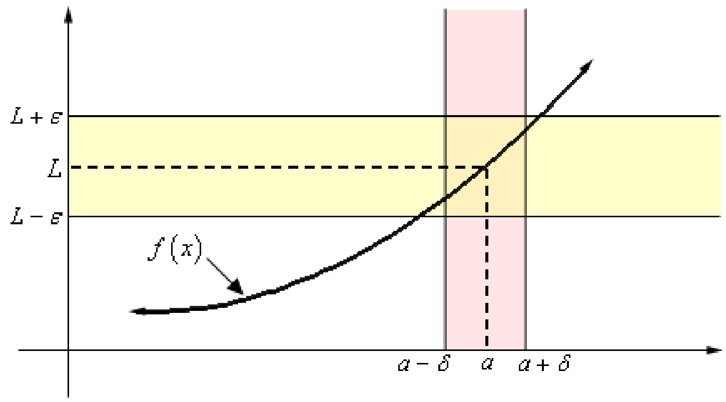
<u>**Limits**</u>

Limit: Let $f(x)$ be a function defined on an interval that contains $x = a(f(x)$ need not be defined at $x = a)$. Then we say that, $\lim_{x\to a} f(x) = L$if for every number $\varepsilon > 0$ there is some number $\delta > 0$ such that

$|f(x) - L| < \varepsilon$ whenever

$0 < |x - a| < \delta$.

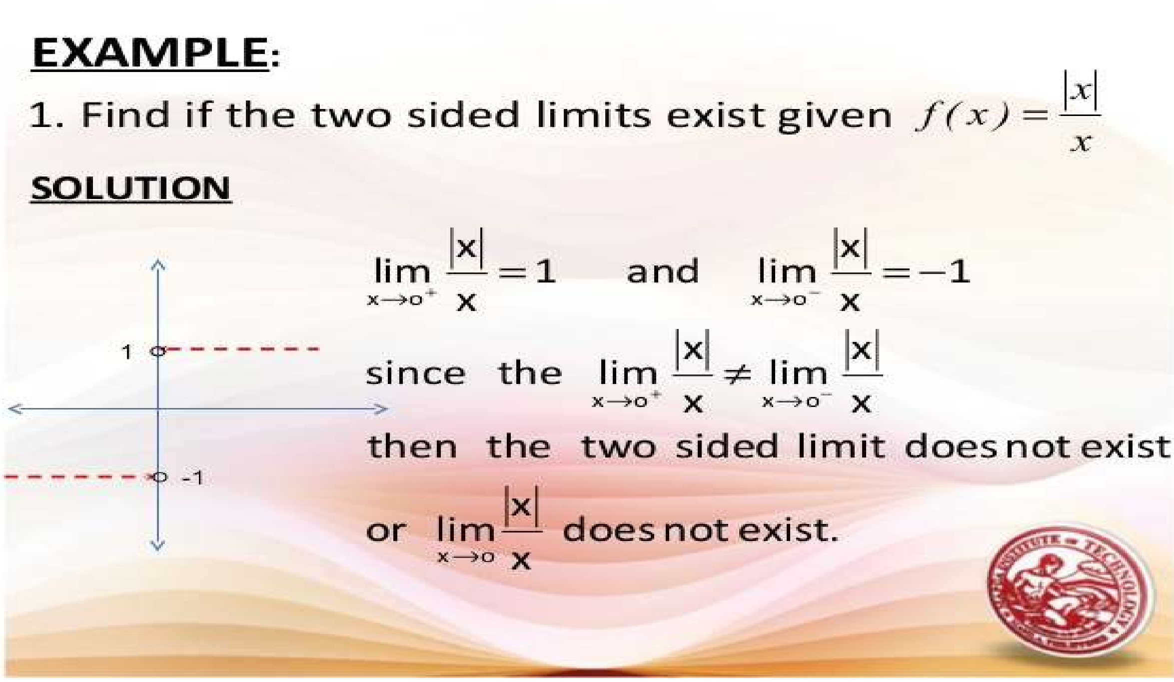


One sided limits:

(i) **Right hand limit:**$\lim_{x\to a^+} f(x) = L$ if for every number $\varepsilon > 0$ there is some number $\delta > 0$ such that $|f(x) - L| < \varepsilon$ whenever $a < x < a + \partial$.

(ii) **Left hand limit:**$\lim_{x\to a^-} f(x) = L$ if for every number $\varepsilon > 0$ there is some number $\delta > 0$ such that $|f(x) - L| < \varepsilon$whenever$a - \partial < x < a$.

Two-sided limit: When we consider both left hand limit and right hand limit at the same time, then they are known as two sided limit.



Properties of limits:

THEOREM 1—Limit Laws If L, M, c, and k are real numbers and

$$\lim_{x \to c} f(x) = L \qquad \text{and} \qquad \lim_{x \to c} g(x) = M, \quad \text{then}$$

1. *Sum Rule:* $\lim_{x \to c} (f(x) + g(x)) = L + M$
2. *Difference Rule:* $\lim_{x \to c} (f(x) - g(x)) = L - M$
3. *Constant Multiple Rule:* $\lim_{x \to c} (k \cdot f(x)) = k \cdot L$
4. *Product Rule:* $\lim_{x \to c} (f(x) \cdot g(x)) = L \cdot M$
5. *Quotient Rule:* $\lim_{x \to c} \dfrac{f(x)}{g(x)} = \dfrac{L}{M}, \quad M \neq 0$
6. *Power Rule:* $\lim_{x \to c} [f(x)]^n = L^n$, n a positive integer
7. *Root Rule:* $\lim_{x \to c} \sqrt[n]{f(x)} = \sqrt[n]{L} = L^{1/n}$, n a positive integer

(If n is even, we assume that $\lim_{x \to c} f(x) = L > 0$.)