ASSIGNMENT 2

(Set N)

MATH110

Differential Calculus and Co-ordinate Geometry

Submitted

By

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Section : 15

1. Differentiate derivative :

$$a. \qquad f(x) = xsinx\frac{1}{x}$$

Answer

$f(x) = xsinx\frac{1}{x}$

$= \frac{d}{dx}(xsinx\frac{1}{x})$

$= x\frac{d}{dx}(sin\frac{1}{x}) + (sin\frac{1}{x})\frac{d}{dx}(x)$

$= x(-cos\frac{1}{x}.\frac{1}{x^2}) + sin\frac{1}{x}$

$= -cos\frac{1}{x}.\frac{1}{x} + sin\frac{1}{x}$

$= sin\frac{1}{x} - cos\frac{1}{x}.\frac{1}{x}$

$$b. \qquad g(x) = xexp(x)cos(x)$$

Answer

$g(x) = xexp(x)cos(x)$

$suppose,$

$g(x) = y = xexp(x)cos(x)$

$=> y = xexp(x)cos(x)$

$=> lny = ln(xexp(x)cos(x))$

$=> \frac{d}{dx}(lny) = \frac{d}{dx}(ln(x) + ln(exp(x)) + ln(cos(x)))$

$=> \frac{1}{y}.\frac{dy}{dx} = \frac{1}{x} + 1 + \frac{1}{cosx}. - sinx$

$=> \frac{dy}{dx} = y(\frac{1}{x} + 1 - tanx)$

$=> \frac{dy}{dx} = xexp(x)cos(x)(1 + \frac{1}{x} - tanx)$

2.Find the 4th derivatives:

$$y = e^{-5z} + 8ln2z^4$$

Answer

$y = e^{-5z} + 8ln2z^4$

$y' = \frac{d}{dz}(e^{-5z}) + 8\frac{d}{dz}(ln2z^4)$

$= -5.e^{-5z} + 8\frac{1}{2z^4}.2.4z^3$

$= -5.e^{-5z} + 32z^{-1}$

$y'' = -5\frac{d}{dz}(e^{-5z}) + 32\frac{d}{dz}(z^{-1})$

$= 25(e^{-5z}) - 32(z^{-2})$

$y''' = 25\frac{d}{dz}(e^{-5z}) - 32\frac{d}{dz}(z^{-2})$

$= -125(e^{-5z}) - 32(-2)(z^{-3})$

$= -125(e^{-5z}) + 64(z^{-3})$

$y'''' = -125\frac{d}{dz}(e^{-5z}) + 64\frac{d}{dz}(z^{-3})$

$= -125.(-5)(e^{-5z}) + 64(-3)(z^{-4})$

$= 625e^{-5z} - 192z^{-4}$

3. Differentiate function :

$$a. \qquad f(x) = cos(ln\frac{2}{x^3})$$

Answer

$f(x) = cos(ln\frac{2}{x^3})$

$suppose,$

$a = cosb, b = lnc, c = \frac{2}{x^3}$

According to chain rule ,

$\frac{da}{dx} = \frac{da}{db}.\frac{db}{dc}.\frac{dc}{dx}$

$= \frac{d}{db}(cosb).\frac{d}{dc}(lnc).\frac{d}{dx}(\frac{2}{x^3})$

$= -sinb.\frac{1}{c}.2.(-3).x^{-4}$

$= sinb.\frac{1}{c}.2.3.x^{-4}$

$= sinln\frac{2}{x^3}\frac{x^3}{2}.2.3.x^{-4}$

$= sinln\frac{2}{x^3}.\frac{3}{x}$

b. $\qquad h(x) = (coshx^3).(sinh^2x + 3)$

Answer

$h(x) = (coshx^3).(sinh^2x + 3)$

$= (coshx^3)\frac{d}{dx}(sinh^2x + 3) + (sinh^2x + 3)\frac{d}{dx}(coshx^3)$

$= coshx^3.2sinhx.coshx + (sinh^2x + 3).sinhx^3.3x^2$

$= 2coshx^3.sinhx.coshx + sinh^2x.sinhx^3.3x^2 + 3sinhx^3.3x^2$

$= 2sinhx.coshx.coshx^3 + 3x^2sinh^2x.sinhx^3 + 9x^2.sinh^3$

4.Analyze the differentiability at x = 2 of the function

$$\text{If } f(x) = \begin{cases} x^2 - 4x - 2, & x < 2 \\ -2x^2 + 4x, & x > 2. \end{cases}$$

Answer

for $x > 2$(RHD)

$lim_{h\to 0^+} \frac{f(x+h)-f(x)}{h}$

$= lim_{h\to 0^+} \frac{-2(x+h)^2+4(x+h)-(-2x^2+4x)}{h}$

$= lim_{h\to 0^+} \frac{-2(x^2+2hx+h^2)+4(x+h)-(-2x^2+4x)}{h}$

$= lim_{h\to 0^+} \frac{-2x^2-4hx-2h^2+4x+4h+2x^2-4x}{h}$

$= lim_{h\to 0^+} \frac{-4hx-2h^2+4h}{h}$

$= lim_{h\to 0^+} \frac{h(-4x-2h+4)}{h}$

$= lim_{h\to 0^+}(-4x - 2h + 4)$

$= -4.2 - 2.0 + 4$

$= -4$

for $x < 2$(LHD)

$lim_{h\to 0^-} \frac{f(x+h)-f(x)}{h}$

$= lim_{h\to 0^-} \frac{(x+h)^2-4(x+h)-2-x^2+4x+2}{h}$

$= lim_{h\to 0^-} \frac{x^2+2hx+h^2-4x-4h-2-x^2+4x+2}{h}$

$= lim_{h\to 0^-} \frac{2hx+h^2-4h}{h}$

$= lim_{h\to 0^-} \frac{h(2x+h-4)}{h}$

$= lim_{h\to 0^-}(2x + h - 4)$

$= 2.2 + 0 - 4$

$= 0$

$Here, LHD \neq RHD$

The function is not continuous as well as not differentiable.

BONUS

5. Use the chain rule to prove the following:

(a) The derivative of an even function is an odd function.

(b) The derivative of an odd function is an even function.

Answer : A

let,

f(x) is a even function.

$f(-x) = f(x)$

Differentiate both sides by using chain rule,

$f(-x) = f(x)$

$=> f'(-x).(-1) = f'(x).1$

$=> -f'(-x) = f'(x)$

$=> f'(-x) = -f'(x)$

So,The derivative of an even function is an odd function.

[proved]

Answer : B

let,

f(x) is a odd function.

$f(-x) = -f(x)$

Differentiate both sides by using chain rule,

$f(-x) = -f(x)$

$=> f'(-x).(-1) = -f'(x).1$

$=> -f'(-x) = -f'(x)$

$=> f'(-x) = f'(x)$

So,The derivative of an odd function is an even function.

[proved]

6.A polynomial of m degree is defines as

$p(x) = a_0 + a_1x + a_2x^2 + ... + a_nx^m$

a.Find $p'(x)$ and $p'''(x)$

b.Find $p^{(m)}$

c.Find $p^{(n)}$ when $n > m$

Answer

(a)Diffrentiating p(x) on basis of x,

Given, $p(x) = a_0 + a_1x + a_2x^2 + ... + a_nx^m$

$\frac{d}{dx}p(x) = \frac{d}{dx}(a_0 + a_1x + a_2x^2 + ... + a_nx^m)$

$p'(x) = a_1 + 2a_2x + 3a_3x^2 + ... + ma_nx^{m-1}$

Diffrentiating p'(x) on basis of x,

$\frac{d}{dx}p(x) = \frac{d}{dx}(a_1 + 2a_2x + 3a_3x^2 + ... + ma_nx^{m-1})$

$p''(x) = 2a_2 + 6a_3x + 12a_4x^2 + ... + m(m-1)a_nx^{m-2}$

Diffrentiating p''(x) on basis of x,

$p''(x) = 2a_2 + 6a_3x + 12a_4x^2 + ... + m(m-1)a_nx^{m-2}$

$\frac{d}{dx}p''(x) = \frac{d}{dx}(2a_2 + 6a_3x + 12a_4x^2 + ... + m(m-1)a_nx^{m-2})$

$p'''(x) = 6a_3 + 24a_4x + 60a_5x^2 + ... + m^2 - m(m-2)a_nx^{m-3}$

$p'(x) = a_1x + 2a_2x + 3a_3x^2 + ... + ma_nx^{m-1}$

$p'''(x) = 6a_3 + 24a_4x + 60a_5x^2 + ... + m^2 - m(m-2)a_nx^{m-3}$

(b) find $p^{(m)}$

$p^{(m)} = (\frac{d}{dx})^m.p(x)$

$= \frac{d^m}{dx^m}(a_0 + a_1x + a_2x^2 + ... + a_nx^m)$

$= 0 + 0 + 0 + ... + \frac{d^m}{dx^m}(a_nx^m)$

$= [m(m-1)(m-2)......3.2.1]a_n x^{m-m}$

$= m!.a_n x^0$

$= a_n m!$

(C) $p^{(n)}$WHEN $n > m$

Let,
$p^{(n)} = \frac{d}{dx}p^{(m)}(x)$ [since $n > m$]

$=\frac{d}{dx}(a_n m!)$

$=0$ [since $(a_n m!)$ is a constant, $\frac{d}{dx}(c) = 0$]

THE END