Proposition

- It is a declarative sentence (a sentence that declares a fact) that is either true or false, but not both.

Example:

Are the following sentences propositions?

1) Dhaka is the capital of Bangladesh. (Yes)
2) Read this carefully. (No)
3) 1+2=3 (Yes)
4) x+1=2 (No)
5) What is your name? (No)

Propositional Logic

- the area of logic that deals with propositions.
- Also called propositional calculus.

Propositional Variables

- variables that represent propositions.
- Just as letters are used to denote numerical variables: p, q, r, s
- E.g. Proposition p – "Today is Friday."

Truth values – T, F

Logical operators

- The logical operators are used to form new propositions from two or more existing propositions.
- The logical operators are also called connectives.
- ¬, /\, V, →,↔ are logical operators.

Negation

- Let p be a proposition.
- The negation of p, denoted by ¬p.
- is the statement “It is not the case that p.”
- The proposition ¬p is read “not p.”
- The truth value of the negation of p, ¬p is the opposite of the truth value of p.

Examples:

- Negation of the proposition “Today is Friday.” and express this in simple English.

Solution: The negation is “It is not the case that today is Friday.” In simple English, “Today is not Friday.” or “It is not Friday today.”

Conjunction (AND)

- Let p and q be propositions.

- The conjunction of p and q, denoted by $p \land q$, is the proposition "p and q".

- The conjunction $p \land q$ is true when both p and q are true and is false otherwise.

Examples:

- Conjunction of the propositions p and q where p is the proposition "Today is Friday." and q is the proposition "It is raining today.", and the truth value of the conjunction.

Solution: The conjunction is the proposition "Today is Friday and it is raining today." The proposition is true on rainy Fridays.

Disjunction (OR)

- Let p and q be propositions. The disjunction of p and q, denoted by $p \lor q$, is the proposition "p or q".

- The conjunction p v q is false when both p and q are false and is true otherwise.

E.g. - "Students who have taken calculus or computer science can take this class." – those who take one or both classes.

Exclusive or (X-OR)

- Let p and q be propositions. The exclusive or of p and q, denoted by p, q.
- is the proposition that is true when exactly one of p and q is true and is false otherwise.

E.g. "Students who have taken calculus or computer science, but not both, can take this class." – only those who take one of them.

Conditional Statements

- Let p and q be propositions. The conditional statement $p \rightarrow q$, is the proposition "if p, then q."

- The conditional statement is false when p is true and q is false, and true otherwise.

- In the conditional statement $p \rightarrow q$, p is called the hypothesis (or antecedent or premise) and q is called the conclusion (or consequence).

- A conditional statement is also called an implication.

Example:

"If I am elected, then I will lower taxes."

Converse of $p \rightarrow q$: $q \rightarrow p$

Contrapositive of $p \rightarrow q$: $\neg q \rightarrow \neg p$

Inverse of $p \rightarrow q$: $\neg p \rightarrow \neg q$

Bi-conditional

- Let p and q be propositions. The biconditional statement $p \leftrightarrow q$ is the proposition "p if and only if q."

- The bi-conditional statement $p \leftrightarrow q$ is true when p and q have the same truth values, and is false otherwise.

- Bi-conditional statements are also called bi-implications.

- $p \leftrightarrow q$ has the same truth value as $(p \rightarrow q) \wedge (q \rightarrow p)$
- "if and only if" can be expressed by "iff"

Example:

Let p be the statement "You can take the flight" and let q be the statement "You buy a ticket." Then $p \leftrightarrow q$ is the statement "You can take the flight if and only if you buy a ticket."

Implication:

If you buy a ticket you can take the flight.
If you don't buy a ticket you can't take the flight.