{-5} \\ f'''(x) &= 240x^{-6} \\ f''''(x) &= 1440x^{-7} \\ &= \frac{1440}{x^7} \textbf{(Ans)} \end{aligned}$$

ANSWER TO THE QUESTION NO: 3(a)

$$
\begin{aligned}
y &= \left(3x^2+2x\right)^2 \\
\Rightarrow \frac{dy}{dx} &= 2\left(3x^2+2x\right)\frac{d}{dy}\left(3x^2+2x\right) \\
&= \left(6x^2+4x\right)(6x+2) \\
&= 36x^3+24x^2+12x^2+8x \\
&= 36x^3+36x^2+8x \\
\therefore \frac{d^2y}{dx^2} &= 108x^2+72x+8\textbf{(Ans)}
\end{aligned}
$$

ANSWER TO THE QUESTION NO: 3(b)

Given that,

$$y=\left(\cos^{-1}x\right)^2$$

$$
\begin{aligned}
\frac{dy}{dx} &= \frac{d}{dx}\left(\cos^{-1}x\right)^2 \\
&= 2\cos^{-1}x\cdot\frac{d}{dx}\cos^{-1}x \\
&= 2\cos^{-1}x\left(\frac{-1}{\sqrt{1-x^2}}\right) \\
&= -2\cos^{-1}x\left(1-x^2\right)^{-\frac{1}{2}}
\end{aligned}
$$

$$\frac{d^2y}{dx^2}=\frac{d}{dx}\left[-2\cos^{-1}x\left(1-x^2\right)^{-1/2}\right]$$

$$
\begin{aligned}
&= -2\cos^{-1}x\left(-\tfrac{1}{2}\right)\left(1-x^2\right)^{-\frac{1}{2}-1}\cdot\tfrac{d}{dx}\left(1-x^2\right)+2\left(1-x^2\right)^{-\frac{1}{2}}\cdot\left(1-x^2\right)^{-\frac{1}{2}} \\
&= \tfrac{\cos^{-1}x}{\sqrt{\left(1-x^2\right)^3}}\cdot(-2x)+2\left(1-x^2\right)^{-1} \\
&= \tfrac{2}{1-x^2}-\tfrac{2x\cos^{-1}x}{\sqrt{\left(1-x^2\right)^3}}\textbf{(Ans)}
\end{aligned}
$$

ANSWER TO THE QUESTION NO: 4

$$
\begin{aligned}
f(x) &= \cos\left(\ln\frac{2}{x^3}\right) \\
f'(x) &= -\sin\left(\ln\frac{2}{x^3}\right)\cdot\frac{d}{dx}\ln\frac{2}{x^3} \\
&= -\sin\left(\ln\frac{2}{x^3}\right)\cdot\frac{x^3}{2}\cdot\frac{d}{dx}2x^{-3} \\
&= -\sin\left(\ln\frac{2}{x^3}\right)\cdot\frac{x^3}{2}\left(-6x^{-4}\right) \\
&= \sin\left(\ln\frac{2}{x^3}\right)\cdot\frac{x^3}{2}\cdot\frac{6}{x^4} \\
&= \frac{3}{x}\sin\left(\ln\frac{2}{x^3}\right)\textbf{(Ans)}
\end{aligned}
$$

ANSWER TO THE QUESTION NO: 5

$y=Ax^2+Bx+C$
$y'=2Ax+B$
$y''=2A$
$y''+y'-2y=x^2$
$2A+2Ax+B-2Ax^2+2Bx-2C=x^2$
$\Rightarrow(2A+B-2C)+x(2A-2B)-2Ax^2=x^2$
By equalizing co-efficient,
$2A+B-2C=0---(i)$

$2A-2B=0---(ii)$
$-2A=1---(iii)$
from equation(iii)
$A=-\frac{1}{2}$ **(Ans)**
from equation(ii)
$2B=2A\ B=A$
$B=-\frac{1}{2}$ **(Ans)**
from equation(i)
$2\left(-\frac{1}{2}\right)+\left(-\frac{1}{2}\right)-2C=0$
$\Rightarrow -1-\frac{1}{2}-2C=0$
$\Rightarrow 1+\frac{1}{2}+2C=0$
$\Rightarrow 2C=-\frac{3}{2}$
$\Rightarrow C=-\frac{3}{4}$ **(Ans)**

ANSWER TO THE QUESTION NO: 6

Let,

$$\begin{aligned} f(x) &= \cos\left(\frac{r}{2}\left[\frac{b^4}{4}\left(1-\frac{2\sin h^2(8\pi l_s Q)}{\sin h^2(9\pi l_s Q)}\right)\right]^{\frac{1}{4}}\right) \\ &= \cos\left[\frac{r}{2}\cdot b\cdot\left(\frac{1}{4}-\frac{\sin h^2(8\pi l_s Q)}{2\sinh^2(9\pi l_s Q)}\right)^{\frac{1}{4}}\right]---(1) \end{aligned}$$

And, $z=\left(\frac{1}{4}-\frac{\sinh^2(8\pi l_s Q)}{2\sinh^2(9\pi l_s Q)}\right)$
From equation (1),
$f(x)=\cos\left(\frac{r}{2}\cdot b\cdot z^{\frac{1}{4}}\right)$
$f'(x)=-\sin\left(\frac{r}{2}\cdot b\cdot z^{\frac{1}{4}}\right)\left(\frac{r}{2}\cdot z^{\frac{1}{4}}\right)$
$=-\left(\frac{r}{2}\cdot z^{\frac{1}{4}}\right)\sin\left(\frac{r}{2}\cdot b\cdot z^{\frac{1}{4}}\right)$ -----------(2)
$=-\frac{r}{2}\left(\frac{1}{4}-\frac{\sinh^2(8\pi l_s Q)}{2\sin^2(9\pi l_s Q)}\right)^{\frac{1}{4}}\sin\left\{\frac{r}{2}\cdot b\left(\frac{1}{4}-\frac{\sinh^2(8\pi l_s Q)}{2\sin^2(9\pi l_s Q)}\right)^{\frac{1}{4}}\right\}$ **(Ans)**
Again,
$f(x)=-\left(\frac{r}{2}\cdot z^{\frac{1}{4}}\right)\cos\left(\frac{r}{2}\cdot b\cdot z^{\frac{1}{4}}\right)\left(\frac{r}{2}\cdot z^{\frac{1}{4}}\right)----$[From equation(2)]
$\quad=-\frac{r^2}{4}\cdot z^{\frac{1}{2}}\cdot\cos\left(\frac{r}{2}\cdot b\cdot z^{\frac{1}{4}}\right)$
$=-\frac{r^2}{4}\sqrt{\left(\frac{1}{4}-\frac{\sinh^2(8\pi l_s Q)}{2\sinh^2(9\pi l_s Q)}\right)}\cos\left[\frac{r}{2}\cdot b\left(\frac{1}{4}-\frac{\sinh^2(8\pi l_s Q)}{2\sinh^2(9\pi l_s Q)}\right)^{\frac{1}{4}}\right]$ **(Ans)**