

Formula Sheet for PHY111 (Summer 2023)

Principles of Physics-I

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DISCLAIMER: I hope this work won't be used as a means of slacking off and just banally memorizing them. Do so, and it will catch up to You one way or another. Please don't mistake it for a mode of cheating.

CONTENTS

1	Translational Kinematics	3
1.1	Position Vector	3
1.2	Displacement Vector	3
1.3	Average Velocity	3
1.4	Instantaneous Velocity/Speed	3
1.5	Average Acceleration	3
1.6	Instantaneous Acceleration	3
1.7	Primitive Laws of Translational Motion ($a = \text{constant}$)	3
1.8	Uniform Circular Motion	3
1.9	Vertical Projection	4
1.10	Slant Projection	4
1.11	Everything about Projectile	4
2	Translational Dynamics	4
2.1	Newton's 2 nd Law of Motion (For Inertial System)	4
2.2	Friction Force	5
3	Work, Energy, Power	5
3.1	Everything about Work	5
3.2	Everything about Energy	6
3.3	Everything about Power	6
4	Rotational Kinematics	6
4.1	Bare Essential Variables	6
4.2	Primitive Laws of Rotational Motion ($\alpha = \text{constant}$)	8
5	Rotational Dynamics	8
5.1	Newton's 2 nd Law of Motion (For Inertial System)	8
5.2	Work and energy in Rotation	8
6	Many Particle System and Rigid Bodies	9
6.1	Everything about a Many-Particle System	9
6.2	Everything about a Rigid Body System	9
6.3	Momentum related Stuff	9
6.4	1D Collision	10
6.5	2D Collision	10
7	State of Equilibrium	10
7.1	Tranlational Equilibrium	10
7.2	Rotational Equilibrium	10
8	Gravitation	11
8.1	Newton's law of Universal Gravitation	11
8.2	Orbits and Gravitational Energies	11
8.3	Kepler's Laws of Planetary Motion	11

1 Translational Kinematics

1.1 Position Vector

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k} \quad (1.1)$$

1.2 Displacement Vector

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 \quad (1.2)$$

1.3 Average Velocity

$$v_{\text{avg}} = \frac{\Delta\vec{r}}{\Delta t} \quad (1.3)$$

1.4 Instantaneous Velocity/Speed

$$v = \lim_{\Delta t \rightarrow 0} \frac{\Delta r}{\Delta t} = \frac{dr}{dt} \quad (1.4)$$

$$\vec{v} = \frac{d\vec{r}}{dt} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k} \quad (1.4.2)$$

1.5 Average Acceleration

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t} \quad (1.5)$$

1.6 Instantaneous Acceleration

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} \quad (1.6)$$

$$\vec{a} = \frac{d\vec{v}}{dt} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k} \quad (1.6.2)$$

1.7 Primitive Laws of Translational Motion (a = constant)

$$\vec{v} = \vec{v}_0 + \vec{a}t \quad (1.7)$$

$$\vec{r} - \vec{r}_0 = \frac{1}{2}(\vec{v}_0 + \vec{v})t \quad (1.7.2)$$

$$\vec{r} - \vec{r}_0 = \vec{v}_0t + \frac{1}{2}\vec{a}t^2 \quad (1.7.3)$$

$$\vec{v}^2 = \vec{v}_0^2 + 2\vec{a}(\vec{r} - \vec{r}_0) \quad (1.7.4)$$

1.8 Uniform Circular Motion

Tangential/Parallel acceleration

$$a_{\parallel} = \frac{dv}{dt} \quad (1.8)$$

Radial/Perpendicular/Centripetal acceleration

$$a_c = \frac{v^2}{R} \quad (1.8.2)$$

Linear Speed of circular motion

$$v = \frac{2\pi R}{T} \quad (1.8.3)$$

Period of a circular motion

$$T = \frac{2\pi R}{v} \quad (1.8.4)$$

1.9 VERTICAL PROJECTION

1.9 Vertical Projection

Height at any point of the trajectory

$$y = y_0 + v_0 t - \frac{gt^2}{2} \quad (1.9)$$

Vertical speed at any instant in time

$$v_y = -gt \quad (1.9.2)$$

1.10 Slant Projection

Height at any point of the trajectory

$$y = x \tan \alpha_0 - \frac{gx^2}{2(v_0 \cos \alpha_0)^2} \quad (1.10)$$

Horizontal component of initial velocity

$$v_x = v_0 \cos \alpha_0 = \text{constant} \quad (1.10.2)$$

Vertical velocity at maximum height

$$v_y = 0 \quad (1.10.3)$$

Vertical speed at any instant in time

$$v_y = v_0 \sin \alpha_0 - gt \quad (1.10.4)$$

1.11 Everything about Projectile

Horizontal Component of the Initial Velocity

$$v_x = v_0 \cos \alpha_0 \quad (1.11)$$

Vertical Component of the Initial Velocity

$$v_{0y} = v_0 \sin \alpha_0 \quad (1.11.2)$$

Time elapsed to reach Maximum Height

$$t_{\max} = \frac{v_{0y}}{g} = \frac{v_0 \sin \alpha_0}{g} \quad (1.11.3)$$

Time elapsed for the Projectile Range (Symmetric Motion/No Air Resistance)

$$t_{\text{range}} = 2t_{\max} = 2\frac{v_{0y}}{g} = \frac{2v_0 \sin \alpha_0}{g} \quad (1.11.4)$$

Maximum height of a Projectile from ground

$$h_{\max} = v_{0y} t_{\max} = \frac{(v_0 \sin \alpha_0)^2}{2g} \quad (1.11.5)$$

Horizontal Range at any time t

$$l = v_x t \quad (1.11.6)$$

Maximum Horizontal Range (Only works for the same height)

$$R = v_x t_{\text{range}} = 2v_0 \cos \alpha_0 \frac{v_{0y}}{g} = \frac{v_0^2 \sin 2\alpha_0}{g} \quad (1.11.7)$$

2 Translational Dynamics

2.1 Newton's 2nd Law of Motion (For Inertial System)

$$\sum F = m\vec{a} \quad (2.1)$$

$$\sum F_x = ma_x \quad (2.1.2)$$

2.2 FRICTION FORCE

$$\sum F_y = ma_y \quad (2.1.3)$$

$$\sum F_z = ma_z \quad (2.1.4)$$

2.2 Friction Force

Static Friction Force

$$f_s^{\max} = \mu_s N \quad (2.2)$$

Kinetic Friction Force

$$f_k = \mu_k N \quad (2.2.2)$$

Speed on a level road with friction (No banking)

$$v = \sqrt{g\mu_s R} \quad (2.2.3)$$

Coefficient of friction on a banked road with friction (No banking)

$$\mu_s = \frac{v^2}{gR} \quad (2.2.4)$$

Banking angle of a road without friction

$$\theta = \tan^{-1} \left(\frac{a_{\text{rad}}}{g} \right) = \tan^{-1} \left(\frac{v^2}{gR} \right) \quad (2.2.5)$$

Coefficient of static friction of banked road

$$\mu_s = \frac{v^2 - gR \tan \theta}{v^2 \tan \theta + gR} \quad (2.2.6)$$

Banking angle of a road with friction

$$\theta = \tan^{-1} \left(\frac{v^2 - \mu_s gR}{\mu_s v^2 + gR} \right) \quad (2.2.7)$$

Maximum speed on a banked road with friction

$$v_{\max} = \sqrt{gR \times \frac{\mu_s + \tan \theta}{1 - \mu_s \tan \theta}} \quad (2.2.8)$$

Minimum speed on a banked road with friction

$$v_{\min} = \sqrt{gR \times \frac{\mu_s - \tan \theta}{1 + \mu_s \tan \theta}} \quad (2.2.9)$$

3 Work, Energy, Power

3.1 Everything about Work

Work done by a constant force

$$W = \vec{F} \cdot \vec{s} \quad (3.1)$$

Work done by a variable force

$$W = \int \vec{F} \cdot \vec{r} d\vec{r} \quad (3.1.2)$$

$$W_s = \vec{F} \cdot \vec{r} = \int_{x_i}^{x_f} F_x(x) dx + \int_{y_i}^{y_f} F_y(x) dy + \int_{z_i}^{z_f} F_z(x) dz \quad (3.1.3)$$

Net work done (on a loop) by a Conservative force

$$\Delta W = 0 \quad (3.1.4)$$

Net work done (on a loop) by a Non-Conservative force

$$\Delta W \neq 0 \quad (3.1.5)$$

3.2 EVERYTHING ABOUT ENERGY

3.2 Everything about Energy

Kinetic Energy

$$E_k = \frac{1}{2}mv^2 \quad (3.2)$$

Work-Energy Theorem

$$W = \Delta E = \vec{F} \cdot \vec{s} = Fs \cos \theta \quad (3.2.2)$$

$W_g \rightarrow$ Gravitational Force

$$W_g = mg\Delta y = -\Delta E_p \quad (3.2.3)$$

Potential Energy

$$\Delta E_p = U = -W = -\int_{r_i}^{r_f} F(r)dr \quad (3.2.4)$$

where

$$F(r) = -\frac{dE_p(r)}{dr} \quad (3.2.5)$$

$\vec{F}_s \rightarrow$ Variable force; $k \rightarrow$ Elastic (Spring) constant

$$\vec{F}_{\text{elas}} = -k\vec{d} \quad (3.2.6)$$

$$W_{\text{elas}} = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2 \quad (3.2.7)$$

Mechanical Energy

$$E_{\text{mech}} = \Delta E_k + \Delta E_p = E + U \quad (3.2.8)$$

3.3 Everything about Power

Average Power

$$P_{\text{avg}} = \frac{W}{\Delta t} \quad (3.3)$$

Instantaneous Power

$$P = \frac{dW}{dt} = \vec{F} \cdot \vec{v} \quad (3.3.2)$$

4 Rotational Kinematics

4.1 Bare Essential Variables

All angles must be calculated in Radian

Angular Position

$$\theta = \frac{s}{r} \quad (4.1)$$

Angular Displacement

$$\Delta\theta = \theta_f - \theta_i \quad (4.1.2)$$

Average Angular Velocity

$$\omega_{\text{avg}} = \frac{\Delta\theta}{\Delta t} \quad (4.1.3)$$

Instantaneous Angular Velocity

$$\omega = \lim_{\Delta t \rightarrow 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt} \quad (4.1.4)$$

Average Angular Acceleration

4.1 BARE ESSENTIAL VARIABLES

$$\alpha_{\text{avg}} = \frac{\Delta\omega}{\Delta t} \quad (4.1.5)$$

Instantaneous Angular Acceleration

$$\alpha = \lim_{\Delta t \rightarrow 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} \quad (4.1.6)$$

$s \rightarrow \theta$: Relating circular displacement to angular position

$$s = \theta r \quad (4.1.7)$$

$v \rightarrow \omega$: Relating angular speed to linear speed

$$v = \omega r \quad (4.1.8)$$

Vector form of angular velocity

$$\vec{v} = \vec{\omega} \times \vec{r} \quad (4.1.9)$$

$a \rightarrow \alpha$: Relating angular acceleration to linear acceleration

$$a = \alpha r \quad (4.1.10)$$

Vector form of angular acceleration

$$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v} \quad (4.1.11)$$

Period of a Rotational motion

$$T = \frac{\pi r}{v} = \frac{2\pi}{\omega} \quad (4.1.12)$$

Centripetal Acceleration, $a_{\text{rad}} \rightarrow$ radial component of a

$$a_{\text{rad}} = \frac{v^2}{r} = \omega^2 r \quad (4.1.13)$$

$I \rightarrow$ Moment of Inertia

$$E_k = \sum \frac{1}{2} m_i v_i^2 = \left(\sum \frac{1}{2} m_i r_i^2 \right) \omega^2 = \frac{1}{2} I \omega^2 \quad (4.1.14)$$

$$I = \sum m_i r_i^2 = \int r^2 dm \quad (4.1.15)$$

Torque

$$\vec{\tau} = \vec{r} \times \vec{F} \quad (4.1.16)$$

$r \rightarrow$ length of the moment arm—perpendicular distance from the center of rotation to the action line; $F_{\perp} \rightarrow$ Perpendicular component of force; $r_{\perp} \rightarrow$ Perpendicular component of the moment arm length, the magnitude of total force

$$\tau = (r)(F \sin \phi) = r F_{\perp} = (r \sin \phi)(F) = (r_{\perp})(F) \quad (4.1.17)$$

Angular Momentum

$$\vec{L} = \vec{r} \times \vec{v} \quad (4.1.18)$$

$r \rightarrow$ length of the moment arm—perpendicular distance from the center of rotation to the action line; $F_{\perp} \rightarrow$ Perpendicular component of force

$$L = (r)(mv \sin \phi) = r m v_{\perp} \quad (4.1.19)$$

Conservation of Angular Momentum

$$\text{If } \sum \vec{\tau} = 0, \quad \Delta \vec{L} = 0 \quad (4.1.20)$$

$$\vec{L} = \text{constant} \quad (4.1.21)$$

where

$$\tau = I \alpha \quad (4.1.22)$$

With no net torque, the angular momentum of an isolated system remains constant

When orbit size changes of a circular motion

$$I_{\text{initial}} \omega_{\text{initial}} = I_{\text{final}} \omega_{\text{final}} \quad (4.1.23)$$

4.2 PRIMITIVE LAWS OF ROTATIONAL MOTION ($\alpha = \text{CONSTANT}$)

With r and ω , You may wanna use

$$r_{\text{initial}}^2 \omega_{\text{initial}} = r_{\text{final}}^2 \omega_{\text{final}} \quad (4.1.24)$$

$$\omega_{\text{final}} = \left(\frac{r_{\text{initial}}}{r_{\text{final}}} \right)^2 \omega_{\text{initial}} \quad (4.1.25)$$

With r and v , You may wanna use

$$v_{\text{initial}} r_{\text{initial}} = v_{\text{final}} r_{\text{final}} \quad (4.1.26)$$

$$v_{\text{final}} = \left(\frac{r_{\text{initial}}}{r_{\text{final}}} \right) v_{\text{initial}} \quad (4.1.27)$$

4.2 Primitive Laws of Rotational Motion ($\alpha = \text{constant}$)

$$\omega = \omega_0 + \vec{\alpha}t \quad (4.2)$$

$$\theta - \theta_0 = \frac{1}{2}(\vec{\omega}_0 + \vec{\omega})t \quad (4.2.2)$$

$$\theta - \theta_0 = \vec{\omega}_0 t + \frac{1}{2}\vec{\alpha}t^2 \quad (4.2.3)$$

$$\vec{\omega}^2 = \vec{\omega}_0^2 + 2\vec{\alpha}(\theta - \theta_0) \quad (4.2.4)$$

5 Rotational Dynamics

5.1 Newton's 2nd Law of Motion (For Inertial System)

$$\sum \vec{\tau} = I\vec{\alpha} \quad (5.1)$$

$$\sum \tau_x = I\alpha_x \quad (5.1.2)$$

$$\sum \tau_y = I\alpha_y \quad (5.1.3)$$

$$\sum \tau_z = I\alpha_z \quad (5.1.4)$$

5.2 Work and energy in Rotation

Rotational Kinetic Energy

$$E_k = \frac{1}{2}I\omega^2 \quad (5.2)$$

Rotational Work done

$$W = \int_{\theta_{\text{initial}}}^{\theta_{\text{final}}} \tau d\theta \quad (5.2.2)$$

Rotational Work-Energy Theorem

$$\Delta E_k = \frac{1}{2}I(\Delta\omega)^2 = \int_{\theta_{\text{initial}}}^{\theta_{\text{final}}} \tau d\theta \quad (5.2.3)$$

For a constant torque τ

$$\Delta E_k = \tau(\theta_{\text{final}} - \theta_{\text{initial}}) \quad (5.2.4)$$

Total Kinetic Energy of a body in both translational and rotational motion

$$E_k = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 \quad (5.2.5)$$

$$E_{\text{rotational}} = \frac{1}{2}I\omega^2 \quad (5.2.6)$$

$$E_{\text{translational}} = \frac{1}{2}mv^2 \quad (5.2.7)$$

Rotational Power

$$P = \tau\omega \quad (5.2.8)$$

6 Many Particle System and Rigid Bodies

6.1 Everything about a Many-Particle System

Center of Mass of n particle system

$$x_{\text{com}} = \frac{1}{M} \sum_{i=1}^n x_i m_i; \quad y_{\text{com}} = \frac{1}{M} \sum_{i=1}^n y_i m_i; \quad z_{\text{com}} = \frac{1}{M} \sum_{i=1}^n z_i m_i \quad (6.1)$$

Position of the center of mass of n particle system

$$\vec{r}_{\text{com}} = \hat{i}x_{\text{com}} + \hat{j}y_{\text{com}} + \hat{k}z_{\text{com}} \quad (6.1.2)$$

Velocity of the center of mass of n particle system

$$\vec{v}_{\text{com}} = \hat{i} \sum_{i=1}^n v_{xi}^{\text{com}} + \hat{j} \sum_{i=1}^n v_{yi}^{\text{com}} + \hat{k} \sum_{i=1}^n v_{zi}^{\text{com}} = \vec{v}_1 + \vec{v}_2 + \vec{v}_3 + \dots \dots + \vec{v}_n \quad (6.1.3)$$

Acceleration of the center of mass of n particle system

$$\vec{a}_{\text{com}} = \hat{i} \sum_{i=1}^n a_{xi}^{\text{com}} + \hat{j} \sum_{i=1}^n a_{yi}^{\text{com}} + \hat{k} \sum_{i=1}^n a_{zi}^{\text{com}} = \vec{a}_1 + \vec{a}_2 + \vec{a}_3 + \dots \dots + \vec{a}_n \quad (6.1.4)$$

Momentum of the center of mass of n particle system

$$\vec{p}_{\text{com}} = \hat{i} \sum_{i=1}^n m_i v_{xi}^{\text{com}} + \hat{j} \sum_{i=1}^n m_i v_{yi}^{\text{com}} + \hat{k} \sum_{i=1}^n m_i v_{zi}^{\text{com}} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \dots \dots + \vec{p}_n \quad (6.1.5)$$

Newton's 2nd law for a n particle system

$$\vec{F}_{\text{net}} = \sum_{i=1}^n \vec{F} = M\vec{a}_{\text{com}} = \frac{d\vec{p}_{\text{com}}}{dt} \quad (6.1.6)$$

6.2 Everything about a Rigid Body System

Center of Mass of a Rigid Body system

$$x_{\text{com}} = \frac{1}{M} \int x dm, \quad y_{\text{com}} = \frac{1}{M} \int y dm, \quad z_{\text{com}} = \frac{1}{M} \int z dm \quad (6.2)$$

Position of the center of mass of a Rigid Body system

$$\vec{r}_{\text{com}} = \frac{1}{M} \left(\hat{i} \int x dm + \hat{j} \int y dm + \hat{k} \int z dm \right) \quad (6.2.2)$$

Velocity of the center of mass of a Rigid Body system

$$\vec{v}_{\text{com}} = \hat{i}v_x + \hat{j}v_y + \hat{k}v_z \quad (6.2.3)$$

Acceleration of the center of mass of a Rigid Body system

$$\vec{a}_{\text{com}} = \hat{i}a_x + \hat{j}a_y + \hat{k}a_z \quad (6.2.4)$$

Momentum of the center of mass of a Rigid Body system

$$\vec{p}_{\text{com}} = \hat{i}p_x + \hat{j}p_y + \hat{k}p_z \quad (6.2.5)$$

Newton's 2nd law for a n particle system

$$\vec{F}_{\text{net}} = \int d\vec{F} = M\vec{a}_{\text{com}} = \frac{d\vec{p}_{\text{com}}}{dt} \quad (6.2.6)$$

6.3 Momentum related Stuff

Linear Momentum

$$\vec{p} = m\vec{v} \quad (6.3)$$

Impulse of a Force

$$\vec{J} = \Delta\vec{p} = \vec{F}_{\text{avg}} = \int_{t_i}^{t_f} \vec{F}(t) dt \quad (6.3.2)$$

6.4 1D COLLISION

Conservation of Linear Momentum

$$\text{If } \sum \vec{F} = 0, \quad \Delta \vec{p} = 0 \quad (6.3.3)$$

$$\vec{p} = \text{constant} \quad (6.3.4)$$

With no net force, the linear momentum of an isolated system remains constant

6.4 1D Collision

Elastic Collision

$$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f} \quad (\text{Conservation of Linear Momentum}) \quad (6.4)$$

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \quad (\text{Conservation of Energy}) \quad (6.4.2)$$

where

$$\vec{v}_{1f} = \frac{m_1 - m_2}{m_1 + m_2} \vec{v}_{1i} + \frac{2m_2}{m_1 + m_2} \vec{v}_{2i} \quad (6.4.3)$$

$$\vec{v}_{2f} = \frac{2m_1}{m_1 + m_2} \vec{v}_{1i} + \frac{m_2 - m_1}{m_1 + m_2} \vec{v}_{2i} \quad (6.4.4)$$

Inelastic Collision

$$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = (m_1 + m_2) \vec{v} \quad (\text{Conservation of Linear Momentum}) \quad (6.4.5)$$

6.5 2D Collision

$\alpha \rightarrow$ Initial Angle; $\theta \rightarrow$ Recoil Angle

x -axis analysis

$$m_1 v_{1i} \cos \alpha_1 + m_2 v_{2i} \cos \alpha_2 = m_1 v_{1f} \cos \theta_1 + m_2 v_{2f} \cos \theta_2 \quad (6.5)$$

y -axis analysis

$$m_1 v_{1i} \sin \alpha_1 + m_2 v_{2i} \sin \alpha_2 = m_1 v_{1f} \sin \theta_1 + m_2 v_{2f} \sin \theta_2 \quad (6.5.2)$$

7 State of Equilibrium

7.1 Translational Equilibrium

Translational Equilibrium

$$\sum \vec{F}_{\text{net}} = 0 \quad (7.1)$$

3 constraints on Force

$$\sum F_x = 0, \quad \sum F_y = 0, \quad \sum F_z = 0 \quad (7.1.2)$$

7.2 Rotational Equilibrium

Rotational Equilibrium

$$\sum \vec{\tau}_{\text{net}} = 0 \quad (7.2)$$

3 constraints on Torque

$$\sum \tau_x = 0, \quad \sum \tau_y = 0, \quad \sum \tau_z = 0 \quad (7.2.2)$$

Total 6 constraints for a complete equilibrium

$$\Delta \vec{p} = 0, \quad \text{and} \quad \Delta \vec{L} = 0 \quad (7.2.3)$$

8 Gravitation

8.1 Newton's law of Universal Gravitation

Force of gravity between two bodies

$$F = G \frac{m_1 m_2}{r^2} \quad (8.1)$$

where Universal constant of Gravitation

$$G = 6.673 \times 10^{-11} \text{ N} \frac{\text{m}^2}{\text{kg}^2} \quad (8.1.2)$$

Gravitational acceleration due to an astronomical body of mass M and radius R

$$a_g = \frac{GM}{r^2} \quad (8.1.3)$$

Gravitational acceleration felt by a body

$$g = a_g - \omega^2 R \quad (8.1.4)$$

Vector form of the force of Gravity, $R \rightarrow$ distance between two bodies

$$\vec{F} = \frac{GMm}{R^3} \vec{R} = \frac{GMm}{R^2} \hat{R} \quad (8.1.5)$$

Gravitational Potential Energy

$$E_p = -\frac{GMm}{r} \quad (8.1.6)$$

8.2 Orbits and Gravitational Energies

Potential Energy of an orbit

$$E_p = -\frac{GMm}{r} \quad (8.2)$$

Kinetic Energy of an orbit

$$E_k = \frac{GMm}{2r} = -\frac{E_p}{2} \quad (8.2.2)$$

Mechanical Energy of an orbit

$$E = E_p + E_k = -\frac{GMm}{2r} \quad (8.2.3)$$

Escape Velocity

$$v = \sqrt{\frac{2GM}{R}} = \sqrt{2gR} \quad (8.2.4)$$

8.3 Kepler's Laws of Planetary Motion

Kepler's 1st Law: **Law of Orbits:** All planets move in elliptical orbits. They move around their host star in elliptical orbits with the star at one of the ellipse's foci.

Kepler's 2nd Law: **Law of Areas:** All planets move so that the line connecting it to the host star sweeps out equal areas at equal times.

$$\text{constant} = \frac{dA}{dt} = \frac{1}{2} a^2 \omega = \frac{L}{2m} \quad (8.3)$$

A planet body speeds up as it gets closer to its host star, and slows down as it goes further

$$a_1^2 \omega_1 = a_2^2 \omega_2 = \text{constant} \quad (8.3.2)$$

Kepler's 3rd Law: **Law of Periods:** The square of the period of a planet is proportional to the cube of the semi-major axis of its orbit.

$$T^2 = \left(\frac{4\pi^2}{GM} \right) a^3 \quad (8.3.3)$$

The square of the orbital period of a planet is directly proportional to the cube of its orbit size