L'hospital's Rule

L' Hospital's Rule

$$\lim_{x\to\infty} \frac{x^5+x^4+x^3+x^2+x+1}{2x^5+x^4+x^3+x^2+x+1}$$

$$= \lim_{x\to\infty} \frac{5x^4+4x^3+3x^2+2x+1}{10x^4+4x^3+2x^2+2x+1} \quad \boxed{D}$$

$$= \lim_{x\to\infty} \frac{20x^3+12x^2+6x+2}{40x^3+12x^2+4x+2} \quad \boxed{D}$$

$$= \lim_{x\to\infty} \frac{60x^2+24x+6}{120x^2+24x+4} \quad \boxed{D}$$

$$= \cancel{\lim_{x\to\infty} \frac{240x^2+24}{480x^2+24}} \quad \cancel{\boxed{D}}$$

$$= \lim_{x\to\infty} \frac{120x+24}{240x+24} \quad \boxed{D}$$

$$= \lim_{x\to\infty} \frac{120}{240} \quad \boxed{D}$$

$$= \frac{1}{2}$$

⊛

$$\lim_{x \to -2} \frac{x+2}{\ln(x+3)}$$

$$= \lim_{x \to -2} \frac{1}{\frac{1}{x+3}} \quad [D]$$

$$= \lim_{x \to -2} \frac{x+3}{1}$$

$= 1$ Ans.

⊛ $$\lim_{x \to \infty} \frac{3^x}{x^2+x-1}$$

$$= \lim_{x \to \infty} \frac{3^x \ln 3}{2x+1} \quad \boxed{D}$$

$$= \lim_{x \to \infty} \frac{\ln 3 \cdot 3^x \cdot \ln 3}{2} \quad [D]$$

$= \infty$

L' Hospital's Rule

$$\lim_{x \to \infty} \frac{\ln x}{\sqrt{x}}$$

$$= \frac{\frac{1}{x}}{\frac{1}{2\sqrt{x}}} \quad [D]$$

$$= \frac{1}{x} \times \frac{2\sqrt{x}}{1}$$

$$\lim_{x \to \infty} = \frac{2}{\sqrt{x}}$$

$$= 0 \quad \underline{\underline{Ans}}$$

$$\lim_{x \to 0} \frac{5x - \tan 5x}{x^3}$$

$$= \lim_{x \to 0} \frac{5 - 5\sec^2 5x}{3x^2} \quad \boxed{D}$$

$$\cancel{= \lim_{x \to 0} \frac{0 - 5.5.2\sec(5x).}{6x}}$$

$$= \lim_{x \to 0} \frac{5(1 - \sec^2 5x)}{3x^2}$$

$$= \frac{5}{3} \lim_{x \to 0} \frac{1 - \sec^2 5x}{x^2}$$

$$= \frac{5}{3} \lim_{x \to 0} \frac{-\tan^2 5x}{x^2}$$

$$= \frac{-5}{3} \lim_{x \to 0} \frac{\tan^2 5x}{x^2}$$

$$= -\frac{5}{3} \lim_{x \to 0} \frac{2\tan 5x \cdot \sec^2 5x \cdot 5}{2x} \quad [D]$$

$$= -\frac{5}{3} \lim_{x \to 0} \frac{\tan 5x \cdot \sec^2 5x \cdot 5}{x}$$

$$= -\frac{5}{3} \lim_{x \to 0} \frac{\frac{\sin 5x}{\cos 5x} \cdot \frac{1}{\cos^2 5x} \cdot 5}{x}$$

$$\cancel{= -\frac{5}{3} \lim_{x \to 0} \frac{\sin 5x}{\cos^3 5x} \cdot \frac{1}{x}}$$

$$= -\frac{5.5}{3} \lim_{x \to 0} \frac{\frac{\sin 5x}{\cos^3 5x}}{x}$$

$$= -\frac{25}{3} \lim_{x \to 0} \frac{\frac{\cos^3 5x \cdot 5\cos 5x - \sin 5x \cdot 3\cos^2 5x \cdot 5\sin 5x}{\cos^6 5x}}{1}$$

$$= -\frac{25}{3} \cdot \frac{\cos^3 0 \cdot 5\cos(0) - \sin 0 \cdot 3\cos^2 0 \cdot 5\sin 0}{\cos^6(5 \cdot 0)}$$

L Hospital's Rule

$$= -\frac{25}{3} \cdot \frac{5-0}{1}$$

$$= -\frac{25}{3} \cdot 5$$

$$= -\frac{125}{3}$$ **Ans.**

⊛

$$\lim_{x\to\infty} (x - \ln x)$$

$$= \lim_{x\to\infty} \left(1 - \frac{1}{x}\right) \quad [D]$$

$$= \lim_{x\to\infty} \left(\frac{x-1}{x}\right)$$

$$= \lim_{x\to\infty} (1-1) \quad [D]$$

$$= 0$$

Another step

$$\lim_{x\to\infty} \left(x - x \cdot \ln x/x\right)$$

$$= \lim_{x\to\infty} x\left(1 - \frac{\ln x}{x}\right)$$

note that,

$$\lim_{x\to\infty} \frac{\ln x}{x} = \infty/\infty$$

$$\therefore \lim_{x\to\infty} \frac{\frac{1}{x}}{1} \quad [D]$$

$$= \lim_{x\to\infty} \frac{1}{x} = 0$$

Therefore,

$$\lim_{x\to\infty} (x - \ln x) = \lim_{x\to\infty} x\left(1 - \frac{\ln x}{x}\right)$$

$$= \lim_{x\to\infty} x(1-0) = \infty \quad (D.N.E)$$

⊛ $$\lim_{x\to 1} \left(\frac{1}{\ln x} - \frac{1}{x-1}\right) \to [\infty - \infty]$$

$$= \lim_{x\to 1} \left(\frac{x-1-\ln x}{\ln x\,(x-1)}\right)$$

$$= \lim_{x\to 1} \frac{1-0-\frac{1}{x}}{\frac{1}{x}(x-1) + \ln x.\,(1-0)}$$

$$= \lim_{x \to 1} \frac{1 - \frac{1}{x}}{\frac{x-1}{x} + \ln x}$$

$$= \lim_{x \to 1} \frac{\frac{x-1}{x}}{\frac{x-1+x\ln x}{x}}$$

$$= \lim_{x \to 1} \frac{x-1}{x-1+x\ln x}$$

$$= \lim_{x \to 1} \frac{1}{1+\ln x + x \cdot \frac{1}{x}}$$

$$= \lim_{x \to 1} \frac{1}{1+\ln x + 1}$$

$$= \lim_{x \to 1} \frac{1}{2+\ln x} = \frac{1}{2+0} = \frac{1}{2}$$ Ans

⊛ $\lim\limits_{x \to 0^+} (\sin x \ln x)$

$$= \lim_{x \to 0^+} \frac{\sin x \cdot \frac{1}{\sin x} \ln x}{\frac{1}{\sin x}}$$

$$= \lim_{x \to 0^+} \frac{\ln x}{(\sin x)^{-1}}$$

$$= \lim_{x \to 0^+} \frac{\frac{1}{x}}{\frac{-1}{(\sin x)^2} \cdot \cos x} \quad [D]$$

$$= \lim_{x \to 0^+} \frac{-1}{x} \times \frac{(\sin x)^2}{\cos x}$$

$$= -\lim_{x \to 0^+} \frac{(\sin x)^2}{x \cos x}$$

$$= -\lim_{x \to 0^+} \frac{2 \sin x \cos x}{\cos x - x \sin x} \quad [D]$$

$$= -\lim_{x \to 0^+} \frac{2 \sin 0 \cos 0}{\cos 0 - x \sin 0} = 0$$

L' Hospital's rule

~~= 0 Ans~~ Type : ∞^0, 0^0, 1^∞

$$\lim_{x \to \infty} x^{\frac{1}{x}}$$

Here,

$$y = x^{\frac{1}{x}}$$

$$\Rightarrow \ln y = \frac{1}{x} \ln x = \frac{\ln x}{x}$$

$$\therefore \lim_{x \to \infty} \frac{\ln x}{x}$$

$$= \lim_{x \to \infty} \frac{\frac{1}{x}}{1} \quad [D]$$

$$= \lim_{x \to \infty} \frac{1}{x} = 0$$

Therefore,

$$\lim_{x \to \infty} x^{1/x} = e^0 = 1$$

$\circledast$ $\lim_{x \to \frac{\pi}{2}} (\tan x)^{2x-\pi}$

Here,

$$y = (\tan x)^{2x-\pi}$$

$$\ln y = (2x-\pi)\ln(\tan x) = \frac{\ln(\tan x)}{(2x-\pi)^{-1}}$$

$$\therefore \lim_{x \to \frac{\pi}{2}} \frac{\ln(\tan x)}{(2x-\pi)^{-1}}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{\frac{\sec^2 x}{\tan x}}{\frac{-1}{(2x-\pi)^2} \cdot (2-0)} \quad [D]$$

$$= \lim_{x \to \pi/2} \frac{\cancel{\tan x}\ \frac{1}{\cos^2 x} \cdot \frac{\cos x}{\sin x}}{\frac{-2}{(2x-\pi)^2}}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{\frac{1}{\cos x \sin x}}{\frac{-2}{(2x-\pi)^2}}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{1}{\cos x \sin x} \times \frac{(2x-\pi)^2}{-2}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{(2x-\pi)^2}{-2 \sin x \cos x}$$

$$= - \lim_{x \to \frac{\pi}{2}} \frac{(2x-\pi)^2}{\sin 2x}$$

$$= - \lim_{x \to \frac{\pi}{2}} \frac{\cancel{2}(2x-\pi)(2-0)}{\cancel{2}\cos 2x} \quad [D]$$

$$= - \lim_{x \to \frac{\pi}{2}} \frac{2(2x-\pi)}{\cos 2x}$$

$$= - \frac{2(2 \cdot \frac{\pi}{2} - \pi)}{\cos(2 \cdot \frac{\pi}{2})} = - \frac{2(\pi - \pi)}{\cos \pi}$$

$$= \frac{-2 \cdot 0}{-1} = 0$$

$$\therefore \lim_{x \to \frac{\pi}{2}} (\tan x)^{2x-\pi} = e^0 = 1$$

$$\lim_{x \to 0} x^x$$

Here,

$$y = x^x$$

$$\ln y = x \ln x = \frac{\ln x}{x^{-1}}$$

$$\therefore \lim_{x \to 0} \frac{\ln x}{x^{-1}}$$

$$= \lim_{x \to 0} \frac{\frac{1}{x}}{\frac{-1}{x^2}} \quad [D]$$

$$= \lim_{x \to 0} \frac{1}{x} \cdot \frac{-x^2}{1}$$

$$= \lim_{x \to 0} -x^2$$

$$= -\lim_{x \to 0} x^2 = 0$$

$$\therefore \lim_{x \to 0} x^x = e^0 = 1$$

$$\lim_{x \to 0^+} (\tan 5x)^x$$

Here,

$$y = (\tan 5x)^x$$

$$\ln y = x \ln(\tan 5x)$$

$$\therefore \lim_{x \to 0^+} \frac{\ln(\tan 5x)}{x^{-1}}$$

$$= \lim_{x \to 0^+} \frac{\frac{1 \cdot 5}{\tan 5x} \cdot \sec^2 5x}{\frac{-1}{x^2}} \quad \boxed{D}$$

$$= \lim_{x \to 0^+} \frac{-5}{\cos^2 5x} \cdot \frac{\cos 5x}{\sin 5x} \cdot x^2$$

$$= \lim_{x \to 0^+} \frac{-5x^2 \times 2}{2 \sin 5x \cos 5x}$$

$$= -10 \lim_{x \to 0^+} \frac{x^2}{\sin 10x}$$

$$= -10 \lim_{x \to 0^+} \frac{2x}{10 \cos 10} \quad \boxed{D}$$

$$= -10 \cdot \frac{0}{10} = 0$$

$$\therefore \lim_{x \to 0^+} (\tan 5x)^x = e^0 = 1$$

$$\circledast \quad \lim_{x \to 0^+} (\sin 2x)^{\tan 3x}$$

Here,

$$y = (\sin 2x)^{\tan 3x}$$

$$\ln y = \tan 3x \ln (\sin 2x)$$

$$= \frac{\ln (\sin 2x)}{(\tan 3x)^{-1}}$$

$$\therefore \lim_{x \to 0^+} \frac{\ln (\sin 2x)}{(\tan 3x)^{-1}}$$

$$= \lim_{x \to 0^+} \frac{\ln (\sin 2x)}{\cot 3x}$$

L' Hospital's Rule

$$y = \left(\frac{x+1}{x+2}\right)^x$$

Here,

$$\ln y = x\ln\left(\frac{x+1}{x+2}\right)$$

$$= \frac{\ln\left(\frac{x+1}{x+2}\right)}{x^{-1}}$$

$$\therefore \lim_{x\to\infty} \frac{\ln\left(\frac{x+1}{x+2}\right)}{x^{-1}}$$

$$= \lim_{x\to\infty} \frac{\frac{x+2}{x+1}\cdot\frac{(1+0)(x+2)-(1+0)(x+1)}{(x+2)^2}}{\frac{-1}{x^2}}$$

$$= \lim_{x\to\infty} \frac{\frac{x+2-x-1}{(x+1)(x+2)}}{\frac{-1}{x^2}}$$

$$= \lim_{x\to\infty} \frac{-1}{(x+1)(x+2)}\cdot x^2$$

$$= -\lim_{x\to\infty} \frac{x^2}{x^2+3x+2}$$

$$= -\lim_{x\to\infty} \frac{2x}{2x+3}$$

$$= -\lim_{x\to\infty} \frac{2}{2}$$

$$= -1$$

$$\therefore \lim_{x\to\infty} \left(\frac{1+x}{x+2}\right)^x = e^{-1}$$

(*)

$$\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x$$

here,

$$y = \left(1 + \frac{1}{x}\right)^x$$

$$\ln y = x \ln\left(\frac{x+1}{x}\right)$$

$$\lim_{x \to \infty} x \ln\left(\frac{x+1}{x}\right)$$

$$= \lim_{x \to \infty} \frac{\ln\left(\frac{x+1}{x}\right)}{x^{-1}}$$

$$= \lim_{x \to \infty} \frac{\frac{x}{x+1} \cdot \frac{x(1+0) - (x+1)}{x^2}}{\frac{-1}{x^2}}$$

$$= \lim_{x \to \infty} \frac{\frac{x}{1+x} \cdot \frac{x - x - 1}{x^2}}{\frac{-1}{x^2}}$$

$$= \lim_{x\to\infty} \frac{\frac{1}{1+x}\cdot\frac{-1}{x}}{\frac{-1}{x^2}}$$

$$= \lim_{x\to\infty} \frac{1}{x(1+x)}\cdot x^2$$

$$= \lim_{x\to\infty} \frac{x}{1+x}$$

$$= \lim_{x\to\infty} \frac{1}{1} = e$$

$$\therefore \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e^1 = e$$

⊛ $$\lim_{x\to 0^+} (1+\sin 7x)^{\cot 5x}$$

Here,

$$y = (1+\sin 7x)^{\cot 5x}$$

$$\ln y = \cot 5x \cdot \ln(1+\sin 7x)$$

$$= \frac{\cos 5x \ln(1+\sin 7x)}{\sin 5x}$$

Now,

$$\lim_{x \to 0^+} \frac{\cos 5x \ln(1 + \sin 7x)}{\sin 5x}$$

$$= \lim_{x \to 0^+} \cos 5x \cdot \lim_{x \to 0^+} \frac{\ln(1 + \sin 7x)}{\sin 5x}$$

$$= 1 \cdot \lim_{x \to 0^+} \frac{\frac{1}{1 + \sin 7x} \cdot (0 + 7\cos 7x)}{5\cos 5x}$$

$$= \lim_{x \to 0^+} \frac{\frac{7\cos 7x}{1 + \sin 7x}}{5\cos 5x}$$

$$= \lim_{x \to 0^+} \frac{7\cos 7x}{1 + \sin 7x} \cdot \frac{1}{5\cos 5x}$$

$$= \lim_{x \to 0^+} \frac{7\cos 7x}{5\cos 5x} \cdot \frac{1}{1 + \sin 7x}$$

$$= \lim_{x \to 0^+} \frac{7\cos 7x}{5\cos 5x} \cdot \lim_{x \to 0^+} \frac{1}{1 + \sin 7x}$$

$$= \cancel{\frac{7}{5} \cdot \lim}$$

$$= \frac{7}{5} \cdot \frac{1}{1 + 0} = \frac{7}{5}$$

$$\therefore \lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x = e^{7/5} \quad \underline{\text{Ans}}$$

$$\circledast \quad \lim_{x \to 0} \frac{\sin 2x}{\cancel{\cos 2x}\ \sin}$$

$$= \lim_{x \to 0} \frac{1}{\cos 2x} \cdot \lim_{x \to 0} \sin 2x$$

$$= 1 \cdot \lim_{x \to 0} 2\cos 2x$$

$$=$$

$$\circledast \quad \lim_{x \to 0} \frac{\sin 2x}{\sin 5x}$$

$$= \lim_{x \to 0} \frac{2\cos 2x}{5\cos 5x}$$

$$= \frac{2}{5}$$

Worksheet [1 - 6]

L' Hospital's rule

1. $\lim_{x \to 1} \dfrac{\ln x}{x-1}$

$= \lim_{x\to 1} \dfrac{\frac{d}{dx}(\ln x)}{\frac{d}{dx}(x-1)}$

$= \lim_{x\to 1} \dfrac{\frac{1}{x}}{1}$

$= \lim_{x\to 1} \dfrac{1}{x} = 1$ (Ans.)

2. $\lim_{x\to 3} \dfrac{x-3}{3x^2-13x+12}$

$= \lim_{x\to 3} \dfrac{x-3}{3x^2-9x-4x+12}$

$= \lim_{x\to 3} \dfrac{x-3}{3x(x-3)-4(x-3)}$

$= \lim_{x\to 3} \dfrac{\cancel{x-3}\,1}{3x-4}$

$= \dfrac{1}{9-4} = \dfrac{1}{5}$

L'Hospit's

$\lim_{x\to 3} \dfrac{x-3}{3x^2-13x+12}$

$= \lim_{x\to 3} \dfrac{1}{6x-13}$

$= \dfrac{1}{18-13}$

$= \dfrac{1}{5}$

(3) $$\lim_{x\to 0}\left(\frac{1}{x^2}-\frac{\cos 3x}{x^2}\right)$$

$$= \lim_{x\to 0}\left(\frac{1-\cos 3x}{x^2}\right)$$

$$= \lim_{x\to 0}\left(\frac{0+3\sin 3x}{2x}\right)$$

$$= \lim_{x\to 0}\frac{3\sin 3x}{2x}$$

$$= \lim_{x\to 0}\frac{3.3\cos 3x}{2} = \frac{9\cos 3x}{2} = \frac{9}{2}$$ Ans.

(4) $$\lim_{x\to \pi}\frac{\sin x}{x-\pi}$$

$$= \lim_{x\to \pi}\frac{\cos x}{1-0}$$

$$= \lim_{x\to \pi}\cos x$$

$$= \cos(\pi)$$

$$= -1$$ Ans

⑤ $\lim_{x \to 0} \dfrac{x - \tan^{-1}x}{x^3}$

$= \lim_{x \to 0} \dfrac{1 - \dfrac{1}{1+x^2}}{3x^2}$ ⓓ

$= \lim_{x \to 0} \dfrac{\dfrac{1+x^2-1}{1+x^2}}{3x^2}$

$= \lim_{x \to 0} \dfrac{\dfrac{x^2}{1+x^2}}{3x^2}$

$= \lim_{x \to 0} \dfrac{\cancel{x^2}}{1+x^2} \times \dfrac{1}{3\cancel{x^2}}$

$= \lim_{x \to 0} \dfrac{1}{3+3x^2}$

$= \lim_{x \to 0} \dfrac{1}{3+0} = \dfrac{1}{3}$ Ans.

(6) $$\lim_{x\to+\infty} \frac{e^{3x}}{x^2}$$

$$= \lim_{x\to+\infty} \frac{3e^{3x}}{2x} \quad (D)$$

$$= \lim_{x\to+\infty} \frac{3.3e^{3x}}{2} \quad (D)$$

$$= \lim_{x\to+\infty} \frac{9e^{3x}}{2}$$

$$= +\infty$$

(7) $$\lim_{x\to 0} \frac{a^x - 1 - x\log a}{x^2}$$

$$= \lim_{x\to 0} \frac{a^x \ln a - 0 - \log a - x.0}{2x} \quad [D]$$

$$= \lim_{x\to 0} \frac{a^x \ln a - \log a}{2x}$$

$$= \lim_{x\to 0} \frac{(a^x \ln a)\ln a + a^x.0 - 0}{2}$$

$$= \lim_{x\to 0} \frac{a^0 (\ln a)^2}{2} = \frac{(\ln a)^2}{2}$$

Ans.

⊛ exponent rules

$$a^x = e^{\ln(a^x)} = e^{x\ln(a)}$$

⑧ $\lim_{x\to 0}(e^x+x)^{1/x}$

$$= \lim_{x\to 0} e^{\frac{1}{x}(\ln(e^x+x))}$$

If we can find the limit of $\frac{1}{x}\ln(e^x+x)$, we can find the limit of this function simply replacinge the value of the limit.

$$\therefore \lim_{x\to 0}\frac{\ln(e^x+x)}{x}$$

$$= \lim_{x\to 0}\frac{\frac{1}{e^x+x}\cdot(e^x+1)}{1}$$

$$= \lim_{x\to 0}\frac{1+e^x}{x+e^x} = \frac{1+e^0}{0+e^0} = \frac{1+1}{1} = 2$$

As $x\to 0$, the exponent $\ln(e^x+x)^{1/x} \to 2$

$$\therefore \lim_{x\to 0}(e^x+x)^{1/x} = \lim_{x\to 0} e^{\ln(e^x+x)^{1/x}} = e^2$$

Ans.

(9) $\lim_{x \to 0} \left(\frac{1}{x} - \frac{1}{\sin x} \right)$

$= \lim_{x \to 0} \left(\frac{\sin x - x}{x \sin x} \right)$

$= \lim_{x \to 0} \frac{\cos x - 1}{\sin x + x \cos x}$ (D)

$= \lim_{x \to 0} \frac{-\sin x}{\cos x + \cos x - x \sin x}$ (D)

$= \lim_{x \to 0} \frac{-\sin x}{2\cos x - x \sin x}$

$= \frac{-\sin(0)}{2\cos(0) - 0 \cdot \sin(0)} = 0$

(10)

$$\lim_{x \to \pi} (x-\pi) \cot x$$

$$= \lim_{x \to \pi} \frac{x-\pi}{\frac{1}{\cot x}}$$

$$= \lim_{x \to \pi} \frac{x-\pi}{\tan x}$$

$$= \lim_{x \to \pi} \frac{1-0}{\sec^2 x}$$

(D)

$$= \lim_{x \to \pi} \frac{1}{\sec^2 x}$$

$$= \lim_{x \to \pi} \cos^2 x$$

$$= \cos^2(\pi)$$

$$= 1$$

9

$$\lim_{x\to 0}\left(\frac{1}{x}-\frac{1}{\sin x}\right)$$

$$= \lim_{x\to 0}\left(\frac{\sin x - x}{x\sin x}\right)$$

$$= \lim_{x\to 0}\frac{\cos x - 1}{\sin x + x\cos x} \quad \text{D}$$

$$= \lim_{x\to 0}\frac{-\sin x}{\cos x + \cos x - x\sin x} \quad \text{D}$$

$$= \lim_{x\to 0}\frac{-\sin x}{2\cos x - x\sin x}$$

$$= \frac{-\sin(0)}{2\cos(0) - 0\cdot\sin(0)} = 0$$

(10)

$$\lim_{x \to \pi} (x-\pi)\cot x$$

$$= \lim_{x \to \pi} \frac{x-\pi}{\frac{1}{\cot x}}$$

$$= \lim_{x \to \pi} \frac{x-\pi}{\tan x}$$

$$= \lim_{x \to \pi} \frac{1-0}{\sec^2 x}$$ (D)

$$= \lim_{x \to \pi} \frac{1}{\sec^2 x}$$

$$= \lim_{x \to \pi} \cos^2 x$$

$$= \cos^2(\pi)$$

$$= 1$$

(11)

$$\lim_{x\to 0} \frac{\ln(\sin x)}{\ln(\tan x)}$$

$$= \lim_{x\to 0} \frac{\frac{1}{\sin x}}{\frac{1}{\tan x}} \quad \text{D}$$

$$= \lim_{x\to 0} \frac{\cos x}{\sin^2 x}$$

$$= \lim_{x\to 0} \frac{-\sin x}{2\sin x \cos x}$$

$$= \lim_{x\to 0} \frac{-1}{2\cos x}$$

$$= \frac{-1}{2\cdot\cos(0)} = \frac{-1}{2}$$

$$= \lim_{x\to 0} \frac{1}{\cos x}$$

$$= 1$$

Ans

(12)

$$\lim_{x\to\infty} xe^{-x}$$

$$= \lim_{x\to\infty} \frac{x}{e^x}$$

$$= \lim_{x\to\infty} \frac{1}{e^x} \quad \boxed{D}$$

$$= \lim_{x\to\infty} \frac{0}{e^x} \quad \boxed{D}$$

$$= 0$$

(13)

$$\lim_{x\to 0} \frac{\sin 2x}{x}$$

$$= \lim_{x\to 0} \frac{2\cos 2x}{1} \quad \boxed{D}$$

$$= 2\cos(0)$$

$$= 2$$

[D] (14)

$$\lim_{x \to 0} \frac{\sin x}{x^2}$$

$$= \lim_{x \to 0} \frac{\cos x}{2x} \quad (D)$$

$$= \lim_{x \to 0} \frac{-\sin x}{2} \quad (D)$$

$$= \frac{-\sin 0}{2} = 0$$

[D] (15)

$$\lim_{x \to \infty} \frac{x}{e^x}$$

$$= \lim_{x \to \infty} \frac{1}{e^x} \quad [D]$$

$$= \lim_{x \to \infty} \frac{0}{e^x} \quad [D]$$

$$= 0$$

(16)

$$\lim_{x\to 0}\left(\frac{1}{x}-\frac{1}{xe^x}\right)$$

$$=\lim_{x\to 0}\left(\frac{e^x-1}{xe^x}\right)$$

$$=\lim_{x\to 0}\left(\frac{e^x}{e^x+xe^x}\right) \quad \boxed{D}$$

$$=\lim_{x\to 0}\frac{e^x}{e^x(1+x)}$$

$$=\lim_{x\to 0}\frac{1}{1+x}$$

$$=1$$

(17) $$\lim_{x\to 0}\frac{\sin 2x}{\sin 5x}$$

$$=\lim_{x\to 0}\frac{2\cos 2x}{\cancel{5\sin 5}\ 5\cos 5x} \quad (D)$$

$$=\cancel{\lim_{x\to 0}}\ \frac{2}{5}$$ Ans