Robotics and Intelligent Systems Lab II
Report 6

Professor Fangning Hu
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Sabeeh ur Rehman Sulehri

Task 1.25. *Show that under the assumption of negligible* L_a,

$$H_{er}(s) = \frac{s^2 D(s)}{Q(s)} \qquad (1.49d)$$

$$Q(s) = R_a J s^3 + s^2(R_a b + k_e k_t + k_t K_D) + k_t K_P s + k_t K_I \qquad (1.49e)$$

$$H_{er}(s) = \frac{1}{1 + H_{\omega v}(s) G(s)} = \frac{s^2 D(s)}{s^2 D(s) + k_t(K_D s^2 + K_p s + K_I)}$$

By putting D(s)= $(R_a(Js + b) + k_e k_t)$ into the formula where L_a = 0, we can prove $H_{er}(s) = \frac{s^2 D(s)}{Q(s)}$;

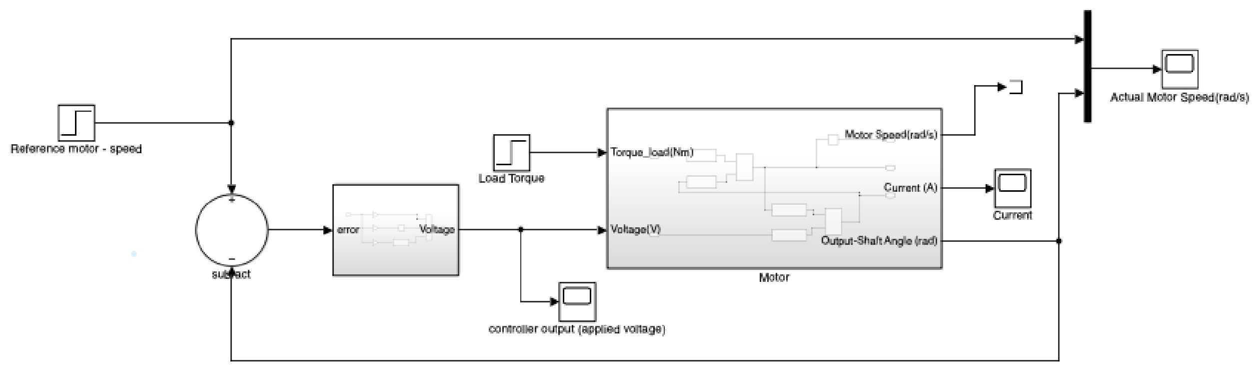
$$H_{er}(s) = \frac{s^2(R_a(Js + b) + k_e k_t)}{s^2(R_a(Js + b) + k_e k_t) + k_t\left(K_D s^2 + K_p s + K_I\right)}$$

$$\begin{aligned} &s^2(R_a(Js + b) + k_e k_t) + k_t\left(K_D s^2 + K_p s + K_I\right) \\ &\qquad = R_a J s^3 + s^2(R_a b + k_e k_t + k_t K_D) + k_t K_P s + k_t K_I = Q(s) \end{aligned}$$

$$\therefore\ H_{er}(s) = \frac{s^2 D(s)}{s^2(R_a(Js + b) + k_e k_t) + k_t\left(K_D s^2 + K_p s + K_I\right)}$$

$$H_{er}(s) = \frac{s^2 D(s)}{R_a J s^3 + s^2(R_a b + k_e k_t + k_t K_D) + k_t K_P s + k_t K_I} = \frac{s^2 D(s)}{Q(s)}$$

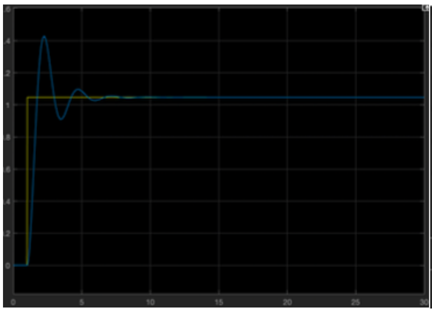
Task 1.26. *Copy the model-file for the PID speed-controller to a new file. Now change the speed-controller to a servo-controller as shown in Fig. 1.19.*



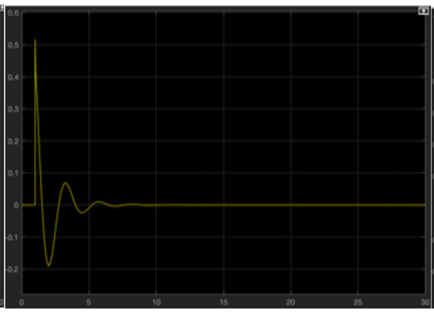
Fig(1)

Task 1.27. *Proceed as follows:*

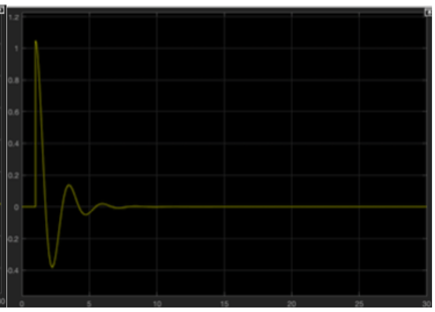
1. *Start with some relatively small value of K_P such as 1, keeping K_D and K_I zero. Set the reference input to fi/3 rad and T_L © 0. Simulate the sytem for 30 seconds. Create a table with the following columns and jot down your observations: K_P, K_I, K_D, max absolute value of current, max absolute value of voltage, max overshoot of the angle above the reference, settle-time (approx. time it takes for the angle to settle near its steady-state value).*



Actual Motor Speed Fig (2)



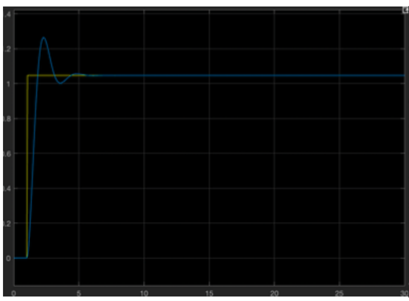
Current Fig (3)



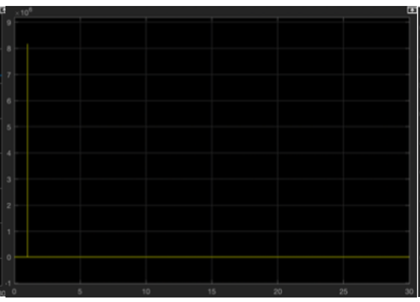
Controller output (applied voltage) Fig (4)

K_p	K_I	K_D	Max abs Current	Max abs Voltage	Max overshoot angle	Settle-time
1	0	0	0.5169	1.047	0.372	8.006

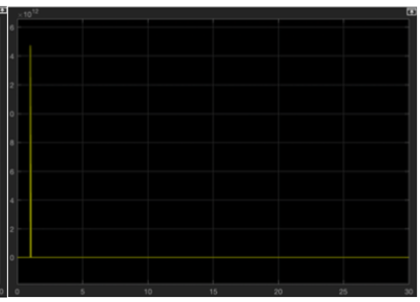
2. *Now add some damping by setting K_D = 0.1. Enter your observations in the table.*



Actual Motor Speed Fig (5)



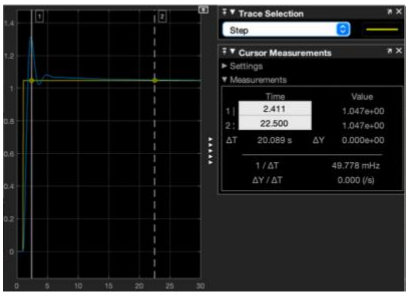
Current Fig (6)

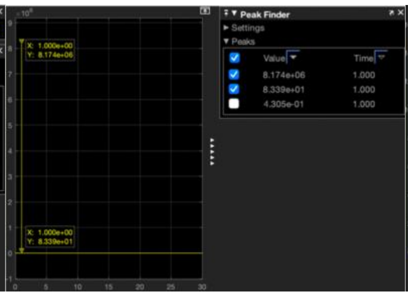


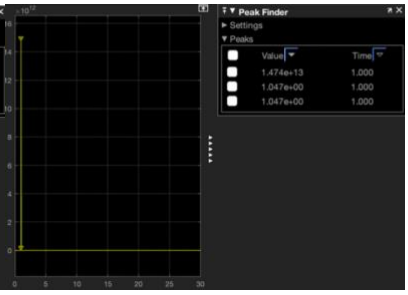
Controller output (applied voltage) Fig (7)

K_p	K_I	K_D	Max abs Current	Max abs Voltage	Max overshoot angle	Settle-time
1	0	0.11	8.174×10^6	1.474×10^{13}	0.207	4.911

3. *Now add some integral control K_I = 0.1. Enter your observations in the table.*

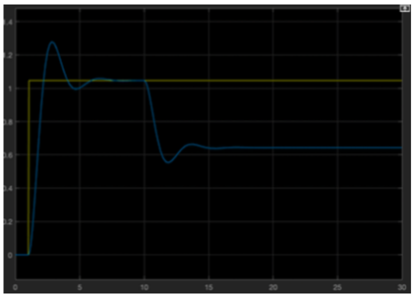

Actual Motor Speed Fig (8)


Current Fig (9)


Controller output (applied voltage) Fig (10)

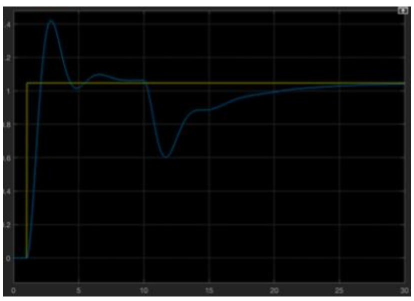
K_p	K_I	K_D	Max abs Current	Max abs Voltage	Max overshoot angle	Settle-time
1	0.1	0.11	8.174×10^6	1.474×10^{13}	0.268	20.976

4. *Now we will see what the controller can do if we add a step load torque. First reset K_I = 0. Reduce K_P to 0.5 and K_D to 0.001. Make a step load of T_L =0.01 Nm at t=10 seconds. Did the system reach its reference value?*


Actual Motor Speed Fig (11)

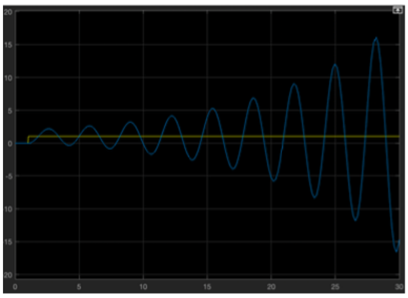
The system did not reach its reference value, that is 1.047.

5. *Now increase K_I to 0.1 and simulate again. Do you see any dierence in the steady-state error?*


Actual Motor Speed Fig (12)

The steady-stat error decreased, leading the system to reach its reference value.

6. *Although it is now tempting to increase K_I even more, we should be careful. Making K_I too big can easily make the closed-loop system unstable. Let us see how an unstable system reacts: set K_I = 1 and simulate.*



Actual Motor Speed Fig (13)

The unstable system reacts in oscillations.

7. *Does our model also predict this instability? Find the poles of the system by finding the roots of Q(s) for these controller parameters. Are they in LHP?*

$$Q(s) = (2.4 \times 0.07)s^3 + (2.4 \times 0.00679 + 0.2154 \times 0.1186 + 0.001 \times 0.1186)s^2 + (0.1186 \times 0.5)s + 0.1186$$

$$Q(s) = 0.0168s^3 + 0.02729464s^2 + 0.0593s + 0.1186 = 0$$

Roots = 0.0969 + 1.9679i, 0.0969 - 1.9679i, -1.8185 + 0.0000i

Our model predicts instability because there's 1 pole in LHS and 2 poles in RHS.