COMPUTER SCIENCE AND ENGINEERING

MAT110 : Differential Calculus and Co - ordinate Geometry
Final Exam

Submitted By

Team 'Limit doesnt exist'

Declaration

This assignment represents our own work in accordance with university regulation

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Answer to the question no 01

⇒ (a) Given that ;

$$f(x) = x\cos x$$

We know the Taylor polynomial for P(x) at $x = x_0$ as:

$$\sum_{i=0}^{n} \frac{P^{(i)}(x_0)}{i!}(x-x_0)^i = P(x_0) + P'(x_0)(x-x_0) + \frac{P''(x_0)}{2!}(x-x_0)^2 + \cdots + \frac{P^{(n)}(x_0)}{n!}(x-x_0)^n \tag{1}$$

The values of $f(x)$ and its derivatives at $x = \pi$ are as follows:

$$\begin{aligned} f(x) &= x\cos(x) & f(\pi) &= -\pi \\ f'(x) &= -x\sin x + \cos x & f'(\pi) &= -1 \\ f''(x) &= -x\cos x - 2\sin x & f''(\pi) &= -1 \\ f'''(x) &= x\sin x - 3\cos x & f'''(\pi) &= 3 \\ f^{(4)}(x) &= x\cos x + 4\sin x & f^{(4)}(\pi) &= -1 \end{aligned}$$

Thus, substituting the values into Formula yields the nth Taylor polynomial we obtain,

$$f(\pi) + f'(\pi)(x-\pi) + \frac{f''(\pi)}{2!}(x-\pi)^2 + \frac{f''(\pi)}{3!}(x-\pi)^3 + \frac{f''(\pi)}{4!}(x-\pi)^4 + \cdots$$

$$= -\pi + (-1)\cdot(x-\pi)^1 + \frac{-1}{2!}(x-\pi)^2 + \frac{3}{3!}(x-\pi)^3 + \frac{-1}{4!}(x-4) + \cdots$$

$$= -\pi - (x-\pi)^1 + -\frac{1}{2!}(x-\pi)^2 + \frac{1}{2}(x-\pi)^3 - \frac{1}{4!}(x-\pi)^4 + \cdots$$

⇒ (b) Given that ;

$$f(x) = x^7 e^x$$

We know the Taylor polynomial for P(x) at $x = x_0$ as:

$$\sum_{i=0}^{n} \frac{P^{(i)}(x_0)}{i!}(x - x_0)^i = P(x_0) + P'(x_0)(x - x_0) + \frac{P''(x_0)}{2!}(x - x_0)^2 + \cdots + \frac{P^{(n)}(x_0)}{n!}(x - x_0)^n \tag{1}$$

The values of $f(x)$ and its derivatives at $x = \pi$ are as follows:

$$f(x) = x^7 e^x \qquad f(0) = 0$$

$$f'(x) = x^7 e^x + 7x^6 e^x \qquad f'(0) = 0$$

$$f''(x) = x^7 e^x + 14x^7 e^x + 42x^5 e^x \sin x \qquad f''(0) = 0$$

Thus, substituting the values into Formula yields the nth Taylor polynomial we obtain,

$$f(0) + f'(0)(x - 0) + \frac{f''(0)}{2!}(x - 0)^2 + \frac{f''(0)}{3!}(x - 0)^3 + \frac{f''(\pi)}{4!}(x - 0)^4 + \cdots$$

$$= 0 + \cdots$$

Answer to the question no 02

⇒ (a) Given that,

$$f(x,y,z) = y^2\cos(6zx) + x^3\sin(2y-5z)$$

Therefore,

$$\begin{aligned}
f_x &= \frac{\partial}{\partial x}\{y^2\cos(6zx) + x^3\sin(2y-5z)\}\\
&= -6y^2z\sin(6zx) + 3x^2\sin(2y-5z)\\
f_{xx} &= \frac{\partial}{\partial xx}\{-6y^2z\sin(6zx) + 3x^2\sin(2y-5z)\}\\
&= -36y^2z^2\cos(6zx) + 6x\sin(2y-5z)\\
f_{xxy} &= \frac{\partial}{\partial xx\partial y}\{-36y^2z^2\cos(6zx) + 6x\sin(2y-5z)\}\\
&= -72yz^2\cos(6zx) + 12x\cos(2y-5z)
\end{aligned}$$

Again ,

$$\begin{aligned}
f_{xy} &= \frac{\partial}{\partial x\partial y}\{-6y^2z\sin(6zx) + 3x^2\sin(2y-5z)\}\\
&= -12yz\sin(6zx) + 6x^2\cos(2y-5z)
\end{aligned}$$

$$\begin{aligned}
f_{xyz} &= \frac{\partial}{\partial xx\partial y}\{-12yz\sin(6zx) + 6x^2\cos(2y-5z)\}\\
&= -72xyz\cos(6zx) - 12y\sin(6zx) + 30x^2\sin(2y-5)
\end{aligned}$$

⇒ (b) Given that,

$$f(x,y) = 4x^2 + 4xy + 4y^2 - 12x$$

Therefore ,

$$f_x = 8x + 4y - 12 = 0 \qquad (1)$$
$$fy = 4x + 8y = 0 \qquad (2)$$

Now ,

$$f_{xx} = 8$$
$$f_{yy} = 8$$
$$f_{xy} = (f_y)_x = 4$$

from equation (1),

$$\rightarrow 4x - 8y = 0$$
$$\rightarrow 4(x + 2y) = 0$$
$$\rightarrow x + 2y = 0$$
$$\rightarrow x = -2y$$

from equation (2),

$$\rightarrow 4(2x + y - 3) = 0$$
$$\rightarrow -4y + y - 3 = 0$$
$$\rightarrow -3y - 3 = 0$$
$$\rightarrow y = -1$$
$$\therefore x = 2$$

So , the critical point (2,-1)

At critieal point (2,-1),

$$f_{xx} = 8, \quad f_{yy} = 8 \quad f_{xx} \cdot f_{yy} = 64$$
$$(f_{xy})^2 = (4)^2 = 16$$
$$\therefore (f_{xy})^2 < f_{xx} \cdot f_{yy}$$

There is a relative minima at (2,-1)

<u>Answer to the question no 03</u>

→(a)Given ,

$$\begin{aligned}
&f(x,y) = e^{3x}\cos 2y\\
&f_x(0,0) = \cos 2y \cdot e^x \cdot 3 = 3\\
&f_{xy}(0,0) = -2e^{3x}\sin(2y) = 0\\
&f_{xx}(0,0) = 9e^{3x}\cdot\cos(2y) = 9\\
&f_{yy}(0,0) = -4e^{3x}\cos 2y = -4\\
&f_{xy}(0,0) = -6e^{3x}\sin 2y = 0\\
&f(0,0) = e^{3.0}\cdot\cos 2\cdot 0 = 1
\end{aligned}$$

$$\begin{aligned}
L(x,y) &= f(0.0) + f_x(0,0)\cdot(x-0) + f_y(0,0)(y-0)\\
&= 1+3+0 = 4
\end{aligned}$$

$$\begin{aligned}
Q(x,y) = L(x,y) + &\frac{f_x(0,0)}{2!}(x-0)^2 + f_{xy}(0,0)\\
&(x-0)(y-0) + \frac{f_{xy}(0,0)}{2!}(y-0)^2
\end{aligned}$$

$$= \frac{9}{2}x^2 - 2y^2 + xy$$

So , the first and second degree taylor polynomial approximation are 4 and $\frac{9}{2}x^2 - 2y^2 + xy$

⇒ (b) Given that,

$$f(x,y) = \cos x \cdot \cos y$$

We know the taylore expansion of multivariable function as

$$f(x,y) = \sum_{n=0}^{\infty}\frac{1}{n!}\left((x-0)\frac{\partial}{\partial x} + (y-0)\frac{\partial}{\partial y}\right)^n \cdot f(x,y)$$

Therefore,

$$f_x(x,y) = -\cos y\sin x \qquad f_x(0,0) = 0$$

$$f_y(x, y) = -\cos x \sin y \qquad f_y(0, 0) = 0$$

$$f_{xx}(x, y) = -\cos x \cos y \qquad f_{xx}(0, 0) = -1$$

$$f_{yy}(x, y) = -\cos x \cos y \qquad f_{yy}(0, 0) = -1$$

$$f_{xy}(x, y) = \sin x \sin y \qquad f_{xy}(0, 0) = 0$$

Now substituting the values into the formula we obtain,

$$\begin{aligned} L(x, y) &= f(0.0) + f_x(0, 0) \cdot (x - 0) + f_y(0, 0)(y - 0) \\ &= 0 + 0 \cdot x + 0.y = 0 \end{aligned}$$

$$\begin{aligned} Q(x, y) &= L(x, y) + \frac{f_x(0, 0)}{2!}(x - 0)^2 + f_{xy}(0, 0) \\ &\qquad (x - 0)(y - 0) + \frac{f_{xy}(0, 0)}{2!}(y - 0)^2 \\ &= 0 + \frac{-1}{2!}x^2 + (0) \cdot x \cdot y + \frac{-1}{2!}y^2 \\ &= -\left(\frac{1}{2}x^2 + \frac{1}{2}y^2\right) \end{aligned}$$

So , the first and second degree taylor polynomial approximation are 0 and $-\left(\frac{1}{2}x^2 + \frac{1}{2}y^2\right)$

Answer to the question no 04(a)

$$\vec{F} = (4z - cos(2x))\vec{i} - z^3e^{5x}\vec{j} + (y^3 + 8z^2)\vec{k}$$

$$\text{div } \vec{V} = \vec{\nabla}.\vec{F}$$

$$= (\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}).((4z - cos(2x))\vec{i} - z^3e^{5x}\vec{j} + (y^3 + 8z^2)\vec{k})$$

$$= \frac{\partial}{\partial x}(4z - cos(2x)) + \frac{\partial}{\partial y}(-z^3e^{5x}) + \frac{\partial}{\partial z}(y^3 + 8z^2)$$

$$= 2sin(2x) + 16z$$

The Divergence of the following vector $\vec{F} = 2sin(2x) + 16z$

Again,

$$\text{Curl,}\vec{\nabla} \times \vec{F} =$$

$$(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}) \times ((4z - cos(2x))\vec{i} - z^3e^{5x}\vec{j} + (y^3 + 8z^2)\vec{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial 2} \\ (4z - cos(2x)) & (-z^3e^{5x}) & (y^3 + 8z^2) \end{vmatrix}$$

$$= \vec{i}(\frac{\partial}{\partial y}(y^3 + 8z^2) + \frac{\partial}{\partial z}(z^3e^{5x})) - \vec{j}(\frac{\partial}{\partial x}(y^3 + 8z^2) - \frac{\partial}{\partial z}(4z - cos(2x))) + \vec{k}(\frac{\partial}{\partial x}(-z^3e^{5x}) - \frac{\partial}{\partial y}(4z - cos(2x)))$$

$$= \vec{i}(3y^2 + 3e^{5x}z^2) + 4\vec{j} - 5e^{5x}z^3\vec{k}$$

$$= 3y^2\vec{i} + 3e^{5x}z^2\vec{i} + 4\vec{j} - 5e^{5x}z^3\vec{k}$$

The Divergence and Curl of the following vector

$$\vec{F} = 3y^2\vec{i} + 3e^{5x}z^2\vec{i} + 4\vec{j} - 5e^{5x}z^3\vec{k}$$

Answer to the question no 04(b)

Here,

$\vec{F} = -(4y+z)\vec{i} - y^2 sinx\vec{j} + (3x+3y)\vec{k}$

$div\ \vec{V} = \vec{\nabla}.\vec{F}$

$= (\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}).(-(4y+z)\vec{i} - y^2 sinx\vec{j} + (3x+3y)\vec{k})$

$= \frac{\partial}{\partial x}(-(4y+z)) + \frac{\partial}{\partial y}(y^2 sinx) + \frac{\partial}{\partial z}(3x+3y)$

$= 2ysinx$

The Divergence of the following vector $\vec{F} = 2ysinx$

Again,

Curl,$\vec{\nabla} \times \vec{F} =$
$(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}) \times ((4z - cos(2x))\vec{i} - z^3 e^{5x}\vec{j} + (y^3 + 8z^2)\vec{k})$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -(4y+z) & y^2 \sin x & 3x+3y \end{vmatrix}$$

$= \vec{i}(\frac{\partial}{\partial y}(3x+3y) - \frac{\partial}{\partial z}(y^2 sinx)) - \vec{j}(\frac{\partial}{\partial x}(3x+3y) - \frac{\partial}{\partial z}(-4y-z) +$
$\vec{k}(\frac{\partial}{\partial x}(y^2 sinx) - \frac{\partial}{\partial y}(-4y-z)$

$= 3\vec{i} - (3+1)\vec{j} + (y^2 cosx + 4)\vec{k}$

$= 3\vec{i} - 4\vec{j} + (y^2 cosx + 4)\vec{k}$

The Curl of the following vector $\vec{F} = 3\vec{i} - 4\vec{j} + (y^2 cosx + 4)\vec{k}$

Answer to the question no 05(a)

(a)
Given equation,
$2x^2 - 4x + 2y^2 + 6y - 10 = 0$
$\rightarrow 2x^2 - 4x + 2y^2 + 6y = 10$
$\rightarrow x^2 - 2x + y^2 + 3y = 5$
$\rightarrow x^2 - 2.1.x + 1^2 + y^2 + 2\frac{3}{2}y + \frac{9}{4} = 5 + 1 + \frac{9}{4}$
$\rightarrow (x-1)^2 + (y+\frac{3}{2})^2 = \frac{33}{4}$
$\rightarrow \frac{(x-1)^2}{\frac{33}{4}} + \frac{(y-(-\frac{3}{2})^2)}{\frac{33}{4}} = 1$
The standard form of the equation of the ellipse:
$\frac{(x-1)^2}{\frac{33}{4}} + \frac{(y-(-\frac{3}{2}))^2}{\frac{33}{4}} = 1$

(b)
Given equation,

$$32x^2 - 2y^2 - 64x - 12y = 114$$

$$\rightarrow 16x^2 - y^2 - 32x - 6y = 57$$

$$\rightarrow (4x)^2 - 2.4.4x + (4)^2 - (y^2 + 2.3y + 3^2) = 57 + 4^2 - 3^2$$

$$\rightarrow (4x-4)^2 - (y+3)^2 = 64$$

$$\rightarrow \frac{16(x-1)^2}{64} - \frac{(y-(-3))^2}{64} =$$

$$\rightarrow \frac{(x-1)^2}{2^2} - \frac{(y-(-3))^2}{8^2} = 1$$

The standard form of the equation of the hyper-bola:

$$\frac{(x-1)^2}{2^2} - \frac{(y-(-3))^2}{8^2} = 1$$

The End