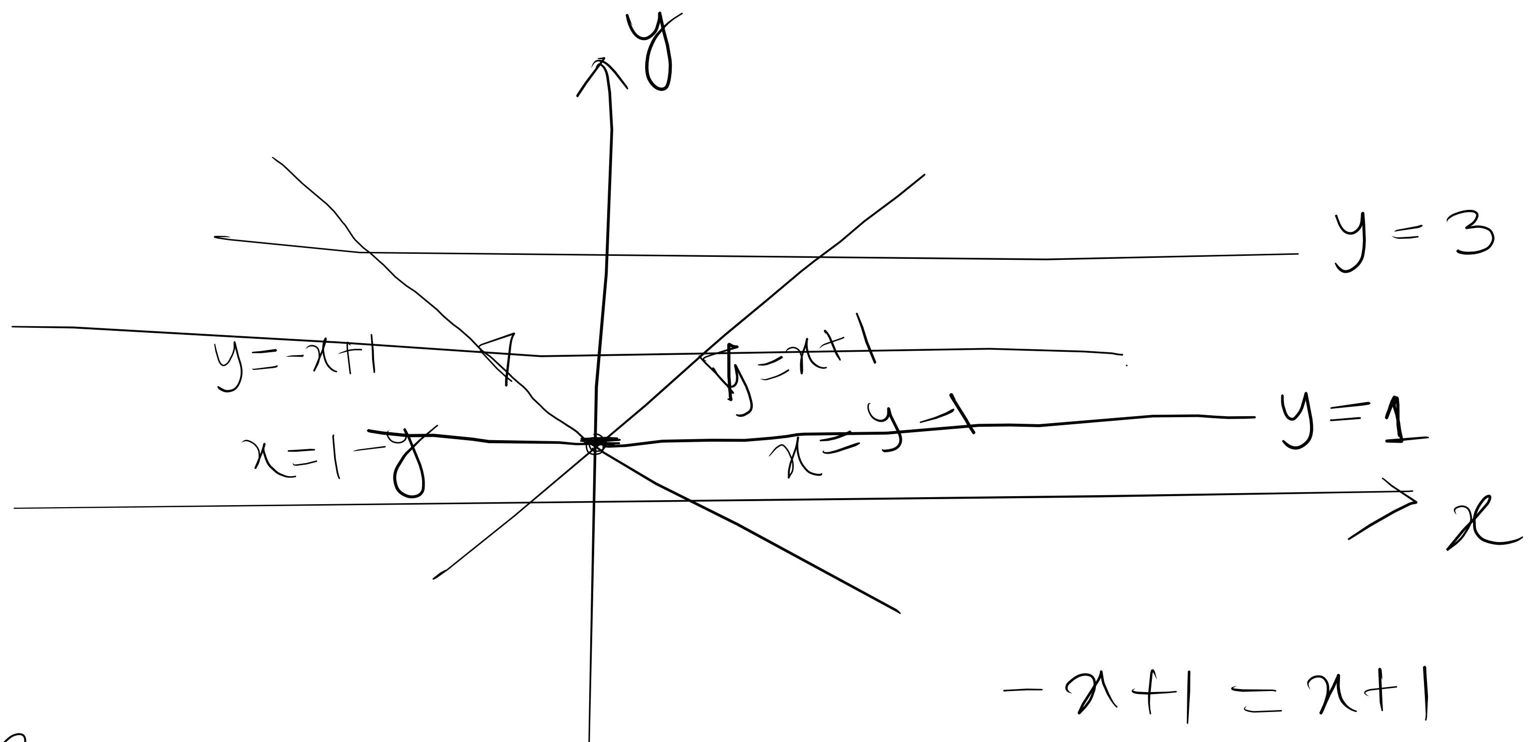
Find $V = \iint_R \boxed{(2x - y^2)}\, dA$ $\xrightarrow{} f(x,y)$ over the region (triangular)

enclosed between the lines $\boxed{y = -x+1,\ y = x+1,\ y = 3}$

Solution:




$-x+1 = x+1$

$\Rightarrow 2x = 0$

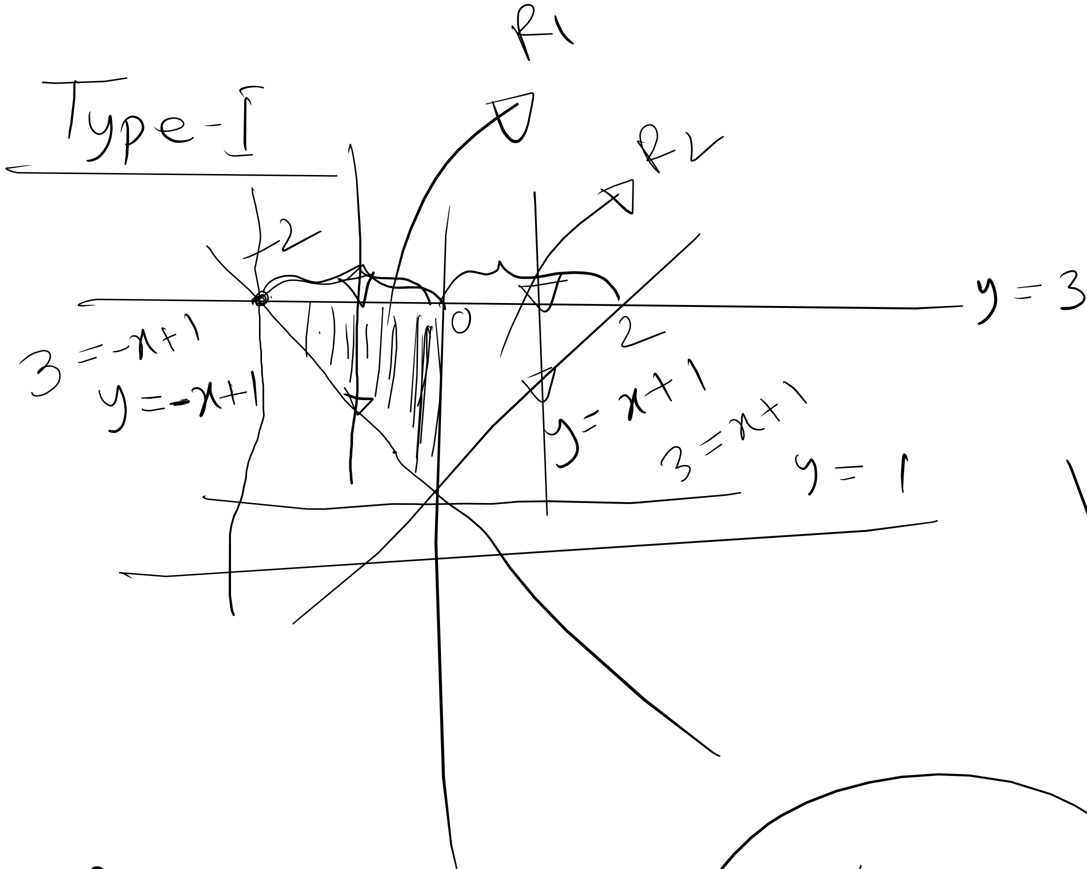
$\Rightarrow x = 0$

H.W.

Type-II

$$V = \int_1^3 \int_{1-y}^{y-1} (2x - y^2)\, dx\, dy$$

## Type-I




$R = R_1 + R_2$

$$V = \iint_{R_1} (2x - y^2)\,dy\,dx + \iint_{R_2} (2x - y^2)\,dy\,dx$$

$$V = \int_{-2}^{0}\int_{-x+1}^{3} (2x - y^2)\,dy\,dx + \int_{0}^{2}\int_{x+1}^{3} (2x - y^2)\,dy\,dx$$

✓

?? H.W.

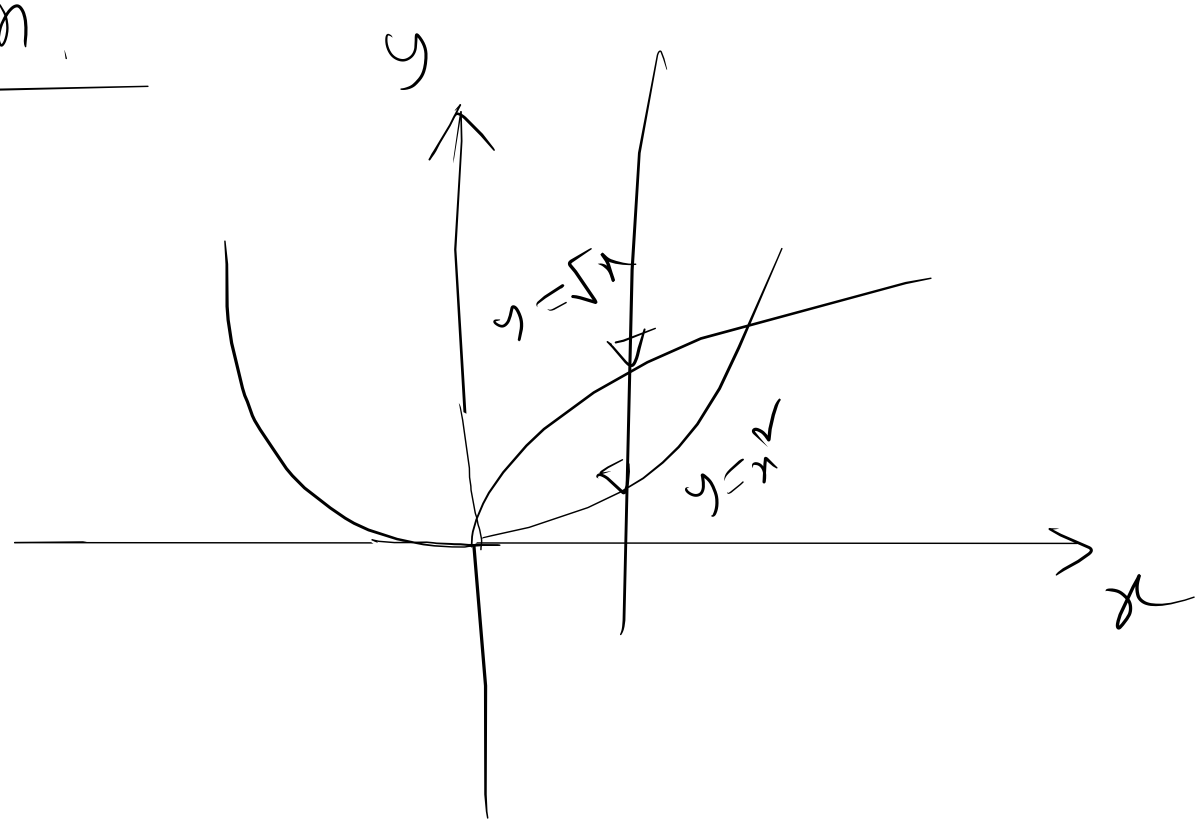
Verify

14.2 Exercise (1-26)

$\Rightarrow y^2 = x$

13. Find $V = \iint_R xy\,dA$ enclosed by $y=\sqrt{x}$, $y = x^2$.

**Solution:**




$$V = \int_0^1 \int_{x^2}^{\sqrt{x}} xy\,dy\,dx$$

$$\sqrt{x} = x^2$$

$$\Rightarrow x = x^4$$

$$\Rightarrow x^4 - x = 0$$

$$\Rightarrow x(x^3 - 1) = 0$$

$$x = 0, 1, \boxed{x^2 + x + 1 \neq 0}$$

# Double Integration in Polar Coordinates:

$x = r\cos\theta$

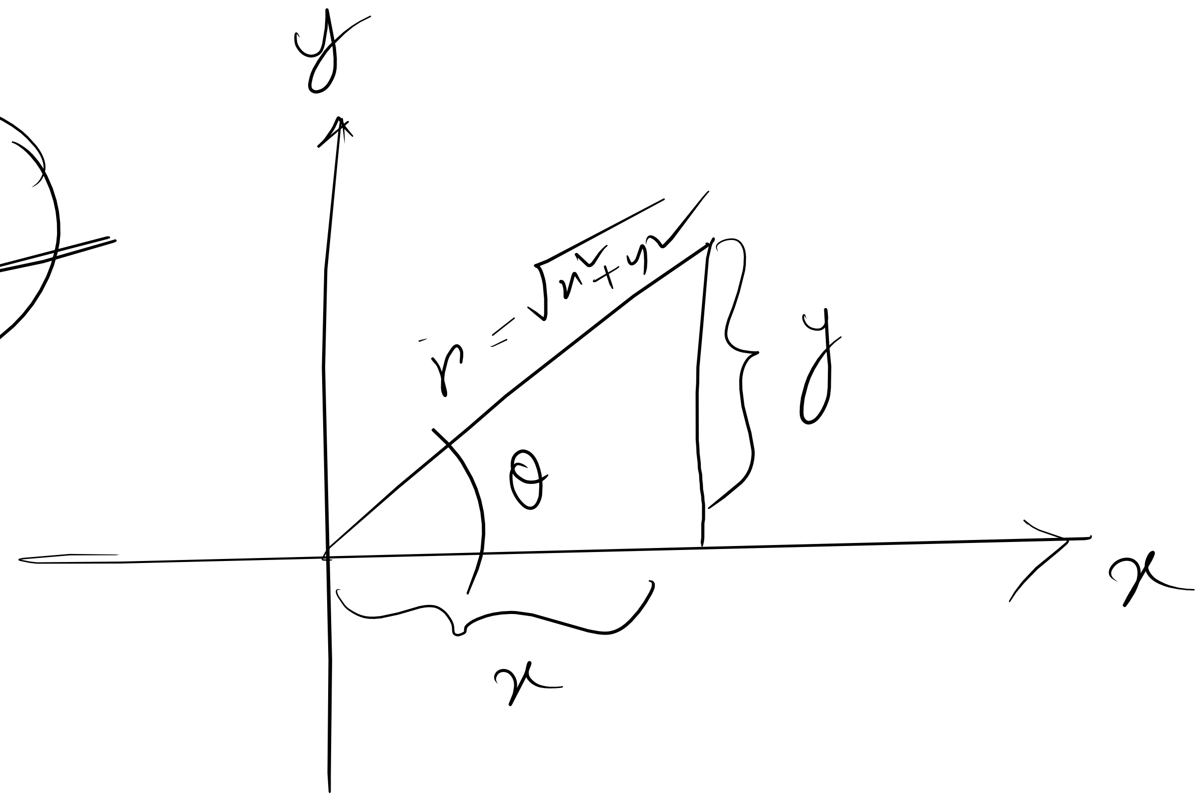
$y = r\sin\theta$

$r \geq 0$

$r^2 = x^2 + y^2$

$\theta = \tan^{-1}\left|\frac{y}{x}\right|$

$r = \sqrt{x^2+y^2}$




$$dA = dy\,dx = dx\,dy = r\,dr\,d\theta$$

$$V = \iint_R f(x,y)\,dA = \iint_R f(r,\theta)\,r\,dr\,d\theta = \int_{\theta_1}^{\theta_2}\int_{r_1}^{r_2} f(r,\theta)\,r\,dr\,d\theta.$$

Use polar coordinate to evaluate the double integration $\int_{-a}^{a}\int_{0}^{\sqrt{a^2-x^2}} (x^2+y^2)^{1/2}\,dy\,dx$.

$0 \le y \le \sqrt{a^2-x^2}$

$-a \le x \le a$

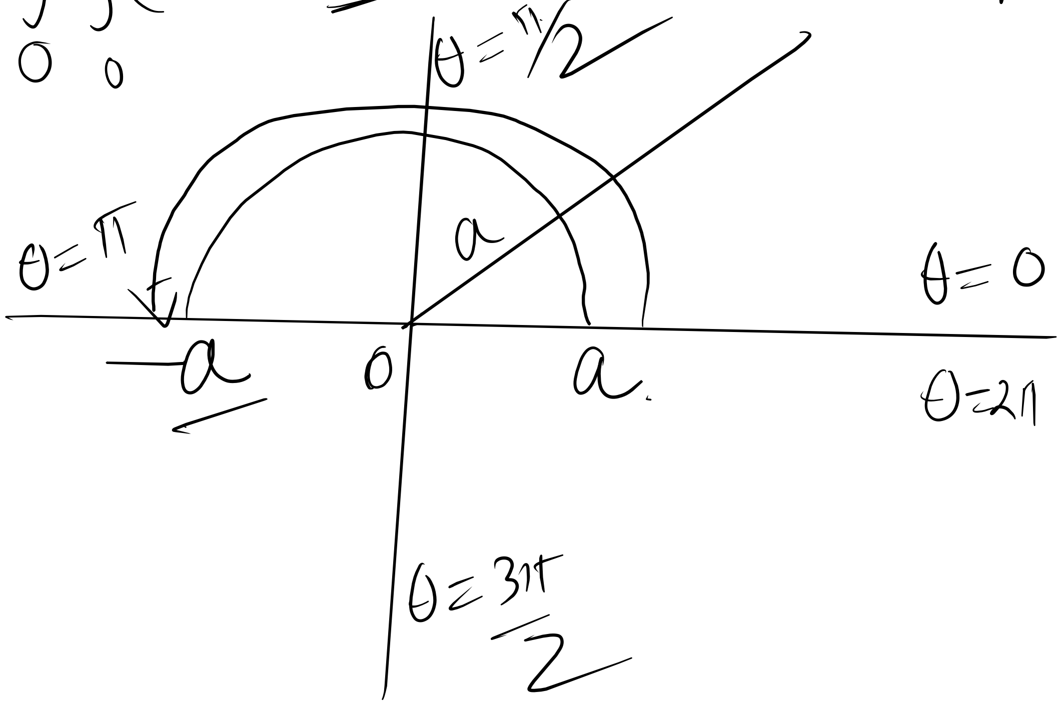
$$V = \int_0^{\pi}\int_0^{a} (r^2)^{1/2}\, r\,dr\,d\theta$$

$r = ??$

$y = \sqrt{a^2-x^2}$

$y^2 = a^2-x^2$

$x^2+y^2 = a^2$



Evaluate $V = \int_{-3}^{0}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} x \, dy \, dx$ ( in Polar Coordinate).

$dA = r\,dr\,d\theta$

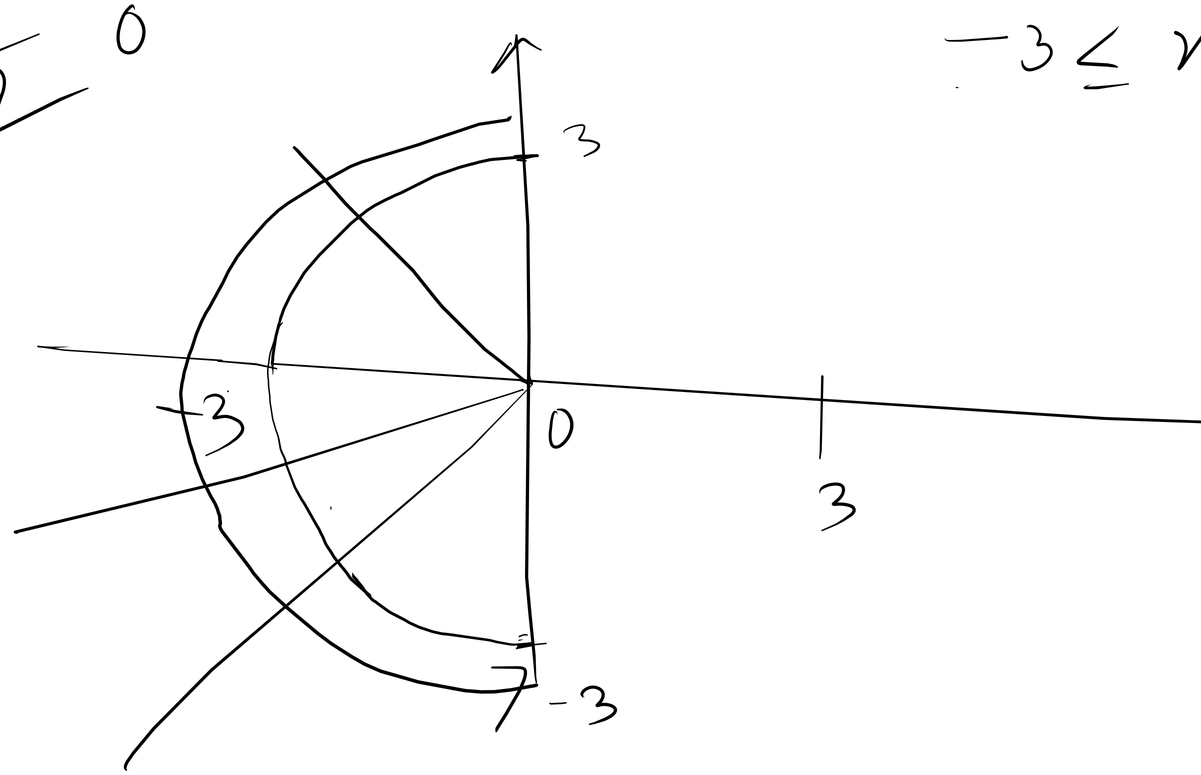
$y^2 = 9 - x^2$

$\Rightarrow x^2 + y^2 = 9$

$V = \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}\int_{0}^{3} r\cos\theta \; r\,dr\,d\theta$

$-\sqrt{9-x^2} \leq y \leq \sqrt{9-x^2}$

$-3 \leq x \leq 0$

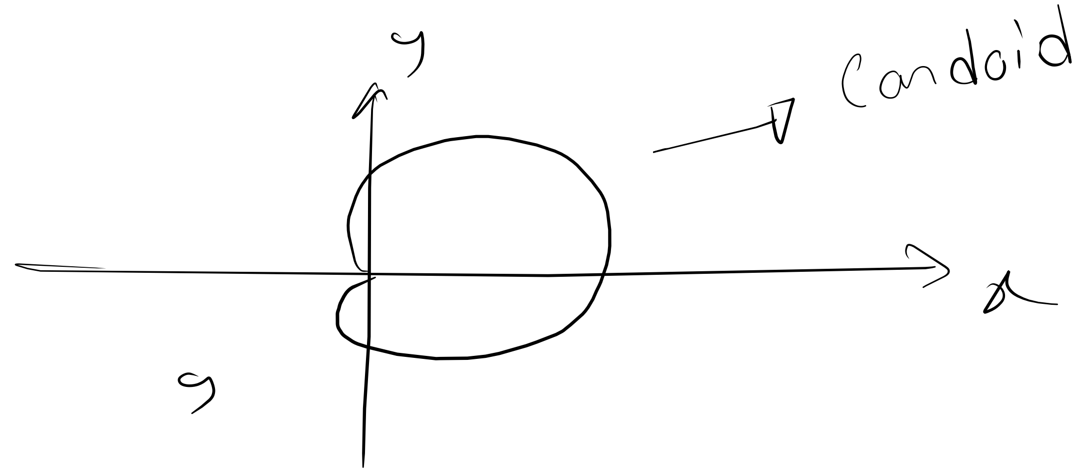



$r = \sqrt{x^2+y^2}$

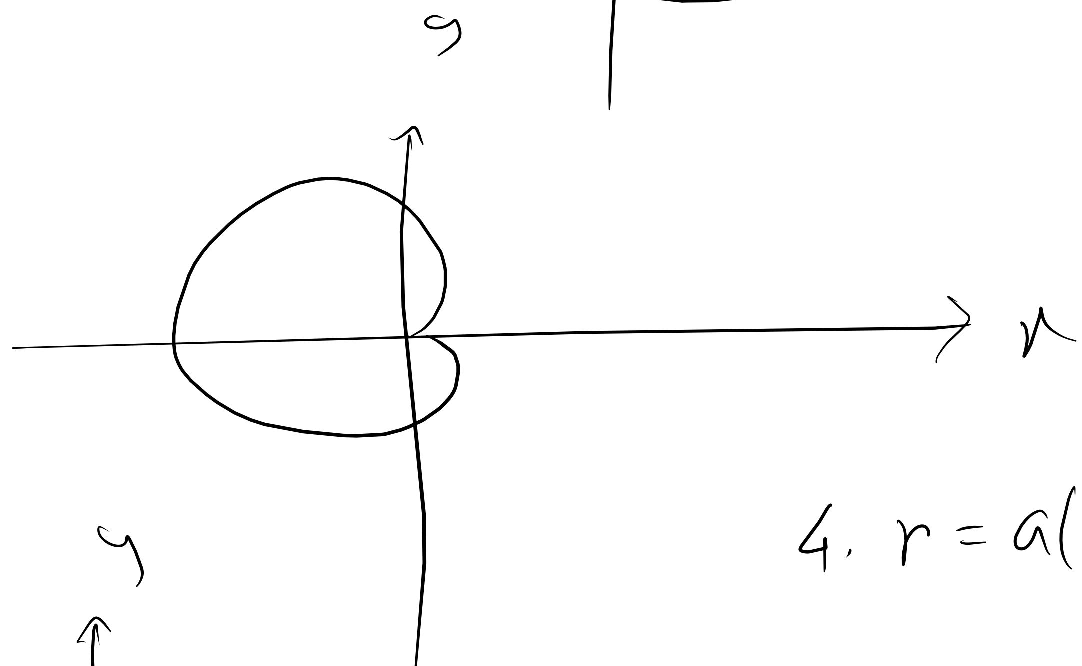
$$V = \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \int_{0}^{3} r^2 \cos\theta \, dr \, d\theta$$

$$= \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \cos\theta \left[\frac{r^3}{3}\right]_0^3 d\theta = 9\left[\sin\theta\right]_{\frac{3\pi}{2}}^{\frac{\pi}{2}}$$

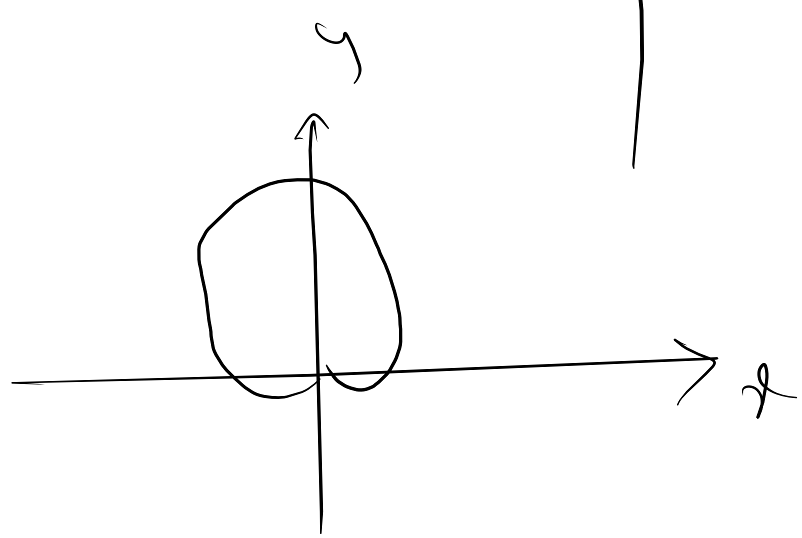
1. $r = a(1+\cos\theta)$



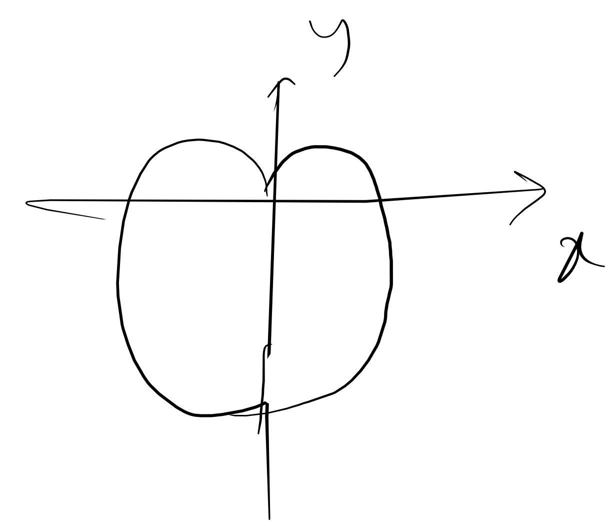

2. $r = a(1-\cos\theta)$



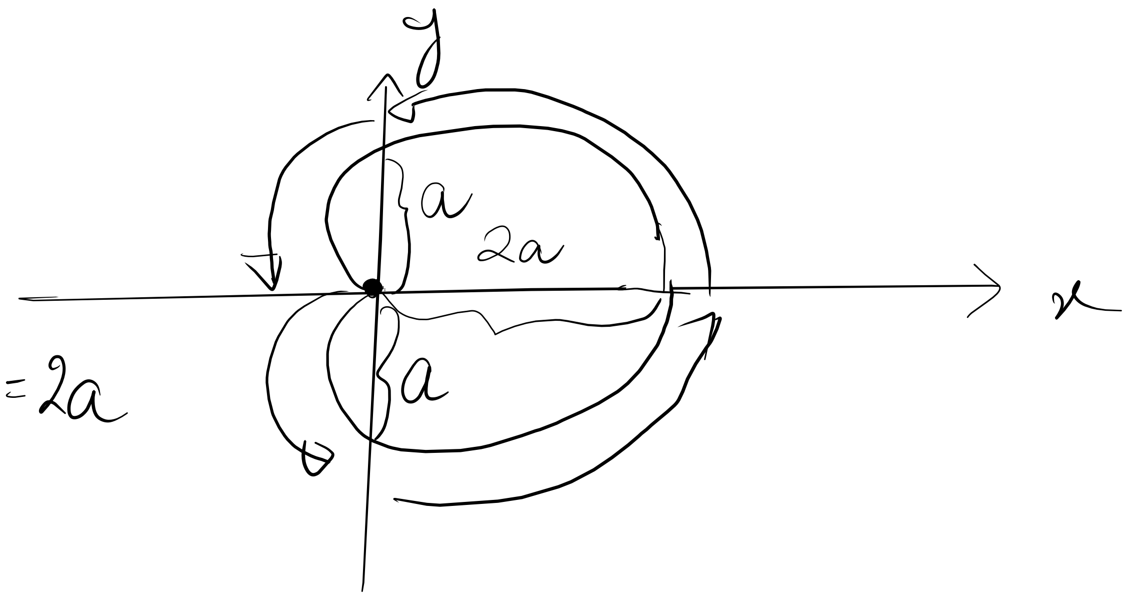

3. $r = a(1+\sin\theta)$




4. $r = a(1-\sin\theta)$



$r = a(1+\cos\theta)$




$\theta = 0, \quad r = a(1+\cos 0) = a(1+1) = 2a$

$\theta = \frac{\pi}{2}, \quad r = a$

$\theta = \frac{3\pi}{2}, \quad r = a\left(1+\cos\frac{3\pi}{2}\right) = a$

$\theta = \pi, \quad r = a(1-1) = 0$

$\theta = 2\pi, \quad r = 2a$

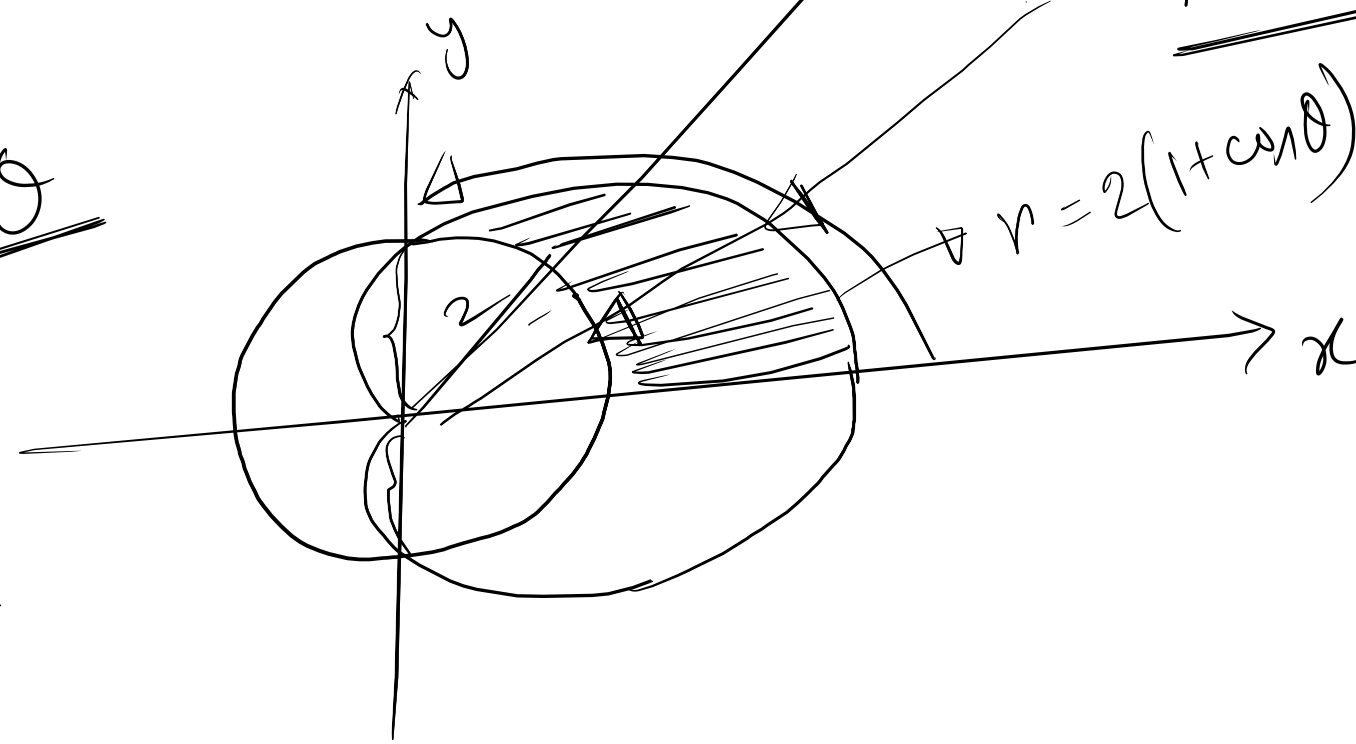
14.3, (1-12)

(23-34)

Evaluate $\iint_R \sin\theta \, dA$, where $R$ is the region in the first quadrant that's outside the circle $r=2$ & inside the cardioid $r = 2(1+\cos\theta)$ $\to a=2$

$x^2+y^2=4$

$x^2+y^2=2^2$




$$V = \iint_R \sin\theta \, r\,dr\,d\theta$$

$$= \int_0^{\frac{\pi}{2}} \int_2^{2(1+\cos\theta)} \sin\theta \, r\,dr\,d\theta$$