Robotics and Intelligent Systems Lab II
Report 4

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Spring 2023

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Task 1.17. *First assume that the disturbace* T_L (s) = 0. *Assume that a unit-step reference input* r(t) = k u(t) *is applied to the system, i.e. we want the controller to take the motor to a constant speed* k *rad/s. Apply the FVT to* (1.32g) *to find the steady state error* e(t) = r(t) -ω(t).

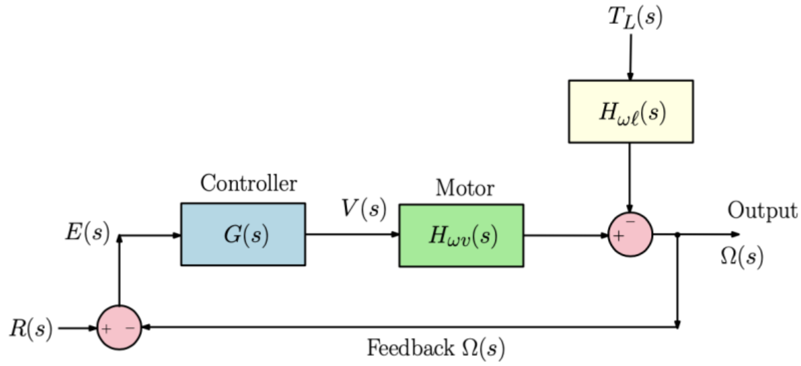


Fig (1)

Assuming that a unit step reference input $r(t) = ku(t)$ is applied,
Laplace transform of $r(t) = ku(t) \rightarrow R(t) = {}^{k}/_{s}$.

$$E(s) = \frac{1}{1+H_{wv}(s)G(s)}R(s) + \frac{H_{wl}(s)}{1+H_{wv}(s)G(s)}T_L(s) \quad (1.32g)$$

$$E(s) \triangleq H_{er}(s)R(s) + H_{el}(s)T_L(s) \quad (1.32h)$$

Since $T_L(s) = 0$,

$$E(s) = H_{er}(s)R(s)$$

During steady-state, the PID controller reduces the error to zero, and this needs to be proven using FVT. To apply FVT to the given equation, we must first determine $H_{er}(s)$.

$$H_{er}(s) = \frac{sD(s)}{P(s)} = \frac{s((R_a + L_a s)(Js + b) + k_e k_t)}{L_a J s^3 + (R_a J + L_a b + k_t K_D)s^2 + (R_a b + k_e k_t + K_P k_t)s + k_t K_I}$$

Now we substitute $H_{er}(s)$ with ${}^{E(s)}/_{R(s)}$ into the equation,

$$\frac{E(s)}{R(s)} = \frac{s((R_a + L_a s)(Js + b) + k_e k_t)}{L_a J s^3 + (R_a J + L_a b + k_t K_D)s^2 + (R_a b + k_e k_t + K_P k_t)s + k_t K_I}$$

$$E(s) = \frac{s\left((R_a + L_a s)(Js + b) + k_e k_t\right)}{L_a J s^3 + (R_a J + L_a b + k_t K_D)s^2 + (R_a b + k_e k_t + K_P k_t)s + k_t K_I} \times \frac{k}{s}$$

For the sake of simplifying our analysis, we make the assumption that L_a is negligible and can be disregarded.

$$= \frac{s\left((R_a)(Js + b) + k_e k_t\right)}{(R_a J + k_t K_D)s^2 + (R_a b + k_e k_t + K_P k_t)s + k_t K_I} \times \frac{k}{s}$$

Now applying FVT,

$$\lim_{s \to 0} sE(s) = 0$$

Task 1.18. *Next assume that* r(t) = ku(t) *and* T_L (t) =,u(t), *i.e. the system is now also disturbed by a load-torque l Nm. Using the FVT, determine the effect of this disturbance on the system steady-state error.*

Assuming $r(t) = ku(t)$ and $T_L(t) = lu(t)$,
Laplace transform of $r(t) = ku(t) \rightarrow R(t) = {}^{k}/_{s}$ and $T_L(t) = lu(t) \rightarrow T_L(t) = {}^{1}/_{s}$.

$$E(s) = \frac{1}{1 + H_{wv}(s)G(s)} R(s) + \frac{H_{wl}(s)}{1 + H_{wv}(s)G(s)} T_L(s) = H_{er}(s)R(s) + H_{el}(s)T_L(s)$$

Substituting $T_L(t) = \frac{1}{s}$, $H_{el}(s) = \frac{R_a + L_a s}{D(s)}$, and $H_{er}(s)R(s)$,

$$E(s) = \left(\frac{s\big((R_a)(Js + b) + k_e k_t\big)}{(R_a J + k_t K_D)s^2 + (R_a b + k_e k_t + K_P k_t)s + k_t K_I} \times \frac{k}{s}\right) + \frac{(I)(R_a + L_a s)}{s((R_a + L_a s)(Js + b) + k_e k_t)}$$

Now applying FVT,

$$\lim_{s \to 0} sE(s) = 0$$

There is no effect as the steady-state remains the same.

Task 1.19. *Copy your previous motor-model .slx file to another file and open it in Simulink. In this new model, create the controller subsystem connected to the motor as shown in Fig. 1.12.*

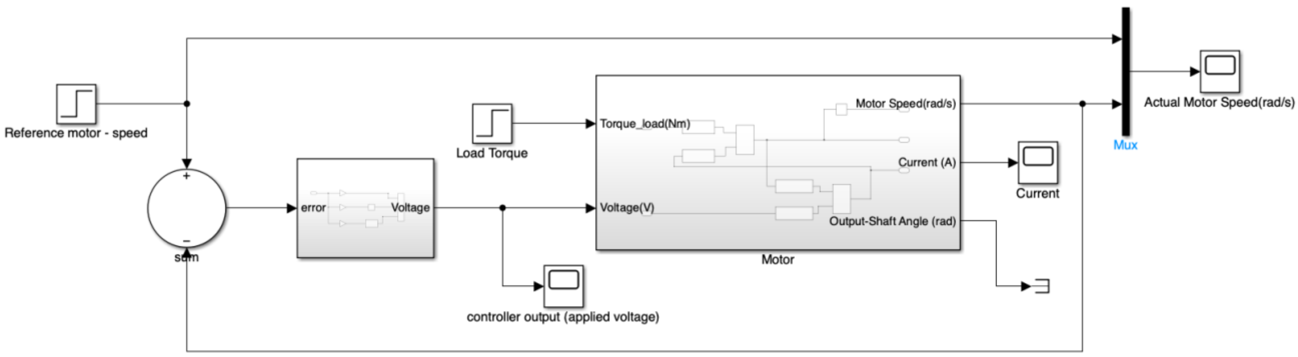


Fig (2)

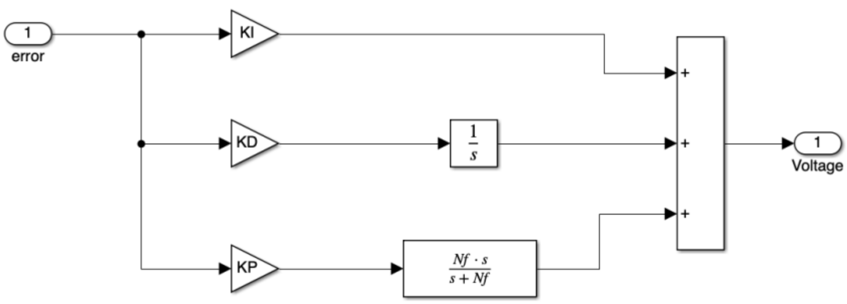


Fig (3)

Task 1.20. *Proceed as follows:*

1. In our controller, let us set $K_I = K_D = 0$ *and* $K_P = 1$*. Let us select a reference speed* $r(t) = 300$ *RPM (convert to rad/s before applying to the model) which is less than* ω_0 *and hence doable. Set the load-torque input to* 0*. Look at the scope: Did* $\omega(t)$ *reach* $r(t)$*? If not, measure the steady-state error* $e(t)$*.*

After setting K_I and K_D equal to 0, $K_P = 1$, reference speed $r(t) = 300\ RPM = 31.416\ {}^{rad}/_{s}$ and load-torque = 0, we get the following results:

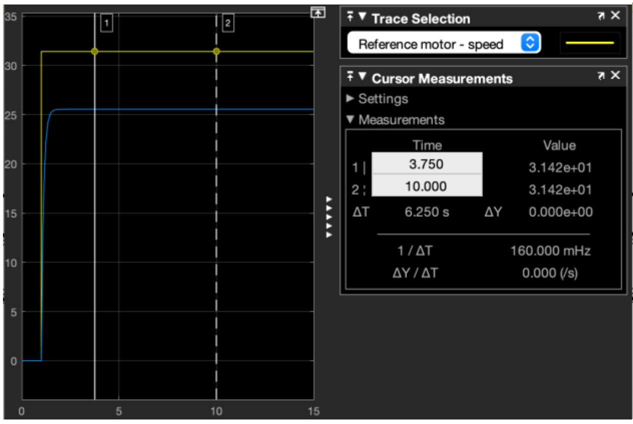


Fig (4)

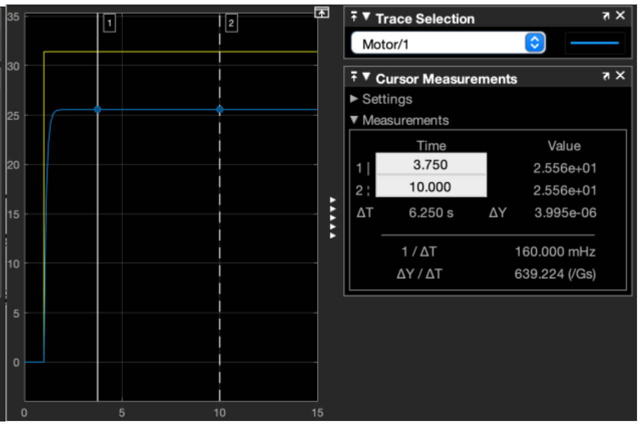


Fig (5)

As we can see in Fig (4) and (5)'s scope, $\omega(t)$ doesn't reach r(t).

$$e(t) = r(t) - \omega(t) = 31.42 - 25.56 = 9.86\ {}^{rad}/_{s}$$

2. Since there is some steady-state error, let us increase KP to 10 and simulate again. What is the steady-state error now? Since it decreased, we could just keep increasing KP till the steady-state error is smaller than some threshold, right?

After increasing K_P to 10, we get

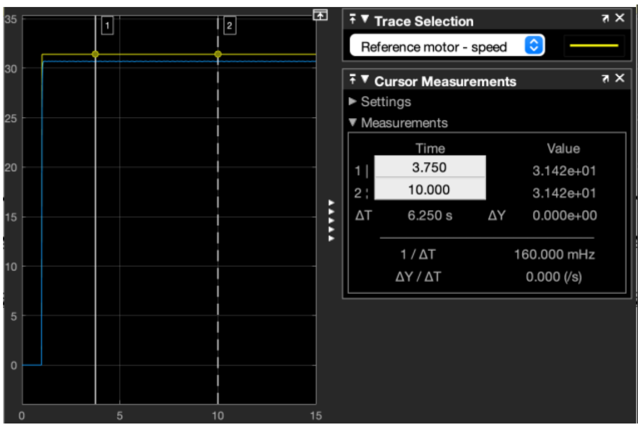


Fig (6)

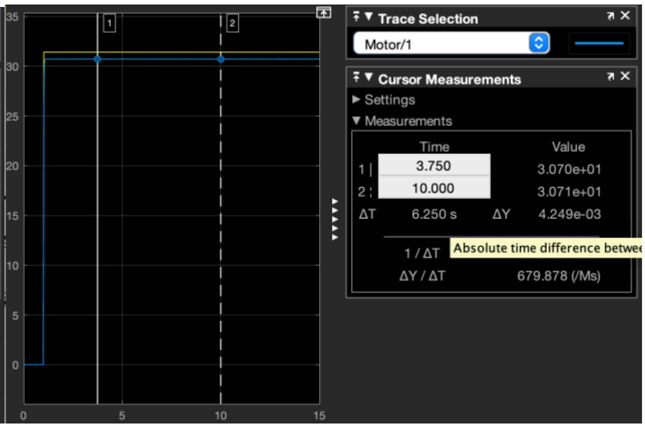


Fig (7)

$$e(t) = r(t) - \omega(t) = 31.42 - 30.70 = 0.72\ {}^{rad}/_{s}$$

3. Wrong. Some part of system just went berserk at $K_P = 10$. Look at the scopes to figure out what went wrong. The moral of the story is that if we just use K_P, there will always be a steady-state error. To have it converge to zero, we need a non-zero K_I.

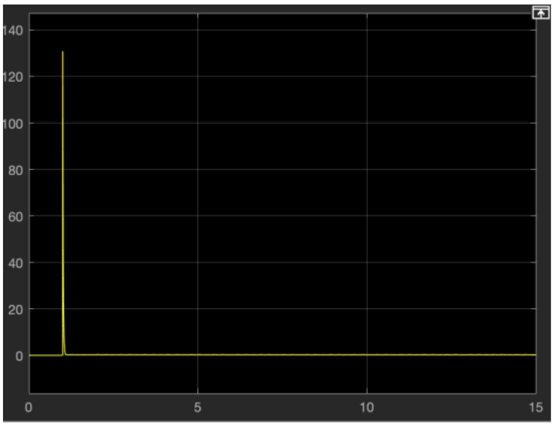


Fig (8)

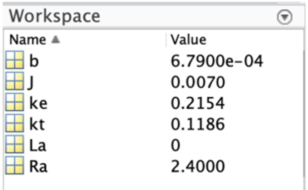
The current scope indicates that the current drawn by the motor is spiking, suggesting that achieving a steady-state error of zero is not possible.

Task 1.21. *Proceed as follows:*

1. *Let us set K_D = 0 as before but set K_P = 0.0084. Select a value of K_I where the system is stable but just begins to show some oscillations. You can find this by writing an expression for the two roots of (1.37) in terms of K_I and find the value of K_I when the discriminant just turns 0. Simulate the system for 30 seconds with this value. Is there a steady-state error?*

To find a suitable value for K_I, we use the formula

$$P(s) = (R_a J + k_t K_D)s^2 + (R_a b + k_e k_t + K_P k_t)s + k_t K_I \qquad (1.37)$$



Workspace

Name	Value
b	6.7900e-04
J	0.0070
ke	0.2154
kt	0.1186
La	0
Ra	2.4000

Fig (9)

Substituting the values from Fig (9) into the function (1.37) and also $K_D = 0$ and $K_P = 0.0084$,

$$P(s) = ((2.4)(0.007) + (0.1186)(0))s^2 + ((2.4)(0.000679) + (0.2154)(0.1186) + (0.0084)(0.1186))s + 0.1186K_I$$

$$P(s) = 0.0168s^2 + (0.0016296 + 0.02554644 + 0.00099624)s + 0.1186K_I$$

$$P(s) = 0.0168s^2 + 0.028172228s + 0.1186K_I$$

Now to find the discriminant of P(s) we use the discriminant formula: $b^2 - 4ac$,

$$\therefore b^2 - 4ac = 0.028172228^2 - 4(0.0168)(0.1186K_I) = 0.00079146 - 0.00797K_I = 0$$

$$K_I = 0.1$$

Fig (10)

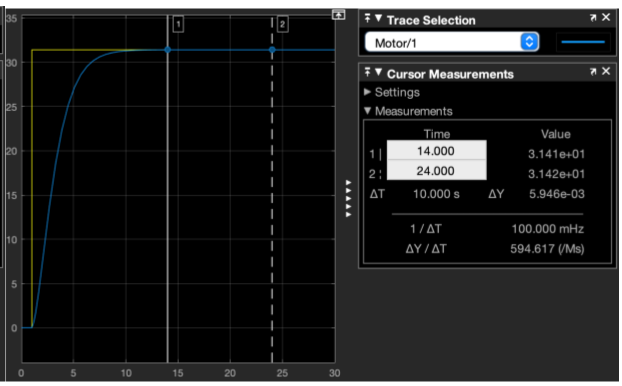
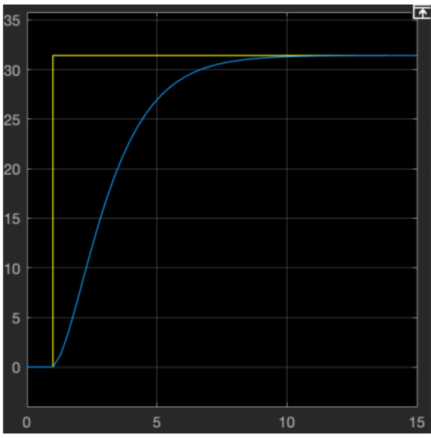


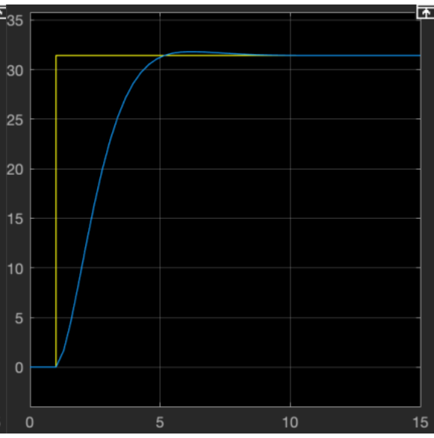
Fig (11)

After setting the variables to their suitable values, the scope tells us that there is no steady-state error now.

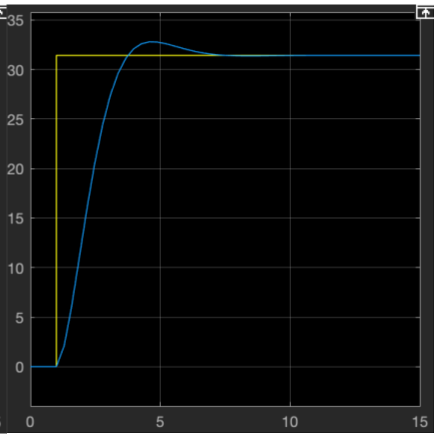
2. *Now select a value of K_I slightly above this threshold, and simulate again. Do you see a ripple before the system settles down?*



$K_I = 0.1$ Fig (12)



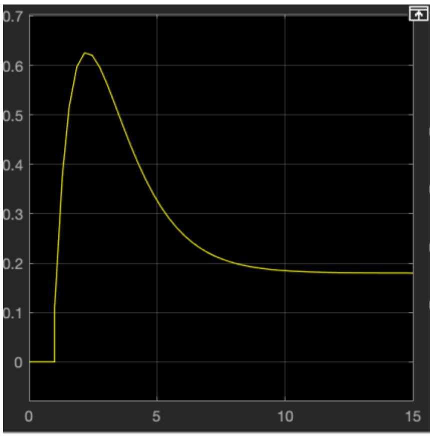
$K_I = 0.15$ Fig (13)



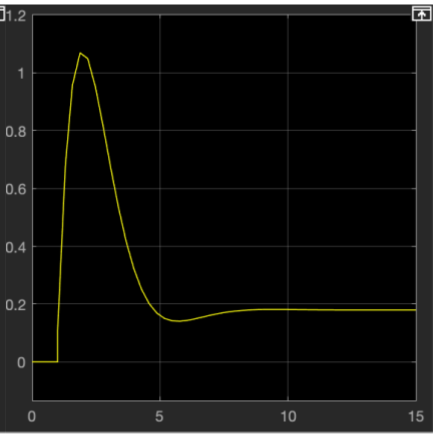
$K_I = 0.2$ Fig (14)

As we can see from the scopes given above, values above a certain value of K_I makes a ripple before the system settles and as the value of K_I increases beyond that certain value, the ripple's amplitude increases too.

3. *Now look at the current values in Amperes. Are they in the safe range? The nice thing about our controller is that it even damps the current-spike when a step-input is applied.*



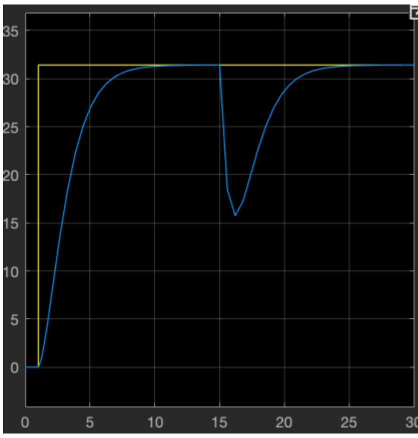
$K_I = 0.1$ Fig (15)



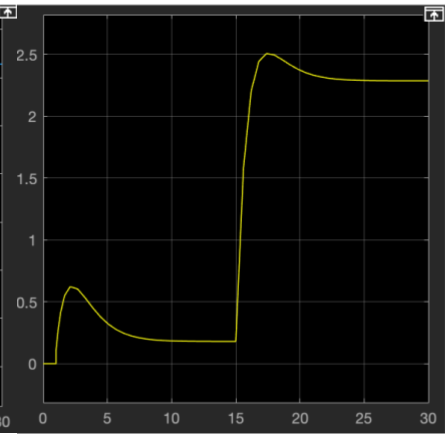
$K_I = 0.2$ Fig (16)

As we can see in Fig (15) and Fig (16), the current values are in a safe range.

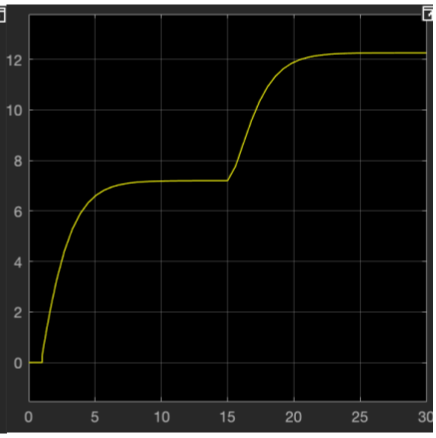
4. *Now add a step load-torque disturbance which is half of the stall torque at t = 15 seconds. Did the steady-state error still go to zero? Also look at the other scopes: Are all values within limits? Refer to Fig. 1.13 for an example.*



Actual Motor Speed Fig (17)

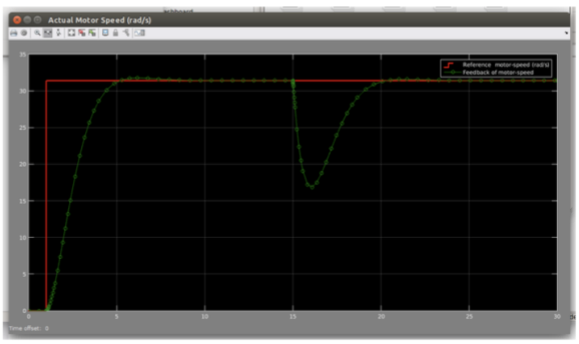


Current Fig (18)

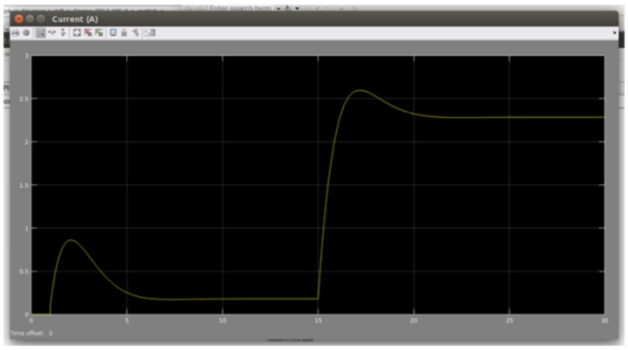


Controller Output Fig (19)

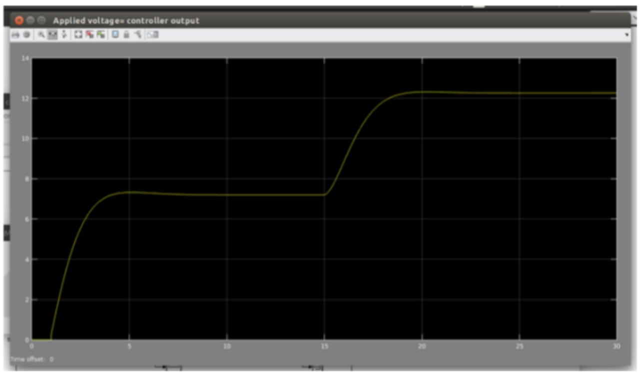
The result on scopes after setting the load-torque to 0.25Nm at t=15s.



(a) The output response to the load-torque step at $t = 15$.



(b) The current response to the load-torque step at $t = 15$.



(c) The voltage output produced by the controller to overcome the load-torque and maintain the reference-speed. Since the voltage is slightly more than the supply-voltage of 12V, we see that in practice, the controller will saturate and will not be able to maintain the reference-speed exactly at this load-torque.

The above three scopes are taken from Fig 1.13 of the Lab Manual.
After examining both set of scopes, we can clearly see that all values are within their respective limits and the steady-state error still goes to zero.

5. *You are, of course, not restricted to step-inputs. Let us apply a reference-speed signal which is sinusoidal with amplitude of 5 rad/s and a bias of 15 rad/s and the frequency of the sine-wave is 1 rad/s. Keep the step torque-load at t = 15 sec. Your scope should look like Fig. 1.14. Are the voltages and currents within limits?*

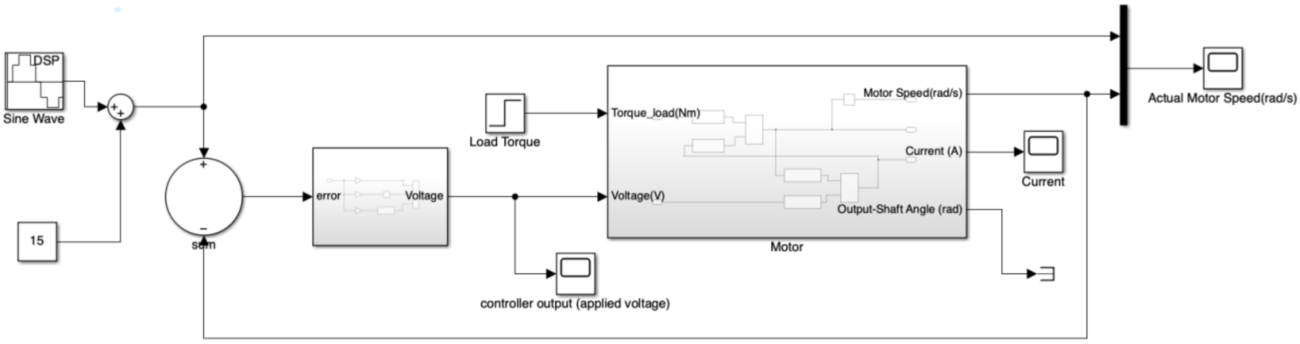


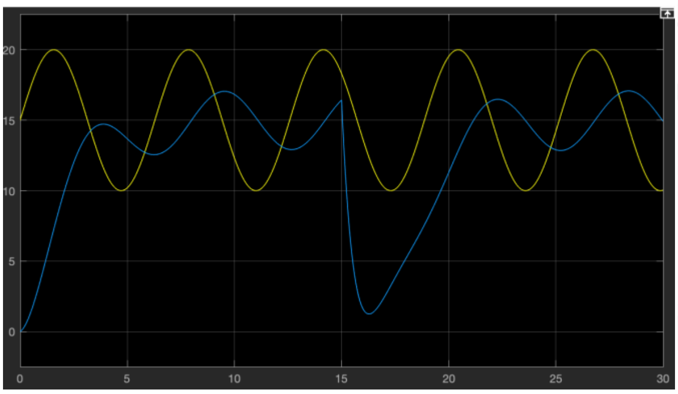
Fig (20)

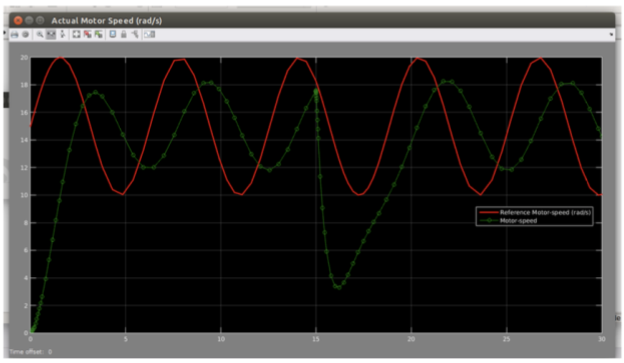
Fig (20) shows the Simulink diagram of the circuit after changing the reference-speed signal to a sinusoidal one. In addition to the sine wave input, there's a 'constant block' with a 'sum block' to create a bias. In order to convert the frequency of the sine-wave from rad/s to Hz, the following formula was used:

$$f(Hz) = \frac{\omega({}^{rad}/_{s})}{2\pi}$$

$$= \frac{5\,{}^{rad}/_{s}}{2\pi} \approx 0.796Hz$$

The following results are shown below.

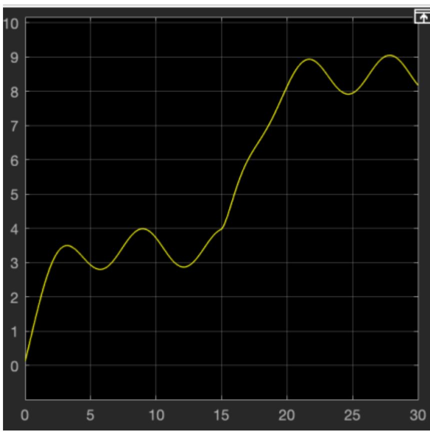


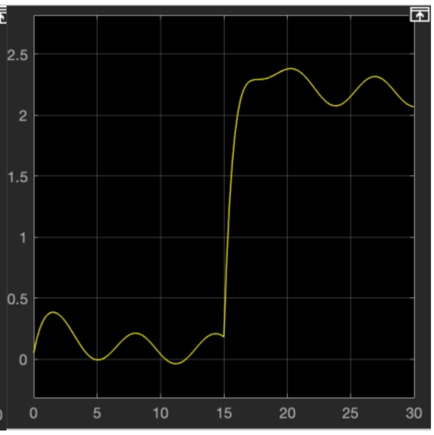


Actual Motor Speed (rad/s) Fig (21)

Figure 1.14: Response of the controller to a sinusoidal reference speed.

Our scope is identical as of the one mentioned in the question.





Applied Voltage Fig (22) Current Fig (23)

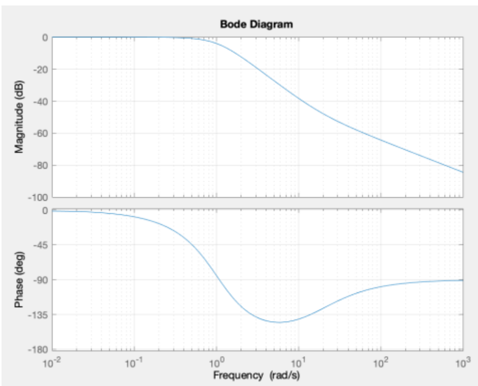
Fig (22) and Fig (23) shows that the voltage and the current are within the limits.

6. *In the previous example, let's try to see if the observed amplitude reduction and phase-difference between the reference and the actual speed sinusoidal signals can be also predicted by the Bode plot. The Bode plot of which transfer function will you have to plot?*

$$\Omega(s) = \frac{H_{\omega v}(s)G(s)}{1+H_{\omega v}(s)G(s)}R(s) - \frac{H_{\omega l}(s)}{1+H_{\omega v}(s)G(s)}T_L(s) \qquad (1.32f)$$

The equation establishes a clear relationship between the output, reference input, and disturbance. Examining the influence of the input and disturbance on the error between the reference input and output may provide further insight and clarity.

Task 1.22. *To plot the Bode plot proceed as shown in the code listing below. Make sure you use your chosen controller constants rather than the ones shown. Do the magnitude and phase-dierence shown by the Bode plot at* 1 *rad/s match with what you observe in the speed scope?*



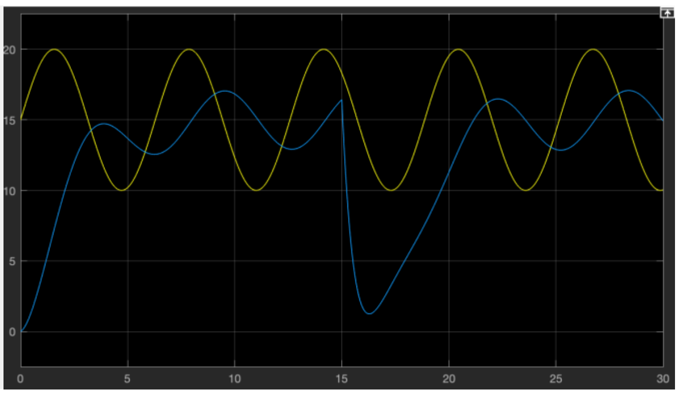


Fig (24) Fig (21.b)

mag = 0.6321, phase = -84.7706

The magnitude and phase difference observed at 1 rad/s on the Bode plot (Fig (24)) correspond to those displayed on the speed scope (Fig (21.b)).

Task 1.23. *How will you realize the derivative filter using a gain (multiplication by a constant) block and negative feedback with an integrator block?* ***Hint****: On the Matlab command line try Hsp= feedback(Nf, 1/s, -1). Now draw this as a block diagram using only gain, integrator and a negative feedback.*

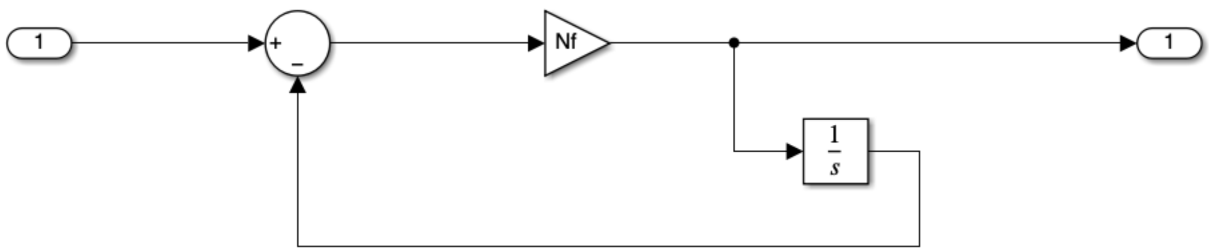


Fig (25)