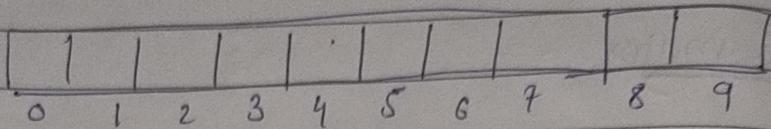


Array ADT

ADT (Abstract Datatype) → Representation of Data
Operations we can perform on Data

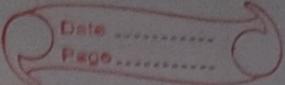
*



int A[10]; or int *A
A = new int [size]

* Operations

- 1) Display()
- 2) Add(x) / Append(x)
- 3) Insert(index, x)
- 4) Delete(index)
- 5) Search(x)
- 6) Get(index)
- 7) Set(index)
- 8) Max() / Min()
- 9) Reverse()
- 10) Shift() / Rotate()



Searching \rightarrow linear Search
 \rightarrow Binary Search.

* Linear Search.

key: Successful Search \rightarrow Best $O(1)$
 \rightarrow Worst $O(n)$

Unsuccessful search \rightarrow Worst $O(n)$

```
for (i=0; i<length ; i++)
{
    if (A[i] = value)
        return value;
}
return -1
```

\rightarrow Improving linear search

- Transposition \rightarrow Reduce the position of the element by 1
- Move to front/Head \rightarrow Replace element with index [0]

Binary Search

→ Here array is sorted

→ if value is equal to ~~middle~~ value of array
middle position is returned.

→ if middle position value is less than key
value then put ~~l =~~ $l = \text{middle} + 1$

→ else $h = m - 1$

initial

$l = 0$

~~h = arr.length~~

$m = (l + h) / 2$

→ Time complexity :-

$$n_0 = n/2 \quad \text{for } I=1$$

$$n_1 = (n/2)/2 = n/2^2 \quad \text{for } I=2$$

$$n_2 = ((n/2)/2)/2 = n/2^3 \quad \text{for } I=3$$

⋮

$$n_{I-K} = \frac{n}{2^K} \quad \text{for } I=K$$

So Here the loop is run K times

i.e time complexity

$$n = n/2^K \quad \text{for } I=K$$

$n = 1$, taking log on b:s

$$\log(1) = \log(n) - \cancel{\log 2} K \log 2$$

$$K \approx \log(n) -- \text{not considering } \log 2$$

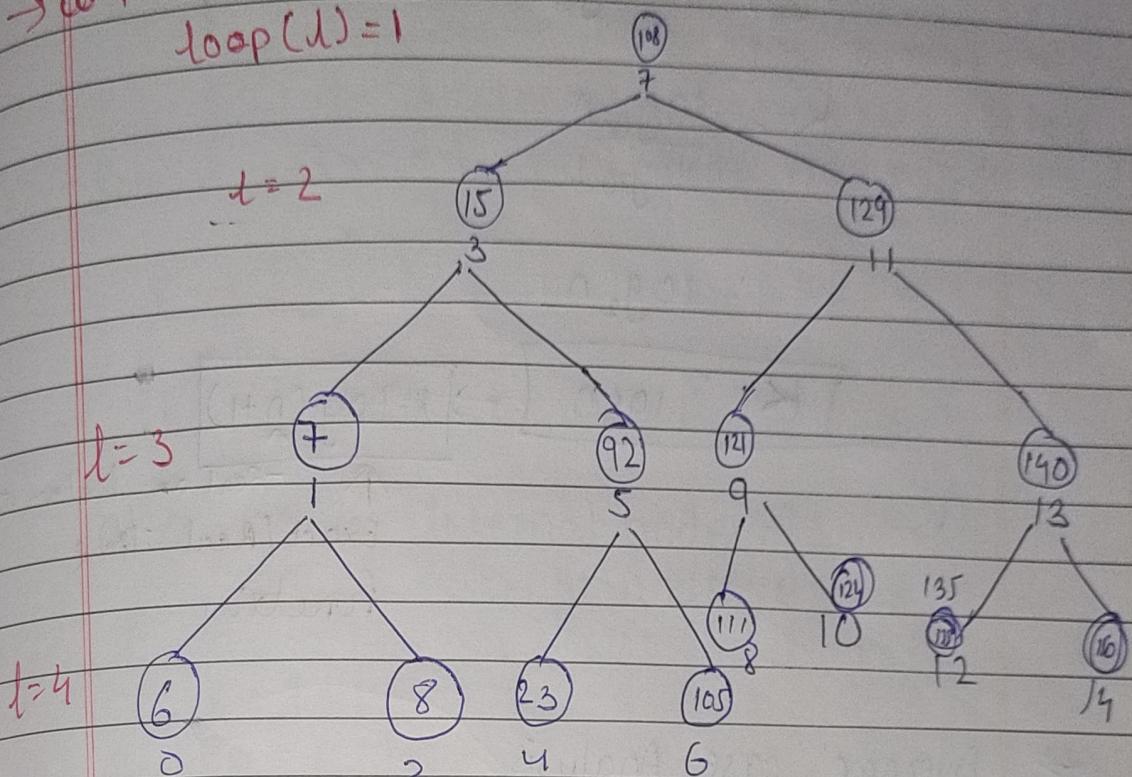
∴ Time

Time complexity - (worst case $\rightarrow \log(n)$)

175														
6	7	8	15	23	92	105	108	101	121	124	129	135	140	150
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14

\rightarrow middle on first loop

$$\text{loop}(l) = 1$$



no of elements $\downarrow l$

So for $l=1 \rightarrow$ loop run & time $n/2$

$$\text{for } l=2 \rightarrow (n/2)/2 = n/2^2$$

$$\text{for } l=3 \rightarrow n/2^3$$

$$\text{for } l=k \rightarrow n/2^k$$

∴ for k times time complexity is k here

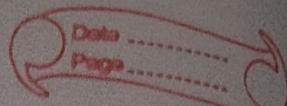
$$\frac{N}{2^K}$$

$$\log N - k \log 2 = 0$$

$$\therefore k = \log n$$

∴ Time complexity is $\log n$

Best case time $\rightarrow 1$
Worst case time $\rightarrow \log(n)$
Average case time $\rightarrow \log(n)$



\therefore for n element time complexity is calculated by

$$\frac{n}{2^k} = 1 \rightarrow 1 \text{ element}$$

Actual no of element

+1 Here k is time complexity

$$\log_{10} n = \log_{10} 2^k$$

$$k = \frac{\log_{10} n}{\log_{10} 2}$$

$$= \log_2 n$$

$$k = \log n \rightarrow k = \log(n+1)$$

↑
Real no of elements
Exact/Real Time
function

\Rightarrow Average case Analysis
from behind example.

$$1 + 1 \times 2 + 2 \times 4 + 3 \times 8$$

$$1 + 2^1 + 2 \times 2^2 + 3 \times 2^3$$

$$\sum_{i=1}^{\log(n)} \underbrace{i \times 2^i}_{n} = \frac{\log n \times 2^{\log n}}{n}$$

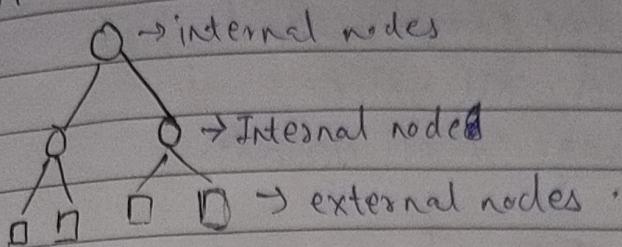
$$= \frac{\log n \times n}{n}$$

$$= \log n$$

E = sum of path of ^{external} nodes.

I = sum of path of internal nodes.

N = number of nodes. i.e. height of tree



$$\text{Sum of path of internal nodes} = 1 \times 0 + 2 \times 1 = 2$$

$$\text{Sum of path of External node} = 4 \times 2 = 8$$

$$E = I + 2N$$

$$= 2 + 2(3)$$

$$E = 8 = 8$$

also no of internal nodes = i

no of external nodes = e

$$e = i + 1$$

$$E = n \log n$$

$$\text{Average Successful search } As(n) = 1 + \frac{I}{n} = \log n$$

$$\begin{aligned} \text{Average unsuccessful search } Au(n) &= \frac{E}{n+1} = \frac{n \log n}{n+1} \\ &= \log n \end{aligned}$$



`Get()` → Reading a value

`Set()` → writing a value.

`Max()` → Reads maximum value.

`Min()` → Reads minimum value.

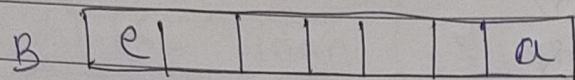
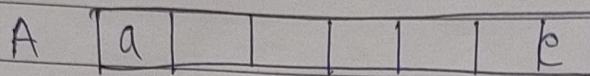
`Sum()` → Sum of all value.

`Average()` → Average of all elements.

* Reverse and shift of an array.

1st Method

→ Using an auxiliary array B

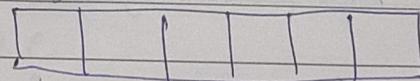


Firstly last element of array A will be copied to B (reversely) i.e as first element

then B elements are copied to A

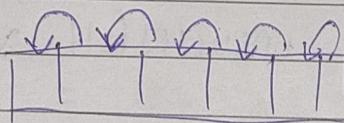
2nd Method

→ Inter changing →



3rd

→ left shift



- * Checking if the list is sorted.
- * Inserting element in a sorted list.

A	8	10	12	22	24	26	28	32	34	36	
	0	1	2	3	4	5	6	7	8	9	10

insert 25

∴ let $x = 25$

$i = \text{length of } A - 1$ i.e. $\text{arr} \rightarrow A[\text{length}]$

→ while ($A[i] > x$)

{ $A[i+1] = A[i];$

$i--;$ }

$A[i+1] = x;$

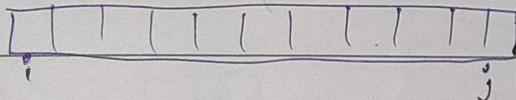
 length++;

- * Check if list is sorted.

if ($A[i] > A[i+1]$)

 return false;

- * -ve on left side



while ($i < j$)

{

 while ($A[i] < 0$) { $i++$ }

 while ($A[j] > 0$) { $j--$ }

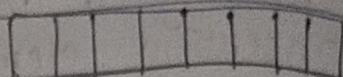
 if ($i < j$)

 swap ($A[i], A[j]$)

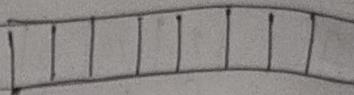
}

* Merging:

Merge array A of m elements



Merge array B of n elements



TOP Here write

i=0;

j=0;

k=0;

while (i < m && j < n)
{

if (A[i] < B[j])

C[k++] = A[i++];

else

C[k++] = B[j++];

}

c[k++] = A[i++];

For (; i < m ; i++)

c[k++] = B[j++];

* Set operations

* Union

for unsorted

A [3 | 5 | 10 | 14 | 6] m

B [12 | 4 | 7 | 2 | 5] n

C [| | | | | | |]

for sorted.

Same as merge

$$\Theta = (m+n)$$

first copy m elements to

C then compare n elements with m elements

$$m + n \times m$$

$$m + m \times n$$

$$n + m \times n$$

$$n + n^2$$

\therefore Order $\Theta(n^2)$

* Intersection

Same as merge

A [3 | 5 | 10 | 14 | 6] m

$$\Theta = (m+n)$$

B [12 | 4 | 7 | 2 | 5] n

C [| | | | | | |]

\therefore Time complexity

$$\Theta(m \times n)$$

* Difference.

unsorted

A	3	5	10	14	6	m
---	---	---	----	----	---	---

B	12	4	7	2	5	n
---	----	---	---	---	---	---

C	8	10	6					
	0	1	2	3	4	5	6	7

Sorted

A	3	4	5	10	14	m
---	---	---	---	----	----	---

B	2	4	5	7	12	n
---	---	---	---	---	----	---

C	3	6	10			
---	---	---	----	--	--	--

$$\Theta = mxn$$

x
+/
4p

* find missing element

→ Q: find missing element in sum of first n natural numbers

firstly find sum for $i=1; i \leq n-1; i++$
 $\text{sum} = \text{sum} + A[i]$

$$\text{then } S = \frac{n \times n+1}{2}$$

Now missing element is $S - \text{sum}$

→ If array doesn't start from 1

6	7	8	10	11	12
0	1	2	3	4	5

$$\text{diff} = 6 - 0$$

if (for $i=0; i \leq 6; i++$)

if $(A[i] - i) \neq \text{difference}$

print (~~and~~ it + diff is missing)
 break

Q) find Multiple missing.

~~$$\text{diff} = A[0] - 0$$~~

~~```
for (i=0 ; i < A.length ; i++)
 if (A[i] - i != diff)
 { print(i+diff) is missing;
 diff += 1; }
```~~

Or

```
for (i=0 ; i < A.length ; i++)
 if (A[i] - i != diff)
 { while (diff < A[i] - i)
 { print (i+diff)
 diff++; }
```

\* Using Hash table i.e Hashing

B [ 3 | 5 | 2 | 1 ]  $\rightarrow$  n elements

Here d = 1 & h = 5

for (i=0 ; i < n ; i++)
 A[B[i]]++;

Hash table  $\rightarrow$  A [ 0 | 0 | 0 | 0 | 0 | 0 ]  $\rightarrow$  m element

for (i=0 ; i < h ; i++)
 {

if (H[i] == 0)
 printf(i);

}

## Q) finding Duplicates -

→ for sorted -

|   |   |   |   |   |   |
|---|---|---|---|---|---|
| 3 | 2 | 2 | 5 | 5 | 5 |
|---|---|---|---|---|---|

$n = 10$

last duplicate = 0

for ( $i=0$ ;  $i < n$ ;  $i++$ ) (Time complexity  $O(n)$ )

{

if ( $A[i] == A[i+1]$  &&  $A[i] != \text{last Duplicate}$ )

{

printf( $A[i]$ );

last Duplicate =  $A[i]$ ;

}

}

## ② → Using two variable (Time complexity $O(n)$ )

when  $A[i] == A[i+1]$  don't print -

( $j = i+1$ )

while ( $A[i] == A[j]$ )  $j++$ ;

print  $A[j]$  appeared  $j-i$  times

$q = j-1$

## → Using Hashing .

B 1 | 0 | 3 | 1 | 4 | 1 | 5      for ( $i=0$ ;  $i < n$ ;  $i++$ )

$n=5$

|   |   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|---|
| A | [ | 0 | 0 | 0 | 0 | 0 | 0 | ] |
|   | 0 | 1 | 2 | 3 | 4 | 5 |   |   |

$A[B[i]]++$ ;

for ( $i=1$ ;  $i < m$ ;  $i++$ )

if ( $A[i] > 1$ )

Point i appears  $A[i] - 1$  times

## \* Finding Duplicates for unsorted Array.

~~for (i=0; i<n-1; i++) → first loop for i  
 for (j=i+1; j<n; j++) → for second loop for j  
 {  
 }~~

|                          |   |   |   |   |   |   |   |
|--------------------------|---|---|---|---|---|---|---|
| 1                        | 8 | 7 | 8 | 6 | 2 | 6 | 6 |
| Time Complexity $O(n^2)$ |   |   |   |   |   |   |   |

- first place i at index 0
- then search for 0 index element by matching with each element of array using j
- if element is same increase count
- mark duplicate as 1 & increment i value

## → Using Hash table

Time complexity  $O(n)$

## \* Finding a pair of Element with sum K

- Using two variable
- Using Hash table

## → in sorted array

- Compare beginning array element with array end elements.
- If sum of element = K then print
- If sum of element < K increment the variable at begin
- Else decrement the variable at end

## \* Finding Max and Min in single scan.

- Assign first element as max & min
- Now for every element  
if its less than min set it as min  
else if its more than max set it as max
- Here time complexity is  $O(n)$
- and to be specific in best case its time is  $n-1$
- worst case time is  $2(n-1)$