2.4 Exact Differential Equations

An differential equation of the form

$$M(x,y)\,dx + N(x,y)\,dy = 0$$

is said to be exact differential equation if

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \Rightarrow M_y = N_x$$

solution of an exact equation can obtain

$$\int M\,dx + \int N_{(\underline{x\text{-free terms only}})}\,dy = c$$

Example:

Solve $(e^{2y} - y\cos xy)\,dx + (2xe^{2y} - x\cos xy + 2y)\,dy =$

Solution:

$$M = e^{2y} - y\cos xy \quad \& \quad N = 2xe^{2y} - x\cos xy + 2y$$

$$M_y = \frac{\partial M}{\partial y} = 2e^{2y} - (\cos xy + y(-x\sin xy))$$

$$= 2e^{2y} - \cos xy + xy\sin(xy)$$

$$N_x = \frac{\partial N}{\partial x} = 2e^{2y} - (\cos xy - xy\sin xy)$$

$$= 2e^{2y} - \cos xy + xy\sin xy.$$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

so the given equation is an exact DE.

$$\therefore \int M\,dx + \int N_{(x\text{-free terms only})}\,dy = c$$

$$\Rightarrow \int e^{2y} - y\cos xy\,dx + \int 2y\,dy = c$$

$$\Rightarrow xe^{2y} - y\frac{\sin xy}{y} + 2\frac{y^2}{2} = c$$

$$\Rightarrow xe^{2y} - \sin xy + y^2 = c$$

which is required solution

 # Example: Solve the IVP

$$\frac{dy}{dx} = \frac{xy^2 - \cos x \sin x}{y(1-x^2)}\ ; \quad y(0) = 2$$

solution:

Given

$$\frac{dy}{dx} = \frac{xy^2 - \cos x \sin x}{y(1-x^2)}\ ; \quad y(0) = 2$$

$$\Rightarrow (xy^2 - \cos x \sin x)\,dx = y(1-x^2)\,dy$$

$$\Rightarrow (\cos x \sin x - xy^2)\,dx + y(1-x^2)\,dy = 0$$

$$M = \cos x \sin x - xy^2 \quad \& \quad N = y(1-x^2)$$

Now,

$$M_y = \frac{\partial M}{\partial y} = -2xy\ ; \quad N_x = \frac{\partial N}{\partial x} = -2xy$$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

So it is an exact DE

Solution given by

$$\int M\,dx + \int N_{(x\text{-free terms only})}\,dy = c$$

$$\Rightarrow \int \cos x \sin x - xy^2\,dx + \int y\,dy = c$$

$$\Rightarrow \frac{1}{2}\int \sin 2x - xy^2\,dx + \int y\,dy = c$$

$$\Rightarrow \frac{1}{2}\,\frac{-\cos 2x}{2} - \frac{1}{2}x^2y^2 + \frac{y^2}{2} = c$$

$$\Rightarrow -\cos 2x - 2x^2y^2 + 2y^2 = 4c$$

$$\Rightarrow 2y^2 - 2x^2y^2 - \cos 2x = 4c \quad \text{(ii)}$$

From initial condition $y(0) = 2$

$x = 0$, then $y = 2$

From (ii)

$$2 \cdot 2^2 - 2 \cdot 0 \cdot 2^2 - \cos 0 = 4c$$

$$\Rightarrow 8 - 1 = 4c$$

$$\therefore c = 7/4$$

Putting this value in (ii)

$2y^2 - 2x^2y^2 - \cos 2x = 7$ which is particular solution.

A non-exact DE made exact:

Consider a non-exact DE $M(x,y)dx + N(x,y)dy = 0$

that is $M_y \neq N_x$

- $\dfrac{M_y - N_x}{N}$ is a function of x alone, then an integrating factor $\mu(x) = e^{\int \frac{M_y - N_x}{N} dx}$

- $\dfrac{N_x - M_y}{M}$ is a function of y-alone, then an integrating factor
$$\mu(y) = e^{\int \frac{N_x - M_y}{M} dy}$$

Then multiply the given non-exact DE with the integrating factor $\mu(x)$ or $\mu(y)$. The resulting DE will be an exact DE.

For example:

Solve $xy\,dx + (2x^2 + 3y^2 - 20)dy = 0$

Solution: $M = xy$ and $N = 2x^2 + 3y^2 - 20$

$M_y = x$ $\qquad N_x = 4x$

$\therefore M_y \neq N_x$

$$\frac{N_x - M_y}{M} = \frac{4x - x}{xy} = \frac{3x}{xy} = \frac{3}{y}$$

Now

$$\mu(y) = e^{\int \frac{N_x - M_y}{M} dy} = e^{3\int \frac{1}{y} dy} = e^{3\ln y}$$

$$= e^{\ln y^3} = y^3$$

$$\therefore \mu(y) = y^3$$

Multiply the given DE by y^3,

$$xy^4 dx + (2x^2y^3 + 3y^5 - 20y^3) dy = 0$$

which is an exact DE

The solution is given by

$$\int M dx + \int N (x\text{-free terms only}) dy = c$$

$$\Rightarrow \int xy^4 dx + \int 3y^5 - 20y^3 dy = c$$

$$\Rightarrow \frac{1}{2}x^2y^4 + \frac{1}{2}y^6 - 5y^4 = c$$

Ans.

Exercise 2.4 : 4, 9, 17, 13, 22, 23
31, 37,

4.3 Higher Order DE's
Homogeneous

Homogeneous Linear Equation with constant coefficients.

Second order linear DE with constant coefficients.

$$ay'' + by' + cy = 0 \quad \text{(I)} \quad ; \; a, b \text{ and } c \text{ are constants}$$

If $y = e^{mx}$ is a solution of (I) then

$$y' = me^{mx}$$
$$y'' = m^2 e^{mx}$$

(I) $\Rightarrow$

$$am^2 e^{mx} + bme^{mx} + ce^{mx} = 0$$

$$\Rightarrow (am^2 + bm + c) e^{mx} = 0$$

Auxiliary equation of given DE is

$$am^2 + bm + c = 0 \quad \text{———— (II)}$$

<u>Case-I:</u> If (II) has two unequal real roots m_1 and m_2 then general solution given by

$$y = c_1 e^{m_1 x} + c_2 e^{m_2 x}$$

<u>Case-II: Equal real roots :</u> If two roots are equal then general solution given by $(m_1 = m_2 = m)$

$$y = c_1 e^{mx} + c_2 x e^{mx}$$

Case III : Complex roots : If the roots are complex

$m_{1,2} = \alpha \pm i\beta$ then

$$y = e^{\alpha x}(c_1 \cos\beta x + c_2 \sin\beta x)$$

Example: Solve $4y'' + 4y' + 17y = 0$, $y(0) = -1$, $y'(0) = 2$

Solution:

Given DE

$$4y'' + 4y' + 17y = 0 \quad \text{——— (1)}$$

After substituting $y = e^{mx}$, $y' = me^{mx}$ and $y'' = m^2 e^{mx}$

the auxiliary equation given by

$$\cancel{4m^2 e^{mx} + 4}$$

$$4m^2 + 4m + 17 = 0$$

$$\Rightarrow m_{1,2} = \frac{-4 \pm \sqrt{4^2 - 4 \cdot 4 \cdot 17}}{2 \cdot 4}$$

$$= -\frac{1}{2} \pm 2i$$

(using calculator you can find the values)

$\therefore \alpha = -\frac{1}{2}$, $\beta = 2$

General solution given by

$$y = e^{-\frac{1}{2}x}(c_1 \cos 2x + c_2 \sin 2x) \quad \text{——— (11)}$$

$$\Rightarrow \cancel{y'(x) = -\frac{1}{2} e^{-\frac{1}{2}x}(c_1 \cos 2x + c_2 \sin 2x) + e^{-\frac{1}{2}x}}$$

$$\cancel{+ c_1 (-\sin 2x (2)}$$

$$y'(x) = e^{-\frac{1}{2}x}(-2c_1 \sin 2x + 2c_2 \cos 2x) - \frac{1}{2}e^{-\frac{1}{2}x}$$
$$\times (c_1 \cos 2x + c_2 \sin 2x) \quad \text{(iii)}$$

$\left[\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}\right]$

Now from initial conditions

$$y(0) = -1$$

$$\text{(ii)} \Rightarrow e^0 (c_1 \cos 0 + c_2 \sin 0) = -1$$

$$\therefore \boxed{c_1 = -1}$$

And $\quad y'(0) = 2$

$$\text{(iii)} \Rightarrow e^0(-2c_1 \sin 0 + 2c_2 \cos 0) - \frac{1}{2}e^0 (c_1 \cos 0 + c_2 \sin 0) = 2$$

$$\Rightarrow 1(0 + 2c_2) - \frac{1}{2}(c_1 + 0) = 2$$

$$\Rightarrow 2c_2 + \frac{1}{2} = 2 \quad [\because c_1 = -1]$$

$$\therefore \boxed{c_2 = \frac{3}{4}}$$

Then the particular solution given by

$$y(x) = e^{-x/2}\left(-\cos 2x + \frac{3}{4}\sin 2x\right)$$

Ans.

Example: Solve $y''' + 3y'' - 4y = 0$

Solution: Given DE

$$y''' + 3y'' - 4y = 0$$

Note
$y'' = m^2$
$y' = m$
$y = m^0 = 1$

Auxiliary given by

$$m^3 + 3m^2 - 4 = 0$$

$$\Rightarrow m_1 = 1, \; m_{2,3} = -2$$

Thus general solution of the DE is

$$y(x) = c_1 e^{m_1 x} + \underbrace{c_2 e^{m_2 x} + c_3 x e^{m_3 x}}_{\text{repeated roots}}$$

$$\therefore \; y(x) = c_1 e^{x} + c_2 e^{-2x} + c_3 x e^{-2x}$$

Ans.

(order of DE = No of arbitrary constant in sol^n)

Exercise: 4.3: 09, 11, 15, 23, 29, 30

4.4 Non-Homogeneous Higher order DE

Undetermined coefficients :

Non-homogeneous linear DE with constant coefficient

$$a_n y^{(n)} + a_{n-1} y^{(n-1)} + \cdots + a_1 y' + a_0 y = g(x)$$

For solving the DE we must do two things

- find the complementary function y_c
  (considering the DE as homogeneous)
- Any particular solution y_p of the DE
  (y_p from $g(x)$)

<u>Trial Particular solution (y_p)</u>

$g(x)$ ... form of y_p

1. 1 (any constant) $\longrightarrow A$
2. $5x+7 \longrightarrow Ax+B$
3. $x^2 / 3x^2+2 / 2x^2+2x+1 \longrightarrow Ax^2+Bx+C$
4. $\sin 4x / \cos 4x \rightarrow A\cos 4x + B\sin 4x$

$g(x)$	form of y_p
5. e^{5x} $\longrightarrow$	Ae^{5x}
6. x^2e^{5x} $\longrightarrow$	$(Ax^2+Bx+c)e^{5x}$

Example: Solve $y''+4y'-2y = 2x^2-3x+6$

<u>Solution:</u> Given $y''+4y'-2y = 2x^2-3$ ——— (1)

For complementary function y_c

$$y''+4y'-2y = 0$$

The corresponding Auxiliary equation given by

$$m^2+4m-2 = 0$$

Solving this we get

$$m_1 = -2-\sqrt{6} \qquad m_2 = -2+\sqrt{6}$$

(use calculator to find the roots)

Then the complementary function is

$$y_c = c_1 e^{(-2-\sqrt{6})x} + c_2 e^{(-2+\sqrt{6})x}.$$

For particular solution; here $g(x) = 2x^2-3x+6$

Let $y_p = Ax^2+Bx+C$

$\Rightarrow y_p' = 2Ax + B$ and $y'' = 2A$

Substituting these in (1)

$$y_p'' + 4y_p' - 2y_p = 2x^2 - 3x + 6$$

$\Rightarrow$ ~~$2A + 4(8\,2Ax$~~

$$\Rightarrow 2A + 4(2Ax + B) - 2Ax^2 - 2Bx - 2C = 2x^2 - 3x + 6$$

$$\Rightarrow -2Ax^2 + (8A - 2B)x + (2A + 4B - 2C) = 2x^2 - 3x + 6$$

Equating the coefficients of x^2, x and constant term

$$-2A = 2, \quad 8A - 2B = -3, \quad 2A + 4B - 2C = 6$$

Solving these system of equations (also can use calculator to solve these)

$$A = -1, \quad B = -\frac{5}{2} \text{ and } C = -9$$

$$\therefore y_p = -x^2 - \frac{5}{2}x - 9$$

Then the general solution of the given equation is

$$y = y_c + y_p$$

$$= c_1 e^{-(2+\sqrt{6})x} + c_2 e^{-(2-\sqrt{6})x} - x^2 - \frac{5}{2}x - 9$$

Ans.

Example: Solve $y'' - y' + y = 2\sin 3x$

Solution :

Given

$$y'' - y' + y = 2\sin 3x \quad \text{———— (1)}$$

For complementary function y_c

$$y'' - y' + y = 0$$

The corresponding auxiliary equation is

$$m^2 - m + 1 = 0$$

Solving this, $m_1 = \frac{1}{2} + \frac{\sqrt{3}}{2} i$

$m_2 = \frac{1}{2} - \frac{\sqrt{3}}{2} i$

$\therefore \alpha = \frac{1}{2}, \ \beta = \frac{\sqrt{3}}{2}$

The complementary function given by

$$y_c = e^{x/2}\left(c_1 \cos\frac{\sqrt{3}}{2}x + c_2 \sin\frac{\sqrt{3}}{2}x\right)$$

———— (ii)

For particular solution, here $g(x) = 2\sin 3x$

Let $y_p = A\cos 3x + B\sin 3x$ be particular solution

then,

$$y_p' = -3A\sin 3x + 3B\cos 3x$$

$$y_p'' = -9A\cos 3x - 9B\sin 3x$$

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Then y_p will satisfy the DE (1)

$$y_p'' - y_p' + y_p = 2\sin 3x$$

$$\Rightarrow (-9A\cos 3x - 9B\sin 3x) - (3A\sin 3x + 3B\cos 3x)$$
$$+ (A\cos 3x + B\sin 3x) = 2\sin 3x$$

$$\Rightarrow (-9A - 3B + A)\cos 3x + (-9B + 3A + B)\sin 3x$$
$$= 2\sin 3x$$

$$\Rightarrow (-8A - 3B)\cos 3x + (3A - 8B)\sin 3x = 2\sin 3x$$

Equating coefficients

$$-8A - 3B = 0 \quad , \quad 3A - 8B = 2$$

Solving, $\quad A = \frac{6}{73} \quad , \quad B = -\frac{16}{73}$

So the particular solution is

$$y_p = \frac{6}{73}\cos 3x - \frac{16}{73}\sin 3x \quad \text{——— (iii)}$$

General solution given by

$$y = e^{x/2}\left(c_1\cos\frac{\sqrt{3}}{2}x + c_2\sin\frac{\sqrt{3}}{2}x\right)$$
$$+ \frac{6}{73}\cos 3x - \frac{16}{73}\sin 3x$$

Ans.

Example: Solve $y'' - 2y' + y = e^x$

Solution:

Given DE

$$y'' - 2y' + y = e^x \text{ ------ (I)}$$

For complementary function y_c

$$y'' - 2y' + y = 0$$

Corresponding auxiliary equation

$$m^2 - 2m + 1 = 0$$

$$\Rightarrow (m-1)^2 = 0$$

$$\therefore m_1 = 1, \quad m_2 = 1 \text{ (repeated)}$$

So the complementary function

$$y_c = c_1 e^x + c_2 x e^x \text{ ------ (II)}$$

For particular solution, here $g(x) = e^x$

Let $y_p = Ax^2 e^x$

Now $y_p' = A(x^2 e^x + 2x e^x)$

$$y_p'' = A(x^2 e^x + 2xe^x + 2e^x + 2xe^x)$$

$$\therefore y_p'' = A(x^2 e^x + 4xe^x + 2e^x)$$

if $g(x) = e^x$ then we should choose $y_p = Ae^x$ but in y_c there is an Ae^x and Axe^x term so we choice $y_p = Ax^2 e^x$

From (1)

$$y_p'' - 2y_p' + y_p = e^x$$

$$\Rightarrow A(x^2e^x + 4xe^x + 2e^x) - 2(x^2e^x + 2xe^x)A$$
$$+ Ax^2e^x = e^x$$

$$\Rightarrow 2Ae^x = e^x$$

$$\therefore A = \frac{1}{2}$$

$$\therefore y_p = \frac{1}{2}x^2e^x$$

General solution given by

$$y = y_c + y_p = c_1e^x + c_2xe^x + \frac{1}{2}x^2e^x$$

Ans

Exercise 4.4 : 03, 05, 07

Modeling with first order DE

Ch-03

Malthusian Population Model: The rate at which population of a country grows at a certain time is proportional to total population of the country at that time.

In mathematically, if $P(t)$ denotes the population at time t then

$$\frac{dP}{dt} \propto P$$

$$\Rightarrow \frac{dP}{dt} = KP$$

where K is the proportionality constant. For growth $K > 0$ and decay $K < 0$

Newton's Law of Cooling/Warming: The rate at which temperature of a body changes is proportional to the difference between the ~~that~~ temperature of the body and the temperature of the surrounding medium.

If $T(t)$ is the temperature of the body at time t, T_m, temperature of the medium then,

$$\frac{dT}{dt} \propto T - T_m$$

$$\Rightarrow \frac{dT}{dt} = k(T - T_m)$$

where k is the constant of proportionality.

<u>Example:</u> A culture initially has P_0 number of bacteria. At $t = 1\,h$ the number of bacteria is masured to be $\frac{3}{2}P_0$. If the rate of growth is proportional to the number of bacteria $P(t)$ present at time t. Determine the time necessary for the number of bacteria to triple.

<u>solution:</u>

As the rate of growth is the proportional to the number of bacteria then,

$$\frac{dP}{dt} = k\cancel{t}P \; ; \quad k \text{ is any constant.} \qquad \text{——— (I)}$$

Given

$$\left.\begin{aligned} P(0) &= P_0 \\ P(1) &= \tfrac{3}{2}P_0 \end{aligned}\right\} \qquad \text{——— (II)}$$

we need to find t for which $P(t) = 3P_0$.

Now from (1)

$$\frac{dP}{dt} = kP$$

$$\Rightarrow \frac{1}{P} dp = k\,dt$$

$$\Rightarrow \ln P = kt + \ln c \quad \text{[integrating]}$$

$$\Rightarrow \ln P = \ln e^{kt} + \ln c$$

$$\Rightarrow \ln P = \ln (c e^{kt})$$

$$\therefore P = c e^{kt} \text{ ——————— (III)}$$

From initial condition $t=0$, $P = P_0$

$$P_0 = ce^0$$

$$\therefore c = P_0$$

and also $t=1$, $P = \frac{3}{2} P_0$

$$\therefore \frac{3}{2} P_0 = P_0 e^k$$

$$\Rightarrow e^k = 3/2$$

$$\Rightarrow \ln (e^k) = \ln (3/2)$$

$$\Rightarrow k = \ln (3/2)$$

~~From (III), P(t)=~~

So (iii) becomes

$$P(t) = P_0\, e^{t\ln(3/2)} \quad \text{———} (iv)$$

Let for $t = t'$, $P = 3P_0$ from (iv)

$$3P_0 = P_0\, e^{t'\ln(3/2)}$$

$$\Rightarrow \quad e^{t'\ln(3/2)} = 3$$

$$\Rightarrow \quad t'\ln(3/2) = \ln 3$$

$$\Rightarrow \quad t' = \frac{\ln 3}{\ln 3/2} = 2.71 \text{ h}$$ Ans.

Example: ~~A fossilized bone is found~~ to ~~contain~~ ~~one thousandth of~~

Example: A breeder reactor converts relatively stable Uranium 238 into the isotope plutonium 239. After 15 years it is determined that 0.043% of the initial amount A_0 of plotonium has disintegrated. Find half-life time of this isotope if the rate of distintegration is proportional to the amount remaining.

Solution:

Let $A(t)$ denote the amount of plutonium present at time t.

The mathematical model given by

$$\frac{dA}{dt} = KA \quad ; A(0) = A_0 \text{ ——— (I)}$$

Now as 0.043% of atoms of A_0 have ~~dist~~ disintegrated, then $(100-0.043)\% = 99.957\%$ of the substance remains.

$$\therefore \quad A(15) = 99.957 \text{ of } A_0$$

$$= \frac{99.957}{100} A_0$$

$$\therefore A(15) = 0.99957 A_0 \text{ ——— (II)}$$

Now from (I)

$$\frac{dA}{dt} = kA$$

$$\Rightarrow \frac{1}{A} dA = k\,dt$$

$$\Rightarrow \int \frac{1}{A} dA = \int k\,dt$$

$$\Rightarrow \ln(A) = kt + \ln c$$

$$\Rightarrow \ln(A) = \ln e^{kt} + \ln c$$

$$\Rightarrow \ln(A) = \ln(ce^{kt})$$

$$\therefore A = ce^{kt} \quad \text{———— (III)}$$

From initial condition $A(0) = A_0$ $(t=0, A = A_0)$

$$A_0 = ce^0 \quad \therefore c = A_0$$

Now from (II), $t = 15$, $A = 0.99957 A_0$

$$\text{(III)} \Rightarrow 0.99957 A_0 = A_0 e^{15K} \quad [\because c = A_0]$$

$$\Rightarrow e^{15K} = 0.99957$$

$$\Rightarrow \ln(e^{15K}) = \ln(0.99957)$$

$$\Rightarrow 15K = \ln(0.99957)$$

$$\Rightarrow K = \frac{\ln(0.99957)}{15}$$

Substituting the values of c and k in (III)

$$A(t) = A_0 e^{\frac{t}{15}\ln(0.99957)} \quad \text{———— (IV)}$$

Let for $t = t' \Rightarrow A = \frac{1}{2}A_0$ (half life time)

$$\text{(IV)} \Rightarrow \frac{1}{2}A_0 = A_0 e^{t'/15 \ln(0.99957)}$$

$$\Rightarrow \frac{1}{2} = e^{-0.00002867t}$$

$$\Rightarrow \ln\left(\frac{1}{2}\right) = -0.00002867\,t$$

$$\Rightarrow \ln(2^{-1}) = -0.00002867\,t$$

$$\Rightarrow -\ln 2 = -0.00002867\,t$$

$$\Rightarrow t = \frac{\ln 2}{0.00002867} \approx 24,180 \text{ yr.}$$

Ans.

Example

When a cake is removed from an oven, its temperature is measured at 300°F. Three minutes later its temperature is 200°F. How long will it take for the cake to cool off to a room temperature of 70°F.

Solution: Given

$$\left.\begin{aligned} T(0) &= 300^\circ F \\ T_m &= 70^\circ F \end{aligned}\right\} \text{—— (i)}$$

The mathematical model given by

$$\frac{dT}{dt} = k(T - T_m)$$

or $$\frac{dT}{dt} = k(T - 70) \text{ —— (ii)}$$

Now solving (ii)

$$\Rightarrow \frac{dT}{T-70} = k\,dt$$

$$\Rightarrow \ln|T-70| = kt + \ln c$$

$$\Rightarrow \ln|T-70| = \ln e^{kt} + \ln c$$

$$\Rightarrow T-70 = ce^{kt}$$

$$\therefore T(t) = ce^{kt} + 70 \quad \text{——— (III)}$$

From initial condition $t=0$, $T=300$ so from (III)

$$300 = ce^{0} + 70$$

$$\therefore c = \cancel{230}\ 230$$

From the condition $t=3$, $T=200\,F$

$$200 = 230e^{3k} + 70 \quad [\because c = 230]$$

$$\Rightarrow e^{3k} = \frac{130}{230}$$

$$\Rightarrow \ln e^{3k} = \ln\left(\frac{13}{23}\right)$$

$$\Rightarrow 3k = \ln\left(\frac{13}{23}\right)$$

$$\Rightarrow k = \frac{1}{3}\ln\left(\frac{13}{23}\right)$$

Substituting the values of c and k in (III) we get,

$$T(t) = 230e^{t\frac{1}{3}\ln\left(\frac{13}{23}\right)} + 70$$

$$\Rightarrow T(t) = 70 + 230\, e^{-0.19018t} \quad \text{—— (iv)}$$

Let for $t = t'$, $T = 70$

from (iv) $\Rightarrow$

$$\therefore \quad 70 = 70 + 230\, e^{-0.19018t'}$$

There is no finite solution of this equation since $\lim_{t\to\infty} T(t) = 70$

Let us calculate the time for $T = 70.5$

$$70.5 = 70 + 230\, e^{-0.19018t'}$$

$$\Rightarrow e^{-0.19018t'} = \frac{0.5}{230}$$

$$\Rightarrow \ln\left(e^{-0.19018t'}\right) = \ln\left(\frac{0.5}{230}\right)$$

$$\Rightarrow -0.19018t' = \ln\left(\frac{0.5}{230}\right)$$

$$\therefore t' = 32.3$$

So it will take about 32.3 minutes to cool off to a room temperature.

<u>Exercise 3.1</u>

1-3, 6, 7, 13-17