Robotics and Intelligent Systems Lab II
Report 2

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Spring 2023

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Task 1.4. *Estimate* k_t *in SI units for this motor (Table 1.1) from the data provided using* (1.4), *the stall current, and the stall torque.*

Parameter	Value	In SI Units	Description
Free-run speed	500 RPM	52.36 rad/s	ω_0, the shaft-speed at no-load
Free-run current	300 mA	0.3 A	i_0, the no-load current
Stall current	5000 mA	5 A	i_s
Stall torque	84 oz-in	0.5932 Nm	τ_s

Table 1.1

$$\tau_g = \eta \hat{k}_t i \triangleq k_t i \qquad \text{Formula (1.4)}$$

Since these values are at the output-shaft, we would modify Formula (1.4) to be:

$$\tau_g = k_t i$$

By substituting the values of τ_s and i_s into the equation, we get

$$0.5932Nm = k_t \times 5A$$

$$k_t = \frac{0.5932\ Nm}{5\ A}$$

$$\therefore effective\ motor\ torque\ constant = 0.1186\ ^{Nm}/_{A}$$

Task 1.5. *Estimate* k_e *in SI units for this motor from the data provided using* (1.7). *We will get a better estimate later.*

$$k_e \leq \frac{v_a}{\omega_0} \qquad \text{Formula (1.7)}$$

Using the formula (1.7) to calculate the upper bound of k_e given with applied voltage and shaft-speed at no-load:

$$k_e \leq \frac{v_a}{\omega_0} = \frac{12\ V}{52.36\ ^{rad}/_{s}} = 0.2292\ ^{Vs}/_{rad}$$

Task 1.6. *Estimate* b *in SI units for the motor in Table 1.1.*

The formula to estimate b is:

$$b = \frac{k_t i_0}{\omega_0}$$

After putting the respective values in the formula:

$$= \frac{(0.1186\ ^{Nm}/_{A}) \times (0.3A)}{52.36\ ^{rad}/_{s}}$$

$$= 0.000679\ ^{Nms}/_{rad}$$

Task 1.7. *Compute* $\mathrm{R_a}$ *from the motor data given. Using this value, obtain a better estimate for* $\mathrm{k_e}$ *than the one we obtained earlier in* (1.7)*? Hint: Consider* (1.18) *in the no-load steady-state condition.*

$$R_a = \frac{v_a}{i_s} = \frac{12\ V}{5\ A} = 2.4\ {}^{V}/_{A}$$

Calculating k_e by using the value found above:

$$v_a - iR_a - L_o\frac{di}{dt} = e = k_e\acute{\theta}_g$$

In no load, i_o = 0.3A, and at steady-state $\frac{di}{dt} = 0$.

$$v_a - i_oR_a = e = k_e\acute{\theta}_g$$

Where $\acute{\theta}_g = \omega_o$, so

$$k_e = \frac{v_a - i_oR_a}{\omega_o}$$

$$= \frac{12V - (0.3A \times 2.4\ {}^{V}/_{A})}{52.36\ {}^{rad}/_{s}}$$

$$= 0.2154\ {}^{Vs}/_{rad}$$

Task 1.8. *On a sheet of paper, derive* (1.21b) *from* (1.17) *and* (1.20) *as explained. These transfer-functions* $H_{\theta v}(s)$ *and* $H_{\theta \ell}(s)$ *are important for position-control (a.k.a. servo mode) of the motor.*

$$(Js^2 + bs)\theta_g(s) = k_tI(s) - T_L(s) \qquad (1.17)$$
$$V_a(s) - (R_a + L_as)I(s) = k_es\theta_g(s) \qquad (1.20)$$
$$\theta_g(s) = \frac{k_t}{sD(s)}V_a(s) - \frac{(R_a+L_as)}{sD(s)}T_L(s) \qquad (1.21b)$$

Solving with the substitution method:

$$(Js^2 + bs)\theta_g(s) = (k_t)\frac{V_a(s) - k_es\theta_g(s)}{R_a + L_as} - T_L(s)$$

$$(Js^2 + bs)\theta_g(s) = \frac{k_t \times V_a(s)}{R_a + L_as} - \frac{k_t \times k_es\theta_g(s)}{R_a + L_as} - T_L(s)$$

$$\theta_g(s)\left(Js^2 + bs + \frac{k_t \times k_es}{R_a + L_as}\right) = \frac{k_t \times V_a(s)}{R_a + L_as} - T_L(s)$$

$$\theta_g(s) = \left(\frac{k_t \times V_a(s)}{R_a + L_as} - T_L(s)\right) \Big/ \left(Js^2 + bs + \frac{k_t \times k_es}{R_a + L_as}\right)$$

$$\theta_g(s) = \left(\frac{k_t \times V_a(s)}{R_a + L_a s} - T_L(s)\right)\left(\frac{R_a + L_a s}{(Js^2 + bs)(R_a + L_a s) + (k_t \times k_e s)}\right)$$

Substituting $D(s) = (Js + b)(R_a + L_a s) + (k_t k_e)$

$$\theta_g(s) = \left(\frac{k_t \times V_a(s)}{R_a + L_a s} - T_L(s)\right)\left(\frac{R_a + L_a s}{sD(s)}\right)$$

$$\theta_g(s) = \frac{k_t \times V_a(s)}{sD(s)} - \frac{T_L(s)(R_a + L_a s)}{sD(s)}$$

Which is equal to the formula (1.21b):

$$\theta_g(s) = \frac{k_t}{sD(s)} V_a(s) - \frac{(R_a + L_a s)}{sD(s)} T_L(s) \qquad (1.21b)$$

Task 1.9. *Show that if the inductance* L_a *is small and can be ignored,* $H_{\omega v}(s)$ *can be written as a first order system*

$$H_{\omega v}(s) \approx \frac{K}{s+a} \qquad (1.23)$$

What are a *and* K *in terms of the other parameters?*

$$H_{\omega v} = \frac{k_t}{D(s)} = \frac{k_t}{(Js + b)(R_a + L_a s) + (k_t k_e)}$$

Taking L_a as 0,

$$H_{\omega v} = \frac{k_t}{(Js + b)(R_a) + (k_t k_e)}$$

Simplifying the equation to get a and K in terms of other parameters:

$$H_{\omega v} = \frac{k_t}{R_a J s + R_a b + k_t k_e} = \frac{\frac{k_t}{R_a J}}{s + \frac{b}{J} + \frac{k_t k_e}{R_a}}$$

$$K = \frac{k_t}{R_a J}, a = \frac{b}{J} + \frac{k_t k_e}{R_a}$$