Kinematic Analysis

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The output matrices of this solution are too long to be pasted here in this file as it is to be printed to A4 page, that's why solutions in the mathematica's standard output format are pasted here one can access the file online through the link or by scanning QR code given.

https://drive.google.com/file/d/1__iQeQPAIeVHLXbl8l8a0g72aKbBC7bd/view?usp=sharing

Functions Definitions: -

Pure Rotation Transformation Matrices: -

In[ ]:= $Tx[theta\_] := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & Cos[theta] & -Sin[theta] & 0 \\ 0 & Sin[theta] & Cos[theta] & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix};$

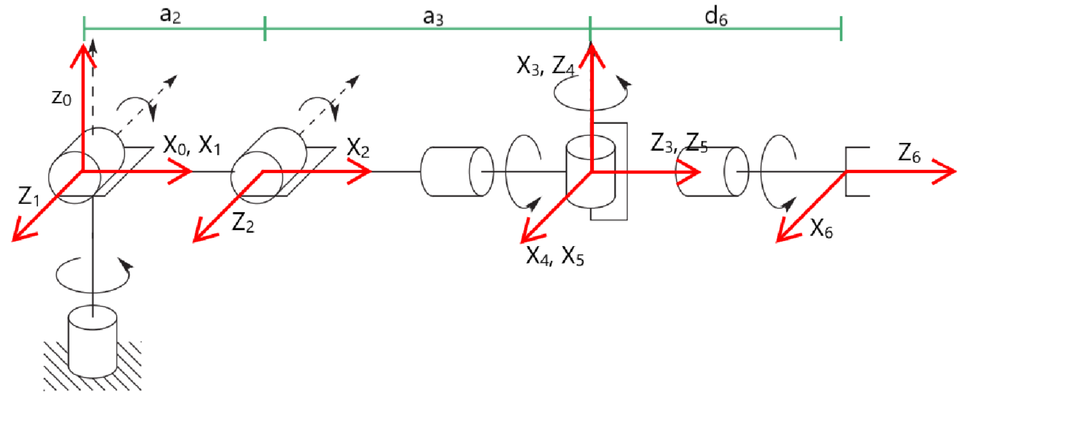
$Ty[theta\_] := \begin{pmatrix} Cos[theta] & 0 & Sin[theta] & 0 \\ 0 & 1 & 0 & 0 \\ -Sin[theta] & 0 & Cos[theta] & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix};$

$Tz[theta\_] := \begin{pmatrix} Cos[theta] & -Sin[theta] & 0 & 0 \\ Sin[theta] & Cos[theta] & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix};$

Pure Translation Transformation Matrix: -

In[ ]:= $Ta[x\_, y\_, z\_] := \begin{pmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{pmatrix};$

Forward Kinematics: -



i	θ	d	a	α
1	θ1	0	0	90
2	θ2	0	a2	0
3	θ3	0	a3	90
4	θ4	0	0	90
5	θ5	0	0	90
6	θ6	d6	0	0

```
In[ ]:= T10 = Tz[t1f].Tx[Pi / 2];
T21 = Tz[t2f].Ta[a2, 0, 0];
T32 = Tz[t3f].Ta[a3, 0, 0].Tx[Pi / 2];
T43 = Tz[t4f].Tx[Pi / 2];
T54 = Tz[t5f].Tx[Pi / 2];
T65 = Tz[t6f].Ta[0, 0, d6];
T20 = T10.T21;
T30 = T10.T21.T32;
T40 = T10.T21.T32.T43;
T50 = T10.T21.T32.T43.T54;
T60 = T10.T21.T32.T43.T54.T65;
```

```
In[ ]:= T60
Out[ ]= {{Cos[t6f] (Cos[t5f] (Cos[t4f] (Cos[t1f] Cos[t2f] Cos[t3f] - Cos[t1f] Sin[t2f] Sin[t3f]) +
            Sin[t1f] Sin[t4f]) +
          (Cos[t1f] Cos[t3f] Sin[t2f] + Cos[t1f] Cos[t2f] Sin[t3f]) Sin[t5f]) +
      (-Cos[t4f] Sin[t1f] + (Cos[t1f] Cos[t2f] Cos[t3f] - Cos[t1f] Sin[t2f] Sin[t3f])
           Sin[t4f]) Sin[t6f], Cos[t6f] (-Cos[t4f] Sin[t1f] +
          (Cos[t1f] Cos[t2f] Cos[t3f] - Cos[t1f] Sin[t2f] Sin[t3f]) Sin[t4f]) -
      (Cos[t5f] (Cos[t4f] (Cos[t1f] Cos[t2f] Cos[t3f] - Cos[t1f] Sin[t2f] Sin[t3f]) +
            Sin[t1f] Sin[t4f]) +
          (Cos[t1f] Cos[t3f] Sin[t2f] + Cos[t1f] Cos[t2f] Sin[t3f]) Sin[t5f]) Sin[t6f],
    -Cos[t5f] (Cos[t1f] Cos[t3f] Sin[t2f] + Cos[t1f] Cos[t2f] Sin[t3f]) +
      (Cos[t4f] (Cos[t1f] Cos[t2f] Cos[t3f] - Cos[t1f] Sin[t2f] Sin[t3f]) +
          Sin[t1f] Sin[t4f]) Sin[t5f],
    a2 Cos[t1f] Cos[t2f] + a3 Cos[t1f] Cos[t2f] Cos[t3f] -
     a3 Cos[t1f] Sin[t2f] Sin[t3f] +
     d6 (-Cos[t5f] (Cos[t1f] Cos[t3f] Sin[t2f] + Cos[t1f] Cos[t2f] Sin[t3f]) +
          (Cos[t4f] (Cos[t1f] Cos[t2f] Cos[t3f] - Cos[t1f] Sin[t2f] Sin[t3f]) +
            Sin[t1f] Sin[t4f]) Sin[t5f])},
   {Cos[t6f] (Cos[t5f] (Cos[t4f] (Cos[t2f] Cos[t3f] Sin[t1f] - Sin[t1f] Sin[t2f] Sin[t3f]) -
            Cos[t1f] Sin[t4f]) +
          (Cos[t3f] Sin[t1f] Sin[t2f] + Cos[t2f] Sin[t1f] Sin[t3f]) Sin[t5f]) +
      (Cos[t1f] Cos[t4f] + (Cos[t2f] Cos[t3f] Sin[t1f] - Sin[t1f] Sin[t2f] Sin[t3f])
           Sin[t4f]) Sin[t6f], Cos[t6f] (Cos[t1f] Cos[t4f] +
          (Cos[t2f] Cos[t3f] Sin[t1f] - Sin[t1f] Sin[t2f] Sin[t3f]) Sin[t4f]) -
      (Cos[t5f] (Cos[t4f] (Cos[t2f] Cos[t3f] Sin[t1f] - Sin[t1f] Sin[t2f] Sin[t3f]) -
            Cos[t1f] Sin[t4f]) +
          (Cos[t3f] Sin[t1f] Sin[t2f] + Cos[t2f] Sin[t1f] Sin[t3f]) Sin[t5f]) Sin[t6f],
    -Cos[t5f] (Cos[t3f] Sin[t1f] Sin[t2f] + Cos[t2f] Sin[t1f] Sin[t3f]) +
      (Cos[t4f] (Cos[t2f] Cos[t3f] Sin[t1f] - Sin[t1f] Sin[t2f] Sin[t3f]) -
          Cos[t1f] Sin[t4f]) Sin[t5f],
    a2 Cos[t2f] Sin[t1f] + a3 Cos[t2f] Cos[t3f] Sin[t1f] -
     a3 Sin[t1f] Sin[t2f] Sin[t3f] +
     d6 (-Cos[t5f] (Cos[t3f] Sin[t1f] Sin[t2f] + Cos[t2f] Sin[t1f] Sin[t3f]) +
          (Cos[t4f] (Cos[t2f] Cos[t3f] Sin[t1f] - Sin[t1f] Sin[t2f] Sin[t3f]) -
            Cos[t1f] Sin[t4f]) Sin[t5f])},
   {Cos[t6f] (Cos[t4f] Cos[t5f] (Cos[t3f] Sin[t2f] + Cos[t2f] Sin[t3f]) +
          (-Cos[t2f] Cos[t3f] + Sin[t2f] Sin[t3f]) Sin[t5f]) +
      (Cos[t3f] Sin[t2f] + Cos[t2f] Sin[t3f]) Sin[t4f] Sin[t6f],
    Cos[t6f] (Cos[t3f] Sin[t2f] + Cos[t2f] Sin[t3f]) Sin[t4f] -
      (Cos[t4f] Cos[t5f] (Cos[t3f] Sin[t2f] + Cos[t2f] Sin[t3f]) +
          (-Cos[t2f] Cos[t3f] + Sin[t2f] Sin[t3f]) Sin[t5f]) Sin[t6f],
    -Cos[t5f] (-Cos[t2f] Cos[t3f] + Sin[t2f] Sin[t3f]) +
     Cos[t4f] (Cos[t3f] Sin[t2f] + Cos[t2f] Sin[t3f]) Sin[t5f],
    a2 Sin[t2f] + a3 Cos[t3f] Sin[t2f] + a3 Cos[t2f] Sin[t3f] +
     d6 (-Cos[t5f] (-Cos[t2f] Cos[t3f] + Sin[t2f] Sin[t3f]) +
          Cos[t4f] (Cos[t3f] Sin[t2f] + Cos[t2f] Sin[t3f]) Sin[t5f])}, {0, 0, 0, 1}}
```

Velocity Kinematics: -

Jacobian Calculation : –

```
In[∘ ]:= Jv =
  Simplify[{{Cross[Transpose[{0, 0, 1}], T60[[Range[3], 4]] - Transpose[{0, 0, 0}]],
     Cross[T10[[Range[3], 3]], T60[[Range[3], 4]] - T10[[Range[3], 4]]],
     Cross[T20[[Range[3], 3]], T60[[Range[3], 4]] - T20[[Range[3], 4]]],
     Cross[T30[[Range[3], 3]], T60[[Range[3], 4]] - T30[[Range[3], 4]]],
     Cross[T40[[Range[3], 3]], T60[[Range[3], 4]] - T40[[Range[3], 4]]],
     Cross[T50[[Range[3], 3]], T60[[Range[3], 4]] - T50[[Range[3], 4]]]}}];

In[∘ ]:= Jw = Simplify[{{{0, 0, 1}, T10[[Range[3], 3]], T20[[Range[3], 3]],
     T30[[Range[3], 3]], T40[[Range[3], 3]], T50[[Range[3], 3]]}}];

In[∘ ]:= J = Join[Jv, Jw];
```

Forward Velocity Kinematics : –

If joint velocities are given in the form of a vector $\dot{q}$;

$$qdot = \begin{pmatrix} q1\_dot \\ q2\_dot \\ q3\_dot \\ q4\_dot \\ q5\_dot \\ q6\_dot \end{pmatrix};$$

Then we calculate the velocity of end effector as;

```
V = J.qdot;
```

Inverse Velocity Kinematics : –

If end effector velocities are given in the form of V;

$$V = \begin{pmatrix} V\_x \\ V\_y \\ V\_z \end{pmatrix};$$

Then for the inverse velocity kinematics;

$$Vend = Join\left[V, \begin{pmatrix} V[[1, 1]] / T60[[1, 4]] \\ V[[2, 1]] / T60[[2, 4]] \\ V[[3, 1]] / T60[[3, 4]] \end{pmatrix}\right];$$

Here the end effector velocities are given in the form of linear velocities just or a velocity vector but we need the angular velocities too for the calculation of inverse velocity kinematics. Hence angular velocities are calculated using well known relation $v = r\omega$, where r is the position of the end effector from the base of the robot.

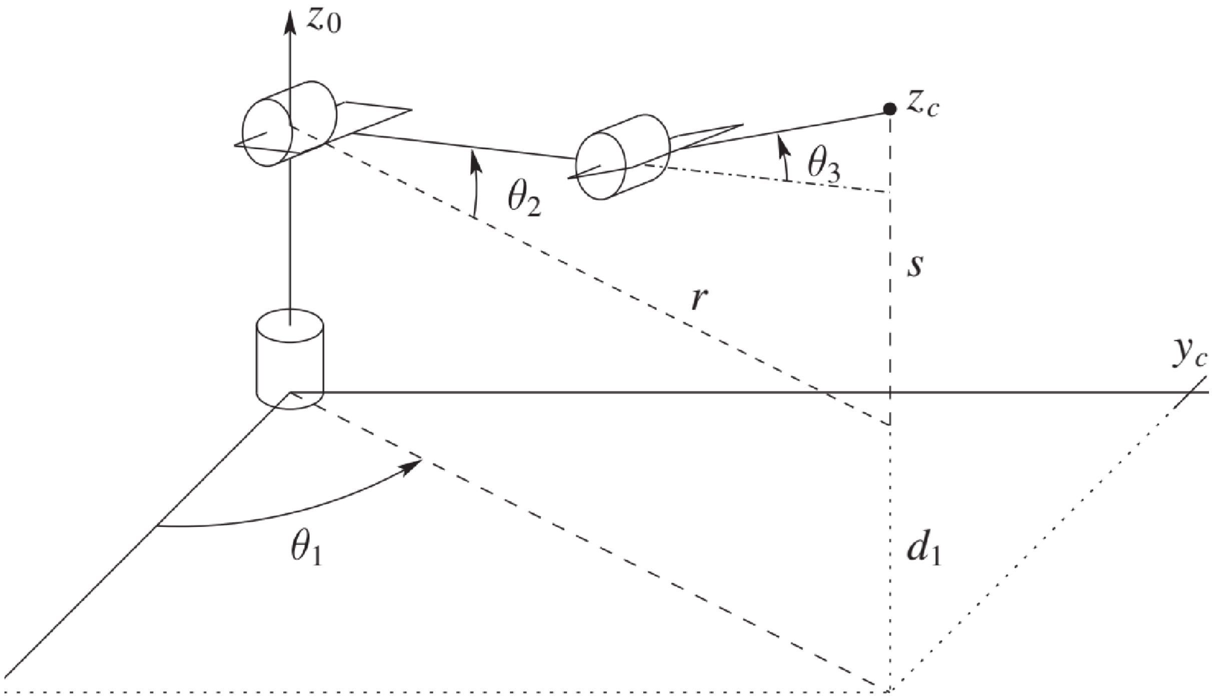
```
qdot = Inverse[J].Vend;
```

```
In[•]:= J

Out[•]= {{{d6 Cos[t3f] Cos[t5f] Sin[t1f] Sin[t2f] + a3 Sin[t1f] Sin[t2f] Sin[t3f] +
      d6 Cos[t4f] Sin[t1f] Sin[t2f] Sin[t3f] Sin[t5f] + d6 Cos[t1f] Sin[t4f] Sin[t5f] -
      Cos[t2f] Sin[t1f] (a2 - d6 Cos[t5f] Sin[t3f] + Cos[t3f] (a3 + d6 Cos[t4f] Sin[t5f])),
     d6 Sin[t1f] Sin[t4f] Sin[t5f] +
      Cos[t1f] (Cos[t2f] (a2 - d6 Cos[t5f] Sin[t3f] + Cos[t3f] (a3 + d6 Cos[t4f] Sin[t5f])) -
         Sin[t2f] (d6 Cos[t3f] Cos[t5f] + Sin[t3f] (a3 + d6 Cos[t4f] Sin[t5f]))), 0},
    {-Cos[t1f] (Sin[t2f] (a2 - d6 Cos[t5f] Sin[t3f] + Cos[t3f] (a3 + d6 Cos[t4f] Sin[t5f])) +
        Cos[t2f] (d6 Cos[t3f] Cos[t5f] + Sin[t3f] (a3 + d6 Cos[t4f] Sin[t5f]))),
     -Sin[t1f] (Sin[t2f] (a2 - d6 Cos[t5f] Sin[t3f] + Cos[t3f] (a3 + d6 Cos[t4f] Sin[t5f])) +
        Cos[t2f] (d6 Cos[t3f] Cos[t5f] + Sin[t3f] (a3 + d6 Cos[t4f] Sin[t5f]))),
     Cos[t2f] (a2 - d6 Cos[t5f] Sin[t3f] + Cos[t3f] (a3 + d6 Cos[t4f] Sin[t5f])) -
      Sin[t2f] (d6 Cos[t3f] Cos[t5f] + Sin[t3f] (a3 + d6 Cos[t4f] Sin[t5f]))},
    {-Cos[t1f] (Sin[t2f] (-d6 Cos[t5f] Sin[t3f] + Cos[t3f] (a3 + d6 Cos[t4f] Sin[t5f])) +
        Cos[t2f] (d6 Cos[t3f] Cos[t5f] + Sin[t3f] (a3 + d6 Cos[t4f] Sin[t5f]))),
     -Sin[t1f] (Sin[t2f] (-d6 Cos[t5f] Sin[t3f] + Cos[t3f] (a3 + d6 Cos[t4f] Sin[t5f])) +
        Cos[t2f] (d6 Cos[t3f] Cos[t5f] + Sin[t3f] (a3 + d6 Cos[t4f] Sin[t5f]))),
     Cos[t2f] (-d6 Cos[t5f] Sin[t3f] + Cos[t3f] (a3 + d6 Cos[t4f] Sin[t5f])) -
      Sin[t2f] (d6 Cos[t3f] Cos[t5f] + Sin[t3f] (a3 + d6 Cos[t4f] Sin[t5f]))},
    {d6 (Cos[t4f] Sin[t1f] - Cos[t1f] Cos[t2f + t3f] Sin[t4f]) Sin[t5f],
     -d6 (Cos[t1f] Cos[t4f] + Cos[t2f + t3f] Sin[t1f] Sin[t4f]) Sin[t5f],
     -d6 Sin[t2f + t3f] Sin[t4f] Sin[t5f]}, {d6 (Cos[t5f] Sin[t1f] Sin[t4f] +
        Cos[t1f] (Sin[t2f] (-Cos[t4f] Cos[t5f] Sin[t3f] + Cos[t3f] Sin[t5f]) +
           Cos[t2f] (Cos[t3f] Cos[t4f] Cos[t5f] + Sin[t3f] Sin[t5f]))),
     d6 (-Cos[t4f] Cos[t5f] Sin[t1f] Sin[t2f] Sin[t3f] - Cos[t1f] Cos[t5f] Sin[t4f] +
        Cos[t3f] Sin[t1f] Sin[t2f] Sin[t5f] +
        Cos[t2f] Sin[t1f] (Cos[t3f] Cos[t4f] Cos[t5f] + Sin[t3f] Sin[t5f])),
     d6 (Cos[t3f] (Cos[t4f] Cos[t5f] Sin[t2f] - Cos[t2f] Sin[t5f]) +
        Sin[t3f] (Cos[t2f] Cos[t4f] Cos[t5f] + Sin[t2f] Sin[t5f]))}, {0, 0, 0}},
   {{0, 0, 1}, {Sin[t1f], -Cos[t1f], 0}, {Sin[t1f], -Cos[t1f], 0},
    {Cos[t1f] Sin[t2f + t3f], Sin[t1f] Sin[t2f + t3f], -Cos[t2f + t3f]},
    {-Cos[t4f] Sin[t1f] + Cos[t1f] Cos[t2f + t3f] Sin[t4f],
     Cos[t1f] Cos[t4f] + Cos[t2f + t3f] Sin[t1f] Sin[t4f],
     Sin[t2f + t3f] Sin[t4f]}, {Sin[t1f] Sin[t4f] Sin[t5f] +
      Cos[t1f] (-Cos[t5f] Sin[t2f + t3f] + Cos[t2f + t3f] Cos[t4f] Sin[t5f]),
     -Cos[t5f] Sin[t1f] Sin[t2f + t3f] + (Cos[t2f] Cos[t3f] Cos[t4f] Sin[t1f] -
         Cos[t4f] Sin[t1f] Sin[t2f] Sin[t3f] - Cos[t1f] Sin[t4f]) Sin[t5f],
     Sin[t2f] (-Cos[t5f] Sin[t3f] + Cos[t3f] Cos[t4f] Sin[t5f]) +
      Cos[t2f] (Cos[t3f] Cos[t5f] + Cos[t4f] Sin[t3f] Sin[t5f])}}}
```

Inverse Kinematics: -

Inverse Position Kinematics: -



)

xc = Oc – d6 T60[[1, 3]];
yc = Oc – d6 T60[[2, 3]];
zc = Oc – d6 T60[[3, 3]];

In[∘]:= d1 = 0;
t1 = $\tan^{-1}$(yc, xc);
t1 = $\tan^{-1}$(yc, xc) + π;
r = a2 cos(t2) + a3 cos(t2 + t3);
zc = a2 sin(t2) + a3 sin(t2 + t3);

In[∘]:= `Simplify[r^2 + zc^2]`

Out[∘]= $a2^2 + 2\, a2\, a3 \cos(t3) + a3^2$

From here we can say that;

$$C3 = \frac{1}{2}\, a2\, a3 \left(-a2^2 + a3^2 + r^2 + zc^2\right);$$

And thus we can say that;

$$S3 = \pm \sqrt{1 - C3}\,;$$

t3 = $\tan^{-1}$(S3, C3);
t3 = $\tan^{-1}$(–S3, C3);

We know that;

$$r = \sqrt{xc^2 + yc^2};$$

$$t2 + t3 = \tan^{-1}(zc, r);$$
$$t2 = \tan^{-1}\left(zc,\ \sqrt{xc^2 + yc^2}\right) - t3;$$

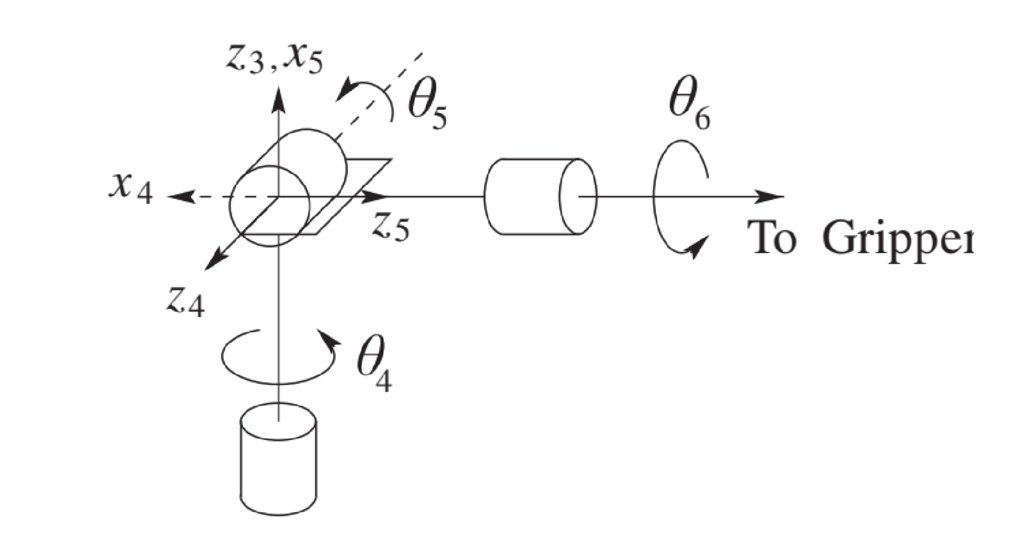
Hence We found all the angles here;

$$t1 = \tan^{-1}(yc, xc);$$
$$t1 = \tan^{-1}(yc, xc) + \pi;$$

$$t3 = \tan^{-1}\left(\pm\sqrt{1 - C3},\ \frac{1}{2}\,a2\,a3\,(-a2^2 + a3^2 + r^2 + zc^2)\right);$$

$$t2 = \tan^{-1}\left(zc,\ \sqrt{xc^2 + yc^2}\right) - t3;$$

Inverse Orientation: -



In[ ]:=
```
Simplify[T63 = Inverse[T30].T60];
T63 // MatrixForm
```

Ans we can also say this;

In[ ]:=
```
Simplify[T63 = T43.T54.T65] // MatrixForm
```

Equating the last columns of the matrices we get that;

$$t4 = \tan^{-1}(A1C23r23 + C1C23r13 + S23r33,\ -c1s23r13 + c23r33 - s1s23r23);$$
$$t5 = \tan^{-1}\left(S1r13 - C1r23,\ \pm\sqrt{1 - (S1r13 - C1r23)^2}\right)$$
$$t6 = \tan^{-1}(C1r21 - S1r11,\ S1r12 - C1r22);$$