

THE PRODUCT RULE

Suppose that a procedure can be broken down into a sequence of two tasks. If there are n_1 ways to do the first task and for each ways of doing the first task there is n_2 ways to do the second task then there are $n_1 n_2$ ways to do the procedure.

Example 1

A company with just two employees, Sanchez and Patel, rents a floor of a building with 12 offices. How many ways are there to assign different offices to these two employees?

Solution: Assigning offices to these two employees consists of assigning an office to Sanchez, which can be done in 12 ways, then assigning an office to Patel different from the office assigned to Sanchez, which can be done in 11 ways. By product rule, there are $12 * 11 = 132$ ways to assign offices to these two employees.

Example 2

The chairs of an auditorium are to be labeled with a letter and a positive integer not exceeding 100. What is the largest number of chairs that can be labeled differently?

Solution: Labeling a chair consists of two tasks, assigning one of the 26 letters and then assigning one of the 100 possible integers to the seat. The product rule shows that there are $26 \cdot 100 = 2600$ different ways that a chair can be labeled.

Example 3

There are 32 microcomputers in a computer center. Each microcomputer has 24 ports. How many different ports to a microcomputer in the center are there?

Solution: The procedure of choosing a port consists of two tasks, first picking a microcomputer and then picking a port on this microcomputer. Because there are 32 ways to choose the microcomputer and 24 ways to choose the port no matter which microcomputer has been selected, the product rule shows that there are $32 \cdot 24 = 768$ ports

Example 4

How many different bit strings of length seven are there?

Solution: Each of the seven bits can be chosen in two ways, because each bit is either 0 or 1. Therefore, the product rule shows there are a total of $2^7 = 128$ different bit strings of length seven.