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A Polytopic Modeling of Aircraft by Using System Identification

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Abstract— **This paper presents a modeling for a linear parameter varying (LPV) aircraft systems by using system identification. The prediction error method for linear time invariant (LTI) models is extended to the parameter estimation of polytopic models. The extended prediction error method is applied to the parameter estimation of an LPV aircraft system whose varying parameter is the flight velocity. The parameters of the initial polytopic model are adjusted so as to make the response and/or the model error small over the entire flight region.**

I. INTRODUCTION

Linearized equations of aircraft are regarded as linear time invariant (LTI) systems if the altitude and the flight velocity are constant, but linear parameter varying (LPV) systems if they are varying. Recently, a number of flight control designs in which the aircraft is treated as LPV systems have been proposed by gain scheduling techniques [1], [2]. In those gain scheduling designs, the LPV system is expressed or sometimes approximated by a polytopic model which is constructed by a linear combination of LTI models at the vertices of the operating region [1]. Then, the constraints in the gain scheduling control design are expressed by linear matrix inequalities (LMIs) [3]. A gain scheduling controller is obtained by solving the LMIs numerically. Unfortunately, in general it is not always possible to exactly transform an LPV system into a polytopic model. It depends on the structure of the LPV system [4]. One of the simplest ways for constructing a polytopic model is that multiple operating points are chosen on the range of the varying parameters. An linear time-invariant model is obtained at each the operating point. A polytopic model is constructed to interpolate the intermediate region between the operating points [1]. However, a model error may be included in the interpolated region although it depends on the used interpolating function. The error may cause the conservativeness of the designed controller. Therefore, it is worthwhile to obtain a polytopic model which is suitable for the original LPV system.

The aim of this paper is to obtain a polytopic model for an LPV aircraft system by using system identification [5]. Since a model is obtained by the measured data in the system identification, it is possible that a desirable polytopic model is obtained by adjusting the coefficients in the model so as to match the data as close as possible. This paper extends the standard prediction error method for LTI models to the parameter estimation of polytopic models. The extended prediction error method is applied to the longitudinal linearized equation of aircraft where the flight velocity is the varying parameter. The estimated polytopic model is evaluated by the time response and the ν-gap metric, which is a criterion associated with the model uncertainty [6].

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II. Polytopic Model and Objective of Parameter Estimation

This paper considers a continuous-time LPV system given by

$$\begin{aligned}\frac{dx(t)}{dt} &= A_c(\tau,\xi(\tau))x(t)+B_c(\tau,\xi(\tau))v(t)\\ y(t) &= C_c(\tau,\xi(\tau))x(t)+D_c(\tau,\xi(\tau))v(t).\end{aligned} \tag{1}$$

$x(t)$, $v(t)$ and $y(t)$ are respectively the n-dimensional state, m-dimensional input and q-dimensional output vectors. $\tau(t)$ is a measurable varying parameter with respect to time t, but the argument t is usually suppressed. $\xi(\tau)$ is the p-dimensional unknown estimation parameter vector which is varying with respect to τ. In particular, when the matrices of (1) are written as the following polytopic form:

$$\begin{aligned}A_c(\tau,\xi(\tau)) &= \sum_{i=1}^{r} w_i(\tau)A_i(\tau_i,\xi_i),\\ B_c(\tau,\xi(\tau)) &= \sum_{i=1}^{r} w_i(\tau)B_i(\tau_i,\xi_i),\\ C_c(\tau,\xi(\tau)) &= \sum_{i=1}^{r} w_i(\tau)C_i(\tau_i,\xi_i),\\ D_c(\tau,\xi(\tau)) &= \sum_{i=1}^{r} w_i(\tau)D_i(\tau_i,\xi_i).\end{aligned} \tag{2}$$

Equation (1) is called a *polytopic model.* $w_i(\tau)$ is the weighting function with respect to τ and the following relations hold.

$$w_i(\tau)\geq 0, \quad \forall i, \quad \sum_{i=1}^{r} w_i(\tau)=1 \tag{3}$$

There are a number of candidates of the weighting function $w_i(\tau)$ which satisfies (3). One of the simplest weight functions is a triangular function whose center is at the operating point. τ_i is a frozen varying parameter and is called the i-th operating point. $A_i(\tau_i,\xi_i)$, $B_i(\tau_i,\xi_i)$, $C_i(\tau_i,\xi_i)$ and $D_i(\tau_i,\xi_i)$ are constant matrices with including an unknown constant estimation parameter vector ξ_i at the i-th operating point. Therefore, the set of (A_i,B_i,C_i,D_i) is called the i-th operating point LTI (OP-LTI) model in this paper. That is, the polytopic form (2) is constructed by an interpolation of r OP-LTIs with the weighting function satisfying (3). When $r=1$, $w_1(\tau)=1$; that is, the polytopic model becomes an LTI model. This paper, off course, considers the case where $r\geq 2$. $A_i(\tau_i,\xi_i)$ preserves the same structure as $A_c(\tau,\xi)$ with respect to the unknown estimation parameters. The same holds for $B_i(\tau_i,\xi_i)$, $C_i(\tau_i,\xi_i)$ and $D_i(\tau_i,\xi_i)$. The element of ξ_i is denoted as

$$\xi_i \triangleq [\xi_{1,i}\cdots\xi_{p,i}]^T. \tag{4}$$

The objective of this paper is summarized as follows; using data obtained from an LPV system $(y(t),\ u(t))$ and the varying parameter τ, estimate p times r unknown parameters $\xi_{l,i}$ $(l=1,\cdots,p,\ i=1,\cdots,r)$ in the polytopic form (2) so that the response of the polytopic model is fitted to the data as close as possible. A simple method of estimating the unknown parameters is that the parameter estimation is done at each OP-LTI model by using data which are obtained at each frozen varying parameter. However, when $\tau(t)$ is varied with time t, there is no guarantee that the response of the constructed polytopic model is fitted to that of the LPV system, especially in the intermediate region because the interpolation of (2) may be an approximated expression of the original LPV system. For such case, the model error contained in the polytopic model should be made as small as possible. This is a motivation why p times r unknown parameters in the polytopic form (2) have to be estimated at the same time by using data of the LPV system.

III. Prediction Error Method for Polytopic Model

This section shows the estimation computation in which the prediction error method for LTI models is modified for the case of polytopic models. Compared to the case of LTI models, there are two novelties in the case of polytopic models: the first is that the number of unknown estimation parameters is proportional to the number of chosen operating points. The second is an assumption on the discretization of the predictor. In fact, both of them are caused to the increase of the computational burden on the parameter estimation.

A. Predictor

A predictor of a polytopic model (1) is given by

$$\begin{aligned}\frac{d\hat{x}(t,\xi)}{dt} =& A_c(\tau,\xi)\hat{x}(t,\xi)+B_c(\tau,\xi)v(t)\\ &+K_c(\tau,\xi)\left(y(t)-\hat{y}(t|\xi)\right)\\ \hat{y}(t|\xi) =& C_c(\tau,\xi)\hat{x}(t,\xi)+D_c(\tau,\xi)v(t),\end{aligned} \tag{5}$$

where $\hat{}$ means the predicted value. $K_c(\tau,\xi)$ is a filter gain which is given so that $A_c(\tau,\xi)-K_c(\tau,\xi)C_c(\tau,\xi)$ becomes a stable matrix. Using Eqs. (2) and (3) and giving $K_c(\tau,\xi)$ by the following polytopic form

$$K_c(\tau,\xi)=\sum_{i=1}^{r} w(\tau)K_i(\tau_i,\xi_i) \tag{6}$$

(5) is then written as

$$\begin{aligned}\frac{d\hat{x}(t,\xi)}{dt} &= F_c(\tau,\xi)\hat{x}(t,\xi)+G_c(\tau,\xi)z(t)\\ \hat{y}(t|\xi) &= C_c(\tau,\xi)\hat{x}(t,\xi)+H_c(\tau,\xi)z(t).\end{aligned} \tag{7}$$

where

$$\begin{aligned} z(t) &\triangleq [y^T(t)\ \ u^T(t)]^T \\ F_c(\tau,\xi) &\triangleq A_c(\tau,\xi)-K_c(\tau,\xi)C_c(\tau,\xi)\\ G_c(\tau,\xi) &\triangleq [K_c(\tau,\xi)\ \ B_c(\tau,\xi)-K_c(\tau,\xi)D_c(\tau,\xi)]\\ H_c(\tau,\xi) &\triangleq [0\ \ D_c(\tau,\xi)]\end{aligned} \tag{8}$$

To converge $\hat{y}(t|\xi)$ to $y(t)$ asymptotically, $K_i(\tau_i,\xi_i)$ $(i=1,\cdots,r)$ must be given so that $F_c(\tau,\xi)$ is a stable matrix. Such $K_i(\tau_i,\xi_i)$ can be obtained by using LMI techniques [3], [8].

B. Discretization

When data $z(t)$ used for estimation are sampled by a constant time period T, a discrete representation of (7) is derived as follows. It is assumed that $\tau(t)$ is constant for each sampling interval; that is,

$$\tau(t)=\tau_k(=const.)\quad (kT\leq t<kT+T). \tag{9}$$

Then, $F_c(\tau,\xi)$, $G_c(\tau,\xi)$, $C_c(\tau,\xi)$ and $H_c(\tau,\xi)$ are constant for a fixed ξ. Applying the zero-order hold discretization to (7), the following piecewise discrete-time prediction representation is obtained.

$$\begin{aligned}\hat{x}(kT+T,\xi) &= F(\tau_k,\xi)x(kT,\xi)+G(\tau_k,\xi)z(kT)\\ \hat{y}(kT|\xi) &= C(\tau_k,\xi)x(kT,\xi)+H(\tau_k,\xi)z(kT)\\ &\qquad (kT\leq t<kT+T)\end{aligned} \tag{10}$$

where

$$\begin{aligned} F(\tau_k,\xi) &\triangleq e^{F_c(\tau_k,\xi)T}\\ G(\tau_k,\xi) &\triangleq \int_0^T e^{F_c(\tau_k,\xi)s}ds\,G_c(\tau_k,\xi)\\ C(\tau_k,\xi) &\triangleq C_c(\tau_k,\xi),\quad H(\tau_k,\xi)\triangleq H_c(\tau_k,\xi)\end{aligned} \tag{11}$$

For convenience, hereafter, the notation 'kT' in $\hat{x}$, $\hat{y}$ and z is replaced by 't' and τ_k is written as τ_t in the discrete-time representation.

C. Quadratic function and Gauss-Newton method

Let N be the number of sampled data. The data set Z^N is defined as

$$Z^N \triangleq [z^T(1) \cdots z^T(N)]. \tag{12}$$

The function to be minimized is given by the following a quadratic function

$$J_N(\xi, Z^N) \triangleq \frac{1}{N} \sum_{t=1}^{N} \frac{1}{2} e^2(t,\xi) \tag{13}$$

where $e(t,\xi)$ is the prediction error vector

$$e(t,\xi) \triangleq y(t) - \hat{y}(t|\xi). \tag{14}$$

Then, the estimated parameter is obtained as

$$\hat{\xi} = \arg\min_{\xi} J_N(\xi, Z^N). \tag{15}$$

Since the estimation parameter ξ is implicitly included in the prediction error, J_N can not be explicitly expressed as a function of ξ. The minimization of J_N is then done by numerically iterative calculation. In this paper, the Gauss-Newton method is used to search a ξ minimizing J_N. Let the superscript (k) be the k-th iteration. The estimation parameter is updated by

$$\hat{\xi}^{(k+1)} = \hat{\xi}^{(k)} - \mu^{(k)} [H_N^{(k)}]^{-1} J'_N(\hat{\xi}^{(k)}, Z^N) \tag{16}$$

where

$$J'_N(\hat{\xi}^{(k)}, Z^N) = -\frac{1}{N} \sum_{t=1}^{N} \psi(t,\hat{\xi}^{(k)}) e(t,\hat{\xi}^{(k)}) \tag{17}$$

$$H_N^{(k)} \triangleq \frac{1}{N} \sum_{t=1}^{N} \psi(t,\hat{\xi}^{(k)}) \psi^T(t,\hat{\xi}^{(k)}) \tag{18}$$

$$\psi(t,\hat{\xi}^{(k)}) \triangleq \frac{\partial \hat{y}(t|\xi^{(k)})}{\partial \xi} \tag{19}$$

$\mu^{(k)}$ is the step size.

D. Gradient

The gradients with respect to the estimation parameter ξ in Eqs. (17) and (18) are calculated as follows. Differentiating the discrete-time predictor (10) with respect to $\xi_{l,j}$, the gradient of the prediction error $\psi(t,\hat{\xi})$ is obtained as

$$\begin{aligned} \frac{\partial \hat{x}(t+1,\xi)}{\partial \xi_{l,i}} &= F(\tau_t,\xi) \frac{\partial \hat{x}(t,\xi)}{\partial \xi_{l,i}} \\ &\quad + \begin{bmatrix} \frac{\partial F(\tau_t,\xi)}{\partial \xi_{l,i}} & \frac{\partial G(\tau_t,\xi)}{\partial \xi_{l,i}} \end{bmatrix} \begin{bmatrix} \hat{x}(t,\xi) \\ z(t,\xi) \end{bmatrix} \\ \psi(t,\hat{\xi}) &= C(\tau_t,\xi) \frac{\partial \hat{x}(t,\xi)}{\partial \xi_{l,i}} \\ &\quad + \begin{bmatrix} \frac{\partial C(\tau_t,\xi)}{\partial \xi_{l,i}} & \frac{\partial H(\tau_t,\xi)}{\partial \xi_{l,i}} \end{bmatrix} \begin{bmatrix} \hat{x}(t,\xi) \\ z(t,\xi) \end{bmatrix} \end{aligned} \tag{20}$$

The procedure of the computation described in this paper is essentially the same as that of LTI model estimation except the increase of the estimation parameters and calculation of the prediction and the gradient by Eqs. (10) and (20). The number of estimation parameters is proportional to the number of the operating points. Since Eqs. (10) and (20) are piecewise LTI equations for each sampling, the discretization has to be done every sampling.

To estimate the parameters accurately, all OP-LTI models have to be excited by the input $v(t)$ and the varying parameter $\tau(t)$. In particular, $\tau(t)$ should be varied over the entire range. This means that all the weighting factor $w_i(\tau)$ $(i = 1, \cdots, r)$ are not constant in getting data. Otherwise, some OP-LTI models are not needed in the polytopic form (2). Let us consider a simple scenario that $w_1(\tau(t))$ is given by a constant. If $w_1(\tau) = 0$, the 1st OP-LTI model (A_1, B_1, C_1, D_1) is not needed because of no contribution to (2). If $w_1(\tau) = 1$, the rest of OP-LTI models are not needed. Moreover, if $0 < w_1(\tau) < 1$, (3) is written as

$$\sum_{i=2}^{r} \frac{w_i}{1 - w_1} = 1. \tag{21}$$

Replacing w_i by $w'_i \triangleq w_i/(1-w_1)$, w'_i $(i = 2, \cdots, r)$ satisfy the properties of the weighting function (3). Then, the polytopic model is constructed by $r-1$ OP-LTI models.

IV. Identification Simulation of LPV Aircraft Model

The estimated parameters in the linearized aircraft equations are the stability and control derivatives (SCDs) which express linear contributions of the perturbed velocities and the angular rates to the aerodynamic forces or moments. The SCDs are varied according to the flight conditions; that is, the flight velocity and the altitude. This section demonstrates the identification simulation of an LPV aircraft system when the flight condition is varying.

A. Longitudinal equations of aircraft

In steady flight, the dynamics of aircraft can be generally divided into two parts; the longitudinal and the lateral motions. This paper considers identification of the longitudinal motion which is regarded as an LPV system

$$\begin{aligned} \frac{dx(t)}{dt} &= A_c(V,\xi(V))x(t) + B_c(V,\xi(V))v(t) \\ y(t) &= C_c x_c(t) + D_c v(t), \end{aligned} \tag{22}$$

where

$$x(t) \triangleq \begin{bmatrix} u \\ \theta \\ \alpha \\ q \end{bmatrix}, \quad y(t) \triangleq \begin{bmatrix} u \\ \theta \\ \alpha \end{bmatrix}, \quad v(t) \triangleq \delta_e \tag{23}$$

$$A_c = E_c^{-1} Q_c, \quad B_c = E_c^{-1} R_c$$

$$E_c \triangleq \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & V & 0 \\ 0 & 0 & -M_{\dot\alpha} & 1 \end{bmatrix},$$

$$Q_c \triangleq \begin{bmatrix} X_u & -g\cos\Theta_0 & X_\alpha & 0 \\ 0 & 0 & 0 & 1 \\ Z_u & -g\sin\Theta_0 & Z_\alpha & V+Z_q \\ -M_u & 0 & M_\alpha & M_q \end{bmatrix},$$

$$R_c \triangleq \begin{bmatrix} 0 \\ 0 \\ Z_{\delta_e} \\ M_{\delta_e} \end{bmatrix}, \quad C_c = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad D_c = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

u is the x-axis velocity, α the angle of attack, θ the pitch angle, q the pitch rate and δ_e the elevator angle. The notations used in (22) are based on the symbols which have been usually used in flight dynamics [7]. The variables denoted by small letters mean the perturbed values. Θ_0 is the pitch angle in the steady-state. g is the center of gravity. In (22), there are nine stability derivatives hich are varied with the flight velocity V and the altitude H. Since V is more considerably influenced on the characteristics of (22) rather than H, the varying parameter considered in this paper is $\tau = V$ and its range is given by

$$V_1 \le V \le V_2 \quad (V_1 < V_2) \tag{24}$$

Collecting the stability and control derivatives, the unknown estimation parameter vector is

$$\xi(V) = [X_u \ X_\alpha \ Z_u \ Z_\alpha \ Z_q \ M_u \ M_\alpha \ M_{\dot\alpha} \ M_q \ Z_{\delta_e} \ M_{\delta_e}]^T. \tag{25}$$

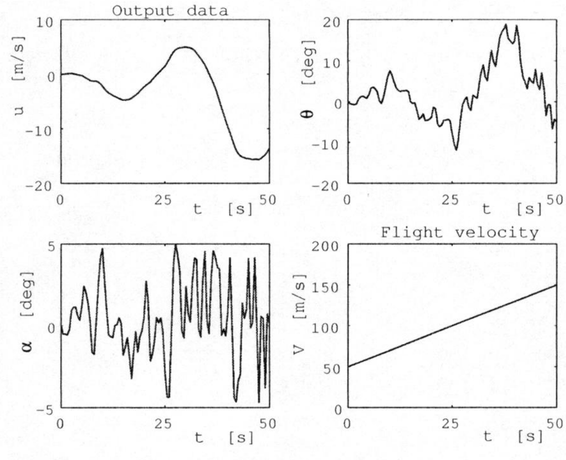

Fig. 1. Output data and flight velocity for parameter estimation.

A polytopic model of the longitudinal motion of aircraft is constructed as follows: two frozen operating points are chosen at the edges of (24); that is, $V = V_1$, V_2 and two OP-LTI models are obtained. Using linear interpolation, A_c and B_c of the polytopic model are then constructed as

$$\begin{aligned} A_c(V,\xi) &= \sum_{i=1}^{2} w_i(V) A_i(V_i,\xi_i), \\ B_c(V,\xi) &= \sum_{i=1}^{2} w_i(V) B_i(V_i,\xi_i) \end{aligned} \tag{26}$$

where

$$w_1(V) \triangleq \frac{V_2 - V}{V_2 - V_1}, \quad w_2(V) \triangleq \frac{V - V_1}{V_2 - V_1} \tag{27}$$

The number of unknown estimation parameters is $11 \times 2 = 22$.

B. Data for parameter estimation

In identification simulation presented in this section, the flight velocity is changed in the range of (24). As an example, consider a situation that the flight velocity $V(t)$ in the continuous-time is constantly accelerated as

$$V(t) = V_1 + a_v t \tag{28}$$

where a_v is the acceleration. The input is given by a random binary signal. Using the flight velocity and the random input, the output data are generated by the LPV system (22) which is converted to the discrete-time representation at each sampling as shown in discretization of the predictor in Section 3.2. The SCDs at each sampling are obtained by an analytical method based on the quasi-steady aerodynamic theory [7]. The number of data was $N = 100$. The sampling time was given by $T = 1$ sec. The acceleration of the flight velocity was given by $a_v = 2$ m/s^2. Figure 1 shows the output

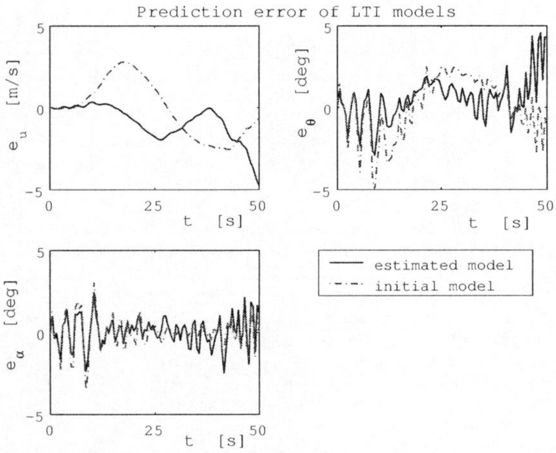

Fig. 2. Prediction error of initial and estimated LTI models.

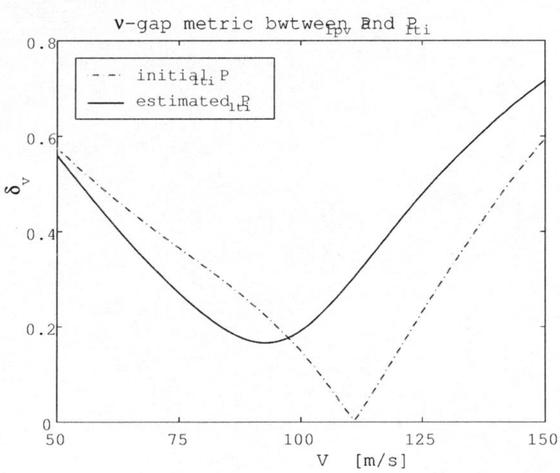

Fig. 3. ν-gap metric between $P_{lpv}(s,V)$ and $P_{lti}(s)$ in flight region.

data and the flight velocity used for estimation. The initial state was given by $x_c(0) = 0$.

C. Parameter estimation results

Using the data obtained above, The estimation of the SCDs was done in the case of LTI and the polytopic models. Figure 2 shows the prediction error, (14), of the initial and the estimated LTI models, where e_u, e_α and e_θ are the errors of u, α and θ, respectively. The solid- and the dashed-dotted-lines mean the prediction error of the initial and the estimated models, respectively.

The ν-gap metric is one of criterion measuring the model error in the frequency domain. It was introduced in consideration of the robust control theories associated with the stability margin [6]. Let $P_{lpv}(s,V)$ be the transfer function of the LPV system where the varying parameter is V. Let $P_{lti}(s)$ be that of the initial or the estimated LTI model. The ν-gap metric between $P_{lti}(s)$ and $P_{lpv}(s,V)$ is defined as

$$\delta_\nu(P_{lti}, P_{lpv}) \triangleq \sup_{\omega} \kappa(P_{lti}(j\omega), P_{lpv}(j\omega, V)) \qquad (29)$$

where

$$\kappa(X,Y) \triangleq \overline{\sigma}\left[(I+YY^*)^{1/2}(Y-X)(I+XX^*)^{1/2}\right]$$

where $\overline{\sigma}[\cdot]$ means the maximum singular value. The range is $\delta_\nu \in [0,1]$. A large δ_ν means that the model error is large. Figure 3 shows the ν-gap metric between $P_{lti}(s)$ and $P_{lpv}(s,V)$. The solid- and the dashed-dotted-lines indicate that $P_{lti}(s)$ was the estimated and the initial LTI models, respectively. Since the initial SCDs were given at $V = 110$ m/s, the ν-gap metric of the initial LTI model was the minimum at $V = 110$ m/s. However, the ν-gap metric was increased when V was shifted from $V = 110$ m/s. On the other hand, the minimum of the estimated LTI model was moved to $V \simeq 93$ m/s. Similar to the initial LTI model, the ν-gap metric was increased in other flight condition. It is seen from the time responses and the ν-gap metric shown above that the LTI model was not enough to express the characteristics of the LPV system from the time and the frequency domain points of view. It might be hard to design a controller which satisfies the robust stability condition over the entire flight region.

The identification simulation of polytopic model is shown in the next. Figure 4 shows the prediction error of the initial and the estimated polytopic models. The response of the estimated polytopic model was better fitted to the data than that of the initial polytopic model.

Letting $P_{poly}(s,V)$ be the transfer function of the initial or the estimated polytopic model, Fig. 5 shows the ν-gap metric between $P_{poly}(s,V)$ and $P_{lpv}(s,V)$, where the solid- and the dashed-dotted-lines indicate that $P_{poly}(s,V)$ was the estimated and the initial LTI models, respectively. The ν-gap metric whose $P_{poly}(s,V)$ was the estimated polytopic model was smaller than that of the initial polytopic model except both edges of the flight region.

Summarizing the identification simulation shown in this section, the polytopic model was more suitable for LPV systems than the LTI model. Applying the prediction error method to the polytopic model, the parameters of the polytopic model are adjusted so as to make the response and/or the model error small over the entire flight region.

V. Concluding Remarks

This paper has presented a modeling for an LPV aircraft system by using system identification. The prediction error

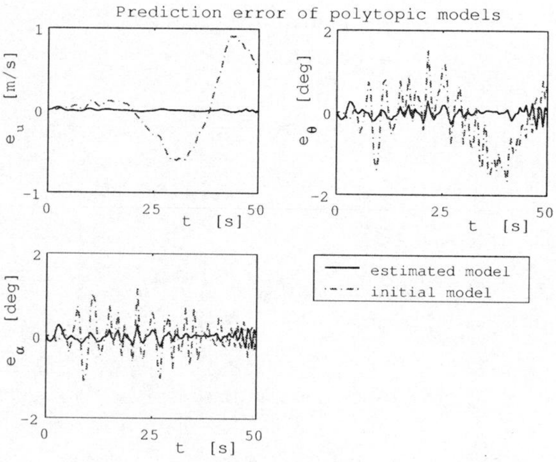

Fig. 4. Prediction error of initial and estimated polytopic models.

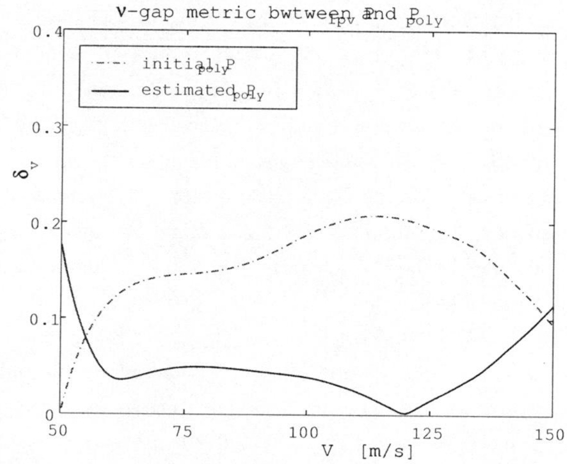

Fig. 5. ν-gap metric between $P_{lpv}(s,V)$ and $P_{poly}(s,V)$ in flight region.

method for LTI models was extended to the parameter estimation of polytopic models. The extended prediction error method was applied to the parameter estimation of an LPV aircraft system whose varying parameter was the flight velocity. It was seen from the identification simulation that the polytopic model was more suitable for LPV systems than the LTI model. The parameters of the initial polytopic model were adjusted so as to make the response and/or the model error small over the entire flight region.

A polytopic model is used for an approximated representation of nonlinear systems in which the reference trajectory is given in advance [8], [9]. The parameter estimation presented in this paper is applicable to the modeling of such nonlinear systems.

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