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Home work sheet #4

(1)

(i) $y = \sin x \sin 2x \sin 3x$

$\Rightarrow y = (\sin x \sin 2x) . \sin 3x$

$\Rightarrow \frac{dy}{dx} = 3 \cos 3x . (\sin x \sin 2x) +$
$\sin 3x \{(2 \cos 2x . \sin x) + (\cos x \sin 2x)\}$

$= 3 \sin x \sin 2x \cos 3x + 2 \cos 2x \sin x \sin 3x$
$+ \cos x \sin 2x \sin 3x$

Ans

(2) $y = \operatorname{cosec}^3 x$

$\frac{dy}{dx} = 3 \operatorname{cosec}^2 x . (- \operatorname{cosec} x \cot x)$

$= -3 \operatorname{cosec}^3 x \cot x$

(3) $y = \cos 2x \cos 3x$

$\frac{dy}{dx} = -3 \sin 3x \cos 2x - \cancel{2 \cos 2x}\ 2 \sin 2x \cos 3x$

Ans

(iv) $y = \sin^{-1}(x^2)$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^4}} \cdot 2x$$

$$= \frac{2x}{\sqrt{1-x^4}}$$

(v) $y = \tan(\sin^{-1}x)$

Let, $\sin^{-1}x = u$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}\tan(\sin^{-1}x) \cdot \frac{d}{dx}\sin^{-1}x$$

$$= \frac{d}{dx}\tan u \cdot \frac{d}{dx}\sin^{-1}x$$

$$= \sec^2(u) \cdot \frac{1}{\sqrt{1-x^2}}$$

$$= \sec^2(\sin^{-1}x) \cdot \frac{1}{\sqrt{1-x^2}}$$

$$= \left(\frac{1}{\sqrt{1-x^2}}\right)^2 \times \frac{1}{\sqrt{1-x^2}}$$

$$= \frac{1}{1-x^2} \cdot \frac{1}{\sqrt{1-x^2}}$$

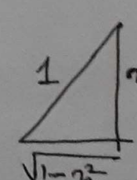
$$= \frac{1}{(1-x^2)^{1+\frac{1}{2}}} = \frac{1}{(1-x^2)^{\frac{3}{2}}}$$

$\sec(\sin^{-1}x)$

Let, $\theta = \sin^{-1}x$

$\sin\theta = \sin\sin^{-1}x$

$\sin\theta = x$



$\therefore \sec\theta = \frac{1}{\sqrt{1-x^2}}$

$\sec(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$

$(\sec^2(\sin^{-1}x)) = \left(\frac{1}{\sqrt{1-x^2}}\right)^2$

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(vi) $\cot^{-1}\left(\frac{1+x}{1-x}\right)$

$$= -\frac{1}{\left(\frac{1+x}{1-x}\right)^2+1} \cdot \frac{(1-x)(0+1)-(1+x)(0-1)}{(1-x)^2}$$

$$= -\frac{1}{\frac{(1+x)^2+(1-x)^2}{(1-x)^2}} \cdot \frac{1-x+1+x}{(1-x)^2}$$

$$= -\frac{(1-x)^2}{(1+x)^2+(1-x)^2} \cdot \frac{2}{(1-x)^2}$$

$$= -\frac{2}{1+2x+x^2+1-2x+x^2}$$

$$= -\frac{2}{2+2x^2} = -\frac{1}{1+x^2}$$

(vii) $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

$$= -\frac{1}{\sqrt{1-\left(\frac{1-x^2}{1+x^2}\right)^2}} \cdot \frac{(1+x^2)(0-2x)-(1-x^2)(0+2x)}{(1+x^2)^2}$$

$$= -\frac{1}{\sqrt{1-\frac{(1-x^2)^2}{(1+x^2)^2}}} \cdot \frac{-2x(1+x^2)-2x(1-x^2)}{(1+x^2)^2}$$

$$= -\frac{1}{\sqrt{\frac{(1+x^2)^2-(1-x^2)^2}{(1+x^2)^2}}} \cdot \frac{-2x(1+\cancel{x^2}+1-\cancel{x^2})}{(1+x^2)^2}$$

$$= -\frac{1}{\sqrt{\frac{(1+\cancel{x^2}+1-\cancel{x^2})(\cancel{1}+x^2-\cancel{1}+x^2)}{(1+x^2)^2}}} \cdot \frac{-4x}{(1+x^2)^2}$$

$$= \frac{1}{\frac{\sqrt{2\cdot 2x^2}}{1+x^2}} \cdot \frac{4x}{(1+x^2)^2}$$

$$= \frac{1+x^2}{\cancel{2x}} \cdot \frac{\overset{2}{\cancel{4x}}}{(1+x^2)^2} = \frac{2}{1+x^2}$$

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(viii)

$$\sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

$$= \frac{1}{\sqrt{1-\frac{4x^2}{(1+x^2)^2}}} \cdot \frac{2(1+x^2) - 2x(0+2x)}{(1+x^2)^2}$$

$$= \frac{1}{\sqrt{\frac{1+2x^2+x^4-4x^2}{(1+x^2)^2}}} \cdot \frac{2+2x^2-4x^2}{(1+x^2)^2}$$

$$= \frac{1}{\sqrt{\frac{1-2x^2+x^4}{(1+x^2)^2}}} \cdot \frac{2-2x^2}{(1+x^2)^2}$$

$$= \frac{1}{\sqrt{\frac{(1-x^2)^2}{(1+x^2)^2}}} \cdot \frac{2(1-x^2)}{(1+x^2)^2}$$

$$= \frac{1+x^2}{1-x^2} \cdot \frac{2(1-x^2)}{(1+x^2)^2}$$

$$= \frac{2}{1+x^2}$$

(ix) $\tan^{-1}\left(\dfrac{2x}{1-x^2}\right)$

$$= \frac{1}{1+\dfrac{4x^2}{1-2x^2+x^4}} \cdot \frac{2(1-x^2)-2x(0-2x)}{(1-x^2)^2}$$

$$= \frac{1}{\dfrac{1-2x^2+x^4+4x^2}{(1-x^2)^2}} \cdot \frac{2-2x^2+4x^2}{(1-x^2)^2}$$

$$= \frac{(1-x^2)^2}{(1+x^2)^2} \cdot \frac{\cancel{(1-x^2)^2}\ 2+2x^2}{\cancel{2+2x^2}\ (1-x^2)^2}$$

$$= \frac{\cancel{(1-x^2)^2}}{(1+x^2)^2} \cdot \frac{2(1+x^2)}{\cancel{(1-x^2)^2}}$$

$$= \frac{2}{1+x^2}$$ Ans.

H·W- sheet #4

(x) $\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$

Suppose,

$$x = \sin\theta$$

$$\theta = \sin^{-1}x$$

Here,

$$\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$$

$$= \tan^{-1}\left(\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}\right)$$

$$= \tan^{-1}\left(\frac{\sin\theta}{\cos\theta}\right)$$

$$= \tan^{-1}\tan\theta$$

$$= \theta$$

$$\therefore \frac{d}{dx}(\theta)$$

$$= \cancel{\frac{d}{dx}\Big(}$$

$$= \frac{d}{dx}(\sin^{-1}x)$$

$$= \frac{1}{\sqrt{1-x^2}} \quad \underline{\text{Ans.}}$$

$\circledast$ $\sin\left(2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right)$

Suppose,

$x = \cos 2\theta$

$\therefore 2\theta = \cos^{-1}x$

$\theta = \frac{1}{2}\cos^{-1}x$

Here,

$\sin\left(2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right)$

$= \sin\left(2\tan^{-1}\sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}\right)$

$= \sin\left(2\tan^{-1}\sqrt{\frac{2\sin^2\theta}{2\cos^2\theta}}\right) \left[\begin{array}{l}\because 2\cos^2\theta = 1+\cos 2\theta \\ \because 2\sin^2\theta = 1-\sin 2\theta\end{array}\right]$

$= \sin\left(2\tan^{-1}\tan\theta\right)$

$= \sin(2\theta)$

$= \sin\left(\cancel{2}\cdot\frac{1}{\cancel{2}}\cos^{-1}x\right)$

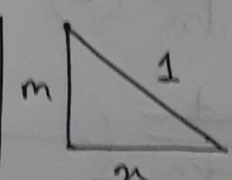
$= \sin\cos^{-1}x$

$= \sin\sin^{-1}\sqrt{1-x^2}$

$= \sqrt{1-x^2}$

$\therefore \frac{d}{dx}\left(\sqrt{1-x^2}\right)$

$= \frac{1}{2\sqrt{1-x^2}}\cdot(0-2x)$



$1^2 = x^2 + m^2$

$m^2 = 1-x^2$

$m = \sqrt{1-x^2}$

H.W sheet # 4

$$\text{or} = \frac{-2x}{2\sqrt{1-x^2}} = \frac{-x}{\sqrt{1-x^2}} \text{ Ans.}$$

(xv) $\ln\sqrt{\frac{1-x}{1+x}}$

Suppose,

$$x = \cos 2\theta$$

$$\theta = \frac{1}{2}\cos^{-1}x \longrightarrow 2\theta = \cos^{-1}x$$

$$\therefore \ln\sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}$$

$$= \ln\sqrt{\frac{2\sin^2\theta}{2\cos^2\theta}}$$

$$= \ln(\tan\theta)$$

$$\cancel{= \frac{1}{\tan\theta}}$$

Now,

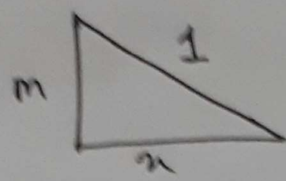
$$\frac{dy}{dx}(\ln\tan\theta) = \frac{1}{\tan\theta}\cdot\sec^2\theta$$

$$= \frac{\cos\theta}{\sin\theta}\cdot\frac{1}{\cos^2\theta}$$

$$= \frac{1}{\sin\theta\cos\theta}$$

$$= \frac{1}{\frac{1}{2}\sin 2\theta} = \frac{1}{\frac{1}{2}\sin\cos^{-1}x}$$

Now,



$$m^2 + x^2 = 1$$

$$m = \sqrt{1-x^2}$$

$$\therefore \frac{d}{dx}(\ln \tan\theta) = \frac{1}{\frac{1}{2}\sin\sin^{-1}\frac{\sqrt{1-x^2}}{1}}$$

$$= \frac{1}{\frac{1}{2}\cdot\sqrt{1-x^2}}$$

$$= \frac{1}{\frac{\sqrt{1-x^2}}{2}}$$

$$= \frac{2}{\sqrt{1-x^2}}$$ **Ans.**

H.W sheet #4

(2)

(i) $(\sin x)^{\ln x}$

Here,

$$y = (\sin x)^{\ln x}$$

$$\cancel{\frac{dy}{dx}} \ln y = \ln\left[(\sin x)^{\ln x}\right]$$

$$\ln y = \ln x \cdot \ln(\sin x)$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{x}\ln(\sin x) + \ln x \cdot \frac{1}{\sin x}\cdot \cos x$$

$$\frac{dy}{dx} = y\left\{\frac{\ln(\sin x)}{x} + \ln x \cdot \cot x\right\}$$

$$= (\sin x)^{\ln x}\left\{\frac{\ln(\sin x)}{x} + \ln x \cot x\right\}$$

Ans.

(11) $(\sin x)^{\cos x} + (\cos x)^{\sin x}$

Suppose,

$$y = (\sin x)^{\cos x} + (\cos x)^{\sin x}$$

$$\ln y = \ln\left\{(\sin x)^{\cos x} + (\cos x)^{\sin x}\right\}$$

$$\ln y = \ln (\sin x)^{\cos x} + \ln (\cos x)^{\sin x}$$

$$\ln y = \cos x \ln (\sin x) + \sin x \ln (\cos x)$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = -\sin x \ln (\sin x) + \cos x \cdot \frac{1}{\sin x} \cdot \cos x$$
$$+ \cos x \ln (\cos x) + \sin x \cdot \frac{-1}{\cos x} \cdot \sin x$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = -\sin x \ln (\sin x) + \cos x \cot x$$
$$+ \cos x \ln (\cos x) - \sin x \tan x$$

$$\frac{dy}{dx} = y\left\{\cos x \cot x + \cos x \ln (\cos x) - \sin x \ln (\sin x) - \sin x \tan x\right\}$$

$$= (\sin x^{\cos x} + \cos x^{\sin x})\left\{\cos x \cot x + \cos x \ln (\cos x) - \sin x \ln (\sin x) - \sin x \tan x\right\}$$

Ans.

H.W sheet #4

(3)

$$3x^4 - x^2y + 2y^3 = 0$$

$$\frac{dy}{dx}(3x^4 - x^2y + 2y^3) = 0$$

$$12x^3 - 2xy - x^2 \cdot \frac{dy}{dx} + 6y^2 \cdot \frac{dy}{dx} = 0$$

$$6y^2 \cdot \frac{dy}{dx} - x^2 \frac{dy}{dx} = 2xy - 12x^3$$

$$\frac{dy}{dx}(6y^2 - x^2) = 2xy - 12x^3$$

$$\frac{dy}{dx} = \frac{2xy - 12x^3}{6y^2 - x^2}$$

Ans.

(II) $x^3+y^3+4x^2y-25=0$

$$3x^2+3y^2\cdot\frac{dy}{dx}+4\left(2xy+x^2\cdot\frac{dy}{dx}\right)-0=0$$

$$\Rightarrow 3x^2+3y^2\frac{dy}{dx}+8xy+4x^2\frac{dy}{dx}=0$$

$$(3y^2+4x^2)\frac{dy}{dx}=-(8xy+3x^2)$$

$$\frac{dy}{dx}=\frac{-(8xy+3x^2)}{3y^2+4x^2}$$

(III)

$$x^y=y^x$$

$$\ln x^y=\ln y^x$$

$$y\ln x=x\ln y$$

$$\cancel{\frac{dy}{dx}\ln x=\ln}$$

$$\frac{dy}{dx}\ln x+\frac{y}{x}=\ln y+x\cdot\frac{1}{y}\cdot\frac{dy}{dx}$$

$$\frac{dy}{dx}\ln x-\frac{x}{y}\frac{dy}{dx}=\ln y-\frac{y}{x}$$

$$\frac{dy}{dx}\left(\ln x-\frac{x}{y}\right)=\ln y-\frac{y}{x}$$

$$\frac{dy}{dx}=\frac{\ln y-\frac{y}{x}}{\ln x-\frac{x}{y}}$$

H.W sheet #4

(4)

$$x = a\cos^3\theta \text{ ——— (I)}$$

$$y = a\sin^3\theta \text{ ——— (II)}$$

(I) $\Rightarrow$

$$\frac{dx}{d\theta} = -a.3\cos^2\theta.\sin\theta$$

$$= -3a\sin\theta\cos^2\theta$$

(II) $\Rightarrow$

$$\frac{dy}{d\theta} = a.3\sin^2\theta.\cos\theta$$

$$= 3a\sin^2\theta\cos\theta$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{3a\sin^2\theta\cos\theta}{-3a\sin\theta\cos^2\theta} = -\frac{\sin\theta}{\cos\theta}$$

$$= -\tan\theta$$ Ans.

(III)

$x = a\sec^2\theta$ ——— (I)

$y = a\tan^2\theta$ ——— (II)

(1) $\Rightarrow$

$$\frac{dx}{d\theta} = a\cdot 2\sec\theta \cdot \sec\theta\tan\theta$$

(2) $\Rightarrow$

$$\frac{dy}{d\theta} = a\cdot 2\tan\theta\cdot\sec^2\theta$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{2a\,\cancel{\tan\theta\cdot\sec^2\theta}}{2a\,\cancel{\sec\theta\cdot\sec\theta}\cdot\cancel{\tan\theta}}$$

$$= 1 \quad \text{Ans.}$$