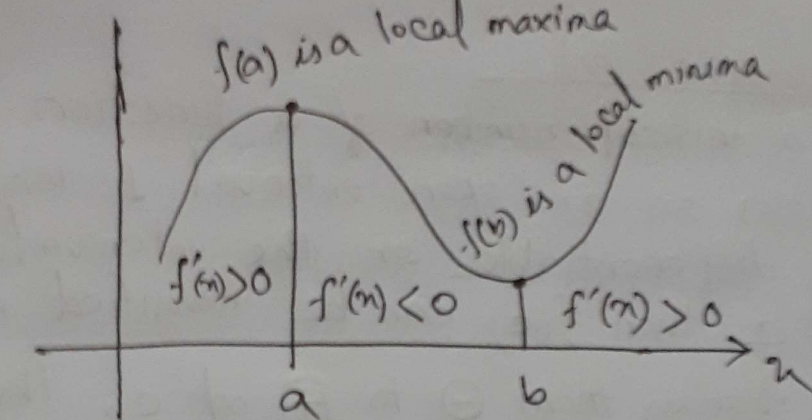
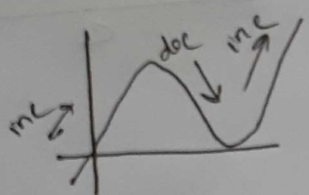


Defination : A function is increasing on an interval if for any two numbers x_1 and x_2 in the interval $x_1 < x_2$ implies $f(x_1) < f(x_2)$.

A function is decreasing on an interval if for any two numbers x_1 and x_2 in the interval $x_1 < x_2$ implies $f(x_1) > f(x_2)$.



Let f be a function that is continuous on the closed interval $[a,b]$ and differentiable on the open interval (a,b).

① If $f'(x) > 0$ for all x in (a,b) then f is increasing on $[a,b]$.

② If $f'(x) < 0$ foll all x in (a,b) then f is decreasing on $[a,b]$.

③ If $f'(x) = 0$ for all x in (a,b) then f is constant on $[a,b]$.

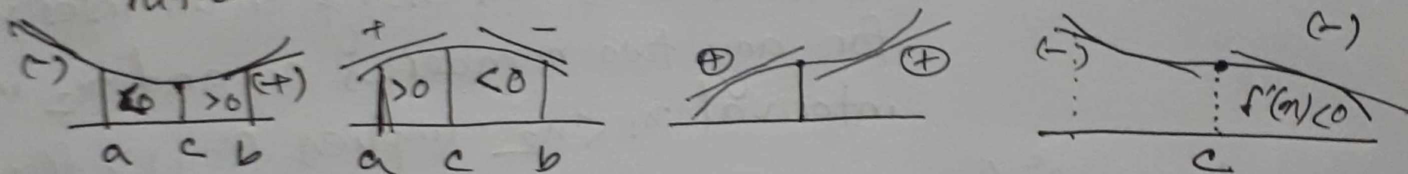
First derivative test

Let c be a critical number of a function f that is continious on an open interval I containing c. If f is differentiable on the interval, except possibly at c, then f(c) can be classified as follows.

① If $f'(x)$ changes from ⊖ to ⊕ at c, then f has a relative minimum at $(c, f(c))$.

② If $f'(x)$ changes from positive to negative at c, then f has a relative maximum at $(c, f(c))$.

③ If $f'(x)$ is positive on both sides of c or negative on both sides of c, then $f(c)$ is neither a relative minimum nor a relative maximum



Home work Sheet #5

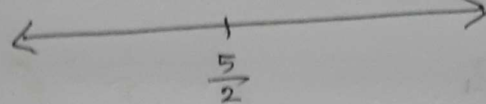
(1)

(i) $f(x) = x^2 - 5x + 6$

$\therefore f'(x) = 2x - 5$

$2x - 5 = 0$

$x = \frac{5}{2}$



Interval	$f'(x) = 2x-5$	conclusion
$x < 5/2$	$(-)$	$\therefore f$ is decreasing on $(-\infty, \frac{5}{2})$
$x > 5/2$	$(+)$	f is increasing on $(5/2, +\infty)$

Again,

$f'(x) = 2x - 5$

$f''(x) = 2 \therefore f''(x) > 0$

~~2 is the inflection point~~

$\therefore f''(x)$ is concave up at $(-\infty, \infty)$

No inflection point.

interval	$f''(x)$	
$x<2$	$+$	concave up

(11) $f(x) = 5 + 12x - x^3$

$f'(x) = 0 + 12 - 3x^2$

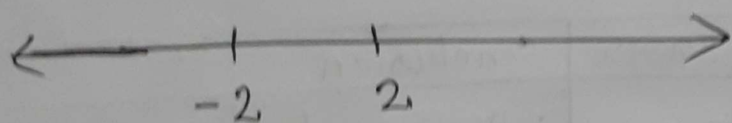
$= 12 - 3x^2$

$\therefore 12 - 3x^2 = 0$

$4 - x^2 = 0$

$x^2 = 4$

$x = \pm 2$



interval	$12-3x^2$	Sign	conclusion
$x<-2$	$\ominus$	$\ominus$	decreasing at $(-\infty, -2)$
$-2<x<2$	$\oplus$	$\oplus$	increasing at $(-2, 2)$
$x>2$	$\ominus$	$\ominus$	decreasing at $(2, \infty)$

$f''(x) = 0 - 6x = -6x$

$-6x = 0$

$x = 0 \rightarrow$ IP

interval	$f'' = -6x$	
$x<0$	$\oplus$	up
$x>0$	$\ominus$	down

(iii) $f(x) = x^4 - 8x^2 + 16$

$f'(x) = 4x^3 - 16x$

$4x^3 - 16x = 0$

$x(4x^2 - 16) = 0$

$4x(x^2 - 4) = 0$

$\therefore x = 0;\ +2;\ -2$

$-2 \quad 0 \quad 2$

interval	$f'(x) = 4x^3 - 16x$	Conclusion
$x < -2$	Decreasing	f is decreasing at $(-\infty, -2)$
$-2 < x < 0$	$+$	f is increasing at $(-2, 0)$
$x > 2$	$+$	f is increasing at $(2, \infty)$
$0 < x < 2$	$-$	f is decreasing at $(0, 2)$

$f'(x) = 4x^3 - 16x$

$f''(x) = 12x^2 - 16$

$12x^2 - 16 = 0$

$x^2 = \frac{16}{12} = \frac{8}{6} = \frac{4}{3}$

$x = \pm \frac{2}{\sqrt{3}}$

interval	$f''(x) = 12x^2 - 16$	conclusion
$x < 2/\sqrt{3}$	$+$	~~down~~ up
$x > 2/\sqrt{3}$	$+$	up

$\therefore$ No point.

$-\frac{2}{\sqrt{3}} \qquad \frac{2}{\sqrt{3}}$

Inter		
$x < \frac{-2}{\sqrt{3}}$	$+$	up
$\frac{-2}{\sqrt{3}} < x < \frac{2}{\sqrt{3}}$	$-$	down
$x > \frac{2}{\sqrt{3}}$	$+$	up

point of inflection: $\frac{-2}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

(iv) $f(x) = \dfrac{x^2}{x^2+2}$

$$f'(x) = \frac{2x(x^2+2) - x^2(2x+0)}{(x^2+2)^2}$$

$$= \frac{2x^3+4x-2x^3}{(x^2+2)^2}$$

$$= \frac{4x}{x^4+4x^2+4}$$

$$\frac{4x}{x^4+4x^2+4} = 0$$

$$x = 0$$

interval	$f''(x) = 4x/x^4+4x^2+4$	
$x<0$	$-$	f is decreasing at $(-\infty, 0)$
$x>0$	$+$	f is increasing at $(0, +\infty)$

again,

$$f'(x) = \frac{4x}{x^4+4x^2+4}$$

$$f''(x) = \frac{4(x^4+4x^2+4) - 4x(4x^3+8x)}{(x^4+4x^2+4)^2}$$

$$= \frac{4x^4+16x^2+16-16x^4-32x^2}{(x^4+4x^2+4)^2}$$

$$= \frac{-12x^4-16x^2+16}{(x^4+4x^2+4)^2}$$

Here,

$$\frac{-12x^4 - 16x^2 + 16}{(x^4 + 4x^2 + 4)^2} = 0$$

$$-12x^4 - 16x^2 + 16 = 0$$

$$-4(3x^4 + 4x^2 - 4) = 0$$

$$3x^4 + 4x^2 - 4 = 0$$

$$3x^4 + 6x^2 - 2x^2 - 4 = 0$$

$$3x^2(x^2 + 2) - 2(x^2 + 2) = 0$$

$$(x^2 + 2)(3x^2 - 2) = 0$$

$x \neq 2$ | $x^2 = \frac{2}{3}$

$$x = \pm\sqrt{\frac{2}{3}}$$

	f''	
$x < -\sqrt{\frac{2}{3}}$	$-$	down
$-\sqrt{2/3} < x < \sqrt{2/3}$	$+$	~~down~~ up
$x < \sqrt{2/3}$	$(-)$	down

$f(x) = \sqrt[3]{x+2}$

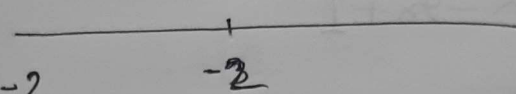
$f'(x) = (x+2)^{\frac{1}{3}}$

$= \frac{-1}{3} \cdot (x+2)^{\frac{1}{3}-1}$

$= \frac{-1}{3} \cdot (x+2)^{\frac{-2}{3}}$

$= \frac{-1}{3} \cdot \frac{1}{(x+2)^{\frac{2}{3}}}$

$= \frac{-1}{3} \cdot \frac{1}{\sqrt[3]{(x+2)^2}}$



$\therefore x = -2$

	$f'(x) = \frac{-1}{3} \cdot \frac{1}{\sqrt[3]{(x+3)^2}}$	Sign	Conclusion
$x < -2$	$(+)\,(+)$	$(+)$	f is increasing $(-\infty, -2)$
$x > -2$	$(+)(+)$	$(+)$	f is increasing at $(-2, +\infty)$

Again,

$f'(x) = \frac{1}{3} \cdot (x+2)^{\frac{-2}{3}}$

$f''(x) = \frac{1}{3} \cdot \frac{-2}{3} \cdot (x+2)^{\frac{-2}{3}-1}$

$= \frac{-2}{9} \cdot (x+2)^{-\frac{5}{3}}$

$= \frac{-2}{9} \cdot \frac{1}{\sqrt[3]{(x+2)^5}}$

interval	f''	Siagn	conclusion
$x<-2$	$(-)(-)$	$(+)$	f is concave up at $(-\infty,-2)$
$x>-2$	$(-)(+)$	$(-)$	f is concave down at ~~$(-\infty$~~ $(-2,+\infty)$

Since f is concave up and down at the point -2, $\therefore$ $x=-2$ is a point of inflaction.

(2)

(1) $f(x) = x^3+3x^2-9x+1$

$f'(x) = 3x^2+6x-9$

$f'(x)=0$

$3x^2+6x-9=0$

$3(x^2+2x-3)=0$

$x^2-3x+x-3=0$

$x(x-3)+1(x-3)=0$

$(x-3)(x+1)=0$

$x=3,-1$

So, $x=3,-1$ are stationary point.

(ii) $f(x) = x^4 - 6x^2 - 3$

$f'(x) = 4x^3 - 12x$

$f'(x) = 4x(x^2 - 3)$

$\therefore x = 0, \pm\sqrt{3}$ are the stationary point.

(iii) $f(x) = \dfrac{x}{x^2+2}$

$$f'(x) = \frac{x^2+2 - x(2x+0)}{(x^2+2)^2}$$

$$= \frac{x^2+2-2x^2}{(x^2+2)^2}$$

$$= \frac{2-x^2}{(x^2+2)^2}$$

$f'(x) = 0$

$$\frac{2-x^2}{(x^2+2)^2} = 0$$

$2 - x^2 = 0$

$x = \pm\sqrt{2}$ are stationary point.

$\therefore$

(iv)

$f(x) = x^{\frac{2}{3}}$

$f'(x) = \frac{2}{3} \cdot x^{\frac{2}{3}-1}$

$= \frac{2}{3} \cdot x^{\frac{-1}{3}}$

$= \frac{2}{3} \cdot \frac{1}{\sqrt[3]{x}}$

No stationary point $\boxed{\frac{1}{x} = 0}$

$x = 3$ is a undefined point.

(v) $f(x) = x^{\frac{1}{3}} \cdot (x+4)$

$f'(x) = \frac{1}{3} x^{\frac{1}{3}-1} (x+4) + x^{\frac{1}{3}} \cdot (1+0)$

$= \frac{1}{3} \cdot x^{\frac{-2}{3}} (x+4) + x^{\frac{1}{3}}$

$= \frac{1}{3} \cdot x^{-1} \cdot x^{\frac{1}{3}} (x+4) + x^{\frac{1}{3}}$

$= \frac{1}{3} x^{\frac{1}{3}} \left\{ x^{-1} (x+4) + 1 \right\}$

$\cancel{= \frac{1}{3} \frac{1}{x}}$

$= \frac{\sqrt[3]{x}}{3} \left\{ \frac{x+4}{x} + 1 \right\}$

$= \frac{\sqrt[3]{x}}{3} \cdot \frac{x+4+x}{x}$

$= \frac{\sqrt[3]{x} \cdot (2x+4)}{3x}$

$$= \frac{x+4}{3x^{\frac{2}{3}}} + x^{\frac{1}{3}}$$

$$= \frac{x+4 + x^{\frac{1}{3}} \cdot 3x^{\frac{2}{3}}}{3x^{\frac{2}{3}}}$$

$$= \frac{x+4 + 3x}{3x^{\frac{2}{3}}}$$

$$= \frac{4x + 4}{3x^{\frac{2}{3}}}$$

$$= \frac{4(x+1)}{3x^{\frac{2}{3}}}$$

critical number $= 0, -1$

Stationary point $= -1$

(13) (1)

$$f(x) = 2x^3 - 9x^2 + 12x$$

$$f'(x) = 6x^2 - 18x + 12$$

$$f'(x) = 0$$

$$6x^2 - 18x + 12 = 0$$

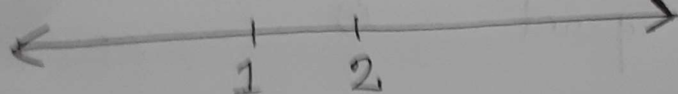
$$6(x^2 - 3x + 2) = 0$$

$$6(x^2 - 2x - x + 2) = 0$$

$$6\{x(x-2) - 1(x-2)\} = 0$$

$$6(x-2)(x-1) = 0$$

$$\boxed{x = 1 ; 2}$$



Interval	$f'(x) = 6x^2 - 18x + 12$	Conclusion
$x < 1$	(+)	$x = 1$ local maximum
$1 < x < 2$	$-$	$x = 2$ local minimum
$x > 2$	(+)	

(vi) $f(x) = \cos 3x$

$f'(x) = \cancel{3\sin 3x}\ -3\sin 3x$

$f'(x) = 0$

$-3\sin 3x = 0$

$\sin 3x = 0$

$\sin 3x = \cancel{\sin^{-1}(0)}\ \sin(0^\circ)$

$3x = 0$

$x = 0$

$\therefore$ Stationary point $= 0$

Critical point $= 0$

Again,

$f'(x) = 6x^2 - 18x + 12$

$f''(x) = 12x - 18$

	$12x-18$	Conclusion
$x = 1$	$\ominus$	local ~~minimum~~ maximum
$x = 2$	$\oplus$	local minimum

(ii) $f(x) = \frac{x}{2} - \sin x$

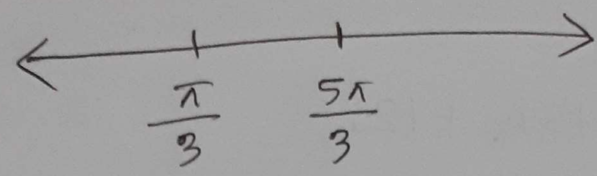
$f'(x) = \frac{1}{2} - \cos x$

$\frac{1}{2} - \cos x = 0$

$\cos x = \frac{1}{2}$

$x = \cos^{-1}\left(\frac{1}{2}\right)$

$x = 60^\circ, 300 \Rightarrow \frac{\pi}{3}, \frac{5\pi}{3}$



interval	$f'(x) = \frac{1}{2} - \cos x$	Conclusion
$x < \frac{\pi}{3}$	$-$	$x = \frac{\pi}{3}$ local minima
$\frac{\pi}{3} < x < \frac{5\pi}{3}$	$+$	$x = \frac{5\pi}{3}$ local maximum
$x > \frac{5\pi}{3}$	$-$	

Again,

$$f'(x) = \frac{1}{2} - \cos x$$

$$f''(x) = 0 + \sin x = \sin x$$

	$f''(x) = \sin x$	
$x = \frac{\pi}{3}$	$+$	$x = \frac{\pi}{3}$ is ~~nima~~ minima
$x = \frac{5\pi}{3}$	$-$	$x = \frac{5\pi}{3}$ is maxima

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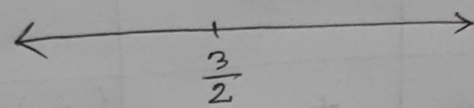
$f(x) = x^2 - 3x + 8$

$f'(x) = 2x - 3$

Here,

$2x - 3 = 0$

$x = \frac{3}{2}$



Range	Sign $2x-3$	I/D	max min	
$x < \frac{3}{2}$	$-$	Decreasing	f has a local minima at $x = \frac{3}{2}$	$(-\infty, \frac{3}{2})$
$x > \frac{2}{3}$	$+$	Increasing		$(\frac{2}{3}, +\infty)$

Again,

$f''(x) = 2$

$\therefore$ f ~~has no~~ is ~~concave up or co~~ always concave up. No inflection point.

(16)

$f(x) = 5 - 4x - x^2$

$f'(x) = 0 - 4 - 2x$

$= -2x - 4$

$-2x - 4 = 0$

$2x = -4$

$x = \frac{-4}{2} = -2$

interval	$f'(x) = -2x-4$ Sign	I/D	max/min
$x < -2$	+	f is increasing at $(-\infty, -2)$	$x = -2$ is local maxima
$x > -2$	−	f is decreasing at $(-2, \infty)$	

Again,

$f''(x) = -2$

$\therefore$ f is decreasing at $(-\infty, +\infty)$. No Inflaction point.

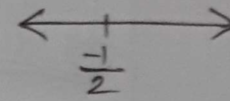
(17)

$f(x) = (2x+1)^3$

$6(2x+1)^2 = 0$

$2x+1 = 0$

$x = \frac{-1}{2}$



$f'(x) = 3(2x+1)^2 \cdot (2+0)$

$= 6(2x+1)^2$

Interval	Sign	I/D	~~c/u~~ Max min
$x < \frac{-1}{2}$	+	~~Decreasing~~ Increasing at $(-\infty, \frac{-1}{2})$	
$x > \frac{-1}{2}$	+	Increasing at $(\frac{-1}{2}, +\infty)$	

again,

$f'(x) = 6(2x+1)^2$

$f''(x) = 12(2x+1)(2+0)$

$= 24(2x+1)$

$x = \frac{-1}{2}$

~~f is concave down at $(-\infty, +\infty)$.~~

~~No concave up. No Inlection point~~

interval	Sign	Concave up/down	max min
$x < \frac{-1}{2}$	−	concave down at $(-\infty, \frac{-1}{2})$	
$x > \frac{1}{2}$	+	concave up at $(\frac{1}{2}, +\infty)$	

f has an inflection point

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$f(x) = 5 + 12x - x^3$

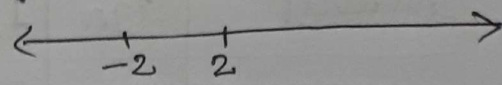
$f'(x) = 0 + 12 - 3x^2$

$= 12 - 3x^2$

$3x^2 = 12$

$x^2 = 4$

$x = \pm 2$



interval	$f'(x)$ Sign	I/D	Max / min 1st d test.
$x < -2$	$-$	f is decreasing at $(-\infty, -2)$	$x = -2$; local minima
$-2 < x < 2$	$+$	f is increasing at $(-2, 2)$	local maxima at
$x > 2$	$-$	f is decreasing at $(2, +\infty)$	$x = 2$

Again,

$f''(x) = 0 - 6x = -6x$

$x = 0$

inflection point $= 0$

Interval	$f''(x)$ Sign	Concave up/down	2nd d. test
$x < 0$	$+$	concave up at $(-\infty, 0)$	$x = -2$; $f''(x) = 12$ minima ~~decreasing~~ at $x = -2$
$x > 0$	$-$	concave down at $(0, +\infty)$	$x = 2$; $f''(x) = -12$ maxima at $x = 2$

(19) $f(x) = 3x^4 - 4x^3$

$f'(x) = 12x^3 - 12x^2$

$12x^3 - 12x^2 = 0$

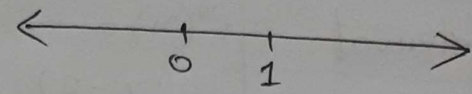
$x^3 - x^2 = 0$

$x(x^2 - x) = 0$

$x\{x(x-1)\} = 0$

$x^2(x-1) = 0$

$x = 0 ; 1$



Interval	Sign	I/D	Max/min
$x<0$	$-$	f is decreasing at $(-\infty, 0)$	
$0<x<1$	$-$	f is decreasing at $(0, 1)$	$x = 1$; local minima
$x>1$	$+$	f is increasing at $(1, +\infty)$	

again,

$f''(x) = 36x^2 - 24x$

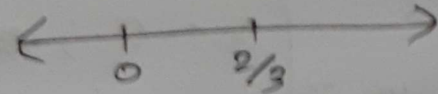
$36x^2 - 24x = 0$

$12x(3x - 2) = 0$

$x = 0 ; \frac{2}{3}$

$f''(x) = \cancel{36x^2 - 24x}\ 12x(3x-2)$

$x = 0, \frac{2}{3}$



Interval	$f''(x) = 12(3x-2)$	Concave up/down	Max-min 2nd dt.
$x<0$	$(-)(-) \rightarrow +$	concave up at $(-\infty, 0)$	$x=0$; $f''(x) = 0$ ~~constant~~
$0<x<2/3$	$(+)(-) \rightarrow -$	concave down at $(0, 2/3)$	$x=1$; $f''(x) = 12$ local minuma.
$x>2/3$	$(+)(+) \rightarrow +$	concave up at $(2/3, \infty)$	

Inflaction point $= 0, 2/3$

(21)

$$f(x) = \frac{x-2}{(x^2-x+1)^2}$$

$$f'(x) = \frac{(x^2-x+1)^2 - 2(x-2)(x^2-x+1)(2x-1)}{\{(x^2-x+1)^2\}^2}$$

$$= \frac{(x^2-x+1)\{(x^2-x+1) - 2(2x^2-x-4x+2)\}}{(x^2-x+1)^3(x^2-x+1)}$$

$$= \frac{x^2-x+1-4x^2+2x+8x-4}{(x^2-x+1)^3}$$

$$= \frac{-3x^2+9x-3}{(x^2-x+1)^3}$$

$$= \frac{-3(x^2-3x+1)}{(x^2-x+1)^3}$$

Here,

$$f'(x) = 0$$

$$\therefore -3(x^2-3x+1) = 0$$

$$x^2-3x+1 = 0$$

$$x = \frac{-(-3) \pm \sqrt{9-4}}{2} = \frac{3 \pm \sqrt{5}}{2}$$

$\frac{3-\sqrt{5}}{2}$ $\qquad$ $\frac{3+\sqrt{5}}{2}$

	Sign	d	Max/min
$x<\frac{3-\sqrt{5}}{2}$	$-$	decreasing at $\left(-\infty, \frac{3-\sqrt{5}}{2}\right)$	$x=\frac{3-\sqrt{5}}{2}$ is local minima
$\frac{3-\sqrt{5}}{2}<x<\frac{3+\sqrt{5}}{2}$	$+$	Increasing at $\left(\frac{3-\sqrt{5}}{2}, \frac{3+\sqrt{5}}{2}\right)$	$x=\frac{3+\sqrt{5}}{2}$ is local maxima
$x>\frac{3+\sqrt{5}}{2}$	$-$	decreasing at $\left(\frac{3+\sqrt{5}}{2}, \infty\right)$	

(23)

$$f(x) = \sqrt[3]{x^2+x+1}$$

$$f'(x) = \frac{1}{3}(x^2+x+1)^{\frac{-2}{3}} \cdot (2x+1)$$

$$= \frac{2x+1}{3(x^2+x+1)^{\frac{2}{3}}}$$

Here,

$$f'(x) = 0$$

$$2x+1 = 0$$

$$x = \frac{-1}{2}$$

$\frac{-1}{2}$

interval	sign		
$x < \frac{-1}{2}$	$-$	f is decreasing at $(-\infty, \frac{-1}{2})$	local minima
$x > \frac{-1}{2}$	$+$	f is increasing at $(\frac{-1}{2}, \infty)$	at $x = \frac{1}{2}$

Again,

$$f'(x) = \frac{2x+1}{3(x^2+x+1)^{\frac{2}{3}}}$$

Here,

$$\frac{d}{dx}\left\{3(x^2+x+1)^{\frac{2}{3}}\right\}$$

$$= 3 \cdot \frac{2}{3}(x^2+x+1)^{\frac{2}{3}-1} \cdot (2x+1)$$

$$= \cancel{3} \cdot \frac{2}{\cancel{3}} \cdot \frac{2x+1}{(x^2+x+1)^{\frac{1}{3}}} = \frac{2(2x+1)}{(x^2+x+1)^{\frac{1}{3}}}$$

$$\therefore f''(x) = \frac{2\left\{3(x^2+x+1)^{\frac{2}{3}}\right\} - (2x+1)\left\{9(x^2+x+1)^{\frac{4}{3}}\right\}}{9(x^2+x+1)^{\frac{4}{3}}}$$

$$= \frac{6(x^2+x+1)^{\frac{2}{3}} - 9(2x+1)(x^2+x+1)^{\frac{4}{3}}}{9(x^2+x+1)^{\frac{4}{3}}}$$

$$f''(x) = \frac{2\left\{3(x^2+x+1)^{\frac{2}{3}}\right\} - (2x+1)\cdot\dfrac{2(2x+1)}{3(x^2+x+1)^{\frac{1}{3}}}}{9(x^2+x+1)^{\frac{4}{3}}}$$

$$= \frac{6(x^2+x+1)^{\frac{2}{3}} - \dfrac{2(2x+1)^2}{(x^2+x+1)^{\frac{1}{3}}}}{9(x^2+x+1)^{\frac{4}{3}}}$$

$$= \frac{\dfrac{6(x^2+x+1)^{\frac{2}{3}+\frac{1}{3}} - 2(2x+1)^2}{(x^2+x+1)^{\frac{1}{3}}}}{9(x^2+x+1)^{\frac{4}{3}}}$$

$$= \frac{6(x^2+x+1) - 2(2x+1)^2}{(x^2+x+1)^{\frac{1}{3}}} \cdot \frac{1}{9(x^2+x+1)^{\frac{4}{3}}}$$

$$= \frac{6x^2+6x+6-2(4x^2+4x+1)}{9(x^2+x+1)^{\frac{5}{3}}}$$

$$= \frac{6x^2+6x+6-8x^2-8x-2}{9(x^2+x+1)^{\frac{5}{3}}}$$

$$= \frac{\cancel{10x^2+10x+16}-2x^2-2x+4}{9(x^2+x+5)^{\frac{5}{3}}}$$

$$= \frac{-2(x^2 + x - 2)}{9(x^2 + x + 5)^{\frac{5}{3}}}$$

$$= \frac{-2(x^2 + 2x - x - 2)}{9(x^2 + x + 5)^{\frac{5}{3}}}$$

$$= \frac{-2\{x(x+2) - 1(x+2)\}}{9(x^2 + x + 5)^{\frac{5}{3}}}$$

$$= \frac{-2(x+2)(x-1)}{9(x^2 + x + 5)^{\frac{5}{3}}}$$

Here,
$x = -2, 1$

Range	Sign	up/down	
$x < -2$	$-$	concave down at $(-\infty, -2)$	-2 is local minima
$-2 < x < 1$	$+$	concave up at $(-2, 1)$	1 is local maxima
$x > 1$	$-$	concave down at $(1, \infty)$	

(24)

$$f(x) = x^{\frac{4}{3}} - x^{\frac{1}{3}}$$

$$f'(x) = \frac{4}{3}x^{\frac{4}{3}-1} - \frac{1}{3}x^{\frac{1}{3}-1}$$

$$= \frac{4}{3}\cdot x^{\frac{1}{3}} - \frac{1}{3}x^{-\frac{2}{3}}$$

$$= \cancel{\frac{4x^{\frac{1}{3}}}{3} - \frac{1}{3x^{\frac{2}{3}}}}$$

Here,

$$\cancel{\frac{4}{3}\left(x^{\frac{1}{3}} - \frac{1}{3x^{\frac{2}{3}}}\right)}$$

$$= \frac{4x^{\frac{1}{3}}}{3} - \frac{1}{3x^{\frac{2}{3}}}$$

$$= \frac{4\cdot x^{\frac{1}{3}}\cdot x^{\frac{2}{3}} - 1}{3x^{\frac{2}{3}}}$$

$$= \frac{4x^{\frac{1}{3}+\frac{2}{3}} - 1}{3x^{\frac{2}{3}}}$$

$$= \frac{4x - 1}{3x^{\frac{2}{3}}}$$

$\therefore$ $4x^{\frac{4}{3}} + 2x^{\frac{1}{3}} = 0$

$2x^{\frac{1}{3}}(2x+1) = 0$

$x = 0 \; ; \; \frac{-1}{2}$

$\frac{-1}{2}$ $\quad 0$

$x < \frac{-1}{2}$	$+$	Concave up at $(-\infty, \frac{-1}{2})$
$\frac{-1}{2} < x < 0$	$-$ $x = 0.1$	concave ~~up~~ down at $(\frac{-1}{2}, 0)$
$x > \frac{-1}{2}$	$+$	concave up $(\frac{-1}{2}, \infty)$

$x = \frac{1}{4}$

interval	sign	I/D	M/m
$x < \frac{1}{4}$	$-$	f is decreasing at $(-\infty, \frac{1}{4})$	local minima at $x = \frac{1}{4}$
$x > \frac{1}{4}$	$+$	f is increasing at $(\frac{1}{4}, \infty)$	

Again,

$$f'(x) = \frac{4}{3}x^{\frac{1}{3}} - \frac{1}{3}x^{-\frac{2}{3}}$$

$$= \frac{4}{3}\cdot\frac{1}{3}x^{\frac{1}{3}-1} - \frac{1}{3}\cdot\left(\frac{-2}{3}\right)\cdot x^{-\frac{2}{3}-1}$$

$$f''(x) = \frac{4}{9}x^{-\frac{2}{3}} + \frac{2}{9}\cdot x^{-\frac{5}{3}}$$

$$= \frac{4}{9x^{\frac{2}{3}}} + \frac{2}{9x^{\frac{5}{3}}}$$

$$= \frac{4}{9x^{\frac{1}{3}}\cdot x^{\frac{1}{3}}} + \frac{2}{9x^{\frac{1}{3}}\cdot x^{\frac{4}{3}}}$$

$$= \frac{4x^{\frac{4}{3}} + 2x^{\frac{1}{3}}}{9x^{\frac{1}{3}}\cdot x^{\frac{1}{3}}\cdot x^{\frac{4}{3}}}$$