Synchronization of Kuramoto Oscillators via Cutset Projections

Saber Jafarpour, *Member, IEEE*, and Francesco Bullo, *Fellow, IEEE*

***Abstract*—Synchronization in coupled oscillators networks is a remarkable phenomenon of relevance in numerous fields. For Kuramoto oscillators, the loss of synchronization is determined by a tradeoff between coupling strength and oscillator heterogeneity. Despite extensive prior work, the existing sufficient conditions for synchronization are either very conservative or heuristic and approximate. Using a novel cutset projection operator, we propose a new family of sufficient synchronization conditions; these conditions rigorously identify the correct functional form of the tradeoff between coupling strength and oscillator heterogeneity. To overcome the need to solve a nonconvex optimization problem, we then provide two explicit bounding methods, thereby obtaining 1) the best-known sufficient condition for unweighted graphs based on the 2-norm, and 2) the first-known generally applicable sufficient condition based on the ∞-norm. We conclude with a comparative study of our novel ∞-norm condition for specific topologies and IEEE test cases; for most IEEE test cases, our new sufficient condition is one to two orders of magnitude more accurate than previous rigorous tests.**

***Index Terms*—Cutset projection, frequency synchronization, Kuramoto oscillators, synchronization manifold.**

is no interaction between oscillators, the dynamics of ith oscillator is governed by the differential equation $\dot{\theta}_i = \omega_i$. One can model the coupling between oscillators using a weighted undirected graph G, where the interaction between oscillators i and j is proportional to sin of the phase difference between angles θ_i and θ_j. This model, often referred to as the Kuramoto model, is one of the most widely used model for studying synchronization of finite population of coupled oscillators. The Kuramoto model and its generalizations appear in various applications including the study of pacemaker cells in heart [31], neural oscillators [8], deep brain simulation [41], spin glass models [25], oscillating neutrinos [36], chemical oscillators [26], multivehicle coordination [39], synchronization of smart grids [17], security analysis of power flow equations [4], optimal generation dispatch [28], and droop-controlled inverters in microgrids [10], [40].

Despite its apparent simplicity, the Kuramoto model gives rise to very complex and fascinating behaviors [16]. A fundamental question about the synchronization of coupled-oscillators networks is whether the network achieves synchronization for a given set of natural frequencies, graph topology, and edge