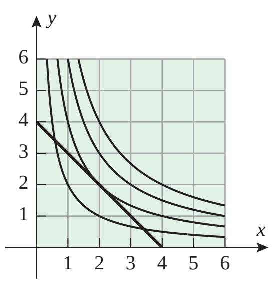
EXERCISE SET 13.9 Graphing Utility C CAS

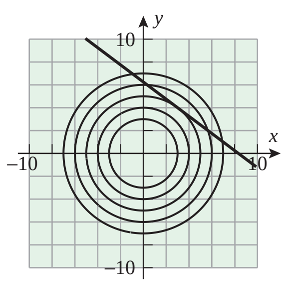
FOCUS ON CONCEPTS

1. The accompanying figure shows graphs of the line $x + y = 4$ and the level curves of height $c = 2, 4, 6$, and 8 for the function $f(x, y) = xy$.
(a) Use the figure to find the maximum value of the function $f(x, y) = xy$ subject to $x + y = 4$, and explain your reasoning.
(b) How can you tell from the figure that your answer to part (a) is not the minimum value of f subject to the constraint?
(c) Use Lagrange multipliers to check your work.

2. The accompanying figure shows the graphs of the line $3x + 4y = 25$ and the level curves of height $c = 9, 16, 25, 36$, and 49 for the function $f(x, y) = x^2 + y^2$.
(a) Use the accompanying figure to find the minimum value of the function $f(x, y) = x^2 + y^2$ subject to $3x + 4y = 25$, and explain your reasoning.
(b) How can you tell from the accompanying figure that your answer to part (a) is not the maximum value of f subject to the constraint?
(c) Use Lagrange multipliers to check your work.



▲ **Figure Ex-1**



▲ **Figure Ex-2**

3. (a) On a graphing utility, graph the circle $x^2 + y^2 = 25$ and two distinct level curves of $f(x, y) = x^2 - y$ that just touch the circle in a single point.
(b) Use the results you obtained in part (a) to approximate the maximum and minimum values of f subject to the constraint $x^2 + y^2 = 25$.
(c) Check your approximations in part (b) using Lagrange multipliers.

C **4.** (a) If you have a CAS with implicit plotting capability, use it to graph the circle $(x - 4)^2 + (y - 4)^2 = 4$ and two level curves of $f(x, y) = x^3 + y^3 - 3xy$ that just touch the circle.
(b) Use the result you obtained in part (a) to approximate the minimum value of f subject to the constraint $(x - 4)^2 + (y - 4)^2 = 4$.
(c) Confirm graphically that you have found a minimum and not a maximum.
(d) Check your approximation using Lagrange multipliers and solving the required equations numerically.

5–12 Use Lagrange multipliers to find the maximum and minimum values of f subject to the given constraint. Also, find the points at which these extreme values occur. ■

5. $f(x, y) = xy;\ 4x^2 + 8y^2 = 16$
6. $f(x, y) = x^2 - y^2;\ x^2 + y^2 = 25$
7. $f(x, y) = 4x^3 + y^2;\ 2x^2 + y^2 = 1$
8. $f(x, y) = x - 3y - 1;\ x^2 + 3y^2 = 16$
9. $f(x, y, z) = 2x + y - 2z;\ x^2 + y^2 + z^2 = 4$
10. $f(x, y, z) = 3x + 6y + 2z;\ 2x^2 + 4y^2 + z^2 = 70$
11. $f(x, y, z) = xyz;\ x^2 + y^2 + z^2 = 1$
12. $f(x, y, z) = x^4 + y^4 + z^4;\ x^2 + y^2 + z^2 = 1$

13–16 True–False Determine whether the statement is true or false. Explain your answer. ■

13. A “Lagrange multiplier” is a special type of gradient vector.
14. The extrema of $f(x, y)$ subject to the constraint $g(x, y) = 0$ occur at those points for which $\nabla f = \nabla g$.
15. In the method of Lagrange mutlipliers it is necessary to solve a constraint equation $g(x, y) = 0$ for y in terms of x.
16. The extrema of $f(x, y)$ subject to the constraint $g(x, y) = 0$ occur at those points at which a contour of f is tangent to the constraint curve $g(x, y) = 0$.

17–24 Solve using Lagrange multipliers. ■

17. Find the point on the line $2x - 4y = 3$ that is closest to the origin.
18. Find the point on the line $y = 2x + 3$ that is closest to $(4, 2)$.
19. Find the point on the plane $x + 2y + z = 1$ that is closest to the origin.
20. Find the point on the plane $4x + 3y + z = 2$ that is closest to $(1, -1, 1)$.
21. Find the points on the circle $x^2 + y^2 = 45$ that are closest to and farthest from $(1, 2)$.
22. Find the points on the surface $xy - z^2 = 1$ that are closest to the origin.
23. Find a vector in 3-space whose length is 5 and whose components have the largest possible sum.
24. Suppose that the temperature at a point (x, y) on a metal plate is $T(x, y) = 4x^2 - 4xy + y^2$. An ant, walking on the plate, traverses a circle of radius 5 centered at the origin. What are the highest and lowest temperatures encountered by the ant?

25–32 Use Lagrange multipliers to solve the indicated exercises from Section 13.8. ■

25. Exercise 38
26. Exercise 39