BRAC University

MAT110:Differential Calculus and Co-ordinate Geometry

Assignment 02

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Section:14

Set-B

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1.Analyze the differentiability at x=2 of the function,

$$f(x)=\begin{cases} x^2-4x-2 & x<2 \\ -2x+4x & x>2 \end{cases}$$

1 Answer:

Here,

LHL$=\lim_{h\to 0^{-1}} \frac{f(x+h)-f(x)}{h}$

$=\lim_{h\to 0^{-1}} \frac{f(2+h)-f(2)}{h}$

$=\lim_{h\to 0^{-1}} \frac{(2+h)^2-4(2+h)-2-(2^2-4.2-2)}{h}$

$=\lim_{h\to 0^{-1}} \frac{4+4h+h^2-8-4h-2-4+8}{h}$

$=\lim_{h\ddot{0}^{-1}\frac{h^2-2}{h}}$

$=\frac{0-2}{0}$

$=\frac{-2}{0}$

$=\infty$

RHL$=\lim_{h\to 0^{+1}} \frac{f(x+h)-f(x)}{h}$

$=\lim_{h\to 0^{+1}} \frac{f(2+h)-f(2)}{h}$

$=\lim_{h\to 0^{+1}} \frac{-2(2+h)^2+4(2+h)-(-2.2^2+4.2)}{h}$

$=\lim_{h\to 0^{+1}} \frac{-2(4+4h+h^2)+8+4h+8-8}{h}$

$=\lim_{h\to 0^{+1}} \frac{-8-8h-2h^2+8+4h+8-8}{h}$

$=\lim_{h\to 0^{+1}} \frac{h(-8-2h)}{h}$

$=\lim_{h\to 0^{+1}}(-8-2h)$

$= -8$

$LHL \neq RHL$

So, at x=2 , the function is not differentiable.

(Ans.)

2.Using the limit definition, find the derivative of:

$f(x) = \frac{x^2-1}{2x-3}$

2 Answer:

Here,

$f(x) = \frac{x^2-1}{2x-3}$

Now,

f'(x)=$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

=$\lim_{h\to 0} \frac{\frac{(x+h)^2-1}{2(x+h)-3}-\frac{x^2-1}{2X-3}}{h}$

$$=\lim_{h\to 0}\frac{\frac{x^2+2xh+h^2-1}{2x+2h-3}-\frac{x^2-1}{2X-3}}{h}$$

$$=\lim_{h\to 0}\frac{(2x-3)(x^2+2xh+h^2-1)-(x^2-1)(2x+2h-3)}{(2x+2h-3)(2x-3)h}$$

$$=\lim_{h\to 0}\frac{2x^3+4x^2h+2xh^2-3x^2-6xh-3h^2+3-(2x^3+2x^2h-3x^2-2x-2h+3)}{(2x+2h-3)(2x-3)h}$$

$$=\lim_{h\to 0}\frac{2x^3+4x^2h+2xh^2-3x^2-6xh-3h^2+3-2x^3-2x^3-2x^2h+3x^2+2x+2h-3}{(2x+2h-3)(2x-3)h}$$

$$=\lim_{h\to 0}\frac{2x^2h+2xh^2-6xh-3h^2+2h}{(2x+2h-3)(2x-3)h}$$

$$=\lim_{h\to 0}\frac{2x^2-6x+2}{(2x+2h-3)(2x-3)}$$

(Ans.)

3.If

f(x) = $\sqrt{x}g(x)$

$with\ g(4)=2, g'(4)=3\ then\ find\ f'(4).$

3 Answer:

Here,

f(x) = $\sqrt{x}g(x)$

$f'(x) = \sqrt{x}g'(x) + \frac{1}{2\sqrt{x}}$ g(x)

f'(4) =$\sqrt{4}g'(4) + \frac{1}{2\sqrt{4}}$ g(4)

f'(4)=$2.3 + \frac{1}{4}.2$

f'(4)=$6 + \frac{1}{2}$

f'(4)=$\frac{13}{2}$

(Ans.)

4.Use the limit definition to find the derivative of each of the following:

(a) f(x)=e^{x}

$(b) f(x) = x^3$

$(c) f(x) = cosx$

4 Answer:

(a) f(x)=e^x

$Using\ the\ limit\ definition,$

$f'(x)$ =$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

=$\lim_{h\to 0} \frac{e^{x+h}-e^x}{h}$

=$\lim_{h\to 0} \frac{e^x.e^h-e^x}{h}$

=$\lim_{h\to 0} \frac{e^x(e^h-1)}{h}$

=$\lim_{h\to 0} \frac{e^x(1+\frac{h^2}{2!}+\frac{h^3}{3!}+.........-1)}{h}$

=$\lim_{h\to 0} \frac{e^x(h+\frac{h^2}{2!}+\frac{h^3}{3!}+.........)}{h}$

=$\lim_{h\to 0} e^x(1+\frac{h}{2!}+\frac{h^2}{3!}+\frac{h^3}{4!}+....)$

=$e^x(1+0+0+0+.....)$

=e^x

(Ans.)

(b) f(x)=x^3

$Using\ the\ limit\ definition,$

$f'(x)$ =$\lim_{h\to0}\frac{f(x+h)-f(x)}{h}$

=$\lim_{h\to0}\frac{(x+h)^3-x^3}{h}$

=$\lim_{h\to0}\frac{x^3+3x^2h+3xh^2+h^3-x^3}{h}$

=$\lim_{h\to0}\frac{h(3x^3+3xh+h^2}{h}$

=3 $x^2+3x.0+0^2$

=3 x^2

(Ans.)

(c) f(x)=cosx

Using the limit definition,

f'(x)=$\lim_{h\to0}\frac{f(x+h)-f(x)}{h}$

=$\lim_{h\to0}\frac{cos(x+h)-cos(x)}{h}$

$=\lim_{h\ 0} \frac{2sin\frac{x+h+x}{2}sin\frac{x-x-h}{2}}{h}$

$=\lim_{h\to 0} \frac{2sin\frac{2x+h}{2}sin\frac{h}{2}}{h}$

$=\lim_{h\to 0} \frac{2sin\frac{2x+h}{2}sin\frac{h}{2}}{\frac{h}{2}.2}$

$-\lim_{h\to 0} sin(x+\frac{h}{2})$

=-sin x

(Ans.)

5.Use the chain rule to prove the following:

(a) The derivative of an even function is an odd function.

(b) The derivative of an odd function is an even function.

5 Answer:

(a) A function is even if f(-1)$\equiv f(x) for\ all\ x$,

$Let\ f\ is\ even:$

f'(x)=$\lim_{h\to 0} \frac{f(-x+h)-f(-x)}{h}$

=$\lim_{h\to 0} \frac{f(-(x-h))-f(-x)}{h}$

=$\lim_{h\to 0} \frac{f(x-h)-f(-x)}{h}$

=-f'(x)

$- f'(x)\ is\ an\ odd\ function.$

$So,\ the\ derivative\ of\ an\ even\ function\ is\ an\ odd\ function.$

$(Showed.)$

(b) A function is even if f(-1)$\equiv -f(x) for\ all\ x,$

Let f is odd:

f'(x)=$\lim_{h\to 0} \frac{f(-x+h)-f(-x)}{h}$

$=\lim_{h\to 0} \frac{-f(x-h)+f(x)}{h}$

$=-\lim_{h\to 0} \frac{f(x-h)-f(x)}{h}$

=-(-f'(x))

=f'(x)

-f'(x) is an even function.

So, the derivative of an odd function is an even function.

(Showed.)

6. A polynomial of m degree is defines as:

p(x) = a0 + a1x + a2x^2 + ... + anx^m

$(a) Find\ p'(x)\ and\ p'''(x)$

$(b) Find\ p^{(m)}$

$(c) Find\ p^{(n)}\ when\ n > m$

6 Answer:

(a)Here,

p(x) = a0 + a1x + a2x^2 + ... + anx^m

$Now,$

$$p\prime(x) = \frac{d}{dx}p(x)$$

$$=\frac{d}{dx}(a_0 + a_1x + a_2x^2 + ... + a_nx^m)$$

$$=\mathrm{a}_1 + 2a_2x + 3a_3x^2 + ... + ma_nx^{m-1}$$

$$p\prime\prime(x) = p'(x)$$

$$=\frac{d}{dx}(a_1 + 2a_2x + 3a_3x^2 + ... + ma_nx^{m-1}$$

$$=2\mathrm{a}_2 + 6a_3x + 12a_4x^2 + ... + m(m-1)a_nx^{m-2}$$

$p'''(x) = \frac{d}{dx}p''(x)$

$=\frac{d}{dx}(2a_2 + 6a_3x + 12a_4x^2 + ... + m(m-1)a_nx^{m-2}$

$=6\mathrm{a}_3 + 24a_4x + 60a_5x^2 + ... + m(m-1)(m-2)a_nx^{m-3}$

(Ans.)

(b) Here,

$p^{(m)} = (\frac{d}{dx})^m\ p(x)$

$= \frac{d^m}{dx^m}\ p(a_0 + a_1x + a_2x^2 + ... + a_nx^m)$

$= 0 + 0 + 0 + ... + \frac{d^m}{dx^m}\ a_nx^m$

$=(\mathrm{m(m-1)(m-2)....3.2.1})\ \mathrm{a}_nx^{m-m}$

$=\mathrm{m!}\ \mathrm{a}_nx^0$

$(Ans.)$

$=\mathrm{a}_nm!$

(c) Let,

$p^n(x) = \frac{d}{dx}\, p^m(x) \qquad [Since\ n > m]$

$= \frac{d}{dx}\, (a_n m!)$

$=0 \qquad$ [Since $a_n m!$ *is a constant*]

(Ans.)