

Vectors

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Vector Diagram

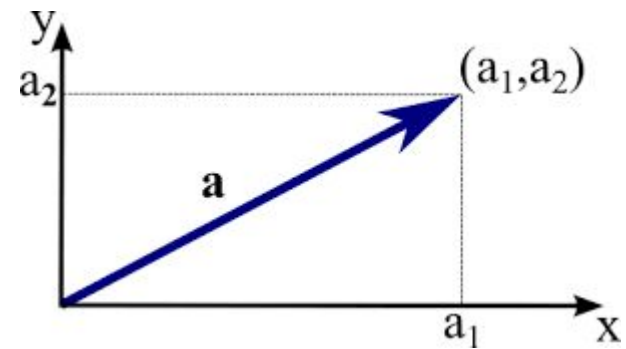
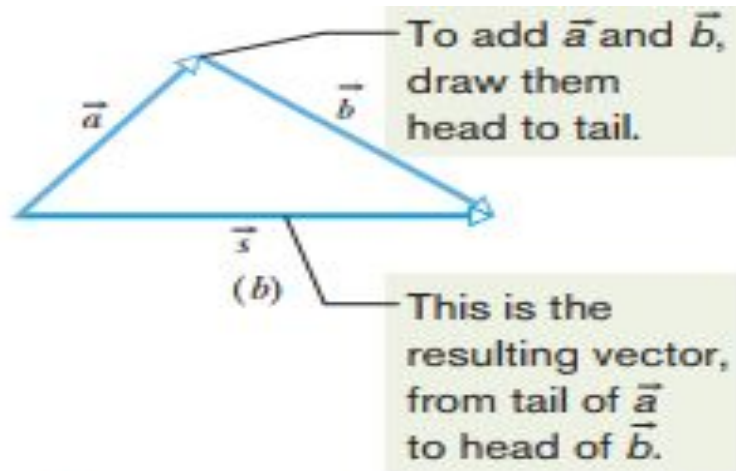
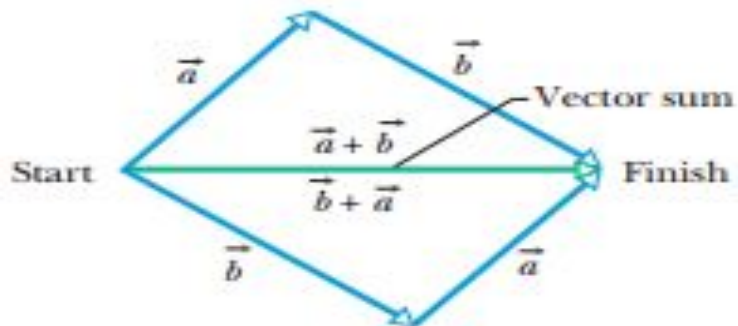


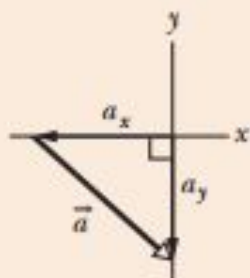
Fig. 3-2 (a) AC is the vector sum of the vectors AB and BC . (b) The same vectors relabeled.



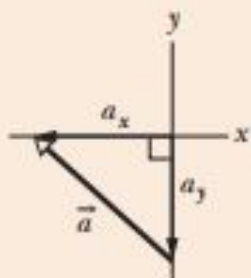


CHECKPOINT 2

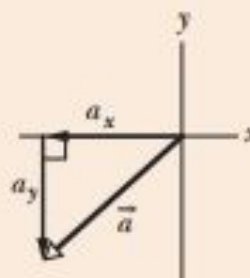
In the figure, which of the indicated methods for combining the x and y components of vector \vec{a} are proper to determine that vector?



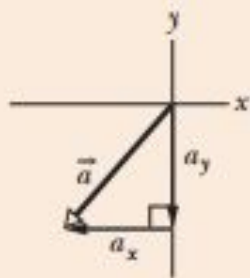
(a)



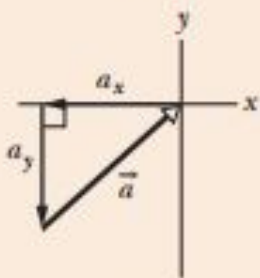
(b)



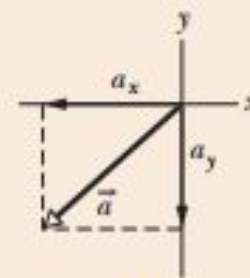
(c)



(d)

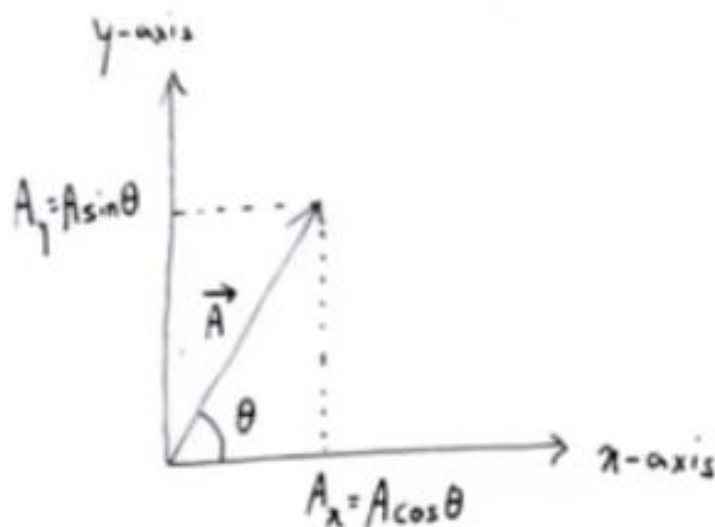


(e)



(f)

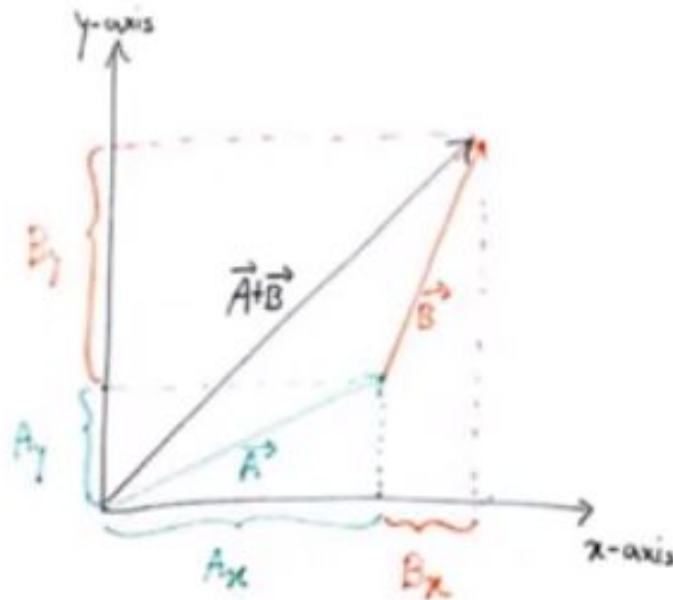
- Once you select a coordinate system, resolving is only a matter of trigonometry



- After resolving into components, the vector can be expressed *in the coordinate system* as $\vec{A} = A_x \hat{i} + A_y \hat{j}$ or simply $\vec{A} = \begin{bmatrix} A_x \\ A_y \end{bmatrix}$

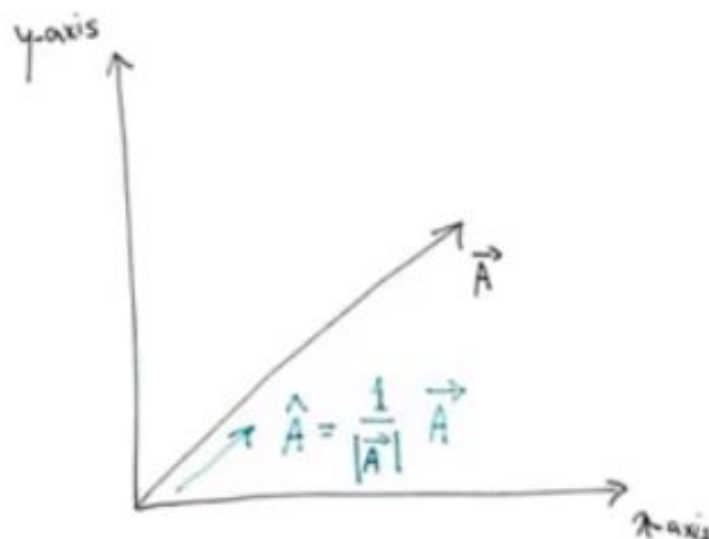


- Among other things, selecting a coordinate system simplifies the job of adding vectors.



- Hence, for $\vec{A} = A_x\hat{i} + A_y\hat{j}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j}$;
 $\vec{A} + \vec{B} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j}$

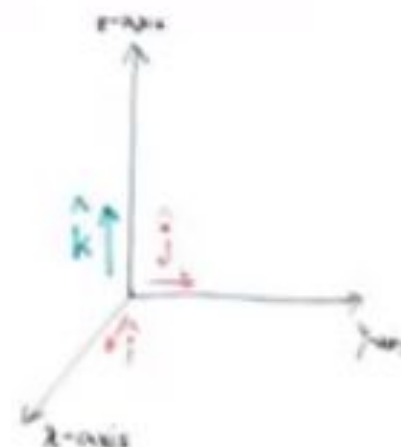
$$\vec{A} + \vec{B} = (A\cos\theta_1 + B\cos\theta_2)\hat{i} + (A\sin\theta_1 + B\sin\theta_2)\hat{j}$$



$$\hat{A} = \frac{1}{|\vec{A}|} \vec{A} = \left(\frac{1}{\sqrt{A_x^2 + A_y^2}} \right) (A_x \hat{i} + A_y \hat{j})$$

Use this unit vector \hat{A} to talk about the direction in which \vec{A} points.

- Resolving a vector in three dimensions is now intuitive



- $\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$
- Norm: $|\vec{A}| = A = \sqrt{A_x^2 + A_y^2 + A_z^2}$
- Unit vector: $\hat{A} = \left(\frac{1}{\sqrt{A_x^2 + A_y^2 + A_z^2}} \right) (A_x \hat{i} + A_y \hat{j} + A_z \hat{k})$

Angle measurement

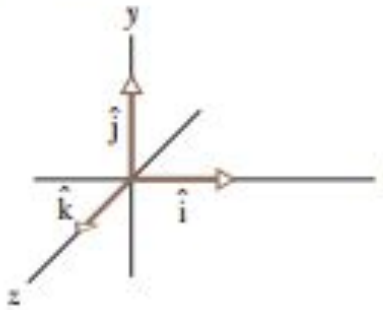
Tactic 1: Angles—Degrees and Radians Angles that are measured relative to the positive direction of the x axis are positive if they are measured in the counterclockwise direction and negative if measured clockwise. For example, 210° and -150° are the same angle.

Angles may be measured in degrees or radians (rad). To relate the two measures, recall that a full circle is 360° and 2π rad. To convert, say, 40° to radians, write

$$40^\circ \frac{2\pi \text{ rad}}{360^\circ} = 0.70 \text{ rad.}$$

Unit Vector and Vector Addition

The unit vectors point along axes.



$$\vec{a} = a_x \hat{i} + a_y \hat{j} \quad (3-7)$$

and

$$\vec{b} = b_x \hat{i} + b_y \hat{j}. \quad (3-8)$$

These two equations are illustrated in Fig. 3-14. The quantities $a_x \hat{i}$ and $a_y \hat{j}$ are vectors, called the **vector components** of \vec{a} . The quantities a_x and a_y are scalars, called the **scalar components** of \vec{a} (or, as before, simply its **components**).

To start, consider the statement

$$\vec{r} = \vec{a} + \vec{b}, \quad (3-9)$$

which says that the vector \vec{r} is the same as the vector $(\vec{a} + \vec{b})$. Thus, each component of \vec{r} must be the same as the corresponding component of $(\vec{a} + \vec{b})$:

$$r_x = a_x + b_x \quad (3-10)$$

$$r_y = a_y + b_y \quad (3-11)$$

$$r_z = a_z + b_z. \quad (3-12)$$

Adding vectors, unit-vector components

Figure 3-15a shows the following three vectors:

$$\vec{a} = (4.2 \text{ m})\hat{i} - (1.5 \text{ m})\hat{j},$$

$$\vec{b} = (-1.6 \text{ m})\hat{i} + (2.9 \text{ m})\hat{j},$$

and

$$\vec{c} = (-3.7 \text{ m})\hat{j}.$$

What is their vector sum \vec{r} which is also shown?

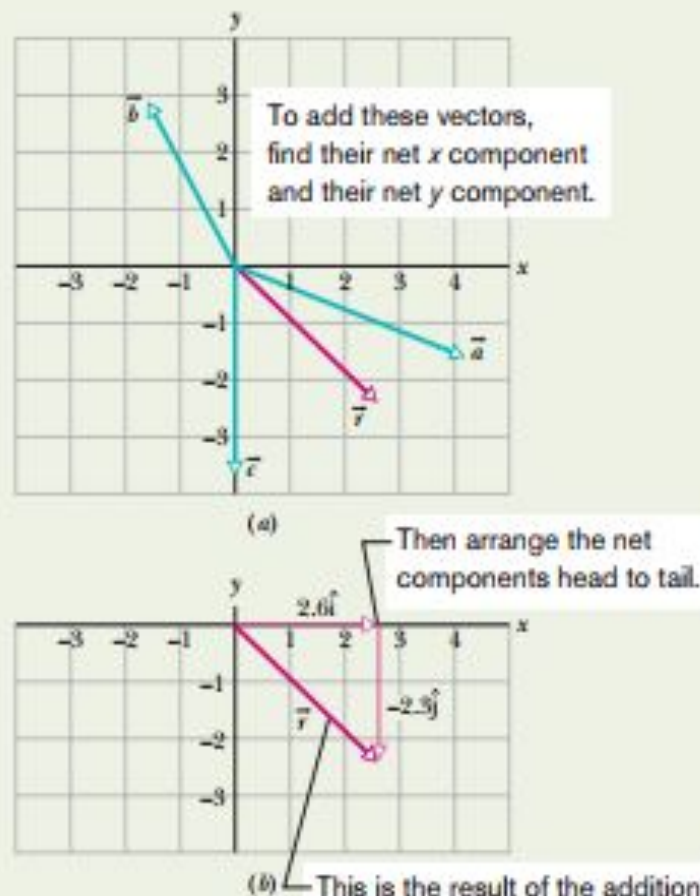


Fig. 3-15 Vector \vec{r} is the vector sum of the other three vectors.

KEY IDEA

We can add the three vectors by components, axis by axis, and then combine the components to write the vector sum \vec{r} .

Calculations: For the x axis, we add the x components of \vec{a} , \vec{b} , and \vec{c} , to get the x component of the vector sum \vec{r} :

$$\begin{aligned} r_x &= a_x + b_x + c_x \\ &= 4.2 \text{ m} - 1.6 \text{ m} + 0 = 2.6 \text{ m}. \end{aligned}$$

Similarly, for the y axis,

$$\begin{aligned} r_y &= a_y + b_y + c_y \\ &= -1.5 \text{ m} + 2.9 \text{ m} - 3.7 \text{ m} = -2.3 \text{ m}. \end{aligned}$$

We then combine these components of \vec{r} to write the vector in unit-vector notation:

$$\vec{r} = (2.6 \text{ m})\hat{i} - (2.3 \text{ m})\hat{j}, \quad (\text{Answer})$$

where $(2.6 \text{ m})\hat{i}$ is the vector component of \vec{r} along the x axis and $-(2.3 \text{ m})\hat{j}$ is that along the y axis. Figure 3-15b shows one way to arrange these vector components to form \vec{r} . (Can you sketch the other way?)

We can also answer the question by giving the magnitude and an angle for \vec{r} . From Eq. 3-6, the magnitude is

$$r = \sqrt{(2.6 \text{ m})^2 + (-2.3 \text{ m})^2} \approx 3.5 \text{ m} \quad (\text{Answer})$$

and the angle (measured from the $+x$ direction) is

$$\theta = \tan^{-1}\left(\frac{-2.3 \text{ m}}{2.6 \text{ m}}\right) = -41^\circ, \quad (\text{Answer})$$

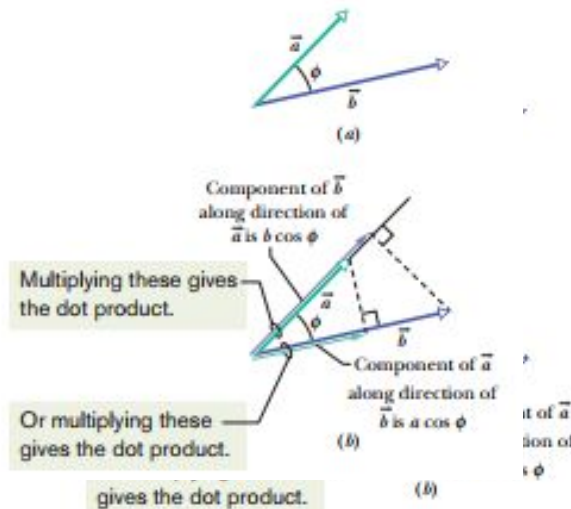
where the minus sign means clockwise.

Vector Multiplication

The Scalar Product

The scalar product of the vectors \vec{a} and \vec{b} in Fig. 3-18a is written as $\vec{a} \cdot \vec{b}$ and defined to be

$$\vec{a} \cdot \vec{b} = ab \cos \phi, \quad (3-20)$$

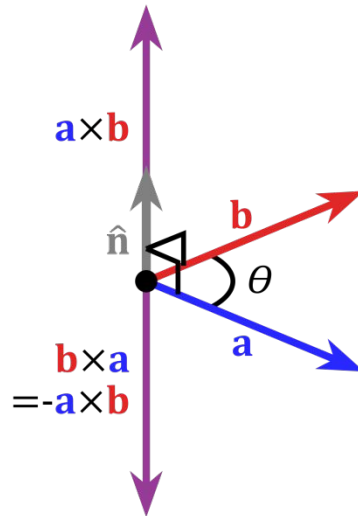


Vector Cross Product Formula

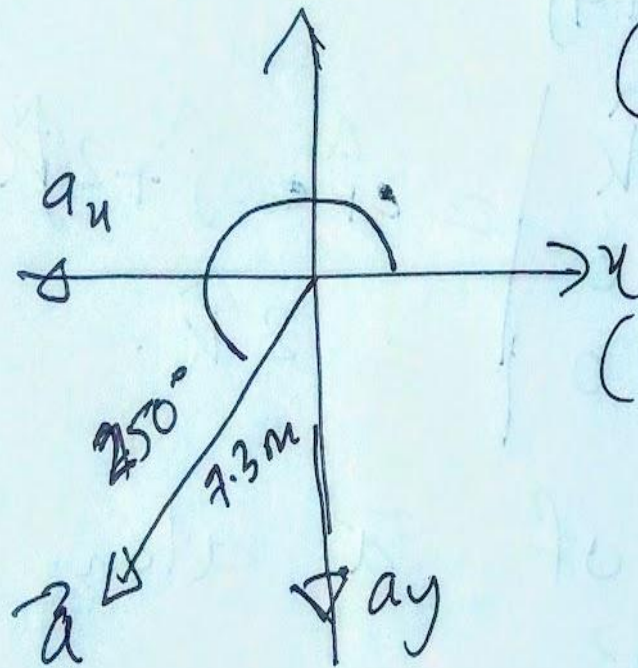


$$\vec{a} \times \vec{b} = |\vec{a}| |\vec{b}| \sin \theta \hat{n}$$

$$\vec{a} \times \vec{b} = i (a_2 b_3 - a_3 b_2) + j (a_3 b_1 - a_1 b_3) + k (a_1 b_2 - a_2 b_1)$$



•1 SSM What are (a) the x component and (b) the y component of a vector \vec{a} in the xy plane if its direction is 250° counterclockwise from the positive direction of the x axis and its magnitude is 7.3 m ?



$$(a) \ a_x = |\vec{a}| \cos \theta = 7.3 \cos 250^\circ = -2.5 \text{ m}$$

$$(b) \ a_y = |\vec{a}| \sin \theta = 7.3 \sin 250^\circ = -6.86 \text{ m}$$

•3 SSM The x component of vector \vec{A} is -25.0 m and the y component is $+40.0$ m.

Problem 2.

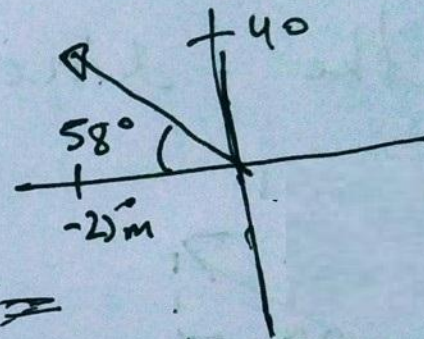
(a) What is the magnitude of \vec{A} ? (b) What is the angle between the direction of \vec{A} and the positive direction of x?

$$\vec{A} = -25\hat{i} + 40\hat{j}$$

$$(a) |\vec{A}| = \sqrt{(-25)^2 + (40)^2}$$

$$(b) \tan \theta = \frac{A_y}{A_x} = \frac{40\text{ m}}{25\text{ m}} = 58^\circ$$

$$\text{Actual Angle} = 180^\circ - 58^\circ = 122^\circ$$



22 (a) What is the sum of the following four vectors in unit-vector notation? For that sum, what are (b) the magnitude, (c) angle in degrees, and (d) the angle in radians?

$$\vec{E}: 6.00 \text{ m at } +0.900 \text{ rad} \quad \vec{F}: 5.00 \text{ m at } -75.0^\circ$$

$$\vec{G}: 4.00 \text{ m at } +1.20 \text{ rad} \quad \vec{H}: 6.00 \text{ m at } -210^\circ$$

$$\checkmark \vec{E} = 6 \text{ m at } +0.9 \text{ rad}$$

$$\theta = \frac{0.9}{2\pi} \times 360^\circ = 51.5^\circ$$

$$\begin{aligned} E &= 6 \cos 51.5^\circ \hat{i} + 6 \sin 51.5^\circ \hat{j} \\ &= 3.73 \hat{i} + 4.77 \hat{j} \end{aligned}$$

$$\checkmark \vec{F} = 5 \text{ m at } -75^\circ$$

$$\begin{aligned} F &= 5 \times \cos(-75^\circ) \hat{i} + 5 \sin(-75^\circ) \hat{j} \\ &= 1.29 \hat{i} - 4.82 \hat{j} \end{aligned}$$

$$\checkmark \vec{G} = 4 \text{ m at } 1.2 \text{ rad}$$

$$\begin{aligned} \theta &= \frac{1.2}{2\pi} \times 360^\circ = 68.75^\circ, \quad \vec{G} = 4(\cos 68.7^\circ \hat{i} + \sin 68.7^\circ \hat{j}) \\ &= 1.45 \hat{i} + 3.73 \hat{j} \end{aligned}$$

$$\checkmark \vec{H} = 6 \text{ m at } (-210^\circ)$$

$$\begin{aligned} H &= 6 \cos(-210^\circ) \hat{i} + 6 \sin(-210^\circ) \hat{j} \\ &= -5.20 \hat{i} + 3 \hat{j} \end{aligned}$$

$$\text{Sum} = \vec{E} + \vec{F} + \vec{G} + \vec{H}$$

$$= 1.27 \hat{i} + 6.61 \hat{j}$$

$$\begin{aligned} \text{(a)} \quad 6.73 \text{ m} \quad \text{(b)} \quad \tan \theta &= \frac{6.61}{1.27} \\ \theta &= 79.1^\circ \end{aligned}$$

•15 SSM ILW WWW The two vectors \vec{a} and \vec{b} in Fig. 3-28 have equal magnitudes of 10.0 m and the angles are $\theta_1 = 30^\circ$ and $\theta_2 = 105^\circ$. Find the (a) x and (b) y components of their vector sum \vec{r} , (c) the magnitude of \vec{r} , and (d) the angle \vec{r} makes with the positive direction of the x axis.

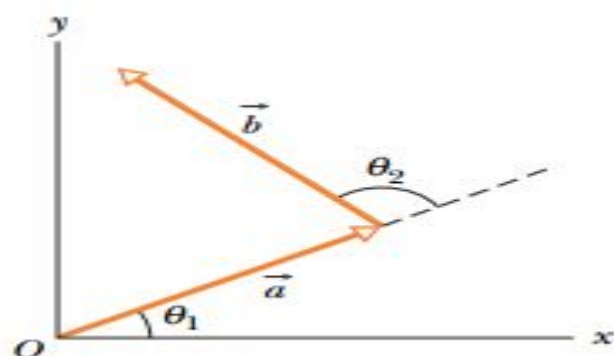
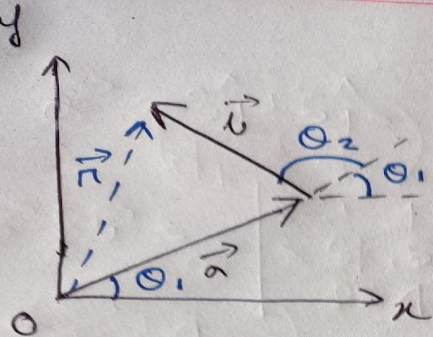


Fig. 3-28 Problem 15.



$$\vec{a} = |\vec{a}| [\cos \theta_1 \hat{i} + \sin \theta_1 \hat{j}]$$

$$= 10 [\cos 30^\circ \hat{i} + \sin 30^\circ \hat{j}] = 8.66 \hat{i} + 5 \hat{j} \text{ m}$$

$$\vec{b} = |\vec{b}| [\cos (\theta_1 + \theta_2) \hat{i} + \sin (\theta_1 + \theta_2) \hat{j}]$$

$$= 10 [\cos (30^\circ + 105^\circ) \hat{i} + \sin (30^\circ + 105^\circ) \hat{j}]$$

$$= [-7.07 \hat{i} + 7.07 \hat{j}] \text{ m}$$

$$\begin{aligned} \vec{r} &= \vec{a} + \vec{b} = 8.66 \hat{i} + 5 \hat{j} - 7.07 \hat{i} + 7.07 \hat{j} \\ &= [1.59 \hat{i} + 12.07 \hat{j}] \text{ m} \end{aligned}$$

a) 1.59 m

b) 12.07 m

c) $|\vec{r}| = \sqrt{(1.59)^2 + (12.07)^2} = 12.17 \text{ m}$

d) $\tan \theta = \frac{12.07}{1.59}$
 $= 82.5^\circ$

•34 Two vectors are presented as

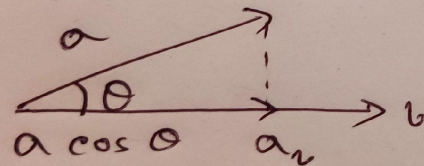
$$\vec{a} = 3.0\hat{i} + 5.0\hat{j} \quad \text{and} \quad \vec{b} = 2.0\hat{i} + 4.0\hat{j}.$$

Find (a) $\vec{a} \times \vec{b}$, (b) $\vec{a} \cdot \vec{b}$, (c) $(\vec{a} + \vec{b}) \cdot \vec{b}$, and (d) the component of \vec{a} along the direction of \vec{b} . (Hint: For (d), consider Eq. 3-20 and Fig. 3-18.)

$$\vec{a} = 3\hat{i} + 5\hat{j} \quad \vec{b} = 2\hat{i} + 4\hat{j}$$

$$\begin{aligned} \text{a) } \vec{a} \times \vec{b} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 5 & 0 \\ 2 & 4 & 0 \end{vmatrix} \\ &= \hat{i}(3 \times 0 - 4 \times 0) - \hat{j}(3 \times 0 - 2 \times 0) \\ &\quad + \hat{k}(3 \times 4 - 2 \times 5) \\ &= 0\hat{i} - 0\hat{j} + 2\hat{k} \end{aligned}$$

d) The component of \vec{a} along the direction of \vec{b}



$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$$

$$a_b = a \cos \theta = \frac{\vec{a} \cdot \vec{b}}{b}$$

$$= \frac{(3\hat{i} + 5\hat{j}) \cdot (2\hat{i} + 4\hat{j})}{\sqrt{2^2 + 4^2}}$$

$$= 5.81$$

*36 If $\vec{d}_1 = 3\hat{i} - 2\hat{j} + 4\hat{k}$ and $\vec{d}_2 = -5\hat{i} + 2\hat{j} - \hat{k}$, then what is $(\vec{d}_1 + \vec{d}_2) \cdot (\vec{d}_1 \times 4\vec{d}_2)$?

$$(\vec{d}_1 + \vec{d}_2) \cdot (\vec{d}_1 \times 4\vec{d}_2) = 0$$

Since $(\vec{d}_1 \times 4\vec{d}_2)$ is perpendicular to $(\vec{d}_1 + \vec{d}_2)$, so the dot product

$$[\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos 90^\circ] \text{ is zero}$$

$$[\cos 90^\circ = 0]$$

Vector \vec{A} has a magnitude of 50 cm and a direction of 30° , and vector \vec{B} has a magnitude of 35 cm and a direction of 110° . Both angles are measured counterclockwise from the positive x axis. Use components to calculate the magnitude and direction of the vector sum (i.e., the resultant) $\vec{R} = \vec{A} + \vec{B}$.

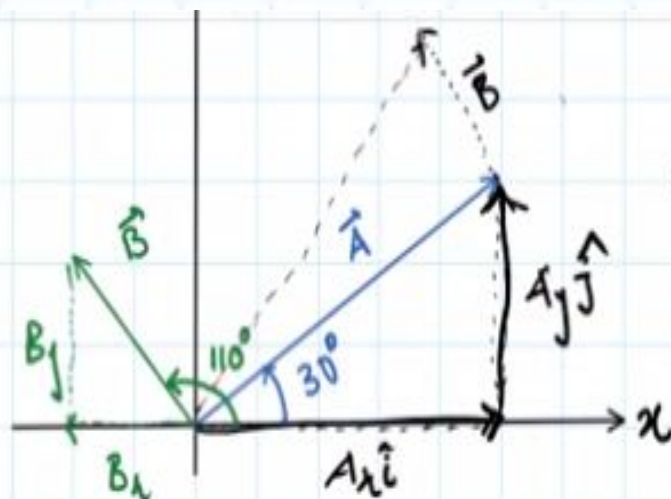
Solⁿ:-

$$\vec{A} = A_x \hat{i} + A_y \hat{j}$$

$$\Rightarrow \vec{A} = A \cos \theta \hat{i} + A \sin \theta \hat{j}$$

$$\Rightarrow \vec{A} = 0.5 \cos 30^\circ \hat{i} + 0.5 \sin 30^\circ \hat{j}$$

$$\Rightarrow \boxed{\vec{A} = 0.43 \hat{i} + 0.25 \hat{j}}$$



$$A_x \hat{i} = A \cos \theta \hat{i}$$

$$\frac{A_x}{A} = \cos \theta = \frac{A_x}{A}$$

$$\boxed{A_x = A \cos \theta}$$

$$\frac{A_y}{A} = \sin \theta = \frac{A_y}{A}$$

$$\Rightarrow \boxed{A_y = A \sin \theta}$$

$$\vec{B} = B \cos \theta \hat{i} + B \sin \theta \hat{j}$$

$$\Rightarrow \vec{B} = 0.35 \cos 110^\circ \hat{i} + 0.35 \sin 110^\circ \hat{j}$$

$$\Rightarrow \vec{B} = 0.35 \cos 110^\circ \hat{i} + 0.35 \sin 110^\circ \hat{j}$$

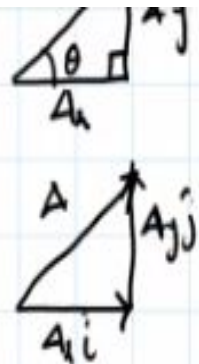
$$\Rightarrow \boxed{\vec{B} = -0.119 \hat{i} + 0.32 \hat{j}}$$

$$\text{Now, } \vec{R} = \vec{A} + \vec{B} = 0.43 \hat{i} + 0.24 \hat{j} - 0.119 \hat{i} + 0.32 \hat{j}$$

$$\Rightarrow \boxed{\vec{R} = 0.311 \hat{i} + 0.57 \hat{j}}$$

$$|R| = \sqrt{(0.311)^2 + (0.57)^2} = 0.64 \text{ m}$$

$$\theta = \tan^{-1} \frac{0.57}{0.311} = 61.38^\circ$$



$$\vec{A} = A_x \hat{i} + A_y \hat{j}$$

$$|A| = \sqrt{A_x^2 + A_y^2}$$

$$\theta = \tan^{-1} \frac{A_y}{A_x}$$

Problem 7 (Finding component using multiplication)

In unit vector notation, find (a) $\vec{r} = \vec{a} - \vec{b} + \vec{c}$ if $\vec{a} = 5.0\hat{i} + 4.0\hat{j} - 6.0\hat{k}$, $\vec{b} = -2.0\hat{i} + 2.0\hat{j} + 3.0\hat{k}$, and $\vec{c} = 4.0\hat{i} + 3.0\hat{j} + 2.0\hat{k}$?

(b) Calculate the angle between \vec{r} and $+z$ axis.

(c) What is the component of \vec{a} along the direction of \vec{b} ?

(d) What is the component of \vec{a} perpendicular to the direction of \vec{b} but in the plane of \vec{a} and \vec{b} ?

Calculation: (a)

$$\begin{aligned}\vec{r} &= \vec{a} - \vec{b} + \vec{c} \\ &= (5 - (-2) + 4)\hat{i} + (4 - 2 + 3)\hat{j} + (-6 - 3 + 2)\hat{k} \\ &= 11\hat{i} + 5\hat{j} - 7\hat{k} \text{ (answer)}\end{aligned}$$

Calculation: (b)

To calculate the angle from $+z$ axis, we will take scalar product of \vec{r} with \hat{k} .

$$\vec{r} \cdot \hat{k} = (r)1 \cos \theta$$

$$\text{Or } (11\hat{i} + 5\hat{j} - 7\hat{k}) \cdot \hat{k} = (\sqrt{11^2 + 5^2 + (-7)^2})(1) \cos \theta$$

$$\text{Or } -7 = (14)(1) \cos \theta \text{ or } \theta = \cos^{-1}(-0.5) = 120^\circ \text{ (answer)}$$

Calculation: (c) It is the very similar problem discussed in Problem 5 (d).

The component of \vec{a} along the direction of \vec{b} is

$$\begin{aligned}a_b &= a \cos \theta \\ &= \vec{a} \cdot \vec{b} / b \\ &= \frac{5(-2) + 4(2) + (-6)3}{\sqrt{(-2)^2 + 2^2 + 3^2}} \\ &= -4.9 \text{ (answer)}\end{aligned}$$



Problem 7 (Continued)

In unit vector notation, find (a) $\vec{r} = \vec{a} - \vec{b} + \vec{c}$ if $\vec{a} = 5.0\hat{i} + 4.0\hat{j} - 6.0\hat{k}$,
 $\vec{b} = -2.0\hat{i} + 2.0\hat{j} + 3.0\hat{k}$, and $\vec{c} = 4.0\hat{i} + 3.0\hat{j} + 2.0\hat{k}$?

(b) Calculate the angle between \vec{r} and $+z$ axis.

(c) What is the component of \vec{a} along the direction of \vec{b} ?

(d) What is the component of \vec{a} perpendicular to the direction of \vec{b} but in the plane of \vec{a} and \vec{b} ?

We note that $a \cos \theta$ gives the component of \vec{a} along the direction of \vec{b} in part(c) then we can say $a \sin \theta$ will yield the orthogonal component.

Calculation: (d) The magnitude of vector product

$$|\vec{a} \times \vec{b}| = ab \sin \theta$$

$$\begin{aligned} \text{Or } a \sin \theta &= \frac{|\vec{a} \times \vec{b}|}{b} = \frac{\sqrt{(24)^2 + (-3)^2 + (18)^2}}{\sqrt{(-2)^2 + 2^2 + 3^2}} \\ &= \frac{\sqrt{909}}{\sqrt{17}} \\ &= 7.3 \text{ (answer)} \end{aligned}$$

$$\begin{aligned} \vec{a} \times \vec{b} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 4 & -6 \\ -2 & 2 & 3 \end{vmatrix} \\ &= \hat{i}(4(3) - 2(-6)) - \hat{j}(5(3) - (-2)(-6)) + \hat{k}(5(2) - (-2)(4)) \\ &= (12 + 12)\hat{i} - (15 - 12)\hat{j} + (10 + 8)\hat{k} \\ &= 24\hat{i} - 3\hat{j} + 18\hat{k} \end{aligned}$$



CHECKPOINT 4

Vectors \vec{C} and \vec{D} have magnitudes of 3 units and 4 units, respectively. What is the angle between the directions of \vec{C} and \vec{D} if $\vec{C} \cdot \vec{D}$ equals (a) zero, (b) 12 units, and (c) -12 units?

Sample Problem

Adding vectors by components, desert ant

The desert ant *Cataglyphis fortis* lives in the plains of the Sahara desert. When one of the ants forages for food, it travels from its home nest along a haphazard search path, over flat, featureless sand that contains no landmarks. Yet, when the ant decides to return home, it turns and then runs directly home. According to experiments, the ant keeps track of its movements along a mental coordinate system. When it wants to return to its home nest, it effectively sums its displacements along the axes of the system to calculate a vector that points directly home. As an example of the calculation, let's consider an ant making five runs of 6.0 cm each on an xy coordinate system, in the directions shown in

Fig. 3-16a, starting from home. At the end of the fifth run, what are the magnitude and angle of the ant's net displacement vector \vec{d}_{net} , and what are those of the homeward vector \vec{d}_{home} that extends from the ant's final position back to home? In a real situation, such vector calculations might involve thousands of such runs.



KEY IDEAS

(1) To find the net displacement \vec{d}_{net} , we need to sum the five individual displacement vectors:

$$\vec{d}_{\text{net}} = \vec{d}_1 + \vec{d}_2 + \vec{d}_3 + \vec{d}_4 + \vec{d}_5.$$

(2) We evaluate this sum for the x components alone,

$$d_{\text{net},x} = d_{1x} + d_{2x} + d_{3x} + d_{4x} + d_{5x}, \tag{3-14}$$

and for the y components alone,

$$d_{\text{net},y} = d_{1y} + d_{2y} + d_{3y} + d_{4y} + d_{5y}. \tag{3-15}$$

(3) We construct \vec{d}_{net} from its x and y components.

Calculations: To evaluate Eq. 3-14, we apply the x part of Eq. 3-5 to each run:

$$\begin{aligned} d_{1x} &= (6.0 \text{ cm}) \cos 0^\circ = +6.0 \text{ cm} \\ d_{2x} &= (6.0 \text{ cm}) \cos 150^\circ = -5.2 \text{ cm} \\ d_{3x} &= (6.0 \text{ cm}) \cos 180^\circ = -6.0 \text{ cm} \\ d_{4x} &= (6.0 \text{ cm}) \cos(-120^\circ) = -3.0 \text{ cm} \\ d_{5x} &= (6.0 \text{ cm}) \cos 90^\circ = 0. \end{aligned}$$

Equation 3-14 then gives us

$$\begin{aligned} d_{\text{net},x} &= +6.0 \text{ cm} + (-5.2 \text{ cm}) + (-6.0 \text{ cm}) \\ &\quad + (-3.0 \text{ cm}) + 0 \\ &= -8.2 \text{ cm}. \end{aligned}$$

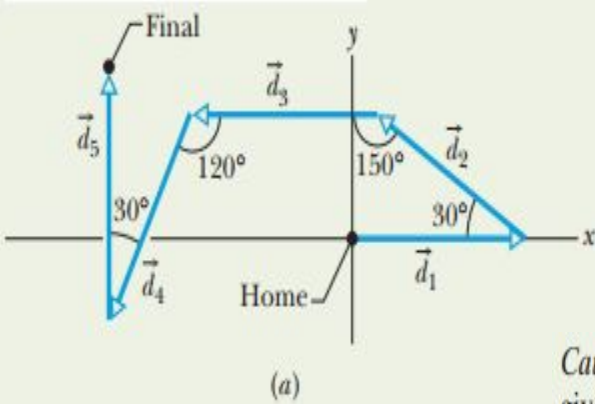
Similarly, we evaluate the individual y components of the five runs using the y part of Eq. 3-5. The results are shown in Table 3-1. Substituting the results into Eq. 3-15 then gives us

$$d_{\text{net},y} = +3.8 \text{ cm}.$$

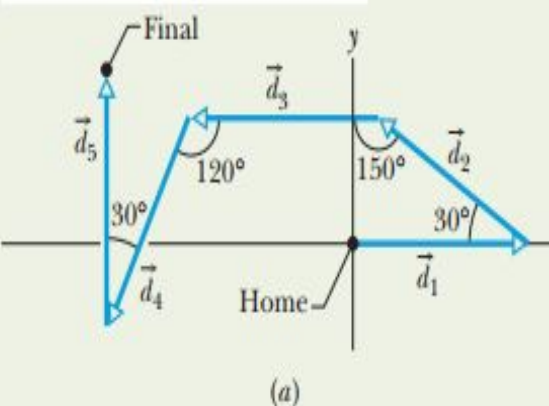
Table 3-1

Run	d_x (cm)	d_y (cm)
1	+6.0	0
2	-5.2	+3.0
3	-6.0	0
4	-3.0	-5.2
5	0	+6.0
net	-8.2	+3.8

To add these vectors, find their net x component and their net y component.



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$$\begin{aligned} d_{\text{net}} &= \sqrt{d_{\text{net},x}^2 + d_{\text{net},y}^2} \\ &= \sqrt{(-8.2 \text{ cm})^2 + (3.8 \text{ cm})^2} = 9.0 \text{ cm}. \end{aligned}$$

To find the angle (measured from the positive direction of x), we take an inverse tangent:

$$\begin{aligned} \theta &= \tan^{-1}\left(\frac{d_{\text{net},y}}{d_{\text{net},x}}\right) \\ &= \tan^{-1}\left(\frac{3.8 \text{ cm}}{-8.2 \text{ cm}}\right) = -24.86^\circ. \end{aligned} \tag{3-16}$$

Caution: Taking an inverse tangent on a calculator may not give the correct answer. The answer -24.86° indicates that the direction of \vec{d}_{net} is in the fourth quadrant of our xy coordinate system. However, when we construct the vector from its components (Fig. 3-16b), we see that the direction of \vec{d}_{net} is in the second quadrant. Thus, we must “fix” the calculator’s answer by adding 180° :

$$\theta = -24.86^\circ + 180^\circ = 155.14^\circ \approx 155^\circ. \tag{3-17}$$

Thus, the ant’s displacement \vec{d}_{net} has magnitude and angle

$$d_{\text{net}} = 9.0 \text{ cm at } 155^\circ. \tag{Answer}$$

Vector \vec{d}_{home} directed from the ant to its home has the same magnitude as \vec{d}_{net} but the opposite direction (Fig. 3-16c). We already have the angle ($-24.86^\circ \approx -25^\circ$) for the direction opposite \vec{d}_{net} . Thus, \vec{d}_{home} has magnitude and angle

$$d_{\text{home}} = 9.0 \text{ cm at } -25^\circ. \tag{Answer}$$