(Chapter - 6)

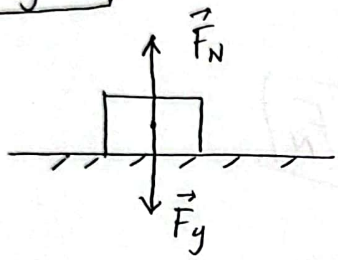
"Force and Motion - II"

Friction

When a force $\vec{F}$ tends to slide a body along a surface, a frictional force from the surface acts on the body. The frictional force is parallel to the surface and directed so as to oppose the sliding.

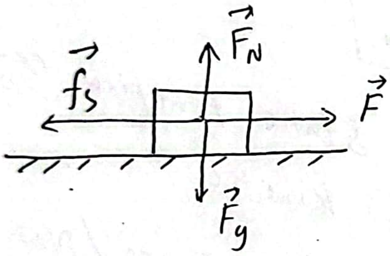
see - Fig 6-1

(a)



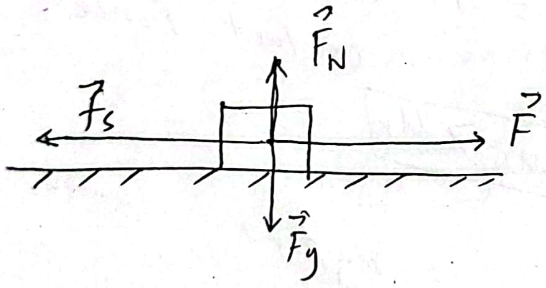
For no motion, and No applied Force → then → Frictional Force = 0

(b)



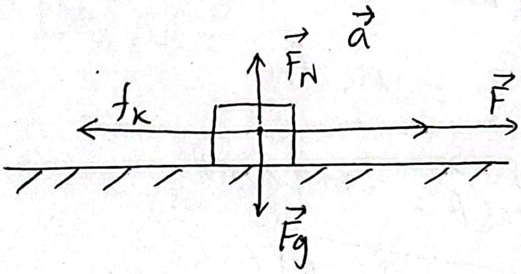
For applied force, F And No motion → then → Magnitude of Frictional Force and applied Force are equal

(C)

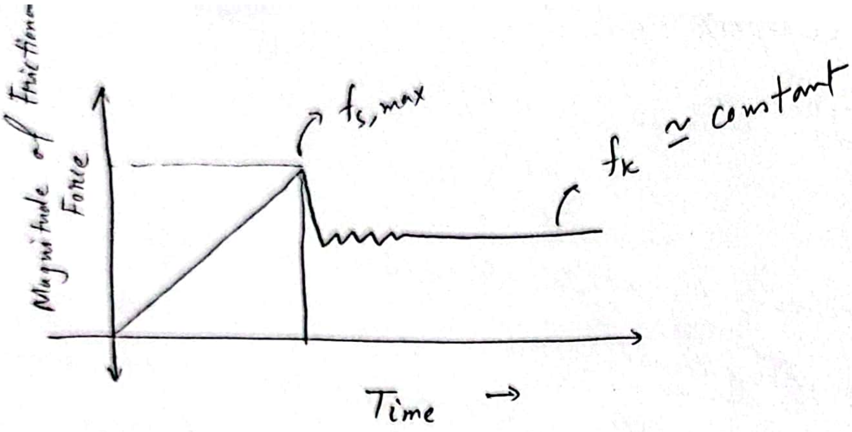


Now, applied Force increased but no motion → Then → Magnitude of Frictional Force and applied Force are equal

(d)



applied Force is greater than maximum static Friction, then : has motion → then → Frictional Force, is Kinetic Frictional Force



Two types of Friction,

i) Static Friction → when $\boxed{\text{no motion}}$

ii) kinetic Friction → when $\boxed{\text{has motion}}$

Maximum Static friction, $\boxed{f_{s,max} = \mu_s F_N}$

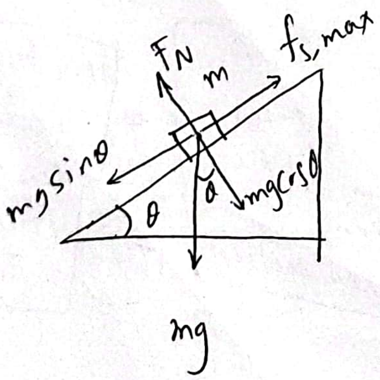
Kinetic Friction, $\boxed{f_k = \mu_k F_N}$

μ_s = Static Friction co-efficient

μ_k = Kinetic " "

F_N = Normal Force/Normal contact Force

$\boxed{\mu_s > \mu_k}$



$f_{s,max} = mg\sin\theta$ —(1)

$f_{s,max} = \mu_s F_N = \mu_s mg\cos\theta$ —(2)

(1) and (2) =>

$\therefore \mu_s mg\cos\theta = mg\sin\theta$

$\Rightarrow \boxed{\mu_s = \tan\theta}$

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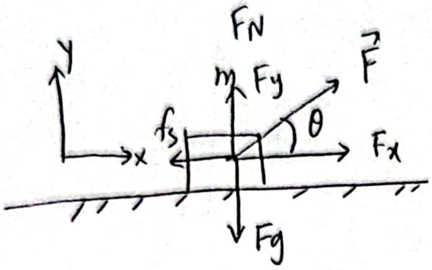
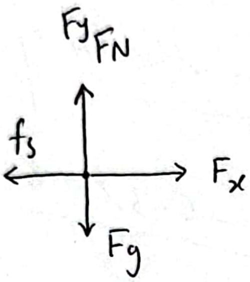


$F = 0.5\,mg$, $\theta = 20°$

(a) $\mu_s = 0.6$, $\mu_k = 0.5$, $a = ?$

Applying Newton's 2nd law along y-axis,



$$F_y + F_N - F_g = m(0)$$

$$\Rightarrow F\sin\theta + F_N - mg = 0$$

$$\Rightarrow F_N = mg - F\sin\theta$$

$$= mg - 0.5\,mg\,\sin(20°)$$

$$\Rightarrow F_N = 0.83\,mg \quad \text{—(1)}$$

$$f_{s,max} = \mu_s F_N$$

$$= 0.6 \times 0.83\,mg$$

$$= 0.498\,mg$$

$$F_x = F\cos\theta = 0.5\,mg\cos(20°) = 0.47\,mg < f_{s,max}$$

So, $\boxed{a = 0}\ m/s^2$

(b) $\mu_s = 0.4$, $\mu_k = 0.3$, $a = ?$

In this case,

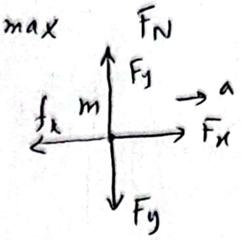
$$f_{s,max} = \mu_s F_N \qquad (F_N \text{ From eq}^n\ 1)$$

$$= 0.4 \times 0.83\,mg$$

$$= 0.332\,mg$$

$F_x = F\cos\theta = 0.5\, mg\cos(20^\circ)$

$= 0.47\, mg > f_{s,max}$



So, block will move.

Applying Newton's 2nd Law

at x-axis, $F_x - f_x = ma$

$$\Rightarrow a = \frac{F_x - f_x}{m}$$

$$= \left[\frac{0.47\, mg - \mu_k F_N}{m}\right] m/s^2$$

$$= \left[\frac{0.47\, mg - 0.3 \times 0.83}{m}\right] m/s^2$$

$$= \left[\frac{0.47\, mg - 0.25\, mg}{m}\right] m/s^2$$

$$= \frac{(0.47 - 0.25)\cancel{m}g}{\cancel{m}} \; m/s^2$$

$$= 0.221 \times 9.8 \; m/s^2$$

$$= \boxed{2.16\; m/s^2}$$

Chapter - 6

SP: 6.01, 6.02, 6.04, 6.05, 6.06

QP: 5, 10*, 16, 23*, 27, 47,
57*, 70, 88

CP: 1, 2

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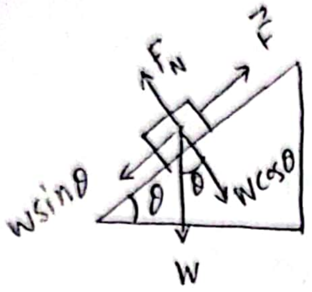


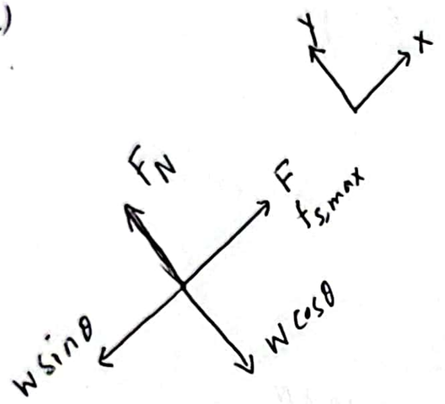
$W = 80N$

$\theta = 20^\circ$

$\mu_s = 0.25$

$\mu_k = 0.15$

(a)



Applying Newton's 2nd Law along x and y axis, respectively,

$$F + f_{s,max} - W\sin\theta = 0 \quad \text{—(1)}$$

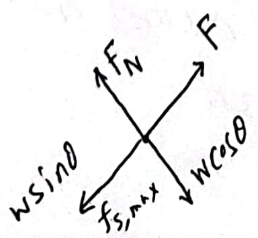
$$F_N - W\cos\theta = 0 \quad \text{—(2)}$$

$$\Rightarrow F_N = W\cos\theta$$
$$= 80\cos(20^\circ)\ N$$
$$= 75.18N \quad \text{—(3)}$$

From (1), $F = W\sin\theta - f_{s,max}$

$$= 80\sin(20^\circ) - \mu_s F_N$$
$$= 27.36\ N - 0.25 \times 75.18\ N$$
$$= (27.36 - 18.80)N$$
$$= \boxed{8.56\ N}$$

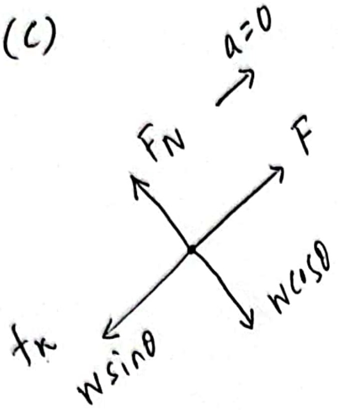
(b)



$$F - f_{s,max} - W\sin\theta = 0$$

$$\Rightarrow F = f_{s,max} + W\sin\theta$$
$$= (18.80 + 27.36)N = \boxed{46.16N}$$

(c)



$$F - f_k - w\sin\theta = 0$$

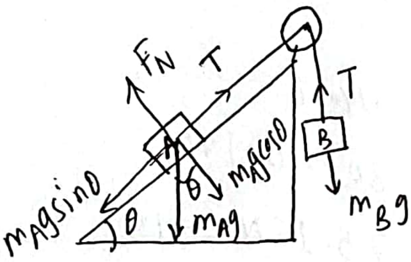
$$\Rightarrow F = f_k + w\sin\theta$$

$$= \mu_k F_N + w\sin\theta$$

$$= [0.15 \times 75.18 + 80 \times \sin(20^\circ)]\ N$$

$$= \boxed{38.64N}$$

$\boxed{27}$



$m_A g = 102N$

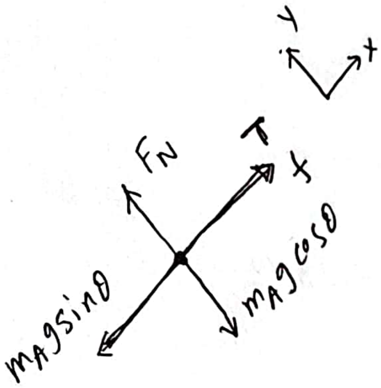
$m_B g = 32N$

$\mu_s = 0.56$

$\mu_k = 0.25$

$\theta = 40^\circ$

(a)



Applying Newton's 2nd Law along x and y axis respectively,

$$T + f - m_A g\sin\theta = 0 \quad -(1)$$

$$F_N - m_A g\cos\theta = 0$$

$$\Rightarrow F_N = m_A g\cos\theta$$

$$= 102 \times \cos(40^\circ)\ N$$

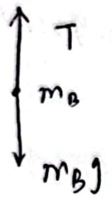
$$= 78.13N$$

Now,

$$f_{s,max} = \mu_s F_N$$

$$= 0.56 \times 78.13 \text{ N}$$

$$= 43.78 \text{N}$$



$$T - m_B g = 0$$

$$\Rightarrow T = m_B g = 32 \text{ N}$$

From equation, (1) $\quad f = m_A g \sin\theta - T$

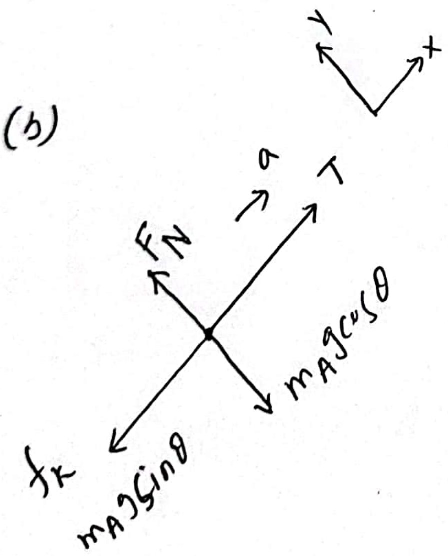
$$= 102 \times \sin(40^\circ)\text{N} - 32\text{N}$$

$$= 65.57\text{N} - 32\text{N}$$

$$= 33.57\text{N} < f_{s,max}$$

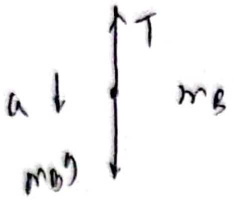
So, acceleration $\boxed{a = 0 \ m/s^2}$

(b)



$$T - f_k - m_A g \sin\theta = m_A a \quad \text{—(3)}$$

$$\Rightarrow a = \frac{T - f_k - m_A g \sin\theta}{m_A}$$



$$m_B g - T = m_B a$$

$$\Rightarrow T = m_B g - m_B a \quad —(4)$$

Putting (4) into (3),

$$m_B g - m_B a - \mu_k F_N - m_A g \sin\theta = m_A a$$

$$\Rightarrow m_B g - \mu_k F_N - m_A g \sin\theta = m_A a + m_B a$$

$$\Rightarrow a = \frac{m_B g - \mu_k F_N - m_A g \sin\theta}{m_A + m_B}$$

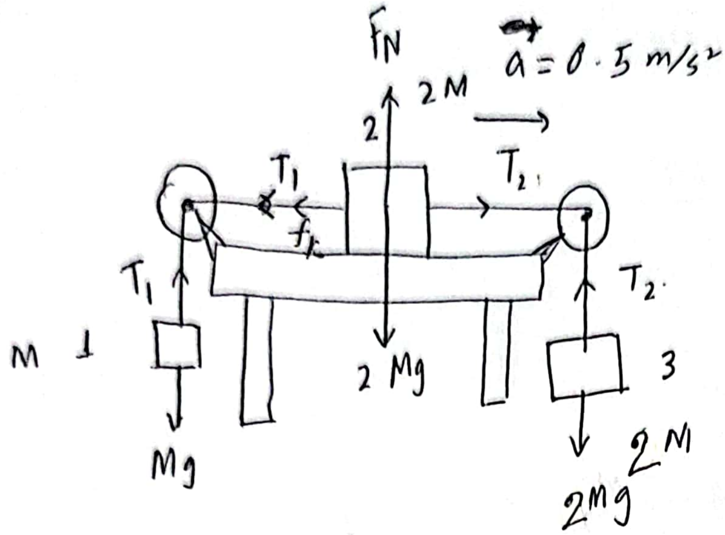
$$= \frac{32 - 6.12 - 102 \sin(40^\circ)}{\left(\frac{102 + 32}{9.8}\right)}$$

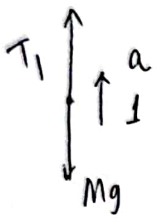
$$= -3.9 \, m/s^2$$

$$\boxed{\vec{a} = -3.9 \, (m/s^2) \, \hat{i}}$$

(c) Do it yourself

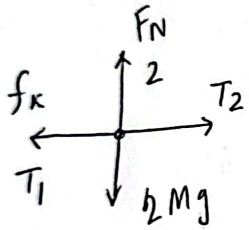
[2]





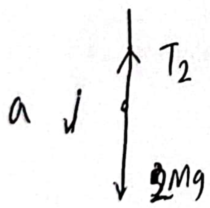
$$T_1 - Mg = Ma$$

$$\Rightarrow \quad T_1 = Ma + Mg \quad —(1)$$



$$F_N = 2Mg \quad —(2)$$

$$T_2 - T_1 - f_k = 2Ma \quad —(3)$$



$$3Mg - T_2 = 2Ma$$

$$\Rightarrow \quad T_2 = 2Mg - 2Ma \quad —(4)$$

Putting (1), (4) into (3),

$$2Mg - 2Ma - Ma - Mg - \mu_k 2Mg = 2Ma$$

$$\Rightarrow \quad \mu_k 2Mg = 2Mg - 2Ma - Ma - Mg - 2Ma$$

$$\Rightarrow \quad \mu_k 2Mg = Mg - 5Ma$$

$$\boxed{\begin{aligned} \mu_k &= \frac{2Mg - 6Ma}{2Mg} \\ &= \frac{\cancel{M}(2g-6a)}{2\cancel{M}g} \\ &= \frac{g-3a}{g} = \frac{9.8 - 3\times 0.5}{9.8} \end{aligned}}$$

$$\Rightarrow \mu_k = \frac{Mg - 5Ma}{2Mg} \Rightarrow \frac{g-5a}{2g} = \frac{9.8 - 5\times 0.5}{2\times 9.8}$$

$$= \boxed{0.372}$$

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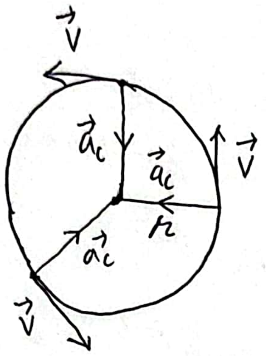
"Uniform Circular Motion"



In uniform circular motion,
$|\vec{v}| = \text{constant}, \quad r = \text{constant}$

Centripetal acceleration, $a_c = \dfrac{v^2}{r}$

In unit vector notation, $\vec{a}_c = \dfrac{v^2}{r}(-\hat{r})$
$= -\dfrac{v^2}{r}\hat{r}$

From Newton's 2nd Law, $\vec{F}_c = m\vec{a}_c$

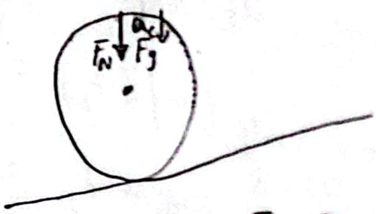
$\Rightarrow \vec{F}_c = -m\dfrac{v^2}{r}\hat{r}$

$$\boxed{F_c = m\frac{v^2}{r}}$$

centripetal force → it can be gravitational force, frictional force, tension force of string, or any other force.

See : Fig 6-8

Sample problem 6.04



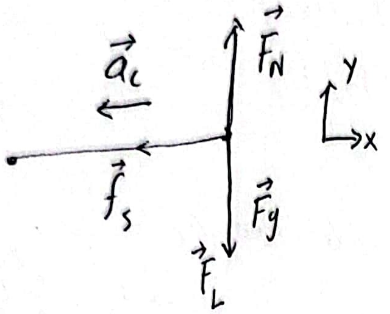
$R = 2.7$ m
least speed $v = ?$

$F_N + F_g = m\dfrac{v^2}{R}$

For least speed → $\Rightarrow \cancel{F_N}^{0} + mg = m\dfrac{v^2}{R}$

$\Rightarrow v = \sqrt{gR} = \sqrt{(9.8)(2.7)}$ m
$= \boxed{5.1 \text{ m}}$

Sample problem: 6.04.



$m = 600\ kg$

$R = 100\ m$

$\mu_s = 0.75$

$V = 28.6\ m/s$

$F_L = ?$

(a)

Applying Newton's 2nd Law along x-axis,

$$-f_s = -ma$$

$$\Rightarrow f_s = m\frac{v^2}{R} \quad \text{--- (1)}$$

Applying Newton's 2nd Law along y-axis,

$$F_N - F_g - F_L = 0$$

$$\Rightarrow F_N = mg + F_L \quad \text{--- (2)}$$

From equation (1),

$$\Rightarrow \mu_s F_N = ma$$

$$\Rightarrow \mu_s (mg + F_L) = m\frac{v^2}{R}$$

$$\Rightarrow mg + F_L = \frac{mv^2}{\mu_s R}$$

$$\Rightarrow F_L = m\frac{v^2}{\mu_s R} - mg$$

$$= m\left(\frac{v^2}{\mu_s R} - g\right)$$

$$= 600 \times \left(\frac{28.6^2}{0.75 \times 100} - 9.8\right) N$$

$\therefore F_L = 663.7N \simeq \boxed{660N}$

(b) $F_L \sim v^2$

$V_1 = 28.6$ m/s

$V_2 = 90$ m/s

$F_{L_1} = 663.7N$

$F_{L_2} = ?$

$$\therefore \frac{F_{L_1}}{F_{L_2}} = \frac{V_1^2}{V_2^2}$$

$$\Rightarrow F_{L_2} = \frac{V_2^2}{V_1^2} \times F_{L_1}$$

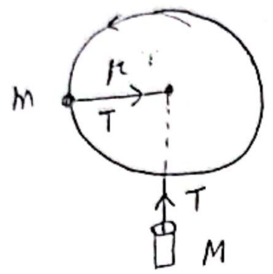
$$= \frac{(90)^2}{(28.6)^2} \times (663.N)$$

$$= 6572\,N \simeq \boxed{6600\,N}$$

See: Sample problem 6.06

Banking $\boxed{\tan\theta = \frac{v^2}{Rg}}$

(57)



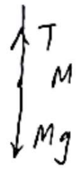
$m = 1.5$ kg

$r = 20.$ cm $= 0.2$ m

$M = 2.5$ kg , $v = ?$

For mass m, applying Newton's 2nd Law,

$$T = m\frac{v^2}{r} \quad —(1)$$



For mass M, applying Newton's 2nd Law,

$$T - Mg = 0$$

$$\Rightarrow T = Mg \quad —(2)$$

Combining equation (1) and (2),

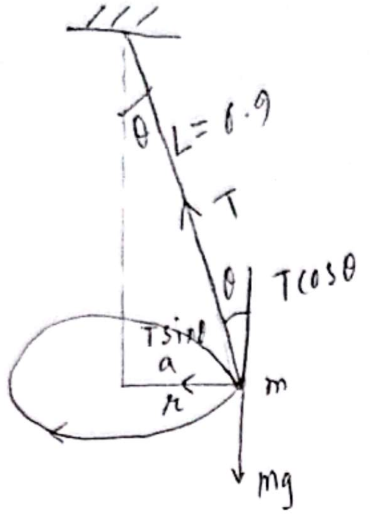
$$m\frac{v^2}{r} = Mg$$

$$\Rightarrow v = \sqrt{\frac{Mgr}{m}}$$

$$= \sqrt{\frac{2.5 \times 9.8 \times 0.2}{1.5}} \text{ m/s}$$

$$= \boxed{1.81 \text{ m/s}}$$

10



$L = 0.9\ m$

$m = 0.04\ kg$

$2\pi r = 0.94\ m$

$\Rightarrow r = \frac{0.94}{2\pi}\ m$

$= 0.15\ m$

(a) $T = ?$ (b) time period of the motion, $t = ?$

$\sin\theta = \frac{r}{L}$

$\Rightarrow \theta = \sin^{-1}\left(\frac{0.15}{0.9}\right)$

$\simeq 9.59°$

(a) $T\cos\theta = mg$

$\Rightarrow T = \frac{mg}{\cos\theta} = \frac{0.04 \times 9.8}{\cos(9.59°)} = \boxed{0.4\ N}$

(b) $T\sin\theta = m\frac{v^2}{r}$

$\Rightarrow v = \sqrt{\frac{(T\sin\theta)\, r}{m}}$

$= \sqrt{\frac{0.4 \times \sin(9.59°) \times 0.15}{0.04}}$

$= 0.5\ m/s$

time period $\Rightarrow$ $t = \frac{2\pi r}{v} = \frac{0.94}{0.5}$ sec

$= \boxed{1.88\ sec}$

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