Vector Resolution: Geometric and Analytical

30/05/2024
Thursday

$\boxed{\vec{A} = \sqrt{A_x^2 + A_y^2}}$

- Sometimes -ve due to direction

$\boxed{\vec{A} = A_x\hat{i} + A_y\hat{j}}$

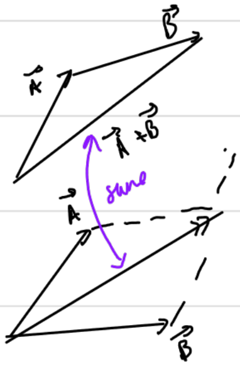
Unit Vector: magnitude 1 and shows direction

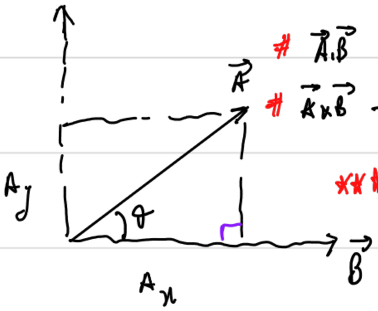
$\boxed{\hat{\eta}_A = \frac{\vec{A}}{|\vec{A}|} = \left(\frac{1}{\sqrt{\ }}\right)\hat{i} + (\quad)\hat{j}}$

vector component, unit-vector → same notation → in terms of i, j, k

*** As long as direction and magnitude remains unchanged, a vector remains same

*** Free transform property of vectors





$\vec{A}.\vec{B}$ → Parallel component

$\vec{A}\times\vec{B}$ → Perpendicular component

*** Direction changes but sign remains same for dot product but changes for cross product

- Can we call x & y-axis as vectors?

= The x and y axes themselves are NOT vectors, but they can be represented by unit vectors. So, while the axes themselves are not vectors, their directions are represented by vectors.

*** Vectors can't be divided but can be added, subtracted & multiplied. → vector product

Vector Field: In every field there is a magnitude & direction eg. wind direction

Scalar Field: e.g. Temp. of room

*** In electric field, intensity is higher tho closer. Direction is shown   charge has field & ____

- In Scalar field: Direction is not shown

Vector Calculus: Gradient, Divergence, Curl

$\vec{\nabla}$ → del, nabla	∇S → Gradient (vector)
$\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$	$\vec{\nabla}.\vec{V}$ → Divergence (Scalar)
Vector Operator	$\vec{\nabla}\times\vec{V}$ → Curl (vector)

Work Done

$\boxed{W = \int \vec{F}.d\vec{r}}$ [Dot product bet^n Force and displacement]

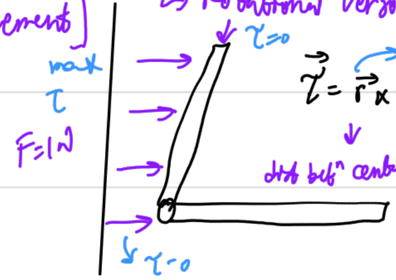
$\boxed{W = \Delta k}$ [Work-Energy Theorem]

Conservation of Energy

Torque and Rotational Motion

↳ Rotational version of Force

$\vec{\tau} = \vec{r}\times\vec{F}$ ($\vec{r}$: moment arm) ↓ dist bet^n centre of rotation and Force



L-2

Electric Field

01/06/2024
Saturday

Electric (Field) → Electrostatics (Microscopic scale) · We are in macroscopic scale
↳ Reason (Electric Charge) ⟶ Electron, Proton, Neutron

Charge: $-1e$ $+1e$ $0e$ [Two types of charge, Not 3]

all of them are in: $\boxed{q = ne}$ $n = +1/0/-1$

⟶ 4 Fundamental Forces

$\boxed{e = 1.602 \times 10^{-19} C \text{ [Scalar]}}$

$e^- = -1.602 \times 10^{-19} C$

electron $(-1\bar{e})$ ✗

The magnitude of the charge of the electron or proton is a natural unit of charge called the electron number

Properties:

1) Conservation of electric Charge: Can't create or destroy them
2) Quantization of electric charge: At microscopic scale
3) Additivity of charges: Charge is Scalar
4) Charge Interactions: 3 types; $++$, $+-$, $--$. Opposites attract, like charges ~~attract~~ same
5) Coulomb's Law of electrostatics: Measures Electrostatic Force [Electric, Electrostatic, Coulomb] $\vec{F}_E$

(From Slides)

Problem 1: Sweater balloon

Sweater	balloon
0	0
$0-10\bar{e}$	$0+10\bar{e}$ [Total charge 0]

∴ They will stick

· charges move from contact point but does not jump

Q) Why does this happen?

Problem 2: # $1p^+ \longrightarrow 1.67 \times 10^{-27}$ kg

1.67×10^{-27} kg $\longrightarrow 1p^+$

1 kg $\longrightarrow \frac{1}{m_p}(\ )$

$\boxed{\frac{q_p}{m_p} \text{ Charge to mass ratio}}$

$q_p = +1.602 \times 10^{-19} C$

$q = ne$
can't be quantized at this scale

H.W.

$1\bar{e} \longrightarrow 9.11 \times 10^{-31}$ kg

1 kg $\rightarrow \frac{1}{9.11 \times 10^{-31}} = \frac{q_e}{m_e}$ (is it -ve??)

Electrostatic / Coulomb Force

→ For Coulomb's Law:

1) Two point charges: One is a source, one is an observer
2) Magnitude is positive [Force → vector → magnitude never -ve]
3) Distance needs to be not atomic

Q) What is point charge?

Q) What is a charge?

$F \propto \frac{q_1 q_2}{r^2}$

$$\boxed{\vec{F} = \Big(\underbrace{\frac{1}{4\pi\varepsilon_0} \times \frac{|q_1 q_2|}{r^2}}_{\text{magnitude}}\Big)\hat{r}}$$

direction of force

$\vec{F}_G = \left(\frac{G\,M_m}{r^2}\right)\hat{r}$

$F = \boxed{\frac{1}{4\pi\varepsilon_0}} \frac{|q_1 q_2|}{r^2}$

[inverse square law] and an action-reaction pair

$\boxed{8.988 \times 10^9 \; Nm^2 C^{-2}}$

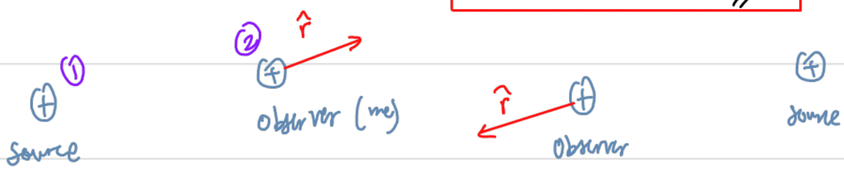
Wrong concept: $F = C\frac{q_1 q_2}{r}$

+ repulsion
− Attraction
} but for two charges what if more than two.

$\boxed{C = \frac{F r^2}{q_1 q_2} \sim Nm^2 C^{-2}}$

$\hat{n}_A = \frac{\vec{A}}{A}$ $\therefore \hat{r} = \frac{\vec{r}}{r}$ $\therefore \boxed{\vec{F}_E = \left(\frac{1}{4\pi\varepsilon_0}\frac{|q_1 q_2|}{r^3}\right)\vec{r}}$ //

- Forms an inverse square law and action-reaction pair
- Source creates electric field



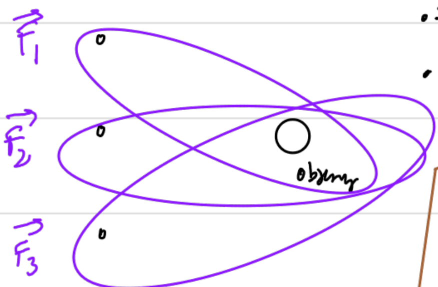
$\vec{F}_{2 \text{ due to } 1} = (\ )\,\hat{r}_{21}$ # $\vec{F}_{1 \text{ due to } 2} = (\ )\,\hat{r}_{12}$

or 1 on 2

$\boxed{\hat{r}_{12} = -\hat{r}_{21}}$ $\therefore \boxed{\vec{F}_{21} = -\vec{F}_{12} \text{ [Action-Reaction pair]}}$

Coulomb Forces for Discrete Charge Distribution

- Multiple source, 1 observer



- Find individual force
- vector add then
  ↓
  Superposition Principle

for $\vec{F}_E$: Discrete

One Observer multiple source

The total force on Q q_i:

$$\vec{F}_Q = \vec{F}_{1Q} + \vec{F}_{2Q} + \vec{F}_{3Q} + \dots \vec{F}_{nQ}$$

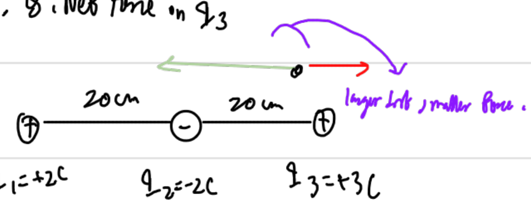
$$= \frac{Qq_1}{4\pi\varepsilon_0 r_{1Q}^2}\hat{r}_{1Q} + \frac{Qq_2}{4\pi\varepsilon_0 r_{2Q}^2}\hat{r}_{2Q} + \dots \frac{Qq_n}{4\pi\varepsilon_0 r_{nQ}^2}\hat{r}_{nQ}$$

$$= \sum_i^N \frac{Qq_i}{4\pi\varepsilon_0 r_{iQ}^2}\hat{r}_{iQ}, \quad \vec{r}_{iQ} = \vec{r}_Q - \vec{r}_i$$

$\vec{r}_i$ → position vector of the i^{th} charge in the distribution

$\vec{r}_Q$: Position vector of observer charge

Problem 3: Q: Net Force on q_3

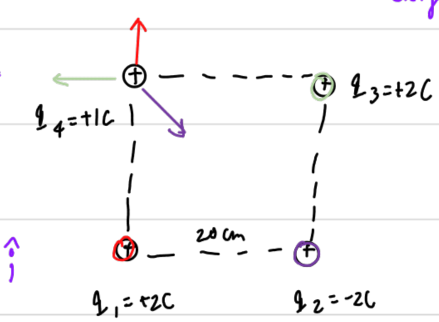


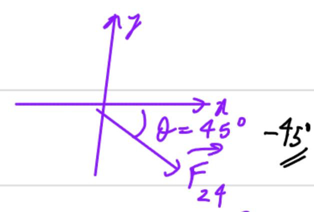
$$\vec{F}_3 = \vec{F}_{13} + \vec{F}_{23} = \left(\frac{C\, q_1 q_3}{(40\times10^{-2}\text{m})^2}\right)\hat{i} - \left(\quad\right)\hat{i}$$

effect right side

= +ve [left side]

Problem 4) Q: Net force on q_4





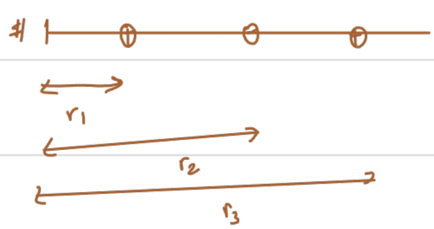
$$\vec{F}_4 = \vec{F}_{14} + \vec{F}_{34} + \vec{F}_{24}$$

$$= \frac{C q_1 q_4}{(20\times10^{-2})^2}(\hat{j}) + \frac{C q_3 q_4}{(20\times10^{-2})^2}(-\hat{i}) + (\ )\hat{i} + (\ )\,{-\hat{j}}$$

$$\frac{C q_2 q_4}{(\sqrt{2}\times 20\times10^{-2}\text{m})^2}$$

$F_{24,x} = F_{24}\cos(-45) = $ +ve

$F_{24,y} = F_{24}\sin(-45) = $ -ve



06/06/2024

Thursday

Problem-2 (i) $F_G = \frac{G m_1 m_2}{r^2}$, $F_E = \frac{C q_1 q_2}{r^2}$, $q = 1.602\times10^{-19}\,C$

$$\frac{F_E}{F_G} = \frac{c|q|^2}{G m_p m_e} \sim 10^{39}$$

$m_p = 1.67\times10^{-27}\,kg$

$m_e = 9.1\times10^{-31}\,kg$

$\therefore F_E >>> F_G$ $\therefore F_G$ is insignificant compared to F_E. Thus gravitational force can be safely ignored.

(ii) ⊕ —— r —— ⊕

$F_g = mg$

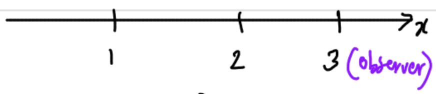
$F_E = m_p g$

$\frac{C q_p^2}{r^2} = m_p g$ $\rightarrow$ $\therefore r = \sqrt{\frac{C q_p^2}{m_p g}}$

$\sim 0.128\,m$

(wrong in Ques: $L_{23} = 3L_{13}$)

Problem-3: Condition for force cancellation



F_{13} — source, observer

★ $\vec{F}_{13} = -\vec{F}_{23}$ (vector, thus −ve sign)

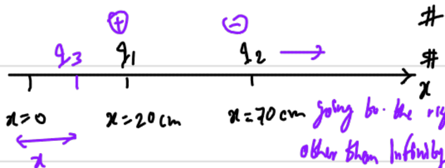
$F_{13} = F_{23}$ (Scalar, thus no −ve sign)

$F_{13} = F_{23}$

$$\frac{c q_1 q_3}{r_{13}^2} = \frac{c q_2 q_3}{r_{23}^2}$$

$$\Rightarrow \left|\frac{q_1}{q_2}\right| = \frac{r_{13}^2}{r_{23}^2} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

Problem-4: H.W Fixed Particles → Source charge



$q_2 = -4.00\,q_1$ (wrong in Question)

$q_3 = 2q_1$

going to the right other than infinity dist. force won't cancel

$F_{13} = F_{23}$

$$\frac{C q_1 q_3}{(20-x)^2} = \frac{C q_2 q_3}{(70-x)^2}$$ → $x = \pm 30\,cm$

$x = 30$ → q_3 will be in betn q_1 & q_2

$x = -30$

The Superposition Principle for $\vec{F}_E$: Continuous

① $q_i \rightarrow dq$

② $\Sigma \rightarrow \int$

$$\boxed{\vec{F}_Q = \int \frac{Q\,dq}{4\pi\varepsilon_0 r^2}\,\hat{r}_{dqQ}}$$

$Q = \sum_i^N q_i$ [Discrete]

$Q = \int dq$ [Continuous]

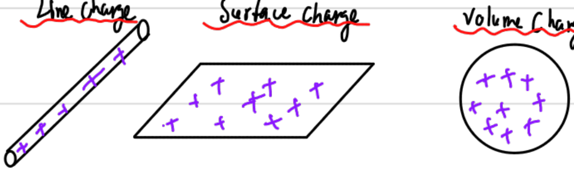
Coulomb Forces for Continuous Charge Distribution

$$\boxed{F_E = \frac{C\,q_{source}\,q_{observer}}{r^2}}$$, Charge element dq (not a single charge)

$q_i \rightarrow dq$, $\Sigma \rightarrow \int$

→ The charge is distributed into a shape. The shape has dimensions

Line charge — Surface charge — Volume Charge



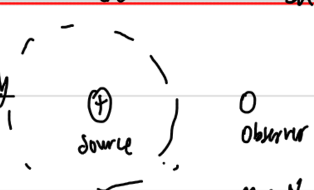
why +ve charge??

Charge Density

(1D) $\lambda = \frac{\text{charge}}{\text{line}}$, $C\,m^{-1}$ $= \frac{dq}{dl}$	(2D) $\sigma = \frac{\text{charge}}{\text{area}}$, $C\,m^{-2} = \frac{dq}{da}$	(3D) $\rho = \frac{\text{charge}}{\text{Volume}}$, $C\,m^{-3} = \frac{dq}{dv}$

How does charge create electric field?

Previously

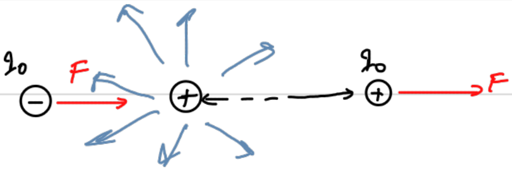


{ Force is completely observer dependent
Field is completely source dependent }

Electric Field: → Magnitude (Intensity)
→ Direction (direction)

Test Charge → Is brought into the electric field of the source.

+1 C or nC depending on the source



Electric Field Intensity $\boxed{\vec{E} = \dfrac{\vec{F}}{q_{observer}}}$ → usually a test charge

Electric Field $\boxed{\vec{E} = \left(\dfrac{C q_{source}}{r^2}\right)\hat{r}}$

$$\boxed{\begin{aligned}\vec{E} &= \lim_{q_0 \to 0} \frac{\vec{F}_E}{q_0} \\ &= \left(\frac{1}{4\pi\varepsilon_0}\frac{|q|}{r^3}\right)\vec{r} \\ &= \left(\frac{1}{4\pi\varepsilon_0}\frac{|q|}{r^2}\right)\hat{r}\end{aligned}}$$

$\boxed{\vec{F}_E = q_{ob}\vec{E}}$ → ⊕ : $\vec{F}$ direction same as $\vec{E}$
⊖ : opposite

$\vec{F} = q\vec{E}$ → $\vec{F}$ exists as q is in field

↑ not source

q_0/q_{ob} did not make this $\vec{E}$.
q_0 only measures its effect

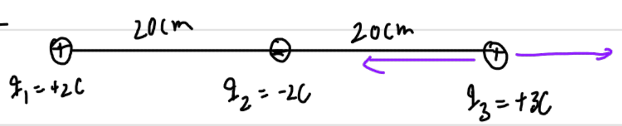
The Superposition Principle for $\vec{E}$: Discrete

$$\boxed{\vec{E} = \frac{\vec{F}}{q_0} = \frac{\vec{F}_1}{q_0} + \cdots = \frac{|q_1|}{4\pi\varepsilon_0 r_{1r}^2} \cdots \frac{|q_n|}{4\pi\varepsilon_0 r_{nr}^2}\hat{r}_{nr} = \sum_i^N \frac{|q_i|}{4\pi\varepsilon_0 r_{ir}^2}\hat{r}_{ir}}$$

L-2 / P-4   now find the net E on q_4

$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$ Nothing to do with observer charge

Problem-5



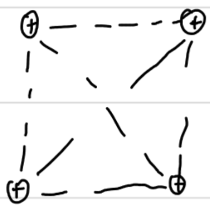
$\vec{E} = \dfrac{\vec{F}}{q_0}$

$\vec{E}_4 = \vec{E}_{14} + \vec{E}_{34} + \vec{E}_{24}$

$= \dfrac{Cq_1}{r_{14}^2}\hat{i} + \cdots$

Problem-6 Calculate the net field at the center. Take $q_1 = q_2 = q_3 = q_4 = +2nC$

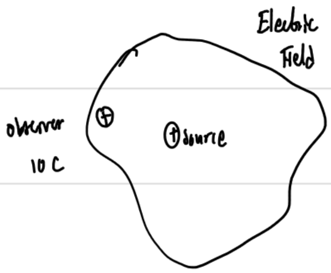
$\vec{F}_E = q_{ob}\vec{E}$
↓
zero → so $\vec{E} = 0$



Problem-7 H.W.

⋮

Electric Field Intensity



$$F = \frac{C q_{source} \times q_{observer}}{r^2} \times \hat{r}$$

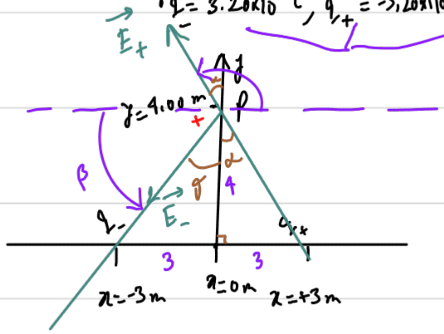
$$\frac{\vec{F}}{q_{observer}} = \vec{E} = \frac{C q_{source}}{r^2} \times \hat{r} \Rightarrow \vec{F} = q_{observer} \vec{E}$$

Electric Field, $\vec{E}$ [NC^{-1} or Vm^{-1}] is defined as the Force per unit positive charge exerted on a test charge.

$$\boxed{\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0} = \left(\frac{1}{4\pi\varepsilon_0} \frac{|q|}{r^3}\right)\vec{r} = \left(\frac{1}{4\pi\varepsilon_0} \frac{|q|}{r^2}\right)\hat{r}}$$

Problem-1

- fixed → source charge
- $q_- = 3.20 \times 10^{-19}$ C, $q_+ = -3.20 \times 10^{-19}$ C → Electric Dipole



(a) Put one unit of test charge as there is no observer charge

$\vec{E} = \vec{E}_+ + \vec{E}_-$ → no need for +, − for quadrants as angle for +ve axis

$E_+ \cos(126.86)\hat{i} + E_+ \sin(126.86)\hat{j} + E_- \cos(233.13)\hat{i} + E_- \sin(233.13)\hat{j}$

$\tan\alpha = \left(\frac{3}{4}\right)$ $\tan^{-1} \frac{4}{3}$

$\alpha = \tan^{-1}\left(\frac{3}{4}\right)$

$= 36.86°$

$\therefore 90 + 36.86$

$\vec{E} = (\ )\hat{i} + (\ )\hat{j}$

magnitude $= \sqrt{(\ )^2 + (\ )^2}$

For direction look at sign of $\hat{i}$ & $\hat{j}$ then according to quadrants find angle

(c) $\vec{F}_E = q_e \vec{E}$ (−ve)

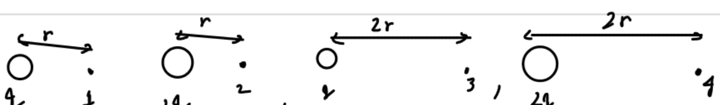
$\therefore \sum \vec{F} = m\vec{a}$

$\therefore \vec{a} = \frac{\vec{F}_E}{m_e}$

- Direction of e^- in the direction of Force

// −charge always opposite to $\vec{E}$ direction

Problem-2

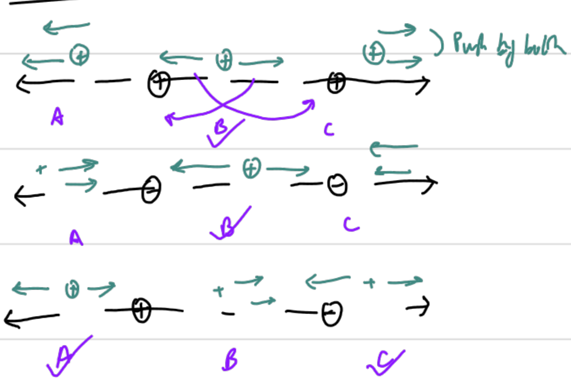


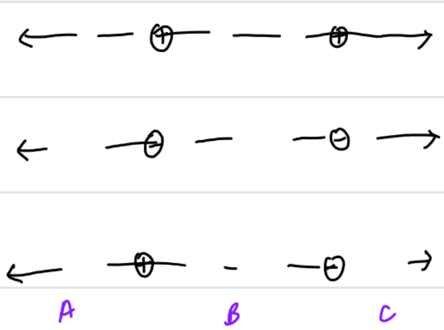
$\therefore E_2 > E_1 > E_4 > E_3$

$E_1 = \frac{Cq}{r^2}$, $E_2 = \frac{C \times 2q}{r^2} = 2E_1$, $E_3 = \frac{C \times q}{(2r)^2} = \frac{1}{4}\frac{Cq}{r^2} = \frac{1}{4}E_1$, $E_4 = \frac{C \times 2q}{(2r)^2} = \frac{1}{2}\frac{Cq}{r^2} = \frac{1}{2}E_1$

Ascending order: $E_3 < E_4 < E_1 < E_2$

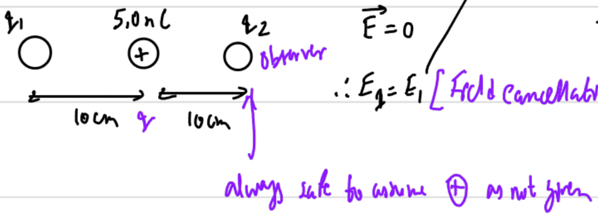
Problem-3





Problem-4

Q) q_2 is in equilibrium. What is q_1? What type?



$\vec{E} = 0$

$\therefore E_q = E_1$ [Field cancellation]

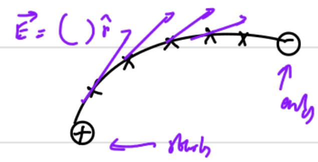
$E_q = E_1$

$$\frac{c\,q_1}{(20\times10^{-2}\,m)^2} = \frac{c\times5\times10^{-9}\,C}{(10\times10^{-2}\,m)^2}$$

$q_1 = (\quad)$; negative

Q) How to see Electric Field?

- Electric field Lines [not real.]
- They are continuous curves tangent to the electric field vectors



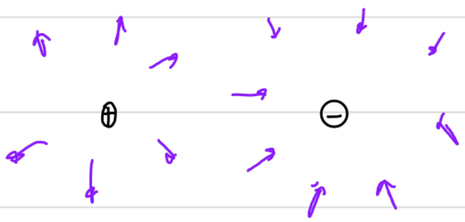
Trajectory

Path of motion

- The motion is projectile

*field lines are not vectors

*** Field lines are not trajectory



*** radially outwards, inwards

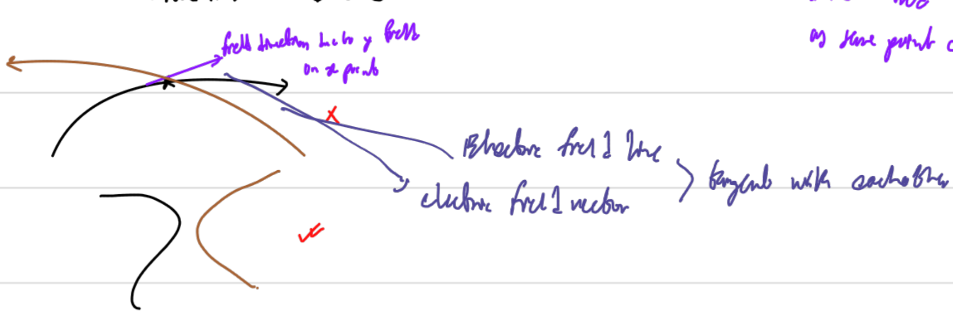
So far discrete

* The principle of superposition! Vector adding. Nothing to do with $\vec{F}$ or $\vec{E}$

Electric Field Lines:

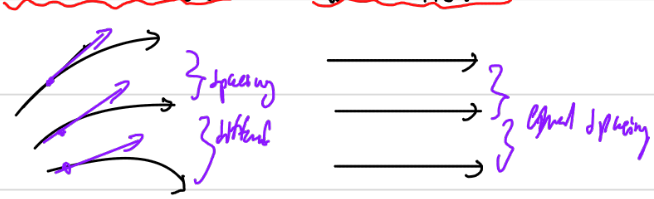
- They are continuous curves tangent to the electric field vectors
- These are not trajectories
- Field lines from ⊕ to ⊖

- Electric field lines can't cross eachother as same point can't have two Electric fields



Non-uniform field

uniform field



Electric Field lines of Charged Particles 3D

Individually they create non-uniform field

less field lines in same area so non-uniform

E-f lines same dist away from the source

radially outward   radially inward

Problem-1 (a) ⊕ ⊖ ⊕

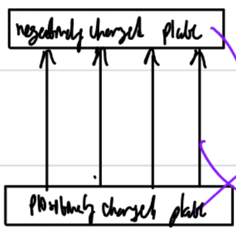
reasoning! ⊕ charge radially outward, ⊖ charge radially inward

(b) vectors have direction. We're doing vector sum. So it's possible to have smaller value. This is superposition principle

Problem-2 $E_3 = E_4 > E_2 > E_1$ Ascending order! $E_1 < E_2 < E_3 = E_4$

Non uniform

Motion in Uniform $\vec{E}$ Field: Straight



• If we put charged particles instead of plate, Electric field lines would be non-uniform

Continuous charge distribution

field lines density very high, have equal dist.

scalar = +ve

$m\vec{a} = \vec{F}_E = q\vec{E}$ • Force in the direction of $\vec{E}$

• $\vec{a}$ direction

$q(+ve)$: $\hat{j}$ $\hat{j}$ $\hat{j}$

$q(-ve)$: $-\hat{j}$ $-\hat{j}$ $\hat{j}$

work with ⊕ & ⊖ when working with direction/vectors

Problem-3: (i) (1-D) $E = \frac{F_E}{q_e} = \frac{ma}{q_e} = \frac{m\left(\frac{v^2-v_0^2}{2\Delta d}\right)}{q_e} = \frac{9.11\times10^{-31}\times\left(\frac{(4\times10^7)^2-(2\times10^7)^2}{2\times1.5\times10^{-2}}\right)}{1.6\times10^{-19}}$

$v_0 = 2\times10^7\ ms^{-1}$

$v = 4\times10^7\ ms^{-1}$

$\Delta d = 1.5\times10^{-2}\ m$ (final-initial)

$v^2 - v_0^2 = 2a\Delta d$ don't give -ve

$a = \frac{v^2-v_0^2}{2\Delta d}$

$= 22.77\times10^4\ NC^{-1}$

(ii) $v_{0x} = 2\times10^7\ ms^{-1}$ x direction in +ve x-axis → $+\hat{i}$

$v_x = 4\times10^7\ ms^{-1}$

$\Delta d_x = 1.5\times10^{-2}\ m$ ∴ $\vec{E}$ $-\hat{i}$ (final-initial)

$\therefore E = \frac{F_E}{q} = \frac{m_e a_x}{\ominus q_e}$ For y-axis, z-axis $\hat{j}$, $\hat{k}$

Problem-4

$F_E = ma = qE$ Newton's law, Coulomb's law

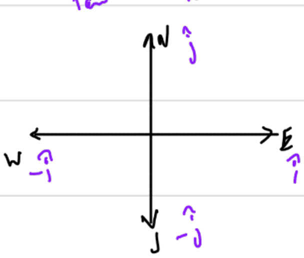
$\vec{E}$ fixed → $\vec{a}$ fixed

$\vec{E} = (50\hat{j})\ NC^{-1}$ y-component so no need to say $\hat{j}$

$v_{0y} = 40\times10^3\ ms^{-1}$

$v_y = ?$

$t = 1.5\times10^{-9}\ s$



technically Cartesian coordinate system

(a) # $a_y = \frac{qE}{m_e}$ (q → -ve) $= -8.80\times10^{12}\ ms^{-2}$ (deceleration)

$v_y = v_{y0} + a_y t = 26.20\ kms^{-1}$ (Speed decreased, so deceleration)

(b) $v_y^2 - v_{0y}^2 = 2a_y\Delta d$ / or $\Delta d = v_0 t + \frac{1}{2}a_y t^2$

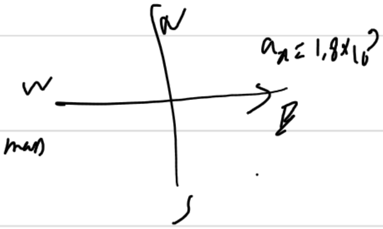
$\therefore \Delta d = 5\times10^{-5}\ m$

always +ve   put a -ve and check

HW Eastward: +ve $\hat{i}$, $\vec{E}$ $-\hat{i}$

positron: +le e^+ } same mass

electron: $\bar{e}$



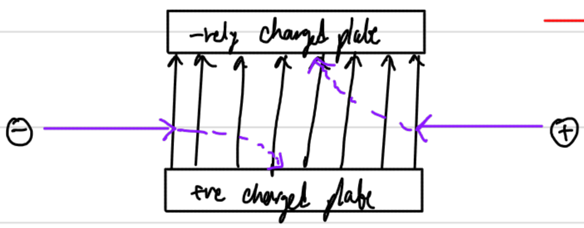
Motion in Uniform $\vec{E}$ Field: Parabolic

L-5

22/06/2024

Saturday

Motion in Uniform Electric Field (Parabolic)



→ The charged particle now perpendicularly enters the field, with uniform velocity.

* Deviation of θ & Θ different?
* Use field lines thought??

Problem-1

* Beam of $\bar{e}$ horizontally

$V_{x_0} = 3.3\times10^7\ ms^{-1}$

$\Delta y = $ (final position - initial position) $= 2\times10^{-2}\ m$

$\vec{E} = (5\times10^4\ NC^{-1})\hat{j}$ (up)

$V_{y_0} = 0\ ms^{-1}$

$a_x = 0$, $a_y = -g$ } from projectile motion

$V_x = V_{x_0} + \cancel{a_x t}$ (same)

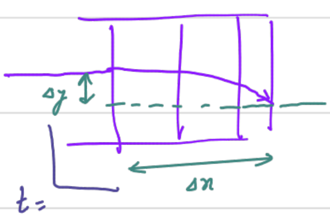
$\Delta y = V_{x_0} t$

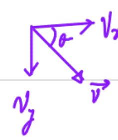
$t = \frac{\Delta y}{V_{x_0}}$

$V_y = \cancel{V_{y_0}} + a_y t$

$\frac{qE}{m} \to -ve$ → meaning $\bar{e}$ moving opposite to $\vec{E}$

$\boxed{\text{Time to cross horizontal dist} = \text{Time to cross vertical dist}}$





$\boxed{\text{Deflection angle, } \theta = \tan^{-1}\left(\frac{V_y}{V_x}\right)}$

$a_y = \frac{qE}{m} = -8.8\times10^{15}\ ms^{-2}$

$t = \frac{\Delta y}{V_{x_0}} = 6.06\times10^{-10}\ s$

$V_y = a_y t$

$= (-8.8\times10^{15}\ ms^{-2})(6.06\times10^{-10}\ s)$

$V_y = -5.3\times10^6\ ms^{-1}$

$V_x = 3.3\times10^7\ ms^{-1}$

HW SP: sample problem

Problem-2

$m = 1.3\times10^{-10}\ kg$

given $Q = -1.5\times10^{-13}\ C$

$V_{x_0} = -18\ ms^{-1}$

$L = \Delta x = 1.6\times10^{-2}\ m$

Uniform electric field = Constant acceleration acting downward $= -\hat{j}$

$\vec{E} = (1.4\times10^6\ NC^{-1})(-\hat{j})$, $\Delta y = ?$

$a_x = 0$, $a_y = \frac{\overset{-ve}{Q}\ \overset{-ve}{E_y}}{m} = +1615.385\ ms^{-2}$

$\Delta x = V_{x_0} t$

$t = \left|\frac{-1.6\times10^{-2}\ m}{-18\ ms^{-1}}\right|$

$= 8.89\times10^{-4}\ s$

$\Delta y = \cancel{V_{y_0} t} + \frac{1}{2} a_y t^2$

$= 6.382\times10^{-4}\ m$

Going Continuous

Discrete $\longrightarrow$ Countable

Continuous $\longrightarrow$ Not countable

Discrete Calculation

Start with $\vec{E} = \left(\frac{cq}{r^2}\right)\hat{r}$ add up the individual $\vec{E}$

Superpose them: $\vec{E} = \sum_i^N \left(\frac{cq_i}{r_i^2}\right)\hat{r}_i$ [Discrete]

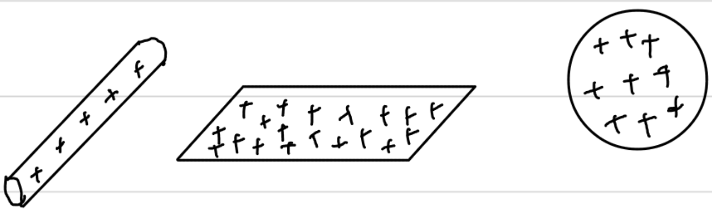
Continuous Calculation

Start with $d\vec{E} = \left(\frac{c\,dq}{r^2_{dq}}\right)\hat{r}_{dq}$

(continuous)

Integrate then $\vec{E} = \int \left(\frac{c\,dq}{r^2_{dq}}\right)\hat{r}_{dq}$

3 Key $\vec{E}$ Field Sources



λ	1D
σ	2D
ρ	3D

Electric Charge Densities $\frac{dq}{dl} = \lambda$, $\frac{dq}{da} = \sigma$, $\frac{dq}{dV} = \rho$ (distribution)

Total Charge, $Q = \int dq = \begin{cases} \lambda\, dl \\ \sigma\, da \\ \rho\, dV \end{cases}$

$dq = (\text{density}) \times (\text{distribution})$ dimension

$E = \int \frac{c\,dq}{r^2}$ $\leftarrow$ continuous

$E = \frac{cq}{r^2}$, $E = \int \frac{c\,dq}{r^2}$, $dq = \lambda\, dl$

$\therefore E = \left(c\int \frac{\lambda\, dl}{r^2}\right)\hat{r}_{dq}$

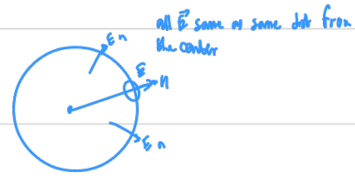
Electric Flux and Gauss' Law

Gauss's Law

Electric flux, Φ_E (scalar)

Net electric flux = $\frac{\text{charge enclosed}}{\varepsilon_0}$

→ flow of electric field



Flux: Way to measure the intensity of a source flow (without touching the source)

Field: Along the direction of your net

Area vector: $A\hat{n} = \hat{A}$ — dot product

$$\boxed{\Phi_E = \vec{E} \cdot \vec{A}}$$ $[N m^2 C^{-1}]$ or $[V m]$ — just a value

Flat surface → $\hat{n}$ same

scalar as dot product $= \vec{E} \cdot (A\hat{n})$ — cosine

so can be +ve, -ve, zero

Non-Uniform

$$\boxed{\Phi_E = \int \vec{E} \cdot d\vec{a}}$$

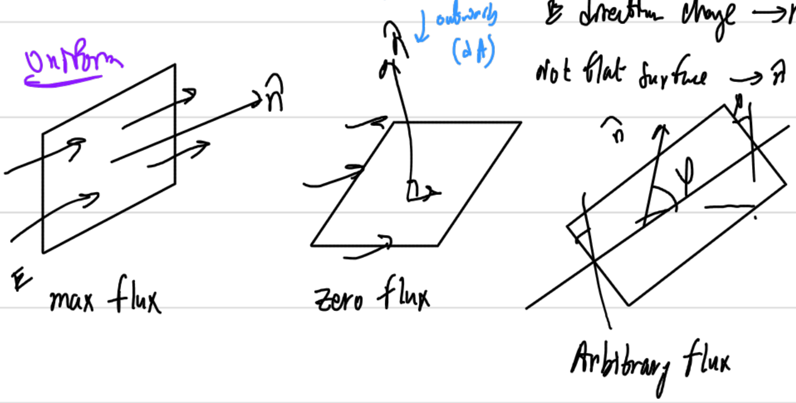
not thinking about the source so don't think about from where $\vec{E}$ comes from

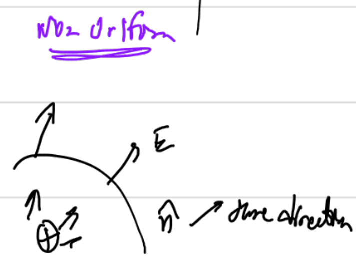
Uniform Electric Field

convention is outwards (dA)

$\vec{E}$ direction change → non-uniform

not flat surface → $\hat{n}$ direction different





Flux Measure

Visually: The no. of electric field lines passing through a given surface

Numerically: The surface integral of $\vec{E}$-field

$\boxed{\text{same } q\text{, so flux same. Shape doesn't matter}}$

$\Phi_E = \vec{E} \cdot \vec{A}$ (Uniform)

$\Phi_E = \int \vec{E} \cdot \vec{A}$ (Non Uniform)

Unit: $[N m^2 C^{-1}]$ $[V m]$

scalar

L-6

27/06/2024
Thursday

Problem-1

Problem-2

MCQ 23-1

Similar Prob

Ex: 1, 2, 3, p' 677

$\vec{A} = A \cdot \hat{n} = 5 \times (10^{-2})^2 \hat{k}$

$\Phi = \vec{E} \cdot \vec{A}$

$= (2000\hat{i} - 4000\hat{k}) \cdot (5 \times 10^{-4} \hat{k}) = -2\hat{k}$

$$\boxed{\Phi_E = \oint \vec{E}\cdot d\vec{a} = \frac{q_{enc}}{\epsilon_0}}$$ $$\boxed{\epsilon_0 = 8.854\times10^{-12}\ F\,m^{-1}}$$

Permittivity of free space

★ q_{enc} is the net charge calculated by taking the algebraic sum of all charges enclosed by the Gaussian surface

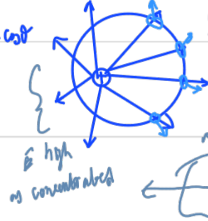
$\phi = \int \vec{E}\cdot d\vec{A} = \int E\,dA\cos 0$

$= E\int dA$

$= \frac{q}{4\pi\epsilon_0 r^2}\times 4\pi r^2$

$\phi = q/\epsilon_0$ — doesn't depend on R → derived from sphere but same for any closed surface

if $R\uparrow$, $A\uparrow$, $\frac{1}{r^2}$ $E\downarrow$



$\phi = \int \vec{E}\cdot d\vec{A} = \int E\,dA\cos\theta$ ← not 0

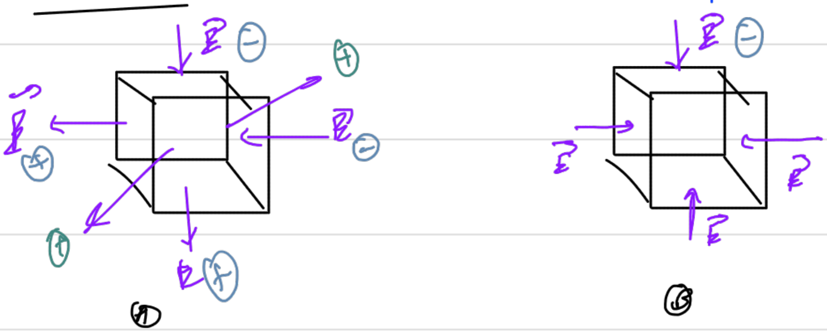
not uniform anymore

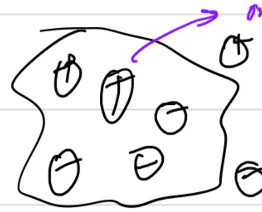
still ϕ same

• 2 enters but also leaves so overall net flux the flux = 0

• So external charges don't affect

Problem-3





☆ Problem-4

Φ doesn't depend on R

$\Phi = \vec{E}\cdot\vec{A} = \frac{q_{enc}}{\epsilon_0}$

(B) = (E) > (A) = (C) = D

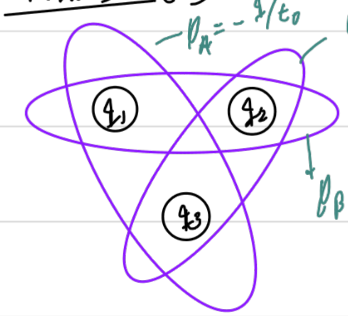
$\Phi = \frac{q_{enc}}{\epsilon_0}$

→ $q_1 + q_3$

A $q_{enc} = \Phi_A \times \epsilon_0 = \frac{-q}{\epsilon_0}\times\epsilon_0 = -q$ (i)

A $q_B =$

Problem-5 (H.W)



$q = -\Phi_A\epsilon_0$ (i)

$q = -\frac{1}{2}\Phi_C\epsilon_0$ (ii)

$q = \frac{1}{3}\Phi_B\epsilon_0$ (iii)

$\phi_A = \frac{q_1+q_3}{\epsilon_0} = \frac{-q}{\epsilon_0}$, $\phi_B = \frac{q_1+q_2}{\epsilon_0} = \frac{3q}{\epsilon_0}$

$\phi_C = \frac{q_2+q_3}{\epsilon_0} = \frac{-2q}{\epsilon_0}$

$q_1+q_3 = -q$ (i)

$q_1+q_2 = 3q$ (ii) — In terms of q

$q_2+q_3 = -2q$ (iii)

$-\Phi_A\cancel{\epsilon_0} = -\frac{1}{2}\Phi_C\cancel{\epsilon_0}$

$\Phi_A = \frac{1}{2}\phi_C$ (iv)

$-\Phi_A\epsilon_0 = \frac{1}{3}\phi_B\epsilon_0$

$\Phi_A = -\frac{1}{3}\phi_B$

$-\frac{1}{2}\phi_C = -\frac{1}{3}\phi_B$

$\Phi_C = \frac{2}{3}\phi_B$ (v)

$-\frac{1}{2}\Phi_C = \frac{1}{3}\phi_B$

$\phi_C = -\frac{2}{3}\phi_B$

$\phi_B = -\frac{3}{2}\phi_C$

$\Phi_C = \frac{2}{3}\left(-\frac{3}{2}\right)$

L-7

29/06/2024
Saturday

$$\boxed{\Phi_E = \oint \vec{E}\cdot d\vec{a} = \frac{q_{enc}}{\epsilon_0}}$$

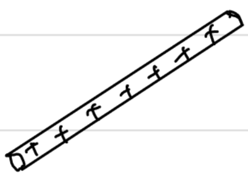
$$\boxed{\epsilon_0 = 8.854\times 10^{-12}\ Fm^{-1}}$$

Permittivity of free space

We want to choose the Gaussian Surface to be symmetric with the source field
These symmetric surfaces are called Equipotential Surfaces.
** It works for all shapes but easy in some selected shapes
* If there is no flux, no Φ_E

* If q_{enc} doesn't change Φ_E remains same
* Doesn't matter what shape or radius it is

Line Charge



$$\oint_S \Phi_E = \oint_E \vec{E}\cdot d\vec{a} = \frac{q_{enc}}{\epsilon_0}$$

(no vector sign)

$$\Rightarrow \oint E\,da\cos\phi = \frac{q_{enc}}{\epsilon_0}$$

$$\pm E\oint da = \frac{q_{enc}}{\epsilon_0}\ ;\text{provided } \cos\phi = \pm 1$$

$$EA = \frac{q_{enc}}{\epsilon_0}$$

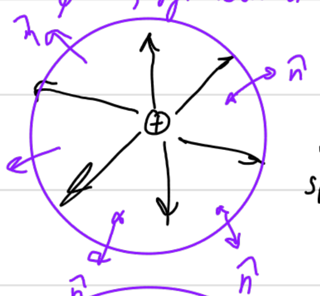
$$E = \frac{1}{A}\times\frac{q_{enc}}{\epsilon_0}$$

where $q_{enc} = \int dq_{enc}$ = charge density x distribution element

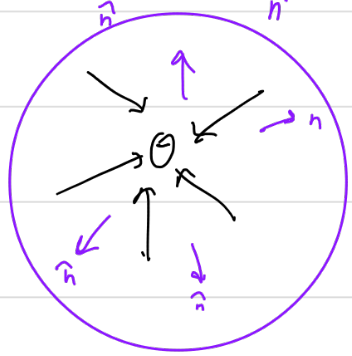
$\int dx = x$

$\oint E\,da\cos\phi = (\ \ )$

$\phi = 0$, symmetrical



$\phi = 0°$
spherical shape



$\phi = 180°$
spherical shape

$\oint E\,da\cos\phi = (\ \ )$

$E\oint = (\ \ )$

$EA = (\ \ )$

$E = \frac{1}{A}(\ \ )$

* We chose the Gaussian Surfaces to be symmetric surfaces called Equipotential Surfaces.

$\vec{E}$ field for an infinitely long line charge

Charge density $= \boxed{\lambda = \frac{q}{L}}$,

charge is uniformly distributed

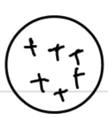
Area

field vectors into the plane

$\boxed{\sigma = \frac{q}{A}}$ ± depends on q

End view = side view

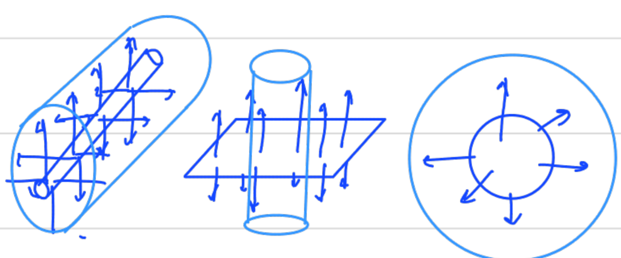
Volume



Sphere

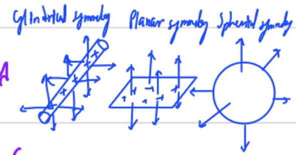
$\boxed{\rho = \frac{q}{V}}$

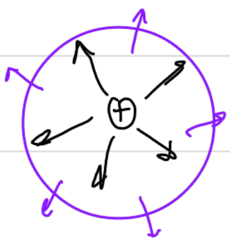
contribution distribution of charge



Line Charge → Current Wire → Long Cylindrical
Surface Charge → Capacitors → Wide Planar
Volume Charge → Electrodes/Shell Charges → Spherical

Suitable Gaussian Surface : Cylindrical symmetry, Planar Symmetry, Spherical Symmetry

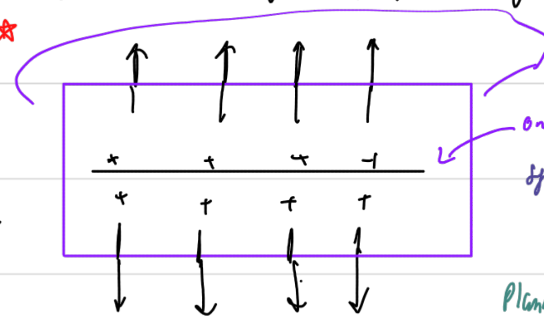




a circle

* it is the end of the line
the whole line makes the shape of a cylinder

cylindrical symmetry

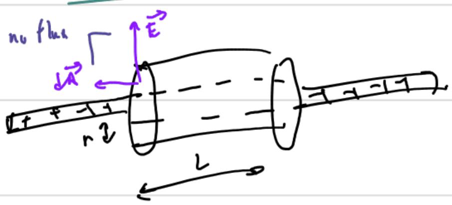


charges are not taken as spray

square / rectangle [no thickness]

Planar Symmetry

only arrows are not in other direction



Fringing effects

Gaussian surface is imaginary

Area of cylinder $= 2\pi r L + 2\pi r^2$ (no flux)

$\Phi_E = \oint \vec{E} \cdot d\vec{a} = \frac{q_{enc}}{\epsilon_0}$, $\lambda = \frac{q}{L}$

[line charge]

$= E \oint da = \frac{\lambda L}{\epsilon_0}$ $\Rightarrow q = \lambda L$ $\longrightarrow E \times 2\pi r L = \frac{\lambda L}{\epsilon_0}$

only curved area has flux

$$\therefore \boxed{E = \frac{\lambda}{2\pi \epsilon_0 r}}$$

* This is not theory it is application of gauss's law

For positively charged line charge	For -vely charge line charge
$\vec{E} = \left(\frac{\lambda_+}{2\pi \epsilon_0 r}\right)(\hat{r})$	$\vec{E} = \left(\frac{\lambda_-}{2\pi \epsilon_0 r}\right)(-\hat{r})$

Problem-1

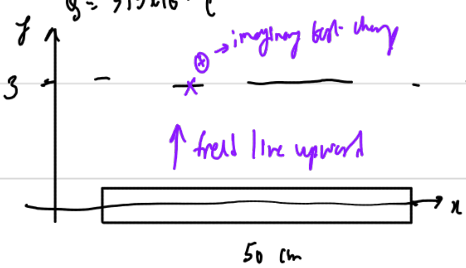
• thin : tells that thickness is zero

$L = 50$ m

$q = 3.5 \times 10^{-9}$ C

$\lambda = \frac{q}{L} = 7 \times 10^{-11}$ C m^{-1}

$\boxed{L \gg r}$



anywhere on x-axis as $x \gg y$

(a) $E = \left(\frac{\lambda}{2\pi \epsilon_0 r}\right)\hat{r} = (419.1407)(\hat{j})$

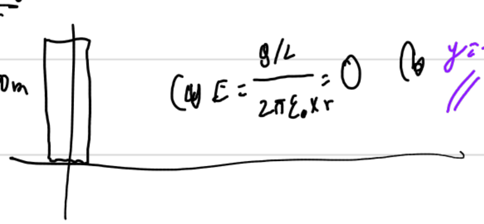
(b) $r = 4$ mm $(-\hat{j})$

(c) same

(d) same

x-axis distance doesn't matter

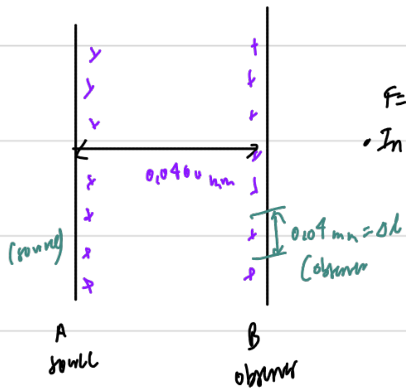
Alternative



(a) $E = \frac{q/L}{2\pi \epsilon_0 \times r} = 0$ (b) $y = -4$ mm (c) $x = 3$, (d) $x = 3$ same

y doesn't matter!?

Problem-2



$\lambda = +5.20 \times 10^{-6}\ C\,m^{-1}$

$F = qE$

- In line charge we don't know q

$Q = \lambda L$

$F = (\lambda\, \Delta l)\, E$, $E = \frac{\lambda}{2\pi\varepsilon_0 r}$ → 0,0400 mm

observer, source

$= (5.20 \times 10^{-6} \times 0.04 \times 10^{-3}) \times E$

$= 0.486\,N$

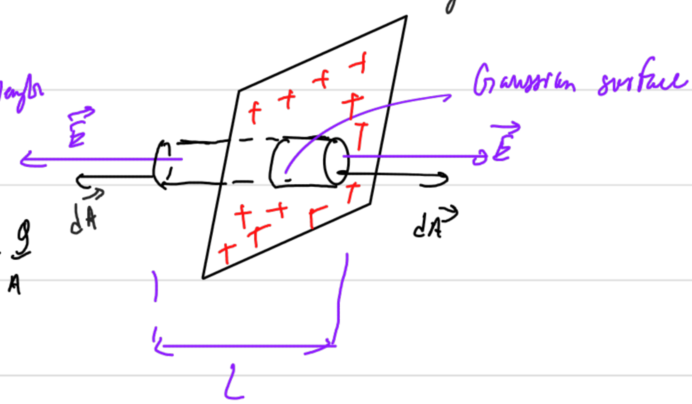
Alternate Problem

Force direction different but same magnitude

Problem-3

(+) → outward

Electric field for a sheet charge



Gauss's Law

Surface Charge

$\Phi_E = \oint \vec{E} \cdot d\vec{a} = \frac{q_{enc}}{\varepsilon_0}$, $q_{enc} = \lambda \times L$

density, length

$E \oint da = \frac{\sigma A}{\varepsilon_0}$ $\qquad \lambda = \frac{q}{L}$, $\sigma = \frac{q}{A}$

$E \times \underbrace{2A}_{} = \frac{\sigma A}{\varepsilon_0}$

two sides

$E = \frac{\sigma}{2\varepsilon_0}$ $\qquad \therefore \boxed{\vec{E} = \left(\frac{\sigma}{2\varepsilon_0}\right)\hat{n}}$

$L \gg r$

└ distance of the observer

For +ve charge σ +ve

For -ve charge σ -ve

sign is for direction

E only on the ends not on the surface

Problem-1

3 planes vertically stacked

one test charge

Anything above plane-1 is A

Dimension is fixed

Plane-1 $\sigma_1 = -\frac{\sigma}{2}$ $\vec{E}_2 \vec{E}_1$ — — — A B

Plane-2 $\sigma_2 = \sigma$ + + + + x $\vec{E}_3$ C

Plane-3 $\sigma_3 = -\frac{\sigma}{2}$ — — — — D

- Any point inside the region has the same $\vec{E}$

$\vec{E} = \left(\frac{\sigma}{2\varepsilon_0}\right)\hat{n}$

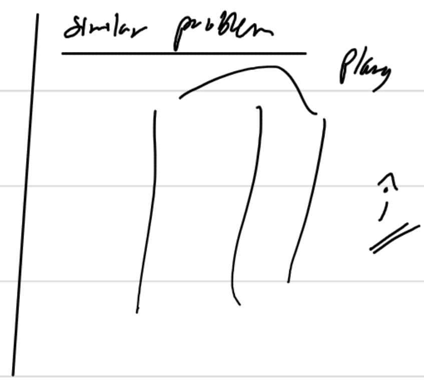
$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = \left(\frac{\sigma_1}{2\varepsilon_0}\right)\hat{j} + \left(\frac{\sigma_2}{2\varepsilon_0}\right)\hat{j} - \left(\frac{\sigma_3}{2\varepsilon_0}\right)\hat{j}$

$= \frac{-\sigma/2}{2\varepsilon_0}\hat{j} + \frac{\sigma}{2\varepsilon_0}\hat{j} + \cancel{\frac{\sigma/2}{2\varepsilon_0}}\hat{j} = \cancel{\frac{\sigma}{2\varepsilon_0}}\hat{j}$

$= \frac{\sigma}{2\varepsilon_0}\hat{j}$ //

Similar problem

Plang

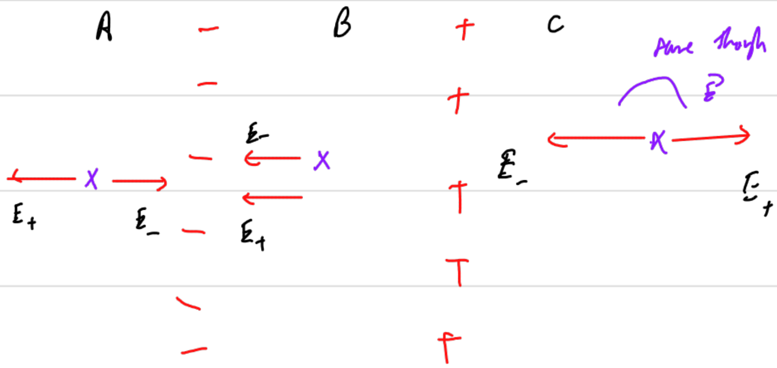


$\vec{E}$ Field for a pair of Oppositely Charged Sheets

-vely charged plate +vely charged plate

$\frac{Q}{A} = \sigma$

σ same for both



Parallel plates with equal σ

$$\vec{E} = \left(\frac{\sigma}{\epsilon_0}\right)\hat{n}$$

Not $2\epsilon_0$

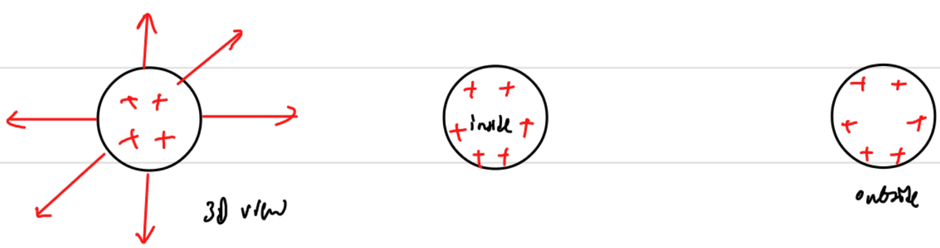
$$\vec{E}_{net} = \vec{E}_+ + \vec{E}_-$$

$$= \left(\frac{\sigma}{2\epsilon_0}\right)(-\hat{i}) + \frac{\sigma}{2\epsilon_0}(-\hat{i})$$

$$= \left(\frac{\sigma}{\epsilon_0}\right)(-\hat{i})$$

point charge, line charge dist imply
surface charge dist doesn't matter

$\vec{E}$ Field for a Spherical Charge (Uniform charge distribution)



$E \times A = \frac{Q_{enc}}{\epsilon_0}$ $\rho = \frac{Q_{enc}}{V}$

$E \times 4\pi r^2 = \frac{Q_{enc}}{\epsilon_0}$

sphere R $E = \frac{q}{4\pi R^2 \epsilon_0}$

$\frac{q}{V} = \rho = \frac{q}{\frac{4}{3}\pi R^3}$

- Radius of the sphere fixed, R
- Radius of Gaussian surface is adjustable, r

radius of gaussian surface radius of sphere

$q_{enc} = \lambda L$
$q_{enc} = \sigma A$
$q_{enc} = \rho V$

3 Scenarios: Inside: $r < R$

on surface: $r = R$

outside: $r > R$

Charges in enclosed space is different

$\theta = 0°$, $\cos\theta = 1$ as $\vec{E}$, $d\vec{A}$ same direction

*** Coulomb force only on surface

$$\Phi_E = \oint \vec{E}\cdot d\vec{a} = \frac{q_{enc}}{\epsilon_0} = \rho V \times \frac{1}{\epsilon_0} = \underbrace{\left(\frac{q}{\frac{4}{3}\pi R^3}\right)}_{\text{of whole sphere}} \times \underbrace{\left(\frac{4}{3}\pi r^3\right)}_{\text{gaussian surface}} \times \frac{1}{\epsilon_0}$$

closed surface

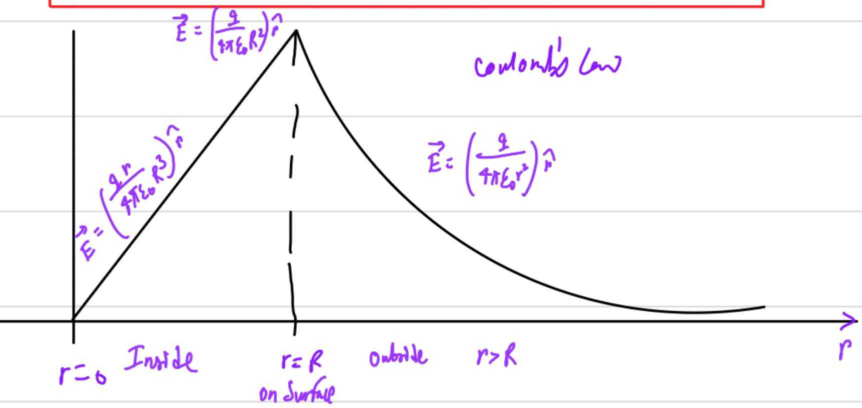
$$\Rightarrow E \times \boxed{4\pi r^2} = \frac{q r^3}{\epsilon_0 R^3}$$

$$\therefore \boxed{E_{inside} = \frac{q r}{4\pi\epsilon_0 R^3}\hat{r}}$$

$$\therefore \boxed{E_{inside} = \left(\frac{q}{4\pi\epsilon_0 R^3}\right) r, \quad E_{on\ surface} = \frac{q}{4\pi\epsilon_0 R^2}, \quad E_{outside} = \frac{q}{4\pi\epsilon_0 r^2}}$$

constant variable

$\boxed{r=0, E=0, \quad r \to R, \; E \text{ increases}, \quad r>R, \; E \text{ decreases}}$



+charge $\vec{E}$ direction radially outward

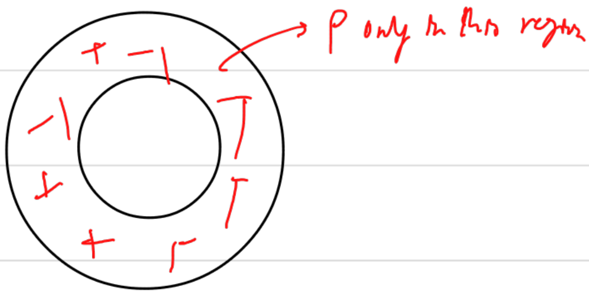
Line Charge: $\vec{E} = \left(\frac{\lambda}{2\pi\varepsilon_0 r}\right)\hat{r}$, $\lambda = \frac{q}{L}$

Surface Charge: $\vec{E} = \left(\frac{\sigma}{2\varepsilon_0}\right)\hat{n}$; $\sigma = \frac{q}{A}$

Volume Charge: $\vec{E} = \begin{cases} \left(\frac{qr}{4\pi\varepsilon_0 R^3}\right)\hat{r} = \left(\frac{r\rho}{3\varepsilon_0}\right)\hat{r} & \text{[inside]} \\ \left(\frac{q}{4\pi\varepsilon_0 R^2}\right)\hat{r} & \text{[on surface]} \\ \left(\frac{q}{4\pi\varepsilon_0 r^2}\right)\hat{r} & \text{[outside]} \end{cases}$

$\boxed{\rho = \frac{q}{\frac{4}{3}\pi R^3}}$

Problem-3



(i) $Q_{enc} = \rho V$
$= \rho\left(\frac{4\pi}{3}b^3 - \frac{4\pi}{3}a^3\right)$

(ii) $r = b$, $E = 0$ (ii) $r = 0.5a$, $E = 0$ (iii) $r = a$, $E = 0$

$\vec{E}$ increase when we

Problem 4 (i) $Q_{enc} = \rho \times V = \rho \times \frac{4}{3}\pi R^3$

(ii) $R = 0$, $E = 0$, (iii) $r = 0.5$, use inside formula (iv) $r = R$ on surface eq (v) $r = 1.5R$ outside (vi) $r = 3R$ (not $r = 3b$) outside

★★ same place if differently charge → ρ changes but we deal with same charge

06/07/2024
Saturday

Maxwell's Eqⁿ

It is Gauss's Law for Electric Fields

→ The total ϕ_E passing through a closed surface is proportional to the total electric charge q_{enc} enclosed within the surface $\boxed{\oint \vec{E} \cdot d\vec{a} = \frac{q_{enclosed}}{\varepsilon_0}}$ [Integral form]

→ The Divergence of $\vec{E}$-fields through a closed surface is directly proportional to the charge distribution within that surface $\boxed{\vec{\nabla} \cdot \vec{E} = \frac{\rho_{enclosed}}{\varepsilon_0}}$ [Differential Form]

Gauss's Divergence Theorem

→ Surface Integral (vector Field) or Flux (vector Field) = Volume Integral (Divergence of the same Vector Field)

$$\oint_S \vec{V} \cdot d\vec{a} = \int_V (\vec{\nabla} \cdot \vec{V}) \cdot dV$$

$$\oint_S \vec{E} \cdot d\vec{a} = \int_V (\vec{\nabla} \cdot \vec{E}) \cdot dV$$

· closed surface area → since closed surface volume

*** $\oint$ no need as dealing with ∇ which is already closed

$\oint \vec{E} \cdot d\vec{a} = \frac{q_{enc}}{\varepsilon_0}$ (Integral form) $\quad \vec{E}) \underline{W} = \frac{q_{enc}}{\epsilon} \quad \frac{1}{V}$

Flux +ve → divergence +ve

Gauss → Electric Charge

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho_{enc}}{\varepsilon_0}$$ (Differential form)

'Leak', 'Sink'

Work Done due to Electrostatic Force

→ When a charge moves or is is moved in the electric field, work is done by or against the Coulomb force to accelerate it, thus displacing it. This work is stored as potential energy in the system.

movement in Electric field is due to $\vec{F}_E (= q\vec{E})$

Hence there is a displacement, hence Work Done

System

$W = \int_{r_i}^{r_f} \vec{F} \cdot d\vec{r}$, $\vec{F}_E = q\vec{E}$

$W = q_0 \int_{r_i}^{r_f} \vec{E} \cdot d\vec{l} = E\, \underbrace{dl \cos\theta}_{dr}$ $\quad , \quad \vec{E} = \frac{W}{q_0}$

$\Rightarrow \boxed{W = q_0 \int_{r_i}^{r_f} E\, dr}$ Uniform Field

Types of Work Done

+ve WD → By the system (releasing pen due to gravity)

−ve WD → On the system (throwing pen upwards)

Problem-1

W.D by the system occurs on the charges

$$W = q_0 \int_{r_i}^{r_f} E\,dr$$

- E direction in −ve x-axis, proton same direction so dot product = 1 [$\cos\theta = 1$] and q_0 is also +ve so W +ve
- For $\bar{e}$ dot product −ve and q_0 also −ve so W +ve

For proton, $W_p = (1.602\times10^{-19}C)(-2.85\times10^{5}\,NC^{-1})\times\left(0 - \frac{2\times10^{-3}}{2}\right) = 4.56\times10^{-17}J$

For electron, $W_e = (-q_e)(-2.85\times10^{5}\,NC^{-1})\times\left(d - \frac{d}{2}\right) = 4.56\times10^{-17}J$

W.D by the system as both +ve

Energy with Electric Fields

Work-Energy Theorem : $W = \Delta K$

Conservation of Mechanical Energy : $\Delta U + \Delta K = 0$ / $\Delta U = -\Delta K$

Now, we'll work with $W = -\Delta U$ [W.D due to displacement in Electric field]

Electric Potential Energy

$$\boxed{\Delta U = -W = -q_0 \int_{r_i}^{r_f} E\,dr}$$

Natural movement : W.D by the system otherwise W.D on the system

W.D by the system:

For both +ve & −ve ; P.E gains, K.E loss

Problem-2

Electric Potential energy with point charge [Non-Uniform field]

$$\Delta U = -W_{i\to f} = -q_0\int_{r_i}^{r_f} E\,dr = -q_0\int_{r_i}^{r_f}\frac{cq}{r^2}\,dr = -c\,q\,q_0\int_{r_i}^{r_f}\frac{dr}{r^2} = -\frac{q q_0}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_{r_i}^{r_f} = U_{final} - U_{initial}$$

*** There is nothing as W_f, W_i as W at a point is zero

$E \propto \frac{1}{r^2}$

$E \to 0$ when $r \to \infty$

$$\therefore \boxed{\Delta U = \frac{-q_0 q}{4\pi\varepsilon_0}\left(0 - \frac{1}{r_f}\right) = \frac{q_0 q}{4\pi\varepsilon_0 r}} \quad (E=0)$$

no -ve as starting is very far ∞

W_∞ → starts with infinity / ends with infinity

$$\boxed{U_{elec} = -W_\infty = \frac{1}{4\pi\varepsilon_0}\frac{q q_0}{r}}$$

$$W_\infty = -\frac{1}{4\pi\varepsilon_0}\frac{q q_0}{r} \Rightarrow ??$$ how W_∞ -ve??

Problem-3

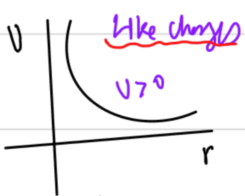
$e = 1.602\times10^{-19}$

(i) $W_{net} = W_{13} + W_{23} = \frac{-q_1 q_3}{4\pi\varepsilon_0 \times 2a} + \frac{-q_2 q_3}{4\pi\varepsilon_0 \times a} = +5.76\times10^{-27}\,J - 1.15\times10^{-26}\,J = \boxed{\ }$ (-ve)

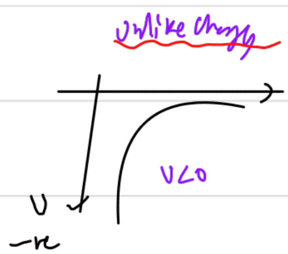
*(ii) For q_1 initially when it entered, there was no field. So P.E of $q_1 = 0$. But q_2 & q_3 has

Incorrect interpretation $U_{net} = U_{12} + (U_{13} + U_{23}) = \frac{q_1 q_2}{4\pi\varepsilon_0 \times a} + \left(\frac{q_1 q_3}{4\pi\varepsilon_0 2a} + \frac{q_2 q_3}{4\pi\varepsilon_0 a}\right)$ this must be $U = -W$

☆☆☆ $U \propto \frac{1}{r}$



r↑ U↓



r↑ U↑

*** Like charges repel to decrease (→ natural) U hence want to increase r

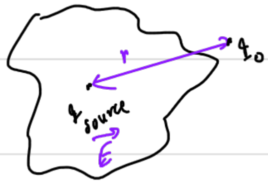
This graph in 3D is a funnel

Final

L-10

05/09/2024

Thursday



- Coulomb Force, $\boxed{F_E = \frac{c\, q_s q_o}{r^2}}$

$\Rightarrow F_E = \left(\frac{c\, q_s}{r^2}\right) q_o$

observer dependent

$\therefore \frac{F_E}{q_o} = \frac{c\, q_s}{r^2} = E$ → The ratio gives a constant = Electric Field

→ Vector

→ Observer Independent

Work done due to moving charge q_o in

$\boxed{\text{Potential energy}, U = \frac{c\, q_s q_o}{r}}$

$$\boxed{\begin{aligned} W_{i\to f} &= q_o \int_{r_{initial}}^{r_{final}} E \cdot d\vec{r} \\ \Delta V_{elec} &= -q_o \int_{initial}^{final} E \cdot d\vec{r} \end{aligned}}$$

$\Rightarrow U = \left(\frac{c\, q_s}{r}\right) q_o$

observer dependent

$\therefore \frac{U}{q_o} = \frac{c\, q_s}{r} = V$ → Electric Potential Field

→ Observer Independent

→ Scalar

$\boxed{\text{V↑ shows higher intensity and vice-versa}}$

$\therefore \boxed{\frac{\Delta U}{q_o} = \Delta V\, [JC^{-1}]}$ → Change in potential / P.D / voltage

NOT potential

$\Rightarrow \Delta V = \frac{\Delta U}{q_o} = \frac{U_f - U_i}{q_o} = \frac{-(W_{i\to f})}{}$

Electric Potential, $\Delta V = \frac{\Delta U}{q_o} = \frac{U_f - U_i}{q_o}$

↓

Scalar

JC^{-1}, volt

$= \frac{-W_{i\to f}}{q_o}$ = P.D ⟺ voltage

W.D by the system

q_+ q_-

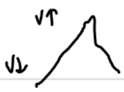
$\Delta U = q \cdot \Delta V$

$\vec{F} = q_o \vec{E}$

$\Delta V = \frac{\Delta U}{q_o}$

Conclusion

Hill convension → positive charge → wants to decrease the potential



going down; $V_f - V_i = -ve$ $\therefore \Delta V < 0$ [Potential Downhill] → $q_o > 0$; $\Delta V < 0$

going up! $V_f - V_i = +ve$ $\therefore \Delta V > 0$ [Potential Uphill] → $q_o < 0$; $\Delta V > 0$

→ negative charge → wants to increase the potential

K.E increases! Conservation of energy

So in circuit, electrons flow from low potential to high potential (uphill)

* Positron goes downhill

// PD is always positive ??

= NO

Problem-1

$V_i = v_{proton} = 2.0 \times 10^5\ ms^{-1}$

$\Delta V = +100V$

$\boxed{\text{proton travels from high V to low V hence loses P.E thus gains K.E}}$

$K.E_i = \frac{1}{2} m_p v_i^2 = \frac{1}{2} \times 1.67 \times 10^{-27} \times (2 \times 10^5)^2$

$= 3.34 \times 10^{-17} J$ //

$\Delta K = -\Delta U = -q_o \Delta V$

$\Delta U = -q_o \Delta V$

Downhill //

Potential decreasing K.E↑

$K_f - K_i = -q_o \Delta V$

$K_f = 3.34 \times 10^{-17} - 1.602 \times 10^{-17} J$ //

For electron

Potential ↑, K.E ↓

Potential of Uniform Sources

E → Discrete → Point

E → Continuous → Line, Plane, Volume

$\vec{E}$

// $\Delta V = ?$

The particle needs to move otherwise no no ΔU no ΔV $[\Delta U = q_o \Delta V]$

$\int \vec{E} \cdot d\vec{l} = \int E\, dr$

$\Delta V = \frac{\Delta U}{q_o} = -\int \vec{E} \cdot d\vec{l}$

$\therefore \Delta E = -\int \vec{E} \cdot d\vec{l} = -\int E\, dr$ //

Potential Change in an Uniform Electric Field

$$\boxed{\Delta V = -\int_{r_i}^{r_f} E\, dr = -E \int_{r_i}^{r_f} dr \quad \therefore \Delta V = -E \Delta r}$$

$V = Ed$ → simplified eqⁿ

↓

not ΔV as initial = 0

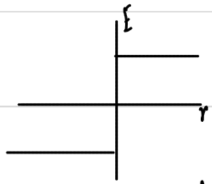
*** E points where Potential ↓

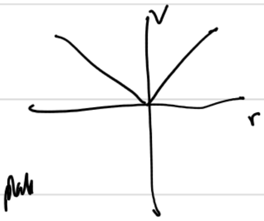
Problem-2

vvv V doesn't depend on charge

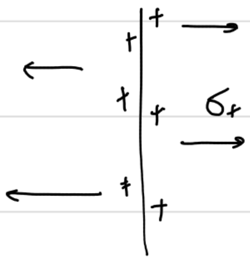
$\Delta V = Ed = 2.85\times10^{5}\ NC^{-1} \times 2\times10^{-3}\ m = 570\ V$

Plot the behaviour of E and V





the charged plate



$\vec{E} = \left(\frac{\sigma_+}{2\varepsilon_0}\right)\hat{n}$

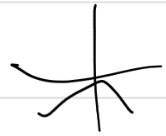
σ_+

$\Delta V = E\Delta r$

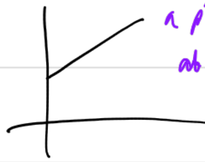
$Y = mx$

$V = E\Delta r$

modulus graph

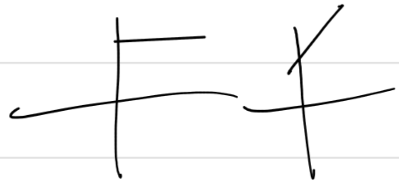


Planar $E = \frac{\sigma}{\varepsilon_0}$ Constantly chng



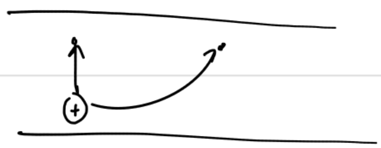
a plate might not start at zero potential

If it does



Problem-3

(2) and (3) have same potential as same dist from -ve



$\Delta U = q_0 \Delta V$ $\quad 1 \rightarrow 2 \quad \Delta V = V_2 - V_1$ $\quad 1-3 \quad \Delta V = V_3 - V_1$ $(= V_2)$

They reach at both points at same energy

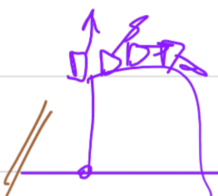
• displacement along the field lines equal

ΔV same, ΔU same > same speed at points

• $\Delta U = -\Delta KE$, $\sqrt{}$ so v same

basically force cancels speed cancel dist.

all hit with same speed # time taken to hit different



Clear concept

4.1 4.1 4.1 4.1

$1\,ms^{-1}$, $2\,ms^{-1}$, $3\,ms^{-1}$, $4\,ms^{-1}$, $5\,ms^{-1}$

$W_\partial = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r}$

$U_{elec} = -W_\partial$

07/09/2024
Saturday

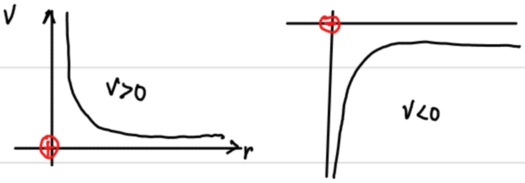
Potential change in a non-uniform Electric field

EPE from class 9

$$\boxed{\Delta V_{\infty} = \frac{\Delta U}{q_0} = \frac{c q_0}{r} [V]} , \quad \boxed{U_{\infty} = \frac{c q_0}{r^2} [J]}$$

~~The potential of Point Charges~~

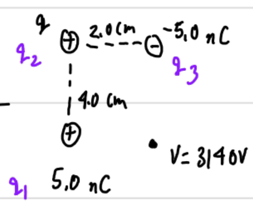
The shape is actually funnel-shaped in 3D



Potential of system of charges

$V_{\infty} =$

Problem-1



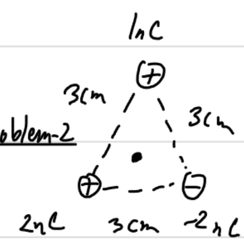
$$\boxed{V = V_1 + V_2 + V_3}$$

$$3140 = \frac{c q_1}{r_1} + \frac{c q_2}{r_2} + \frac{c q_3}{r_3}$$

$$3140 = 2246.888 + \frac{c q_2}{\sqrt{(2\times10^{-2})^2 + (4\times10^{-2})^2}} - 1123.444$$

$\therefore q_2 = +5.590 \times 10^{-9}$ C

Problem-2

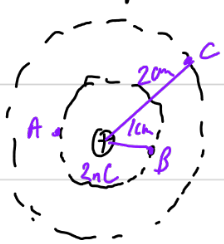


incentre

Point charge is 3D

Problem-3

• It is a sphere



(i) $V_A = V_B = \frac{c \times 2\times10^{-9}}{1\times10^{-2}}$ $= 1797.510$ V

$V_C = \frac{c \times 2 \times 10^{-9}}{2\times10^{-2}}$ $= 898.755$ V

$\Delta V_{CB} = V_B - V_C$ $= 898.755$ V

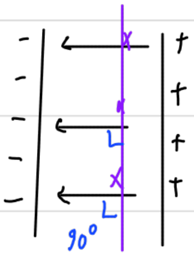
* $\Delta V_{AB} = V_B - V_A = 0$ [Points are equidistant. Thus equipotential surface]

As ⊕ thus we see upward part of funnel
For ⊖ we'll see downward part of funnel

Problem-4

H.W.

Equipotential Surface



- uniform field
- V same in every position
- Q) Find the point of equipotential surface

$\Delta V = \Delta E r$

electric dipole

electric field weak

// are the equipotential at all points in line ??

closed → It is no just a line. But a whole plane

$$\boxed{\Delta V = -\oint \vec{E}\cdot d\vec{l} = 0}$$ → $\perp$ → No potential change

$$\boxed{\text{Thus, claiming } \oint \vec{E}\cdot d\vec{l} = 0 \quad \vec{\nabla}\times\vec{E} = 0 \text{ (Apply Stokes Theorem)}}$$

Conclusion: Electrostatic force is conservative

Conservative property

*** Charge moves in presence of potential → they experience force thus they move

Potential energy can only be defined only if force is Conservative

conservation of energy → $\Delta U = -\Delta K$
Work energy theorem → $W = \Delta K$
$\Big\}$ $W = -\Delta U$

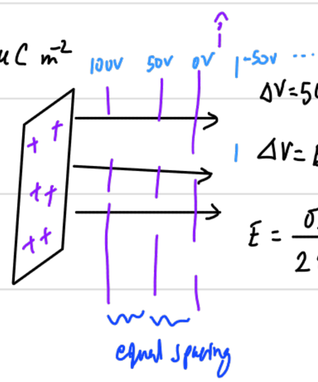
// $\boxed{F_E = -\frac{\partial U_{elec}}{\partial r}}$

↳ Conserves Mechanical energy

Problem-5

$\sigma = 0.10\ \mu C\ m^{-2}$

$\sigma_+ = 0.10\times10^{-6}\ C m^{-2}$



$\Delta V = 50V$

$\Delta V = E\Delta r$ Δx

$E = \frac{\sigma_+}{2\varepsilon_0} = 5647.095\ Vm^{-1}$

$\Delta x = \frac{\Delta V}{E}$

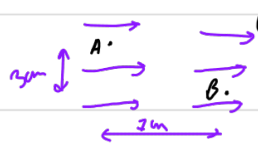
$= \frac{50}{5647.095}$

$= 8.854\times10^{-3}\ m$ //

$$\boxed{\begin{array}{l}\text{Previously, } E = \frac{F}{q}\ [NC^{-1}] \\ \text{Now, } E = \frac{\Delta V}{\Delta r}\ [Vm^{-1}]\end{array}}$$

Problem-6

- Uniform field lines → just see if they have equal dist separation



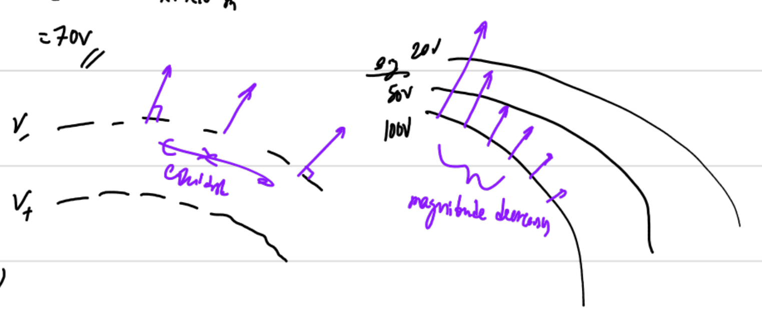
(a) A has high potential
B has low potential

(b) $\Delta V = 1000\ Vm^{-1} \times 7\times10^{-2}\ m$ (horizontal dist)

$= 70V$ //

As we go right V decreases
then where does V go ?!

The Geometry of Electric Fields

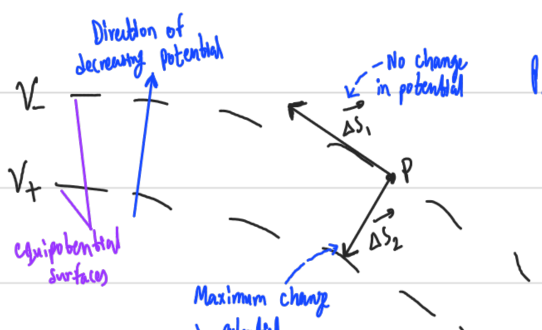


1) E points where Potential is decreasing
2) E lines → 90° to equipotential surfaces (E_s)
3) Closely spaced $E_s \equiv$ Strong Field

The Geometry of Electric Fields

$$\vec{E} = -\vec{\nabla}V = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right) = -\left(E_x\hat{i} + E_y\hat{j} + E_z\hat{k}\right)$$

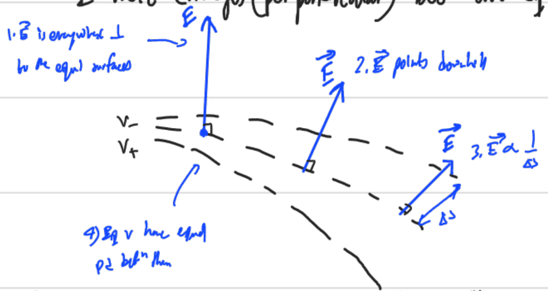
$$E_x = -\frac{\partial V}{\partial x},\ E_y = -\frac{\partial V}{\partial y},\ E_z = -\frac{\partial V}{\partial z}$$



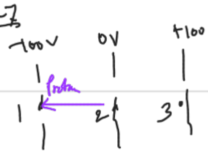
Problem 7

How to diagnose $\vec{E}$ from a diagram

$\vec{E}$-field changes (perpendicular) betn two equipotential surfaces toward decreasing potential



Problem-7



increasing speed → F

Problem-8) (a) $E = \frac{\Delta V}{d}$

(1) $\Delta V = 20$, (2) $\Delta V = 20$, (3) $\Delta V = 20$

E 1 = 2 = 3 ✗

(b) (3)

whole region

$\Delta V_1 = 80$, $\Delta V_2 = 40V$, $\Delta V_3 = 40$

$\therefore E_1 > E_2 = E_3$ //

Problem-9

$E = -\frac{\Delta V}{\Delta s}$

-gradient

✓

L-12

12/09/2024

Thursday

Electric current: flow of charge carriers within a conductor

$$\boxed{I = \frac{\Delta Q}{\Delta t}, \text{ scalar, } A}$$

Drift motion $\rightarrow I$ $\quad 10^{29}$ electrons$/m^3$

$E \rightarrow \Delta V$

$$\boxed{V_{ele} = \frac{-k q_e}{r}, \ \Delta V = \frac{\Delta U}{q} = \frac{-W}{q}}$$

A pd created by separating the + -ve charges → battery
↓
chemical making battery / dead

The steady electric force causes the electrons to move along parabolic trajectories b/w collisions

Internal electric force pushes electron current through the wire

$$\boxed{\text{Electromotive Force}, \ \mathcal{E} = \frac{W_{elec}}{q}}$$

more +ve → more -ve

10^{29} $\bar{e}/m^3$ → conductor is it diff for other materials

Define electric Current

n_e = concentration/density of electron

$$\boxed{EMF, \ \mathcal{E} = \frac{W_{chem}}{q}}$$

$$\boxed{I = n_e q_e A V_d} \rightarrow \boxed{I = i_e q_e} \quad \text{Electron current}$$

$$\boxed{i_e = n_e} \ A V_d \quad \text{mm s}^{-1} \ \sim 10^{29}$$

The electrons hit each other thus they pass the signal, almost at the speed of light
for individual it would take very long

electron current
↓
$i_e = n_e \times A \times V_d$ $\quad I = q_e \times i_e$

$n_e = \frac{Q}{V}$, $Q = n_e \times V = n_e \times A \times \Delta x$
$n_e A \times V_d \times \Delta t$
$\therefore I = n A V_d q_e$

Problem-1

Ans) $d = 2 \times 10^{-3}$, $V_d = 10 \times 10^{-3} \ ms^{-1}$, $n_e = 8.5 \times 10^{28} \ m^{-3}$

→ electron current
$i_e = n_e A V_d$ → rest except q_e, $A = \pi r^2 = \frac{\pi d^2}{4}$

electric currents $[A C^{-1}]$
$I = q_e i_e = [A]$
↳ can't be negative. make it absolute value

Ans 2 $\quad d = 2.6 \times 10^{-3}$
$\Delta Q = 420 C$
$\Delta t = 80 \times 60 = 4800 s$
$n_e = 5.8 \times 10^{28} \ m^{-3}$

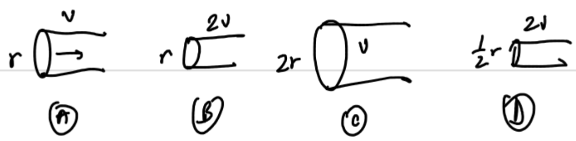
(a) $I = \frac{\Delta Q}{\Delta t} = \frac{420}{4800} = \square \ A$

(b) $V_d = \frac{I}{n_e q_e \times \pi \frac{d^2}{4}} \ ms^{-1}$

If it is not told electron current/electric current then convention is finding I

Problem 2

4 wires, same metal, electric currents I_A to I_D



$I \propto A V_d$
↓
$I_A \propto (\pi r^2) v$, $I_B \propto (\pi r^2)(2v)$, $I_C \propto \pi (2r)^2 v$, $I_D \propto \pi \left(\frac{1}{2} r\right)^2 2v = \frac{1}{2}$
$I_B = 2 I_A \qquad I_C = 4 I_A \qquad I_D = \frac{1}{2} I_A$

$$\boxed{\therefore I_C > I_B > I_A > I_D}$$

The current density in a wire

L-13

14/09/2024

Saturday

Electric current, $\boxed{I = q_e i_e = (n_e q_e v_d) A}$

vector

Electric current density, $\boxed{J = \frac{I}{A} = n_e q_e v_d \ [A\,m^{-2}]}$ $\boxed{I = \int \vec{J} \cdot \vec{A} = JA\cos\theta}$

↳ depends on electric field (at the beginning??)

Problem-1

$J = \dfrac{1A}{\frac{\pi (10^{-3})^2}{4}} = 1273239.54\ A\,m^{-2}$ // $J = \frac{I}{A}$

can be wrong

$I = n_e q_e A v_d$

$v_d = \dfrac{J}{n_e q_e} = 9.335 \times 10^{-5}\ m s^{-1}$

$J = \dfrac{I}{A} = \dfrac{n_e q_e A v_d}{A}$ → E → battery applies this

$$\boxed{\begin{aligned} &\vec{J} \propto \vec{E} \\ &\vec{J} = \sigma \vec{E} \quad \Omega^{-1} m^{-1} \\ &\quad \text{↳ conductivity} \\ &\frac{1}{\sigma} = \rho \to \text{resistivity} \quad \Omega m \\ &\therefore \vec{E} = \rho \vec{J} \end{aligned}}$$

$\Delta V = EL$

$E = \dfrac{\Delta V}{L}$

$E = \rho J$

$\dfrac{\Delta V}{L} = \rho \dfrac{I}{A}$

$\Delta V = \left(\dfrac{\rho L}{A}\right) I$ → R

$\Delta V \propto I$

$R = \rho \dfrac{L}{A}$

$R \propto \dfrac{L}{A}$

$R_A = \dfrac{L}{\pi r^2}$

Problem 2

$R_A \propto \dfrac{L}{\pi r^2}$

$R_B \propto \dfrac{L}{\pi (2r)^2} \to R_B = \frac{1}{4} R_A$

$R_C \propto \dfrac{2L}{\pi (2r)^2} \to R_C = \frac{1}{2} R_A$

$R_D \propto \dfrac{2L}{\pi r^2} \to R_D = 2R_A$

$R_E \propto \dfrac{4L}{\pi (2r)^2} \to R_E = R_A$

$\boxed{\therefore R_D > R_E = R_A > R_C > R_B}$

Problem-3

$d = 2 \times 10^{-3}\ m$

$I = 800 \times 10^{-3}\ A$

$\sigma = 3.5 \times 10^{7}\ \Omega^{-1} m^{-1}$

(a) $E = \rho J$

$= \dfrac{1}{\sigma} = \dfrac{I}{\pi \frac{d^2}{4}}$

$= 0.007275\ V m^{-1}$

(b) $\rho = \dfrac{1}{\sigma} = 2.8571 \times 10^{-8}$

(c) $R = \rho \dfrac{L}{A} = 0.00909$ //

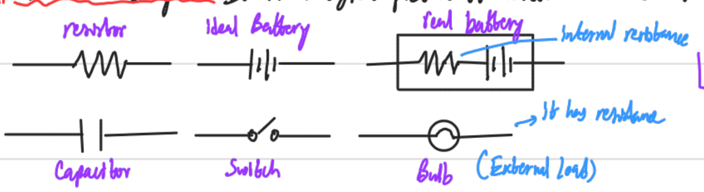
Recording

Electric Circuits   L-14   10,11,12,13   (Capacitor out of part)   19/09/2024 Thursday

Circuit components

Electric Circuit: To extract energy from Electric Field
- Controlled flow of charge = Current

A circuit diagram: It is a logical picture of what is connected to what



↳ default resistance due to chemical reaction/chemical work done (due to electrolytes)
↓
Causes potential drop

Ideal Battery: When EMF = terminal voltage → 100% efficiency

For real battery, $\boxed{\Delta V_{terminal} = \mathcal{E} - Ir}$ $\boxed{V_{terminal} \text{ is what the external circuit receives from the battery ideal or real}}$

$$\boxed{I = \frac{\mathcal{E}}{R+r}}$$

Problem-1

$R = 5.0\,\Omega$
$\mathcal{E} = 2.0\,V$
$r = 1.0\,\Omega$

(a) $2 = I(5) + I(1)$ (external, internal)
$2 = 6I$
$\therefore I = \frac{2}{6} = 0.33A$

(b) $V_{terminal} = IR$
$= 0.33 \times 5$
$= 1.67$

H.W. Solar cell: aka photovoltaic (PV) cell that converts sunlight into electricity through photovoltaic effect

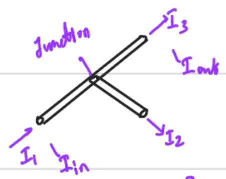
$\Delta V_{term} = 0.10V$, $R = 500\,\Omega$ $0.15 = \mathcal{E} - \left(\frac{0.15}{1000}\right)r$

$\Delta V = 0.10 = \mathcal{E} - \left(\frac{0.1}{500}\right)r$ (i)   (i)-(ii) $0.05V = -\left(\frac{0.15}{1000}\right)r + \left(\frac{0.10}{500}\right)r$ $\therefore r = \square\,\Omega$

Kirchhoff's Two Rules (Laws?) for circuits: (KCL) The total current into the junction must be equal to the total current leaving the junction because electric charge is conserved (Charge conservation)

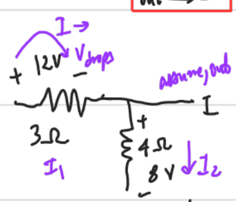
$\boxed{\Sigma I_{in} = \Sigma I_{out}}$

$\boxed{\text{Charge is Conserved Not current}}$



Convention
in → +
out → −

Problem-2



* Potential downhill
$I_1 = \frac{12}{3} = 4A$
$I_2 = \frac{8}{4} = 2A$

$I_1 = I + I_2$
$4 = I + 2$
$I = 2A$ outward

or
$-I_1 + I + I_2 = 0$
$I_1 = I_1 - I_2$
$= +2A$ (left to right)
Charged particle (right to left)

Kirchhoff's 2nd Rule: Voltage Rule/Loop Rule: A charge that moves around a closed path has $\Delta U_{elec} = 0$ because energy is conserved

$\boxed{\Delta V_{loop} = \Sigma(\Delta V)_i = 0}$ $\Delta U_{elec} = q\,\Delta V$

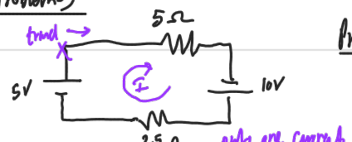
In a circuit, the algebraic sum of all pd sum to zero.

not voltage conservation

* Pick a node → start travelling, if same direction as I (+→−) then it's potential downhill (−IR)

$\boxed{\text{KCL requires min}^m \text{ two loop (a branch)}}$

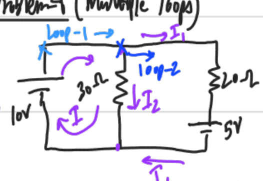
Problem-3



only one current

$-I(5) + 10 - 2.5(I) + 5 = 0$
$I = 2A$
$\Delta V = IR$
For 5Ω: $2 \times 5 = 10V$ } not 15V
2.5: $2 \times 2.5 = 5V$

Problem-4 (multiple loops)

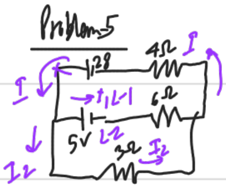


(a) Loop-1 (left)
$-I_2(30) + 10V = 0$
$I_2 = \frac{1}{3} = +0.33A$ (30Ω)

(b) Loop-2 (with ...)
$-I_1(20) - 5 + (0.33 \times 30) = 0$
$I_1 = 0.245A \rightarrow 20\Omega$

(c) $I_{V_{10}} = I_1 + I_2 = 0.575$

Problem-5

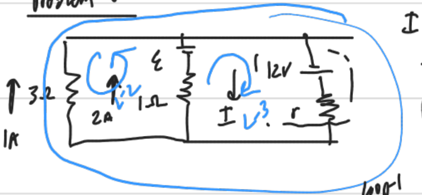


$I = I_1 + I_2$
Loop-1: $23 - 5 - I_1(6) - I(4) = 0$
$I(4) + I_1(6) = 23$
Loop-2: $5 - I_2(3) + I_1(6) = 0$

In (i)
$4\Omega(I_1 + I_2) + I_1(6) = 23$
$4I_2 + I_1(10) = 23$

$I_2 = 3.48A$
$I_1 = 0.908A$
$I = I_1 + I_2$
$= 4.388A$

Problem-6

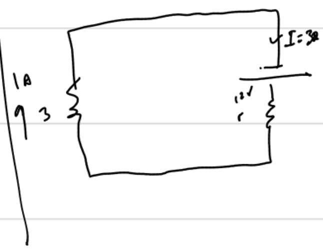


Loop 1

$I = 2A + 1A = 3A$

for r, use KVL on Loop 1

for ε, use KVL on Loop 2 or Loop 3



$12 - 3(r) - 3 = 0$

$3r = 9$

$r = 3\,\Omega$

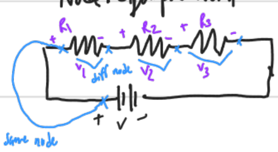
$3 - 2 + \varepsilon = 0$

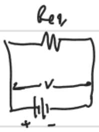
$\varepsilon = 1\,V$

$\varepsilon = -1$

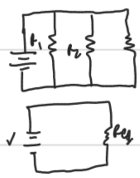
Resistors in Series and Parallel

Node: equipotential





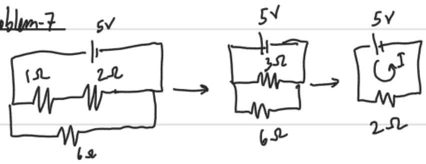
$$\boxed{\begin{gathered} R_{eq} = R_1 + R_2 + R_3 = \sum^{n} R_i \\ V = V_1 + V_2 + V_3 = I R_{eq} = IR_1 + IR_2 + IR_3 \end{gathered}}$$



$$\boxed{\begin{gathered} I = I_1 + I_2 + I_3 \\ \frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} \\ \therefore \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \end{gathered}}$$

When positive node of one resistor is connected with -ve node of other resistor. Then those are series connection.

Problem-7



$I = \frac{5}{2}$ For $6\,\Omega$, $I = \frac{5}{6} = 0.83$

$= 2.5$ For $1\,\Omega, 2\,\Omega, \ldots$

Energy and Power In Electric Circuits

$\frac{W_{chemical}}{q_0} = \varepsilon$ → input

Energy Dissipation in Resistors: $E_{chemical} \to U \to K \to E_{thermal}$ (Drift; heat → output)

$\Delta U = -W$
$= -(-w)$ $\quad \Delta U > 0$

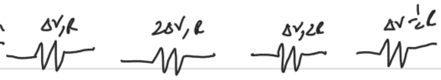
The battery's chemical energy is transferred to the thermal energy of the resistors, raising the temperature

Energy exchanged per unit time is Power

Power delivered by the EMF: $\boxed{P_{battery} = \frac{\Delta U}{\Delta t} = \frac{\Delta q}{\Delta t}\varepsilon = I\varepsilon}$ J/s/W $\boxed{P_{in} = \varepsilon I}$ $(P = VI)$

Power dissipation through Resistors: $P_R = \frac{\Delta U}{\Delta t} = \left|\frac{\Delta q}{\Delta t}\right| V_R = I\Delta V_R = I^2 R = \frac{(\Delta V)^2}{R}$, $P_{out} = I\Delta V_R = I^2 R = \left(\frac{\Delta V}{R}\right)^2 \times R = \frac{(\Delta V)^2}{R}$

Problem



$P_A = \frac{\Delta V^2}{R}$, $P_B = \frac{(2\Delta V)^2}{R}$, $P_C = \frac{(\Delta V)^2}{2R}$, $P_D = \frac{(\Delta V)^2}{R/2}$ $\quad \therefore \boxed{P_B > P_D > P_A > P_C}$

P_A $\quad P_B = 4P_A$ $\quad P_C = \frac{P_A}{2}$ $\quad P_D = 2P_A$

Problem

$I = \frac{\varepsilon}{R + r} = \frac{3}{7}$ $\quad P_R = I^2 R$, $\quad \Delta V_{terminal} = \varepsilon - Ir$

Expected output is < Power delivered

\# Electric Bills are measured in terms of kilowatt hours (kWh)

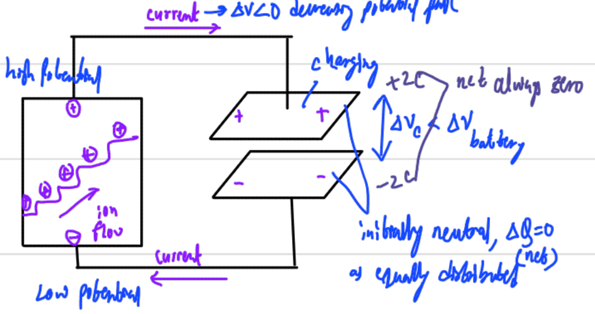
$\boxed{E_{thermal} = P_R \Delta t}$ $\quad$ 1 unit

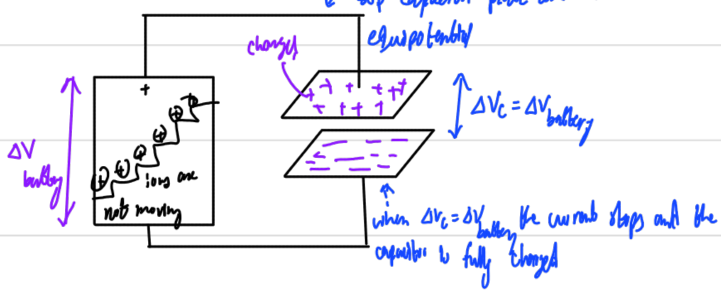
Power Saving: Use key

Capacitance and capacitors

Need idea of: plane charge + uniform field + $\Delta V = E\Delta r$

Use:
- Chest defibrillator
- keyboard stroke
- Touchscreen devices
- Flash camera





- positive charge gets stored in positive plate
- ΔQ at any time is zero, $\Delta Q=0$. [conservation of electric charge]
- Equal and oppositely charged 2 parallel plates generation Electric field from ⊕ to ⊖

$$\boxed{\sigma = \frac{|Q|}{A}}$$

- When $\Delta V_c = \Delta V_{battery}$, charging stops
- After disconnecting, the amount of charge stored in capacitor remains there
- Actually energy is stored not really charge bcs capacitor is a circuit element
- Capacitor: short burst of energy supply
- The energy is bet the 2 plates where electric field is generated

Discharging

- The capacitor connected to a bulb, now it acts as a power source.
- It gives short burst of energy. Thus we can see the intensity going from high to low quickly.
- The field intensity also decreases as energy is passing out

d field line separation increases

storing charge is a step
Main target is storing energy

With time, the charge (current) exponentially decreases/increases

Transient Current

RC-circuit: series connection bet resistor and capacitor.
Charging and discharging

$\Delta Q=0$, $|Q|\uparrow$, $E\uparrow$, $\uparrow\Delta V_c$ $\quad \Delta V = E\Delta r$

$$\boxed{C = \frac{Q}{\Delta V_c}}$$ → common charge

$$\boxed{Q_{enc} = \epsilon_0 \Phi_E = \epsilon_0 EA\cos 0 = \epsilon_0 EA}$$

$$\boxed{C = \frac{Q}{\Delta V_c} = \frac{\epsilon_0 EA}{Ed} = \frac{\epsilon_0 A}{d}}$$

$$\boxed{\text{\# Unit: } C\,V^{-1} / \text{Farad } [F] \quad \epsilon_0 \sim F\,m^{-1}}$$

*** Calculation requires $\mu F / nF / pF$

- Without charging, due to the geometric property of the plates, capacitance can be provided by adjusting the Area & separation
- This adjustment is limited by their geometry. There is a certain limit
- Insulating material: Dielectric material is placed betn the plates. Otherwise there would be only air. This would let charge to flow from +ve to -ve plate.
- Remember charge is Not jumping from +ve plate to -ve plate. The electric field generated stores the energy

Problem-11

1) $Q = VC$

$C = \frac{Q}{V} = \frac{\epsilon_0 A}{d}$

$A = \frac{1\times10^{-3}}{8.854\times10^{-12}}$

$A = 112.943\times10^{6}\ m^2$

requires such a big area. Thus small values are used

2) $C = 1.0\ mF = 1\times10^{-3} F$

(a) $d = 0.050\ mm = 0.05\times10^{-3}$

(i) $C = \frac{\epsilon_0 A}{d}$ $\quad A = \frac{Cd}{\epsilon_0} = \frac{1\times10^{-3}\times0.05\times10^{-3}}{8.854\times10^{-12}} = 5\ km$ ← still large

(ii) $|Q| = C\Delta V = 10^{-3}\times1.5 = 0.0015C$ → stored charge (large)

3) $d = 5\times10^{-3} m$

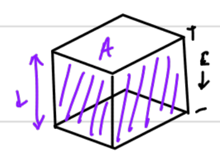
$A = 2\ m^2$

$\Delta V = 10\times10^3\ V$

(i) $C = \frac{\epsilon_0 A}{d} = \frac{8.854\times10^{-12}\times2}{5\times10^{-3}} = 3.54\ nF$

(ii) $|Q| = C\Delta V = 3.5416\times10^{-9}\times10\times10^{3} = 3.5\ \mu C$

(iii) $E = \frac{\Delta V_c}{d} = \frac{10\times10^3}{5\times10^{-3}} = 2\times10^6\ V\,m^{-1}$



The Energy stored

$$\boxed{\begin{aligned} \int dU &= \int dq\,\Delta V_c = \int dq\,\frac{|q|}{C} \\ &= \frac{1}{C}\int_0^Q q\,dq = \frac{1}{C}\left|\frac{q^2}{2}\right|_0^Q \\ U &= \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}Q\Delta V_c = \frac{1}{2}C\Delta V_c^2 \end{aligned}}$$ → Recall $U_{elas} = \frac{1}{2}kx^2$

Problem-12

1) (i) $C = \frac{\epsilon_0 A}{d}$

$C = \frac{Q}{\Delta V_c}$

(A) $(\Delta V_c)_1 = \frac{Q}{C}$

(B) $(\Delta V_c)_2 = \frac{Q}{2C}$

$\frac{1}{2}$(A)

(C) $V = \frac{2Q}{C}$

$= 2$(A)

(D) $V = \frac{2Q}{2C}$

$=$(A)

$V_c > V_4 = V_1 > V_2$

(ii) $U_1 = \frac{Q^2}{2C}$

$U_2 = \frac{Q}{2\times2C} = \frac{1}{2}U_1$

$U_3 = \frac{(2Q)^2}{2C} = 4U_1$

$U_4 = \frac{(2Q)^2}{2\times2C} = 2U_1$

$U_3 > U_4 > U_1 > U_2$

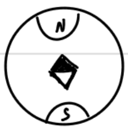
21/09/2024
Saturday

Parmanent Magnet → Just exists

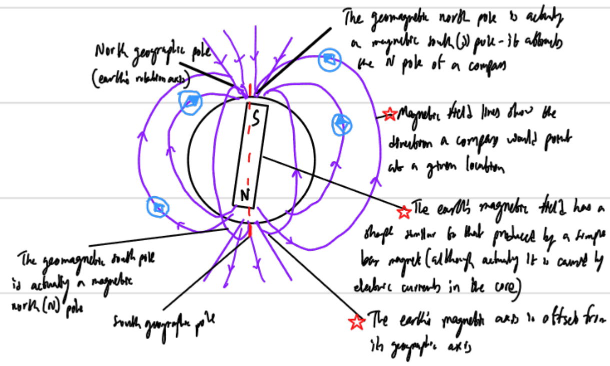
$\vec{B}$ → vector field
$\vec{B}$ → Intensity / strength / magnitude

like ⊕ → ⊖
North pole → South pole

- Aurora happens due to magnetic field
- geo magnet
  ↳ magma (lava) gets ionised → internal current flow → magnetic field
  happens due to high pressure, heat..



- Other places, high intensity of magnetic field (strong field) but not in poles. This radiation causes aurora / solar particle



*** Magnetism is interaction betn moving charges

Notation

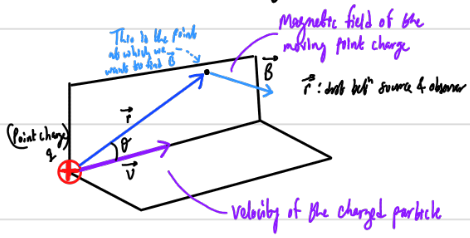
vector		current	
×	•	⊗	⊙
into the figure	out of figure	into the figure	out of figure

Magnetic Field Sources

→ Magnetism is an interaction betn moving charges.
*** One creates it; another experiences it.
*** But both need to be moving

A magnetic field $\vec{B}$ is created by a moving charge
magnetic interactions are understood in terms of Magnetic Poles: North and South
Practical Magnetic fields are created by electric currents - collection of moving charges
[Check Orsted's Experiment]
↳ Observation: A compass needle, normally aligned with Earth's magnetic field, changes direction when placed near a current-carrying wire

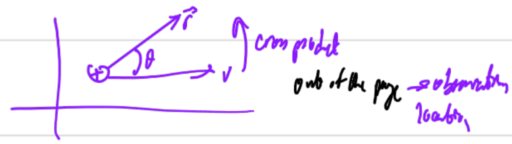
Biot-Savart (Bee-oh-Savah) law: Measures the $\vec{B}$-field produced by a moving point charge.



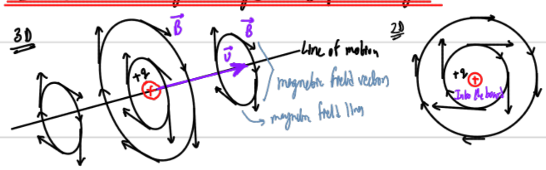
$$\vec{B} \propto \frac{q v \sin\theta}{r^2}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q v \sin\theta \, (\times \hat{r})}{r^2}$$
(source; unit vector $\hat{r}=1$)

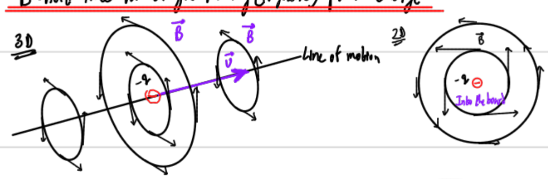
$$|\vec{B}|_{\text{point charge}} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$$



$\vec{B}$-field lines for single moving (Positive) point charge



B-field lines for single moving (Negative) point charge



Problem-1

mm scale
Source (0,0,0)
$(x_1, y_1) \to (x_2, y_2)$
$(x_2-x_1)\hat{i} + (y_2-y_1)\hat{j}$

- The proton is already moving (0,0,0) to a snapshot

$\vec{v} = (1.5\times10^{7}\,\hat{i})\,ms^{-1} + 0\hat{j} + 0\hat{k}$
$\vec{r}_1 = (1-0)\hat{i} + (0-0)\hat{j} + (0-0)\hat{k}$
$\hat{r}_1 = (10^{-3}\,m)\,\hat{i}$
$r_1 = 10^{-3}\,m$

at first position, $\vec{v}$ & $\hat{r}$ direction same (0°) so
at position 1, $\vec{B} = 0$
at position 2, $\vec{r}_2 = (0-0)\hat{i} + (1-0)\hat{j} + (0-0)\hat{k}$
$\hat{r}_2 : (10^{-3}\,m)\hat{j} \quad r : 10^{-3}\,m$

at position 3 //

with unit vector not $\vec{r}$

$$\vec{v}\times\vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1.5\times10^{7} & 0 & 0 \\ 0 & 10^{-3} & 0 \end{vmatrix}$$

$$\vec{B} = \frac{4\pi\times10^{-7}\,(TmA^{-1})}{4\pi} \times \frac{1.602\times10^{-19}\,C \times 1.5\times10^{4}\,ms^{-1}}{(10^{-3}\,m)^2}$$

$= (\ )\,T$

when $\vec{v}$ & $\hat{r}$ opposite direction
$\vec{v}\,r\sin(180) = 0$
⊕ & ⊖ opposite direction
first use right hand to find current direction

Problem-2

Source $\equiv (10^{-2}\,m)\hat{i} + 0\hat{j} + 0\hat{k}$

$\vec{v} = (0\hat{i} + 2\times10^{7}\hat{j} + 0\hat{k})\,ms^{-1}$

$\vec{r} = (0-10^{-2}\,m)\hat{i} + (10^{-2}\,m - 0)\hat{j} + (0-0)\hat{k}$

$\hat{r} = \dfrac{-10^{-2}\hat{i} + 10^{-2}\hat{j}}{\sqrt{(10^{-2})^2 + (10^{-2})^2}}\,m$

$r = \sqrt{(10^{-2})^2 + (10^{-2})^2}\,m$

$B = \dfrac{\mu_0}{4\pi}\dfrac{q(\vec{v}\times\hat{r})}{r^2}$

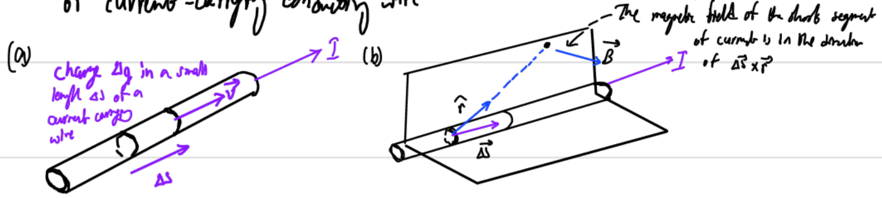
$= \dfrac{\mu_0}{4\pi}\dfrac{-1.6\times10^{-19}}{}$

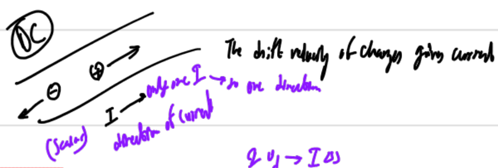
$\vec{v}\times\hat{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 2\times10^{7} & 0 \\ (\,) & (\,) & 0 \end{vmatrix}$

remember to keep $\hat{k} = 0$ for cross product

The source of the $\vec{B}$-field: Electric currents

Use Biot-Savart law to measure the $\vec{B}$-field produced by a very short segment of current-carrying conducting wire





$q\,v_d \rightarrow I\Delta s$

$I = n_e q_e A v_d$

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q(\vec{v}\times\hat{r})}{r^2} = \frac{\mu_0}{4\pi}\frac{q\,v_d \sin\theta}{r^2} = \frac{\mu_0}{4\pi}\frac{I\Delta s\sin\theta}{r^2} = \frac{\mu_0}{4\pi}\frac{I\,\Delta\vec{s}\times\hat{r}}{r^2}$$

from source to observing point

as I is scalar, $I\Delta s$ to make it vector (current element)

$I \times \hat{r} = 1$ ... round

\# Right-Hand method to find field direction

*** Magnetic Field Lines are circular

Problem-3

(a) (i) I (ii) (b)

Problem-4

$|\vec{B}| = \dfrac{\mu_0}{4\pi}\dfrac{I\Delta s\sin\theta}{r^2}$

Gauss's Law for $\vec{E}$-field

$$\boxed{\oint E\cdot d\vec{a} = \frac{Q_{enclosed}}{\varepsilon_0}, \quad \vec{\nabla}\cdot\vec{E} = \frac{\rho_{enclosed}}{\varepsilon_0}}$$

(Integral form) (Differential form)

Gauss's Law for Magnetic Fields

$$\boxed{\oint \vec{B}\cdot d\vec{a} = 0, \quad \vec{\nabla}\cdot\vec{B} = 0}$$

(Integral form) (Differential form)

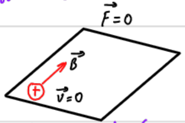
L-16

26/09/2024

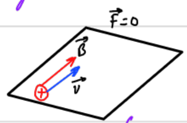
Thursday

Magnetic Forces on Moving Charges

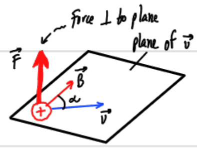
Both source and observer must be moving

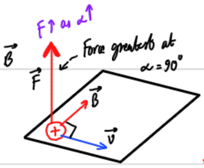


There is no magnetic force on a charged particle at rest



There is no magnetic force on a charged particle moving parallel to a magnetic field.





As the angle α bet^n the velocity and the magnetic field increase, the magnetic force also increases.
The force is greatest when angle is 90°
The magnetic force is always ⊥ to the plane containing $\vec{v}$ & $\vec{B}$.

$$\boxed{|\vec{F}_B| = |q\vec{v}\times\vec{B}| = qvB\sin\phi}$$ (direction of Right-Hand Method) // Right hand for +ve and left hand for -ve

Index: $\vec{B}$, thumb: $\vec{v}$, middle: $\vec{F}_B$

Problem-1

(a) F=0 (b) Down the page (c) ↑F (d) ↓F

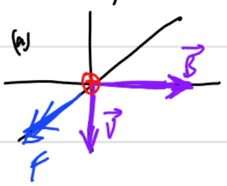
$\vec{v} = 10^7\cos 45\hat{i} + 0\hat{j} + 10^7\sin 45\hat{k}$

$B = 0.50T\ \hat{i} + 0\hat{j} + 0\hat{k}$

Problem-2

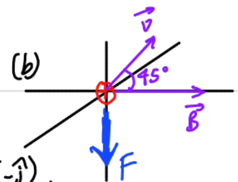
$\vec{B} = 0.5T$, $\vec{v} = 1\times10^7\ ms^{-1}$

(a)



$F = Bqv\sin 90$
$= 0.5\times1.6\times10^{-19}\times1\times10^7$
$= 0.8\times10^{-12} = 8\times10^{-13}N\ (-\hat{j})$ //

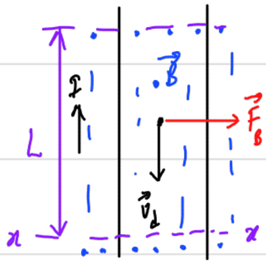
(b)



$\vec{F}_B = qvB\sin\theta$
$= 1.6\times10^{-19}\times1\times10^7\times0.5\times\sin 45$
$= \frac{1}{2\sqrt{2}}\times1.6\times10^{-12}\ N\ (-\hat{k})$ //

Magnetic Force on Current segment

$$\boxed{|\vec{F}_B| = |I\vec{L}\times\vec{B}| = ILB\sin\phi}$$ (direction of Right-Hand Method)



*** Changing the direction of I, changes direction of $\vec{v}_d$ thus force direction opposite

problem-3

// $I = \frac{\Delta V}{R} = \frac{15}{3} = 5A$ //

why ?? it's not parallel

Net zero //

Aspect	Electric Field	Magnetic Field
Source	Electric charges (static or moving)	Moving charges (currents) or changing electric field
Direction	Points from positive to negative charges	Forms closed loops around source
Field Lines	Diverge from +ve to converge on negative charges	Forms closed loops around source with no start or end
Force	$\vec{F}_E = q\vec{E}$	$\vec{F}_B = q\vec{v}\times\vec{B}$
Interaction	On both stationary and moving charges	ONLY on moving charges
Superposition	Yes	Yes
Maxwell's Equations	Two	Two