Assignment - 1 SET: M

Q:- 1 Prove the following

$$\lim_{x \to 0} (1+\sin x)^{\frac{1}{x}} = e$$

considering $\sin x = x$

Let, $\lim_{x \to 0} \left[(1+x)^{1/x}\right] = y$

$$\ln \left[\lim_{x \to 0} (1+x)^{1/x}\right] = \ln (y)$$

$$\lim_{x \to 0} \ln \left[(1+x)^{1/x}\right] = \ln (y)$$

$$\lim_{x \to 0} \frac{1}{x} \ln (1+x) = \ln (y)$$

$$\lim_{x \to 0} \frac{\frac{d}{dx}[\ln(1+x)]}{\frac{d}{dx}[x]} = \ln (y)$$

$$\lim_{x \to 0} \frac{\frac{1}{1+x}}{1} = \ln (y)$$

$$\lim_{x \to 0} \frac{1}{1+x} = \ln (y)$$

$$\frac{1}{1+0} = \ln y \qquad \therefore \lim_{x \to 0} (1+\sin x)^{1/x} = e$$

$$\ln (y) = 1$$

$$y = e^1$$

$$y = e$$

[Proved]

Q:-2

(a) $f(x) = \frac{x^2-4}{x-2}$

at $x=2$ $f(2) = \frac{2^2-4}{2-2}$

$= \frac{0}{0}$

$f(x)$ is in indeterminate format at $x=2$
thats why $f(x)$ is discontinuous. function.

(b) $g(x) = \begin{cases} \frac{x^2-4}{x-2} & x \neq 2 \\ 3 & x=2 \end{cases}$

at $x=2$

$g(2) = \left\{ \frac{2^2-4}{3\ 2-2} \right\}$

$= 3\ \frac{0}{0}$

$g(x)$ is in indeterminate format at $x=2$
thats why $f(x)$ is discontinuous function.

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(c) $h(x) = \begin{cases} 4^{\frac{x^2-4}{x-2}} & x \neq 2 \\ 4 & x = 2 \end{cases}$

at $x=2$

$$h(2) = 4^{\frac{2^2-4}{2-2}}$$

$$= 4^{\frac{0}{0}}$$

$h(x)$ is in indeterminate format at $x=2$

thats why $h(x)$ is discontinuous function.

③

$$\lim_{x\to 0} \frac{x}{\sqrt{x+1}-1}$$

$$= \lim_{x\to 0} \frac{x(\sqrt{x+1}+1)}{(\sqrt{x+1}-1)(\sqrt{x+1}+1)}$$

$$= \lim_{x\to 0} \frac{x(\sqrt{x+1}+1)}{(\sqrt{x+1})^2-(1)^2}$$

$$= \lim_{x\to 0} \frac{x(\sqrt{x+1}+1)}{x+\cancel{1}-\cancel{1}}$$

$$= \lim_{x\to 0} \frac{\cancel{x}(\sqrt{x+1}+1)}{\cancel{x}}$$

$$= \lim_{x\to 0} (\sqrt{x+1}+1)$$

$$= \sqrt{0+1}+1$$

$$= 2$$

Ans.

(4) $$\lim_{x \to 0} \frac{\sqrt{x^2+4} - 2}{x}$$

$$= \lim_{x \to 0} \frac{(\sqrt{x^2+4} - 2)(\sqrt{x^2+4} + 2)}{x(\sqrt{x^2+4} + 2)}$$

$$= \lim_{x \to 0} \frac{(\sqrt{x^2+4})^2 - (2)^2}{x(\sqrt{x^2+4} + 2)}$$

$$= \lim_{x \to 0} \frac{x^2+4-4}{x(\sqrt{x^2+4} + 2)}$$

$$\Rightarrow \lim_{x \to 0} \frac{x}{\sqrt{x^2+4} + 2}$$

$$\Rightarrow \frac{0}{\sqrt{0+4} + 2}$$

$$= \frac{0}{4}$$

$$= 0$$

(5) $$\lim_{t \to -\frac{1}{2}} \frac{4t^2 - 1}{4t^2 + 8t + 3}$$

$$= \lim_{t \to -\frac{1}{2}} \frac{\frac{d}{dt}(4t^2 - 1)}{\frac{d}{dt}(4t^2 + 8t + 3)}$$

$$= \lim_{t \to -\frac{1}{2}} \frac{8t}{8t + 8}$$

$$= \frac{8\left(-\frac{1}{2}\right)}{8\left(-\frac{1}{2}\right) + 8}$$

$$= \frac{-4}{-4+8}$$

$$= -1$$

Ans.