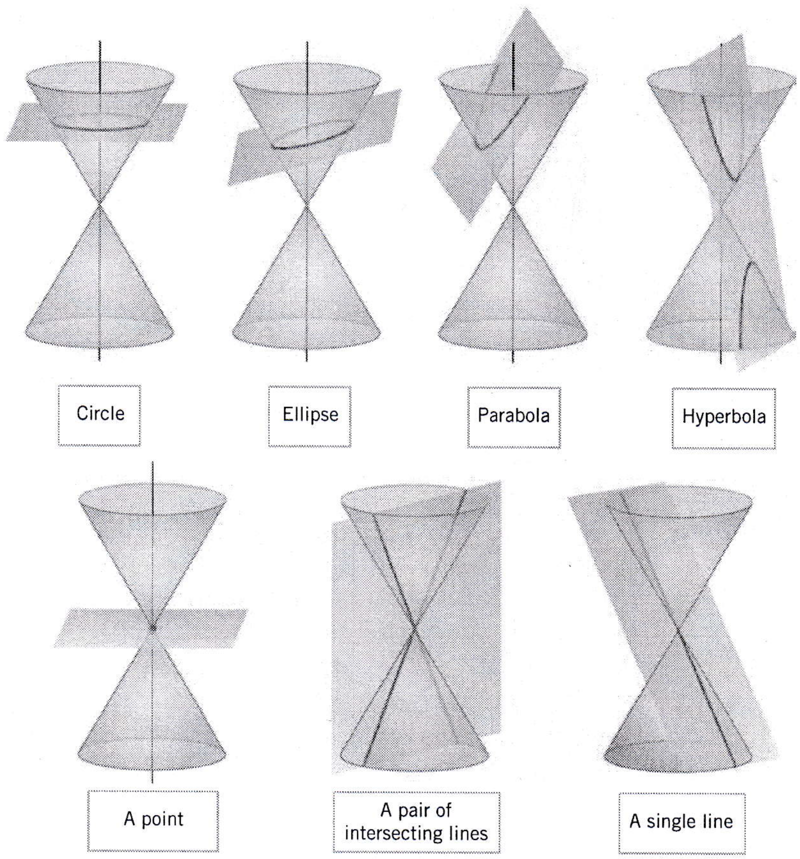
Week 10 — Differential Calculus & Coordinate Geometry

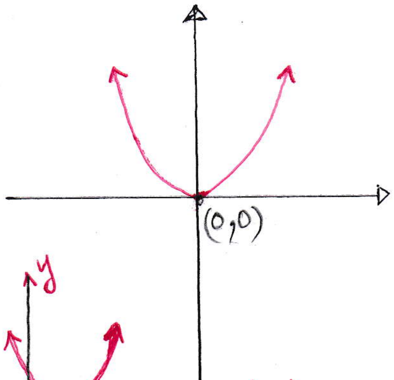
Conic Section

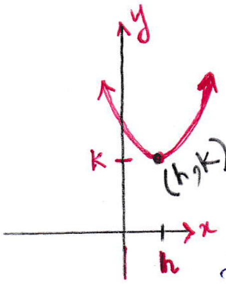
CONIC SECTIONS

Circles, ellipses, parabolas, and hyperbolas are called ***conic sections*** or ***conics*** because they can be obtained as intersections of a plane with a double-napped circular cone (Figure 10.4.1). If the plane passes through the vertex of the double-napped cone, then the intersection is a point, a pair of intersecting lines, or a single line. These are called ***degenerate conic sections***.



▲ **Figure 10.4.1**





parabola

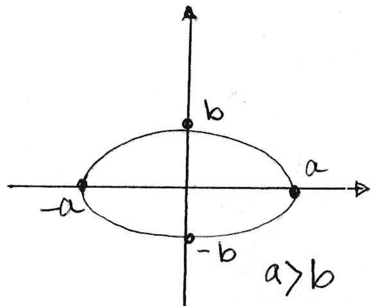
$y = x^2$

$x^2 = 4py \rightarrow$ Standard form

Vertex: (0,0) (turning point)

$$\boxed{(x-h)^2 = \overset{\text{constant}}{\widetilde{4p}}(y-k)}\ ;\ p>0 \text{ or } p<0$$

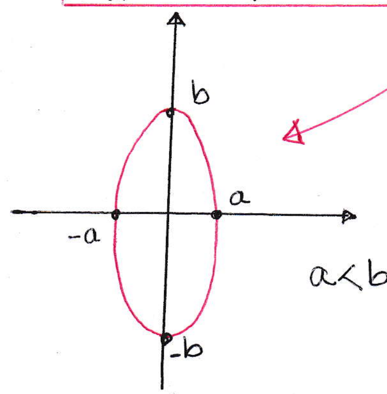
Vertex: (h, k)



Ellipse

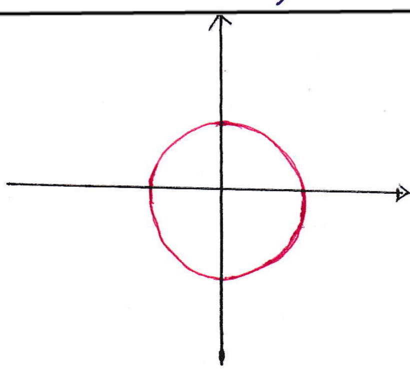
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \longrightarrow \text{Center: } (0,0)$$

$$\boxed{\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1}$$



$$\boxed{\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1} \rightarrow \text{Center: } (h, k)$$

If it was not centered at the origin



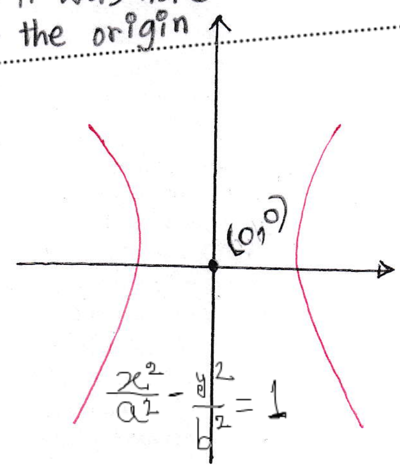
circle

$x^2 + y^2 = r^2$

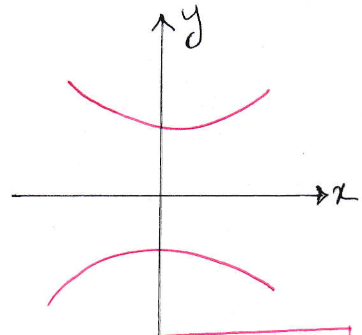
Center: (0,0)

$$\boxed{(x-h)^2 + (y-k)^2 = r^2}$$

Center: (h, k)



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



$$\boxed{\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1}$$

Hyperbola

$$\boxed{\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1} \longrightarrow \text{cencer } (h, k)$$

General Equation of Second Degree

$$Ax^2+Bxy+Cy^2+Dx+Ey+F=0 \quad \text{—— (1)}$$

or $\boxed{ax^2+2hxy+by^2+2gx+2fy+c=0}$ → (2) Relabeling the Constants

$$\begin{bmatrix} x & y & 1 \end{bmatrix}\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}=0$$

$A=a$; $B=2h$; $C=b$
$D=2g$; $E=2f$; $F=c$

We need to find the following invariants

$$\Delta = abc+2fgh-af^2-bg^2-ch^2$$

$C=h^2-ab$ discriminent of Eqn (1)

B^2-4AC
$(2h)^2-4ab$
$4h^2-4ab$
$4(h^2-ab)$
(h^2-ab)
↳ we can ignore the constan "4"

$I=a+b$

Invariants are some unchanged constants which indicates the locus of conic.

Locus — a set of points, the position of which satisfy a set of algebric condition, such as circle, ellipse etc.

"Δ" identifies whether the conic is "Degenerate" (weak conic, quality of conic reduces) or not

$$\Delta=\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = a(bc-f^2)-h(hc-fg)+g(hf-bg)$$
$$= abc-af^2-ch^2+fgh+fgh-bg^2$$
$$= abc+2fgh-af^2-bg^2-ch^2$$

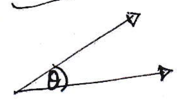
For degenerate case $\Delta=0$; For proper Conic $\Delta \neq 0$

Degenerate Case : $\Delta = 0$

If $h^2 - ab = 0$ then Parallel lines or straight line

If $a + b = 0$ then perpendicular line $\perp$

If $\Delta = 0$ only but the above conditions do not fulfil then $\underbrace{\text{pair of st. lines}}$.



non parallel lines

Non-Degenerate Case : $\Delta \neq 0$

$h^2 - ab = 0 \longrightarrow$ Parabola

$h^2 - ab > 0 \longrightarrow$ hyperbola

$h^2 - ab < 0 \longrightarrow$ Ellipse

$$\left.\begin{array}{l} h^2 - ab < 0 \\ a = b \\ h = 0 \end{array}\right\} \longrightarrow \text{Circle}$$

What is represented by the following equation (s)?

(i) $\underbrace{4}_{a}x^2 \underbrace{-24}_{2h}xy \underbrace{-6}_{b}y^2 \underbrace{+4}_{2g}x \underbrace{-12}_{2f}y \underbrace{+1}_{c} = 0$

$2h = -24$
$h = -12$

$2g = 4$
$g = 2$

$2f = -12$
$f = -6$

$$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$$

$$= (4)(-6)(1) + 2(-6)(2)(-12) - 4(-6)^2 - (-6)(2)^2 - (1)(-12)^2$$

$$= 0$$

Degenerate Case

$$\left\{\begin{array}{l} h^2 - ab = (-12)^2 - (4)(-6) = 168 \neq 0 \\ a + b = 4 + (-6) = -2 \neq 0 \end{array}\right.$$

$\hookrightarrow$ The eqn represents a pair of st. lines

(ii) $x^2-4xy+y^2+8x+2y-5=0$

$a=1$, $2h=-4 \Rightarrow h=-2$; $b=1$, $c=-5$; $2g=8$, $g=4$; $2f=2$, $f=1$

$h^2-ab=(-2)^2-(1)(1)=3>0$

$\Delta = abc+2fgh-af^2-bg^2-ch^2$
$= -5+(-16)-1-16-(-20)$
$= -18 \neq 0$

The eqn represents a hyperbola

(iii) $8x^2+4xy+5y^2-16x-14y+13=0$

$a=8$, $b=5$, $c=13$; $2h=4$, $h=2$; $2g=-16$, $g=-8$; $2f=-14$, $f=-7$

$\Delta = abc+2fgh-af^2-bg^2-ch^2$
$=520+224-392-320-52$
$=-20\neq 0$

$h^2-ab = 2^2-(8)(13) = -100<0$

The eqn represents an ellipse

(iv) $x^2-6xy+9y^2+4x+8y+15=0$

$a=1$, $b=9$, $c=15$; $2h=-6$, $h=-3$; $2g=4$, $g=2$; $2f=8$, $f=4$

$h^2-ab=(-3)^2-(1)(9)=0$

$\therefore h^2-ab=0$, It could be degenerate or parabola

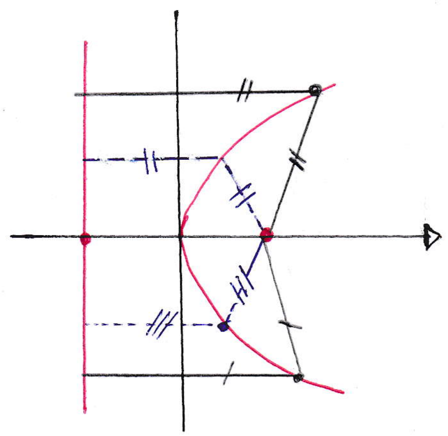
$\Delta = abc+2fgh-af^2-bg^2-ch^2$
$=(1)(9)(15)+2(4)(2)(-3)-(1)(4)^2-(9)(2)^2-(15)(-3)^2$

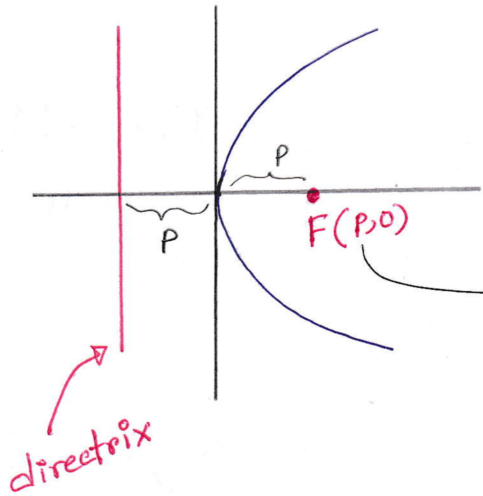
$=-100$

$\therefore \Delta \neq 0$ Hence the eqn represents a parabola.

Parabola

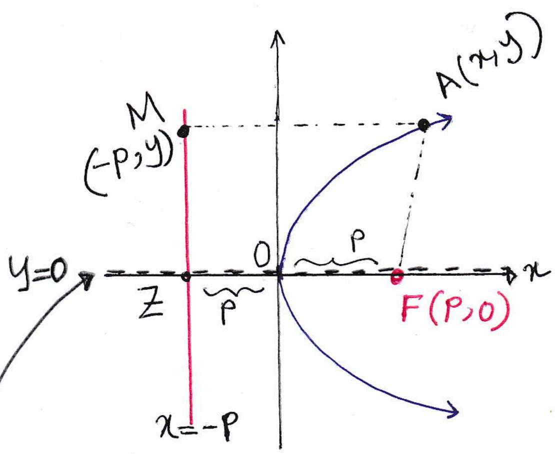
Set of points in a plane that are equidistant from a fixed point F (focus) and a fixed line (directrix)





F → focus

→ $y^2 = 4px$



$OZ = OF$

Vertex, $O = (0,0)$

Focus, $F = (p,0)$

Eqn of directrix, MZ: $x = -p$

$FA = AM$

$$\sqrt{(x-p)^2+(y-0)^2} = \sqrt{(x-(-p))^2+(y-y)^2}$$

$$\sqrt{(x-p)^2+y^2} = \sqrt{(x+p)^2}$$

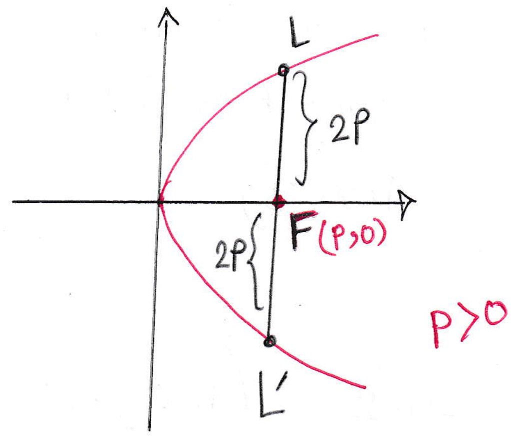
$$(x-p)^2+y^2 = (x+p)^2$$ square both sides

$$x^2-2xp+p^2+y^2 = x^2+2xp+p^2$$

$$y^2 = 4px$$

$$(y-k)^2 = 4p(x-h)$$ → if the Vertex shifts from the origin (0,0)

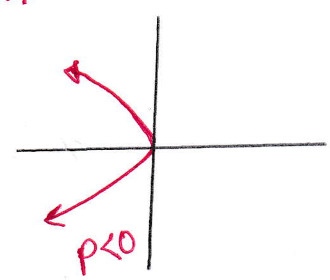
Axis of symmetry:
the line that divides the parabola into two equal parts: $y=0$



LL' : Latus Rectum

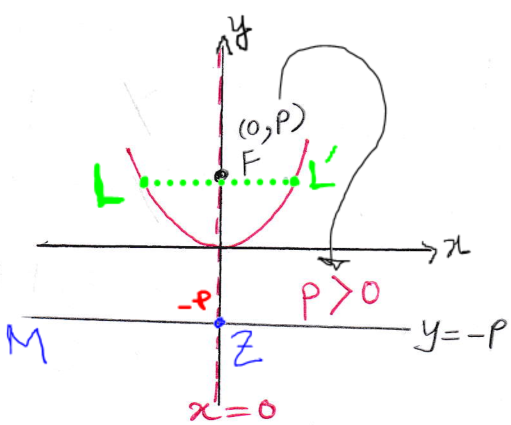
$\therefore LL' = 4p$

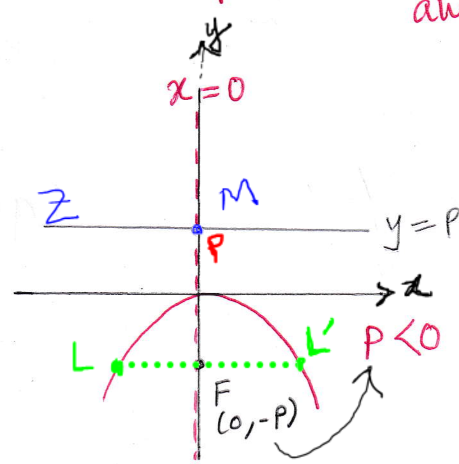
$p > 0$ or $p < 0$



Eqn of Latus Rectum: $x = p$

Length of Latus Rectum: $LL' = |4p|$ → length is always +ve





Vertex: $(0,0)$

Focus: $(0,p)$

(directrix) MZ : $y = -p$

axis of symmetry: $x = 0$

Latus Rectum, LL': $|4p|$

Vertex: $(0,0)$

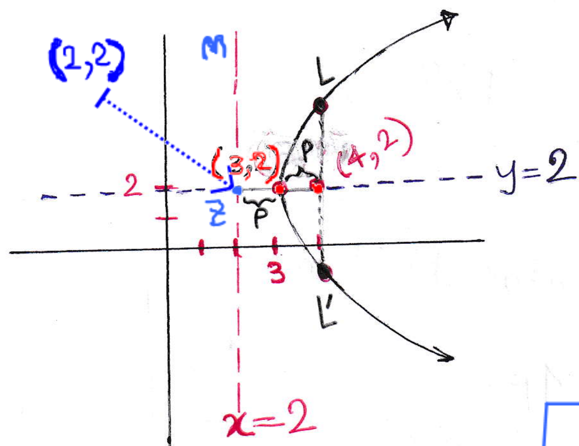
Focus: $(0,-p)$

MZ : $y = p$

axis of symmetry: $x = 0$

Latus Rectum, $LL' = |4p|$

Find the vertex, focus, equation of directrix, equation of axis of symmetry, length of Latus rectum of the parabola: $(y-2)^2 = 4(x-3)$



$(h, k) = (3, 2)$

$4P = 4$

$P = 1$

$(y-2)^2 = 4(x-3)$

$Y^2 = 4pX$

$4p = 4$

$p = 1$

$Y = 0 \Rightarrow y-2=0$
$y = 2$

$X = 0 \Rightarrow x-3=0$
$x = 3$

Vertex: $(3,2)$

Focus: $(p, 0)$

$= (4,2)$

$X = p$	$Y = 0$
$x-3 = p$	$y-2=0$
$x-3 = 1$	$y = 2$
$x = 4$	

Vertex: $(3, 2)$

Focus: $(4, 2)$

Eqn of directrix:

$x = 2$

Eqn of axis:

$Y = 0$

$y - 2 = 0$

$y = 2$

Eqn of directrix: $X = -p$

$\Rightarrow X + p = 0$

$\Rightarrow (x-3) + p = 0$

$\Rightarrow x = 2$

Length of Latus Rectum: $|4p|$
(LL')

$= |4(1)|$

$= 4$

Find the vertex, focus, directrix, axis and length of latus rectum for the parabola, $y^2-2y-4x-3=0$

$y^2-2y-4x-3=0$ ⟶ General form

$y^2-2\cdot y\cdot 1+1^2-1^2-4x-3=0$ completing square

$(y-1)^2-4x-4=0$

$(y-1)^2=4(x+1)$ ⟶ standard form

$(y-1)^2=4p(x+1)$

$Y^2=4pX$

$4p=4$

$\therefore \boxed{p=1}$

(i) Vertex: $(-1,1)$

(ii) Focus: $(p,0)=(0,1)$

$X=p$

$x+1=1$

$x=0$

$Y=0$

$y-1=0$

$y=1$

(iii) Eqn of $\underbrace{\text{directrix}}_{MZ}$: $x=-2$

$X=-p$

$x+1=-1$

$x=-2$

(iv) Eqn of axis: $y=1$

$Y=0$

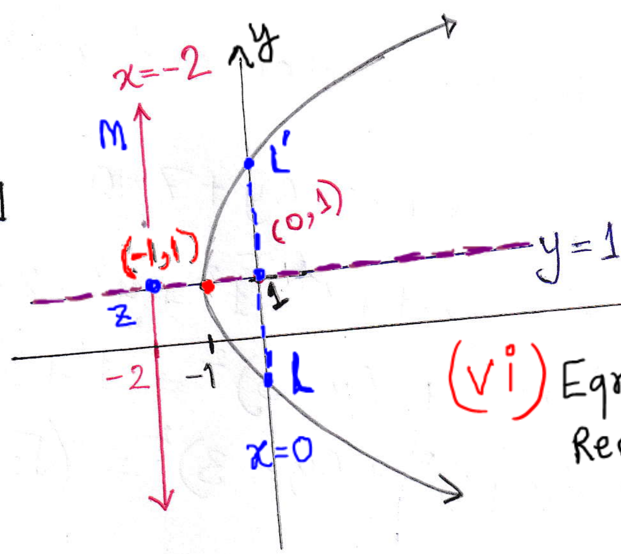
$y-1=0$

$y=1$

(v) Length of Latus Rectum: $|4p|$

$=|4(1)|$

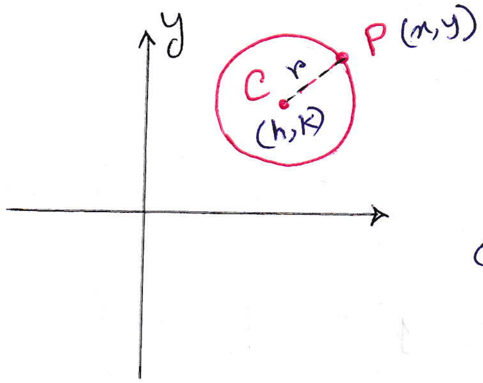
$=4$



(vi) Eqn of Latus Rectum: $x=0$

Circle

Set of points in a plane that are equidistant from a fixed point C (center).



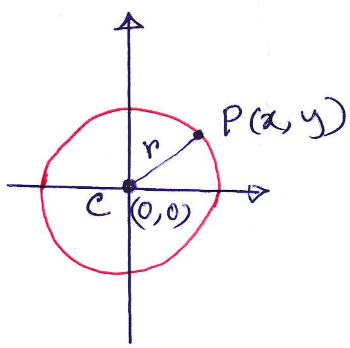
$$|PC| = \sqrt{(x-h)^2+(y-k)^2} = r$$

$$\Rightarrow \boxed{(x-h)^2+(y-k)^2 = r^2}$$

Square both sides

Center: (h,k)

Standard eqn of the circle.

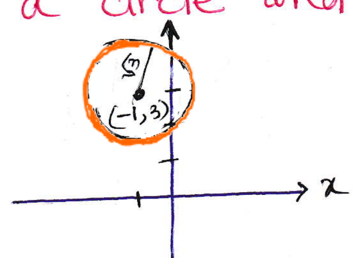


$$|PC| = \sqrt{(x-0)^2+(y-0)^2} = r$$

$$\Rightarrow x^2+y^2 = r^2$$

Center: $(0,0)$

<u>Question:</u> Sketch the graph with the equation $x^2+y^2+2x-6y+7=0$ by first showing that it represents a circle and then finding its center and radius.



$$x^2+2x+y^2-6y+7=0$$

$$\Rightarrow x^2+2\cdot x\cdot 1+1^2-1^2+y^2-2\cdot y\cdot 3+3^2-3^2+7=0$$

$$\Rightarrow (x+1)^2+(y-3)^2 = 1^2+3^2-7 = 3$$

$$\Rightarrow (x+1)^2+(y-3)^2 = (\sqrt{3})^2$$

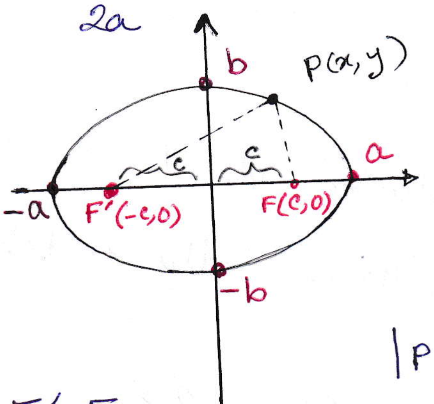
Center: $(-1,3)$

radius: $\sqrt{3}$

Ellipse

An ellipse is the set of all points in the plane the sum of whose distances from two fixed points (the foci) is a $\underbrace{\text{constant}}_{2a}$.

foci $\rightarrow$ plural of focus



$a > b$

$\underbrace{F', F}_{\text{foci}}$

$$|PF'| + |PF| = 2a$$

$$\Rightarrow \sqrt{(x+c)^2+y^2} + \sqrt{(x-c)^2+y^2} = 2a$$

$$\Rightarrow \sqrt{(x-c)^2+y^2} = 2a - \sqrt{(x+c)^2+y^2}$$

$$\Rightarrow \cancel{x^2} - 2cx + \cancel{c^2} + \cancel{y^2} = 4a^2 - 4a\sqrt{(x+c)^2+y^2} + \cancel{x^2} + 2cx + \cancel{c^2} + \cancel{y^2}$$

(sq both sides)

$$\Rightarrow a\sqrt{(x+c)^2+y^2} = a^2 + cx$$

$$\Rightarrow a^2(x^2+2cx+c^2+y^2) = a^4 + 2a^2cx + c^2x^2$$

(sq both sides)

$$\Rightarrow a^2x^2 + \cancel{2a^2cx} + a^2c^2 + a^2y^2 = a^4 + \cancel{2a^2cx} + c^2x^2$$

$$\Rightarrow a^2x^2 - c^2x^2 + a^2y^2 = a^4 - a^2c^2$$

$$\Rightarrow (a^2-c^2)x^2 + a^2y^2 = a^2(a^2-c^2)$$

$$\Rightarrow b^2x^2 + a^2y^2 = a^2b^2$$

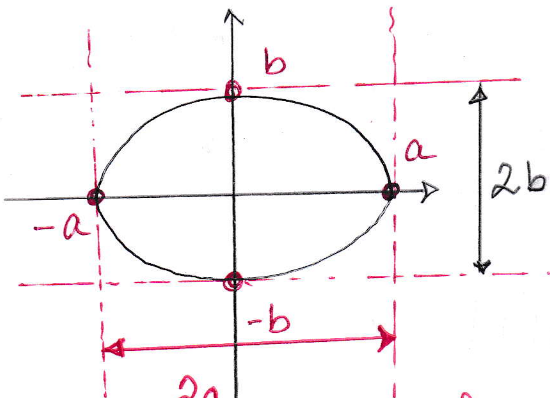
Redefine the constant $a^2 - c^2 = b^2$

$$\Rightarrow \frac{b^2x^2}{a^2b^2} + \frac{a^2y^2}{a^2b^2} = \frac{a^2b^2}{a^2b^2} \quad (\div \text{ by } a^2b^2)$$

$$\Rightarrow \boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1} \rightarrow \text{Eqn of Ellipse}$$

Center $(0,0)$

$a > b$

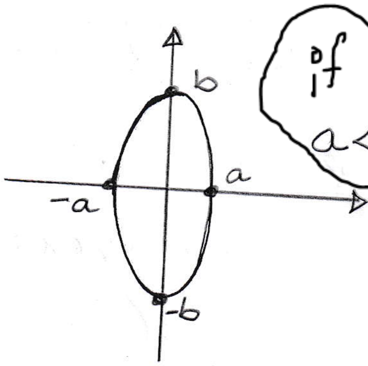


$2b \rightarrow$ distance of minor axis $\because a>b$

(Horizontally stretched)

$2a$

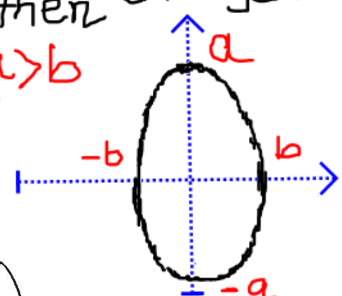
$\hookrightarrow$ distance of the major axis. $\because a>b$

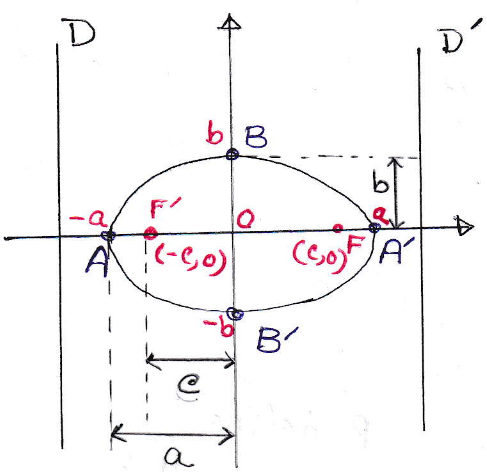


if $a<b$ $\rightarrow$ then change it $a>b$

$$\boxed{\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1}$$

(Vertically strectched)





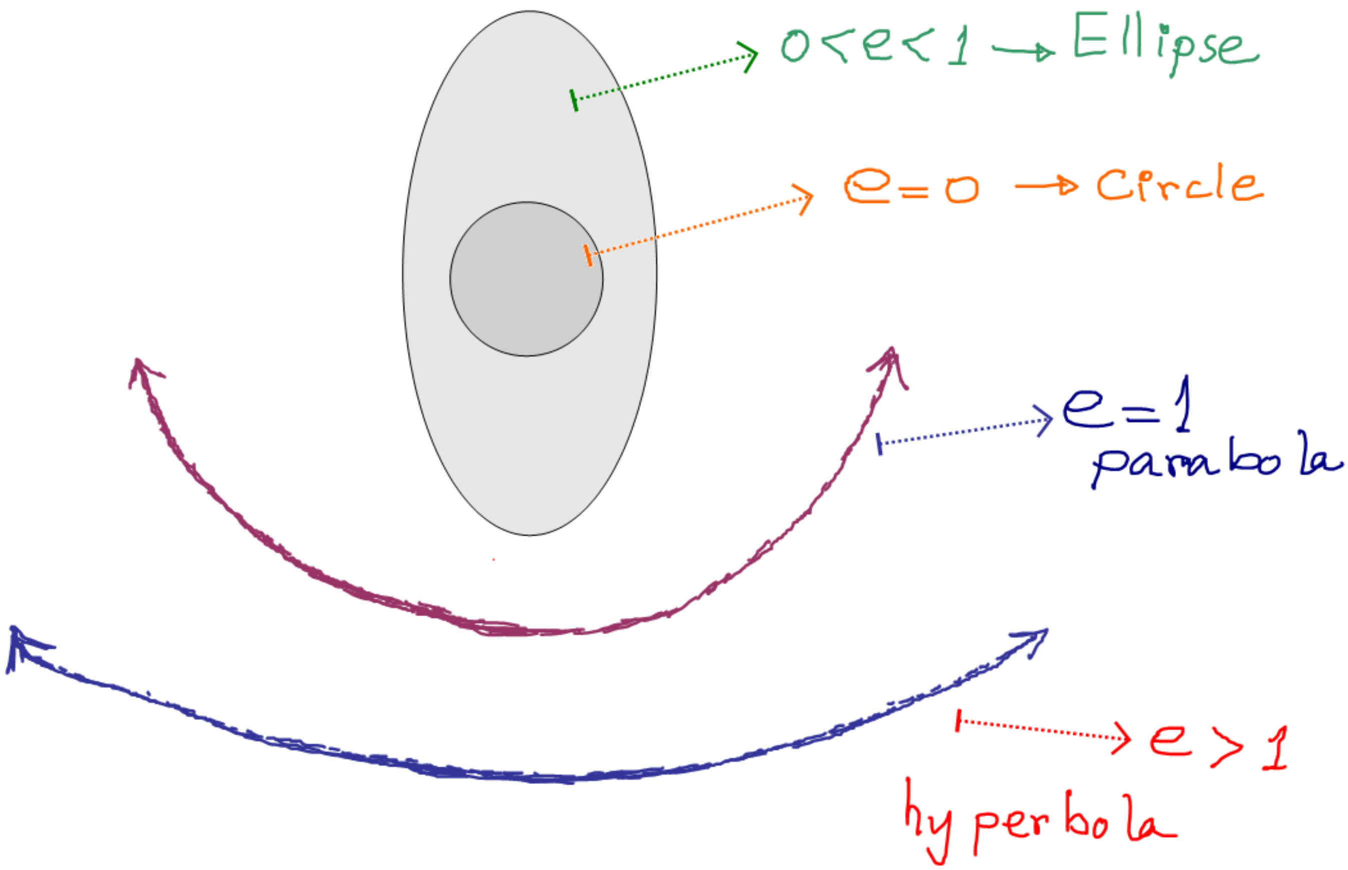
Eccentricity, $e = \frac{c}{a}$ $\rightarrow$ Ratio between the length of the focus and the length of the semi-major axis (one of the vertices)

$\downarrow$

It is a constant $\{$It defines how circular or uncircular the shape of the conic is$\}$

$0 < e < 1$

$\boxed{c<a}$



$$e = \frac{c}{a}$$

$$a^2 - c^2 = b^2$$ ∵ we redefined this constant

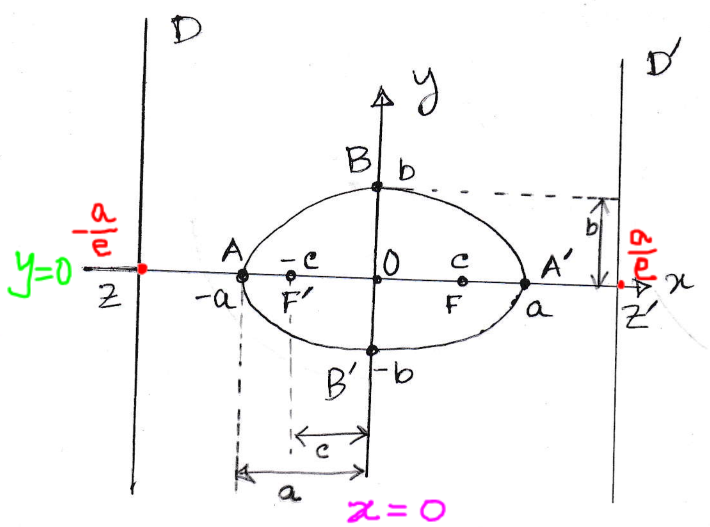
$$e = \frac{\sqrt{a^2-b^2}}{a}$$

$$\Rightarrow c^2 = a^2 - b^2$$

$$\Rightarrow c = \sqrt{a^2-b^2}$$

$$= \sqrt{\frac{a^2-b^2}{a^2}}$$

$$= \sqrt{1 - \frac{b^2}{a^2}}$$



Centre; O : (0,0)

Foci, $(\pm c, 0)$ or $(\pm ae, 0)$
↳ F′ & F

$\begin{cases} e = \frac{c}{a} \\ c = ae \end{cases}$

Vertices $(\pm a, 0) \rightarrow$ A & A′

Covertices $(0, \pm b) \rightarrow$ B & B′

Equation of major axis: $\boxed{y=0} \longrightarrow$ AA′ (the line)

Equation of minor axis: $\boxed{x=0} \longrightarrow$ BB′ (the line)

Length of major axis: $2a$ (AA′)

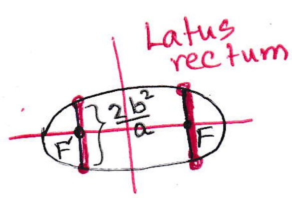
Length of minor axis: $2b$ (BB′)

Equation of directrices: DZ & D′Z′

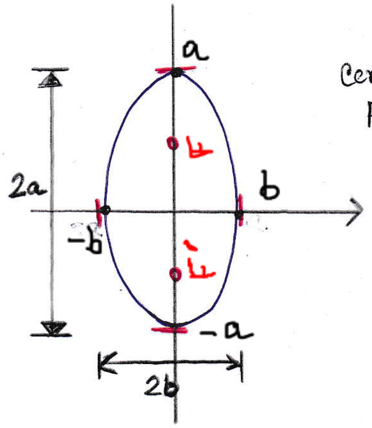
$$\boxed{x = \pm \frac{a}{e}}$$

Equation of latus rectum : $\begin{cases} x = \pm c \\ x = \pm ae \end{cases}$

$\because c = ae$



$$\boxed{\text{Length of Latus rectum: } \frac{2b^2}{a}}$$



center: (0,0)

Foci : $(0, \pm ae)$

Vertices: $(0, \pm a) \longrightarrow$ major axis

Covertices: $(\pm b, 0) \longrightarrow$ minor axis

Example: Find all the characteristics of the graph with equation: $9x^2+4y^2=36$

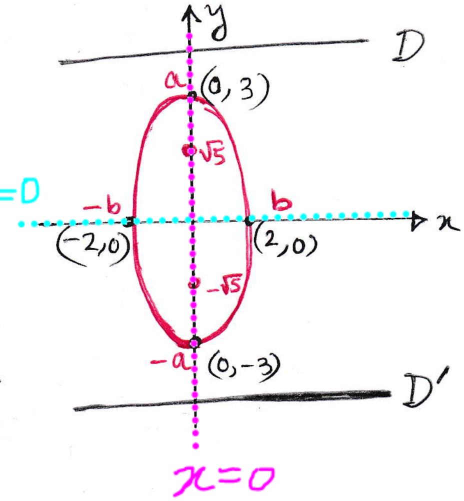
$$\frac{y^2}{a^2}+\frac{x^2}{b^2}=1$$

$$\frac{x^2}{4}+\frac{y^2}{9}=1 \quad (\div \text{ by } 36)$$

$$\Rightarrow \frac{x^2}{2^2}+\frac{y^2}{3^2}=1$$

$$\Rightarrow \frac{y^2}{3^2}+\frac{x^2}{2^2}=1$$

$a>b$
$a=3$
$b=2$



Center: $(0,0)$

Vertices: $(0,\pm a)=(0,\pm 3) \longrightarrow$ major axis

Covertices: $(\pm b,0)=(\pm 2,0) \longrightarrow$ minor axis

$$e=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{4}{9}}=\sqrt{\frac{9-4}{9}}=\frac{\sqrt{5}}{3}$$

Foci: $(0,\pm ae)=(0,\pm\sqrt{5})$ $\qquad ae=3\cdot\frac{\sqrt{5}}{3}=\sqrt{5}$

Equation of major axis: $\boxed{x=0}$

Eqn of minor axis: $\boxed{y=0}$

Length of major axis: $2a=2(3)=6$

Length of minor axis: $2b=4$

Equation of directrices: $y=\pm\frac{a}{e}=\pm\frac{9}{\sqrt{5}}$ $\qquad \begin{cases} a=3 \\ e=\frac{\sqrt{5}}{3} \\ b=2 \end{cases}$

Eqn of latus rectum: $y=\pm ae=\pm\sqrt{5}$

Length of latus rectum: $\frac{2b^2}{a}=\frac{8}{3}$

Compare

$$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$$

$$\frac{Y^2}{a^2} + \frac{X^2}{b^2} = 1$$

if center is not at the origin;
Center: (h, k)

Standard forms of the eqn of an ellipse with center: (h,k)

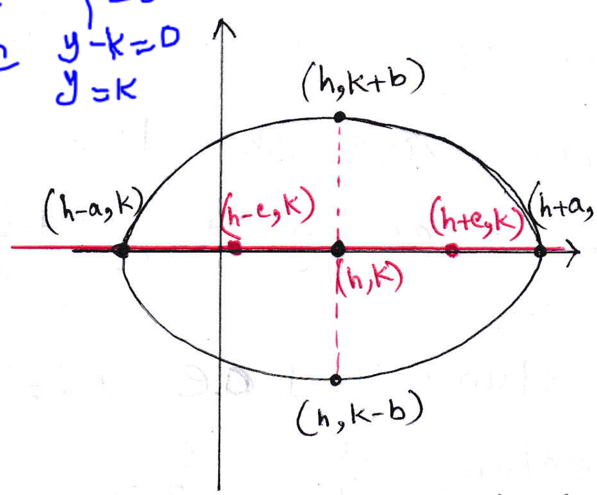
$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

- $a > b$
- length of major axis $= 2a$
- " " minor " $= 2b$
- Coordinates of the vertices: $(h \pm a, k)$
- Coordinates of the covertices: $(h, k \pm b)$
- Coordinate of the foci: $(h \pm c, k)$

$(\pm a, 0)$
$X = \pm a \qquad Y = 0$
$x - h = \pm a \qquad y - k = 0$
$x = h \pm a \qquad y = k$

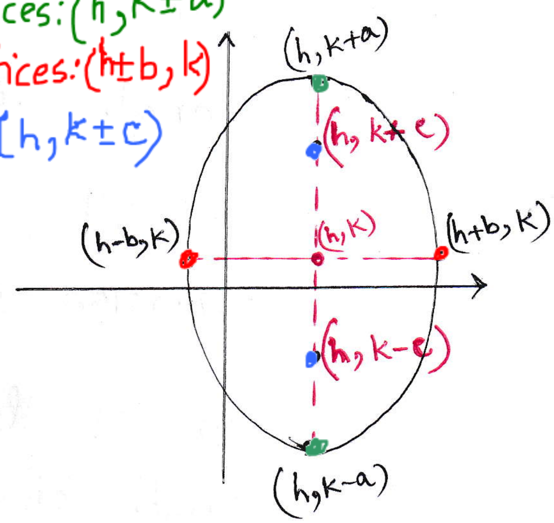
$(0, \pm b)$
$X = 0 \qquad Y = \pm b$
$x - h = 0 \qquad y - k = \pm b$
$x = h \qquad y = k \pm b$

$(\pm c, 0)$
$X = \pm c \qquad Y = 0$
$x - h = \pm c \qquad y - k = 0$
$x = h \pm c \qquad y = k$



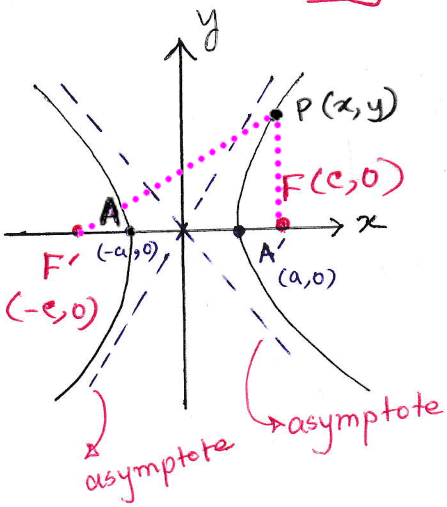
Horizontally stretched

Vertices: $(h, k \pm a)$
Covertices: $(h \pm b, k)$
foci: $(h, k \pm c)$



Vertically stretched

Hyperbola



Set of all points in a plane the difference of whose distances from two fixed points $\underbrace{\text{(the foci)}}_{F\ \&\ F'}$ is a $\underbrace{\text{constant}}_{2a}$.

$$|PF| - |PF'| = \pm 2a \qquad \begin{array}{l} PF > PF' = 2a \\ PF < PF' = -2a \end{array}$$

$$\Rightarrow \sqrt{(x-c)^2 + y^2} - \sqrt{(x+c)^2 + y^2} = \pm 2a$$

$$\Rightarrow \sqrt{(x-c)^2 + y^2} = \pm 2a + \sqrt{(x+c)^2 + y^2}$$

$$\Rightarrow \cancel{x^2} - 2cx + \cancel{c^2} + \cancel{y^2} = 4a^2 \pm 4a\sqrt{(x+c)^2 + y^2} + \cancel{x^2} + 2cx + \cancel{c^2} + \cancel{y^2}$$

$$\Rightarrow -4cx = 4a^2 \pm 4a\sqrt{(x+c)^2 + y^2}$$

$$\Rightarrow \pm a\sqrt{(x+c)^2 + y^2} = a^2 + cx$$

$$\Rightarrow a^2(x^2 + 2cx + c^2 + y^2) = a^4 + 2a^2cx + c^2x^2$$

$$\Rightarrow a^2x^2 + \cancel{2a^2cx} + a^2c^2 + y^2a^2 = a^4 + \cancel{2a^2cx} + c^2x^2$$

$$\Rightarrow a^2x^2 + a^2c^2 + y^2a^2 = a^4 + c^2x^2$$

Redefine the constant

$$c^2 = a^2 + b^2$$

$$\Rightarrow a^2x^2 + a^2(a^2+b^2) + a^2y^2 = a^4 + (a^2+b^2)x^2$$

$$\Rightarrow \cancel{a^2x^2} + \cancel{a^4} + a^2b^2 + a^2y^2 = \cancel{a^4} + \cancel{a^2x^2} + b^2x^2$$

$$\Rightarrow a^2b^2 = b^2x^2 - a^2y^2 \qquad (\div \text{ by } a^2b^2)$$

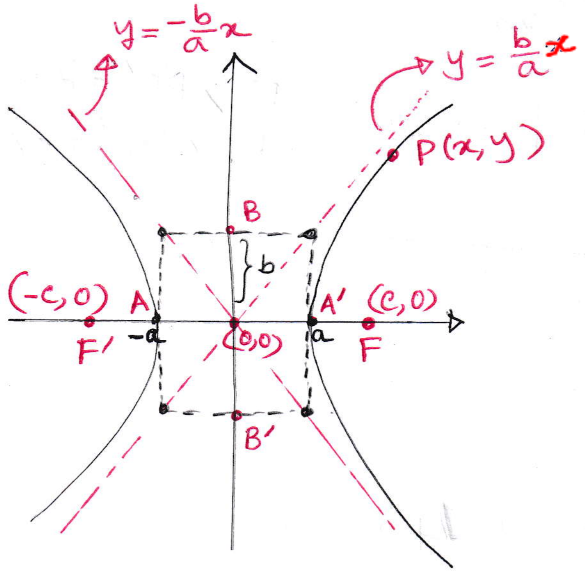
$$\Rightarrow \boxed{\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1}$$

standard form

In Ellipse we relabel $c^2 = a^2 - b^2$

Center: $(0,0) \Rightarrow \dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$

Center: $(h,k) \Rightarrow \dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1$



Center: $C\ (0,0)$

Vertices: A & A'

$\Rightarrow (\pm a, 0)$

Foci: F & F'

$\Rightarrow (\pm c, 0)$ or $(\pm ae, 0)$

$e > 1$

See graph at pg $\boxed{13}$

Eccentricity: $e = \sqrt{1 + \dfrac{b^2}{a^2}}$: $e = \dfrac{c}{a}$, $e > 1$

The sign is opposite to ellipse

Eqn of Asymptoses: $y = \pm\left(\dfrac{b}{a}\right)x$

Length of transverse axis: $2a$ (AA')

Length of conjugate axis: $2b$ (BB')

Eqn of directrices: $x = \pm \dfrac{a}{e}$

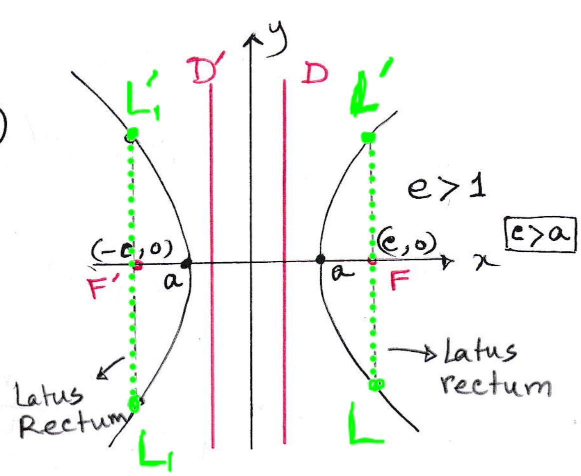
$(D \ \& \ D')$

same as ellipse

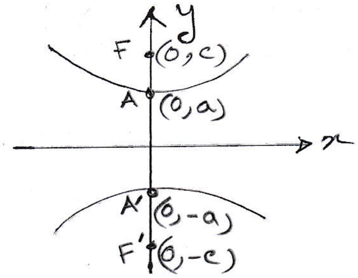
Length of Latus rectum: $\boxed{\dfrac{2b^2}{a}}$

Eqn of Latus rectum:

$\underbrace{x = -c = -ae, \quad x = c = ae}_{\text{Foci}}$



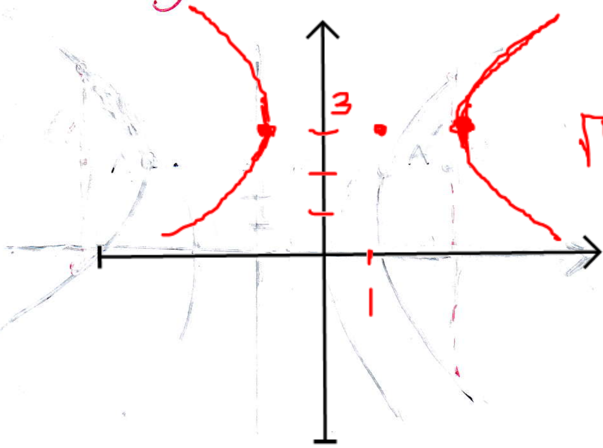
If we have: $\dfrac{y^2}{a^2} - \dfrac{x^2}{b^2} = 1$



Vertices: $(0, \pm a)$

Foci: $(0, \pm c)$ or $(0, \pm ae)$

Example: Find all the characteristics of the hyperbola with equation $2(x-1)^2 - 3(y-3)^2 = 30$



$\sqrt{15} > \sqrt{10}$

$$2(x-1)^2 - 3(y-3)^2 = 30$$

$$\frac{(x-1)^2}{15} - \frac{(y-3)^2}{10} = 1$$

$a = \sqrt{15}$, $b = \sqrt{10}$

Center, $C = (1, 3)$

$$\frac{X^2}{a^2} - \frac{Y^2}{b^2} = 1$$

Vertices: $(\pm a, 0)$

Vertices $= (1 \pm \sqrt{15}, 3)$

$X = \pm a$; $Y = 0$

$\Rightarrow x - 1 = \pm a$; $\Rightarrow y - 3 = 0$

$\Rightarrow x = 1 \pm a$; $\Rightarrow y = 3$

$x = 1 \pm \sqrt{15}$

$$e = \sqrt{1 + \frac{b^2}{a^2}}$$

$$= \sqrt{1 + \frac{10}{15}}$$

$$e = \sqrt{\frac{25}{15}} = \frac{\sqrt{5}}{\sqrt{3}} = \frac{\sqrt{15}}{3}$$

Foci: $(\pm ae, 0)$
$\quad X \quad Y$

$X = \pm ae$

$x - 1 = \pm \sqrt{15}\left(\frac{\sqrt{15}}{3}\right) = \pm\frac{15}{3} = \pm 5$

$x = 1 \pm 5 = -4, 6$

$Y = 0$

$y - 3 = 0$

$y = 3$

Foci: $(6, 3)$ and $(-4, 3)$

Eqn of directrices: $X = \pm \frac{a}{e}$

$$x-1 = \pm \frac{\sqrt{15}}{\sqrt{15}/3} = \pm 3$$

$$x = 1 \pm 3$$

$$x = 4, \quad x = -2$$

Eqn of asymptotes: $Y = \pm \frac{b}{a} X$

$$y-3 = \pm \frac{\sqrt{10}}{\sqrt{15}}(x-1)$$

$$y = 3 \pm \frac{\sqrt{2}}{\sqrt{3}}(x-1)$$

$$= 3 \pm \frac{\sqrt{6}}{3}(x-1)$$

$$y = \frac{\sqrt{6}}{3}(x-1)+3 \; ; \; y = -\frac{\sqrt{6}}{3}(x-1)+3$$

Eqn of latus rectum:

$$\underbrace{x=6 \quad \text{and} \quad x=-4}_{\text{Foci}}$$

Length of Latus rectrum: $\frac{2b^2}{a} = \frac{2(\sqrt{10})^2}{\sqrt{15}}$

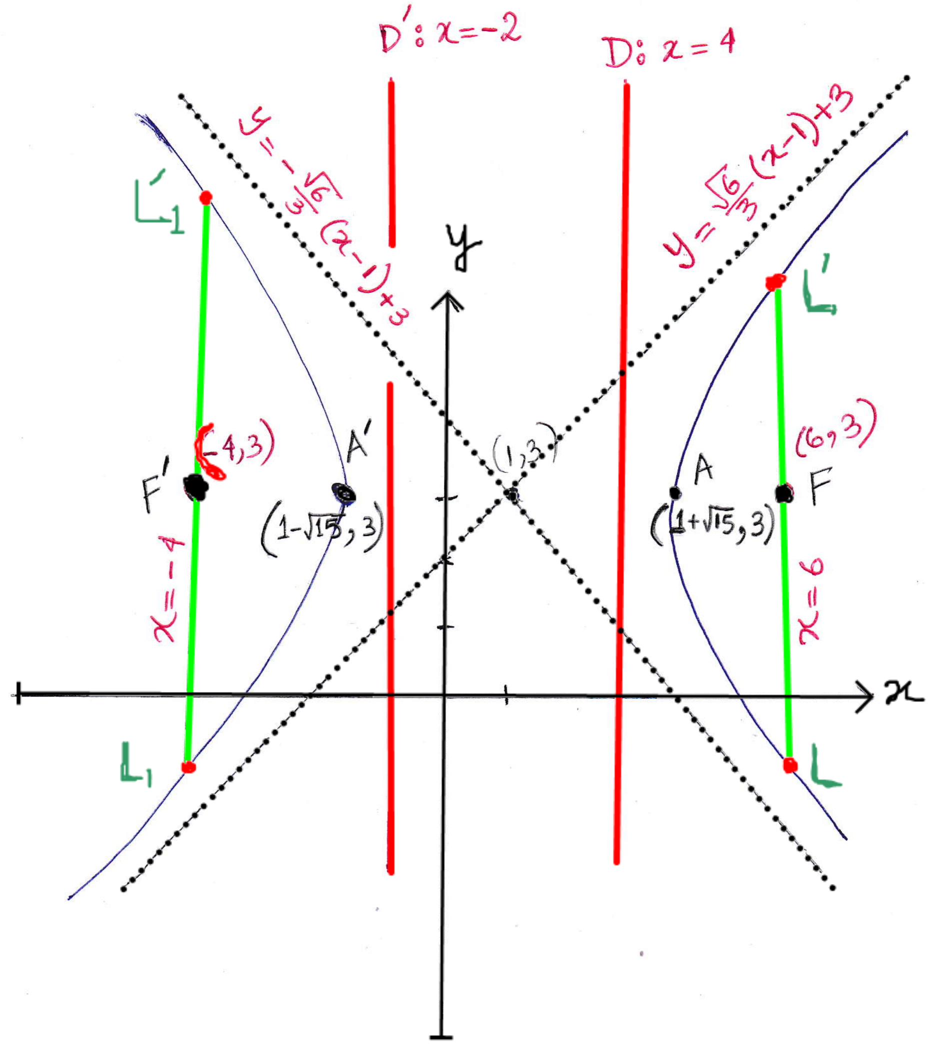
$$= \frac{20}{\sqrt{15}}$$

$$= \frac{20\sqrt{15}}{15}$$

$$= \frac{4\sqrt{15}}{3}$$

Length of tranverse axis: $2a = 2\sqrt{15}$

" " conjugate axis: $2b = 2\sqrt{10}$



D': x=-2
D: x=4
$y=-\frac{\sqrt{6}}{3}(x-1)+3$
$y=\frac{\sqrt{6}}{3}(x-1)+3$
y
x
L'_1
L'
L_1
L
(-4,3)
F'
A'
$(1-\sqrt{15},3)$
(1,3)
A
$(1+\sqrt{15},3)$
(6,3)
F
x=-4
x=6