Mat110:Differential Calculus and Co-ordinate Geometry
Assignment 02

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Introduction

Name: Anika Ebnat Binty
Student ID: 20301075
Set No: E
Section: 14

Brac University

Answer to the question no: 01 (a)

Find the derivative with respect to x of:

$$y = \frac{1}{\sqrt{2x^2 + 5x}}$$

$$\text{Now,} \frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{\sqrt{2x^2 + 5x}}\right)$$

$$= \frac{d}{dx}[(2x^2 + 5x)^{-1/2}]$$

$$= -\frac{1}{2}(2x^2 + 5x)^{-3/2} \frac{d}{dx}(2x^2 + 5x)$$

$$= -\frac{1}{2}(2x^2 + 5x)^{-3/2}(4x + 5)$$

$$= -\frac{1}{2(2x^2 + 5x)^{3/2}}(4x + 5)$$

$$= -\frac{(4x + 5)}{2(2x^2 + 5x)^{3/2}}$$

(Ans)

Answer to the question no: 01 (b)

Find the derivative with respect to x of:

$$y = x^3 sin\frac{1}{x^2}$$

$$\text{Now,} \frac{dy}{dx} = \frac{d}{dx}\left(x^3 sin\frac{1}{x^2}\right)$$

$$= \left(sin\frac{1}{x^2}\right)\frac{d}{dx}(x^3) + x^3 \frac{d}{dx}\left(sin\frac{1}{x^2}\right)$$

$$= \left(sin\frac{1}{x^2}\right)3x^2 + x^3\left(cos\frac{1}{x^2}\right)\frac{d}{dx}\left(\frac{1}{x^2}\right)$$

$$= (sin\frac{1}{x^2})3x^2 + x^3(cos\frac{1}{x^2})(-2)x^{(-2-1)}$$

$$= (sin\frac{1}{x^2})3x^2 - 2x^3(cos\frac{1}{x^2})x^{-3}$$

$$= (sin\frac{1}{x^2})3x^2 - 2(cos\frac{1}{x^2})$$

(Ans)

Answer to the question no: 2

Find the first derivative of:

$y = ln(x^a + x^{-a})$

Now, $\frac{dy}{dx} = \frac{d}{dx}ln(x^a + x^{-a})$

$$= \frac{1}{(x^a + x^{-a})}\frac{d}{dx}(x^a + x^{-a})$$

$$= \frac{1}{(x^a + x^{-a})}(ax^{a-1} - ax^{-a-1})$$

$$= \frac{(ax^{a-1} - ax^{-a-1})}{(x^a + x^{-a})}$$

(Ans)

Answer to the question no: 03 (a)

Use the sum, product and quotient rule to find the derivative of the following functions:

$f(v) = e^v sinv$

$f_1(v) = \frac{d}{dv}(e^v sinv)$

$$= sinv\frac{d}{dv}e^v + e^v\frac{d}{dv}sinv$$

$$= e^v sinv + e^v cosv$$

$$= e^v(sinv + cosv)$$

(Ans)

<u>Answer to the question no: 03 (b)</u>

$$f(x) = \frac{cosx}{1 + 2sinx}$$

$$f_1(x) = \frac{d}{dx}(\frac{cosx}{1 + 2sinx})$$

$$= \frac{(1 + 2sinx)\frac{d}{dx}cosx - cosx\frac{d}{dx}(1 + 2sinx)}{(1 + 2sinx)^2}$$

$$= \frac{(1 + 2sinx)(-sinx) - cosx2cosx}{(1 + 2sinx)^2}$$

$$= \frac{-sinx(1 + 2sinx) - 2cos^2x}{(1 + 2sinx)^2}$$

$$= \frac{-sinx - 2sin^2x - 2cos^2x}{(1 + 2sinx)^2}$$

$$= \frac{-sinx - 2(sin^2x + cos^2x)}{(1 + 2sinx)^2}$$

$$= \frac{-sinx - 2}{(1 + 2sinx)^2}$$

(Ans)

Answer to the question no: 03 (c)

$$f(x) = \frac{e^x lnx}{(x^2+2x^3)}$$

$$f_1(x) = \frac{d}{dx}\frac{e^x lnx}{(x^2+2x^3)}$$

$$= \frac{(x^2+2x^3)\frac{d}{dx}(e^x lnx) - e^x lnx \frac{d}{dx}(x^2+2x^3)}{(x^2+2x^3)^2}$$

$$= \frac{(x^2+2x^3)(e^x \frac{d}{dx} lnx + lnx \frac{d}{dx} e^x) - e^x lnx(2x+6x^2)}{(x^2+2x^3)^2}$$

$$= \frac{(x^2+2x^3)(e^x \frac{1}{x} + e^x lnx) - e^x lnx(2x+6x^2)}{(x^2+2x^3)^2}$$

$$= \frac{(x^2+2x^3)(\frac{e^x}{x} + e^x lnx) - e^x lnx(2x+6x^2)}{(x^2+2x^3)^2}$$

$$= \frac{x^2\frac{e^x}{x} + x^2e^x lnx + 2x^3\frac{e^x}{x} + 2x^3e^x lnx - e^x lnx2x - e^x lnx6x^2}{(x^2+2x^3)^2}$$

$$= \frac{xe^x + x^2e^x lnx + 2x^2e^x + x^3e^x ln(x)^2 + xe^x ln(x)^2 - x^2e^x ln(x)^6}{(x^2+2x^3)^2}$$

$$= \frac{xe^x + x^2e^x lnx + 2x^2e^x - x^2e^x ln(x)^6 - xe^x ln(x)^2 + x^3e^x ln(x)^2}{(x^2+2x^3)^2}$$

(Ans)

Answer to the question no: 04

Find the derivative of following function:
$g(x) = sin(lnx)$

$$g_1(x) = \frac{d}{dx} sin(lnx)$$
$$= cos(lnx)\frac{d}{dx}lnx$$
$$= cos(lnx)\frac{1}{x}$$
$$= \frac{cos(lnx)}{x}$$

(Ans)

Answer to the question no: 05

The equation $y_2 + y_1 - 2y = x^2$is a differential equation. Find the constants A, B, and C such that the function: $y = Ax^2 + Bx + c$

Given that,

$$y = Ax^2 + Bx + c \tag{1}$$
$$\implies y_1 = 2Ax + B \tag{2}$$
$$\implies y_2 = 2A \tag{3}$$

Plugging these into the equation $\implies$

$$(2A) + (2Ax + B) - 2(Ax^2 + Bx + c) = x^2$$
$$\implies x^2 = -2Ax^2 + (2A - 2B)x + (2A + B - 2C) \tag{4}$$

From the equation no (4) $\implies$

$$-2A = 1$$
$$\implies A = -\frac{1}{2}$$

Again, $2A - 2B = 0$

$$\implies 2(-\frac{1}{2}) - 2B = 0 \qquad [A = -\frac{1}{2}]$$
$$\implies -1 - 2B = 0$$

$\implies 2B = -1$
$\implies B = -\frac{1}{2}$
Again, $\quad 2A + B - 2C = 0$
$\implies 2(-\frac{1}{2}) + (-\frac{1}{2}) - 2C = 0 \qquad [A = -\frac{1}{2}, B = -\frac{1}{2}]$
$\implies -1 - \frac{1}{2} - 2C = 0$
$\implies 2C = -1 - \frac{1}{2}$
$\implies 2C = -\frac{3}{2}$
$\implies C = -\frac{3}{4}$

Hence,conclude that $\implies$
$y = -\frac{1}{2}x^2 + \frac{1}{2}x - \frac{3}{4} \qquad [A = -\frac{1}{2}, B = -\frac{1}{2}, C = -\frac{3}{4}] \qquad$ (Ans)

Answer to the question no: 06

Find the first and second derivative of the following function with respect to b:

$$f(x) = cos(\frac{r}{2}\{\frac{b^4}{4}(1 - \frac{2sinh^2(8\pi l_s Q)}{sinh^2(9\pi l_s Q)})\}^{\frac{1}{4}})$$
$$f_1(x) = \frac{d}{db}cos(\frac{r}{2}\{\frac{b^4}{4}(1 - \frac{2sinh^2(8\pi l_s Q)}{sinh^2(9\pi l_s Q)})\}^{\frac{1}{4}})$$
Let,$u = (1 - \frac{2sinh^2(8\pi l_s Q)}{sinh^2(9\pi l_s Q)})\}$
The first derivative is :

Now, $f_1(x) = \frac{d}{db}cos(\frac{r}{2}\{\frac{b^4}{4}u\}^{\frac{1}{4}})$
$= -sin(\frac{r}{2}\{\frac{b^4}{4}u\}^{\frac{1}{4}})\frac{d}{db}(\frac{r}{2}\{\frac{b^4}{4}u\}^{\frac{1}{4}})$
$= -sin(\frac{r}{2}\{\frac{b^4}{4}u\}^{\frac{1}{4}})(\frac{r}{2}\frac{d}{dv}\{\frac{b^4}{4}u\}^{\frac{1}{4}})$

$$= -\frac{rsin(\frac{r}{2}\{\frac{b^4}{4}u\}^{\frac{1}{4}})}{2}\frac{d}{db}[(\frac{b^4u}{4})^{\frac{1}{4}}]$$
$$= -\frac{rsin(\frac{r}{2}\{\frac{b^4}{4}u\}^{\frac{1}{4}})}{2}(\frac{1}{4}[\frac{b^4u}{4}])^{-\frac{3}{4}}\frac{d}{db}(\frac{b^4u}{4})$$
$$= -\frac{rsin(\frac{r}{2}\{\frac{b^4}{4}u\}^{\frac{1}{4}})}{8}[\frac{b^4u}{4}]^{-\frac{3}{4}}\frac{u}{4}4b^3$$
$$= -\frac{ub^3rsin(\frac{r}{2}\{\frac{b^4}{4}u\}^{\frac{1}{4}})}{8}[\frac{b^4u}{4}]^{-\frac{3}{4}}$$
$$= -\frac{4^{\frac{3}{4}}u^{\frac{1}{4}}rsin(\frac{rbu^{\frac{1}{4}}}{2^{\frac{3}{2}}})}{8}$$

The Second Derivative is :

$$f_2(x) = \frac{d}{db}\{-\frac{4^{\frac{3}{4}}u^{\frac{1}{4}}rsin(\frac{rbu^{\frac{1}{4}}}{2^{\frac{3}{2}}})}{8}\}$$
$$= -\frac{4^{\frac{3}{4}}u^{\frac{1}{4}}r}{8}\frac{d}{db}sin(\frac{rbu^{\frac{1}{4}}}{2^{\frac{3}{2}}})$$
$$= -\frac{4^{\frac{3}{4}}u^{\frac{1}{4}}r}{8}[cos(\frac{rbu^{\frac{1}{4}}}{2^{\frac{3}{2}}})](\frac{ru^{\frac{1}{4}}}{2^{\frac{3}{2}}})\frac{d}{db}b$$
$$= -\frac{4^{\frac{3}{4}}u^{\frac{1}{2}}r^2[cos(\frac{rbu^{\frac{1}{4}}}{2^{\frac{3}{2}}})]}{8.2^{\frac{3}{2}}}$$
$$= -\frac{4^{\frac{3}{4}}u^{\frac{1}{2}}r^2[cos(\frac{rbu^{\frac{1}{4}}}{2^{\frac{3}{2}}})]}{2^{\frac{9}{2}}}$$
$$= -(1-\frac{2sinh^2(8\pi l_sQ)}{sinhh^2(9\pi l_sQ)})^{\frac{1}{2}}\frac{r^2}{8}cos(\frac{rb}{2^{\frac{3}{2}}}(1-\frac{2sinh^2(8\pi l_sQ)}{sinhh^2(9\pi l_sQ)})^{\frac{1}{4}})$$
(Ans)

THE END