
SUBSPACE IDENTIFICATION FOR LINEAR SYSTEMS

Theory - Implementation - Applications

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**Peter VAN OVERSCHEE
Bart DE MOOR**

*Katholieke Universiteit Leuven
Belgium*



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PREFACE

Ceci n'est pas une pipe.

René Magritte, Belgian painter, 1898-1967.

The last 30 years or so, system identification has matured from Eykhoff's '*bag of tricks*', over the impressive *Ljungian theory for the user* of so-called *prediction-error methods*, to Willems' *behavioral framework*. Many papers have been written, several excellent textbooks have appeared and hundreds of workshops and conferences have been organized. Specifically for the identification of linear dynamic time-invariant models from given input-output data, the collection of available methods has become immense.

So why write yet another book about this, by now, almost classical problem? Well, to start with, the problem is important! There is a growing interest in manageable mathematical models for all kinds of applications, such as simulation, prediction, fault diagnosis, quality and safety monitoring, state estimation, signal processing (direction-of-arrival algorithms (SDMA)) and last but not least, model-based control system design. And sure enough, *linear* models are very popular because of their utmost simplicity (at least at first sight).

In this book, we do not really solve a new problem. Indeed, the goal is to find dynamical models from input-output data that were generated by so-called combined deterministic-stochastic linear systems. Said in other words, data that are generated by a linear, time-invariant, finite-dimensional, dynamic system, with both deterministic and stochastic input signals (including several special cases).

What is new in this book, are the methods and algorithms for solving this 'classical' problem. The insights that will be developed, originate in a mixture of ideas, facts and algorithms from system theory, statistics, optimization theory and (numerical) linear algebra. They culminate in so-called 'subspace' methods, the name of which reflects the fact that linear models can be obtained from row and column spaces of certain matrices, calculated from input-output data. Typically, the column space of such data matrices contains information about the model, while the row spaces allow to obtain

a (Kalman filter) state sequence, *directly from input-output data* (i.e. without knowing the model a priori)¹. Another important aspect of this book is the development of a *unifying* framework, in which almost all existing subspace methods that have appeared in the literature of the last 10 years or so, have found their place.

Apart from these *conceptual* contributions, there are other advantages to subspace methods. For instance, there is no need for an explicit model parametrization, which, for multi-output linear systems is a rather complicated matter. A second numerical advantage is the elegance and computational efficiency of subspace algorithms. The dimension and numerical representation of the subspaces mentioned before, are calculated using the QR- and the singular value decomposition. These are well-understood techniques from numerical linear algebra, for which numerically robust and efficient algorithms are widely available.

Of course, we should *never* forget that *a (mathematical) model is not the real system* (think of Magritte). Even though there are still missing links in the question of guaranteed optimality of subspace methods, it is now widely accepted that they prove very useful in many applications, in which they often provide excellent models and because of their utmost user-friendliness (limited number of user-choices to deal with). They also provide (often excellent) initial guesses for non-linear iterative optimization algorithms which are used in prediction-error methods, L_2 -optimal system identification, neural nets, etc. . .

Finally, we have paid special attention to the development of easy accessible and user-friendly software packages, which are described in Chapter 6 (Xmath² ISID II) and Appendix B (which describes Matlab files and several demos). This book goes with a diskette that contains all of these .m files and examples.

¹ Have a look at Theorems 2, 8 and 12 of this book.

² A product of Integrated Systems Incorporated, Santa Clara, CA, USA.

Mister Data, there's a subspace communication for you.
Quote from Star Trek, the Next Generation.

This book emanates from the authors' PhD theses at ESAT, the department of Electrical Engineering of the Katholieke Universiteit Leuven in Belgium. Bart's 1988 thesis contained the initial concepts and ideas for subspace identification (of course inspired by the work of many others), linking ideas from system theory (realization algorithms), linear algebra (orthogonal projections and intersections of subspaces) to numerical issues (QR and singular value decompositions). Peter's 1995 thesis, which forms the basis of this book, contains the detailed unification of all these insights, culminating in some robust subspace identification methods, together with other results such as model reduction issues, relations with other identification algorithms, etc. . .

The work reported on in this book would have been impossible without the support, both financial and moral, from many institutions and people.

We would like to mention the financial support from the Flemish Government (Concerted Action GOA-MIPS *Model-Based Information Processing Systems*), the National Fund for Scientific Research, the Federal Government (Interuniversity Attraction Poles IUAP-17 *Modeling and Control of Dynamic Systems* and IUAP-50 *Automation in Design and Production*) and the European Commission (Human Capital and Mobility SIMONET *System Identification and Modeling Network*).

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It's to them all that we dedicate this book.

Peter Van Overschee
Bart De Moor

Leuven, January 1, 1996.