CHAPTER-7 (WORK AND KINETIC ENERGY)

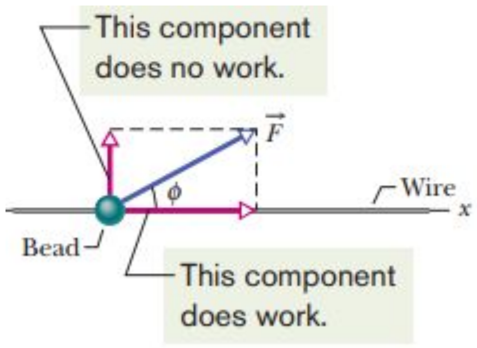
Work done

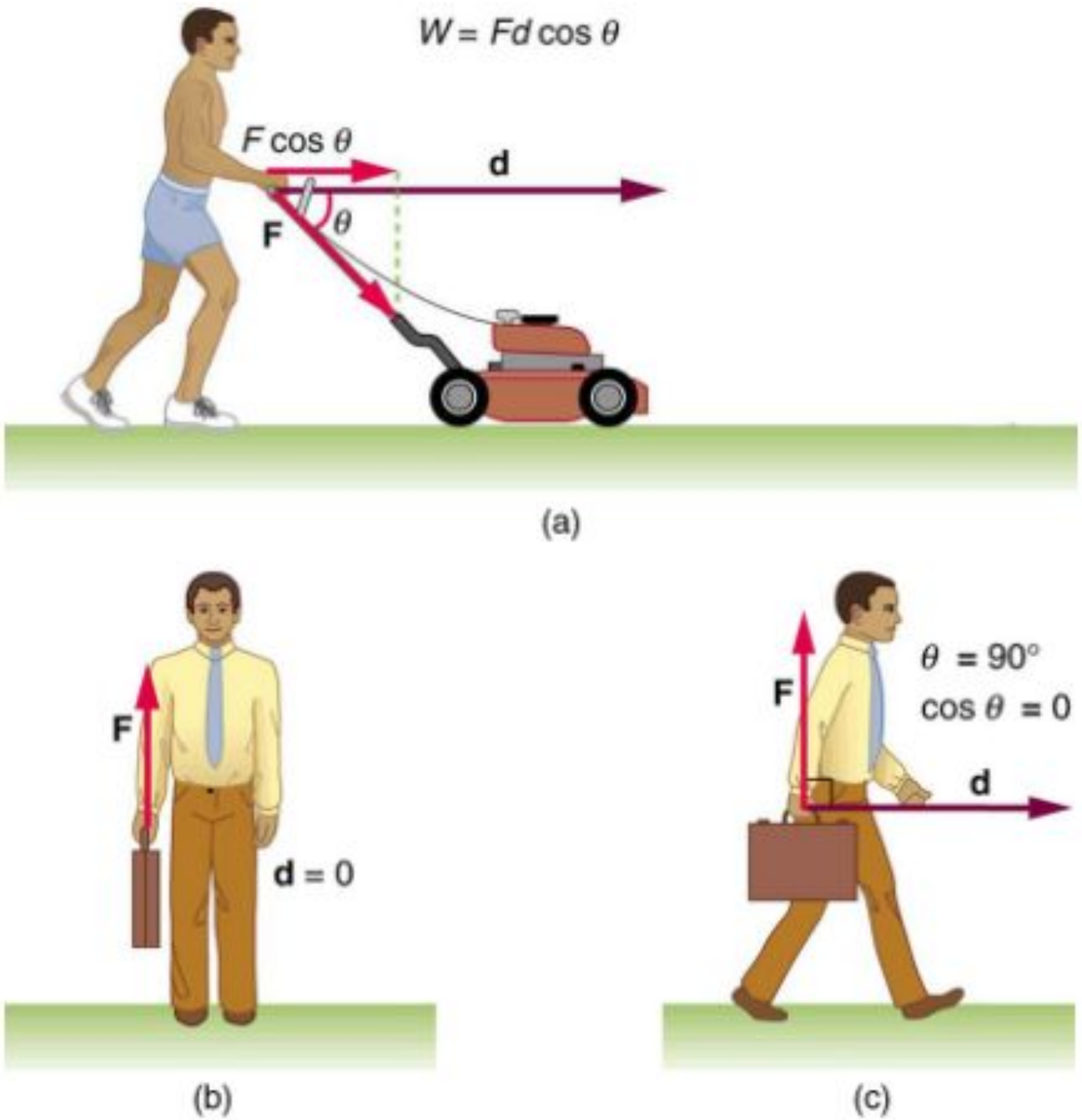
- Work W is energy transferred to or from an object by means of a force acting on the object. Energy transferred to the object is positive work, and energy transferred from the object is negative work.

$$W = Fd\cos\phi \qquad \text{(work done by a constant force)}. \qquad (7\text{-}7)$$

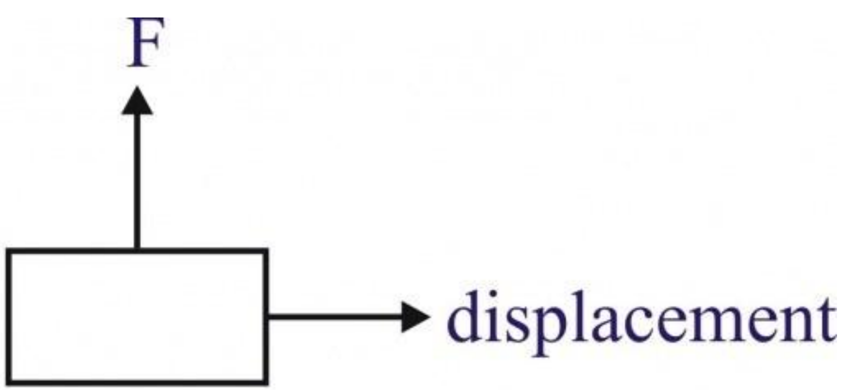
Because the right side of this equation is equivalent to the scalar (dot) product $\vec{F}\cdot\vec{d}$, we can also write

$$W = \vec{F}\cdot\vec{d} \qquad \text{(work done by a constant force)}, \qquad (7\text{-}8)$$

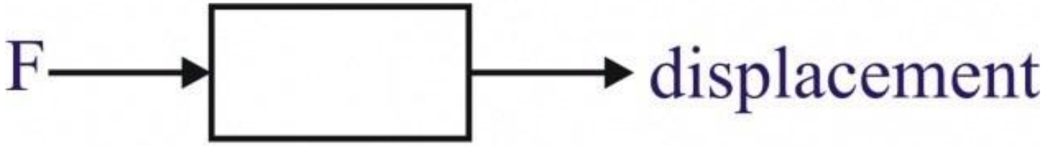




W = Fd cos θ
F cos θ
d
θ
F
(a)
F
d = 0
(b)
F
θ = 90°
cos θ = 0
d
(c)



Work done = 0 as F & d are perpendicular to each other.



Work done is positive.

Work done is negative.

Questions: Positive or Negative Work

Positive Work = Force *along* the direction of motion ($\theta < 90°$)

Negative Work = Force *opposite* to the direction of motion ($\theta > 90°$)

Work W is energy transferred to or from an object by means of a force acting on the object. Energy transferred to the object is positive work, and energy transferred from the object is negative work.

Work is a Scalar Quantity

- +ve Work → energy transferred to the object
- -ve Work → energy transferred from the object

WORK KINETIC ENERGY THEOREM

$$\Delta K = K_f - K_i = W, \qquad (7\text{-}10)$$

which says that

$$\begin{pmatrix}\text{change in the kinetic}\\ \text{energy of a particle}\end{pmatrix} = \begin{pmatrix}\text{net work done on}\\ \text{the particle}\end{pmatrix}.$$

We can also write

$$K_f = K_i + W, \qquad (7\text{-}11)$$

which says that

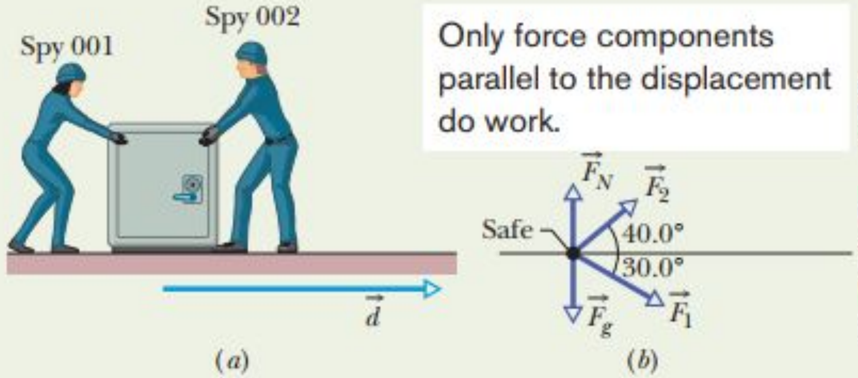
$$\begin{pmatrix}\text{kinetic energy after}\\ \text{the net work is done}\end{pmatrix} = \begin{pmatrix}\text{kinetic energy}\\ \text{before the net work}\end{pmatrix} + \begin{pmatrix}\text{the net}\\ \text{work done}\end{pmatrix}.$$

CHECKPOINT 1

A particle moves along an x axis. Does the kinetic energy of the particle increase, decrease, or remain the same if the particle's velocity changes (a) from -3 m/s to -2 m/s and (b) from -2 m/s to 2 m/s? (c) In each situation, is the work done on the particle positive, negative, or zero?

Figure 7-4a shows two industrial spies sliding an initially stationary 225 kg floor safe a displacement $\vec{d}$ of magnitude 8.50 m, straight toward their truck. The push $\vec{F}_1$ of spy 001 is 12.0 N, directed at an angle of 30.0° downward from the horizontal; the pull $\vec{F}_2$ of spy 002 is 10.0 N, directed at 40.0° above the horizontal. The magnitudes and directions of these forces do not change as the safe moves, and the floor and safe make frictionless contact.

(a) What is the net work done on the safe by forces $\vec{F}_1$ and $\vec{F}_2$ during the displacement $\vec{d}$?



Calculations: From Eq. 7-7 and the free-body diagram for the safe in Fig. 7-4b, the work done by $\vec{F}_1$ is

$$W_1 = F_1 d \cos \phi_1 = (12.0\text{ N})(8.50\text{ m})(\cos 30.0°) = 88.33\text{ J},$$

and the work done by $\vec{F}_2$ is

$$W_2 = F_2 d \cos \phi_2 = (10.0\text{ N})(8.50\text{ m})(\cos 40.0°) = 65.11\text{ J}.$$

Thus, the net work W is

$$W = W_1 + W_2 = 88.33\text{ J} + 65.11\text{ J} = 153.4\text{ J} \approx 153\text{ J}. \quad \text{(Answer)}$$

During the 8.50 m displacement, therefore, the spies transfer 153 J of energy to the kinetic energy of the safe.

(b) During the displacement, what is the work W_g done on the safe by the gravitational force $\vec{F}_g$ and what is the work W_N done on the safe by the normal force $\vec{F}_N$ from the floor?

$$W_g = mgd \cos 90° = mgd(0) = 0 \quad \text{(Answer)}$$

and

$$W_N = F_N d \cos 90° = F_N d(0) = 0. \quad \text{(Answer)}$$

(c) The safe is initially stationary. What is its speed v_f at the end of the 8.50 m displacement?

KEY IDEA

The speed of the safe changes because its kinetic energy is changed when energy is transferred to it by $\vec{F}_1$ and $\vec{F}_2$.

Calculations: We relate the speed to the work done by combining Eqs. 7-10 and 7-1:

$$W = K_f - K_i = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2.$$

The initial speed v_i is zero, and we now know that the work done is 153.4 J. Solving for v_f and then substituting known data, we find that

$$v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2(153.4\text{ J})}{225\text{ kg}}} = 1.17\text{ m/s}. \quad \text{(Answer)}$$

During a storm, a crate of crepe is sliding across a slick, oily parking lot through a displacement $\vec{d} = (-3.0\text{ m})\hat{\text{i}}$ while a steady wind pushes against the crate with a force $\vec{F} = (2.0\text{ N})\hat{\text{i}} + (-6.0\text{ N})\hat{\text{j}}$. The situation and coordinate axes are shown in Fig. 7-5.

(a) How much work does this force do on the crate during the displacement?

Calculations: We write

$$W = \vec{F}\cdot\vec{d} = [(2.0\text{ N})\hat{\text{i}} + (-6.0\text{ N})\hat{\text{j}}]\cdot[(-3.0\text{ m})\hat{\text{i}}].$$

Of the possible unit-vector dot products, only $\hat{\text{i}}\cdot\hat{\text{i}}$, $\hat{\text{j}}\cdot\hat{\text{j}}$, and $\hat{\text{k}}\cdot\hat{\text{k}}$ are nonzero (see Appendix E). Here we obtain

$$W = (2.0\text{ N})(-3.0\text{ m})\hat{\text{i}}\cdot\hat{\text{i}} + (-6.0\text{ N})(-3.0\text{ m})\hat{\text{j}}\cdot\hat{\text{i}}$$
$$= (-6.0\text{ J})(1) + 0 = -6.0\text{ J}. \qquad \text{(Answer)}$$

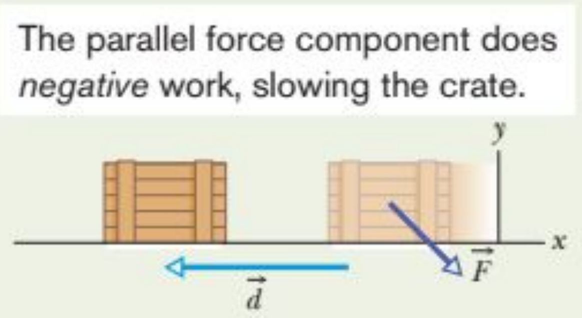


Fig. 7-5 Force $\vec{F}$ slows a crate during displacement $\vec{d}$.

Thus, the force does a negative 6.0 J of work on the crate, transferring 6.0 J of energy from the kinetic energy of the crate.

(b) If the crate has a kinetic energy of 10 J at the beginning of displacement $\vec{d}$, what is its kinetic energy at the end of $\vec{d}$?

KEY IDEA

Because the force does negative work on the crate, it reduces the crate's kinetic energy.

Calculation: Using the work–kinetic energy theorem in the form of Eq. 7-11, we have

$$K_f = K_i + W = 10\text{ J} + (-6.0\text{ J}) = 4.0\text{ J}. \qquad \text{(Answer)}$$

Less kinetic energy means that the crate has been slowed.

LOWERING AND RISING A OBJECT

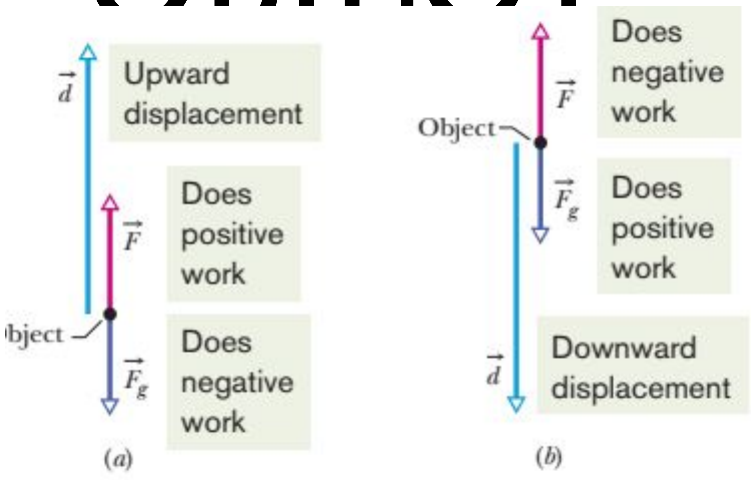


Fig. 7-7 (*a*) An applied force $\vec{F}$ lifts an object. The object's displacement $\vec{d}$ makes an angle $\phi = 180°$ with the gravitational force $\vec{F}_g$ on the object. The applied force does positive work on the object. (*b*) An applied force $\vec{F}$ lowers an object. The displacement $\vec{d}$ of the object makes an angle $\phi = 0°$ with the gravitational force $\vec{F}_g$. The applied force does negative work on the object.

$$\Delta K = K_f - K_i = W_a + W_g,$$

In one common situation, the object is stationary before and after the lift—for example, when you lift a book from the floor to a shelf. Then K_f and K_i are both zero, and Eq. 7-15 reduces to

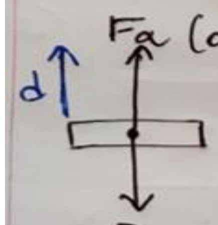
$$W_g = mgd\cos\phi \quad \text{(work done by gravitational force).} \qquad (7\text{-}12)$$

For a rising object, force $\vec{F}_g$ is directed opposite the displacement $\vec{d}$, as indicated in Fig. 7-6. Thus, $\phi = 180°$ and

$$W_g = mgd\cos 180° = mgd(-1) = -mgd. \qquad (7\text{-}13)$$

$$W_a = -mgd\cos\phi \quad \text{(work done in lifting and lowering; } K_f = K_i\text{).}$$

Lifting



$W = \Delta K$

$\Rightarrow W = K_f - K_i$

$\Rightarrow W_a + W_g = K_f - K_i$

W_a: Work done due to applied force

W_g: work done due to gravitational force

Velocity of object is zero both before and after the lift

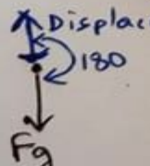
$\therefore K_f = K_i = 0$

So, $W_a + W_g = 0$

$\Rightarrow W_a = -W_g$

$= -F_g d \cos\theta$

Now,

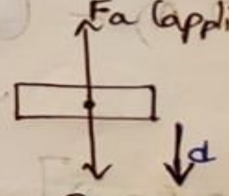


$\therefore W_a = -F_g d \cos 180$

$= -F_g d \cos 180$

$= -F_g d(-1) = F_g d = mgd$

Lowering



$W = \Delta K$

$\Rightarrow W_a + W_g = K_f - K_i$

$\because v_i = v_f = 0 \quad \therefore K_f = K_i = 0$

So, $W_a + W_g = 0$

$\Rightarrow W_a = -W_g$

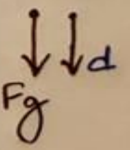
$= -F_g d \cos\theta$

$= -F_g d \cos 0$

$= -F_g d$

$= -mgd$

$\theta = 0$



Work done on an accelerating elevator cab

An elevator cab of mass $m = 500$ kg is descending with speed $v_i = 4.0$ m/s when its supporting cable begins to slip, allowing it to fall with constant acceleration $\vec{a} = \vec{g}/5$ (Fig. 7-8*a*).

(a) During the fall through a distance $d = 12$ m, what is the work W_g done on the cab by the gravitational force $\vec{F}_g$?

KEY IDEA

We can treat the cab as a particle and thus use Eq. 7-12 ($W_g = mgd \cos \phi$) to find the work W_g.

Calculation: From Fig. 7-8*b*, we see that the angle between the directions of $\vec{F}_g$ and the cab's displacement $\vec{d}$ is 0°. Then, from Eq. 7-12, we find

$$W_g = mgd \cos 0° = (500 \text{ kg})(9.8 \text{ m/s}^2)(12 \text{ m})(1)$$
$$= 5.88 \times 10^4 \text{ J} \approx 59 \text{ kJ}. \qquad \text{(Answer)}$$

(b) During the 12 m fall, what is the work W_T done on the cab by the upward pull $\vec{T}$ of the elevator cable?

KEY IDEAS

(1) We can calculate work W_T with Eq. 7-7 ($W = Fd \cos \phi$) if we first find an expression for the magnitude T of the cable's pull. (2) We can find that expression by writing Newton's second law for components along the y axis in Fig. 7-8*b* ($F_{net,y} = ma_y$).

Calculations: We get

$$T - F_g = ma. \qquad (7\text{-}18)$$

Solving for T, substituting mg for F_g, and then substituting the result in Eq. 7-7, we obtain

$$W_T = Td \cos \phi = m(a + g)d \cos \phi. \qquad (7\text{-}19)$$

Next, substituting $-g/5$ for the (downward) acceleration a and then 180° for the angle ϕ between the directions of forces $\vec{T}$ and $m\vec{g}$, we find

$$W_T = m\left(-\frac{g}{5} + g\right) d \cos \phi = \frac{4}{5} mgd \cos \phi$$
$$= \frac{4}{5}(500 \text{ kg})(9.8 \text{ m/s}^2)(12 \text{ m}) \cos 180°$$
$$= -4.70 \times 10^4 \text{ J} \approx -47 \text{ kJ}. \qquad \text{(Answer)}$$

Caution: Note that W_T is not simply the negative of W_g. The reason is that, because the cab accelerates during the fall, its speed changes during the fall, and thus its kinetic energy also changes. Therefore, Eq. 7-16 (which assumes that the initial and final kinetic energies are equal) does *not* apply here.

(c) What is the net work W done on the cab during the fall?

Calculation: The net work is the sum of the works done by the forces acting on the cab:

$$W = W_g + W_T = 5.88 \times 10^4 \text{ J} - 4.70 \times 10^4 \text{ J}$$
$$= 1.18 \times 10^4 \text{ J} \approx 12 \text{ kJ}. \qquad \text{(Answer)}$$

(d) What is the cab's kinetic energy at the end of the 12 m fall?

KEY IDEA

The kinetic energy changes *because* of the net work done on the cab, according to Eq. 7-11 ($K_f = K_i + W$).

Calculation: From Eq. 7-1, we can write the kinetic energy at the start of the fall as $K_i = \frac{1}{2}mv_i^2$. We can then write Eq. 7-11 as

$$K_f = K_i + W = \tfrac{1}{2}mv_i^2 + W$$
$$= \tfrac{1}{2}(500 \text{ kg})(4.0 \text{ m/s})^2 + 1.18 \times 10^4 \text{ J}$$
$$= 1.58 \times 10^4 \text{ J} \approx 16 \text{ kJ}. \qquad \text{(Answer)}$$

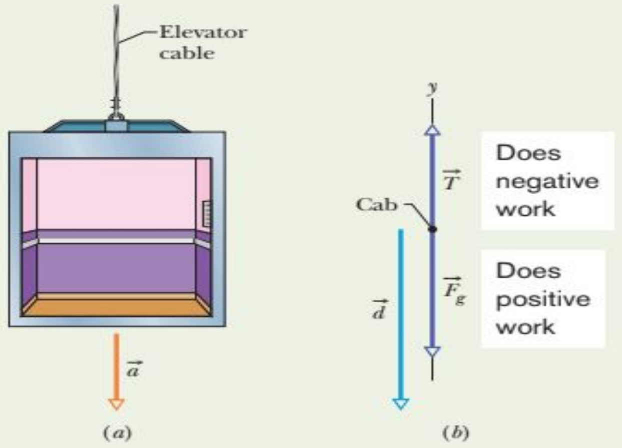
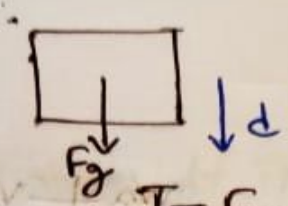


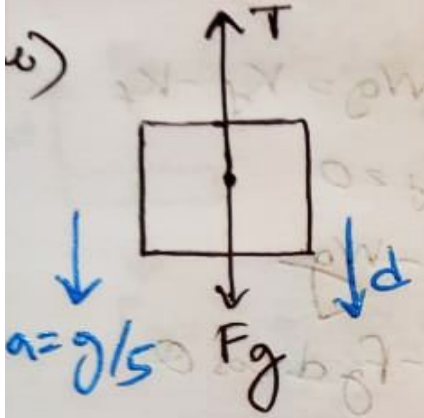
Fig. 7-8 An elevator cab, descending with speed v_i, suddenly begins to accelerate downward. (*a*) It moves through a displacement $\vec{d}$ with constant acceleration $\vec{a} = \vec{g}/5$. (*b*) A free-body diagram for the cab, displacement included.

a)



$$W_g = F_g\, d \cos\theta$$
$$= mg\, d\cos 0$$
$$= mgd$$

b)



$$T - F_g = -ma$$
$$\Rightarrow T - mg = -m\,(g/5)$$
$$\Rightarrow T = mg - mg/5$$
$$= \tfrac{4}{5} mg$$

$$W_T = T\, d\cos\theta$$
$$= \tfrac{4}{5} mg \times d \times \cos 180$$
$$= -\tfrac{4}{5} mgd$$

c)
$$W = W_g + W_T$$

d)
$$W = K_f - K_i$$
$$\Rightarrow W = K_f - K_i$$
$$\Rightarrow K_f = W + K_i$$
$$= W + \tfrac{1}{2} m (v_i)^2$$

Work done by a spring

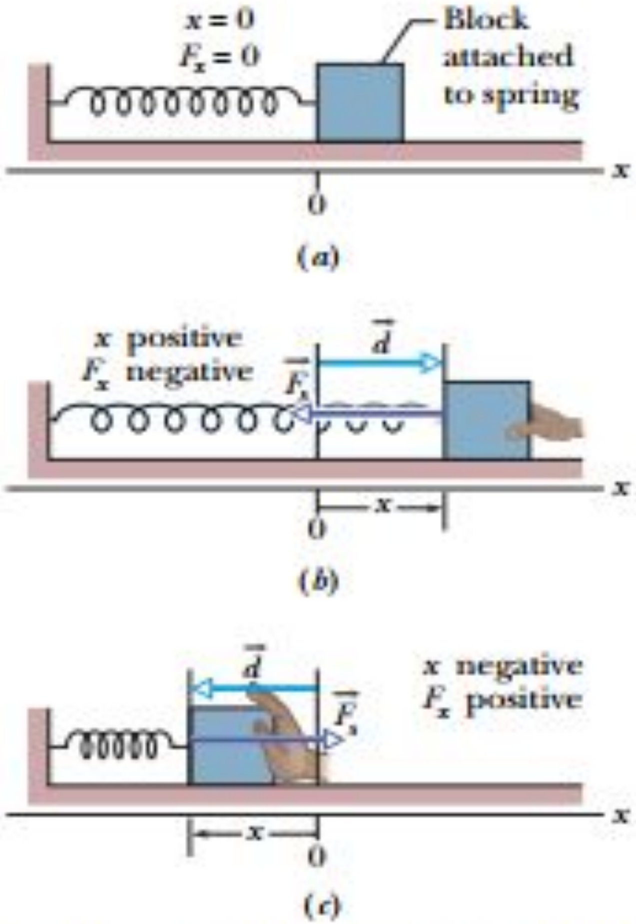


Fig. 7-9 (*a*) A spring in its relaxed state. The origin of an *x* axis has been placed at the end of the spring that is attached to a block. (*b*) The block is displaced by $\vec{d}$, and the spring is stretched by a positive amount *x*. Note the restoring force $\vec{F}_s$ exerted by the spring. (*c*) The spring is compressed by a negative amount *x*. Again, note the restoring force.

$$F_x = -kx \qquad \text{(Hooke's law),}$$

7-7 Work Done by a Spring Force
Formula

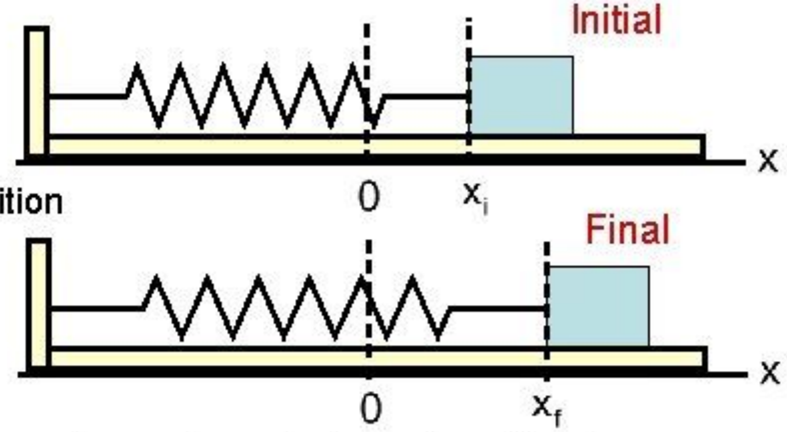
Work done by a spring force

$$W_s = \frac{1}{2} k x_i^2 - \frac{1}{2} k x_f^2$$

Final position

Initial position

Spring constant



Work W_s is positive if the block ends up closer to the relaxed state.
Work W_s is negative if the block ends up farther away form x = 0.

Derivation

$$W_s = \int_{x_i}^{x_f} F(x)dx = \int_{x_i}^{x_f} -k\,x\,dx$$

$$= -k \int_{x_i}^{x_f} x\,dx = \frac{1}{2} k x_i^2 - \frac{1}{2} k x_f^2$$

WORK DONE BY A SPRING

$$W_s = \tfrac{1}{2}kx_i^2 - \tfrac{1}{2}kx_f^2 \qquad \text{(work by a spring force).} \qquad (7\text{-}25)$$

This work W_s done by the spring force can have a positive or negative value, depending on whether the *net* transfer of energy is to or from the block as the block moves from x_i to x_f. *Caution:* The final position x_f appears in the *second* term on the right side of Eq. 7-25. Therefore, Eq. 7-25 tells us:

Work W_s is positive if the block ends up closer to the relaxed position ($x = 0$) than it was initially. It is negative if the block ends up farther away from $x = 0$. It is zero if the block ends up at the same distance from $x = 0$.

If $x_i = 0$ and if we call the final position x, then Eq. 7-25 becomes

$$W_s = -\tfrac{1}{2}kx^2 \qquad \text{(work by a spring force).} \qquad (7\text{-}26)$$

THE WORK DONE BY AN APPLIED FORCE

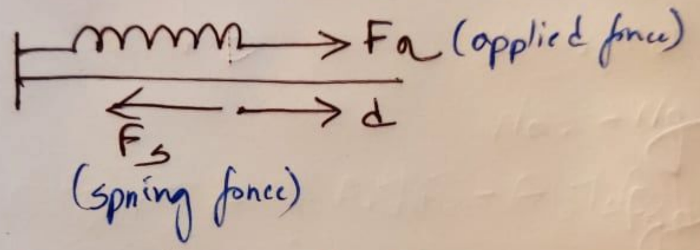
Now suppose that we displace the block along the x axis while continuing to apply a force $\vec{F}_a$ to it. During the displacement, our applied force does work W_a on the block while the spring force does work W_s. By Eq. 7-10, the change ΔK in the kinetic energy of the block due to these two energy transfers is

$$\Delta K = K_f - K_i = W_a + W_s, \tag{7-27}$$

in which K_f is the kinetic energy at the end of the displacement and K_i is that at the start of the displacement. If the block is stationary before and after the displacement, then K_f and K_i are both zero and Eq. 7-27 reduces to

$$W_a = -W_s. \tag{7-28}$$

$\Delta K = K_f - K_i = W_a + W_s$



$V_i = V_f = 0 \quad \therefore \quad K_f = K_i = 0$

$\therefore \quad 0 = W_a + W_s$

$\rightarrow W_a = -W_s$

$= -\left(-\frac{1}{2}Kx^2\right)$

$= \frac{1}{2}Kx^2$

In Fig. 7-10, a cumin canister of mass $m = 0.40$ kg slides across a horizontal frictionless counter with speed $v = 0.50$ m/s. It then runs into and compresses a spring of spring constant $k = 750$ N/m. When the canister is momentarily stopped by the spring, by what distance d is the spring compressed?

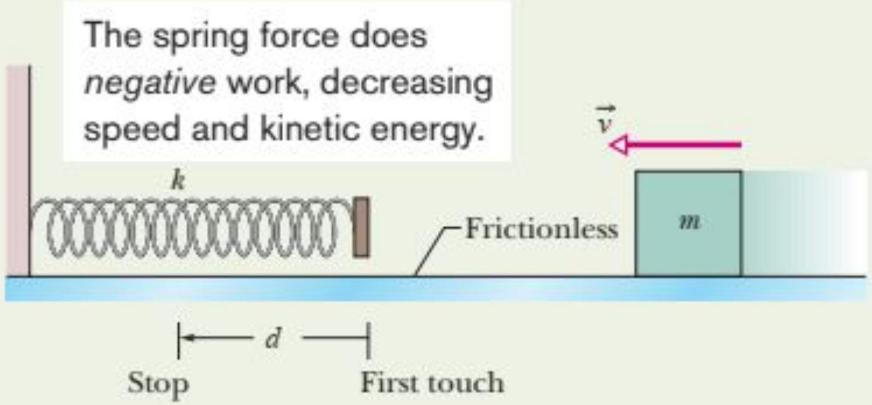


Fig. 7-10 A canister of mass m moves at velocity $\vec{v}$ toward a spring that has spring constant k.

KEY IDEAS

1. The work W_s done on the canister by the spring force is related to the requested distance d by Eq. 7-26 ($W_s = -\frac{1}{2}kx^2$), with d replacing x.
2. The work W_s is also related to the kinetic energy of the canister by Eq. 7-10 ($K_f - K_i = W$).
3. The canister's kinetic energy has an initial value of $K = \frac{1}{2}mv^2$ and a value of zero when the canister is momentarily at rest.

Calculations: Putting the first two of these ideas together, we write the work–kinetic energy theorem for the canister as

$$K_f - K_i = -\tfrac{1}{2}kd^2.$$

Substituting according to the third key idea gives us this expression

$$0 - \tfrac{1}{2}mv^2 = -\tfrac{1}{2}kd^2.$$

Simplifying, solving for d, and substituting known data then give us

$$d = v\sqrt{\frac{m}{k}} = (0.50\text{ m/s})\sqrt{\frac{0.40\text{ kg}}{750\text{ N/m}}}$$
$$= 1.2 \times 10^{-2}\text{ m} = 1.2\text{ cm}. \qquad \text{(Answer)}$$

••15 GO Figure 7-27 shows three forces applied to a trunk that moves leftward by 3.00 m over a frictionless floor. The force magnitudes are $F_1 = 5.00$ N, $F_2 = 9.00$ N, and $F_3 = 3.00$ N, and the indicated angle is $\theta = 60.0°$. During the displacement, (a) what is the net work done on the trunk by the three forces and (b) does the kinetic energy of the trunk increase or decrease?

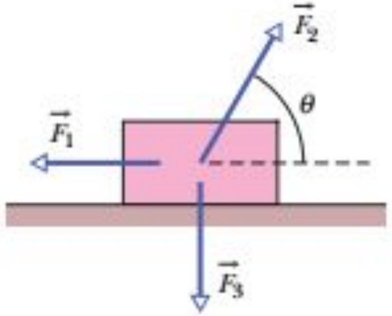
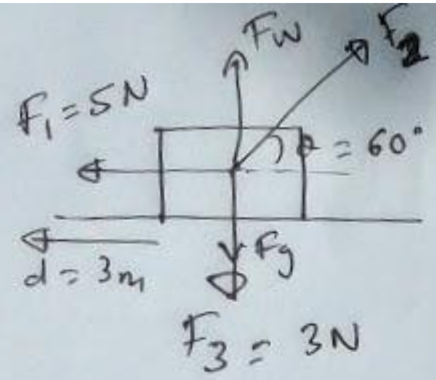


Fig. 7-27 Problem 15.



(a) $W_1 = F_1 d \cos\theta_1$

$= 5 \times 3 \cos 0$

$= 15J$

$W_2 = F_2 d \cos\theta_2$

$= 9 \times 3 \times \cos(120°)$

$= -13.5J$

$W_3 = F_3 d \cos\theta_3$

$= 3.3 \cos 90°$

$= 0$

$W_g = W_N = 0.$

$W_{net} = W_1 + W_2 + W_3 + W_g + W_N$

$= 15 - 13.5 + 0 + 0 + 0$

$= 1.5J$

(b) $W = K_f - K_i$

$\Rightarrow 1.5 = \Delta K$

Kinetic energy increase

••19 In Fig. 7-29, a block of ice slides down a frictionless ramp at angle $\theta = 50°$ while an ice worker pulls on the block (via a rope) with a force $\vec{F}_r$ that has a magnitude of 50 N and is directed up the ramp. As the block slides through distance $d = 0.50$ m along the ramp, its kinetic energy increases by 80 J. How much greater would its kinetic energy have been if the rope had not been attached to the block?

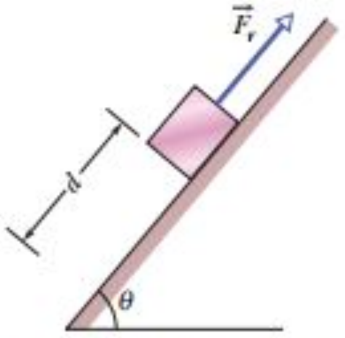
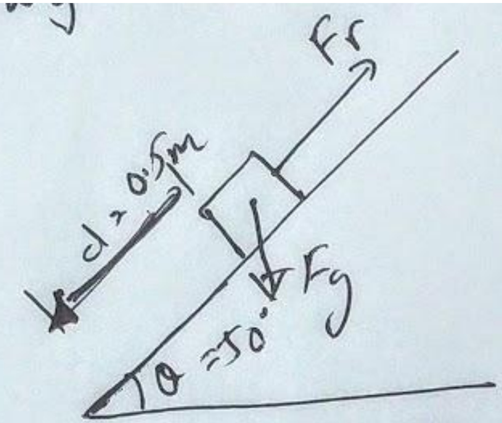


Fig. 7-29 Problem 19.



$W_r = F_r \times d \times \cos 180°$

$= -50 \times 0.5 = -25$ J

$W_{net} = 80$ J.

If the rope had not been attached to the block

The loss of 25 J will not be there so

~~$W_{net} = 80 + 25$~~

So kinetic energy would be 25 J greater

••23 In Fig. 7-31, a constant force $\vec{F}_a$ of magnitude 82.0 N is applied to a 3.00 kg shoe box at angle $\phi = 53.0°$, causing the box to move up a frictionless ramp at constant speed. How much work is done on the box by $\vec{F}_a$ when the box has moved through vertical distance $h = 0.150$ m?

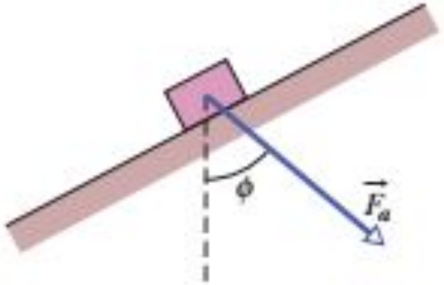
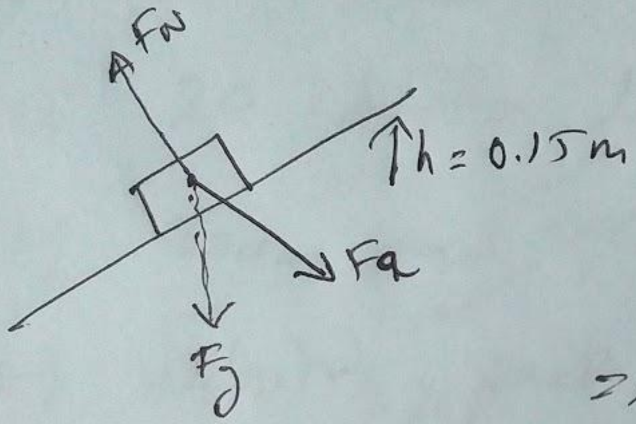


Fig. 7-31 Problem 23.



<u>Lifting concepts</u>

$W_a + W_g = \Delta K$

$\Rightarrow W_a = -W_g$

$\Rightarrow W_a = -mgh \cos\theta$

$\Rightarrow W_a = -3 \times 9.8 \times 0.15 \cos 180°$

$\therefore W_a = 4.415$

$m = 3kg$

$\vec{F_a} = 82N$

$\phi = 53°$