An Application of Floquet Theory to Prediction of Mechanical Instability

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The problem of helicopter mechanical instability is considered for the case where one blade damper is inoperative. It is shown that if the hub is considered to be nonisotropic the equations of motion have periodic coefficients which cannot be eliminated. However, if the hub is isotropic the equations can be transformed to a rotating frame of reference and the periodic coefficients eliminated. The Floquet Transition Matrix method is shown to be an effective way of dealing with the nonisotropic hub and nonisotropic rotor situation. Time history calculations are examined and shown to be inferior to the Floquet technique for determining system stability. A smearing technique used in the past for treating the one damper inoperative case is examined and shown to yield unconservative results. It is shown that instabilities which occur when one blade damper is inoperative may consist of nearly pure blade motion or they may be similar to the classical mechanical instability.

NOTATION

c_i = lag damping rate
c_x = effective hub damping in x-direction
c_y = effective hub damping in y-direction
e = lag hinge offset
I_b = second mass moment of blade about lag hinge
k_i = lag spring rate
k_x = effective hub stiffness in x-direction
k_y = effective hub stiffness in y-direction
m_b = blade mass
m_x = effective hub mass in x-direction
m_y = effective hub mass in y-direction
N = number of blades in rotor
P_x = force acting on hub in x-direction
P_y = force acting on hub in y-direction
S_b = first mass moment of blade about lag hinge
T = period of the periodic coefficients, $T = 2\pi/\Omega$
$\bar{x}, \bar{y}$ = coordinates of hub in rotating reference
x_c, y_c = frame
x_c, y_c = coordinates of rotor center of mass in fixed reference frame
x_h, y_h = coordinates of hub in fixed reference frame
x_i, y_i = coordinates of elemental blade mass dm in fixed reference frame
ζ_i = lag deflection of ith blade
η_h = defined by Equations (18)
η_i = defined by Equations (7)
ν_h = defined by Equations (18)
ν_0 = defined by Equations (7)
ρ = distance from lag hinge to elemental blade mass dm
ψ_i = azimuthal location of ith blade
Ω = rotor speed
ω_h = defined by Equations (18)
ω_{0i} = defined by Equations (7)

Presented at the AHS/NASA Ames Specialists' Meeting on Rotorcraft Dynamics, Moffett Field, Calif., February 13 – 15, 1974.

The problem of mechanical instability of helicopters on the ground has been recognized and understood for many years. The analysis by Coleman and Feingold[1] has become the standard reference on this phenomenon although it was not published until many years after the first incidents of mechanical instability, or ground resonance as it is commonly known, were encountered on the early autogyros. The mechanical instability phenomenon is most commonly associated with helicopters having articulated rotors; however, helicopters using the soft-inplane hingeless rotors which have become popular in recent years are also susceptible to this problem. Machines employing these soft-inplane hingeless rotors are also known to experience a similar problem, commonly known as air resonance, which occurs in flight rather than on the ground and this problem has received much attention in recent years (see, e.g., Refs. 2 and 3).

From the analysis of Reference 1 and others it is known that the ground resonance problem is due primarily to a coupling of the blade inplane motion with the rigid body degrees of freedom of the helicopter on its landing gear. These analyses have shown that with the proper selection of blade lag dampers and landing gear characteristics the problem of mechanical instability can be eliminated within the operating rotor speed range. All of the mechanical instability analyses conducted to date have one assumption in common— all blades are assumed to have identical properties. This is a reasonable assumption under ordinary circumstances; however, the U.S. Army has a requirement on new helicopters which invalidates this assumption. The requirement is that the helicopter be free from ground resonance with one blade damper inoperative. As will be shown later, this one blade damper inoperative requirement has a serious impact on the classical method of analyzing a helicopter for mechanical instability. Further, there is at present no published method available for treating the case where each of the blades is permitted to have different properties. Thus the designer is faced with the dilemma of trying to satisfy the requirement with an analysis method in which one of the basic assumptions is severely violated.

Two methods have been used to circumvent this difficulty. The first of these involves a physical approximation so that the classical analysis becomes applicable. In this approach all blades are still assumed to have identical lag dampers even when one blade damper is removed, but the value of each of the dampers is reduced by the amount c_i/N where N is the number of blades and c_i is the original lag damper rate. As can be seen, with this approach a system is analyzed which is quite different from the actual situation of a rotor with no damping on one blade. The second method which has been used is to reformulate the equations of motion allowing for differing blade characteristics and to obtain the stability characteristics of the system using a time history integration of the equations. This second approach has the drawback that interpretation of stability characteristics from time history calculations is often difficult and open to question. The method will yield correct results, however, provided the equations are integrated over a sufficiently long time period using a reliable numerical integration procedure.

The purpose of this paper is to present a method of obtaining the mechanical stability characteristics directly for a helicopter operating on the ground with one blade damper inoperative. As will be shown later, the equations governing the motion of this system have periodic coefficients. This fact suggests the use of Floquet theory as the means for determining the stability characteristics of the system. In the following, the one-damper-inoperative problem is formulated and the resulting equations are solved using the Floquet Transition Matrix method described by Peters and Hohenemser.[4] Results obtained using this method are compared with results obtained from the two previously used methods and recommendations are made concerning the future use of the three methods described.

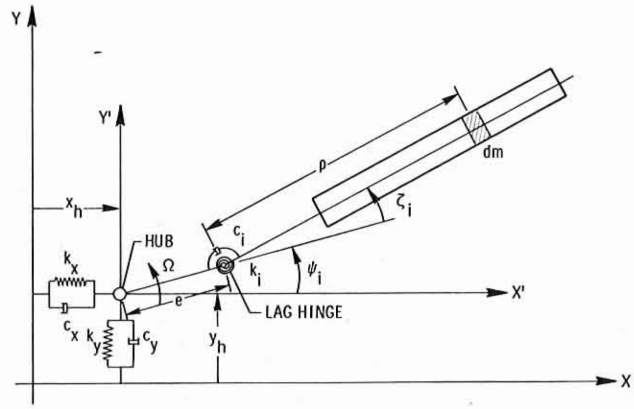

Figure 1. Mathematical representation of rotor and hub.

EQUATIONS OF MOTION

The equations of motion for the mechanical instability problem will be formulated using an Eulerian approach. It will be assumed, as is done in Reference 1, that the helicopter on its landing gear can be represented by effective parameters applied at the rotor hub. It will be further assumed that only inplane motions of the hub and blades are important in determining the ground resonance characteristics of the helicopter. Thus the degrees of freedom to be considered consist of two inplane hub degrees of freedom and a lead-lag degree of freedom for each blade in the rotor. The mathematical model to be used in the analysis is shown in Fig. 1. Note that in the figure only a typical blade is shown. The analysis will be formulated for a rotor having N blades, and each blade is assumed to have a rotational spring and damper which act about the lag hinge. Rotor speed is constant.

The blade equations are developed by summing moments about the lag hinge. The coordinates of the elemental mass dm in the fixed system are

$$\left.\begin{aligned} x_i &= x_h + e\cos\psi_i + \rho\cos(\psi_i + \zeta_i) \\ y_i &= y_h + e\sin\psi_i + \rho\sin(\psi_i + \zeta_i) \end{aligned}\right\} \quad (1)$$

where

$$\psi_i = \Omega t + 2\pi(i - 1)/N \qquad i = 1, 2, \ldots, N$$

These expressions can be differentiated twice with respect to time to yield the accelerations experienced by the differential mass

$$\left.\begin{aligned} \ddot{x}_i &= \ddot{x}_h - e\Omega^2\cos\psi_i - \rho(\Omega + \dot{\zeta}_i)^2 \\ &\quad \cdot\cos(\psi_i + \zeta_i) - \rho\ddot{\zeta}_i\sin(\psi_i + \zeta_i) \\ \ddot{y}_i &= \ddot{y}_h - e\Omega^2\sin\psi_i - \rho(\Omega + \dot{\zeta}_i)^2 \\ &\quad \cdot\sin(\psi_i + \zeta_i) + \rho\ddot{\zeta}_i\cos(\psi_i + \zeta_i) \end{aligned}\right\} \quad (2)$$

Using D'Alembert's principle the summation of moments about the lag hinge can be written as

$$\left.\begin{aligned} S_b &= \int \rho dm \\ I_b &= \int \rho^2 dm \end{aligned}\right\} \quad (4)$$

$$\int \rho\sin(\psi_i + \zeta_i)\ddot{x}_i dm - \int \rho\cos(\psi_i + \zeta_i)\ddot{y}_i dm$$

$$- k_i\zeta_i - c_i\dot{\zeta}_i = 0 \qquad i = 1, 2, \ldots, N \quad (3)$$

where the integrals are evaluated over the length of the blade. Introducing the expressions for $\ddot{x}_i$ and $\ddot{y}_i$ and defining the following

the blade equations become

$$I_b\ddot{\zeta}_i + e\Omega^2 S_b\sin\zeta_i - S_b[\ddot{x}_h\sin(\psi_i + \zeta_i)$$

$$- \ddot{y}_h\cos(\psi_i + \zeta_i)] + k_i\zeta_i + c_i\dot{\zeta}_i = 0 \quad (5)$$

$$i = 1, 2, \ldots, N$$

If small displacements are now assumed the blade equations may be linearized to obtain

$$\ddot{\zeta}_i + \eta_i\dot{\zeta}_i + ({\omega_0}_i^{\,2} + \Omega^2\nu_0^2)\zeta_i = (\nu_0^2/e)[\ddot{x}_h\sin\psi_i$$

$$- \ddot{y}_h\cos\psi_i] \qquad i = 1, 2, \ldots, N \quad (6)$$

where the following parameters have been introduced

$$\left.\begin{aligned} \nu_0^2 &= eS_b/I_b \\ {\omega_0}_i^{\,2} &= k_i/I_b \\ \eta_i &= c_i/I_b \end{aligned}\right\} \quad (7)$$

Under the assumptions stated earlier the hub equations of motion can be written directly as

$$\left.\begin{aligned} m_x\ddot{x}_h + c_x\dot{x}_h + k_x x_h &= P_x \\ m_y\ddot{y}_h + c_y\dot{y}_h + k_y y_h &= P_y \end{aligned}\right\} \quad (8)$$

where the coefficients on the left side of these equations are the effective hub properties in the x- and y-directions, respectively. The determination of these properties depends on an extensive knowledge of the helicopter inertial characteristics and the stiffness, damping, and geometrical characteristics of the landing gear system. These properties may be determined either by ground shake tests of the helicopter, as suggested in Reference 1, or by direct calculations. The right-hand side of the above equations are the forces acting on the hub due to the fact that the rotor is experiencing accelerations in the x- and y-directions. If the accelerations of the rotor center of mass are $\ddot{x}_c$ and $\ddot{y}_c$, respectively, the P_x and P_y are given by

$$\left.\begin{aligned} P_x &= -Nm_b\ddot{x}_c \\ P_y &= -Nm_b\ddot{y}_c \end{aligned}\right\} \quad (9)$$

The equations as written also indicate that in the absence of the rotor the hub degrees of freedom are uncoupled. This is an approximation, but it is an assumption made in Reference 1 and one generally used in helicopter mechanical stability analyses.

If all blades in the rotor are assumed to have the same mass distribution, the coordinates for the total rotor center of mass may be written as

$$\left.\begin{aligned} x_c &= x_h + \frac{1}{N}\sum_{i=1}^{N} x_{i_c} \\ y_c &= y_h + \frac{1}{N}\sum_{i=1}^{N} y_{i_c} \end{aligned}\right\} \quad (10)$$

where x_{i_c} and y_{i_c} are the coordinates of the individual blade center of mass, measured with respect to the hub. If the center of mass of the ith blade is a radial distance ρ_c from the lag hinge

$$\left.\begin{aligned} x_{i_c} &= e\cos\psi_i + \rho_c\cos(\psi_i + \zeta_i) \\ y_{i_c} &= e\sin\psi_i + \rho_c\sin(\psi_i + \zeta_i) \end{aligned}\right\} \quad (11)$$

Making the observation that, for $N > 1$

$$\sum_{k=1}^{N} \cos\psi_k = \sum_{k=1}^{N} \sin\psi_k = 0$$

the rotor center of mass coordinates become

$$\left.\begin{aligned} x_c &= x_h - (\rho_c/N)\sum_{i=1}^{N} \zeta_i \sin\psi_i \\ y_c &= y_h + (\rho_c/N)\sum_{i=1}^{N} \zeta_i \cos\psi_i \end{aligned}\right\} \quad (12)$$

These expressions may now be differentiated twice with respect to time and the forces P_x and P_y obtained as

$$\left.\begin{aligned} P_x &= -Nm_b\ddot{x}_h + S_b\sum_{i=1}^{N}[(\ddot{\zeta}_i - \Omega^2\zeta_i)\sin\psi_i + 2\Omega\dot{\zeta}_i\cos\psi_i] \\ P_y &= -Nm_b\ddot{y}_h - S_b\sum_{i=1}^{N}[(\ddot{\zeta}_i - \Omega^2\zeta_i)\cos\psi_i - 2\Omega\dot{\zeta}_i\sin\psi_i] \end{aligned}\right\} \quad (13)$$

The hub equations of motion thus become

$$\left.\begin{aligned} &(m_x + Nm_b)\ddot{x}_h + c_x\dot{x}_h + k_x x_h \\ &\quad = S_b\sum_{i=1}^{N}[(\ddot{\zeta}_i - \Omega^2\zeta_i)\sin\psi_i + 2\Omega\dot{\zeta}_i\cos\psi_i] \\ &(m_y + Nm_b)\ddot{y}_h + c_y\dot{y}_h + k_y y_h = \\ &\quad -S_b\sum_{i=1}^{N}[(\ddot{\zeta}_i - \Omega^2\zeta_i)\cos\psi_i - 2\Omega\dot{\zeta}_i\sin\psi_i] \end{aligned}\right\} \quad (14)$$

The equations of motion for the system thus consist of $(N + 2)$ coupled second-order differential equations with the coupling terms having periodic coefficients. The periodic coefficients arise because the blade equations are written in a rotating reference system whereas the hub equations are in a fixed system. As is shown in Reference 1 if all the blades have identical lag springs and lag dampers, the periodic coefficients may be eliminated by transforming the blade equations from the rotating to the fixed system of reference. The resulting constant coefficient system of equations is the set normally solved in the classical ground resonance analysis. It should be observed, however, that if the blades are allowed to have different lag springs and dampers, the periodic coefficients cannot be eliminated in the usual manner.

An alternative does exist, however, for eliminating the periodic coefficients even when the blades are allowed to have differing characteristics. The alternative consists of transforming the hub equations into the rotating system of reference. In order to eliminate the periodic coefficients using this approach, the additional assumption must be made that the hub is isotropic. That is

$$m_x = m_y$$

$$c_x = c_y$$

$$k_x = k_y$$

This is the approach used in Reference 1 for treating the two-bladed rotor which is another case where the periodic coefficients in the equations of motion cannot be eliminated by transforming the blade equations to the fixed system.

The transformation from fixed to rotating coordinates is given by

$$\left.\begin{aligned} \overline{x} &= x_h\cos\Omega t + y_h\sin\Omega t \\ \overline{y} &= -x_h\sin\Omega t + y_h\cos\Omega t \end{aligned}\right\} \quad (15)$$

Differentiating these expressions allows the following identities to be established

$$\begin{aligned} \dot{x}_h\cos\Omega t + \dot{y}_h\sin\Omega t &= \dot{\overline{x}} - \Omega\overline{y} \\ -\dot{x}_h\sin\Omega t + \dot{y}_h\cos\Omega t &= \dot{\overline{y}} + \Omega\overline{x} \\ \ddot{x}_h\cos\Omega t + \ddot{y}_h\sin\Omega t &= \ddot{\overline{x}} - \Omega^2\overline{x} - 2\Omega\dot{\overline{y}} \\ -\ddot{x}_h\sin\Omega t + \ddot{y}_h\cos\Omega t &= \ddot{\overline{y}} - \Omega^2\overline{y} + 2\Omega\dot{\overline{x}} \end{aligned}$$

The hub equations in the rotating system are then obtained by appropriate combinations of the x_h and y_h equations, Equations (14). The resulting equations are given below

$$\begin{aligned} &\ddot{\overline{x}} + \eta_h\dot{\overline{x}} + (\omega_h^2 - \Omega^2)\overline{x} - 2\Omega\dot{\overline{y}} - \Omega\eta_h\overline{y} \\ &\quad = \nu_h^2\sum_{j=1}^{N}[(\ddot{\zeta}_j - \Omega^2\zeta_j)\sin\frac{2\pi}{N}(j - 1) \\ &\qquad + 2\Omega\dot{\zeta}_j\cos\frac{2\pi}{N}(j - 1)] \end{aligned} \quad (16)$$

$$\begin{aligned} &\ddot{\overline{y}} + \eta_h\dot{\overline{y}} + (\omega_h^2 - \Omega^2)\overline{y} + 2\Omega\dot{\overline{x}} + \Omega\eta_h\dot{\overline{x}} \\ &\quad = -\nu_h^2\sum_{j=1}^{N}[(\ddot{\zeta}_j - \Omega^2\zeta_j)\cos\frac{2\pi}{N}(j - 1) \\ &\qquad - 2\Omega\dot{\zeta}_j\sin\frac{2\pi}{N}(j - 1)] \end{aligned} \quad (17)$$

where the following parameters have been introduced

$$\left.\begin{aligned} \nu_h^2 &= S_b/(m_x + Nm_b) \\ \omega_h^2 &= k_x/(m_x + Nm_b) \\ \eta_h &= c_x/(m_x + Nm_b) \end{aligned}\right\} \quad (18)$$

TABLE 1

Parameters Used in the Sample Calculations

Number of blades	4
Blade mass, m_b	6.5 slugs (94.9 kg)
Blade mass moment, S_b	65.0 slug-ft (289.1 kg-m)
Blade mass moment of inertia, I_b	800.0 slug-ft^2 (1084.7 kg-m^2)
Lag hinge offset, e	1.0 ft (0.3048 m)
Lag spring, k_i	0.0 ft-lb/rad (0.0 m-N/rad)
Lag damper, c_i	3000.0 ft-lb-sec/rad (4067.5 m-N-s/rad)
Hub mass, m_x	550.0 slugs (8026.6 kg)
Hub mass, m_y	225.0 slugs (3283.6 kg)
Hub spring, k_x	85000.0 lb/ft (1240481.8 N/m)
Hub spring, k_y	85000.0 lb/ft (1240481.8 N/m)
Hub damper, c_x	3500.0 lb-sec/ft (51078.7 N-s/m)
Hub damper, c_y	1750.0 lb-sec/ft (25539.3 N-s/m)

Introducing the rotating coordinates into the blade equations, Equations (6), results in

$$\ddot{\zeta}_j + \eta_j \dot{\zeta}_j + (\omega_{0j}^2 + \Omega^2 \nu_0^2)\zeta_j$$

$$= (\nu_0^2/e)[(\ddot{\bar{x}} - \Omega^2 \bar{x} - 2\Omega\dot{\bar{y}}) \sin \frac{2\pi}{N}(j-1) - (\ddot{\bar{y}} - \Omega^2 \bar{y} + 2\Omega\dot{\bar{x}}) \cos \frac{2\pi}{N}(j-1)] \quad (19)$$

$$j = 1, 2, \ldots, N$$

Since modern helicopters do not in general have isotropic hubs, the above equations can only be used to approximate the effects of a nonisotropic rotor. They are, however, easily solved for the stability characteristics of the system and thus they might be used to obtain a first approximation to the mechanical stability boundary for a helicopter with one blade damper inoperative.

From the foregoing discussion it can be seen that if either the rotor or the hub is isotropic, the mechanical stability characteristics of the system may be obtained using conventional techniques. If both the rotor and hub are nonisotropic the equations of motion of the system contain periodic coefficients and thus the standard eigenvalue techniques cannot be used to determine whether the system is stable or unstable. It is the purpose of this paper to demonstrate that Floquet theory can be used to analyze this general situation of a nonisotropic rotor coupled with a nonisotropic hub.

DISCUSSION OF RESULTS

In order to demonstrate the application of the Floquet theory and to obtain a general understanding of the effect of one blade damper inoperative on mechanical stability, a set of parameters were chosen. The parameters in the mechanical stability analysis were chosen so as to be in the general range of interest for a single rotor helicopter and were such that the system was stable with all dampers functioning up to a rotor speed of 400 rpm. The parameter values chosen for the calculations are shown in Table 1.

The parameters presented in Table 1 correspond to an isotropic rotor and a nonisotropic hub. In the following discussion results are presented for the case of an isotropic hub coupled with a nonisotropic rotor and a nonisotropic hub coupled with an isotropic rotor as well as the case of interest which involves a nonisotropic hub coupled with a nonisotropic rotor. When an isotropic hub is mentioned, this means that the hub parameters in both the x- and y-directions were assigned the values shown in Table 1 for the x-direction. An isotropic rotor implies that all dampers are operational and a nonisotropic rotor is meant to indicate that the lag damper has been removed from blade number 1. The analysis has been formulated in such a way that any number of blade lag dampers or lag springs may be removed to make the rotor nonisotropic. The results presented here, however, only involve the removal of the lag damper from one blade.

The case of an isotropic hub was first run in an effort to become familiar with the nonisotropic rotor results before proceeding with the more complicated Floquet analysis. The isotropic hub permits the equations to be transformed into the rotating reference frame and results in a system of equations with constant coefficients, Equations (16), (17), and (19), even with a nonisotropic rotor.

Figure 2 shows the results of the calculations for the isotropic hub with all blade dampers working. The real parts of the eigenvalues, labeled

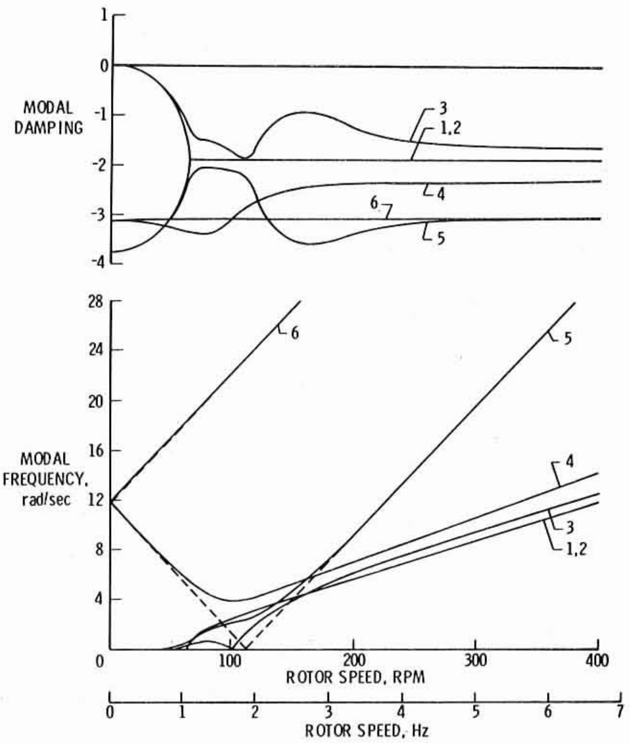

Figure 2. Modal damping and frequencies for isotropic hub in rotating system with all blade dampers working

modal damping, are plotted in the upper portion of the figure and the imaginary parts of the eigenvalues, labeled modal frequency, are plotted in the lower portion of the figure. Note that since the equations were solved in the rotating system, the frequencies in the lower portion of Fig. 2 are plotted in the rotating system. The numbers attached to the different modes in Fig. 2 and in subsequent similar figures have no significance other than to provide a label for the various modes. In Fig. 2 the dashed lines represent the uncoupled hub modes. The uncoupled rotor modes follow along the curves labeled 1,2 which also represent, in the terminology of Reference 5, the rotor collective modes. Note that the uncoupled blade frequencies are zero for rotor speeds less than about 65 rpm. This is due to the fact that the blades are critically damped for these low rotor speeds. At the higher rotor speeds modes 3 and 4 are essentially rotor modes and modes 5 and 6 are essentially hub modes. At the lower speeds, however, due to the coupling between rotor and hub, mode 4 changes to a hub mode and mode 5 changes to a blade mode. Note from the damping plot that all the modes indicate stability over the entire rotor speed range.

The results for one blade damper inoperative and an isotropic hub are plotted in Fig. 3. Note that the removal of a blade damper has caused the appearance of a mode which was not present in Fig. 2, namely the mode labeled 3 in Fig. 3, and that this mode exhibits a mild instability between 160 and 200 rpm. At rotor speeds below about 100 rpm this mode has a frequency which corresponds to the uncoupled frequency of the blade which has no damper. At rotor speeds above 100 rpm this mode begins to deviate in frequency from the uncoupled frequency. Another interesting point is that mode 1 in Fig. 3 is precisely the same as the collective modes of Fig. 2, and in Fig. 3 there is only one such mode. Thus it appears that the unstable mode in Fig. 3 has evolved from one of the two collective modes shown in Fig. 2 because of the removal of one of the blade dampers.

Having examined the case of one blade damper inoperative and an isotropic hub, the next logical step is to examine the more realistic situation of a nonisotropic hub. Before examing the one damper inoperative situation it was first desired to confirm that the system was stable with all dampers working. The modal damping and frequency of the various modes with all dampers working and a nonisotropic hub are shown in Fig. 4. As can be seen from the damping plot, all the modes are stable. In this case the equations of motion are solved in the fixed frame of reference and hence the frequencies are plotted in this frame. The dashed lines on the frequency plot represent the uncoupled

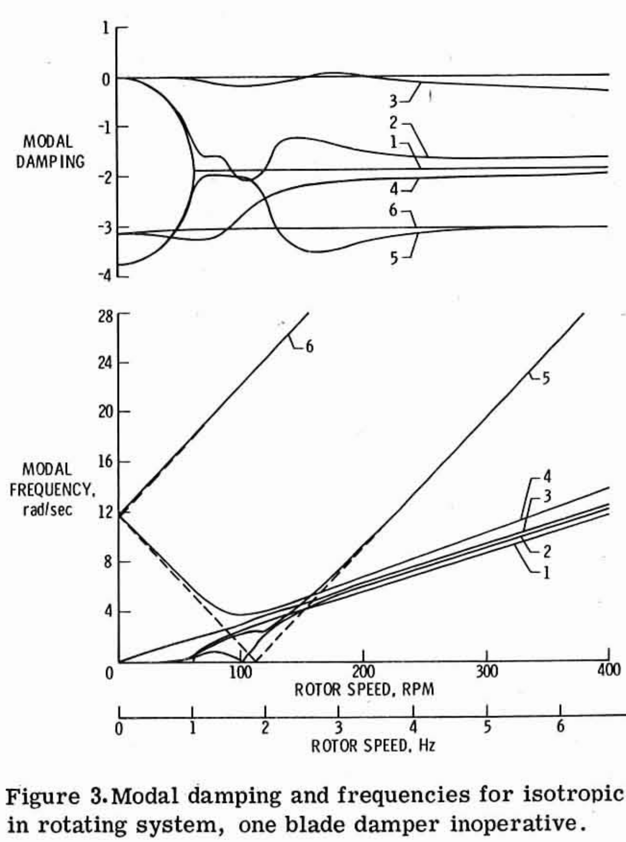

Figure 3. Modal damping and frequencies for isotropic hub in rotating system, one blade damper inoperative.

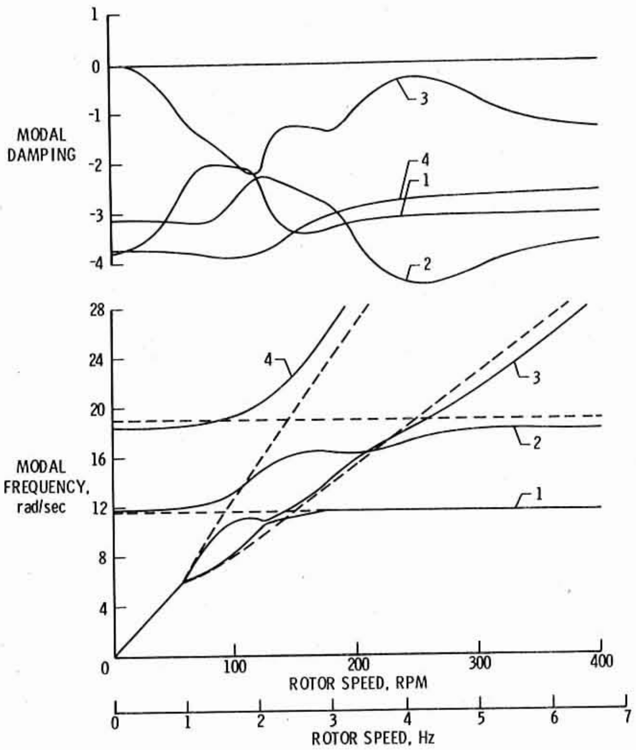

Figure 4. Modal damping and frequencies for nonisotropic hub in fixed system, all blade dampers working.

system: the horizontal dashed lines being the hub modes and the slanted dashed lines being the rotor modes. Note that because the rotor modes become critically damped at low rotor speeds the two uncoupled rotor frequencies come together before reaching the origin. The uncoupled rotor lines also represent the collective modes for the rotor. These modes are completely uncoupled from the other modes and hence are not included in the eigenvalue analysis of the nonisotropic hub coupled with an isotropic rotor. The damping for the collective modes is exactly the same as that shown for modes 1,2 in Fig. 2.

The Floquet Transition Matrix method has been discussed in detail by Peters and Hohenemser,[4] and Hohenemser and Yin.[5] In applying the method to the mechanical stability problem, a fourth order Runge-Kutta numerical integration procedure with Gill coefficients[6] was used to generate the Floquet transition matrix.

The validity of the Floquet analysis was verified by comparing results from this analysis with results from both the rotating system analysis (isotropic hub) and from the fixed system analysis (isotropic rotor). In each case the results from the Floquet analysis were identical to results from the other analyses.

Having thus established the validity of the Floquet analysis, results were obtained for the nonisotropic hub and one blade damper inoperative. These results are shown in Fig. 5. Note that these results are very much similar to those shown in Fig. 4 except that, as was the case with the isotropic hub and one blade damper inoperative, there are additional modes introduced. Also indicated is a relatively strong instability between 210 and 305 rpm. The frequencies of the additional modes which are introduced correspond, at low rotor speeds, to the frequencies of the uncoupled blade which has no damper. In the rotor speed range where the instability occurs, however, the frequency deviates from the uncoupled value as indicated by the mode labeled 3. In this range and at higher rotor speeds the mode labeled 5 is nearer the uncoupled blade frequency. It thus appears that for this case the instability is more a coupled rotor/hub mode than a pure blade mode as was indicated for the isotropic hub.

This conjecture is further strengthened by an examination of the modal functions. The modal functions for a rotor speed of 255 rpm, which is the point of maximum instability, are shown in Fig. 6.

These plots were obtained by converting the complex characteristic functions from the Floquet analysis to real functions using the procedure suggested by Hohenemser and Yin.[5] These real func-

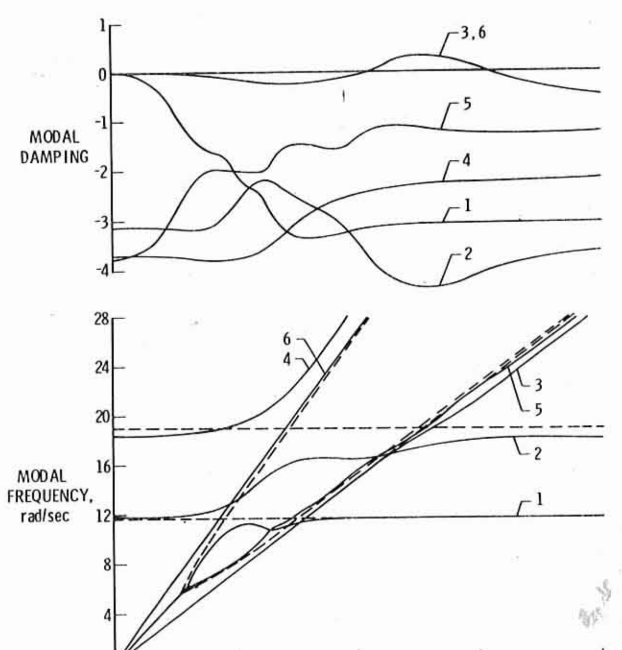

Figure 5. Modal damping and frequencies for nonisotropic hub in fixed system, one blade damper inoperative.

tions are then plotted over a time period corresponding to one rotor revolution. Note from this figure that blade 1, the blade without a damper, has a significantly higher contribution to the mode than the other blades. Also from the plot of hub response it can be seen that the participation of the lateral hub degree of freedom, which has the higher of the uncoupled hub frequencies shown on Fig. 5, is considerable. It is thus concluded from Figs. 5 and 6 that the one damper inoperative situation can lead to a classical mechanical instability.

A time history calculation was made for this same condition and the results are shown in Fig. 7. The top portion of the figure represents the individual blade lag motions whereas the lower portion represents the hub response in the x- and y-directions. The figure indicates that the blades which have lag dampers are well damped, but the blade on which the damper is inoperative experiences large lag excursions. The hub traces show moderate response in both directions, but no indication of instability is observable. From this time history solution one would conclude that the system is stable since the motions of the various degrees of freedom do not increase in amplitude with increasing time.

In attempting to resolve the discrepancy between the time history results and Floquet results, several parameter variations were attempted: numerical integration step size was reduced, period of integration was increased, several combinations of initial conditions were used. Results basically the same as those shown in Fig. 7 were obtained in each case. Since the integration scheme being used was relatively unsophisticated (constant step size and no round-off or truncation error checking) a more elaborate predictor-corrector routine was tried. Results from this calculation agreed identically with the Floquet results. The conclusion to be drawn here is that the simple integration routine which was completely adequate for the short time period integration required for the Floquet transition matrix apparently allowed error build-up over the long time period required for the time histories. This numerical experimentation which was required to obtain a correct time history solution illustrates that whenever an eigenvalue approach to stability calculations is available, it is to be preferred over the time history approach.

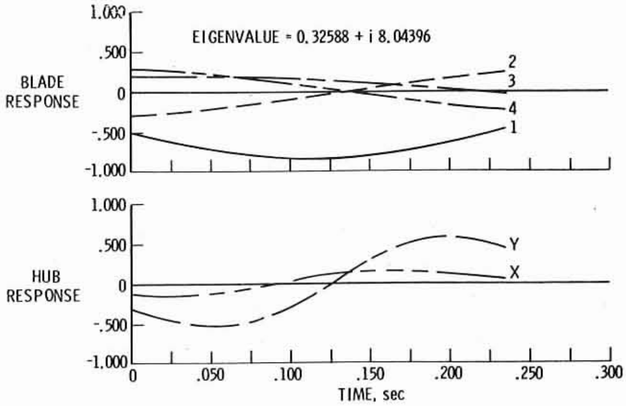

Figure 6. Modal functions for nonisotropic hub, one blade damper inoperative, $\Omega = 255$ rpm.

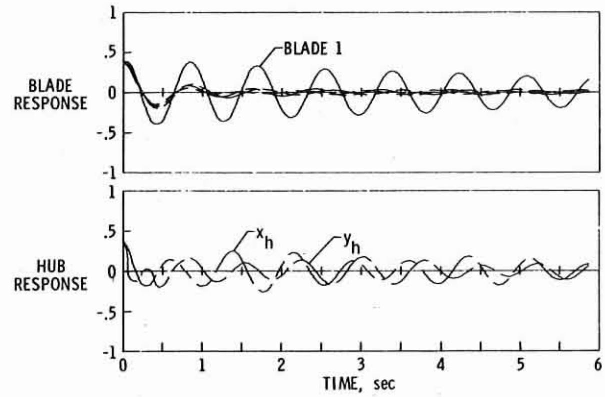

Figure 7. Time history for nonisotropic hub, one blade damper inoperative, $\Omega = 255$ rpm.

One of the methods used in the past for treating the one blade damper inoperative case involves a smearing of the total blade damping. The reasoning for this approach is as follows. If the rotor has N blades then the total damping available in the rotor is Nc_i where c_i is the damping on one blade. If one damper is removed, the total damping becomes $(N - 1)c_i$. Thus, using this approach, each blade in the rotor would be treated as if it had a lag damper equal to $c_i(N - 1)/N$.

After an examination of the preceding one damper inoperative results it would be expected that this approach would lead to unconservative results. This is due to the fact that the instabilities encountered in the previous results involved large motions of the blade which had no damper. The smearing technique results in damping, which is not greatly different from the original value, being applied to each blade and thus the true situation is not adequately modeled.

To illustrate this method, the nonisotropic hub case was analyzed using the smearing approach. The results from these calculations are shown in Fig. 8. Note that although mode 3 becomes lightly damped the system remains stable throughout the rotor speed range considered. The fact that mode 3 approaches instability is attributable to the fact that this mode was not heavily damped in the orig-

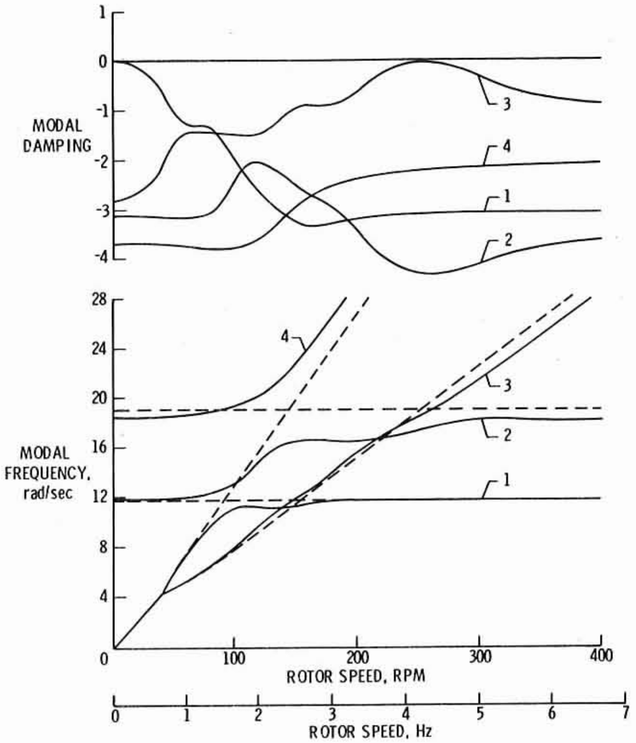

Figure 8. Modal damping and frequencies for nonisotropic hub, one blade damper inoperative, using the smearing technique.

inal calculations. A run of the isotropic hub case, where all the modes were originally well damped, indicated that the smearing technique resulted in well damped modes for one blade damper removed. The smearing technique is thus not recommended for treating the one blade damper inoperative situation since it leads to unconservative results.

Since one way for eliminating the classical mechanical instability is to increase the blade damping, it was decided to attempt this approach on the instability indicated in Fig. 5. The approach was to leave the damping indentically zero on one blade and increase the damping on the remaining three blades. The results of this series of calculations are shown in Fig. 9 where the region of instability is presented as a function of blade lag damping and rotor speed. As can be seen from the figure, increasing the blade damping on three of the blades has very little effect on the stability boundaries when one blade has zero damping. This result was somewhat expected since from the previous calculations it was observed that the blade with zero damping responds more or less independently of the other blades in the rotor.

During the increased damping calculations no attempt was made to determine whether or not the nature of the instability had changed. That is, whether the instability had changed from one involving both blade and hub motion to one consisting of primarily blade motion with only small amounts of hub motion. Further delving into possible corrective actions for the instability which occurs with one blade damper inoperative was beyond the scope of this paper and thus more research is needed to determine how the instability may be eliminated.

CONCLUSIONS

There are several conclusions which may be inferred from the preceding results. First of all, the fact that a helicopter is free from mechanical instability with all blade dampers working does not guarantee that it will be free of instabilities with one blade damper inoperative. The instability encountered with one blade damper inoperative may be an instability with mostly blade motion and little hub motion or it may be the classical mechanical instability.

The Floquet Transition Matrix method can be used effectively in examining the mechanical stability characteristics of helicopters with one blade damper inoperative. When both the hub and rotor are considered to be nonisotropic, the equations of motion contain periodic coefficients and the Floquet approach provides an efficient means for dealing with this situation. Since the Floquet approach yields the stability characteristics directly, it furnishes a more desirable approach to stability problems than time history calculations.

Time history calculations can lead to erroneous conclusions relative to the determination of system stability. The time history approach to stability problems is recommended only when no other re-

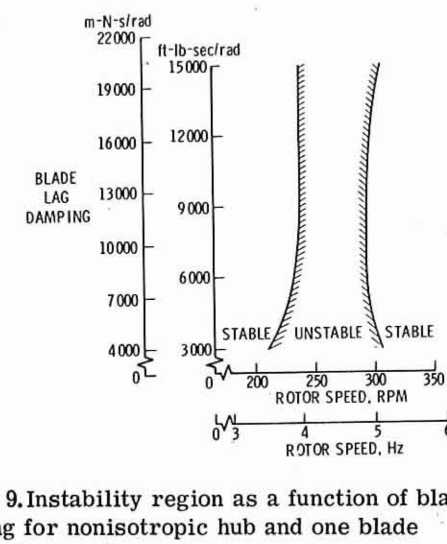

Figure 9. Instability region as a function of blade lag damping for nonisotropic hub and one blade damper inoperative.

course is available. If the time history approach is used, some numerical experimentation with more than one numerical integration scheme on the system of equations under consideration should be undertaken to assure that accurate conclusions regarding stability may be obtained.

The smearing approach which has been used in the past for treating the one blade damper inoperative situation leads to unconservative results. Therefore, this method is considered to be an unacceptable means for determining stability under these conditions.

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