Robotics and Intelligent Systems Lab II
Report 3

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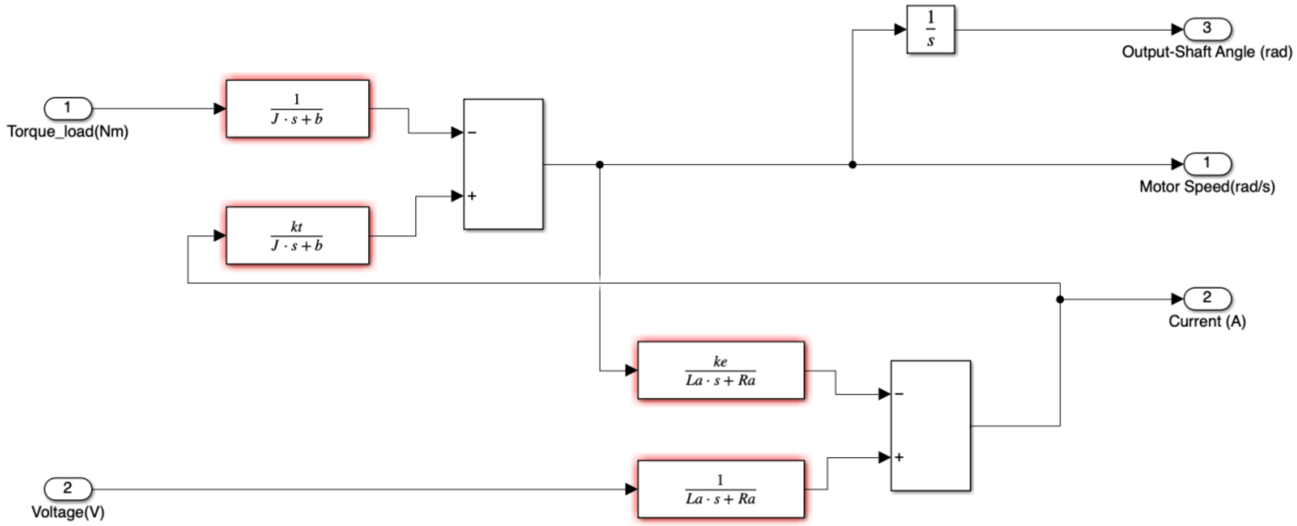
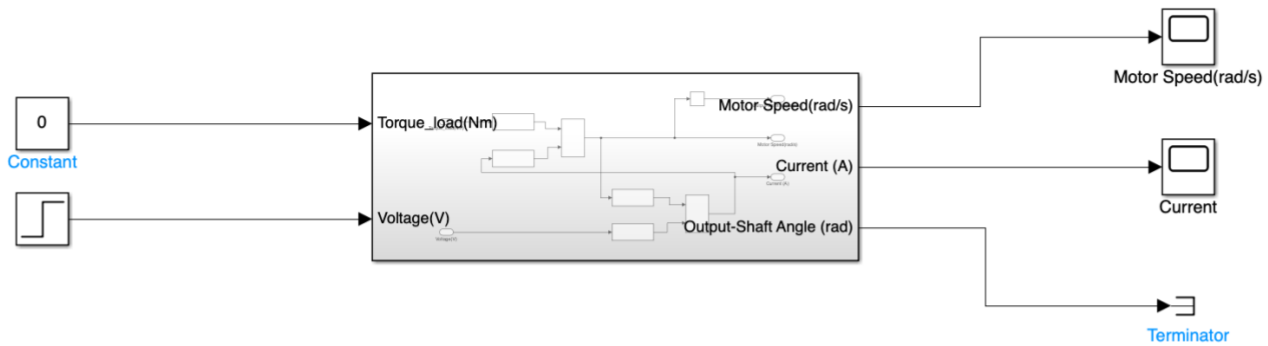
Task 1.10. *Proceed as follows:*

1. *Create the model as shown in Fig. 1.10. Verify that it is indeed implenting the equations (1.24a) and (1.24b).*



(Fig.1)



(Fig.2)

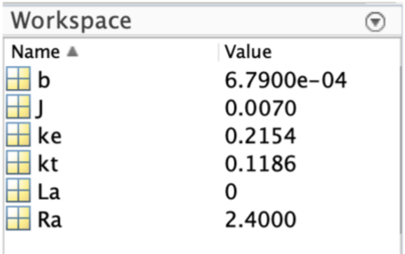
$$\Omega(s) = \frac{k_t}{Js+b} I(s) - \frac{1}{Js+b} T_L(s) \qquad (1.24A)$$

$$I(s) = \frac{1}{L_a s+R_a} V_a(s) - \frac{k_e}{L_a s+R_a} \Omega(s) \qquad (1.24B)$$

As seen in (Fig.1), multiplying Torque_load(Nm) in to Transfer function 1 and subtracting it from Current multiplied in to Transfer function 2 gives us the Motor speed(rad/s) which when written in mathematical terms would turn out to be formula (1.24A).

As seen in (Fig.1), multiplying Motor Speed(rad/s) in to Transfer function 3 and subtracting it from Voltage multiplied in to Transfer function 4 gives us the Current which when written in mathematical terms would turn out to be formula (1.24B).

2. *In the Matlab workspace create the variables representing the motor-parameters such as k_t, k_e, R_a, L_a, J, b, etc.*



(Fig.3)

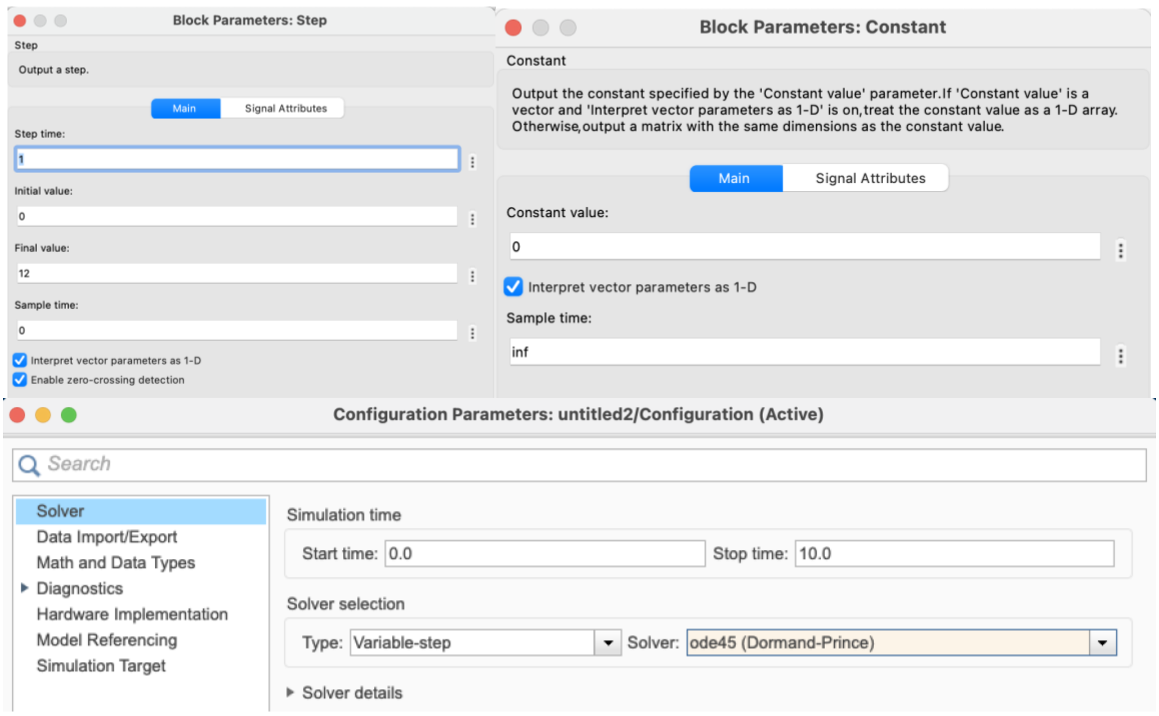
(Fig.3) clearly shows the initialization of the variables representing the motor-parameters which were calculated and reported in Lab Report 2.

motor params.mat

(Fig.4)

(Fig.4) shows the parameter file which consists of the variables shown in (Fig.3). This file will load up every time the Simulink file is opened as we have set it up in a pre-load function.

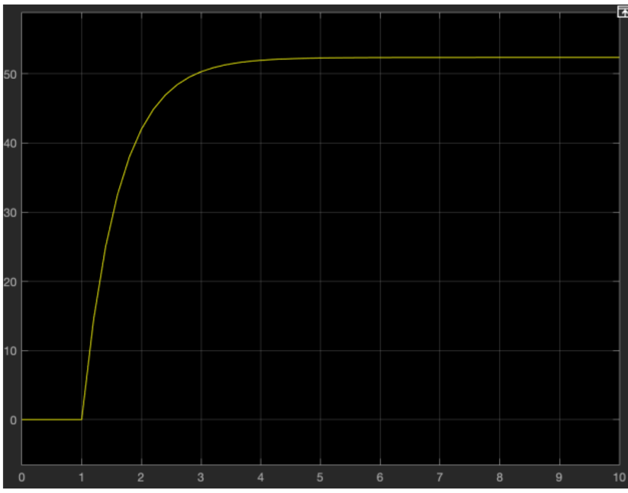
3. *Apply a step of 12 V at t=1. Keep the load-torque to 0. Simulate the system from 0 to 10 seconds using the default solver (ode45).*



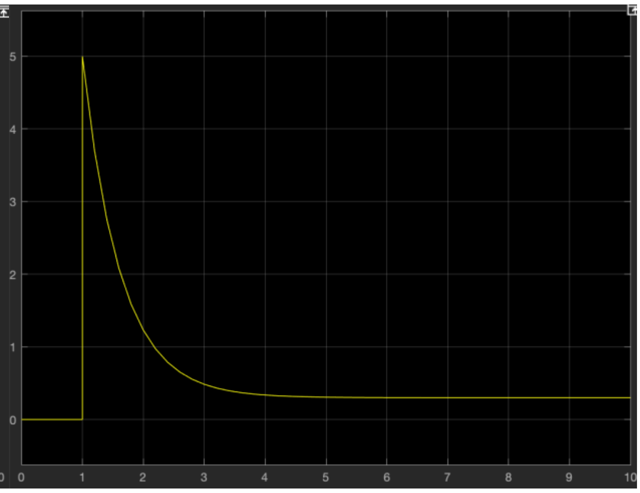
(Fig.5-7)

4. *If the results shown in the scopes do not appear proprly, press Ctrl+E and switch the solver to ode15s (stiff/NDF). Press the “Autoscale” button of the scope to see the whole curve.*

The results were shown properly, as we can see below;



Motor Speed(rad/s) (Fig.8)



Current (Fig.9)

5. *Explain the behavior of the current (the initial peak) and the speed scopes. Hint: Make use of (1.18). Verify that they settle to the no-load values in Table 1.1? You can use the zoom button in the scope.*

$$v_a - IR_a - L_a\frac{di}{dt} = e = k_e\dot{\theta}_g \qquad (1.18)$$

To comprehend the current and motor speed behaviors depicted in Fig.9 and Fig.8 respectively, it is important to bear in mind that the load-torque is zero, and therefore the motor is operating in free-run mode. At t = 1, when the input voltage is transmitted as a step function, the current reaches its peak value. Thereafter, the current gradually decreases and stabilizes at 0.3A, which corresponds to the free-run current or no-load current. On the other hand, the motor speed starts increasing and reaches 52.36rad/s as soon as the input is applied at t = 1, which corresponds to the shaft-speed at no-load or free-run speed.

Verifying if they settle to the no-load values
Current:

$$v_a - IR_a - L_a\frac{di}{dt} = e = k_e\omega_g$$

$$I = \frac{v_a - k_e\omega_g}{R_a}$$

Where according to our calculations, $R_a = 2.4$, $\frac{di}{dt} = 0$, $k_e = 0.2154$, $\omega_g = 52.36\,{}^{rad}/_{s}$ so,

$$I = \frac{12 - (0.2154)(52.36)}{2.4} = 0.3A$$

Motor Speed:

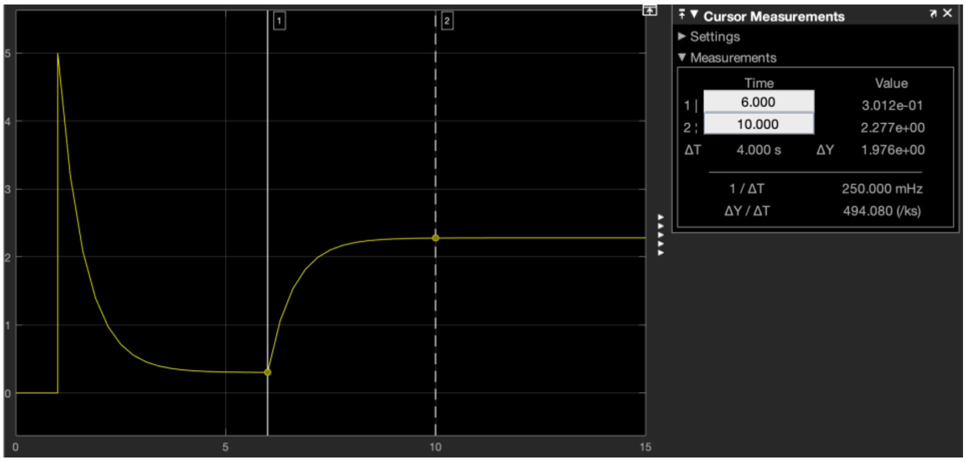
$$v_a - IR_a - L_a\frac{di}{dt} = e = k_e\omega_g$$

$$\frac{v_a - IR_a}{k_e} = \omega_g$$

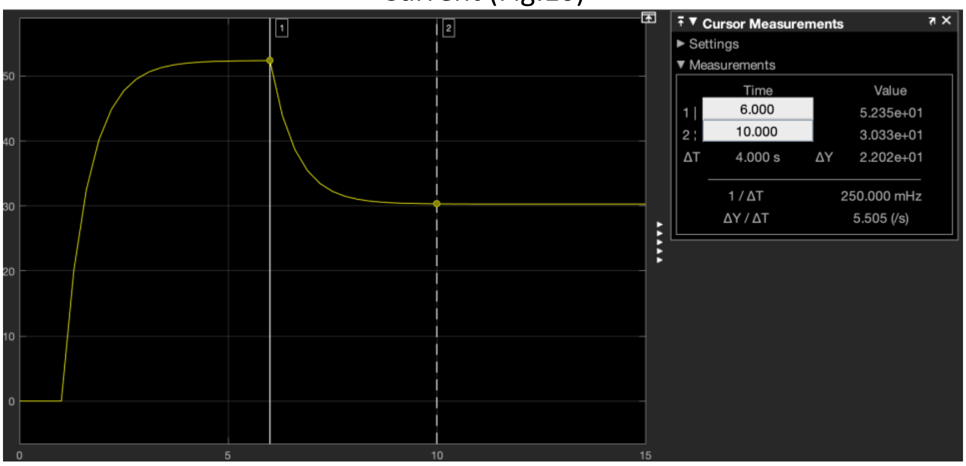
Where according to our calculations, $R_a = 2.4$, $\frac{di}{dt} = 0$, $k_e = 0.2154$, $I = 0.3A$ so,

$$\frac{12 - (0.3 \times 2.4)}{0.2154} = \omega_g = 52.36\ {}^{rad}/_{s}$$

6. *Now at the external load-torque input add a step torque of 0.25 Nm (about half of the stall- torque) at time t = 6 seconds. Increase the simulation interval to 15 seconds. Explain what you see.*



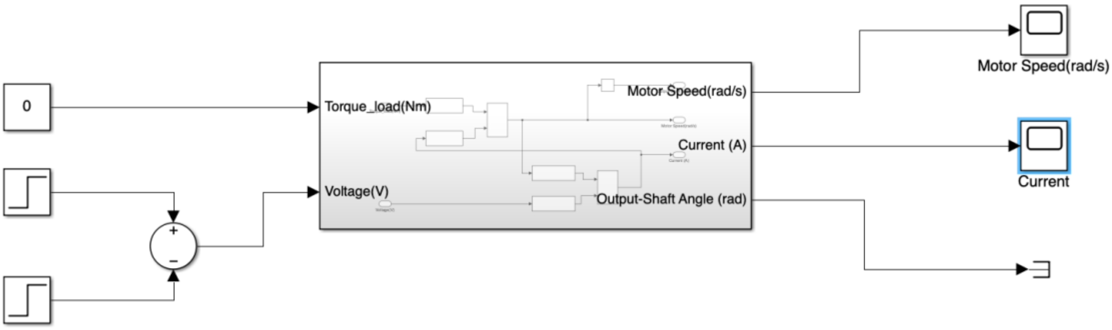
Current (Fig.10)



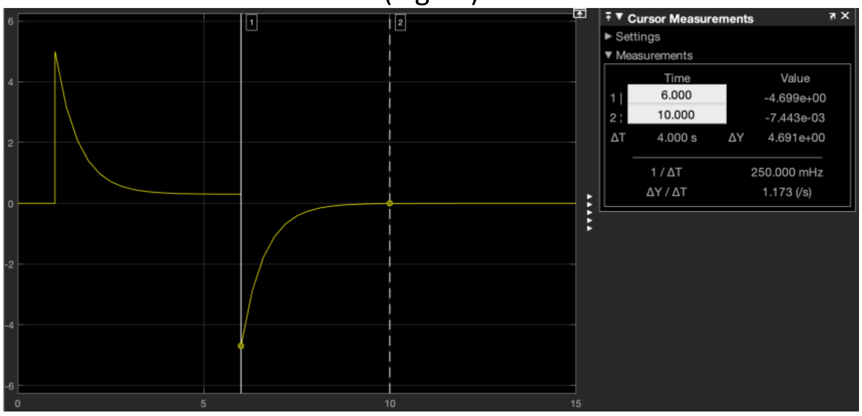
Motor Sped (Fig.11)

Prior to t = 6, the graphs remain unchanged in free-run mode. However, once a load-torque is introduced, the current rises to 2.277A at t = 6, which is approximately half of the stall-current (5A), resulting in a decrease in the motor speed to 30.33 rad/s.

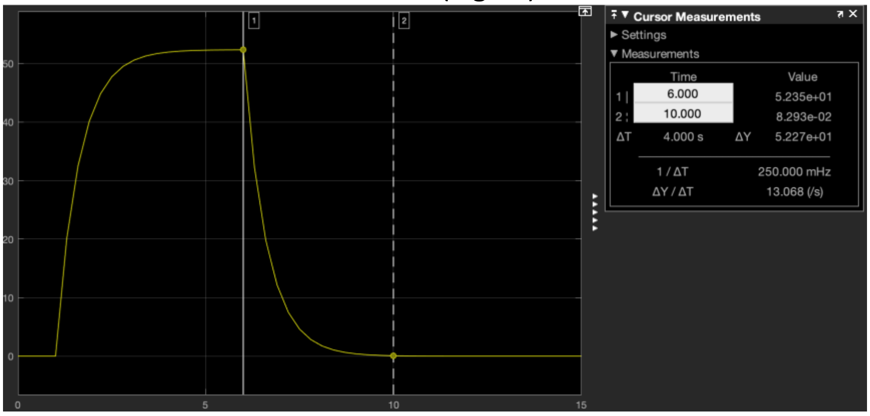
7. *Next, keeping the load-torque zero, apply an input voltage which steps from 0V to 12V at t=1 seconds and then drops to 0V again at t=6 seconds. You can achieve this by adding together the signals of two step sources. Explain why you see a negative current spike at t=6 seconds. Hint: Make use of (1.18). The reverse voltage spike which causes this current can damage other components (like transistors) connected to the circuit – this is why, usually a flyback/snubber diode is used.*



(Fig.12)



Current (Fig.13)



Motor Speed (Fig.14)

A negative current spike can be seen at t=6 which can be explained by the formula:

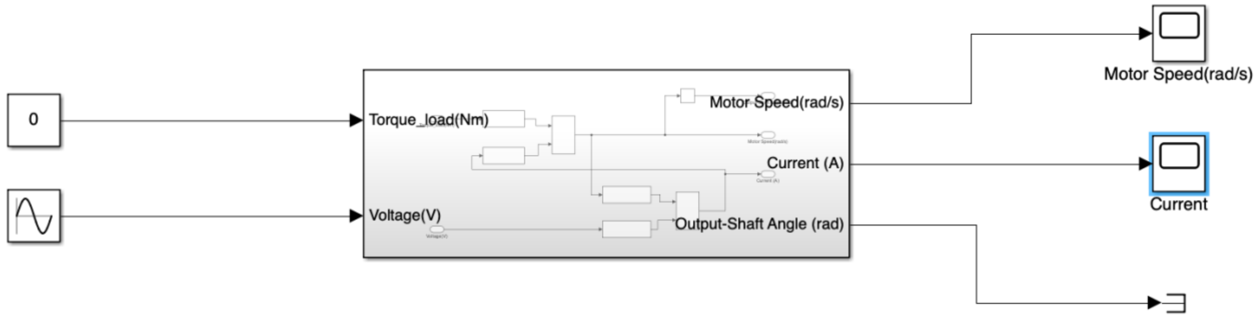
$$v_a - IR_a - L_a\frac{di}{dt} = e = k_e\omega_g$$

$$I = \frac{v_a - k_e\omega_g}{R_a} = \frac{0 - (0.2154) \times (52.36)}{2.4} = -4.7\ A$$

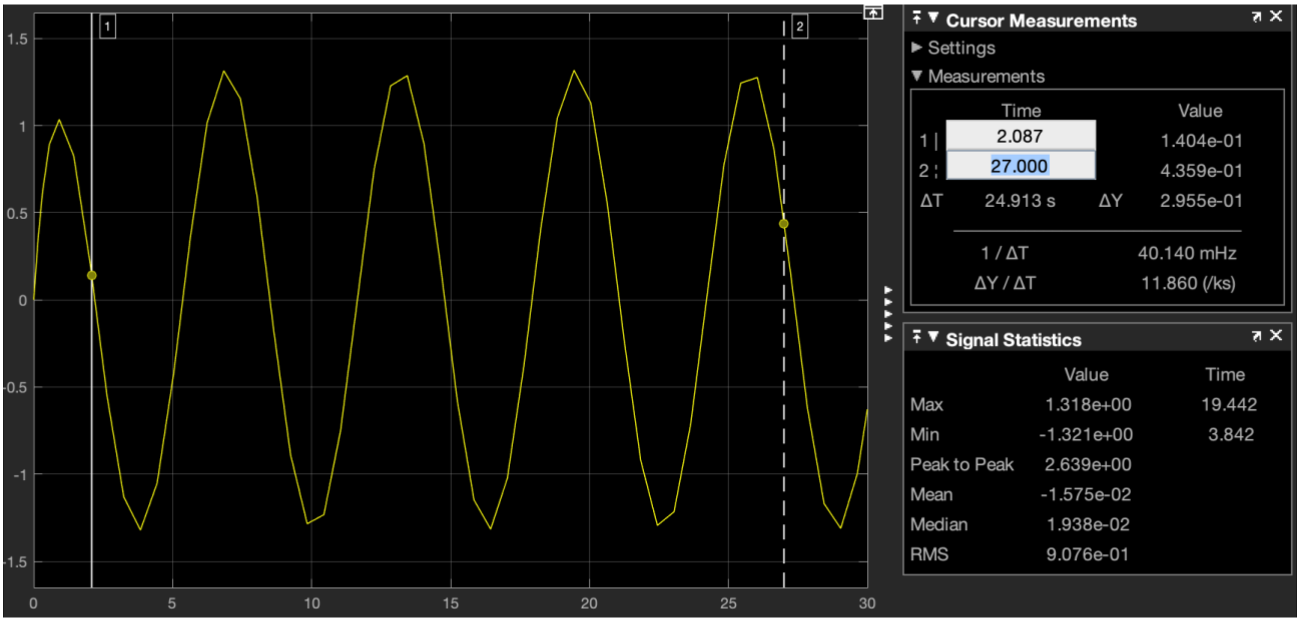
Task 1.11. *Use the Source/Sine-Wave block to input a sine-wave voltage of amplitude 6V and frequency 1 rad/s to the motor-model. Simulate for about 30 seconds. After the initial transients have died down and the system reaches steady-state, note down the amplitude $\hat{I}$ of the sinusoidal current response from the scope – use the zoom-button. Compute the ratio*

$$h(\omega = 1) = \frac{\hat{I}}{V_a} = \frac{\hat{I}}{6}$$

We will need this value later.



(Fig.15)

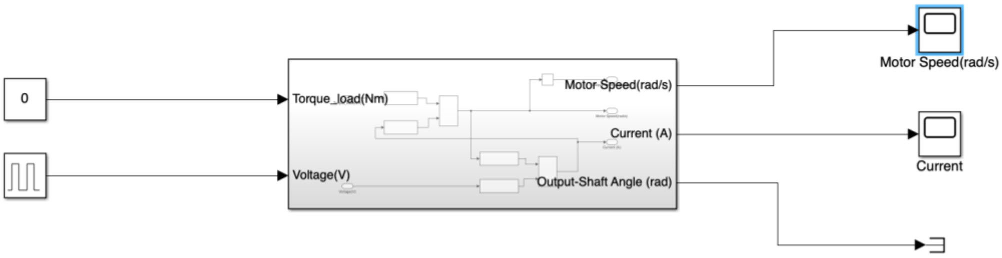


(Fig.16)

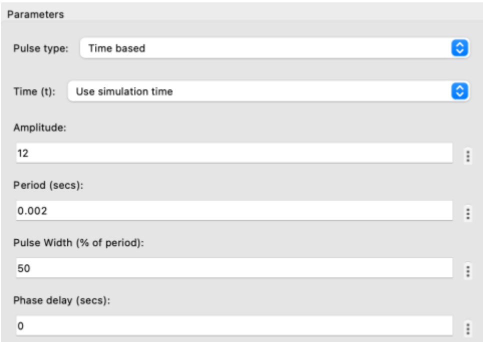
The system reaches steady-state when its initial transient dies, which happens at t= 2.087s. According to Fig.16 the maximum amplitude is 1.318A, which we are going to use to find the ratio:

$$h(\omega = 1) = \frac{\hat{I}}{V_a} = \frac{\hat{I}}{6} = \frac{1.318}{6} = 0.22$$

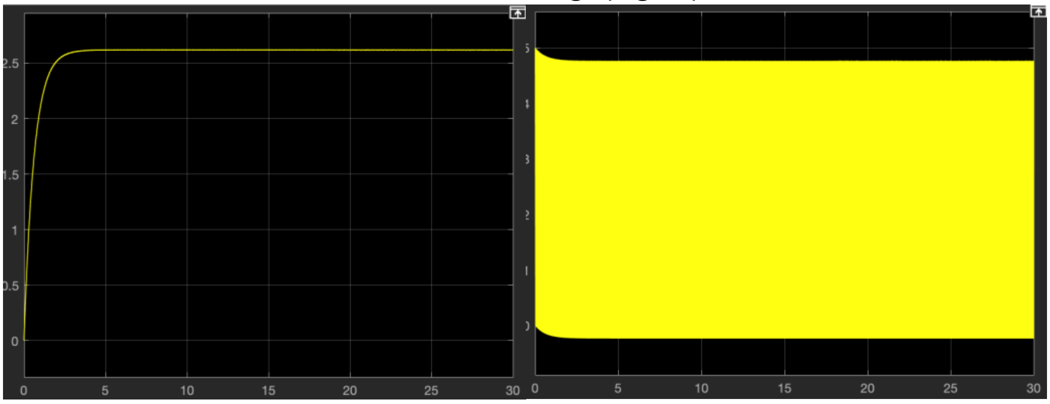
Task 1.12. *Set the time-period of the pulse-generator to correspond to that of an Arduino PWM signal, namely 2 milli-seconds. Set the amplitude to be 12 V. Vary the pulse width parameter to vary the PWM duty-cycle value in percentage. See how the motor's speed and current react to such an input.*



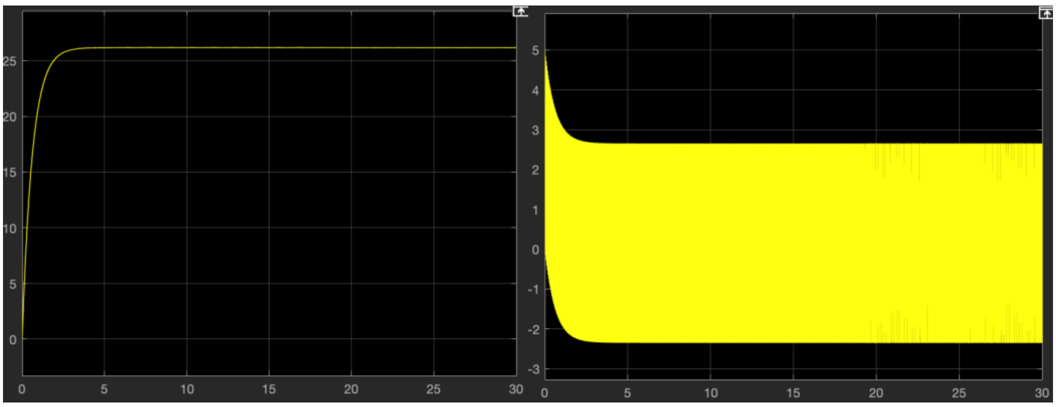
(Fig.17)



Parameter settings (Fig.18)



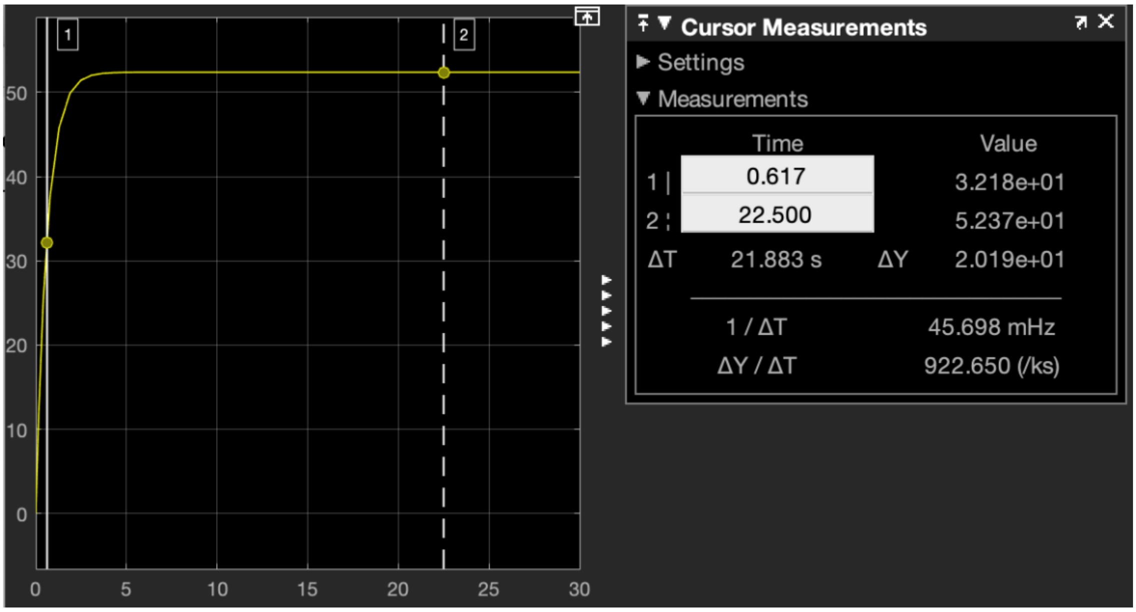
Motor speed and current with 5% pulse width (Fig.19)



Motor speed and current with 50% pulse width (Fig.20)

If the percentage of pulse width is increased, a larger range of current and motor speed is obtained.

Task 1.13. *In your model, set the load-torque input to zero and apply a step voltage input of 12V at* t = 0*. Measure* T_C *from the speed-output scope. Now estimate* J*. Does it match with the value you knew beforehand?*



(Fig.21)

The time constant refers to the duration required for the motor-speed to attain 63% of its steady-state value. This means that when the motor-speed reaches 52.36 rad/s, its time constant would be equivalent to 32.98 rad/s.

$$T_c = 0.617s$$

By using the formula to find J

$$J = \frac{T_c(R_a b + k_t k_e)}{R_a} = \frac{0.617(2.4 \times 0.000679 + 0.2154 \times 0.1186)}{2.4} = 0.00698\ kgm^2$$

The calculated value of J matches our old value of J ≈ $0.007\ kgm^2$.

Task 1.14. *Find the transfer function* H_I (s) = I(s)/Va(s) *from* (1.24) *by assuming no-load and eliminating* $\Omega(s)$ *between the two equations.*

$$\Omega(s) = \frac{k_t}{Js+b} I(s) - \frac{1}{Js+b} T_L(s) \qquad (1.24A)$$

$$I(s) = \frac{1}{L_a s+R_a} V_a(s) - \frac{k_e}{L_a s+R_a} \Omega(s) \qquad (1.24B)$$

Assuming no-load, $T_L(s) = 0$, and by substitution, $\Omega(s)$ can be eliminated.

$$I(s) = \frac{1}{L_a s + R_a} V_a(s) - \frac{k_e}{L_a s + R_a} (\frac{k_t}{Js + b} I(s) - \frac{1}{Js + b} T_L(s))$$

$$I(s) = \frac{1}{L_a s + R_a} V_a(s) - \frac{k_e}{L_a s + R_a} (\frac{k_t}{Js + b} I(s) - \frac{1}{Js + b} (0))$$

$$I(s) = \frac{1}{L_a s + R_a} V_a(s) - \frac{k_e}{L_a s + R_a} \left(\frac{k_t}{Js + b} I(s)\right) = \frac{1}{L_a s + R_a} V_a(s) - \frac{k_e k_t}{(L_a s + R_a)(Js + b)} I(s)$$

$$\frac{1}{L_a s + R_a} V_a(s) = I(s) + \frac{k_e k_t}{(L_a s + R_a)(Js + b)} I(s)$$

$$\frac{1}{L_a s + R_a} V_a(s) = I(s) \left(1 + \frac{k_e k_t}{(L_a s + R_a)(Js + b)}\right)$$

$$H_I(s) = \frac{I(s)}{V_a(s)} = \frac{1}{L_a s + R_a} \Bigg/ \left(1 + \frac{k_e k_t}{(L_a s + R_a)(Js + b)}\right)$$

$$= (\frac{1}{L_a s + R_a}) \times (\frac{(L_a s + R_a)(Js + b)}{(L_a s + R_a)(Js + b) + k_e k_t})$$

$$= (\frac{(Js + b)}{(L_a s + R_a)(Js + b) + k_e k_t})$$

Task 1.15. *We would like to compute* $|H_I(j\omega)|$ *at* $\omega = 1$ *using the known parameter values in Matlab.* $H_I(j\omega)$ *is the complex-number which you get by substituting* $s = j\omega$ *in* $H_I(s)$*. Compare* $|H_I(j\omega)|_{\omega=1}$ *with the ratio* $h(\omega = 1)$ *you found back in* (1.25)*. Are they the same?*

By substituting $j\omega$ into s and ω=1,

$$H_I(j\omega) = \frac{J(j\omega) + b}{(L_a(j\omega) + R_a)(J(j\omega) + b) + k_e k_t}$$

$$= \frac{Jj + b}{(L_a j + R_a)(Jj + b) + k_e k_t}$$

$$= \frac{0.007j + 0.000679}{2.4(0.007j + 0.000679) + 0.2154 \times 0.1186}$$

$$= \left(\frac{0.000679 + 0.007j}{0.2718 + 0.0168j}\right) \times \left(\frac{0.2718 - 0.0168j}{0.2718 - 0.0168j}\right)$$

$$= \frac{0.00013 + 0.000179j}{0.00102} = 0.127 + 0.176j$$

To find the magnitude we take modulus

$$|H_I(j\omega)| = \sqrt{(0.127)^2 + (0.176)^2} = 0.217$$

Which is equal to the ratio found in **Task 1.11.** ≈ 0.22.