MAT110:Differential Calculas and Co-ordinate Geometry

BRAC UNIVERITY

ASSUGNMENT 1

SUBMITTED BY :

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SECTION : 10

1 Evaluate the limits

1 no ques ans:

$$\lim_{x\to -1} \frac{x^2 + 6x + 5}{x^2 - 3x - 4}$$

$$= \lim_{x \to -1} \frac{2x+6}{2x-3}$$
$$= \frac{-2+6}{-2-3}$$
$$= \frac{4}{-5}$$
$$(\text{Ans}: \frac{4}{-5})$$

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2 Evaluate the following limits using L'Hopital's rules:

2 no ques ans(a) :

$$\lim_{x \to 0} \frac{4^x - 1}{x}$$
$$= \lim_{x \to 0} \frac{4^x ln4}{1}$$
$$= \lim_{x \to 0} 4^x ln4$$
$$= ln4 \lim_{x \to 0} 4^x$$
$$= ln4$$
$$(\text{Ans}: \ln 4)$$

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2 no ques ans (b):

$$= \lim_{x\to 2} \frac{4x^2 - 3x - 10}{6 + 5x - x^4}$$
$$= \lim_{x\to 2} \frac{8x - 3}{5 - 4x^3}$$
$$= \frac{-13}{27}$$
$(\text{Ans} : \frac{-13}{27})$

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3 Evaluate the following limits:

3 no ques ans:

$$\lim_{x\to 0} \frac{\sqrt{x+4} - 2}{x}$$
$$= \lim_{x\to 0} \frac{(\sqrt{x+4} - 2)(\sqrt{x+4} + 2)}{x(\sqrt{x+4} + 2)}$$

$$= \lim_{x\to 0} \frac{(\sqrt{x+4})^2 - 4}{x(\sqrt{x+4}+2)}$$
$$= \lim_{x\to 0} \frac{x+4-4}{x(\sqrt{x+4}+2)}$$
$$= \lim_{x\to 0} \frac{1}{\sqrt{x+4}+2}$$
$$= \frac{1}{\sqrt{4}+2}$$
$$= \frac{1}{2+2}$$
$$= \frac{1}{4}$$
(Ans : $\frac{1}{4}$)

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4 4. Prove by squeezing that value of each of the following limit is zero:

4 no ques ans (a) :

Given that,

$$LHS = \lim_{x \to 0} \mid x \mid sin(\frac{1}{x})$$

$$= \lim_{x \to 0} x \lim_{x \to 0} sin(\frac{1}{x})$$

$= -\lim_{x \to 0} x <= \lim_{x \to 0} xsin(\frac{1}{x}) <= \lim_{x \to 0} x$[Squeeze theorem]

$$= 0 <= \lim_{x \to 0} xsin(\frac{1}{x}) <= 0$$

$$= \lim_{x \to 0} xsin(\frac{1}{x}) = 0 = R.H.S$$

So,LHS=RHS

[Proved]

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4 no ques ans (b):

Given that

$$LHS = \lim_{x \to 1} \mid x \mid cos\frac{1}{x-1}$$

$$= \lim_{x\to 1}(x-1)\lim_{x\to 1} cos\frac{1}{x-1}$$

$$= -\lim_{x\to 0}(x-1) <= \lim_{x\to 0}(x-1)cos\frac{1}{x-1} <= \lim_{x\to 0}(x-1$$

[Squeeze theorem]

$$= -1+1 <= \lim_{x\to 0}(x-1)cos\frac{1}{x-1} <=$$

$1-1$

$$= 0 <= \lim_{x\to 0}(x-1)cos\frac{1}{x-1} <= 0$$

$$= \lim_{x\to 0}(x-1)cos\frac{1}{x-1} = 0 = RHS$$

SO,LHS=RHS

[Proved]

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4 no ques ans (c):

Given that,

$$LHS = \lim_{x\to -\infty} e^x sin(x^2+1)$$

$$= \lim_{x\to-\infty} e^x \lim_{x\to-\infty} sin(x^2+1)$$
$$= -\lim_{x\to-\infty} e^x <= \lim_{x\to-\infty} e^x \lim_{x\to-\infty} sin(x^2+1) <= \lim_{x\to-\infty} e$$
[Squeeze theorem]
$$= -e^{-\infty} <= \lim_{x\to-\infty} e^x \lim_{x\to-\infty} sin(x^2+1) <= e^{-\infty}$$
$$= 0 <= \lim_{x\to-\infty} e^x \lim_{x\to-\infty} sin(x^2+1) <= 0$$
$$= \lim_{x\to-\infty} e^x \lim_{x\to-\infty} sin(x^2+1) = 0 = RHS$$

So,LHS=RHS

[Proved]

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5 no ques ans :

Given that,

$$\lim_{x\to 0} \frac{\sqrt{x+25}-5}{x}$$
$$= \lim_{x\to 0} \frac{(\sqrt{x+25}-5)(\sqrt{x+25}+5)}{x(\sqrt{x+25}+5)}$$

$$= \lim_{x\to 0} \frac{(\sqrt{x+25})^2 - 25}{x\sqrt{x+25}+5}$$
$$= \lim_{x\to 0} \frac{1}{(\sqrt{x+25}+5)}$$
$$= \frac{1}{10}$$
Ans : $\frac{1}{10}$

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THE END