BRAC UNIVERSITY

Principles of Physics-I (PHY-111)

Department of Mathematics and Natural Sciences

Assignment: 02 — **Section**: 36

Duration: 7 Days | **Summer 2023 (UB41403)** | Marks: 25

Attempt all three questions. Show Your work in detail. 1:1 plagiarism will be penalized.

1. Astronaut Mark Watney is stranded on Mars with limited resources. He needs to get to his prearranged rescue location, which is 200 km away. To save time, he decides to ride a rover, which he estimates can travel at a maximum speed of $30\,\mathrm{km\,h^{-1}}$. [**Hint**: Use $g = 3.72\,\mathrm{m\,s^{-2}}$ for Mars.]

   (a) A projectile moves in a parabolic path without air resistance. Is there any point at which $\vec{a}$ is parallel to $\vec{v}$? Perpendicular to $\vec{v}$? Explain. **(1)**

   In a projectile motion, $a_x = 0$ and $a_y = -g$. a_y always acts downward. Since during the ascending and descending part of the trajectory, the velocity vector points upward and downward, respectively, nowhere can $\vec{v}$ be parallel to $\vec{a}$. However, $\vec{v}$ be perpendicular to $\vec{a}$ only at the maximum height because $\vec{v} = v_x$ in this location.

   (b) As Mark drives toward his destination, his rover hits an uneven rock, and the rover's engine stops abruptly. If the maximum deceleration of the rover is $5\,\mathrm{m\,s^{-2}}$, what is the minimum time it will take for the rover to come to a complete stop from its current speed of $30\,\mathrm{km\,h^{-1}}$? **(2)**

$$v_x = v_{x0} - a_x t \quad \text{(Deceleration)}$$

$$t = \frac{v_{x0}}{a_x} = \frac{8.33\,\mathrm{m\,s^{-1}}}{5\,\mathrm{m\,s^{-2}}} = 1.666\,\mathrm{s}.$$

   (c) After fixing the rover, Mark resumes his journey to the rescue location but realizes that he is running low on oxygen and needs to conserve it. He decides to drive the rover at a constant speed of $25\,\mathrm{km\,h^{-1}}$ to reduce oxygen consumption the next day. If Mark starts at 8 AM, Martian time, and wants to reach the rescue location by 3 PM, can he make it in time? **(2)**

$$d = 200\,\mathrm{km} = \bar{v}t = 6.94\,\mathrm{m\,s^{-1}} \times t$$

$$t = \frac{200\,\mathrm{km}}{6.94\,\mathrm{m\,s^{-1}}} = 28818.44\,\mathrm{s} \sim 8\,\mathrm{h}.$$

   Since this is an hour short than Mark's targeted 'seven hour' time to reach the rescue location, he won't make it.

2. A teenage kid tosses a ball vertically upwards with an initial speed of $12\,\mathrm{m\,s^{-1}}$.

   (a) Can You move in a particular direction while acceleration acts in the opposite way? Provide an example. **(1)**

   A projectile would be a good example. As a projectile ascends up to a maximum height, the acceleration points in the downward direction. Similarly, any decelerating scenario can be picked as an example here.

   (b) How long does the ball take to reach its maximum height? **(2)**

$$t_H = \frac{v_{0y}}{g} = \frac{12\,\mathrm{m\,s^{-1}}}{9.81\,\mathrm{m\,s^{-2}}} = 1.22\,\mathrm{s}.$$

(c) Find the maximum height of that ball. Now the kid throws the same ball vertically upwards from a platform of height 5 m. Compare the maximum height found in this scenario with the previous one. Find their ratio. **(3)**

When the ball is thrown from the ground

$$H_{\text{ground}} = \frac{v_{0y}^2}{2g} = \frac{(12\,\text{m}\,\text{s}^{-1})^2}{2 \times 9.81\,\text{m}\,\text{s}^{-2}} = 7.34\,\text{m}.$$

When the ball is thrown from a platform $y_0 = 5\,\text{m}$

$$H_{\text{platform}} = y_0 + \frac{v_{0y}^2}{2g} = 5\,\text{m} + \frac{(12\,\text{m}\,\text{s}^{-1})^2}{2 \times 9.81\,\text{m}\,\text{s}^{-2}} = 12.34\,\text{m}$$

$H_{\text{platform}} : H_{\text{ground}} = 12.34 : 7.34 = 1 : 0.6.$

(d) The kid now throws the same ball with the same speed from the same platform, making an angle of 35° upward with the horizontal. How long will the ball stay above the ground, and how far ahead will it hit from the platform's base? **(4)**

Use the 3rd law of motion

$$y - y_0 = v_0 \sin\alpha_0 t - \frac{1}{2} g t^2$$

$$0 - y_0 = v_0 \sin\alpha_0 t_{\text{flight}} - \frac{1}{2} g t_{\text{flight}}^2$$

$$\frac{1}{2} g t_{\text{flight}}^2 - v_0 \sin\alpha_0 t_{\text{flight}} - y_0 = 0$$

$$\frac{1}{2} \times 9.81\,\text{m}\,\text{s}^{-2} t_{\text{flight}}^2 - (12\,\text{m}\,\text{s}^{-1} \sin 35°) t_{\text{flight}} - 5\,\text{m} = 0$$

$$t_{\text{flight}} = 1.932\,\text{s}$$

The horizontal distance is

$$\begin{aligned} d = x - x_0 &= v_0 \cos\alpha_0 t_{\text{flight}} \\ &= (12\,\text{m}\,\text{s}^{-1}) \cos 35° \times 1.932\,\text{s} \\ &= 18.99\,\text{m}. \end{aligned}$$

3. A faulty model rocket moves in the xy-plane (the positive y-direction is vertically upward). The rocket's acceleration has components $a_x(t) = \alpha t^2$ and $a_y(t) = \beta - \gamma t$, where $\alpha = 2.50\,\text{m}\,\text{s}^{-4}$, $\beta = 9.00\,\text{m}\,\text{s}^{-2}$, and $\gamma = 1.40\,\text{m}\,\text{s}^{-3}$. At $t = 0$, the rocket is at the origin and has velocity $\vec{v}$ with $v_{0x} = 1.00\,\text{m}\,\text{s}^{-1}$ and $v_{0y} = 7.00\,\text{m}\,\text{s}^{-1}$.

(a) When you drop an object from a certain height, it takes time T to reach the ground without air resistance. If you dropped it from three times that height, how long (in terms of T) would it take to reach the ground? **(1)**

According to the 2nd law of motion, the distance covered is proportional to the square of the time elapsed. $H = y - y_0 \propto T^2$. This can also be restated as $T \propto \sqrt{H}$. If we make this height three times larger, the time required becomes $t = \sqrt{3}T$.

(b) Calculate the velocity and position vectors as functions of time. **(2)**

$$v_x(t) = \int a_x(t) = v_{0x} + \frac{\alpha}{3} t^3 = 1.00\,\text{m}\,\text{s}^{-1} + \left(\frac{2.50\,\text{m}\,\text{s}^{-4}}{3}\right) t^3$$

$$v_y(t) = \int a_y(t) = v_{0y} + \beta t - \frac{\gamma}{2} t^2 = 7.00\,\text{m}\,\text{s}^{-1} + (9.00\,\text{m}\,\text{s}^{-2}) t - \left(\frac{1.40\,\text{m}\,\text{s}^{-3}}{2}\right) t^2$$

$$x(t) = \int v_x(t) = v_{0x} t + \frac{\alpha}{12} t^4 = (1.00\,\text{m}\,\text{s}^{-1}) t + \left(\frac{2.50\,\text{m}\,\text{s}^{-4}}{12}\right) t^4$$

$$y(t) = \int v_y(t) = v_{0y} t + \frac{\beta}{2} t^2 - \frac{\gamma}{6} t^3 = (7.00\,\text{m}\,\text{s}^{-1}) t + \left(\frac{9.00\,\text{m}\,\text{s}^{-2}}{2}\right) t^2 - \left(\frac{1.40\,\text{m}\,\text{s}^{-3}}{6}\right) t^3$$

Here, x_0 and y_0 are both set to zero.

(c) What is the maximum height reached by the rocket? **(3)**

One can approach this by setting $v_y = 0$ and finding t from there to substitute it into $y_(t)$. That is the expression of the maximum height.

$$v_y = \left(\frac{1.40\,\mathrm{m\,s^{-3}}}{2}\right) t^2 - (9.00\,\mathrm{m\,s^{-2}})t - 7.00\,\mathrm{m\,s^{-1}} = 0$$
$$t_H = 13.59\,\mathrm{s}$$

Plug this into $y(t)$

$$H = (7.00\,\mathrm{m\,s^{-1}}) \times 13.59\,\mathrm{s} + \left(\frac{9.00\,\mathrm{m\,s^{-2}}}{2}\right) \times (13.59\,\mathrm{s})^2 - \left(\frac{1.40\,\mathrm{m\,s^{-3}}}{6}\right) \times (13.59\,\mathrm{s})^3$$
$$\therefore H = 340.58\,\mathrm{m}.$$

(d) What is the horizontal displacement of the rocket when it returns to $y = 0$? **(4)**

Set $y(t) = 0$ first to find t, and substitute it into $x(t)$.

$$y(t) = (7.00\,\mathrm{m\,s^{-1}})t + \left(\frac{9.00\,\mathrm{m\,s^{-2}}}{2}\right) t^2 - \left(\frac{1.40\,\mathrm{m\,s^{-3}}}{6}\right) t^3 = 0$$
$$t_f = 20.73\,\mathrm{s}.$$

Plug this into $x(t)$

$$x(t) = (1.00\,\mathrm{m\,s^{-1}}) \times (20.73\,\mathrm{s}) + \left(\frac{2.50\,\mathrm{m\,s^{-4}}}{12}\right) \times (20.73\,\mathrm{s})^4$$
$$= 38493.72\,\mathrm{m}$$
$$\therefore x \sim 38.5\mathrm{km}.$$