PROJECTILE MOTION

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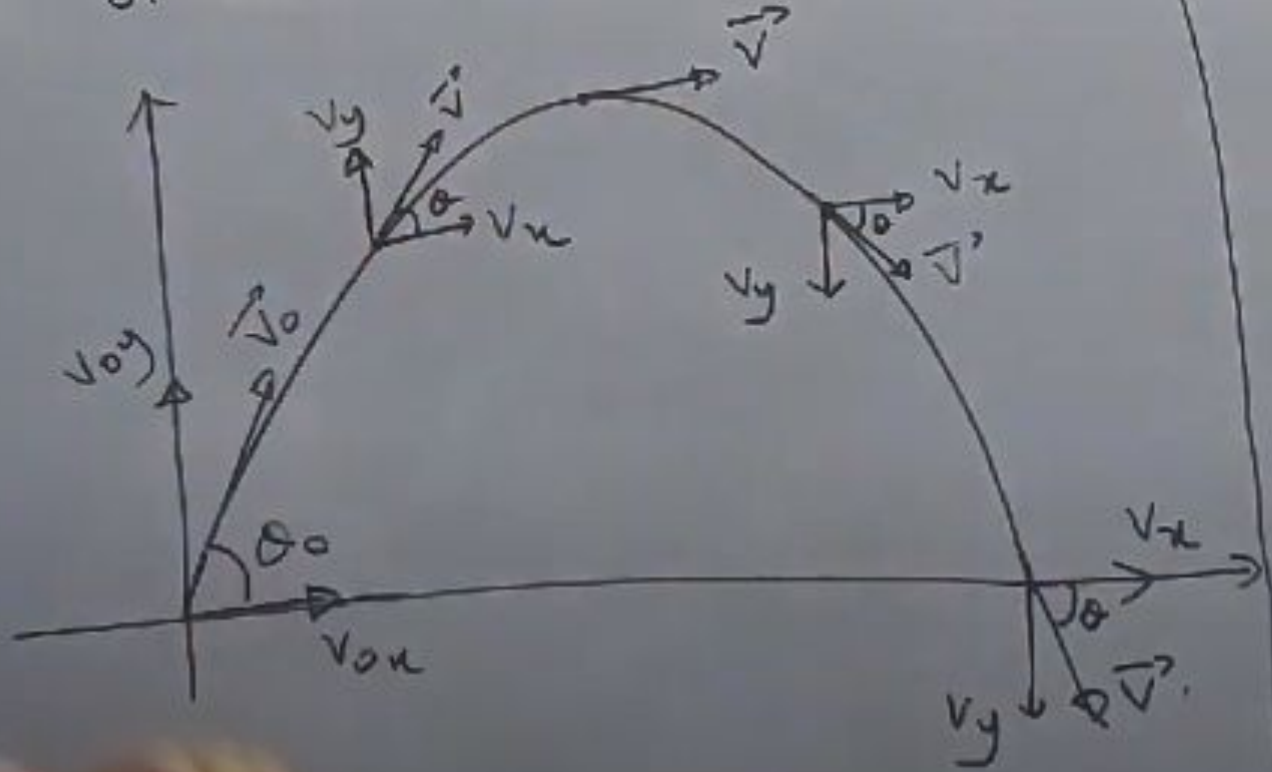
PROJECTILE MOTION

* It is a two dimensional motion.
* The horizontal and vertical motion are independent of each other.



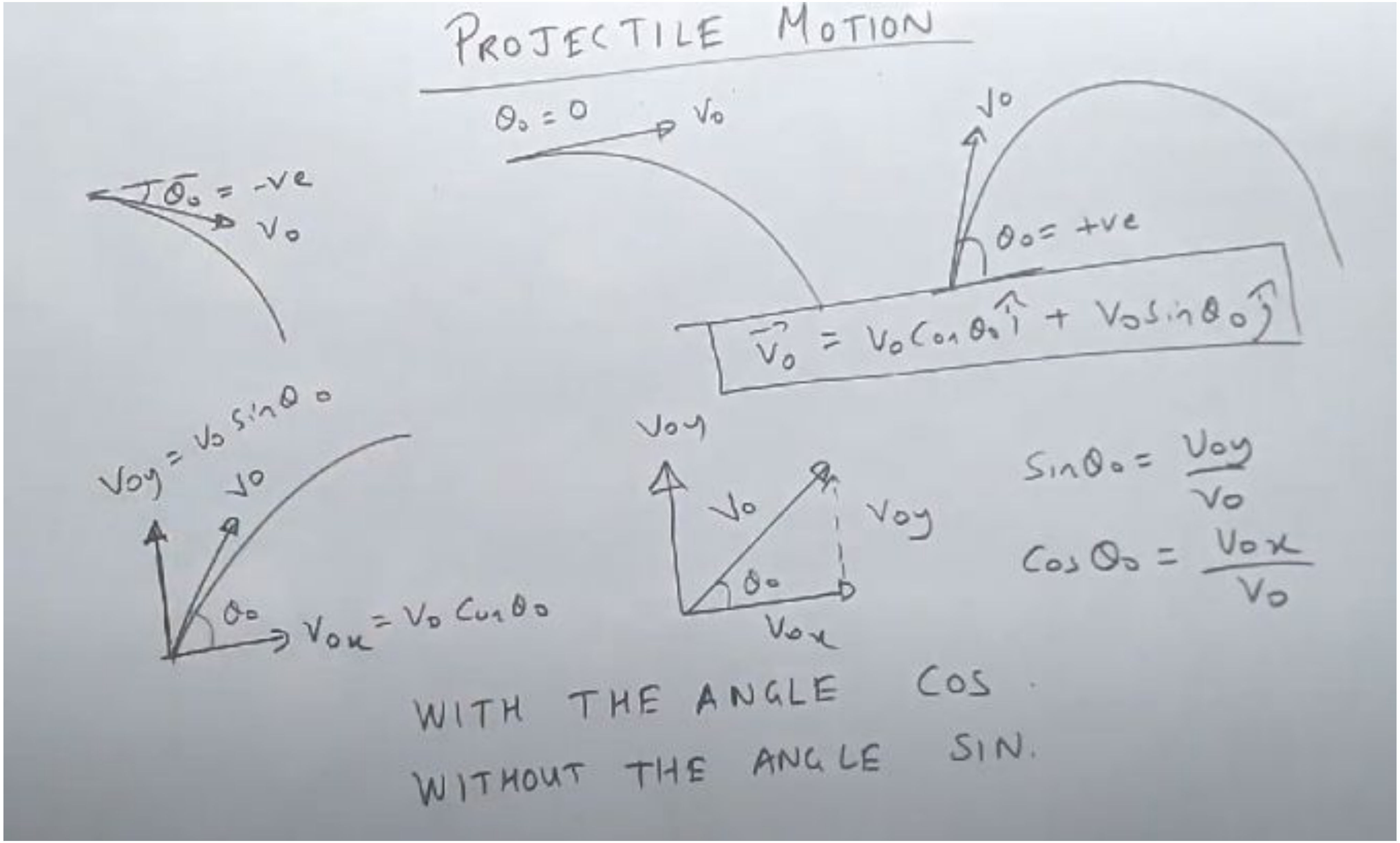
$\vec{v} = v_x\hat{i} + v_y\hat{j} \rightarrow$ rising

$\vec{v}' = v_x\hat{i} \rightarrow$ reached maximum point

$\vec{v}' = v_x\hat{i} - v_y\hat{j} \rightarrow$ falling

$$\boxed{\begin{array}{c} \text{If } v_{ox} = v_x \\ a = 0 \end{array}}$$

$$a_x = \frac{v_x - v_{ox}}{t}$$


PROJECTILE MOTION
$\theta_0 = 0$
V_0
$\theta_0 = -ve$
V_0
V_0
$\theta_0 = +ve$
$\vec{V_0} = V_0 \cos\theta_0 \hat{i} + V_0 \sin\theta_0 \hat{j}$
$V_{0y} = V_0 \sin\theta_0$
V_0
θ_0
$V_{0x} = V_0 \cos\theta_0$
V_{0y}
V_0
V_{0y}
θ_0
V_{0x}
$\sin\theta_0 = \frac{V_{0y}}{V_0}$
$\cos\theta_0 = \frac{V_{0x}}{V_0}$
WITH THE ANGLE COS
WITHOUT THE ANGLE SIN.

PROJECTILE MOTION

$$\vec{V}_0 = V_0 \cos\theta_0 \hat{i} + V_0 \sin\theta_0 \hat{j} = V_{0x}\hat{i} + V_{0y}\hat{j}$$

$$v = u + at$$
$$s = ut + \frac{1}{2}at^2$$
$$v^2 = u^2 + 2as$$

$$V_x = V_{0x} + at$$
$$x - x_0 = V_{0x}t + \frac{1}{2}at^2$$
$$V_x^2 = (V_{0x})^2 + 2a(x - x_0)$$

$$V_y = V_{0y} - gt$$
$$y - y_0 = V_{0y}t - \frac{1}{2}gt^2$$
$$V_y^2 = V_{0y}^2 - 2g(y - y_0)$$

Horizontal

$$a = 0$$

$$x - x_0 = V_{0x}t$$

$$\boxed{x - x_0 = V_0 \cos\theta_0 \times t}$$

$$V_0 = 2i + 3\hat{j}$$
$$V_0 = 5\angle 30^\circ$$

Vertical

$$a = -g$$

$$\boxed{V_y = V_0 \sin\theta_0 - gt}$$

$$\boxed{y - y_0 = V_0 \sin\theta_0 t - \frac{1}{2}gt^2}$$

$$\boxed{V_y^2 = (V_0 \sin\theta_0)^2 - 2g(y - y_0)}$$

••28 In Fig. 4-34, a stone is projected at a cliff of height h with an initial speed of 42.0 m/s directed at angle $\theta_0 = 60.0°$ above the horizontal. The stone strikes at A, 5.50 s after launching. Find (a) the height h of the cliff, (b) the speed of the stone just before impact at A, and (c) the maximum height H reached above the ground.

Fig. 4-33 Problem 27.

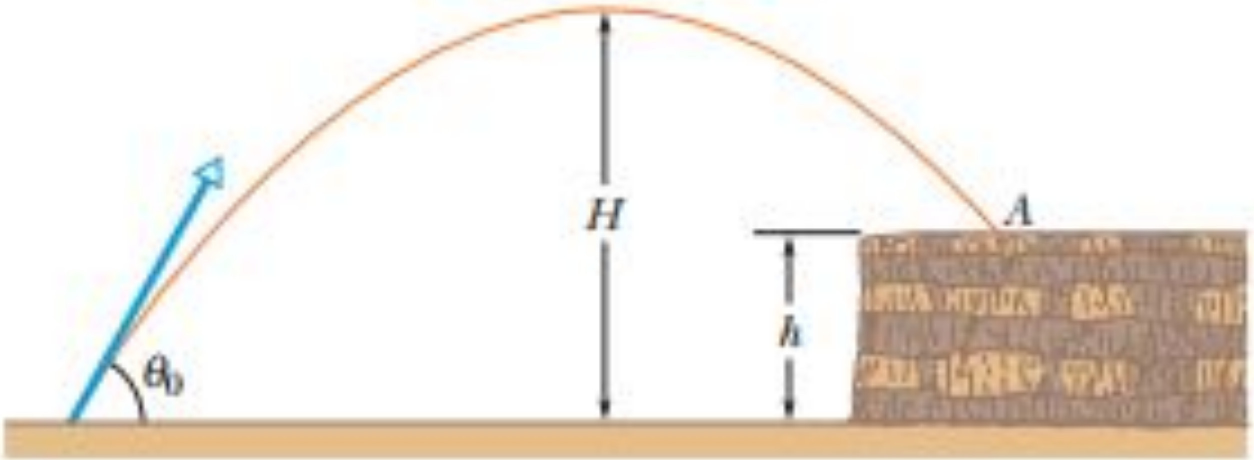


Fig. 4-34 Problem 28.

PROJECTILE MOTION

(28)

$V_0 = 42\,m/s$

$\theta_0 = 60^\circ$

$t = 5.5$ to reach A

(a) $h = ?$

(b) $V_A = ?$

(c) Find the maximum height H.

(a) $y - y_0 = V_0 \sin\theta_0 t - \frac{1}{2} g t^2$

$\Rightarrow h - 0 = 42 \times \sin 60^\circ \times 5.5 - \frac{1}{2} \times 9.8 \times 5.5^2$

$\therefore h = 51.83\,m$

(b) $V_x = V_{0x} = V_0 \cos\theta_0 = 42 \times \cos 60^\circ$
$= 21\,m/s$

(b) $V_y = V_0 \sin\theta_0 - gt$

$\Rightarrow V_y = 42 \times \sin 60^\circ - 9.8 \times 5.5\,s$
$= -17.5\,m/s$

$V_A = V_{Ax}\hat{i} + V_{Ay}\hat{j} = (21\hat{i} - 17.5\hat{j})\,m/s$

(c) At maximum height

$V_y = 0$

$V_y^2 = (V_0 \sin\theta_0)^2 - 2g(y - y_0)$

$\Rightarrow 0 = (42 \times \sin 60^\circ)^2 - 2 \times 9.8 \times (H - 0)$

$\Rightarrow H = \frac{(42 \times \sin 60^\circ)^2}{2 \times 9.8}$

$\therefore H = 67.5\,m$

** Speed → ?

Speed would be the magnitude of velocity

$\overset{\text{vector}}{\vec{V}} = V_x \hat{i} + V_y \hat{j}$

$\underbrace{|\vec{V}|}_{\text{speed}} = \sqrt{V_x^2 + V_y^2}$

**

+ve ↑ ↓ −ve , y component/axis

$\xrightarrow{+ve}$, x component/axis

$\xleftarrow[-ve]{}$

** $V_0 = 2i + 3\hat{j}$ (Unit Vector notation)

$V_0 = 5\angle 30^\circ$ (Polar coordinate)

••27 ILW A certain airplane has a speed of 290.0 km/h and is diving at an angle of $\theta = 30.0°$ below the horizontal when the pilot releases a radar decoy (Fig. 4-33). The horizontal distance between the release point and the point where the decoy strikes the ground is $d = 700$ m. (a) How long is the decoy in the air? (b) How high was the release point?

••28 In Fig. 4-34, a stone is pro-

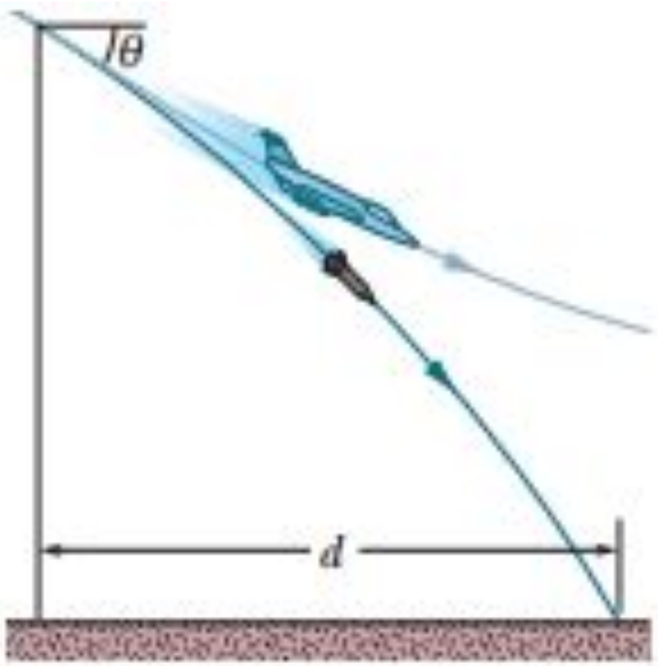
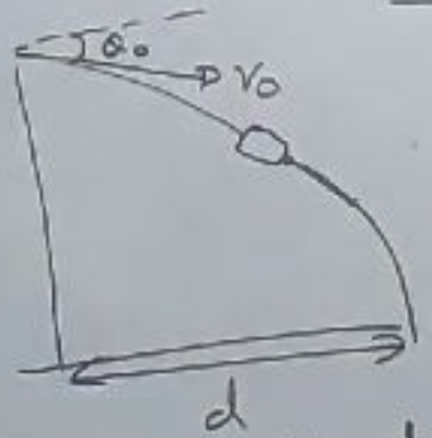


Fig. 4-33 Problem 27.

PROJECTILE MOTION

(27)



$v_0 = 290\,km/h = \frac{290 \times 1000}{3600} = 80.56\,m/s$

$\theta_0 = -30^\circ$

$d = 700\,m$

(a) How long is decoy in the air

(b) How high was the release point

(a) $x - x_0 = v_0 \cos\theta_0 \times t$

$\Rightarrow 700 - 0 = 80.56 \times \cos(-30^\circ) \times t$

$\therefore t = 10.1\,s$

(b) $y - y_0 = v_0 \sin\theta_0 \times t - \frac{1}{2} g t^2$

$\Rightarrow 0 - h = 80.56 \times \sin(-30^\circ) \times 10.1 - \frac{1}{2} \times 9.8 \times (10.1)^2$

$\Rightarrow -h = -406.83 - 499.85$

$\Rightarrow -h = -906.68$

$\therefore h = 906.7\,m$

••32 GO You throw a ball toward a wall at speed 25.0 m/s and at angle $\theta_0 = 40.0°$ above the horizontal (Fig. 4-35). The wall is distance $d = 22.0$ m from the release point of the ball. (a) How far above the release point does the ball hit the wall? What are the (b) horizontal and (c) vertical components of its velocity as it hits the wall? (d) When it hits, has it passed the highest point on its trajectory?

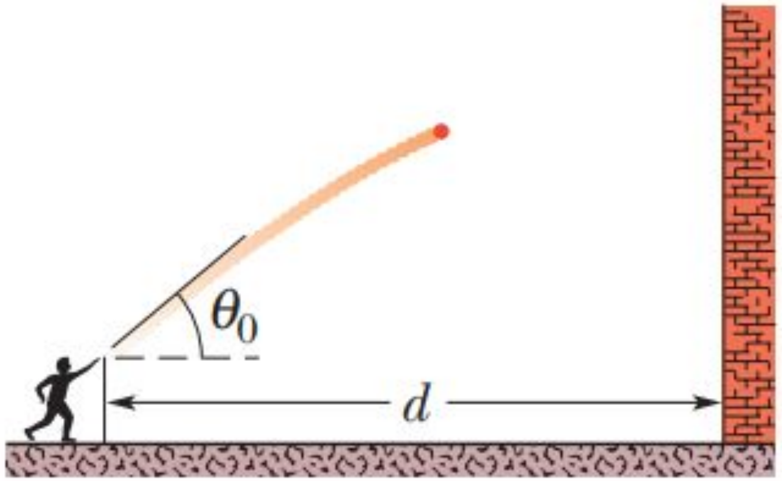
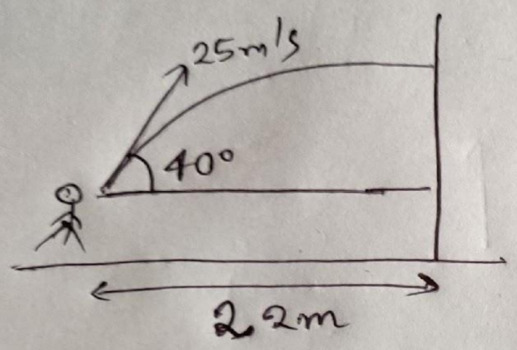
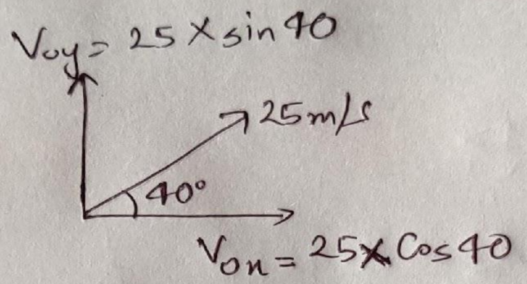


Fig. 4-35 Problem 32.

(32)





a) How far above the release point does the ball hit the wall?
b) horizontal velocity when it hits the wall
c) vertical velocity when it hits the wall
d) When it hits has it passed the highest point on its trajectory?

a) $x - x_0 = V_0 \cos\theta_0 t$

$\Rightarrow 22 - 0 = 25\cos 40 \times t$

$\Rightarrow t = 1.15s$

$y - y_0 = V_0 \sin\theta_0 t - \frac{1}{2} g t^2$

$\Rightarrow y - 0 = 25 \sin 40 \times 1.15 - \frac{1}{2} \times 9.8 \times 1.15^2$

$\Rightarrow y = 12.0m$

b) $V_x = V_{0x} = V_0 \cos\theta_0$

$= 25\cos 40 = 19.15 m/s$

c) $V_y = V_0 \sin\theta_0 - gt$

$= 25\sin 40 - 9.8 \times 1.15$

$= 4.8 m/s$

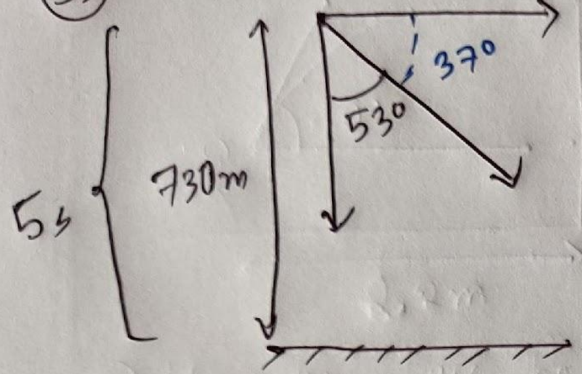
$\therefore \vec{V} = 19.15 \hat{i} + 4.8\hat{j}$

(in unit vector notation)

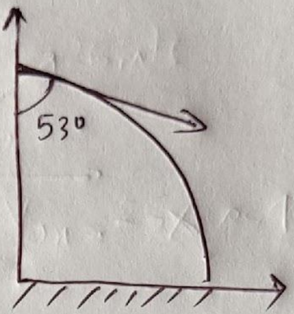
d) Since the y component of velocity is positive, it did not reach its highest point

••33 SSM A plane, diving with constant speed at an angle of 53.0° with the vertical, releases a projectile at an altitude of 730 m. The projectile hits the ground 5.00 s after release. (a) What is the speed of the plane? (b) How far does the projectile travel horizontally during its flight? What are the (c) horizontal and (d) vertical components of its velocity just before striking the ground?

33



$\theta_0 = -37°$



a) What is the speed of the plane?

b) How far does the projectile travel horizontally during its flight?

c) horizontal component of velocity before ~~sth~~ striking the ground

d) vertical component of velocity before striking the ground

a) $y - y_0 = v_0 \sin\theta_0 t - \frac{1}{2} g t^2$

$\Rightarrow 0 - 730 = v_0 \sin(-37) \times 5 - \frac{1}{2} \times 9.8 \times 5^2$

$\Rightarrow v_0 = 202 \text{ m/s}$

b) Horizontal distance travelled

$x - x_0 = v_0 \cos\theta_0 \times t$

$\Rightarrow x - 0 = 202 \times \cos(-37) \times 5$

$\Rightarrow x = 806 \text{ m}$

c) $v_x = v_{0x} = 202 \cos(-37)$

$= 161 \text{ m/s}$

d) $v_y = v_0 \sin\theta_0 - gt$

$= 202 \sin(-37) - 9.8 \times 5$

$= -171 \text{ m/s}$

••39 In Fig. 4-37, a ball is thrown leftward from the left edge of the roof, at height h above the ground. The ball hits the ground 1.50 s later, at distance $d = 25.0$ m from the building and at angle $\theta = 60.0°$ with the horizontal. (a) Find h. (*Hint:* One way is to reverse the motion, as if on video.) What are the (b) magnitude and (c) angle relative to the horizontal of the velocity at which the ball is thrown? (d) Is the angle above or below the horizontal?

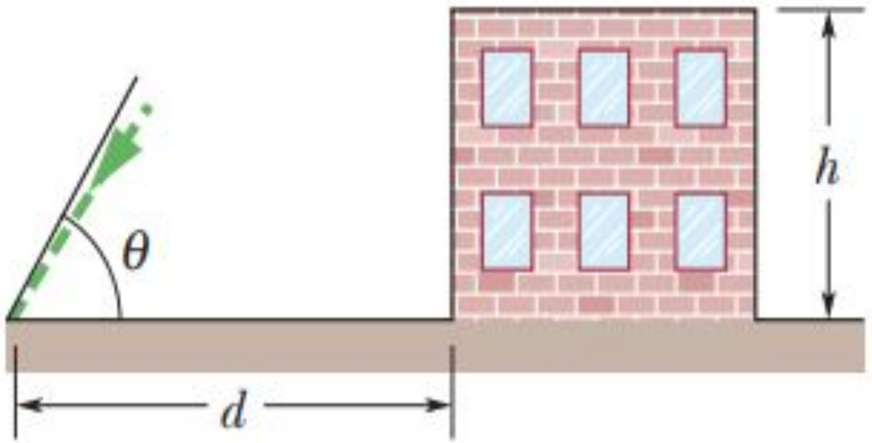
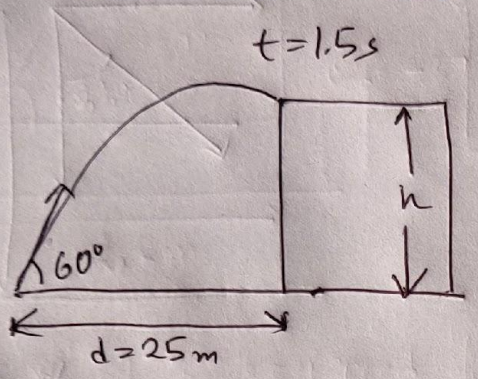


Fig. 4-37 Problem 39.

(39) **Reverse the motion**



a) Find h

b) Magnitude of ~~horizontal~~ velocity

c) angle relative to the horizontal velocity at which ball is thrown?

d) Is the angle above or below the horizontal?

a) $x - x_0 = V_0 \cos \theta_0 \times t$

$\Rightarrow 25 - 0 = V_0 \cos 60 \times 1.5$

$\Rightarrow V_0 = 33.3 \text{ m/s}$

$\xrightarrow{+ve}$ / $\xleftarrow{-ve}$ } Usual approach

$y - y_0 = V_0 \sin \theta_0 t - \frac{1}{2} g t^2$

$\Rightarrow y - 0 = 33.3 \sin 60 \times 1.5 - \frac{1}{2} \times 9.8 \times 1.5^2$

$\Rightarrow y = h = 32.23 \text{ m}$

b) $V_x = V_{0x} = 33.3 \text{ m/s} \cos 60 = 16.7 \text{ m/s}$

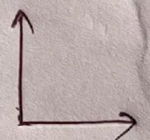
$V_y = V_{0y} - gt = V_0 \sin 60 - 9.8 \times .15$

$= 33.3 \sin 60 - 9.8 \times 1.5$

$= 14.2 \text{ m/s}$

$\therefore |\vec{V}| = \sqrt{V_x^2 + V_y^2} = \sqrt{16.7^2 + 14.2^2}$

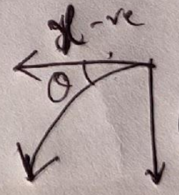
$= 21.9 \text{ m/s}$

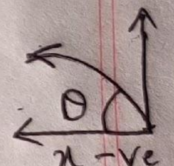
c) 

$\theta = \tan^{-1}\left(\frac{V_y}{V_x}\right)$

$= \tan^{-1}\left(\frac{14.2}{16.7}\right)$

$= 40.4°$

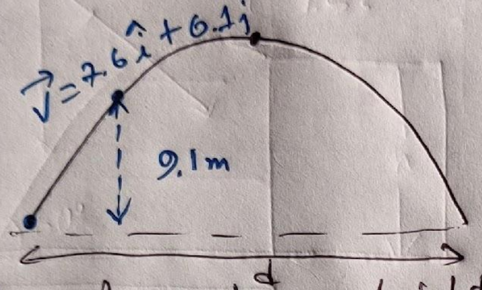
d) 



Down from the left / below the horizontal

••43 ILW A ball is shot from the ground into the air. At a height of 9.1 m, its velocity is $\vec{v} = (7.6\hat{i} + 6.1\hat{j})$ m/s, with $\hat{i}$ horizontal and $\hat{j}$ upward. (a) To what maximum height does the ball rise? (b) What total horizontal distance does the ball travel? What are the (c) magnitude and (d) angle (below the horizontal) of the ball's velocity just before it hits the ground?

(43) $\vec{v} = 7.6\hat{i} + 6.1\hat{j} \rightarrow 9.1\,m$



** Have to avoid $\cos\theta$ and $\sin\theta$

a) to get maximum height
b) total horizontal distance travelled
c) magnitude and d) angle (below the horizontal) of the ball's velocity just before it hits the ground?

$v_x = v_{0x} = 7.6\,m/s$ (always constant)

a) $v_y^2 = v_{0y}^2 - 2g(y - y_0)$

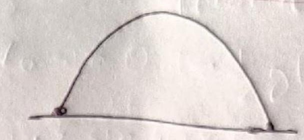
(at highest point) $\Rightarrow 0 = (6.1)^2 - 2 \times 9.8\,(H - 9.1)$

$\Rightarrow H = 11\,m$

b) $v_y^2 = v_{0y}^2 - 2g(y - y_0)$

$\Rightarrow (6.1)^2 = (v_{0y})^2 - 2 \times 9.8\,(9.1 - 0)$

$\Rightarrow v_{0y} = 14.61\,m/s$



$y - y_0 = 0$

$y - y_0 = v_{0y}t - \frac{1}{2}gt^2$

$\Rightarrow 0 = 14.6\,t - \frac{1}{2} \times 9.8 \times t^2$

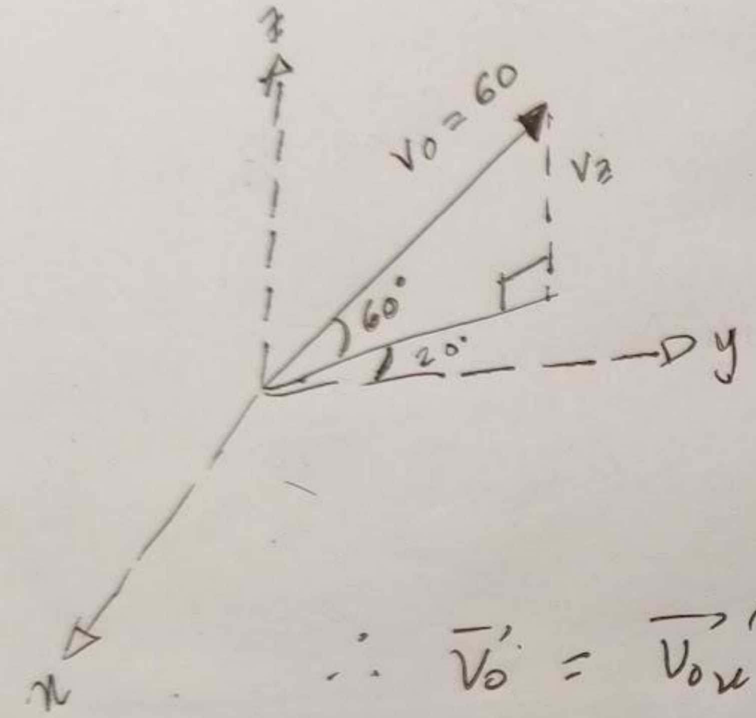
$\Rightarrow 0 = 4.9t^2 - 14.6t$

$\Rightarrow t = \frac{14.6}{4.9} = 2.97\,s$

$x - x_0 = v_{0x} \times t$

$\Rightarrow d - 0 = 7.6 \times 2.97$

$\Rightarrow d = 22.57\,m$



$V_0 = 60\,m/s$, $t = 20s$

$\sin 60° = \frac{V_{0z}}{60}$

$\Rightarrow \vec{V}_{0z} = 51.96\,m/s$

$\cos 60° = \frac{V_{0xy}}{60}$

$\Rightarrow V_{0xy} = 30\,m/s$

$\vec{V}_{0x} = V_{0xy} \cos(90° + 20°)$

$\Rightarrow \vec{V}_{0x} = 30 \times \cos 110°$

$\therefore \vec{V}_{0x} = -10.26\,m/s$

$\vec{V}_{0y} = V_{0xy} \sin 110°$

$\therefore \vec{V}_{0y} = 30 \sin 110° = 28.19\,m/s$

$\therefore \vec{V}_0 = \vec{V}_{0x}\hat{i} + \vec{V}_{0y}\hat{j} + V_{0z}\hat{k} = -10.26\hat{i} + 28.19\hat{j} + 51.96\hat{k}$

$x - x_0 = V_{0x} \times t = -10.26 \times 20 = 205.2\,m$

$y - y_0 = V_{0y} \times t = 28.19 \times 20 = 563.8\,m$

$z - z_0 = V_{0z} \times t - \frac{1}{2} g t^2 = 51.96 \cdot 20 - \frac{1}{2} \times 9.8 \times 20^2$

1. A particle is launched from point O with an initial velocity of magnitude $v0 = 60$ m /sec, directed as shown in Figure-1. Compute the x-, y-, and z-components of position, velocity, and acceleration 20 seconds after launch. Neglect aerodynamic drag.

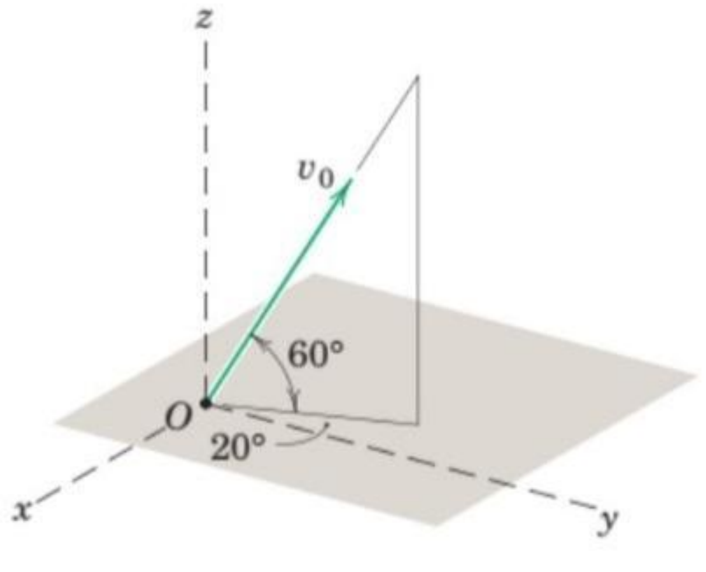


Figure-1

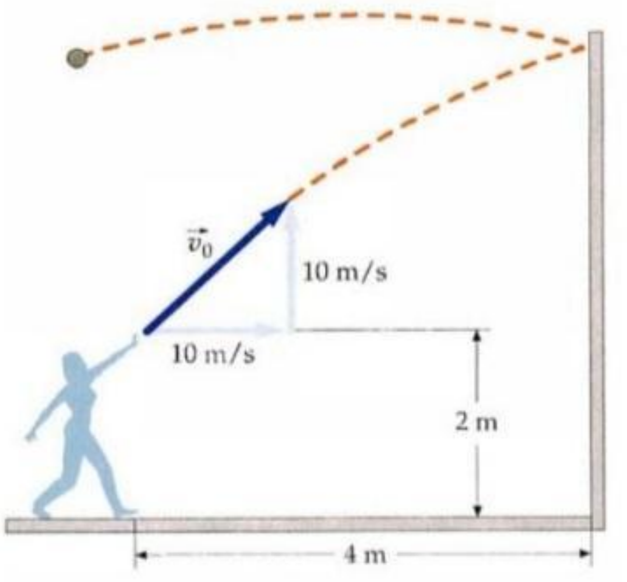


Figure-2

2. A girl throws a ball at a vertical wall 4 m away as shown in Figure-2. The ball is 2 m above ground when it leaves the girl's hand with an initial velocity of $\vec{v}_0 = (10\, m/s)(\hat{\imath} + \hat{\jmath})$ or $10\sqrt{2}\, m/s$ at 45°. When the ball hits the wall, the horizontal component of its velocity is reversed; the vertical component remains unchanged. Where does the ball hit the ground?