IDENTIFICATION OF LINEAR MODELS FOR A HOVERING HELICOPTER ROTOR

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Abstract: Rotor control techniques for active control of vibrations require the availability of dynamic models of the response of the rotor to control inputs. The results of a simulation study of black-box identification trials aiming at the derivation of reduced order, discrete-time, linear models for the response of helicopter rotor loads to individual blade control inputs are presented and discussed. The study has been performed by making use of a simulation program for the dynamics and aerodynamics of the rotor of an existing aircraft, and three identification algorithms have been compared: a frequency domain method and two time domain, subspace based methods.

Keywords: Aerospace engineering, linear identification, vibration control, discrete-time systems.

1. INTRODUCTION

In the last few years, a considerable effort has been devoted to the development of active control systems for the attenuation of vibrations in the fuselage of helicopters (see e.g., (Teves *et al.*, 1995)). For active control of vibrations, only small perturbations to the pilot commands are allowed: this implies that linear models usually provide a good enough representation of the system's dynamics. Furthermore, black box techniques (based on data obtained either via experiment or by simulation trials) play an important role as an alternative to physical modelling in the derivation of the reduced order models which are required for controller design.

The aim of the work presented in this paper is the evaluation of the performance of various identification methods when applied to the identification of such kind of low order models for the dynamics of a helicopter rotor; in particular the flight condition of hovering, in which all the components of the aircraft velocity are zero has been considered.

The identification study has been performed by making use of simulated data, generated by a complex simulation program for the dynamics and aerodynamics of the rotor of the Agusta A109 helicopter.

Such a simulation program has been developed by making use of the ADAMS (Automatic Dynamic Analysis of Mechanical Systems) environment for the multibody simulation and analysis of mechanical systems ((Bertogalli *et al.*, 1995)).

The identification methods which have been compared are a frequency domain method, the N4SID algorithm ((Van Overschee and De Moor, 1994)) and the MOESP algorithm ((Verhaegen and Dewilde, 1992)).

The paper is organized as follows: in Section 2 the rotor model which has been used in the study is described, while the identification algorithms which have been compared are introduced in Section 3.

In Section 4 and Section 5 the performed identification experiments and the results obtained from the various algorithms in the reduced order identification process are compared and discussed.

2. ROTOR MODEL

The main rotor of a helicopter has the function of developing the forces and moments required to fly and control the aircraft; in a rotor of the articulated type, as the one this paper deals with, each blade is attached to the rotor's central hub by a set of hinges which allow it to rotate:

- out of the rotor disk plane (up and down, flapping motion)
- in the disk plane (back and forth, lagging motion)
- around its longitudinal axis (pitching or feathering motion).

The hub rotates at an (almost) constant angular rate Ω (typically Ω ranges from 20 to 50 rad/sec, depending on the particular aircraft), thanks to the torque provided by the engines. The pilot can control the rotor by varying the pitch angle of each blade by means of an actuator called swash-plate; by commanding the blade pitch angle one can indirectly control the amplitude and orientation of the loads the rotor applies to the fuselage, as these are (roughly) proportional to such an angle.

From the modelling point of view, the rotor can be considered as being constituted by four separate subsystems or blocks; this decomposition is represented in Figure 1, which shows a functional block diagram of a helicopter rotor:

The figure puts into evidence the interactions between dynamics and aerodynamics, which are typical of helicopter rotors: the control inputs influence directly the aerodynamic loads acting on the blade, which, in turn, act as forcing inputs for the blade dynamics (both rigid body and flexibility modes) and perturb the distribution of the airflow around the rotor; the loop is closed by taking into account the effects of blade motion (and elastic deformations) and induced flow on the aerodynamic behaviour of the blade's lifting surface.

The ADAMS-based model of the A109c rotor is structured in building blocks following the fundamental and convenient distinction between two classes of phenomena, i.e., mechanical ones and aerodynamic ones.

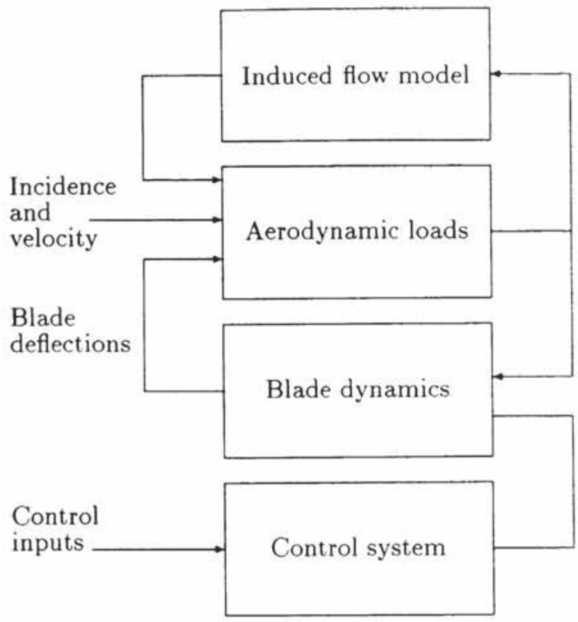

Fig. 1. Block diagram of helicopter rotor dynamics and aerodynamics.

3. IDENTIFICATION ALGORITHMS

Three identification methods have been considered in the present study: a frequency domain method, and two time domain, subspace based methods, namely the N4SID algorithm ((Van Overschee and De Moor, 1994)) and the MOESP algorithm ((Verhaegen, 1994)). This Section provides a brief overview of their characteristics.

3.1 *Frequency domain method*

The frequency domain identification method which has been used in this study can be described as follows: first, an estimate $\hat{G}(j\omega)$ of the frequency response of the system has been obtained, by making use of a nonparametric algorithm ((Wellstead, 1981)) consisting of the following steps:

(1) Sample-pairs of the signals u and y are stored in frames of M sample pairs each.
(2) For each frame, estimates (periodograms) of the spectrum of u and of the cross-spectrum between u and y are computed.
(3) The frequency response of the system is then estimated by the ratio of spectral estimates, computed by averaging the periodograms over the M time frames.

The second step of the frequency-domain identification procedure consists in fitting the estimated frequency response to a rational transfer function model. Once the model order is chosen, the algorithm estimates the parameter vector q that minimizes the following loss function:

$$J(q) = \sum_{k=1}^{K} (\hat{G}(j\omega) - G(j\omega, q))^2 w_k \qquad (1)$$

where

$$w = [w_1 w_2 \dots w_k] \qquad (2)$$

is a proper vector of weights.

3.2 *Subspace methods*

The family of subspace identification algorithms provides methods for obtaining discrete-time state-space linear models for MIMO systems from input-output measurements ((Viberg, 1994; Bittanti and (eds), 1996)); all methods are based on a geometric approach that involves subspaces spanned by rows or columns of matrices built with input-output data. The main starting point of most subspace algorithms is the following equation:

$$Y = \Gamma X + HU + V \qquad (3)$$

where:

- $\mathbf{Y}, \mathbf{U}$ are Hankel matrices formed with the output signals and the input signals:

$$\mathbf{Y} = \begin{bmatrix} \mathbf{y}_k & \mathbf{y}_{k+1} & \cdots & \mathbf{y}_{k+j-1} \\ \mathbf{y}_{k+1} & \mathbf{y}_{k+2} & \cdots & \mathbf{y}_{k+j} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{y}_{k+i-1} & \mathbf{y}_{k+i} & \cdots & \mathbf{y}_{k+i+j-2} \end{bmatrix} \qquad (4)$$

$$\mathbf{U} = \begin{bmatrix} \mathbf{u}_k & \mathbf{u}_{k+1} & \cdots & \mathbf{u}_{k+j-1} \\ \mathbf{u}_{k+1} & \mathbf{u}_{k+2} & \cdots & \mathbf{u}_{k+j} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{u}_{k+i-1} & \mathbf{u}_{k+i} & \cdots & \mathbf{u}_{k+i+j-2} \end{bmatrix} \qquad (5)$$

- Γ is the extended observability matrix of the state space model:

$$\Gamma = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \mathbf{CA}^2 \\ \vdots \\ \mathbf{CA}^{i-1} \end{bmatrix} \qquad (6)$$

- $\mathbf{X}$ is formed by consecutive state vectors:

$$\mathbf{X} = \begin{bmatrix} \mathbf{x}_k & \mathbf{x}_{k+1} & \cdots & \mathbf{x}_{k+j-1} \end{bmatrix} \qquad (7)$$

- $\mathbf{H}$ is a Toeplitz matrix formed with the Markov parameters of the model:

$$\mathbf{H} = \begin{bmatrix} \mathbf{D} & 0 & \cdots & 0 \\ \mathbf{CB} & \mathbf{D} & \cdots & 0 \\ \mathbf{CAB} & \mathbf{CB} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{CA}^{i-2}\mathbf{B} & \mathbf{CA}^{i-3}\mathbf{B} & \cdots & \mathbf{D} \end{bmatrix} \qquad (8)$$

Most subspace identification algorithms proceed from equation (3) in order to estimate from the input/output data a state sequence for the model to be identified; once such a state sequence is available, the system matrices can be easily estimated by solving a least squares problem.

Two subspace identification algorithms have been used in this study: the MOESP algorithm and the N4SID algorithm.

3.2.1. *Multivariable Output Error State sPace identification*

The MOESP algorithm starts from equation (3), and consists of two stages:

(1) firstly, an estimate of the column space of the extended observability matrix for the model to be identified is obtained. From this estimated, the A and C matrices can be determined, by exploiting the shift invariance of the observability subspace.
(2) secondly, the B and D matrices are determined, by minimizing the simulation error for the model wrt to the identification data set.

3.2.2. *Numerical subspace-based state space identificc (N4SID)*

N4SID operates on equation (3) with the aim of computing an estimate of the state sequence matrix X; such an estimate of the X matrix is obtained from a projection of the rows of Y on a subspace spanned by the rows of Hankel matrices containing past input-output data. The Main Projection Theorem (Van Overschee and De Moor, 1994) shows that those computed projection are closely related to the output of a bank of time-varying Kalman filters. The relation obtained in this way can be unraveled so that the whole set of matrices can be calculated.

4. IDENTIFICATION EXPERIMENTS

The simulation program described in Section 2 has been used for black-box identification of reduced order linear models describing the response of rotor hub loads to perturbations in the control inputs. In particular, this study has focused on the analysis of the response of the vertical shear at the blade root to perturbations in the pitch angle of the blade.

This kind of methodologies could be employed in order to derive dynamic models for control system design, oriented to vibration suppression (see, e.g., (Bittanti *et al.*, 1996; Bittanti and Moiraghi, 1994)) As the identified models would be employed in control systems design, it is natural to choose as inputs the same variables that would be used as controls in order to influence rotor dynamics; a very promising control technique is the Individual Blade Control (IBC) ((Jacklin *et al.*, 1994)) which requires actuators located in place of the connecting rods between the swashplate and each blade. A typical IBC control action consists in the application of the same perturbation (input signal u) on the pitch angle commanded to each blade, with a relative phase lag from one to the other. In the identification experiments, multiharmonic input signals have been used, i.e., signals of the form:

$$u(t) = \sum_{k=1}^{N} A\cos(k\omega_f t + \phi_k) \qquad (9)$$

This signal depends on the choice of the amplitude A, the main frequency ω_f, the total number of harmonics N and the phases ϕ_k; to this purpose, the following choices have been made:

- $A = 0.1$;
- $N = 100$;
- $\omega_f = 2\pi 45/100 rad/s$ (corresponding to 0,45 Hz).

With these choices the higher frequency of u becomes $N\omega_f = 282,743 rad/s$, i.e., about 7Ω, where Ω is the angular frequency of the rotor. Having defined the phase ϕ_1 of the first harmonic of the input signal, the phases ϕ_k of the higher harmonics have been selected according to the Schroeder formula (see, e.g., (Schroeder, 1970)): this particular phasing allows to obtain multiharmonic signals with a reduced value for the peak amplitude, so that the system remains in its linear operating region around the selected equilibrium (trim) point. Finally, a sampling period of 0.0016 seconds has been selected, which corresponds to 100Ω.

One such dataset is depicted in Figure 2; the data have been detrended and normalized to unit variance, in order to avoid conditioning problems in the subsequent processing.

5. IDENTIFICATION RESULTS

The results presented herein refer to the identification of linear models for the rigid blade rotor: in

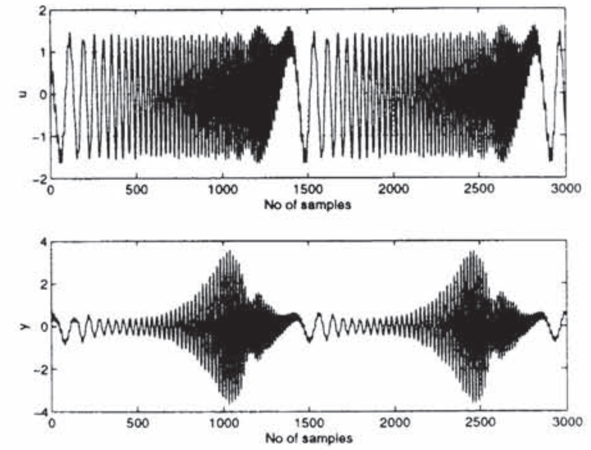

Fig. 2. Simulated data used in the study.

Table 1. Poles of the identified models

Freq. Dom.	N4SID	MOESP
$-14.6 \pm 40.38i$	$-43.5 \quad -96.6$	$-31 \pm 27.3i$
$-12.8 \pm 210i$	$-12.8 \pm 205i$	$-13 \pm 205.4i$

Table 2. Zeros of the identified models

Freq. Dom.	N4SID	MOESP
$-69.9 \pm 65i$	$73 \pm 70.4i$	$-101.3 \pm 42i$
$54.8 \pm 69.3i$	$-116.8 \pm 112.6i$	$80.7 \pm 61.2i$

such a case, prior knowledge is available about the dominant modes affecting the system's response, which can be used for the selection of model order: as a matter of fact, all the identified models are of order four.

In Tables 1 and 2 the location in the complex plane of the poles and zeros of the identified models (all converted to continuous time) are shown. As can be seen, the models obtained by the frequency domain method and by the MOESP algorithm are characterized by two couples of complex conjugate poles, which can be given a physical interpretation: the low frequency poles correspond to the flapping mode of the rotor blade, while the high frequency poles are associated to the poorly damped pitching mode.

Subspace methods implicitly provide a mean for the estimation of model order, via inspection of the singular values of a matrix constructed from input/output data. It is interesting to compare such singular values for the two subspace algorithms, see Figures 3 and 4.

As can be seen, no clear gap appears between the singular values computed by the N4SID algorithm, while the MOESP PO algorithm seems capable of detecting the existence of a low order set of dominant modes in the input/output data.

Concerning the zeros, it is interesting to notice that here the behaviour of the identification algorithms is fairly homogeneous; in particular, this

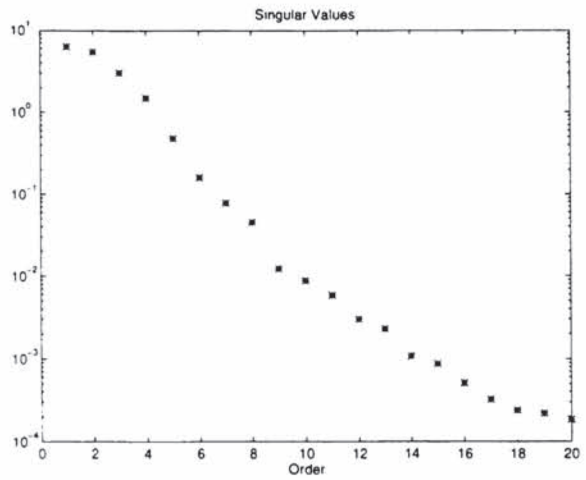

Fig. 3. Singular values computed by the N4SID algorithm.

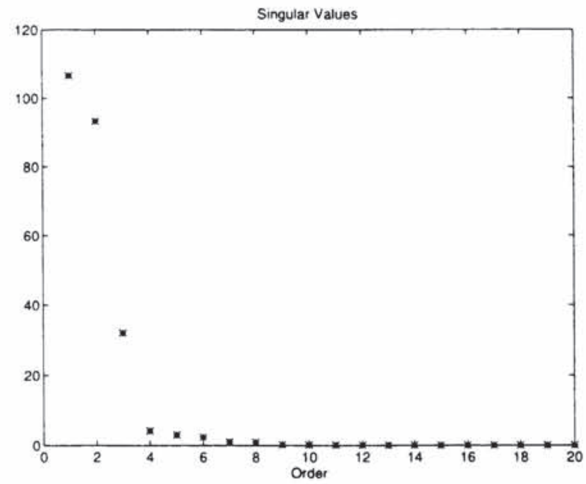

Fig. 4. Singular values computed by the MOESP PO algorithm.

can be expected from the two subspace methods, since they both estimate the B and D matrices of the model in a similar way.

Furthermore, all the identified models are characterized by the presence of zeros in the right-half plane, which is a well known feature of the system. In particular, in (Bittanti and Lovera, 1996) a physical model for the out-of-plane dynamics of a rotor blade was analyzed and the presence of a right half plane zero could be predicted and interpreted. In the present case, the simulation model is based on a more complex physical model than the one analyzed in (Bittanti and Lovera, 1996), which explains the additional zeros which have been determined in the identification trials.

One more point should be mentioned concerning the estimation of the system zeros. The schemes originally proposed in the subspace literature for the estimation of the B and D matrices ((Van Overschee and De Moor, 1994; Verhaegen, 1994)) were known to suffer from robustness problems wrt non-white input signals; such signals can give rise to nearly rank deficient data matrices the inversion of which played an important role in the determination of B and D. Similar problems have been experienced in this study, where poor performance has been obtained when using the original schemes. Since, various alternative methods have been proposed to overcome this difficulty; the one which has been used in this study is based on the minimization of the simulation error for the identified model wrt the identification data set ((Van Overschee, 1995)).

6. CONCLUDING REMARKS

Three identification methods have been applied to the problem of deriving reduced order linear models for the response of a hovering helicopter rotor to individual blade control inputs. All the considered methods provide satisfactory results.

7. ACKNOWLEDGEMENTS

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