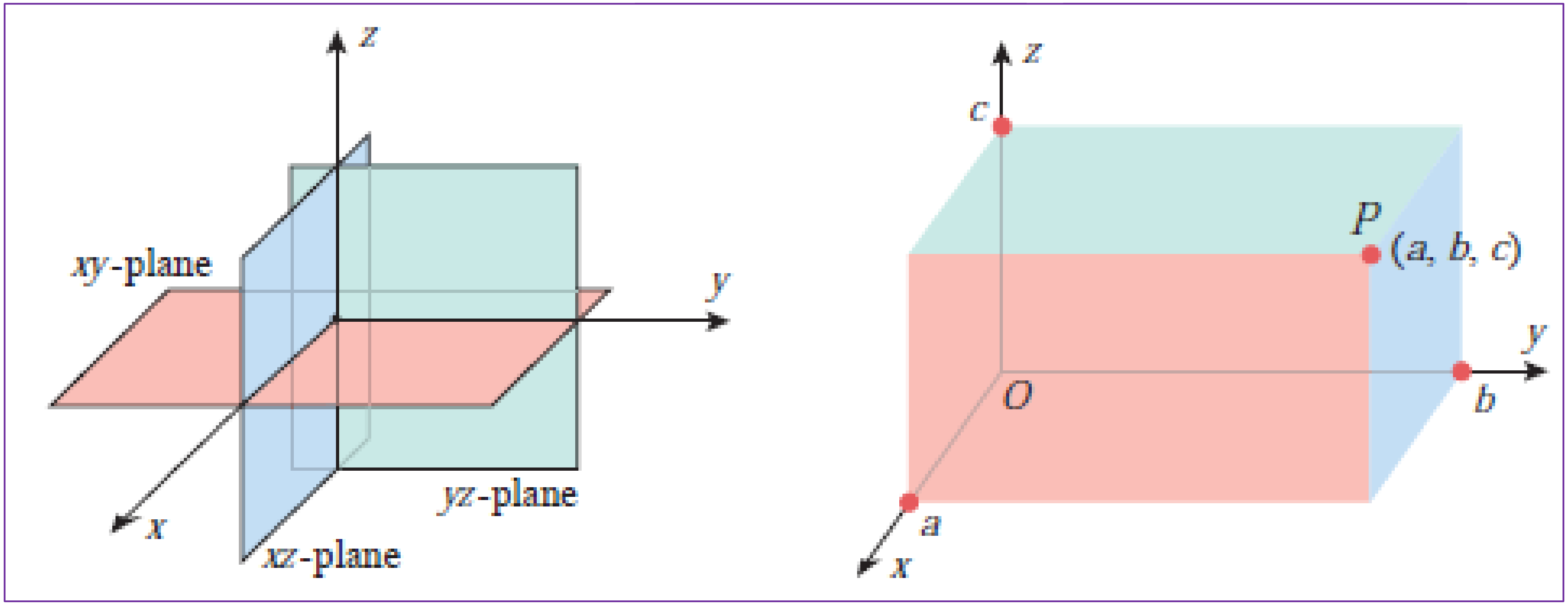
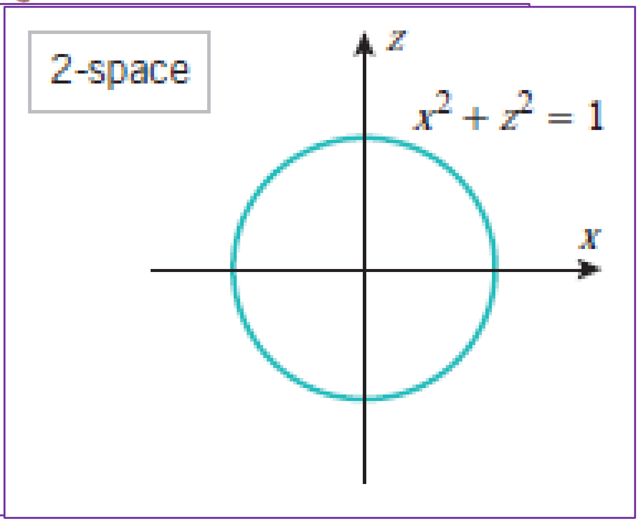
- Three-dimensional coordinate system
  - ✓ Rectangular coordinate systems
  - ✓ Cylindrical surfaces
  - ✓ Cylindrical coordinate systems
  - ✓ Spherical coordinate systems

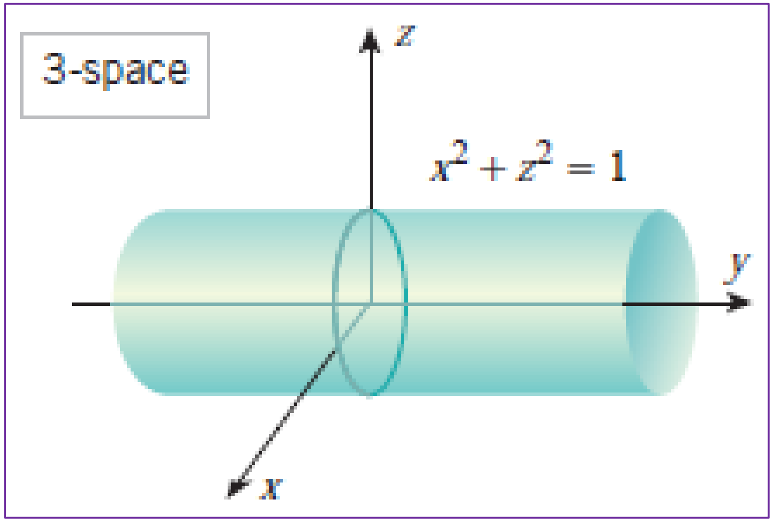
## RECTANGULAR COORDINATE SYSTEMS




| REGION | DESCRIPTION |
|---|---|
| $xy$-plane | Consists of all points of the form $(x, y, 0)$ |
| $xz$-plane | Consists of all points of the form $(x, 0, z)$ |
| $yz$-plane | Consists of all points of the form $(0, y, z)$ |
| $x$-axis | Consists of all points of the form $(x, 0, 0)$ |
| $y$-axis | Consists of all points of the form $(0, y, 0)$ |
| $z$-axis | Consists of all points of the form $(0, 0, z)$ |

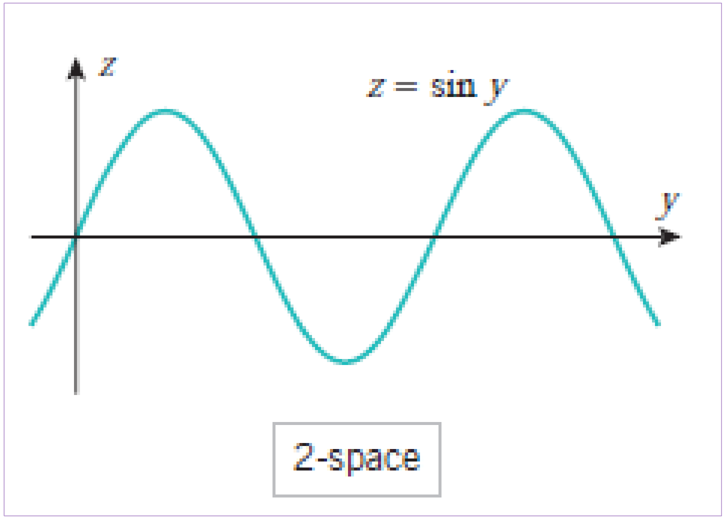
## CYLINDRICAL SURFACES

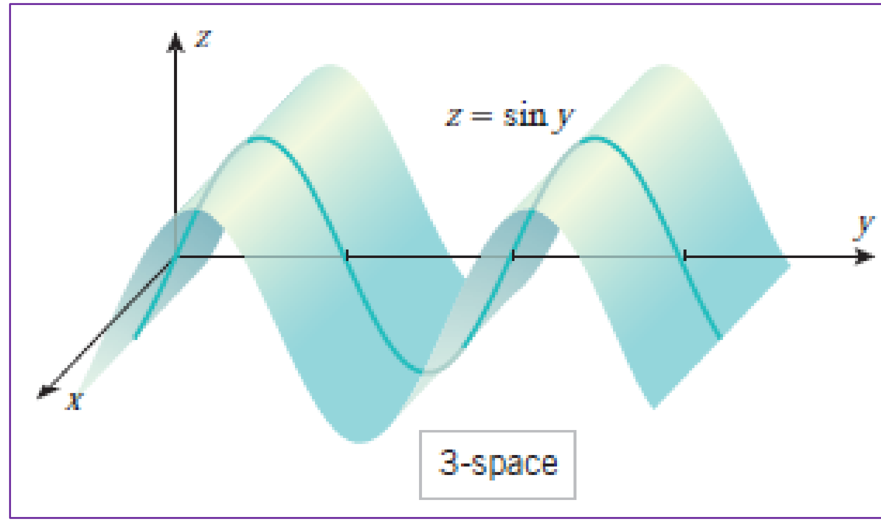


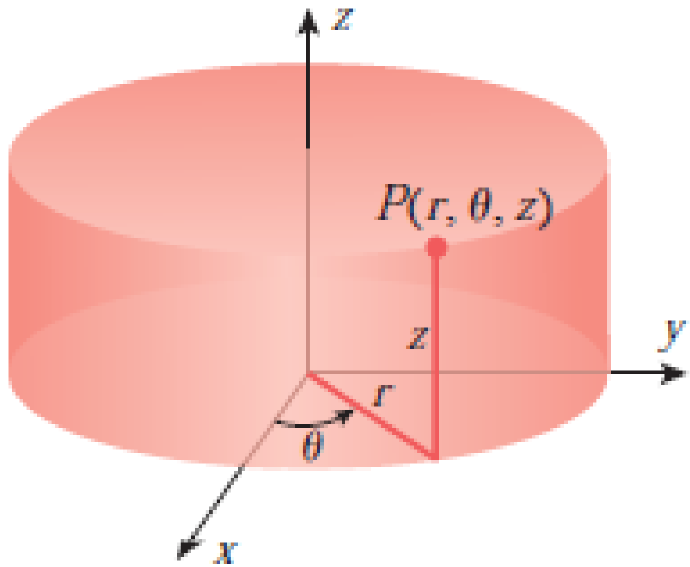

The process of generating a surface by translating a plane curve parallel to some line is called ***extrusion***, and surfaces that are generated by extrusion are called ***cylindrical surfaces***.

**11.1.2 THEOREM** *An equation that contains only two of the variables $x$, $y$, and $z$ represents a cylindrical surface in an $xyz$-coordinate system. The surface can be obtained by graphing the equation in the coordinate plane of the two variables that appear in the equation and then translating that graph parallel to the axis of the missing variable.*

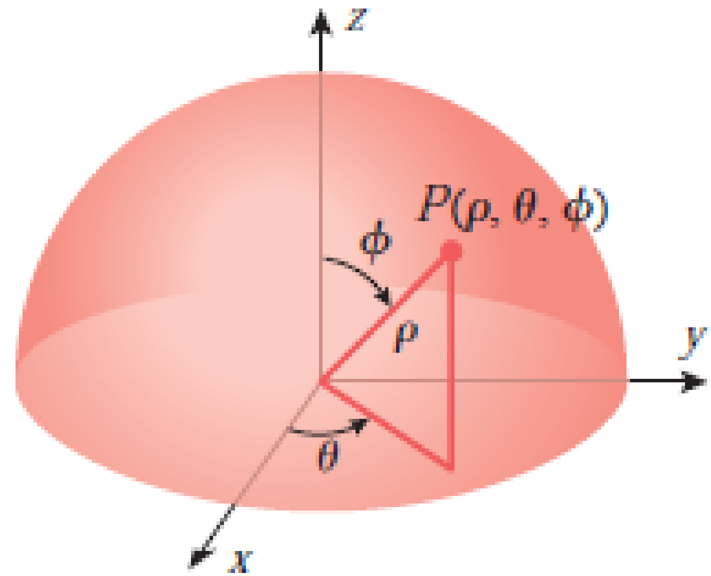

z
z = sin y
y
2-space


z
z = sin y
y
x
3-space

## CYLINDRICAL AND SPHERICAL COORDINATE SYSTEMS




Cylindrical coordinates
$(r, \theta, z)$
$(r \geq 0, 0 \leq \theta < 2\pi)$



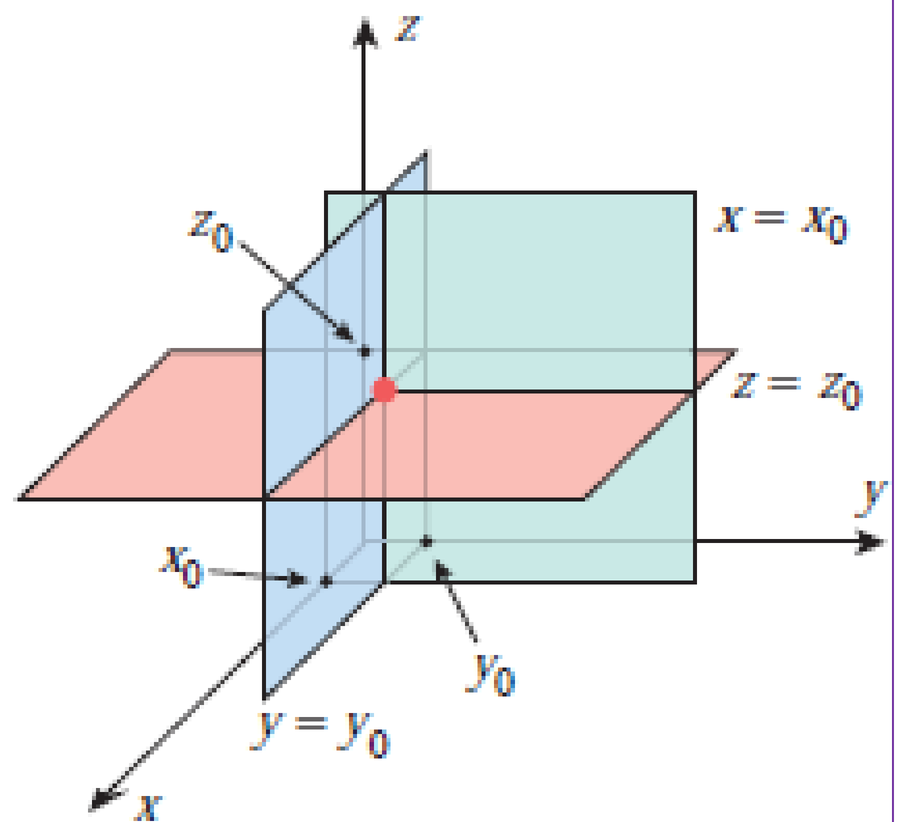

Spherical coordinates
$(\rho, \theta, \phi)$
$(\rho \geq 0, 0 \leq \theta < 2\pi, 0 \leq \phi \leq \pi)$

## CONSTANT SURFACES

In rectangular coordinates the surfaces represented by equations of the form

$$x = x_0, \quad y = y_0, \quad \text{and} \quad z = z_0$$

where $x_0$, $y_0$, and $z_0$ are constants, are planes parallel to the $yz$-plane, $xz$-plane, and $xy$-plane, respectively (Figure 11.8.2).



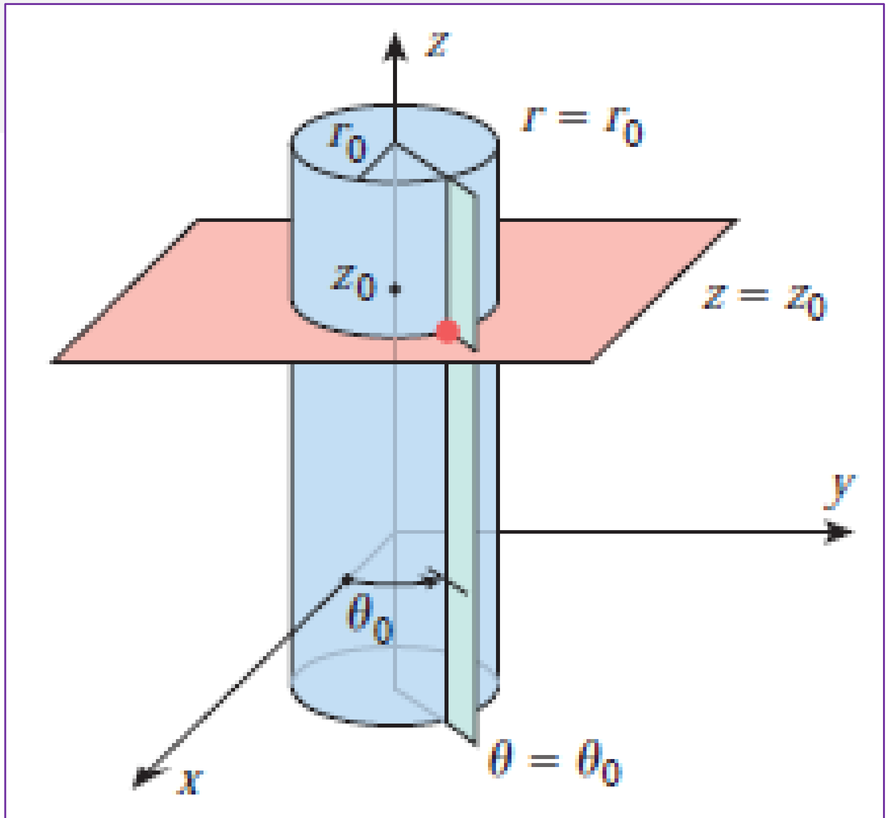

▲ **Figure 11.8.2**

In cylindrical coordinates the surfaces represented by equations of the form

$$r = r_0, \quad \theta = \theta_0, \quad \text{and} \quad z = z_0$$

where $r_0$, $\theta_0$, and $z_0$ are constants, are shown in Figure 11.8.3:

- The surface $r = r_0$ is a right circular cylinder of radius $r_0$ centered on the $z$-axis.
- The surface $\theta = \theta_0$ is a half-plane attached along the $z$-axis and making an angle $\theta_0$ with the positive $x$-axis.
- The surface $z = z_0$ is a horizontal plane.



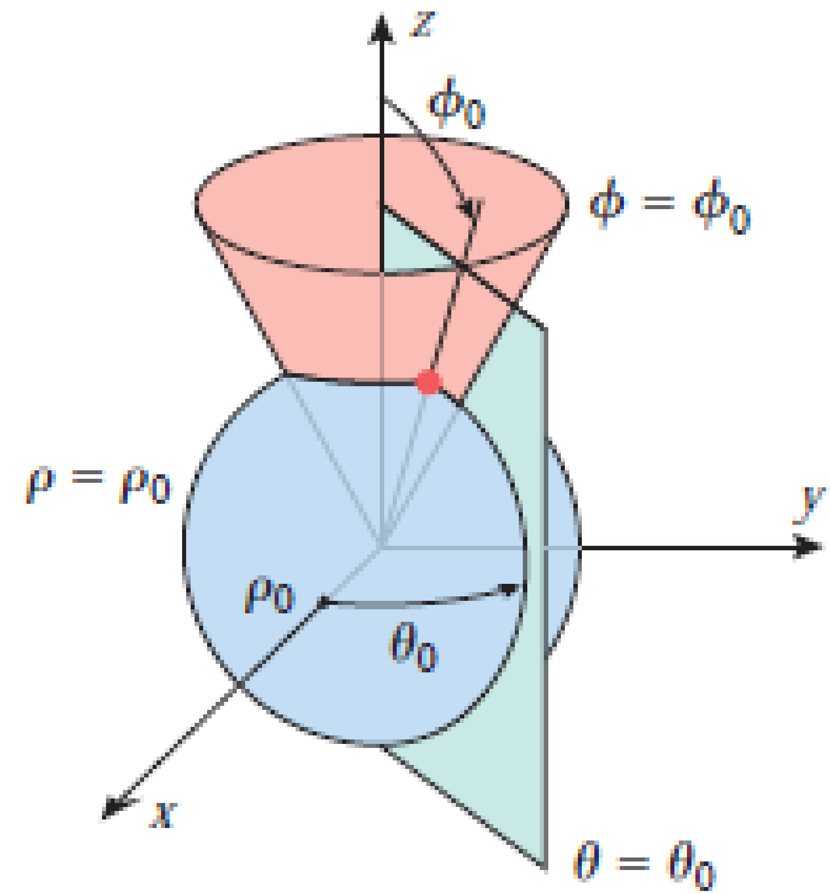

▲ **Figure 11.8.3**

In spherical coordinates the surfaces represented by equations of the form

$$\rho = \rho_0, \quad \theta = \theta_0, \quad \text{and} \quad \phi = \phi_0$$

where $\rho_0$, $\theta_0$, and $\phi_0$ are constants, are shown in Figure 11.8.4:

- The surface $\rho$ … om the origin is $\rho_0$. Assuming $\rho_0$ … ntered at the origin.
- As in cylindri… attached along the $z$-axis, makin…
- The surface $\phi$ … gment to the origin makes an ang… $\pi/2$, this will be the nappe of a co… l be the nappe of a cone opening… d the surface is the $xy$-plane.)




▲ **Figure 11.8.4**

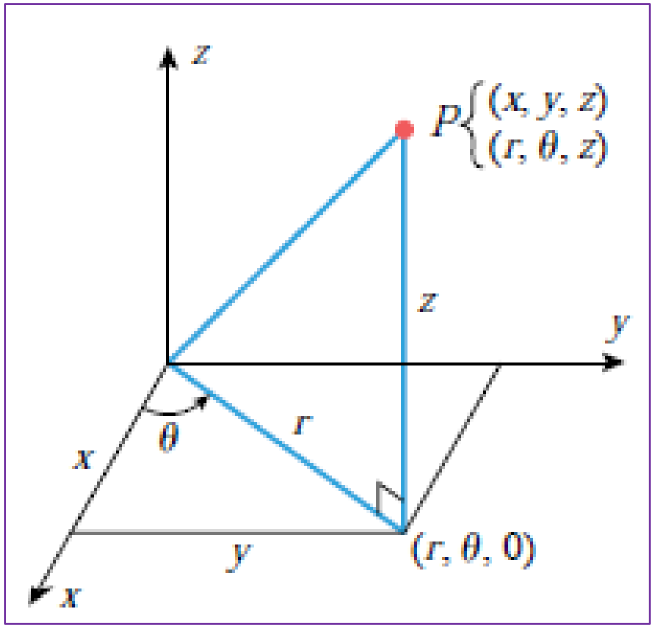
## CONVERTING COORDINATES

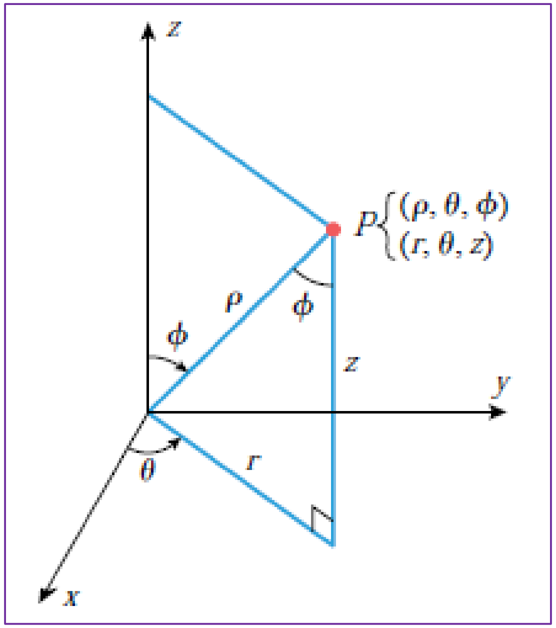
**Table 11.8.1**

CONVERSION FORMULAS FOR COORDINATE SYSTEMS

| CONVERSION | | FORMULAS | RESTRICTIONS |
|---|---|---|---|
| Cylindrical to rectangular<br>Rectangular to cylindrical | $(r, \theta, z) \to (x, y, z)$<br>$(x, y, z) \to (r, \theta, z)$ | $x = r\cos\theta, \quad y = r\sin\theta, \quad z = z$<br>$r = \sqrt{x^2+y^2}, \quad \tan\theta = y/x, \quad z = z$ | |
| Spherical to cylindrical<br>Cylindrical to spherical | $(\rho, \theta, \phi) \to (r, \theta, z)$<br>$(r, \theta, z) \to (\rho, \theta, \phi)$ | $r = \rho\sin\phi, \quad \theta = \theta, \quad z = \rho\cos\phi$<br>$\rho = \sqrt{r^2+z^2}, \quad \theta = \theta, \quad \tan\phi = r/z$ | $r \geq 0, \rho \geq 0$<br>$0 \leq \theta < 2\pi$<br>$0 \leq \phi \leq \pi$ |
| Spherical to rectangular<br>Rectangular to spherical | $(\rho, \theta, \phi) \to (x, y, z)$<br>$(x, y, z) \to (\rho, \theta, \phi)$ | $x = \rho\sin\phi\cos\theta, \quad y = \rho\sin\phi\sin\theta, \quad z = \rho\cos\phi$<br>$\rho = \sqrt{x^2+y^2+z^2}, \quad \tan\theta = y/x, \quad \cos\phi = z/\sqrt{x^2+y^2+z^2}$ | |

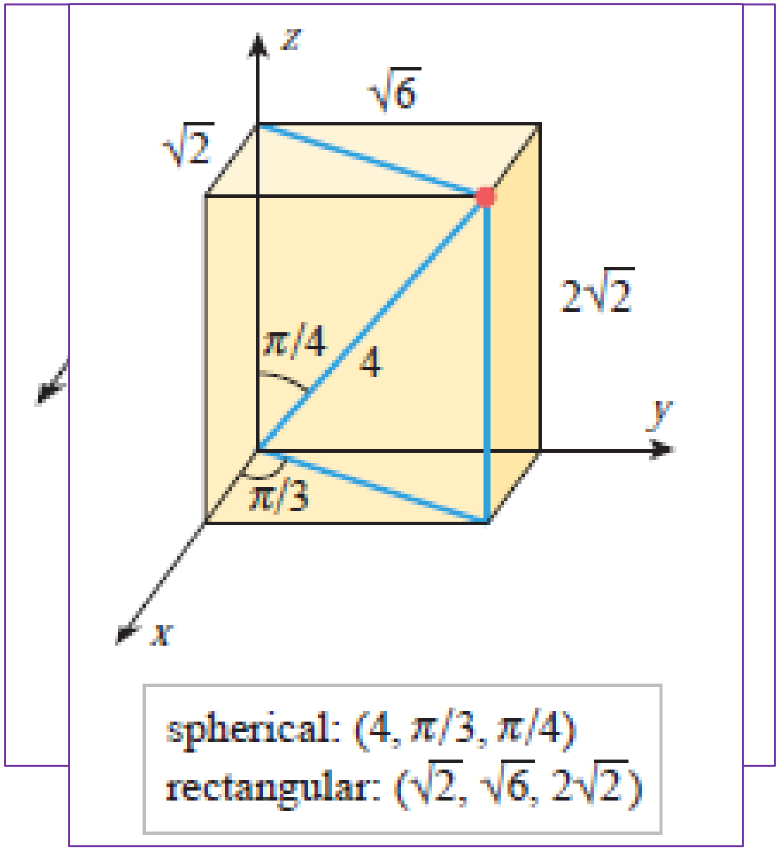
# Comparison of coordinate systems




(b) Find the rectangular coordinates of the point with spherical coordinates

$$(\rho, \theta, \phi) = (4, \pi/3, \pi/4)$$



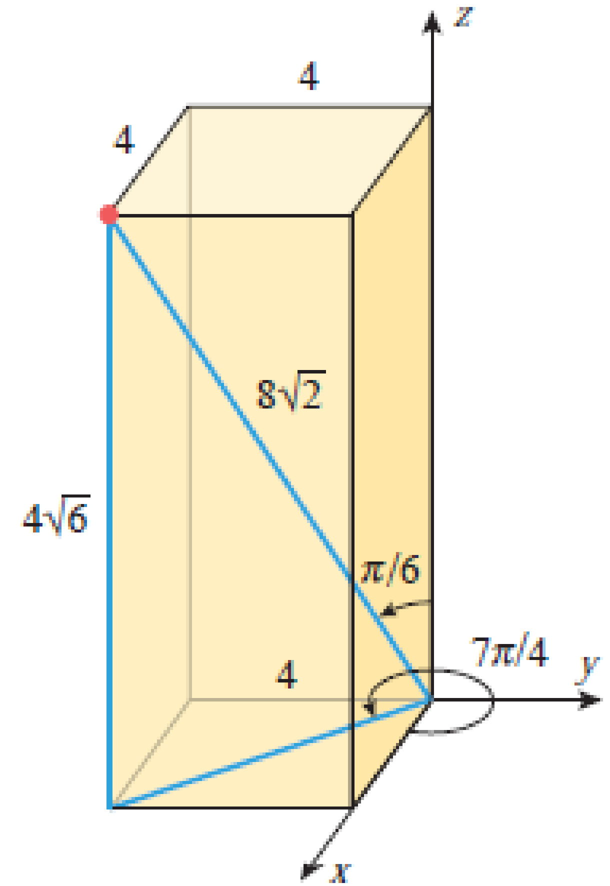
Find the spherical coordinates of the point that has rectangular coordinates

$$(x, y, z) = (4, -4, 4\sqrt{6})$$

$$\rho = \sqrt{x^2 + y^2 + z^2} = \sqrt{16 + 16 + 96} = \sqrt{128} = 8\sqrt{2}$$

$$\tan\theta = \frac{y}{x} = -1$$

$$\cos\phi = \frac{z}{\sqrt{x^2 + y^2 + z^2}} = \frac{4\sqrt{6}}{8\sqrt{2}} = \frac{\sqrt{3}}{2}$$




rectangular: $(4, -4, 4\sqrt{6})$
spherical: $(8\sqrt{2}, 7\pi/4, \pi/6)$


ANY
QUESTIONS
?

**1–2** Convert from rectangular to cylindrical coordinates.

**1.** (a) $(4\sqrt{3}, 4, -4)$ (b) $(-5, 5, 6)$
(c) $(0, 2, 0)$ (d) $(4, -4\sqrt{3}, 6)$

**2.** (a) $(\sqrt{2}, -\sqrt{2}, 1)$ (b) $(0, 1, 1)$
(c) $(-4, 4, -7)$ (d) $(2, -2, -2)$

**3–4** Convert from cylindrical to rectangular coordinates.

**3.** (a) $(4, \pi/6, 3)$ (b) $(8, 3\pi/4, -2)$
(c) $(5, 0, 4)$ (d) $(7, \pi, -9)$

**4.** (a) $(6, 5\pi/3, 7)$ (b) $(1, \pi/2, 0)$
(c) $(3, \pi/2, 5)$ (d) $(4, \pi/2, -1)$

**5–6** Convert from rectangular to spherical coordinates.

**5.** (a) $(1, \sqrt{3}, -2)$ (b) $(1, -1, \sqrt{2})$
(c) $(0, 3\sqrt{3}, 3)$ (d) $(-5\sqrt{3}, 5, 0)$

**6.** (a) $(4, 4, 4\sqrt{6})$ (b) $(1, -\sqrt{3}, -2)$
(c) $(2, 0, 0)$ (d) $(\sqrt{3}, 1, 2\sqrt{3})$

**7–8** Convert from spherical to rectangular coordinates.

**7.** (a) $(5, \pi/6, \pi/4)$ (b) $(7, 0, \pi/2)$
(c) $(1, \pi, 0)$ (d) $(2, 3\pi/2, \pi/2)$

**8.** (a) $(1, 2\pi/3, 3\pi/4)$ (b) $(3, 7\pi/4, 5\pi/6)$
(c) $(8, \pi/6, \pi/4)$ (d) $(4, \pi/2, \pi/3)$


THANK
YOU