Uniform Circular Motion

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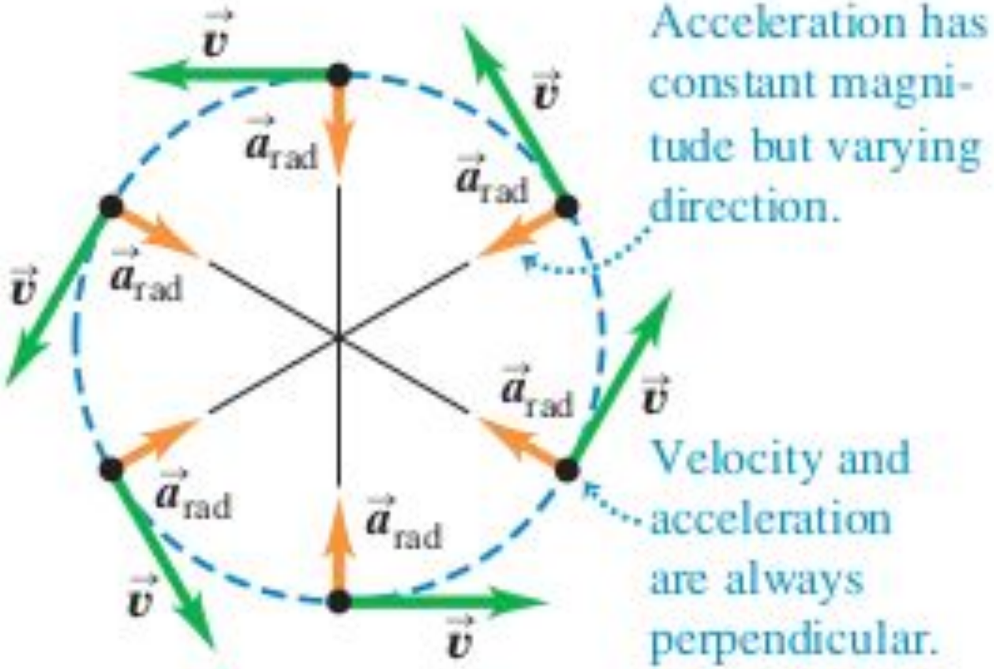


$$a = \frac{v^2}{r}$$

$$v = \omega r$$

$$a = \omega^2 r$$

$$F = ma = \frac{mv^2}{r} = m\omega^2 r$$

$$v = \frac{d}{T}$$

$$= \frac{2\pi r}{T}$$

$$\therefore T = \frac{2\pi r}{v} = \frac{2\pi r}{\omega r} = \frac{2\pi}{\omega}$$

82 In Fig. 6-57, a stuntman drives a car (without negative lift) over the top of a hill, the cross section of which can be approximated by a circle of radius $R = 250$ m. What is the greatest speed at which he can drive without the car leaving the road at the top of the hill?

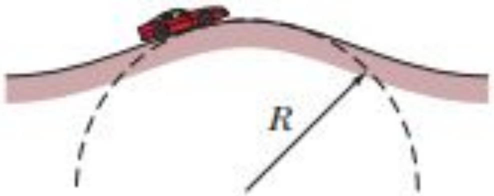
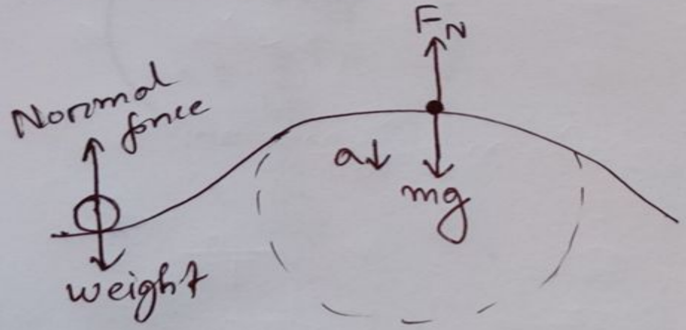


Fig. 6-57 Problem 82.

(82)

Radius = 250 m, Greatest speed = ?



$$F_N - mg = -ma$$

$$\Rightarrow F_N - mg = -\frac{mv^2}{r}$$

At greatest speed, $F_N = 0$

$$\therefore 0 - mg = -\frac{mv^2}{r}$$

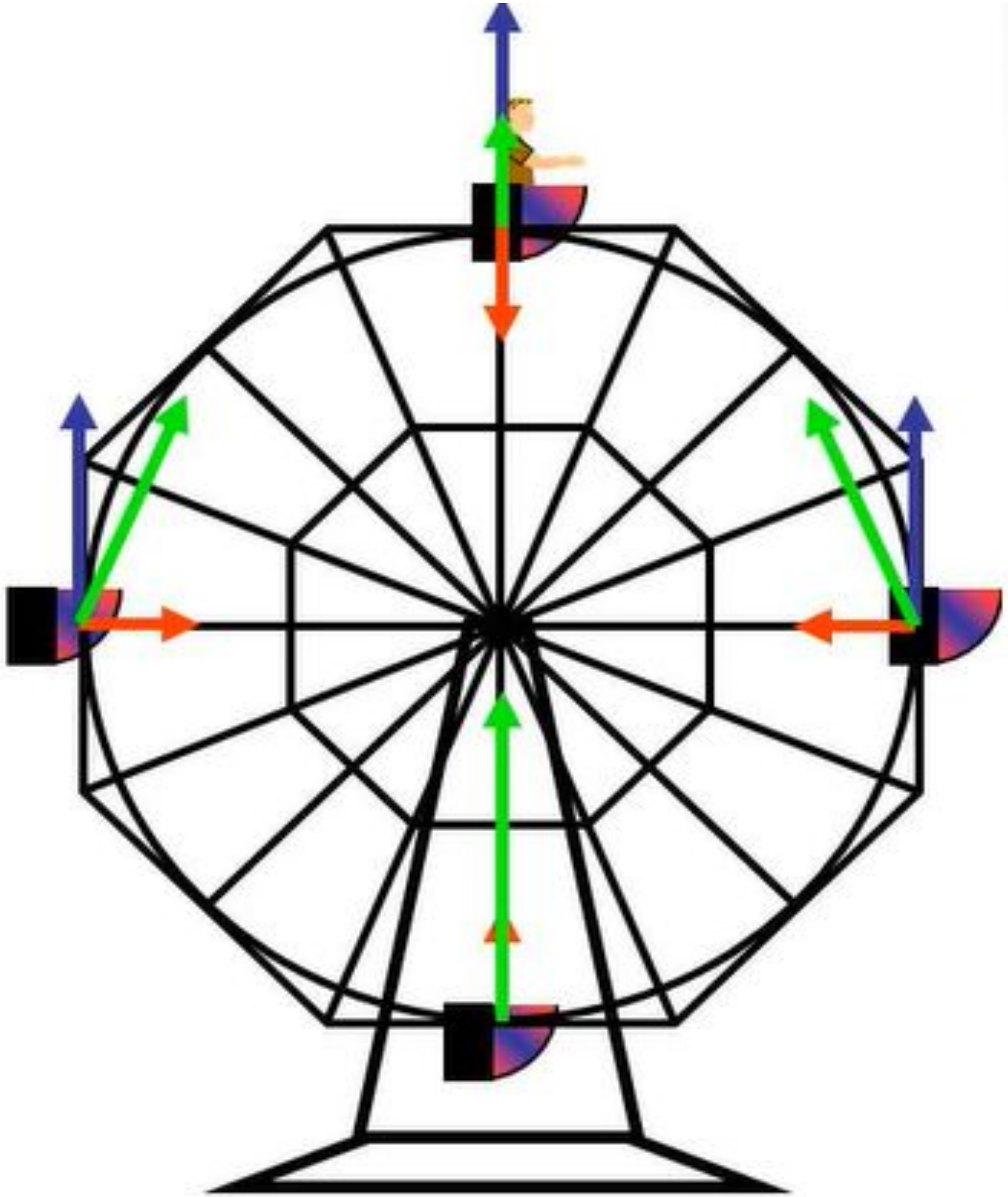
$$\Rightarrow -mg = -\frac{mv^2}{r}$$

$$\Rightarrow g = \frac{v^2}{r}$$

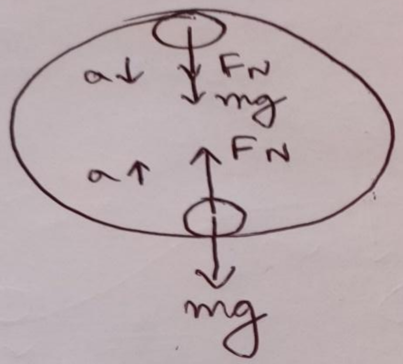
$$\Rightarrow v = \sqrt{rg}$$


FOREPAUGH & SELLS BROTHERS
ENORMOUS SHOWS UNITED
J.A. BAILEY, W.W. COLE, LEWIS SELLS & PETER SELLS: EQUAL OWNERS.
DIAVOLO
LOOPING THE LOOP
THE VERITABLE CAP-SHEAF OF ALL HAZARDOUS EXPLOITS
THE EXTREME AND ABSOLUTE LIMIT OF SENSATIONALISM REACHED AT LAST. BEYOND THE TREMENDOUSLY TERRIBLE TEMERITY AND ILLIMITABLE, INIMITABLE INTREPIDITY OF DIAVOLO, NO MAN MAY GO.

(a)



Diavolo



Top point

$$-F_N - mg = -ma$$

$$\Rightarrow -F_N - mg = -\frac{mv^2}{r}$$

At least speed, $F_N = 0$

$$-mg = -\frac{mv^2}{r}$$

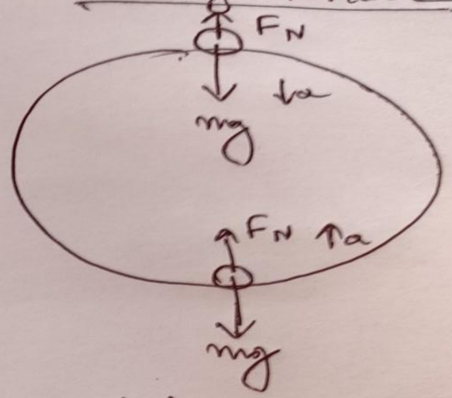
$$\Rightarrow v^2 = rg$$

$$\Rightarrow v = \sqrt{rg}$$

Lowest point

$$F_N - mg = \frac{mv^2}{r}$$

Fenny's Wheel



Top point

$$F_N - mg = -\frac{mv^2}{r}$$

Lowest point

$$F_N - mg = ma$$

$$\Rightarrow F_N - mg = \frac{mv^2}{r}$$

70 Figure 6-53 shows a *conical pendulum,* in which the bob (the small object at the lower end of the cord) moves in a horizontal circle at constant speed. (The cord sweeps out a cone as the bob rotates.) The bob has a mass of 0.040 kg, the string has length $L = 0.90$ m and negligible mass, and the bob follows a circular path of circumference 0.94 m. What are (a) the tension in the string and (b) the period of the motion?

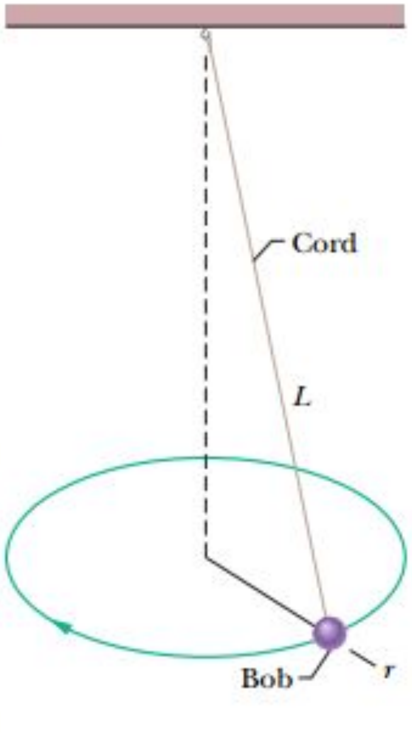


Fig. 6-53 Problem 70.

71 An 8.00 kg block of steel is at rest on a horizontal table. The coefficient of static friction between the block and the table is 0.450. A force is to be applied to the block. To three significant figures, what is the magnitude of that applied force if it puts the block on the verge of sliding when the force is directed (a) horizontally, (b) upward at

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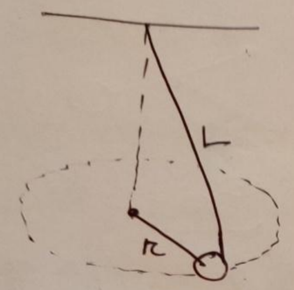


Tension = ?
Period of Motion = ?

Circumference $= 2\pi r$

$\Rightarrow 0.94 = 2\pi r$

$\Rightarrow r = \dfrac{0.94}{2\pi}$

$= 0.15\,m$

(a)

(H) $-T\cos\theta = -\dfrac{mv^2}{r}$

$\Rightarrow T\cos\theta = \dfrac{mv^2}{r}$ —(i)

(V) $T\sin\theta = mg$ —(ii)

$\cancel{\dfrac{T\sin\theta}{T\cos\theta} = \dfrac{mg}{\frac{mv^2}{r}}}$

$\Rightarrow T\sin 80 = 0.040 \times 9.8$

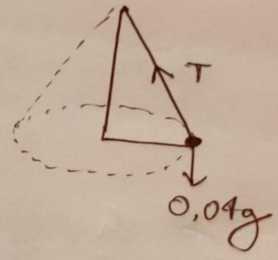
$\Rightarrow T = 0.40\,N$

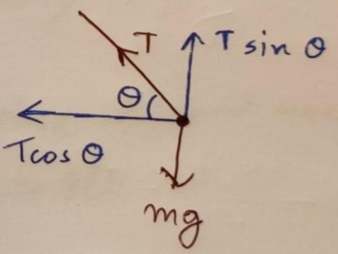
(b) $T\cos\theta = \dfrac{mv^2}{r}$

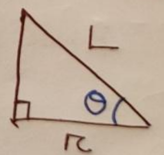
$\Rightarrow 0.40\cos 80 = \dfrac{0.04 \times v^2}{0.15}$

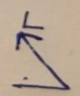
$\Rightarrow v = 0.51\,m/s^2$

$L = 0.90\,m$
$m = 0.040\,kg$
circumference $= 0.94\,m$









$\cos\theta = \dfrac{r}{L}$

$\Rightarrow \cos\theta = \dfrac{0.15}{0.90}$

$\Rightarrow \theta = \cos^{-1}\left(\dfrac{0.15}{0.90}\right)$

$= 80^\circ$

$\therefore t = \dfrac{2\pi r}{v} = \dfrac{2\pi \times 0.15}{0.51} = 1.85\,s$

••57 GO A puck of mass $m = 1.50$ kg slides in a circle of radius $r = 20.0$ cm on a frictionless table while attached to a hanging cylinder of mass $M = 2.50$ kg by means of a cord that extends through a hole in the table (Fig. 6-43). What speed keeps the cylinder at rest?

••58 *Brake or turn?* Figure 6-44 depicts an overhead view of a car's path as the car travels toward a wall. Assume that the driver begins

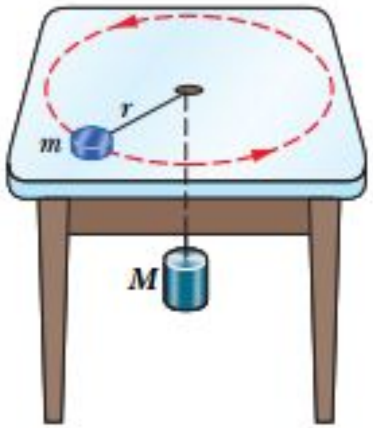


Fig. 6-43 Problem 57.

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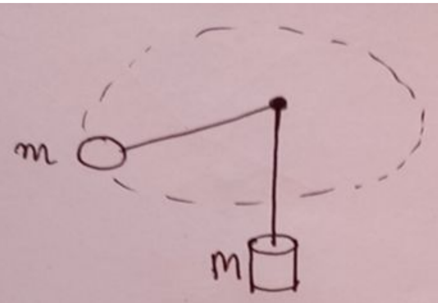
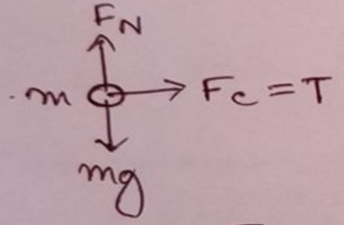
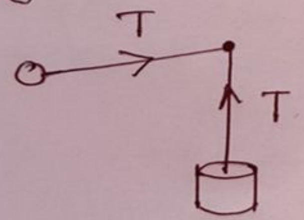




(V) $T - Mg = 0$

$\Rightarrow T = Mg$



(H) $F_c = T = \frac{mv^2}{r}$

$T = \frac{mv^2}{r}$

$\Rightarrow Mg = \frac{mv^2}{r}$

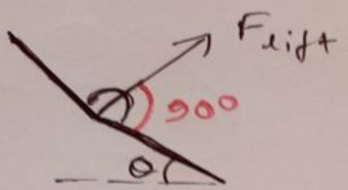
$\Rightarrow v^2 = \frac{Mgr}{m}$

$\Rightarrow v = \sqrt{\frac{Mgr}{m}}$

••51 SSM WWW An airplane is flying in a horizontal circle at a speed of 480 km/h (Fig. 6-41). If its wings are tilted at angle $\theta = 40°$ to the horizontal, what is the radius of the circle in which the plane is flying? Assume that the required force is provided entirely by an "aerodynamic lift" that is perpendicular to the wing surface.

Fig. 6-41 Problem 51.

(51)

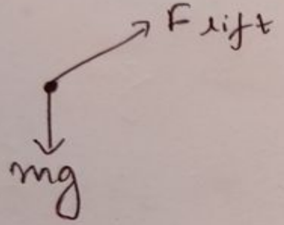


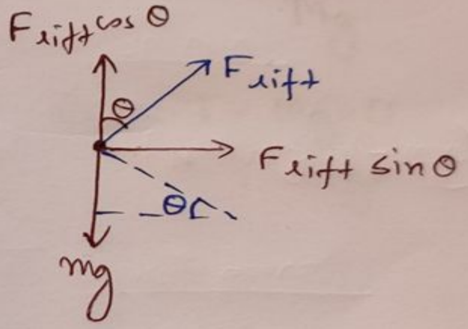
$\theta = 40°$

$v = 480\ \text{km/hr}$

$= \dfrac{480 \times 10^3}{60 \times 60}\ \text{m/s}$

$= 133\ \text{m/s}$





(H) $F_{lift} \sin\theta = \dfrac{mv^2}{r}$ — (i)

(V) $F_{lift} \cos\theta - mg = 0$

$\Rightarrow F_{lift} \cos\theta = mg$ — (ii)

(i) ÷ (ii)

$$\frac{F_{lift} \sin\theta}{F_{lift} \cos\theta} = \frac{\frac{mv^2}{r}}{mg}$$

$\Rightarrow \tan\theta = \dfrac{v^2}{rg}$

$\Rightarrow r = \dfrac{v^2}{g \tan\theta}$

<u>Problem</u>: A rotating fan completes 1200 revolutions every minute. Consider the tip of a blade, at a radius of 0.15m. (a) Through what distance does the tip move in one revolution? What are (b) the tip's speed, and (c) the magnitude of its acceleration? (d) What is the period of motion? (e) Repeat (b) and (c) for a point halfway along the blade.

$f = 1200\,rev/min = \frac{1200}{60}\,rev/s = 20\,rev/s$

$\omega = 2\pi f =$

(a) $s = \theta r = 2\pi \times 0.15 = 0.94\,m$

(b) $v = \omega r = 2\pi f \times r = 2\pi \times 20 \times 0.15 = 18.8\,m/s$

(c) $a = \frac{v^2}{r} = r\omega^2 = r(2\pi f)^2 = 0.15 \times (2\pi \times 20)^2$

$= 2368.7\,m/s^2$

(d) $T = \frac{1}{f} = \frac{1}{20}\,s = 0.05\,s$

(e) $v = \omega r = 0.075 \times (2\pi \times 20)\,m/s$

$= 9.42\,m/s.$