ON THE ADVANTAGES OF PERIODIC EXCITATION IN SYSTEM IDENTIFICATION.

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Abstract. In this paper it is shown that periodic excitations offer significant advantages in system identification compared to noise excitation. The influence on the uncertainty of the estimates, the model validation problem, initial state estimate and the frequency range of the measurements is analysed and illustrated.

Key Words. periodic excitations, frequency range, model validation, initial conditions, signal-to-noise-ratio

1. INTRODUCTION

The choice of the excitation is a very important step in the design of an identification process and its nature is closelely linked to the selected identification method. The most popular methods make use of discrete time models, supposing that the input is piecewice constant (ZOH-excitation) (Ljung, 1987), while frequency domain identification becomes attractive if periodic excitations are used (Schoukens and Pintelon, 1991). Optimum excitation signals can be selected by maximizing some criterion like the determinant of the information matrix (Federov,1972, Goodwin and Payne, 1977, Zarrop, 1979). In practice this design is almost never made because to much a priori information is required and for these reasons it is replaced by an intuitive approach. But even then some general rules are valid which are based on the experimenters experience and physical insight in the problem. The signal-to-noise ratio (SNR) of the measurements can be improved by increasing the power of the excitation and the available power should be injected at those frequencies where it contributes most to the knowledge of the system, for example in the pass band of the system, trying to make the unknown model parameters as sensitive as possible to the excitation (Ljung 1987). The injected power can be maximized by selecting signals with a minimum crest factor. Two classes of excitation signals will be considered in this paper:

- periodic excitations: the most general representation is a multisine which can be designed with a very low crest factor (= ratio of the peak value to the RMS value of the signal) and an arbitrary power spectrum
- noise excitations: in general they have a quite large crest factor (unless very specific noise sequences are used like random binary sequences (Godfrey, 1993), especially after filtering which is required to shape the power spectrum.

In this paper we will show that the use of periodic excitations has some significant advantages over noise excitation, and this for frequency domain identification as well as for time domain identification. The study will be made under the ZOH-assumption. In these conditions the excitation signals are generated from a discrete sequence mostly stored in a digital (computer) memory. This signal is applied to the DUT through a zero-order-hold reconstruction, followed by a signal conditioner and/or actuator. It is clear that in this setup the discrete time excitation signal $u_d(k)$ is exactly known. A discrete time model $G_d(\omega)$ is identified between the exact known input $u_d(k)$ and the measured (sampled) output $y_m(k)$. It includes the transfer characteristics of the ZOH, the signal conditioner/actuator, the DUT, and the output measurement channel as shown in Figure 1.

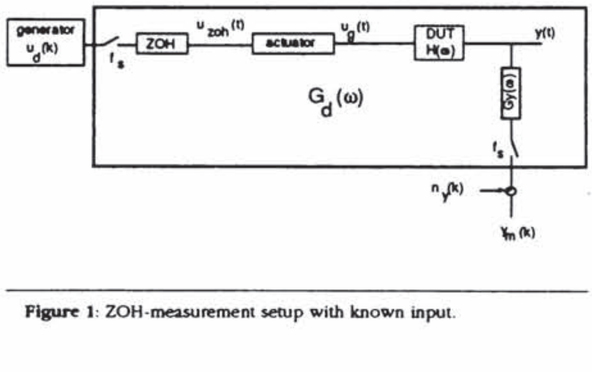

Figure 1: ZOH-measurement setup with known input.

This work is supported by the Belgian National Fund for Scientific Research; the Belgian Government as a part of the Belgian Programme on Interuniversity Poles of Attraction (IUAP 50) initiated by the Belgian State, Prime Minister's Office, Science Policy Programming; and the Flemmish Government (GOA-IMMI).

In this paper it will be shown that periodic excitations allow to

- improve the SNR
- reduce the amount of raw data to be used in the identification process
- increase the frequency range
- eliminate the initial state estimation problem
- simplify the model validation
- easy removal of the DC offset errors in the input and output signals.

These topics will be discussed in detail below. During the study we consider as well ZOH as BL signals which are defined as:

ZOH assumption (zero order hold):
Consider a discrete signal $u_d(k)$. The corresponding continuous time signal under the ZOH assumption is

$$u_{ZOH}(t) = \sum_{-\infty}^{\infty} u_d(k) ZOH(t - kT_s) \tag{1}$$

with

$$ZOH(t) = \begin{cases} 1 & 0 \le t < T_s \\ 0 & \text{elsewhere} \end{cases} \tag{2}$$

and T_s the sampling time.

BL assumption (band limited):
A signal $u(t)$ with power spectrum $\Phi(\omega)$ is called BL if there exists a value ω_{max} such that

$$\Phi(\omega) = 0 \qquad \forall |\omega| > \omega_{max}$$

2. ANALYSIS OF THE ADVANTAGES OF PERIODIC EXCITATIONS.

2.1. Improvement of the SNR

Some general comparisons of the properties of excitation signals with respect to frequency response function measurements are available in the literature (Schoukens and Pintelon, 1991; Schoukens et al., 1988; Schoukens, 1993). Noise excitation, burst noise, swept sine, maximum length binary sequences (Godfrey, 1991), discrete interval binary sequences (Van den Bos and Krol, 1979; Paehlike and Rake, 1979) and multisine signals (Guillaume et al., 1991, Van Der Ouderaa et al., 1988) are compared. It turned out that the multisine is the most flexible signal with optimal properties. A multisine is a finite sum of harmonically related sinusoids

$$u_g(t) = \sum_{k=1}^{F} A_k \sin(\omega_0 l_k t + \varphi_k) \qquad l_k \in \mathbb{N} \tag{3}$$

Inhere ω_o is the basic angular frequency of the signal. A similar definition can also be given for discrete time signals. The properties will be discussed in more detail below.

A periodic excitation can result in a serious improvement of the SNR of the measurements due to the combination of three advantages: crest factor minimization, selection of the frequency lines, time domain averaging.

crest factor minimization
Consider the excitation signal $u_g(t)$ as specified in (1). The information matrix depends only on the amplitudes A_k and not on the phases φ_k (Federov,1972; Goodwin and Payne, 1977; Zarrop, 1979). This freedom can be used to minimize the peak value of the excitation signal. The search of these phases is a highly non-linear problem. Special algorithms are available now to minimize the peak value. Most of them are iterative (Guillaume et al., 1991; Van Der Ouderaa et al., 1988), but also suboptimal closed form solutions are available (Schroeder, 1970). A small crest factor (crest factor = peak value/effective value) can be used to increase the SNR of the measurements or to reduce the required peak value for a given SNR. This can result in a shorter measurement time (for example a factor 10), a higher quality of the measurements (for example 10 dB), or a reduction of the non-linearities which can be hidden in the studied system. It should be clear that all these advantages are independent of the basic assumption (BL or ZOH) which is used. So they apply to a very wide class of problems.

If it is known in advance that the multisine will be used under the ZOH-assumption, it is possible to improve its properties using a simplified optimization. In the crest factor minimization algorithms, usually a high oversampling ratio is internally used ($f_s \approx 32 f_{max}$) in order to avoid the creation of large peaks in between the calculated samples during the BL reconstruction of the analog signal from the discrete sequence. However, if the BL interpolation is replaced by a ZOH reconstruction, it is sufficient to compress the signal at the considered sampling points. This does not only reduce considerably the calculation effort, but it will also result in less severe restrictions because the number of constraints is dropped. This allows a further reduction of the crest factor compared to the BL-assumption and makes the multisine a very attractive signal, also for time domain identification. In Section 3 an example of a ZOH-multisine is given.

selection of the frequency lines
A second advantage of periodic excitation is the full control over the power spectrum of the excitation by the choice of the amplitudes A_k. Also a noise spectrum can be shaped with digital filters but even then the flexibility of a multisine is significantly larger. Again this results in a double advantage: with a multisine the number, frequency and amplitude of the excitation lines can be selected, and this information can be used during the identification process to eliminate all the lines which do not contribute to the experiment. Although this does not influence the information matrix (and hence the asymptotic properties of many estimators), a better convergence of the iterative estimation algorithms is obtained. It is obvious that these advantages are valid essentially for frequency domain identification, although they also can be applied to time domain estimators using an FFT (fast Fourier transform) and an IFT (inverse fast Fourier transform), putting the non-excitation lines equal to zero.

time domain averaging
The periodic nature of the excitation allows to improve the SNR of the measurements using a time domain averaging procedure. This simple and fast procedure which can be applied to many problems is applicable again under the BL and ZOH assumptions.

2.2. Data reduction/increase of the frequency range

The number of measured data points N in an experiment is directly linked to the record length T_m and the sample frequency $f_s=1/T_s$, as $N=T_m f_s$. The minimum record length is mainly imposed by the lowest frequency of interest f_{min} (or the spectral resolution). In order to get sufficient information about the corresponding large time constants a reasonable measurement time should be chosen, for example $T_m \geq 1/f_{min}$ so that at least one period is measured. The sample frequency is imposed by the highest frequencies which have to be chosen so that the frequency band of interest of the DUT is covered, and to avoid folding of the high frequency poles and zeros: $f_s \geq 2 f_{max}$. Hence the minimum number of samples which should be measured and processed is $N \geq 2f_{max}/f_{min}$. It is obvious that the number of measurements increases drastically if a large frequency range should be covered. As an example we can mention the modeling of an electrical machine in a frequency band from 0.01 Hz to 100 Hz resulting in N = 20 000. It is clear that this can lead to impractical situations.

If periodic excitations are applied and combined with a frequency domain estimator, two significant simplifications can be made: a data reduction and experiment simplification.

i) data reduction: the measurement set can be reduced to the Fourier coefficients of the input/output on the corresponding lines of the DFT using the same arguments as before. If for example an (approximately) logarithmic distribution of the frequencies is used, it is possible to cover wide frequency ranges with a reduced set of frequencies. However, even then the very long time records can be disturbing.

ii) experiment simplification: a simplification of the experimental requirements can be obtained by splitting the experiment in a number of sub-experiments, each covering another frequency range. For each of these sub-experiments a much shorter record length can be used while it is still possible to measure all the required Fourier coefficients. A similar approach can not be applied to the ZOH-models [Ljung, 1987] because they strongly depend on the sample frequency and hence combination of the different records is not possible. It should be noted that the time domain errors-in-variables method proposed by Van Hamme et al. (1991) also allows to combine different sampling frequencies in one identification run.

2.3. Elimination of the initial state estimation problem

If no a priori information about the excitation is available (or not used), the initial state of the model should be estimated in addition to the unknown model parameters. In the simulations reported in Section 3. two methods were used to estimate the initial states. The first method is a generalized output error method in which the k initial states of the model are chosen by putting its calculated output at the first k samples equal to the measured outputs (Ljung, 1986). In the second method the initial states are considered as additional parameters which are estimated together with the model parameters (Ljung, 1986). In both cases the uncertainty on the model parameters will increase compared to the situation with known initial states, especially if the data records are short compared to the length of the impulse response. The second method is less noise sensitive than the first one because all data are used to estimate the initial states, but the computation time increases significantly and even then a significant loss in accuracy can occur depending on the complexity of the system and the poles configuration.

The initial state estimation problem disappears if a frequency domain estimator in combination with periodic excitations is used. A Fourier series representation implies that the transients have disappeared and that the system is in steady state at the start of the experiment. This is the price which has to be paid for this advantage. The transfer function model does not depend upon the initial conditions of the system.

The periodic behaviour can also be exploited in the time domain in order to reduce the problem. If the signals are known to be periodic, a concatenated file can be created by repeating the periodic data record. This will de-emphasize the influence of the initial state problem during the estimation process, and if a sufficient number of repetitions is made the same uncertainties are found for the model parameters as with the frequency domain methods. The price which has to be paid for the inefficient use of the a priori knowledge of the periodic nature of the signals is a strong increase of the computation time compared to frequency domain methods.

2.4. Model validation

Each identification process should be followed by a validation process to verify the validity of the proposed model. Among the wide variety of possible techniques, the comparison of the measured and estimated transfer function is an important one. If periodic excitations are used, it is very simple to obtain a good measurement of the non-parametric transfer function with negligible or very small systematic errors (Guillaume et al., 1992). If a noise excitation is used, it is necessary to use correlation methods in order to get a useful measurement (Bendat and Piersol, 1980; Godfrey, 1969; Godfrey K.R., 1980). Compared to the measurements using periodic excitations, the correlation method combined with noise excitation has some severe drawbacks: a bias will be present if there is input and output disturbing noise, the variance will be high due to the stochastic character of the excitation, and finally the results will be disturbed due to leakage (Oppenheim et al.,1983). The variance can be reduced by smoothing the ETFE in a window which is shifted over the DFT lines (Ljung and Glover, 1981; Ljung, 1987). This reduces the variance of the estimate at the cost of an additional bias. So we can conclude that it is not as easy to make a precise comparison between the modelled and the measured transfer function if stochastic excitations are used as it is for periodic excitations.

3. ILLUSTRATIONS OF THE ADVANTAGES OF PERIODIC EXCITATIONS.

3.1. Periodic excitation versus noise excitation

In this section the influence of the nature of the excitation signal (periodic, non-periodic) on the parameter estimates is illustrated. A white uniform excitation signal between [-1,1] is used as noise excitation. The noise sequence is changed in each experiment of the simulation in order to get an idea about its average behaviour. Each experiment consists of 64 time points. For the periodic excitation a multisine with 31 components is used ($f_k = 2\pi\, k/64$, k=1,...,31) and its peak value scaled to 1. The phases are optimized in order to get an optimal ZOH behaviour of the multisine resulting in a crest factor of 1.2 (compare to the crest factor of a sinewave which is $\sqrt{2}$). An example of both signals together with their amplitude spectrum is given in Figure 2 showing

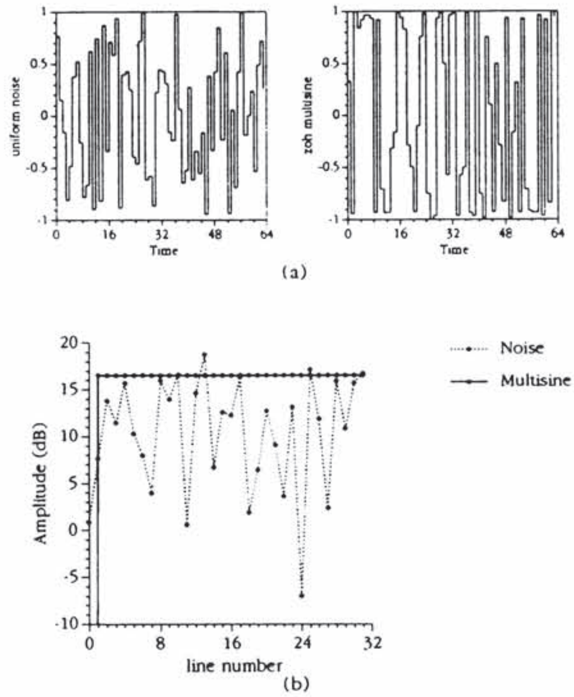

Figure 2: Comparison of a ZOH-multisine and a ZOH-noise sequence (uniform distributed): (a) time domain, (b) amplitude spectrum.

clearly the full control of the amplitude spectrum with a multisine. For each excitation signal, the corresponding output is calculated and disturbed with white normal distributed noise $N(0,\sigma_y)$. On each data set three identifications are made using software packages (Ljung, 1986; Kollar et al., 1991):

i) time domain output error identification with the noise excitation

ii) time domain output error identification with the multisine excitation

(with the simple initial state estimation techniques using the first few samples)

iii) frequency domain output error identification with the multisine excitation

The last simulation is added in order to show the influence of the initial states of the model on the final estimates as discussed before. The transfer function of the simulated system is

$$H(z) = \frac{az^{-1}}{1-bz^{-1}} \tag{4}$$

with a = 0.019801 and b = 0.980199

1000 experiments were processed with σ_y=0.01. The results are given in Table 1.

Table 1

	mean value		standard deviation	
	a	b	a	b
exact value	0.019801	0.98012		
o.e. time + noise	0.020171	0.97733	0.00189	0.00885
o.e. time + multisine	0.019936	0.97911	0.00131	0.00695
o.e. freq. + multisine	0.019794	0.98017	0.00066	0.00404

This table shows that the quality of the estimates increases if the uniform noise excitation is replaced by a good periodic excitation. It is also clear that the elimination of the initial state estimation problem gives also a significant improvement. If the initial states were estimated on the full record together with the model parameters the same results were found on this simple problem as with the frequency domain method, while on more complex problems there still are significant differences.

3.2. Improvement of the finite sample behaviour

If a periodic excitation is used, the a priori knowledge of the excitation frequencies can be used to eliminate the 'zero' lines which contain only disturbing noise contributions. Although this does not change the information matrix and the related asymptotic properties, it can have a large impact on the convergence region of the method and on the finite sample variance as well for time domain as for frequency domain identification. This is illustrated in this simulation. A second order system

$$\frac{0.0977z^{-1}+0.0970z^{-2}}{1-1.7855z^{-1}+0.9802z^{-2}} \tag{5}$$

is excited with a periodic signal with 39 logarithmically distributed frequencies between DC and Nyquist frequency. Two estimates are calculated, the first time using all frequencies, the second time using only the excitation frequencies. 50 runs were made and each time we checked if the method got stuck in a local minimum. The results are given in Table 2.

Table 2 : Elimination of the non-excitation lines and its influence on the convergence

	local minimum	global minimum
all lines	10	40
excitation lines	4	46

This table shows that the risk to find a local minimum is smaller for the second experiment than for the first. It is obvious that the improvement will strongly depend upon the situation and the procedure used to find the starting values.

3.3. Increase of the frequency range: modeling an electrical machine

In this example, it will be shown that periodic excitation combined with a frequency domain identification technique can offer significant advantages. The input impedance of an electrical synchronous machine is modelled from standstill experiments. The excitation is applied to the machine using a thyristor bridge which is used as a power amplifier. A wide frequency range should be covered by the model, typical from 0.01 Hz to 100 Hz. At least 20000 samples are required to cover this range with one experiment. The experiment is split in a low frequency band [0.01Hz - 1 Hz] with a sample frequency of 6 Hz and a high frequency band [1 Hz - 100 Hz] measurement with a sample frequency of 600 Hz. The signals were passed through anti-alias filters in order to obey the BL-assumption. Each time 10 periods of the steady state response were measured resulting in 6000 data points. In each experiment a multisine excitation is used with an almost logarithmically frequency distribution, covering the frequency range with 2 times 37 frequencies. The frequencies were selected in order to avoid the 50 Hz components and its harmonics because these lines are very noisy. In Figure 3 the measured amplitude spectrum of current

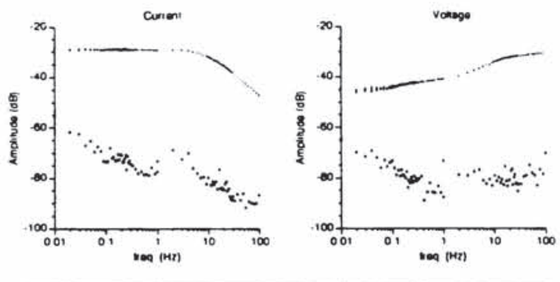

Figure 3: Spectrum of the measured current and voltage (·) and the complex standard deviation (+) on the selected frequencies.

and voltage are given together with the sample standard deviations which are calculated from these records as explained before. The averaged Fourier coefficients are used to model the electrical machine. The identified model (order 4 in numerator, order 3 in denominator, noted as a 4/3 model) is compared to the measured transfer function in Figure 4a. The predicted uncertainty on this model due to stochastic errors is about 0.1% on the amplitude and 0.05 degrees on the phase. The residues are larger than what can be explained from these bounds indicating that there are still modeling errors, most probably due to a slight non-linear behaviour of the electrical machine. It can be remarked that the model validation was very simple because an (almost) unbiased non-parametric measurement of the transfer function is available due to the periodic nature of the excitation.

The measurements were also modelled with a ZOH-method (output error) applied on the BL-data (Andersson and Pucar, 1991). It is obvious that model errors will be present because the measurements are gathered under the BL-assumption. The admittance is modelled instead of the impedance which reduces the ZOH-model errors because the admittance amplitude goes to zero for increasing frequencies (a strictly proper model can be used). The length of the data records is shortened using subsampling methods and reducing the sampling frequency with a factor 2. From (12) it is clear that this will increase the ZOH-model errors because ω_s will be half its original value.

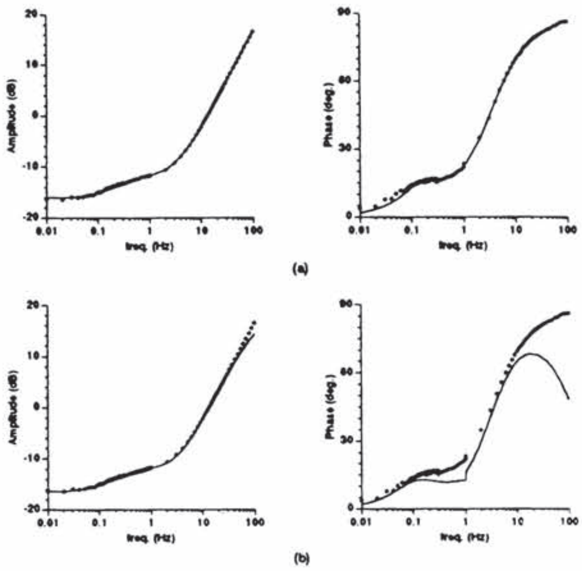

Figure 4: Comparison of the measured and identified transfer function
(a) : results of the frequency domain identification
(b) : combined results of the time domain identificaton

During the frequency domain identification a model order $(n+1)/n$ was used, expressing the inductive behaviour of the electrical machine. It is not possible to do this for the ZOH model because there is no simple relation between the zeros of the continuous time model and those of the discrete time model.

The major problem for the ZOH-model is the combination of two data records at different sampling rate. Because no formal framework exists to deal with this problem two models were proposed by Andersson and Pucar (1992), each covering one frequency range.

In Figure 4b the identified models are compared with the measured transfer function, and all the problems due to the ZOH-model are clearly visible.

4. CONCLUSIONS

In this article it is shown that the use of periodic excitations in system identification not only creates restrictions, but also has some significant advantages with respect to the quality of the estimates, the experimental conditions, and the model validation. The advantages are so impressive that we advise to use periodic excitations whenever it is possible to apply them. Even if the only possibility is to inject a disturbing single on top of the regular (non periodic) control signals, it still desirable to use periodic signals because they give a complete control over the power spectrum combined with a very low crest factor. This will still result in significant improvements of the experimental conditions because it is possible to get maximum information with a minimum disturbance of the process.

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