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# HW 6 – Axiomatic Semantics

## CS 477 – Spring 2025

**Assigned** April 30, 2025, 10:00 PM  
**Due** May 7, 2025, 11:59 PM

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### Policy

**For all theory questions, including proofs, only typewritten solutions in LaTeX are allowed. Handwritten solutions will not be considered and will be awarded zero marks.**

This homework is an individual assignment. In addition, directly copying from LLMs, including ChatGPT and Gemini, is strictly forbidden. In this homework, you can use these tools to gain insights; however, stringent measures will be implemented to detect any instances of direct replication from these sources. Any identified instances of suspicion will prompt an invitation to an oral examination.

**Late policy:** You have a total of four additional days throughout the term to accommodate late submissions for homework assignments. This cumulative allowance can be used at your discretion across different assignments. If you submit more than 5 minutes late for an assignment, then it is considered as using one full day of the late submission allowance. Once you exhaust the four-day late submission allowance, any further late homework submissions will result in a penalty of 33% grade reduction/day.

Please do not submit any code or use code to accomplish the non-programming problems unless we specifically instruct you to do so.

### Turn-In Procedure (PLEASE READ CAREFULLY)

Use <https://www.gradescope.com/> to submit your homework. There are two assignments on Gradescope for this homework; one is designated for theory problems, and the other is for programming assignments. For the theory section of this assignment, you can submit a single PDF file or a set of photos just like HW 1.

For the coding problems, submit **exactly one file** to Gradescope, named `<net_id>.interval.py`, with `<net_id>` replaced by your own NETID. Failure to name your code files correctly will result in a 0 grade for the problem. Some tutorials on how to use Gradescope are available here [https://www.gradescope.com/get\\_started#student-submission](https://www.gradescope.com/get_started#student-submission).

# Hoare Logic (40 pts)

## All Rules on One Slide

The Assignment Rule

$$\frac{}{\{P[e/x]\} x := e \{P\}}$$

Sequencing

$$\frac{\{P\} C_1 \{Q\} \quad \{Q\} C_2 \{R\}}{\{P\} C_1; C_2 \{R\}}$$

If Then Else

$$\frac{\{P \text{ and } B\} C_1 \{Q\} \quad \{P \text{ and } (\text{not } B)\} C_2 \{Q\}}{\{P\} \text{if } B \text{ then } C_1 \text{ else } C_2 \text{ fi } \{Q\}}$$

Precondition Strengthening

$$\frac{P \rightarrow P' \quad \{P'\} C \{Q\}}{\{P\} C \{Q\}}$$

Postcondition Weakening

$$\frac{\{P\} C \{Q'\} \quad Q' \rightarrow Q}{\{P\} C \{Q\}}$$

Rule of Consequence

$$\frac{P \rightarrow P' \quad \{P'\} C \{Q'\} \quad Q' \rightarrow Q}{\{P\} C \{Q\}}$$

While

$$\frac{\{P \text{ and } B\} C \{P\}}{\{P\} \text{while } B \text{ do } C \text{ od } \{P \text{ and not } B\}}$$

Determine the truth value of the following Hoare triples. All values are integers. Show your work and explain your reasoning. The above slide contains some convenient rules you can use; however, we encourage you to read the full lecture slides.

1.  $\{i = j\} i := j - 2 * i \{i + j = 0\}$  [5 pts]
2.  $\{i = a\} i := j + i ; i := j - i \{i = a\}$  [5 pts]
3.  $\{i \neq j\} \text{if } i < j \text{ then } m := j - i \text{ else } m := j + i \text{ fi } \{m > 0\}$  [5 pts]
4.  $\{x < a\} \text{while } x < a \text{ do } x := x + 1 \{x = a\}$  [5 pts]
5.  $\{x = a\} \text{while } x \leq a \text{ do } x := x \{x < a\}$  [5 pts]

6. Prove the given statements about the following programs using Hoare logic. [15 pts]

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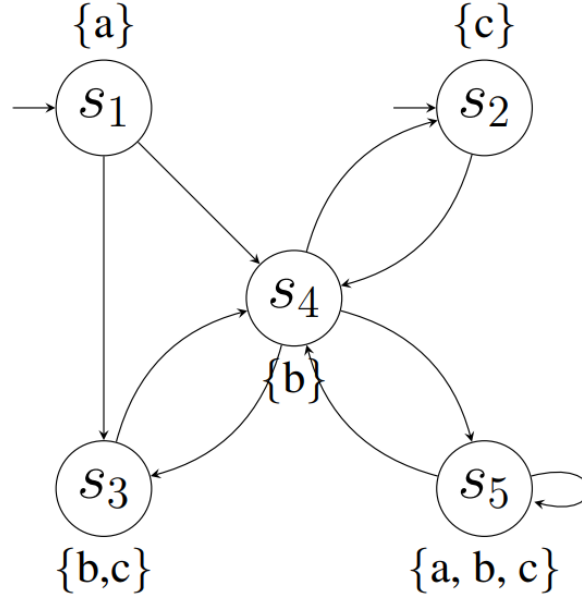
1 {0 ≤ s and u < |a|}
2 bool LinearSearch(int[] a, int s, int u, int e) {
3     int i=s;
4     while(i≤u) {
5         if(a[i]==e) return true;
6         i++;
7     }
8     return false;
9 }
10 {retLinearSearch ⇔ ∃i. s ≤ i ≤ u and a[i] = e}

```

Listing 1: (a)

## LTL - Theory (20 pts)

For the next few questions, we will be working with the following transition system TS over the set of atomic propositions  $\{a, b, c\}$  and starting states  $s_1, s_2$ .



Decide for each of the following LTL formulae  $\varphi_i$  whether  $TS \models \varphi_i$  holds. Provide a brief explanation for your answer in words. There is no requirement to provide a formal proof. If  $TS \not\models \varphi_i$ , provide a path  $\pi \in Paths(TS)$  such that  $\pi \not\models \varphi_i$ .

7.  $\varphi_1 = \neg c \Rightarrow \neg c$  [5 pts]

8.  $\varphi_2 = \Box a$  [5 pts]

9.  $\varphi_3 = a \cup \Box(b \vee c)$  [5 pts]

10.  $\varphi_4 = (\neg b) \cup (b \vee c)$  [5 pts]

## Programming - Invariants (40 pts)

11. This question involves programming in Dafny([documentation](#)). Please begin by downloading the necessary starter codes at [Here](#). The installation instructions can be found in `install.md`. In each of the 4 programs, write an appropriate loop invariant such the assertion holds. See `example.dfy` for an example solution. [4x10 pts]