Lecture 14

Conic Sections:

General equation of Second degree

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0 \quad \text{——— (1)}$$

* Intersection of a plane with a double-napped circular cone represents two types of conic sections.

(1) Degenerate conic sections

(2) Non-degenerate conic sections.

Non-degenerate form is one for which the associated matrix is non-singular i.e. $\det([A]) \neq 0$.

Non-degenerate conic sections or only conic sections represents circle, parabola, ellipse and hyperbola.

Conditions: (1) $\Delta \neq 0$, $B^2 - 4AC = 0$ parabola

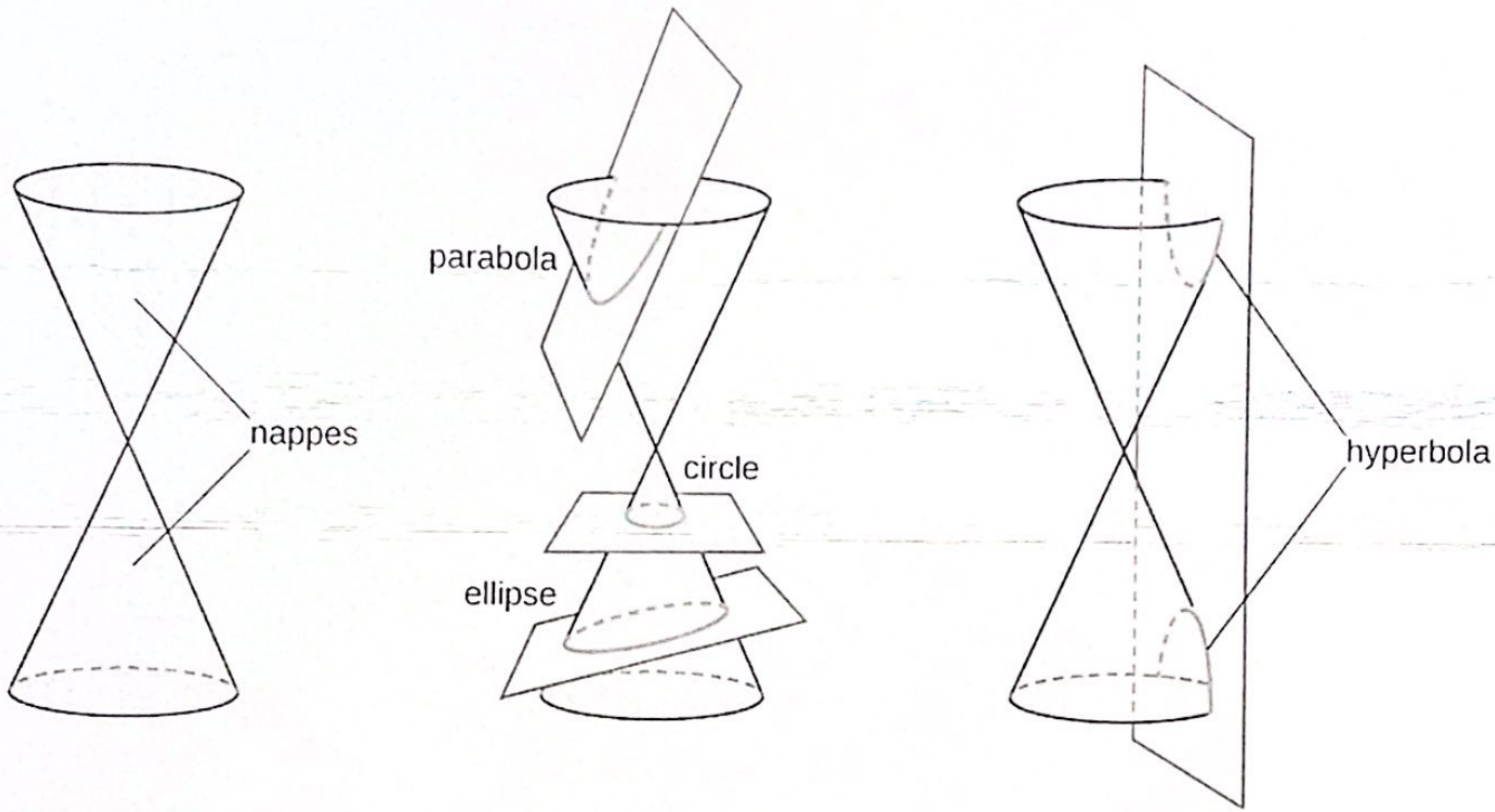
(2) $\Delta \neq 0$, $B^2 - 4AC < 0$ Ellipse

(3) $\Delta \neq 0$, $B^2 - 4AC > 0$ Hyperbola

(4) $\left.\begin{array}{l} \Delta \neq 0,\ B^2 - 4AC < 0 \\ A = C \\ B = 0 \end{array}\right\}$ Circle

where

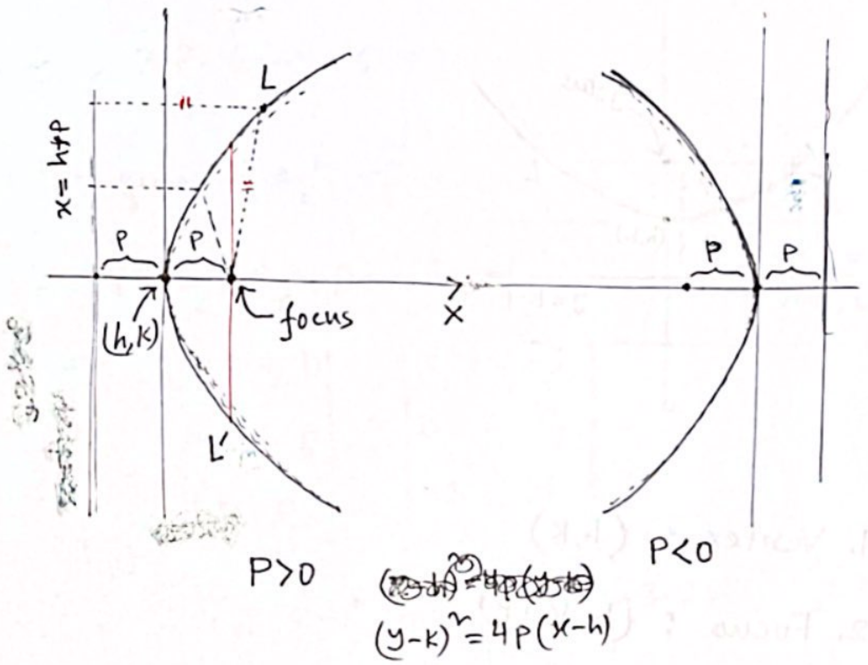
$$\Delta = \begin{vmatrix} A & \frac{1}{2}B & \frac{1}{2}D \\ \frac{1}{2}B & C & \frac{1}{2}E \\ \frac{1}{2}D & \frac{1}{2}E & F \end{vmatrix}$$


nappes
parabola
circle
ellipse
hyperbola

Parabola

Equation of a parabola ~~at~~ centered at (h,k) (Standard form):

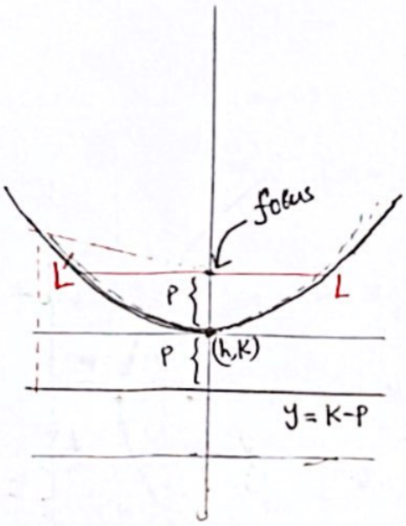
$$\boxed{(y-k)^2 = 4p(x-h)}$$



1. Vertex : (h,k)
2. Focus : $(h+p, k)$
3. Eqn. of directrix, $x = h-p$
4. Length of Latus rectum : $|4p| = LL'$
5. Eqn of latus rectum : $x = h+p$
6. eccentricity : $e = 1$

Standard forms of a parabola centered at (h,k):

$$(x-h)^2 = 4p(y-k)$$



1. Vertex : (h,k)

2. Focus : $(h, k+p)$

3. Eqn of directrix : $y = k-p$

4. Length of latus rectum: $LL' = |4p|$

5. Eqn of latus rectum : $y = k+p$

Example: Transform the conics $3x^2-6x-y+5=0$ into its standard form and find its vertices, ecentricity, foci, equation of directrices, equation of latus rectum, length of latus rectum.

Solⁿ: Given the conics

$$3x^2-6x-y+5=0$$

We know the general eqⁿ. of second degree.

$$Ax^2+Bxy+Cy^2+Dx+Ey+F=0 \quad , \quad \begin{array}{l} A=3, B=0, C=0 \\ D=-6, E=-1, F=5 \end{array}$$

$$\Delta = \begin{vmatrix} A & \frac{1}{2}B & \frac{1}{2}D \\ \frac{1}{2}B & C & \frac{1}{2}E \\ \frac{1}{2}D & \frac{1}{2}E & F \end{vmatrix} = \begin{vmatrix} 3 & 0 & -3 \\ 0 & 0 & -\frac{1}{2} \\ -3 & -\frac{1}{2} & 5 \end{vmatrix}$$

$$= 3(-\tfrac{1}{4})-0-3.0 = -\tfrac{3}{4} \neq 0$$

Now, $B^2-4AC = 0-4.3.0 = 0$

So, the conics represents parabola.

Therefore,

$$3x^2-6x-y+5=0$$
$$x^2-2x=\tfrac{1}{3}(y-5)$$
$$x^2-2x+1=\tfrac{1}{3}(y-2)$$
$$(x-1)^2=\tfrac{1}{3}(y-2)$$
$$(x-1)^2=4.\tfrac{1}{12}(y-2)$$

$$(x-h)^2=4.P(y-K)$$
$$h=1, \; K=2, \; P=\tfrac{1}{12}$$

① vertex : $(1,2)$

② Focus : $(h, K+P)=(1, 2+\frac{1}{12})=(1, \frac{25}{12})$

③ Eqⁿ of directrix: $y=K-P \Rightarrow y = 2-\frac{1}{12}=\frac{23}{12}$ i.e. $y-\frac{23}{12}=0$.

④ Length of latus rectum : $LL' = |4P| = |4.\frac{1}{12}| = \frac{1}{3}$.