Course Title : Simulation and Modeling
Course no: CSC-302
Credit hours : 3

Full Marks: 60+20+20
Pass Marks: 24+8+8

Course Contents:

UNIT 1: Introduction to Simulation - 6 Hrs.

Continuous and discrete systems, System simulation, Real time simulation, Types of Simulation Models, Steps in Simulation Study, Phases of a simulation study, Advantages of simulation, Limitations of the Simulation Technique, Areas of applications, Principles used in modeling

14 page

UNIT 2: Simulation of Continuous Systems - 5 Hrs.

Queuing system, Markov chains, Differential and partial differential equations

15 page

UNIT 3: Random Numbers - 10 Hrs.

Random Numbers, Random Number Tables, Pseudo Random Numbers, Generation of Random Number, Testing Numbers for Randomness, Uniformity Test, Chi-square test, Testing for auto correlation, Poker Test

20 page

UNIT 4: Verification and Validation of Simulation Models - 6 Hrs.

Model building, verification and validation, verification of Simulation Models, Calibration and Validation of Models.

6 page

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5 pags

UNIT 5: Analysis of Simulation Output - 8Hrs.

Estimation methods, Simulation run statistics, Replication of runs, Elimination of internal bias.

UNIT 6: Simulation Languages - 10Hrs.

Basic concept of Simulation tool, Discrete systems modeling and simulation, Continuous systems modeling and simulation, Data and control, Hybrid simulation, Feedback systems: typical applications.

Text Books: Jerry Banks "Discrete Event System Simulation"
References: G. Gorden "System Simulation"

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UNIT 1: INTRODUCTION TO SIMULATION

Simulation :-

A simulation is the imitation of an operation of real world process/system over a time. It involves the generation and observation of an artificial history of system and then to draw inference about characteristics of real system. Behaviour of a system is studied by developing a simulation model. This model contains a set of assumption about the operation of system. A simulation model can be developed that can be solved by mathematical methods. But many real world system are complex and cannot be solved by mathematically. In such case numerical computer based simulation is used to imitate the behaviour of the system.

When simulation is the appropriate tools?

Simulation is appropriate tools for following process:

i) For study and experiment of a complex system.
ii) For informational, organizational and environmental
iii) Simulation model will be useful to suggest the improvement in the system.
iv) By changing simulation inputs and observing outputs, it helps to find which variable is important and how variable interact.
v) It is used to support and verify analystic solution methodologies
vi) It can be used to experiment with new design/policy.
vii) It is used to simulate capabilities of a machine and to determine its requirement.
viii) Many modern systems like water fabrication, plants, service and

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Measure of performance:
The model can be solved by mathematical methods such as differential calculus, probability theory, algebric method or other mathematical techniques. Such solution usually consist of one or more numerical parameters which are called measure of performance of the system.

(2)

organization etc are very complex and can be treated by only through the simulation.

When simulation is not appropriate?

The simulation should not be used when:

i) The problem can be solved just by using the common sense.
ii) It is easy to perform direct experiments.
iii) The cost exceeds saving.
iv) The resource and time are not available.
v) There is not enough data.
vi) There is no enough time and manpower to verify and validate the model.
vii) There is unreasonable expectation.

Advantage of simulation:

The advantages of simulation are:

i) New policy, operating procedures, decision rules, information flow, etc can be explored with out disturbing the ongoing operation of the real system.
ii) New hardware design, physical layout, etc can be tested without committing resource for them.
iii) Time can be compressed or expanded allowing for a speed up or slow down of the phenomena under investigation.
iv) A simulation study can help in understanding how the system operate.
v) Hypothesis about how or why certain phenomena occur can be tested for feasibility.
vi) Insight can be obtained about the interaction of variables.
" " " " " " importance of variables to the performance of the system.

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Disadvantages of simulation:

The disadvantages of simulation are:

i) Model building require special training.
ii) Simulation need time and experience.
iii) There may be more than one model developed by different people and it may become difficult to one among them.
iv) Simulation result may be difficult to interprete.
v) Simulation modeling and analysis can be time consuming and expensive.

Application Areas of Simulation:

The applications of simulation are vast. Some of the application areas of simulation are:

i) Manufacturing Applications:
- Dynamic modeling of continuous manufacturing systems, using analogies to electrical systems.
- Benchmarking of a stochastic production planning model in a simulation test bed.
- Paint line color change reduction in automobile assembly
- Modeling for quality and productivity in steel cord manufacturing.
- Shared resource capacity analysis in biotech manufacturing
- Neutral information model for simulating machine shop operations.

ii) Semiconductor Manufacturing:
- Constant time interval production planning with application to work-in-process control

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- Accelerating products under due-date oriented dispatching rules.
- Design framework for automated material handling systems in 300-mm water fabrication factories
- Making optimal design decisions for next-generation dispensing tools
- Application of cluster tool modeling in a 300-mm water fabrication factory.
- Resident-entity based simulation of batch chamber tools in 300-mm semiconductor manufacturing.

III) Construction Engineering and Project Management:
- Impact of multitasking and merge bias on procurement of complex equipment.
- Application of lean concepts and simulation for drainage operations maintenance crews.
- Building a virtual shop model for steel fabrication
- Simulation of the residential lumber supply chain.

IV) Military Applications:
- Frequency-based design for terminating simulations: A peace-enforcement example.
- A multibased framework for supporting military-based interactive simulations in 3D environments.
- Specifying the behaviour of computer-generated forces without programming.
- Fidelity and validity: Issues of human behavioural representation
- Assessing technology effects on human performance through trade-space development and evaluation.

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- Impact of an automatic logistics system on the sortie-generation process.
- Research plan development for modeling and simulation of military operations in urban terrain.

v) Logistics, Supply Chain, and Distribution Applications:
- Inventory analysis in a server-computer manufacturing environment.
- Comparison of bottleneck detection methods for AGV systems.
- Semi-conductor supply-network simulation
- Analysis of international departure passenger flows in an airport terminal.
- Application of discrete simulation techniques to liquid natural gas supply chains.
- Online simulation of pedestrian flow in public buildings.

vi) Transportation Modes and Traffic:
- Simulating aircraft-delay absorption.
- Runway schedule determination by simulation optimization
- Simulation of freeway merging and diverging behaviour
- Modeling ambulance service of the Austrian Red Cross
- Simulation modeling in support of emergency firefighting in Norfolk.
- Modeling ship arrivals in ports
- Optimization of a barge transportation system for petroleum delivery.
- Iterative optimization and simulation of barge traffic on an inland waterway.

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VII) Business Process Simulation:

- Agent-based modeling and simulation of store performance for presonalized pricing.
- Visualization of probabilistic business models.
- Modeling and simulation of a telephone call center.
- Using simulation to approximate subgradients of convex performance measures in service systems.
- Simulation's role in baggage screening at airports
- Human-fatigue risk simulations in continuous operations.
- Optimization of a telecommunications billing system.
- Segmenting the customer base for maximum returns

VIII) Health care:

- Modeling front office and patient care in ambulatory health care practices.
- Evaluation of hospital operations between the emergency department and a medical telemetry unit.
- Estimating maximum capacity in an emergency room
- Reducing the length of stay in an emergency department.
- Simulating six-sigma improvement ideas for a hospital emergency department.
- A simulation-integer-linear-programming-based tool for scheduling emergency room staff.

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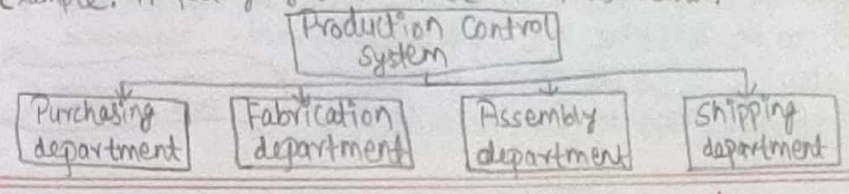
Example: A factory system can be shown as below:



System and System Environment:

A system is defined as a group of objects that are joined together in some regular interaction or interdependence toward the accomplishment of some purpose. An example is a production system manufacturing automobiles. The machines, component parts, and workers operate jointly along an assembly line to produce a high-quality vehicle.

A system is often affected by changes occurring outside the system. In modeling systems, it is necessary to decide on the boundary between the system and its environment.

In the case of the factory system, for example, the factors controlling the arrival of orders may be considered to be outside the influence of the factory and therefore the part of the environment. However, if the effect of supply on demand is to be considered, there will be a relationship between factory output and arrival of orders, and this relationship must be considered an activity of the system.

Components of a System:

In order to understand and analyze a system, a number of terms need to be defined.

I) Entity: An entity is an object of interest in the system.
II) Attribute: An attribute is a property of an entity.
III) Activity: An activity represent a time period of specified length.

⇒ If a bank is being studied, customers might be one of the entities, the balance in their checking accounts might be an attribute, and making deposits might be an activity.

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System environment: The external components which intract with the system and produce necessary changes are said to be constitue the system environment. In modeling system, it is necessary to decide on the boundary between system and its environment. This decision depends upon the purpose of the study.

iv) State: The state of a system is defined to be that collection of variables necessary to describe the system at any time, relative to the objectives of the study.

⇒ In the study of the bank, possible state variables are the number of busy tellers, the number of customers waiting in line or being served, and the arrival time of the next customer.

v) Event: An event is defined as an instantaneous occurrence that might change the state of the system.

vi) Endogenous: The term endogenous is used to describe activities and events occurring within a system.

and, the term exogenous is used to describe activities and events in the environment that affect the system.

⇒ In the bank study, the arrival of a customer is an exogenous event, and the completion of service of a customer is an endogenous event.

Examples of Systems and Components:

System	Entities	Attributes	Activities	Events	State Variables
Banking	Customers	Checking-account balance	Making deposits	Arrival; departure	No. of busy tellers; no. of customers waiting
Inventory	Warehouse	Capacity	Withdrawing	Demand	Levels of inventory; back logged demands
Communication	Messages	Length; destination	Transmitting	Arrival at destination	No. waiting to be transmitted
Production	Machines	Speed; capacity breakdown rate	Welding; Stamping	Breakdown	Status of machine (busy, idle or down)
Rapid rail	Riders	Origination; destination	Travelling	Arrival at station; arrival at destination	No. of riders waiting at each station; no. of riders in transit

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Steps in simulation Study :-

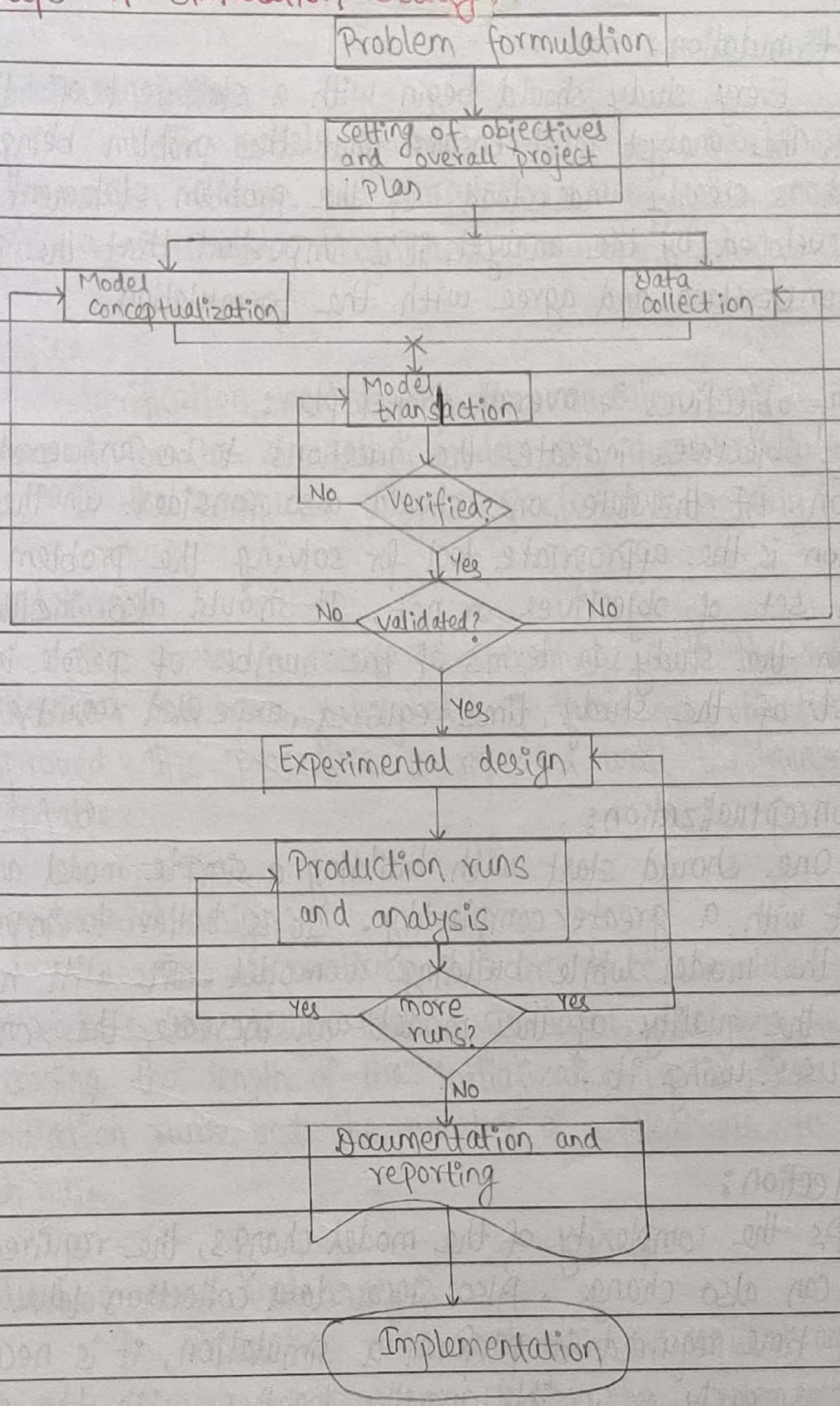

Fig: Steps in simulation study

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I) Problem formulation:

Every study should begin with a statement of the problem. The analyst must ensure that the problem being described is clearly understood. If the problem statement is being developed by the analyst, it is important that the policy maker understand and agree with the formulation.

II) Setting of objectives & overall project plan:

The objectives indicate the questions to be answered by the simulation. At this state, one should also consider whether the simulation is the appropriate tool for solving the problem for a given set of objectives or not. It should also include the plans for the study in terms of the number of people involved, the cost of the study, time required, expected result, etc.

III) Model conceptualization:

One should start with building a simple model and then expand it with a greater complexity. It is better to involve the user of the model while building a model. This will help to improve the quality of the model and increase the confidence of the user using it.

IV) Data collection:

As the complexity of the model changes, the required data elements can also change. Also, since data collection take huge amount of time required to perform a simulation, it is necessary to begin it as early as possible, usually together with the early

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stages of model building.

v) Model translation:

The models of most of the real world system require information storage and computation. Hence, the model must enter into a computer recognizable format.

vi) Verified:

Verification related with the computer program that has been prepared for the simulation model. The questions to be answered by, "Is the computer program performing properly?".

vii) Validation:

The model is compared iteratively with the actual system behaviour and hence validated. If there are objection, model is improved. This procedure is repeated until the accuracy of model is acceptable.

viii) Experimental design:

The alternatives that are to be simulated must be determined. For each system design that is simulated, decisions need to be made concerning the length of the initialization period, the length of the simulation runs, and the number of replications to be made of each run.

ix) Production runs and analysis:

Production runs, and their subsequent analysis, are used to estimate measures of performance for the system designs that are being simulated.

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X) More Runs ? :

After analysing the completed run, the analyst must determine whether additional runs are needed or not.

XI) Documentation and Reporting:

There are two types of documentation : program and progress.

a) Program documentation :

Program documentation can be used again by the same or different analyst to understand how the program operates. Further modification will be easier.

b) Progress documentation :

It gives the history of simulation project. The result of all analysis should be reported clearly in the final report.

XII) Implementation:

The success of the implementation phase depends on how well the previous steps have been performed. If the model user has been thoroughly involved during the model-building process and if the model user understands the nature of the model and its outputs, then the likelihood of a vigorous implementation is enhanced.

The simulation-model building process can be divided into four phases:

Phase 1 : Step 1 & 2
Phase 2 : Step 3 - Step 7
Phase 3 : Step 8 - Step 10
Phase 4 : Step 11 & 12.

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Phases of Simulation Study:

The simulation model building can be broken into four phases:

i) I phase:
- consist of step 1 and step 2.
- It is period of discovery or orientation.
- The analyst may have to re-start the process if it is not fine tuned.
- Recalibration and clearification may occur in this phase.

ii) II phase:
- consist of step 3, 4, 5, 6 and 7.
- A continuing interplaying is required among the steps.
- Exclusion of model user results in implication during implementation.

iii) III Phase:
- consist of step 8, 9 and 10.
- conceives a through plan for experimenting
- Discrete event stochastic is a statistical experiment.
- The output variables are estimated that contains random error and therefore proper statistical analysis is required.

iv) IV phase:
- Consist of steps 11 and 12.
- successful implementation depends on involvement of user and every steps successful completion.

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Discrete and Continuous System

Systems can be categorized as discrete or continuous as explained below:

I) Discrete System:

A discrete system is one in which the state variable(s) change at a discrete set of points in time. For example; banking system in which number of customer (state variable) changes only when customer arrives or when service is provided to the customer i.e. customer depart from the system. Figure below shows how number of customer changes only at discrete points in time.

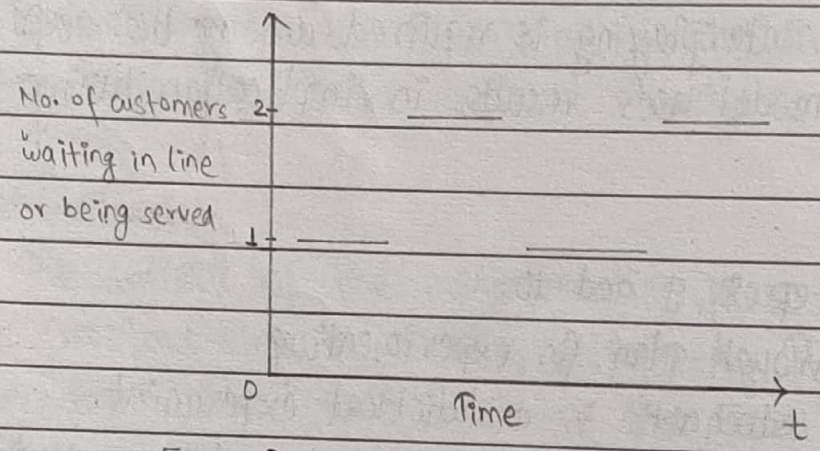

Fig: Discrete system state variable

II) Continuous System:

Continuous system is one in which the state variable changes continuously over time. For example; during winter season level of water decrease gradually and during rainy season level of water increase gradually. The change of water level is continuous. Figure below shows the change of water level over time.

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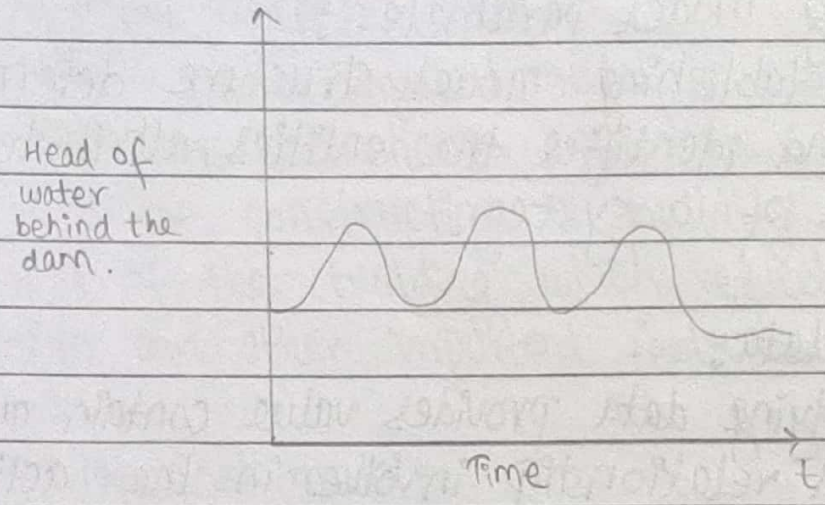

Fig: Continuous system state variable

Model of a system (System modeling):

A model is defined as a representation of a system for the purpose of studying the system. It is necessary to consider only those aspect of the system that affect the problem under investigation. These aspects are represented in a model and by definition it is a simplification of the system.

Aspect of system that affect the problem under investigation are only represented in a model of system. Therefore, model is the simplification of the real system. Therefore, no unique model of a system. Different model of the same system will be produced by different analyst who are interested in different aspect of the system.

The task of deriving a model of a system may be divided broadly into two sub-task:

i) Establishing model parameters
ii) Supplying data.

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i) Establishing model parameters:

Establishing model structure determines system boundary and identifies the entities, attributes, activities and events of a system.

ii) Supplying data:

Supplying data provides value contain an attribute and defined relationship involve in the activities.

Types of model:

The various types of model are:

i) Physical model
ii) Mathematical model
iii) Static model
iv) Dynamic model
v) Deterministic model
vi) Stochastic model
vii) Discrete model
viii) Continuous model
ix) Numerical model
x) Analytical model

i) Physical model:

These models are based on some analogy between mechanical and electrical system. The system attributes represented by physical measures such as voltage. The system activities are represented by physical laws. Physical models are of two types:

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transducer : a device that converts variations in a physical quantity, such as pressure or brightness, into an electrical signal, or vice-versa.

a) Static Physical model:

The static physical model is scaled down model of a system which does not change with time. For example; an architect before constructing a building makes a scaled down model of the building which reflects all its rooms, outer design and other important features.

b) Dynamic Physical model:

They are one in which change with time or which are function of time. For example, in wind tunnel small aircraft models are kept and air is blown over them with different velocities and pressure profiles are measured with the help of transducers embedded in the model. Here wind velocity changes with time and is an example of dynamic physical model.

ii) Mathematical model:

It uses symbolic notation and mathematical equations to represent system. The system attributes are represented by variables and the activities are represented by mathematical functions. For example: $f(x) = mx + c$ is a mathematical model of a line.

iii) Static model:

Static models can show the values that the system attributes value doesn't change over time. For example: Scientist has used models in which sphere represents atom, sheet of metal t connect the sphere to represent atomic bonds. Graphs are used

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to model the various system based on network. A map is also a kind of graph, these models are sometimes said to be iconic models and are of kind of static physical models.

iv) **Dynamic model:**

Dynamic model follow the changes over time that results from system's activities. Mechanical and electrical systems are the example of dynamic system. Generally dynamic model involves the computation of variables value over time and hence they are represented by differential equations.

v) **Deterministic model:**

Simulation models that contain no random variables are classified as deterministic. They have a known set of inputs which will result in a unique set of outputs. For example: arrival of patients to the dentist at the scheduled appointment time.

vi) **Stochastic model:**

A stochastic simulation model has one or more random variables as input. Random inputs lead to random outputs. Since the outputs are random they can be considered only as estimates of the true characteristics of a model. For example: Simulation of a bank involves random inter-arrival time and service time.

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VII) Discrete model:

A discrete model is one in which the state variable change only at a discrete set of points in time. It is analogous to discrete system. However, a discrete simulation model is not always used to model a discrete system.

VIII) Continuous model:

A continuous model is one in which the state variable change continuously over time. It is analogous to continuous system. However it is also not only used to model the continuous system.

IX) Analytical model:

In mathematical model, we can differentiate the model on the basis of solution technique used to solve the model. Analytical technique means using deductive technique reasoning of mathematical theory to solve a model. Such a model are known as analytical model.

X) Numerical model:

Numerical model involves applying computational process to solve the equation. For example; we may solve differential equation numerically when the specific limit of variable is given.

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Hybrid Simulation:-

For most studies, the system under study is clearly either of continuous or discrete nature. And it is a determining factor in deciding whether to use an analog or digital computer system in simulation.

If the system being simulated is an interconnection of continuous and discrete systems then such a system is known as hybrid simulation. Such hybrid simulation can be linked with digital computer.

Hybrid simulation require high speed converters to transform signals from analog to digital form and vice-versa.

Real time Simulation:-

Real time simulation refers to a computer model of a physical system that can execute at the same rate as the physical system. In other words, the computer model runs at the same rate as the actual physical system. In real-time simulation, actual device are used in conjunction with either digital or hybrid computer.

Example: If a tank takes 10 minutes to fill in the real world, then in real time simulation the system must take 10 minutes to fill.

Real time simulation occurs commonly in computer gaming, aircraft design, etc.

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Principle Used in Modeling:

It is not possible to provide rule by which models are built, but a number of guidelines can be stated. The different view-points from which, we can judge whether certain information should be included or excluded in models are:

1) Block Building:

The description of system should be organized as a sequence of blocks. It simplifies the interaction between block within system. Then it will be easy to describe the whole system in terms of interaction between the blocks and can be represented graphically as simple block diagram.

Example: Block diagram of factory system.

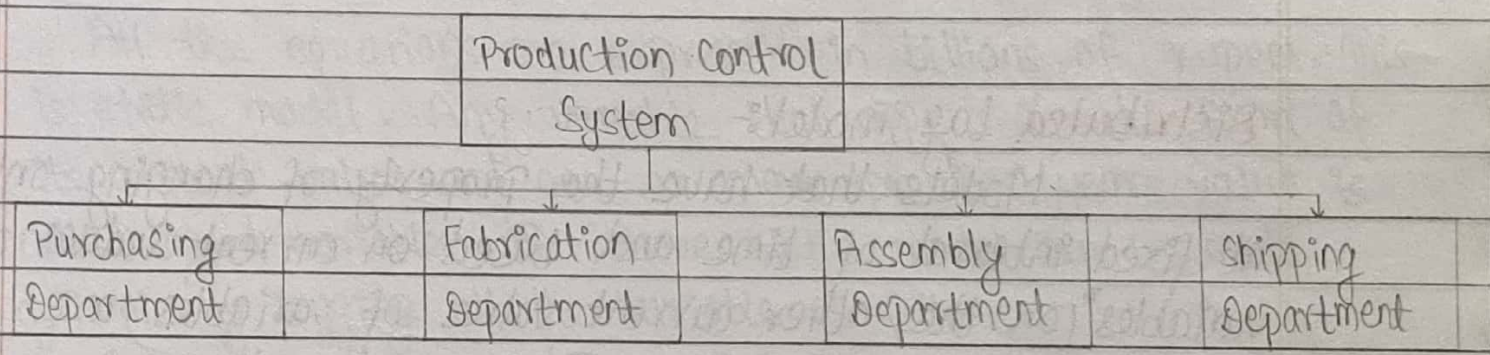

Fig:- Block diagram of factory system

11) Relevance:

The model should only include relevant information. For example: if the factory system study aims to compare the efficiency of different operating rules then it is not relevant to consider the mining of employees as an activity. Irrelevant information should not include in the system and not including

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this doesn't make any harm to the system.

III) Accuracy:

The gathered information should be accurate as well. For example; in aircraft system the accuracy of movement of aircraft depends upon the representation of air frame such as rigid body.

IV) Aggregation:

It should be considered that to which number of individual entities can be grouped into the block. For example; in a factory system different departments are grouped together which is handled by production manager.

Distributed lag model:-

Models that have the property of changing only at fixed interval of time and based on current values of variables* (on the other current value of variables) are called distributed lag model. These are the type of dynamic model because time factor is involved in them. They are extensively used in economic studies where the data are collected over uniform time interval such as a month or year. This model consist of linear algebric equations that represent continuous system but data are available at fixed point in time.

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Example:

Mathematical model for national economy.

Let,

C = consumption
I = investment
T = taxes
G = government expenditure
Y = national income.

Then,

$$\left.\begin{aligned} C &= 20 + 0.7(Y - T) \\ I &= 2 + 0.1Y \\ T &= 0.2Y \\ Y &= C + I + G \end{aligned}\right\} \text{—— (I)}$$

All the equations are expressed in billions of rupees. This is static model. Any variable that appears in the form of its current value and one or more previous years value is called lagged variables. The above static model can be made dynamic by lagging all the variable as follows:

$$\left.\begin{aligned} C &= 20 + 0.7(Y_{-1} - T_{-1}) \\ I &= 2 + 0.1Y_{-1} \\ T &= 0.2Y_{-1} \\ Y &= C_{-1} + I_{-1} + G_{-1} \end{aligned}\right\} \text{—— (II)}$$

In this equation, the previous value is represented by -1 subscript. If the previous value is known then the values for the current event can be computed. Taking these values as input, values for the next year can also be computed.

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Advantages of distributed lag model:

i) Simple to understand and can be computed by hand, computers are extensively used to run them.

ii) There is no need for special programming language to organize simulation task.

Differentiate between dynamic physical model and static physical model with example.

The physical models are based on some analogy between mechanical and electrical system. The system attributes are represented by physical measures such as voltages. The system activities are represented by physical laws. The best known example of physical model is scaled model.

Physical models are of two types:

i) Static physical model:

Static physical model is a scaled down model of a system which does not change with time. An architect before constructing a building makes the scaled down model of a building which reflects all its rooms, outer design and other important features. This is an example of static physical model. Similarly, for conducting trails in water, we make small water tanks which are replica of sea and fire small scaled-down shells in them. This tank can be treated as static physical model of ocean.

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11) Dynamic physical model:

Dynamic physical models are ones which change with time or function of time. In wind tunnel small aircraft models are kept and air is blown over them with different velocities and pressure profiles are measured with the help of transducers embedded in the model. Here, wind velocity changes with time and is an example of dynamic physical model. A model of hanging wheel of vehicle is another case of dynamic physical model. Let us take an example of hanging wheel of stationary truck and analyze its motion under various forces. Consider a wheel of mass 'M', suspended in vertical direction of force 'F(t)' which varies with time, is acting on it. Mass is connected with the spring of stifness k and a piston with dumping factor θ. When force F(t) is applied, mass M oscillates under the action of force. This model can be used to study the oscillation in a motor wheel figure below show such a system.

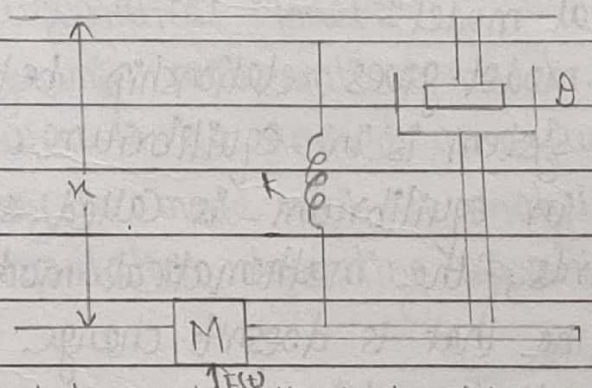

Fig: suspended weight attached with string and piston

This model is a discrete static physical model. Discrete in a sense that one can give discrete values and observed the oscillation of wheel with some measuring equipment. When force is applied on it which is the function of time, this

discrete static physical model becomes dynamic model. Parameters k and δ can also be adjusted in order to get control oscillation of the wheel. This type of system is spring mass system.

Mathematical model:

It uses symbolic notation and mathematical equation to represent system. The system attributes are represented by variables and activities are represented by mathematical functions. Most of the system can in general be transformed into mathematical equations, these equations are called mathematical model of the system. Mathematical model is of two types:

i) Static
ii) Dynamic.

i) Static mathematical model:

A static model gives relationship between the system attribute when the system is in equilibrium. Mathematical model of a system in equilibrium is called static mathematical model. In otherwords, the mathematical model without the consideration of time that is doesn't change with time is called static mathematical model.

Example; Suppose a company wants to optimize the cost of its products by balancing the supply and demand. Generally there should be a balance between the supply and demand of any product in the market. Supply increases if price is higher this is because shopkeeper gets more commission on that

product and that product push to the customer even if the quality is not excellent. Customer generally feels that more cost means better quality but on the otherhand, demand decreases with the increase of price. The aim is to find the optimum price with which demand can match the supply. Let us model this situation mathematically if we denote price by P, supply by S and demand by D and assuming the price equation to be linear we have:

$$D = a - bp$$
$$S = c + dp$$
$$S = D$$

In the above equation a, b, c and d are parameters computed based on previous market data. Equation $S = D$ says supply should be equal to demand so that price should accordingly be adjusted.

ii) Dynamic Mathematical Model:

Dynamic mathematical model allows the change of system attributes to be derived as a function of time. The derivation may be made with an analytical solution or with a numerical computation depending upon the complexity of the model.

Example: Let us consider the mathematical model of hanging wheel of a vehicle using Newton's second law of motions, system of wheel model can be access in the mathematical model K as:

$$M = \frac{d^2x}{dt^2} + D\frac{dx}{dt} + Kx = KF(t)$$

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where,

x = distance moved
M = mass of the wheel
K = stifness of the string
δ = Dumping force on the shock observer.

This system is a function of time and is a dynamic system. The above equations can be solved analytically. This equation can also be solved numerically with the help of Runge-Kutta method.

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UNIT-2: SIMULATION OF CONTINUOUS SYSTEM

Queuing System:-

Simulation is often used in the analysis of queuing models. Most system in a simulation study also contains a process in which there is a demand for service. The system can service entities at the rate which is greater than rate at which entities arrives. The entities are then said to join waiting line. The line where the entities or customers will wait is generally known as queue.

The combination of all entities in the system being served and being waiting for service will be called a queuing system. Queuing model provide a powerful tool for designing and evaluating the performance of queuing system. The general diagram for queuing system is as shown in figure below:

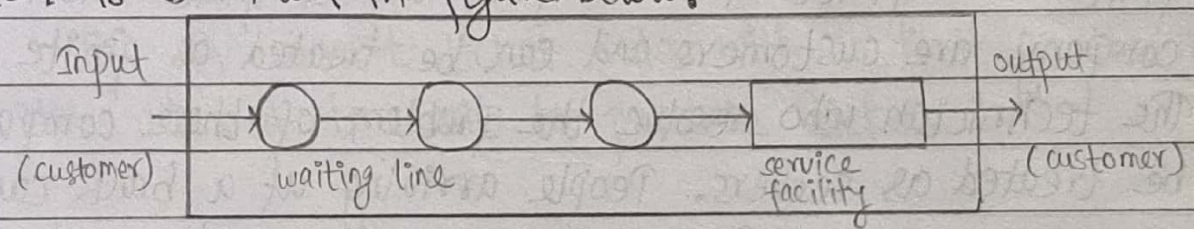

Fig: Single queue - single server queuing system.

A queuing system involves customer's arrival at a constant or variable time rate for service at a service station (server). Customers can be a student waiting for registration in college, aeroplane queuing for landing at air field or jobs waiting in machine shop. If the customer after arriving can enter the service center, it is good otherwise they have to wait for the service and form a queue that is waiting line. They remain in queue till they are provided the service. Sometime queue being too long, then they will leave the queue and go, it result loss of customer. Customers are to be serviced at a constant or variable rate before they leave the service station.

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Characteristics of queuing system (Elements of queuing system)

The key elements of a queuing system are the customers and server. The term customer can refer to anything that arrives at a facility and requires services. The term server refer to any resources that provide the requested service. In order to model queuing systems, we need to be a bit more precised about what constitutes a queuing system.

The other basic components common to all queuing systems are:

1) The calling population:

The population of potential customers is referred to as the calling population. The calling population may be finite or infinite. For example; the computer at a small company are customers and can be treated as finite population. The technician who resolve the problem of these computers can be treated as servers. People arriving at a bank can be considered as a infinite population because there is no exact count of people visiting every day.

a) Finite population model:

If arrival rate depends upon the number of customers being served and waiting then it is called finite population model. For example; model of one corporate jet if it is being repeated, then repaired arrival rate become zero.

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b. B) Infinite population model:

If arrival rate is not affected by the number of customer being served and waiting then it is called infinite ~~populat~~ population model. For eg: System with large population of potential customers.

ii) System capacity:

A limit on the number of customers that may be on the waiting line or system. It may be of two types:

a) Limited capacity:

Eg: An automatic car wash only has rooms for 10 cars to wait in the line to enter the mechanism.

b) Unlimited capacity:

Eg: A concert ticket sales with no limit on the number of peoples allowed to wait to purchase ticket.

iii) Arrival process or pattern:

The arrival process for infinite population model is usually characterized in terms of inter-arrival time of successive customer. Arrivals may occur at scheduled times or at random times. When at random time the inter-arrival times are usually characterized by a probability distribution. In addition, customer may arrive one at a time or in a batches. The batch may be constant size or random size.

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no. of arrival in time interval = p.d
inter-arrival time = exponentially d.

⇒ Poission arrival process:

The most important model for random arrivals is the Poission arrival process. If A_n represents the inter-arrival time between customer (n-1) and (n) then for a Poission arrival process A_n is exponentially distributed with mean $1/\lambda$ time units. The arrival rate is λ customers per time unit. The number of arrivals in a time interval of length 't', say N(t), has the Poission distribution with mean λt customers.

For example, the Poission arrival process has been employed successfully as a model of the arrival of people to restaurant, customers in bank, the arrival of telephone calls to a call centre, the arrival of demands or orders for a service or product and the arrival of failed component or machines to a repair facility.

⇒ Markov arrival process:

Another special arrival process which is highly useful for modeling purpose is the markov arrival process. The name refers to the situation when entities arrive one at a time and the time between arrivals are exponential random variables. This type of arrival process is memory less which means that the likelihood of an arrival within the next 't' minutes is the same no matter how long it has been since in the last arrival.

For eg: this arrival occurs phone calls arriving at an exchange, customer arriving at fast-food restaurant, hits on a website, and many others.

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iv) Service process or pattern:

Once entities have entered to the system, they must be served. The physical meaning of service depends on the system. Customer may go through the checkout process. Parts may go through assembling, patient may go through medical treatment, orders may be filled and so on. From a modeling stand point, the operational characteristics of service matters more than the physical characteristics. Specifically, we care about whether service times are long or short, and whether they are regular or highly variable. We care about whether entities are processed into FCFS order or according to some kind of priority rule. We also care about whether entities are serviced by a single server or by multiple servers working in parallel.

⇒ Poission service Process:

In a queuing system we assume that the service time for customer are also generated by a Poission process. Unfortunately this assumption is not general as the arrival time distribution but it could still be a reasonable assumption when no other data is available about service time. For example; telephone call duration :- telephone call duration define the service time for utilization of various resources in a telephone exchange. If the telephone call duration can be assumed Poission process then the following condition must satisfy:

- total number of customer that are served by a telephone exchange is very large.
- a single telephone call takes a very small fraction of the system resources.

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- decision on how long to talk is independently made by each customer.

From these rules the Poission process can represent the service.

⇒ **Markov service process:**

A special service process is Markov Service Process in which entities are processed one at a time. FCFS order and service time are independent and exponential. As with the case of Markov arrivals, a Markov service process is memoryless which means that the expected time untill an entity is finished remains constant regardless of how long it has been in service.

For example: A Markov service process for hardware maintenance would imply that the time required to resolved caller's problem is 15 minutes no matter how long the technician has already spent talking to customers.

v) **Queue Discipline:**

The logical ordering of customer in a waiting line is called queuing discipline and it determines which customer will be choosen for service. We say that queuing discipline is a rule to choose the customer for service from the waiting line. Common queuing discipline includes the following:

a. **First Come First Serve (FCFS):**

According to this rule, service is offered on the

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basis of arrival time of customer. The customer who come first will get the service first. So in other words, the customer who get the service next will be determined on the basis of longest waiting time.

b. Last In First Out (LIFO):

It is usually abbreviated as LIFO, occurs when service is next offered to the customer that arrive recently or which have waiting time least. In the crowd bus, the passenger getting in or out from the bus is an example of LIFO.

c. Service in Random order (SIRO):

It means that a random choice is made between all waiting customers at the time service is offered i.e. a customer is picked of randomly from the waiting queue for the service.

d. Shortest Processing Time First (SPTF):

It means that the customer with shortest service time will be choosen first for the service i.e. the shortest service time. Customer will get the priority in the selection process.

e. Priority:

A special number is assigned to each customer in the waiting line and it is called priority. Then according to this number the customer is choosen for service.

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VI) Queue Behaviour:

Queue behaviour refers to the action of customer while in a queue waiting for service to begin. The queuing behaviour may be one of the following:

a. Balking: (रोकिनु)

Customers may balk at joining the queue when it is too long. It is called balking. For example: car passes up a drive through restaurant if there are too many cars already waiting.

b. Reneging:

Customer may also exit the system due to impatience or perishability it is called reneging. For eg: Customer kept waiting too long at a bank decide to leave without services.

c. Jockeying: (cheating गर्नु)

When there is more than one line forming for the service or server, the action of moving customers from one line to another line because they think that they have choosen slow line, it is called jockeying.

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Queuing Notation (kendall's notation):

For recognizing the diversity of queuing system, kendall proposed a notational system for parallel server system which has been widely adopted. kendall's notation is the standard system used to describe and classify queuing characteristics. i.e. A | B | C | N | K

where, the above letter represents the following system characteristic

A = arrival pattern
B = service pattern
C = no. of server
N = system capacity
K = calling population

The common symbol used for probability distribution for inter-arrival time and service time are D for deterministic, M for exponential or Markov, H for hyper exponential, G for arbitrary or general and E_K for Earlang.

If the capacity is not specified it is taken as infinite. And if calling population is not specified it is assumed unlimited or infinite.

Example:

1. M | D | 2 | 5 | ∞

Stands for a queuing system having exponential arrival time, deterministic service time, two servers, capacity of five customers and infinite population.

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2. M|D|10

Stands for exponential arrival time deterministic service time, ten server, infinite service capacity and infinite population.

Measures of performance of queuing system:

The primary measures of performance of queuing system are:

i) Traffic intensity (u)
ii) Server utilization (ρ)
iii) Average number of customer in the system (L_s)
iv) Average number of customer in the queue (L_q)
v) Average time of customer spends in the system ($\bar{W}_s$)
vi) Average time of customer spends in the queue ($\bar{W}_q$)

Let the mean inter-arrival time T_a and the mean service time T_s and the corresponding rates;

$$\text{arrival rate } (\lambda) = \frac{1}{T_a}$$

$$\text{service rate } (\mu) = \frac{1}{T_s}$$

Then, the queuing measures are formulated as follows:

i) Traffic intensity:

The ratio of mean service time to the mean inter-arrival time is called traffic intensity i.e. $\left[u = \frac{\mu}{\lambda} \right]$

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Imp. ii) Server utilization:

It is defined as the ~~populat~~ proportion of time that a server is busy, it is denoted by (ρ) i.e. $\left[\rho = \frac{\lambda}{\mu}\right]$ single server utilization

$$\left[\rho = \frac{\lambda}{n\mu}\right]$$ multiple server

This is also the average number of customer in the service facility. Thus the probability of finding service counter free is: $[1-\rho]$.

iii) Average number of customer in the system:

We define average number of customers at time T in the system as $\bar{L}_s$. Here, bar over L depicts average length of queue and subscript s is for system.

$\bar{L}_s$ is given by;

$$\left[\bar{L}_s = \frac{\lambda}{\mu-\lambda} = \frac{\rho}{1-\rho}\right]$$

iv) Average number of customer in the queue:

Expected number of customers in the queue.

$$\bar{L}_q = \bar{L}_s - \rho$$

$$= \frac{\lambda}{\mu-\lambda} - \frac{\lambda}{\mu}$$

$$= \frac{\lambda^2}{\mu(\mu-\lambda)}$$

$$\therefore \left[\bar{L}_q = \frac{\rho^2}{1-\rho}\right]$$

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v) Average time of customer spend in the system:

It is denoted by $\overline{W}_s$ and is equal to expected number of customer in the system at time t, divided by number of customer arrived in unit time.

$$\overline{W}_s = \frac{\lambda}{\mu-\lambda} \cdot \frac{1}{\lambda}$$

$$= \frac{1}{\mu-\lambda}$$

$$\left[\therefore \overline{W}_s = \frac{1}{\mu-\lambda}\right]$$

vi) Average time of customer spend in queue:

It is the average time a customer spends in the queue. It is same as average time a customer spent in the system i.e.

$$\left[\overline{W}_Q = \overline{W}_s - \frac{1}{\mu}\right]$$

Numerical Problems:

Q: 1. At the ticket counter of football stadium, people come in queue and purchase tickets, arrival rate of customer is 1 per minute. It takes at average 20 sec to purchase the ticket. If a sport fan arrives 2 minute before the game starts and if he takes exactly 1.5 minutes to reach the correct seat after he purchase a ticket, can the sport fan expects to be sitted for a tip off.

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Sol^n:

A minute is used as unit of time. Since ticket is sold in 20 seconds. This means three customers enter the stadium per minutes i.e. service rate is 3 customer per minute. Therefore,

$$\lambda = 1/\text{min}$$

$$\mu = 3/\text{min}$$

$$\overline{W}_s = \frac{1}{\mu-\lambda} = \frac{1}{3-1} = \frac{1}{2} \text{ min}$$

The average time to get the ticket and time to reach the correct seat is 2 min exactly so the sportsfan can expect to be sitted (seated) for the tip off.

Q. 2. Customer arrives in a bank according to Poission's process with mean inter-arrival time of 10 minutes. Customer spent an average of 5 minutes in a single available counter and leave. Then,

a, What is the probability that a customer will not have to wait at the counter?

b, What is the expected number of customers in the bank?

c, How much time a customer expect to spend in the bank?

Sol^n:

Let us take an hour as the unit of time then $\lambda = 6$ customer/hr.

$\mu = 12$ customer/hr.

a, The customer will not have to wait if there are no customer in the bank. Thus,

$$P = 1-\rho = 1-\lambda/\mu = 1-6/12 = 1/2 = 0.5$$

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b, Expected number of customers in the bank is given by;

$$L_s = \frac{\lambda}{\mu - \lambda} = \frac{6}{6} = 1$$

c, Expected time to be spent in the bank is given by;

$$W_s = \frac{1}{\mu - \lambda} = \frac{1}{6} \text{ hr} = 10 \text{ min.}$$

Single Server Queuing System:

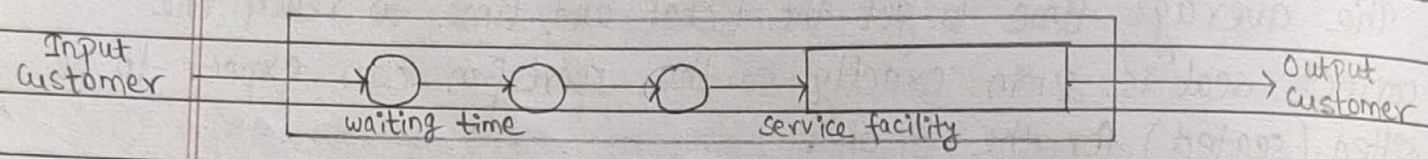

Fig:- Single queue - single server queuing system

Figure above shows the single server queuing system. For the case of simplicity to study the measures of queuing system we assume that there is single queue and only one server serving the customer. To study all characteristics of queuing system, we make the following assumptions:

I) First-In-First Out (FIFO):

Service is provided on the FCFS basis.

II) Random:

Arrival of customer is completely random but at a certain arrival rate.

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iii) Steady state:

The queuing system is at steady state.

The above conditions are very ideal conditions for any queuing system and assumptions are made to model the situation mathematically.

First condition only means irrespective of customer, one who comes first is attended first and no priority is given to anyone.

Second condition says that arrival of a customer is random. This means that an arrival can occur at any time and the time of next arrival is independent of the previous arrival with this assumption it is possible to show that the distribution of inter-arrival time is exponential i.e equivalent to saying that the number of arrival per unit time is a random variable with a Poission distribution. This distribution is used when chances of occurrence of an event out of a large sample is small i.e. if X = number of arrivals per unit time then the probability distribution function (pdf) of arrival is given as:

$$f(x) = P_r(X=x) = \frac{e^{-\lambda}\lambda^{x}}{x!} \quad \begin{cases} x = 0, 1, \ldots \\ \lambda > 0. \end{cases}$$

where,

$E(x) = \lambda$ is the average number of arrivals per units time i.e. $1/\tau$.

x is the no. of customer per unit time.

This arrival pattern is called Poisson's arrival pattern.

Third assumption means queuing system has been operating long enough to be independent of the initial state of the system and is independent of time i.e. the system has

reached a state of equilibrium with respect to time. The distribution of the number of arrivals per unit time and the distribution of service time do not change with time.

Example:

Prepare the simulation table for single channel queuing system for 10 customers using the inter-arrival time and service time as given table below determine the following,

i) Idle time of server
ii) Average waiting time
iii) Average time customer spend in system
iv) Average service time.

Inter-arrival time	-	5	10	1	2	3	8	5	8	4
Service time	5	8	5	6	5	5	8	3	3	5

Sol:

Here,

Inter-arrival time and service time is already given. So we go to make directly the simulation table:

Customer	Inter-arrival time (A)	arrival time (B)	Service time (C)	Time service begin (D)	Waiting time in queue (E = D−C)	Time service end (F = D+C)	Time spend in system (G = F−B)	Idle time of server (H)
1	-	0 (being first)	5	0	0	5	5	0
2	5 +	5	8	5	0	13	8	0
3	10	15	5	15	0	20	5	2
4	1	16	6	20	4	26	10	0
5	2	18	5	26	8	31	13	0
6	3	21	5	31	10	36	15	0
7	8	29	8	36	7	44	15	0
8	5	34	3	44	10	47	13	0
9	8	42	3	47	5	50	8	0
10	4	46	5 (5.3)	50	4	55	9	0

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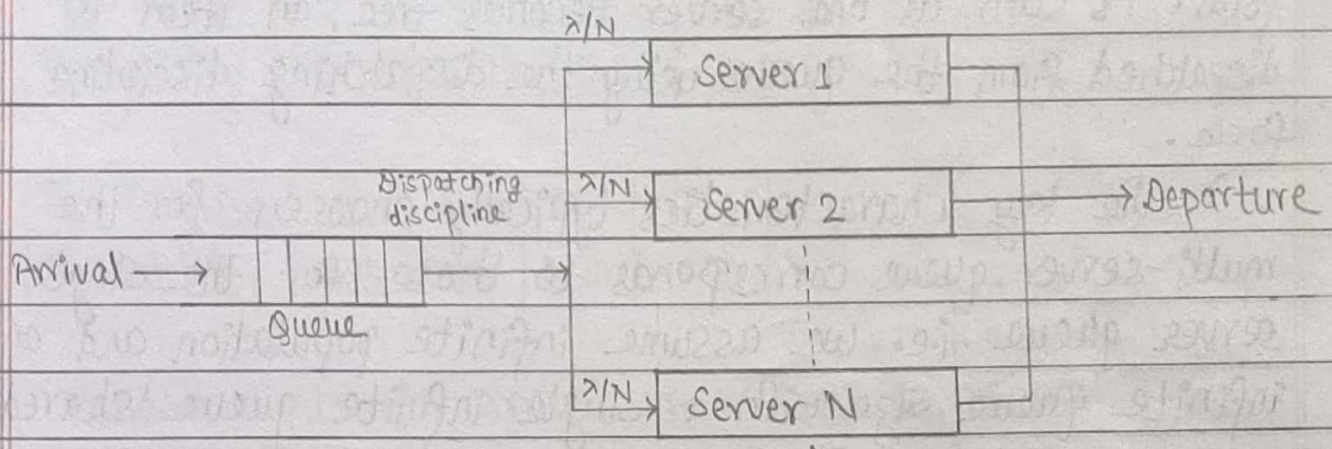
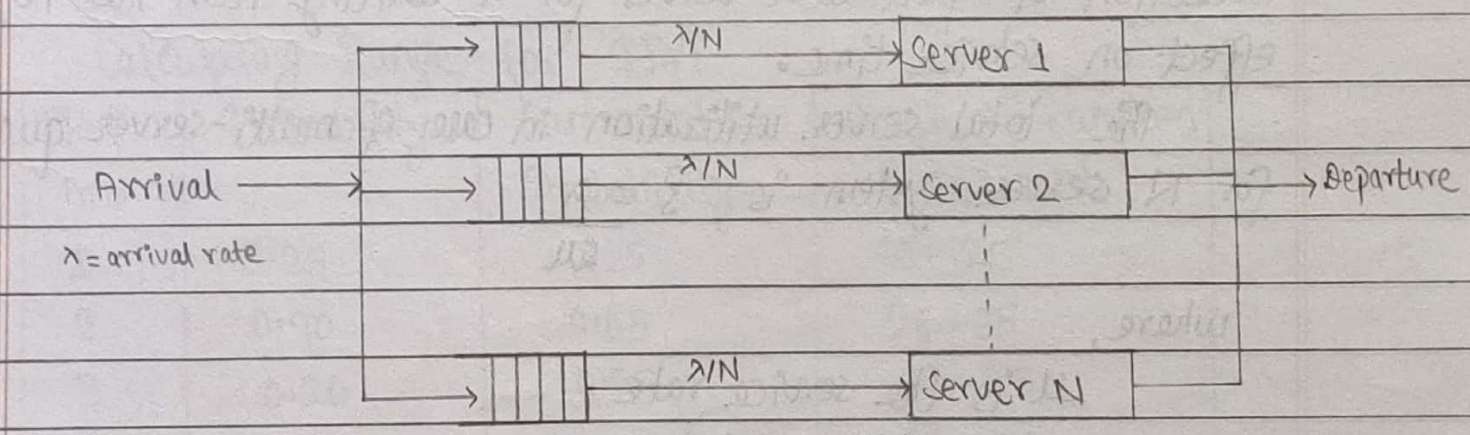
i) Idle time of server = 2
ii) Average waiting time = 4.8
iii) Average time customer spend in system = 10.3
iv) Average service time = 5.3

Multi-Server Queuing System:



Fig(a): Multiple server with single queue



Fig(b): Multiple server with multiple queue

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Figure (a) shows a generalization of simple model for multiple server all sharing a common queue. If an item arrives and at least one server is available then the item is immediately dispatching to that server. It is assumed that all servers are identical, thus if more than one server is available, it makes no difference which server is choosen for the item. If all server are busy, a queue begins to form. As soon as one server becomes free, an item is dispatched from the queue using the dispatching discipline in force.

The key characteristics typically choosen for the multi-server queue corresponds to those for the single server queue. i.e. we assume infinite population and an infinite queue size with a single infinite queue shared among all servers. The dispatching discipline is FIFO. For the multi-server case, if all servers are assumed identical, the selection of particular server for a waiting item has no effect on service time.

The total server utilization in case of multi-server queue for N server system is $\rho = \frac{\lambda}{c\mu}$

where,

μ is the service rate

λ is arrival rate

c is number of server

There is another concept which is called multiple server, multiple queue system as shown in figure (b).

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Example:

Car arrives to the server for service in a manner shown below:

IAT	1	2	3	4
Probability	0.25	0.40	0.20	0.15

There are two cars hops able and baker. Able is working faster than baker. Perform the simulation for six car and construct the simulation table distribution.

For ABLE	2	3	4	5
Probability	0.30	0.28	0.25	0.17

For BAKER	3	4	5	6
Probability	0.35	0.25	0.20	0.20

Random digits for arrival are 26, 98, 90, 26, 42.
Random digits for service are 95, 21, 51, 92, 89, 38.

Sol:

Here,

Calculating Range for IAT

IAT	Probability	Cummulative Probability	Range of Random Assignment
1	0.25	0.25	00 - 25
2	0.40	0.65	26 - 65
3	0.20	0.85	66 - 85
4	0.15	1.00	86 - 99

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Service distribution for ABLE:

Service time	Probability	Cummulative Probability	Range of Random Assignment
2	0.30	0.30	0-30
3	0.28	0.58	31-58
4	0.25	0.83	59-83
5	0.17	1.00	84-99

Service distribution for BAKER:

Service time	Probability	Cummulative Probability	Range of Random Assignment
3	0.35	0.35	0-35
4	0.25	0.60	36-60
5	0.20	0.80	61-80
6	0.20	1.00	81-99

Calculating Inter-arrival Time:

Cars	Random digits	IAT
1	-	-
2	26	2
3	98	4
4	90	4
5	26	2
6	42	2

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			ABLE			BAKER				
Cars	IAT	Arrival time	Time service begin	Service time	Time service end	Time service begin	service time	Time service end	Call delay	Time spend in system
1	-	0	0	5	5	-	-	-	0	5
2	2	2	-	-	-	2	3	5	0	3
3	4	6	6	3	9	-	-	-	0	3
4	4	10	10	5	15	-	-	-	0	5
5	2	12	-	-	-	12	6	18	0	6
6	2	14	15	3	18	-	-	-	1	4

stateless, memoryless, independent

Markov Chains and its Applications:

Markov Process:

Important classes of stochastic processes are Markov chains and Markov processes. If the future states of a process are independent of the past and depends only on the present then the process is called the Markov process. A discrete state of Markov process is called a Markov chain. Many queuing models are infact Markov processes.

Markov Chain:

A Markov chain, named after Andrey Markov is a mathematical system that undergoes transition from one state to another between a finite or countable number of possible states. It is a random process with the property that the next state depends only on

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the current state and not on the sequence of events that preceeded it. This specific kind of memoryless property is called Markov property. Markov chains have many applications in many real world processes.

Formally,

a Markov chain is a sequence of random variable $x_1, x_2, x_3, \ldots$ with the Markov property, namely given the present state the future state can be determined independent of past states i.e.

$$P(X_{n+1} = j \mid X_n = i_n, X_{n-1} = i_{n-1}, \ldots, X_1 = i_0)$$
$$= P(X_{n+1} = j \mid X_n = i_n)$$

The conditional probability above gives us the probability that a process in a state 'in' at time n moves to i(n+1) at time n+1. We call this transition probability for the Markov chain. If the transition probability does not depends on time n then we have a stationary Markov chain with transition probability,

$$P_{ij} = P(X_{n+1} = j \mid X_n = i)$$

Now,

We can write down the whole Markov chain as a matrix 'P' as:

$$P = \begin{bmatrix} P_{11} & P_{12} & P_{13} & \cdots & P_{1n} \\ P_{21} & P_{22} & P_{23} & \cdots & P_{2n} \\ \vdots & & & & \\ \vdots & & & & \\ P_{n1} & P_{n2} & P_{n3} & \cdots & P_{nn} \end{bmatrix}$$

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The probability of going from state i to state j in n time step is :

$$\left[P_{ij}^{n} = P(X_n = j \mid X_0 = i)\right]$$

And the single state transition is:

$$\left[P_{ij} = P(X_1 = j \mid X_0 = i)\right]$$

Example:

A simple weather model the probability of weather condition (model as either raining or sunny). Given the weather on the preceeding daty can be represented by a transiting matrix.

$$P = \begin{array}{c} \\ \text{sunny} \\ \text{rainy} \end{array} \begin{array}{c} \begin{array}{cc} \text{sunny} & \text{rainy} \end{array} \\ \begin{bmatrix} 0.9 & 0.1 \\ 0.5 & 0.5 \end{bmatrix} \end{array}$$

The matrix represent the weather model in which a sunny day is 90% likely to be followed by another sunny day and rainy day is 50% likely to be followed by another rainy day. The column can be level sunny and rainy respectively and rows can be level in the same order.

P_{ij} is the probability that if a given day is of type i, it will be followed by a day of type j. Notice that the rows of P sum to 1 this is because P is a stochastic matrix.

Let, weather on day zero is to be sunny this is represented by a vector in which the sunny entry is 100% and the rainy entry is 0% i.e. $X^{(0)} = [1, 0]$. The weather on day one can be predicted by $X^{(1)} = X^{(0)} P = [1\ 0] \begin{bmatrix} 0.9 & 0.1 \\ 0.5 & 0.5 \end{bmatrix} = [0.9\ \ 0.1]$.

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The weather on day two can be predicted in same way.

$$X^{(2)} = X^{(1)}P = [0.9 \quad 0.1] \begin{bmatrix} 0.9 & 0.1 \\ 0.5 & 0.5 \end{bmatrix} = [0.86 \quad 0.14]$$

OR

or

$$X^{(2)} = X^{(0)}P^2 = [1 \quad 0] \begin{bmatrix} 0.9 & 0.1 \\ 0.5 & 0.5 \end{bmatrix}^2 = [0.86 \quad 0.14]$$

Key features of Markov Chain:-

A sequence of trails of an experiment is Markov chain if;

i) The outcome of each experiment is one of the set descrete state.

ii) The outcome of an experiment depends only on the present state and not any past states.

iii) The transition probabilities remain constant from one transition to the next.

Application of Markov chain or process:

Markov chains are applied in number of ways to many different fields. Often they are used as mathematical model for some random physical processes if the parameter of the chain are known, quantitative prediction can be made. In other cases, they are used to model a more abstract process and are the theoretical understanding of an algorithm. The commonly used application areas are:

i) Physics:

Markovian system appear extensively in thermo-dynamics and statistical mechanics, whenever probabilities are used to

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Applications:-
→ Connection admission control
→ Bandwidth allocation
→ Routing
→ Queuing and scheduling.

represent unknown or unmodeled details of the system, If it can be assumed that the dynamics are time varients and that no history need to be considered which is not already included in the state description.

ii) Chemistry:

Chemistry is often a place where Markov chains and continuous time Markov processes are specially useful because this simple physical systems tends to satisfy the Markov property quite well. The classical model of enzymes activity can be viewed as a Markov chain where at each time step the reaction proceeds in some direction.

iii) Queuing Theory:

Markov chains are the basis for the analytical treatment of queues (Queuing Theory). Agner Kraurup Erlang initiated the subject in 1917. This makes them critical for optimizing the performance of telecommunication networks where messages must often complete for limited resources (such as bandwidth).

iv) Internet Application:

The page rank of a web-page as used by google is defined by a Markov chain. It is a probability to be at page 'i' in the stationary distribution on the following Markov chain in all web-pages.

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v) Statistics:

Markov chain methods have also become very important for generating sequence of random numbers to accurately reflect very complicated desired probability distribution through a process called Markov chain Monte Carlo (MCMC) and many more.

vi) Economic and Finance:

Markov chains are used in Finance and Economics to model a variety of different phenomena including assets price and market crashes.

vii) Social science:

Markov Chains are generally used in describing path dependent arguments where current structural configuration condition future outcomes. An example is the commonly argued link between economic development and rise of capitalism.

viii) Mathematical Biology:

Markov Chains also have many application in biological modeling, particularly population process which are useful in modeling process that are analogous to biological populations.

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Differential and Partial Differential Equations:

Continuous model:

When continuous system is modelled mathematically, the variables of model representing the attributes of system are controlled by continuous functions. The distributed lag model is an example of a continuous model. Since in continuous system, the relationship between variables describe the rate at which the value of variable changed, this system consists of differential equations.

Continuous system simulation uses the notation of differential equation to represent the change on the basic parameter of the system with respect to time. Hence, the mathematical model for continuous system simulation is usually represented by differential and partial differential equation.

Differential Equation:

A differential equation is a mathematical equations that relates some function with its derivatives. In many application the function usually represent physical quantities. The derivatives represent their rate of changes and equation defines a relationship between them. Because such relation are extremely common, differential equations play a prominent role in many disciplines including physical, economics and biology.

The simplest differential equation models have one or more linear differential equation with constant coefficients. It is often possible to solve the model without the use of simulation technique i.e. we can solve such equation using

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analytically. However, when non-linear equations are used as model it frequently becomes impossible or at least very difficult to solve the model.

An example of linear differential equation with constant coefficients to describe the wheel suspension system of an automobile can be given as:

$$M\ddot{x} + D\dot{x} + Kx = kF(t)$$

Here, the dependent variable x appears together with first and second derivatives, single dot and double dot respectively. The quantity $F(t)$ is input to the system depending upon the independent variable t.

Differential equations both linear and non-linear occur frequently in scientific and engineering studies. The reason for this is that most physical and chemical process involve rate of changes which requires differential equations to represent their mathematical descriptions.

Partial Differential Equations:

When more than one independent variable occurs in a differential equation, the equation is said to be a partial differential equation. It can involve the derivatives of same dependent variables with respect to each of the independent variable.

An example is an equation describing the flow of it in a three dimensional body. There are four independent variables representing the three dimensions and time and one dependent variable representing temperature. The general

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method of solving such equations numerically is to use finite differences to convert the equations into the set of ordinary differential equations which can then solve analytically.

Since partial differential equation can represent a growth rate, continuous model can also be applied to the problems of a social or economic nature.

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UNIT 3: RANDOM NUMBERS

Introduction:

Random numbers are numbers generated in an unpredictable haphazard sequence; numbers generated by chance alone. Random numbers are necessary basic ingredients in the simulation of almost all discrete systems. Most computer languages have a sub-routine, object or function that will generate a random number. Random numbers are used to generate event times and other random variables with simulation languages.

Properties of Random Numbers:

A sequence of random numbers $R_1, R_2, R_3, \ldots$ must have two important statistical properties:

- Uniformity
- Independence

Each random number R_i must be an independent sample drawn from a continuous uniform distribution between zero and one. Some sequences of uniformity and independence properties are the following:

i) If the interval $[0,1]$ is divided into n classes or sub-interval of equal length, the expected number of observation in each interval is $\frac{N}{n}$ where,

N = total no. of observation

n = number of intervals

ii) The probability of observing a value in a particular interval is independent of the previous value drawn.

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Random Number Table:

A table of number generated in an unpredictable, haphazard that are uniformly distributed within certain interval are called random number table. The random number in random number table exactly ~~obey~~ obey two random number properties, uniformity and independence so random number generated from random tables are also called true random numbers. Table of random numbers are used to create a random sample. A random number table is also called random sample table.

There are many physical devices or processes that can be used to generate a sequence of uniformly distributed random numbers i.e. true random numbers. For eg: an electrical pulse generator can be made to drive a counter cycling from 0 to 9. Using an electric noise generator or radio active source, the pulse can be generated as random numbers.

Generation of pseudo-random numbers:

Pseudo random number generator (PRNG) is an algorithm for generating a sequence of numbers whose properties approximate the properties of sequence of random numbers. The term pseudo means false so false random numbers are being generated. The goal of any generation scheme is to produce a sequence of numbers between zero and one which simulates the ideal properties of uniform distribution and independence as closely as possible.

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When generating pseudo random numbers certain problems can occur. Some of them are as follows:

i. The generated numbers may not be uniformly distributed.
ii. The generated numbers may be discrete valued instead of continuous valued.
iii. The mean of the generated numbers may be too high or too low.
iv. The varience of the generated numbers might be too high or too low.
v. There may be dependence such as:
   - auto-correlation between numbers
   - numbers successively higher or lower than adjacent number.
   - several numbers above the mean followed by several numbers below the mean.

Properties of good random number generators:

Usually random numbers are generated by a digital computer as a part of the simulation. Numerous methods can be used to generate the values. In selecting between these methods or routines, there are the number of important consideration:

i) The routine should be fast. Individual computation are expensive but simulation could require many millions of random numbers so the total cost can be managed by selecting a computationally efficient method of random number generation.

ii) The routine should be portable to different computers and ideally to different programming languages. This desirable show that the simulation program will produce same result whenever it is executed.

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iii) The routine should have a sufficiently long cycle. The cycle length or period represents the length of the random number sequence before previous number begins to repeat themselves in an earlier order. Thus if 10 thousand events are to be generated, the period should be many times longer than that. A special case of cycling is degenerating. A routine degenerates when the same random number appears repeatedly. Such an occurrence is unacceptable. This can rapidly happen with some methods.

iv) The random number should be replicable. Given the starting point, it should be possible to generate the same set of random numbers completely independent of the system that is being simulated. This is helpful for debugging purpose and is a means of facilitating comparison between systems.

v) The generated random number should closely approximate the ideal statistical property of uniformity and independence.

Techniques for Generation of Random Numbers :-

Random number generation is the generation of a sequence of number or symbols that cannot be reasonably predicted better than by a random chance, usually through a random number generator (RNG).

Random Number Generation technique is basically divided into two types:

1) Uniform Random Number Generator
2) Non-uniform Random Number Generator (Random variate generator).

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Uniform Random Number Generator:

Given a sequence $R_0, R_1, R_2, \ldots, R_n$ of random numbers then the generated numbers must be uniformly distributed in the interval (0,1). This is denoted by U(0,1). Linear congruential method is an example of uniform random number generator.

Imp: Linear Congruential Method:

- D.H Lehmer in 1951 proposed the linear congruential method as (generator) as a source of random numbers.
- The linear congruential produces a sequence of integers $x_1, x_2, x_3, \ldots$ between zero and m-1 according to the following recursive relationship:

$$x_{i+1} = (ax_i + c) \bmod m \quad , i = 0, 1, 2, \ldots$$

- The initial value x_0 is called as seed, a is called constant multiplier, c is called increment and m is the modulus.

Case 1:

⇒ If $c \neq 0$ in the above equation, the form is called the mixed congruential method.

Case 2:

⇒ If $c = 0$ in the above equation, the form is called multiplicative congruential method.

Notice that, random integer are generated rather than random number. These random integer should appear to be uniform distribution on the integer 0 to m-1. But our requirement is random numbers (R_i) in the range of (0-1). Hence, they are calculated as:

$$R_i = \frac{x_i}{m} \quad , i = 1, 2, 3, \ldots$$

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Example:

1) Use the linear congruential method to generate a sequence of five random numbers with $x_0 = 27$, $a = 17$, $c = 43$ & $m = 100$.

Sol:

Here, the integer values generated will be between 0 and 99 because of the value of the modulus (m). These random integers should appear to be uniformly distributed the integer between 0 and 99. Random numbers between 0 and 1 can then be generated by,

$$R_i = \frac{x_i}{m}, \quad i = 1, 2, 3 \ldots$$

The sequence of x_i and sub sequence R_i values are computed as follows:

$$x_0 = 27$$

$$x_1 = (17 * 27 + 43) \bmod 100$$

$$= 502 \bmod 100$$

$$= 2$$

$$R_i = \frac{x_1}{m} = \frac{2}{100} = 0.02$$

Again,

$$x_2 = (a x_1 + c) \bmod m$$

$$= (17 * 2 + 43) \bmod 100$$

$$= 77 \bmod 100$$

$$= 77$$

$$R_2 = \frac{x_2}{m} = \frac{77}{100} = 0.77$$

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Now,

$$x_3 = (ax_2 + c) \bmod m$$
$$= (17 * 77 + 43) \bmod 100$$
$$= 1352 \bmod 100$$
$$= 52$$
$$R_3 = \frac{52}{100} = 0.52$$

For 4th random number,

$$x_4 = (ax_3 + c) \bmod m$$
$$= (17 * 52 + 43) \bmod 100$$
$$= 927 \bmod 100$$
$$= 27$$
$$R_4 = \frac{27}{100} = 0.27$$

For 5th random number,

$$x_5 = (ax_4 + c) \bmod m$$
$$= 502 \bmod 100$$
$$= 2$$
$$R_5 = \frac{2}{100} = 0.02.$$

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2) Using linear congruential method, generate 3 random numbers with $a = 13$ and $m = 64$. Consider seed as 6.

Sol:

Given, $a = 13$, $m = 64$, $x_0 = 6$ & $c = 0$

Here,

$x_0 = 6$

$x_1 = (ax_0 + c) \bmod m$

$= (13 * 6 + 0) \bmod 64$

$= 78 \bmod 64$

$= 14$

$R_1 = \frac{x_1}{m} = \frac{14}{64} = 0.2187$

Now,

$x_2 = (13 * 14 + 0) \bmod 64$

$= 182 \bmod 64$

$= 54$

$R_2 = \frac{54}{64} = 0.84$

Again,

$x_3 = (13 * 54 + 0) \bmod 64$

$= 702 \bmod 64$

$= 62$

$R_3 = \frac{62}{64} = 0.96.$

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3) Using linear congruential method to generate a sequence of five random integers and its corresponding random numbers with $x_0 = 127$, $a = 18$, $c = 148$, $m = 277$.

Soln.

Here,

$x_0 = 127$

$x_1 = (18 * 127 + 148) \bmod 277$

$= 2434 \bmod 277$

$= 218$

$R_1 = \frac{218}{277} = 0.787$

For second random number,

$x_2 = (18 * 218 + 148) \bmod 277$

$= 4072 \bmod 277$

$= 194$

$R_2 = \frac{194}{277} = 0.7$

For 3rd random number,

$x_3 = (18 * 194 + 148) \bmod 277$

$= 3640 \bmod 277$

$= 39$

$R_3 = \frac{39}{277} = 0.14$

For 4th random number,

$x_4 = (18 * 39 + 148) \bmod 277$

$= 850 \bmod 277$

$= 19$

$R_4 = \frac{19}{277} = 0.068$

For 5th random number,

$x_5 = (18 * 19 + 148) \bmod 277$

$= 490 \bmod 277$

$= 213$

$R_5 = \frac{213}{277} = 0.7689.$

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Example: Use the multiplicative congruential generator to generate sequence of random integers and random number under the following circumstances.

$x_0 = 1,23,457$

$a = 7^5 = 16,807$

$m = 2^{31} - 1 = 2,14,74,83,647$

Sol^n:

Here,

$x_0 = 1,23,457$

$a = 16,807$

$m = 2,14,74,83,647$

Now,

$x_1 = (ax_0) \bmod m$

$= (16,807 * 123457) \bmod 2147483647$

$= 2074941799$

Then,

$R_1 = \frac{x_1}{m} = \frac{2074941799}{2147483647} = 0.9662$

For second random number,

$x_2 = (ax_1) \bmod m$

$= (16807 * 2074941799) \bmod 2147483647$

$= 559872160$

$R_2 = \frac{x_2}{m} = \frac{559872160}{2147483647} = 0.2607$

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For 3rd random number,

$x_3 = (a x_2) \bmod m$

$= (16807 * 5598721680) \bmod 2147483647$

$= 1645535613$

$R_3 = \frac{1645535613}{2147483647} = 0.7662$

Non-Uniform Random Number Generation :-
(Random Variate Generation)

In this method variables are modelled as random variables with some specified probability distribution. It means when probability distribution are known, it is possible to generate random variate. There are statistical procedure exist for estimating the parameter of the hypothesis distribution and testing the validity of the assumed statistical model.

There are different technique for generating random variates :

i) Inverse Transform Technique (ITT)

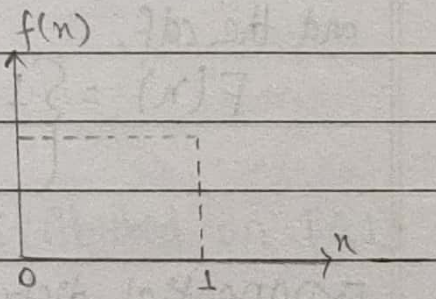
ii) Acception-Rejection Technique (ART)

All the technique in this chapter assume that a solution of uniform [0,1] random numbers exist.

Random numbers $R_1, R_2, R_3 \ldots$ with

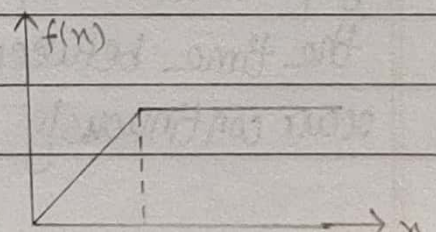
* probability density function (pdf):

$$f_R(x) = \begin{cases} 1 & , 0 \le x \le 1 \\ 0 & , \text{otherwise} \end{cases}$$



* cumulative distribution function (cdf):

$$F_R(x) = \begin{cases} 0 & , x < 0 \\ x & , 0 \le x \le 1 \\ 1 & , x > 1 \end{cases}$$



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1. Inverse Transform Technique:

The inverse transform technique can be used to sample from the exponential, uniform, Weibull, triangular and empirical distributions. Let x be a random variable, F be cdf of x and f be pdf of x.

- Let R be a random number with cdf function

$$R = F(x)$$

- Let R is a random number uniformly distributed on $(0,1)$. Find x:

$$x = F^{-1}(R)$$

- The inverse transform technique can be used in principle of any distribution.

Following are the steps that are used in inverse-transform technique (ITT):

i, Compute the cdf of the desired random variable x i.e. $F(x) = x$.

ii, Set $F(x) = R$ on the range of x.

iii, Solve the equation $F(x) = R$ for x in terms of R. i.e. $x = F^{-1}(R)$.

iv, Generate uniform random number $R_1, R_2, \ldots$ and compute the desired random variate by $x_i = F^{-1}(R_i)$.

Inverse Transform Technique for Exponential Distribution:

The exponential distribution has pdf: $f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$

and the cdf,

$$F(x) = \begin{cases} 1 - e^{-\lambda x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

- Exponential distribution is the probability distribution that describes the time between events in process i.e. a process in which events occur continuously and independently at a constant average rate.

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- The goal is to develop a procedure for generating values $x_1, x_2, x_3, \ldots$ that have an exponential distribution.
- The step by step procedure for the ITT illustrated by exponential distribution is given below:

Step 1: Compute the cdf of desired random variable x.

i.e. $F(x) = 1 - e^{-\lambda x}$, $x \geq 0$

Step 2: Set $F(x) = R$ on the range of x

i.e. $1 - e^{-\lambda x} = R$ on the range $x \geq 0$.

Step 3: Solve the equation $F(x) = R$ for x in term of R

i.e. $F(x) = R$

$$\Rightarrow 1 - e^{-\lambda x} = R$$

$$\Rightarrow e^{-\lambda x} = 1 - R$$

$$\Rightarrow \ln(e^{-\lambda x}) = \ln(1-R)$$

$$\Rightarrow -\lambda x = \ln(1-R)$$

$$\Rightarrow x = \frac{-1}{\lambda} \ln(1-R)$$

This can also be written as $x = F^{-1}(R)$.

Step 4: Generate uniform random number $R_1, R_2, R_3, \ldots$ and compute the desired random variates by:

$$x_i = F^{-1}(R_i)$$

$$x_i = \frac{-1}{\lambda} \ln(1-R_i)$$

Since, R_i and $(1-R_i)$ are uniformly distributed on $[0,1]$ so the formula becomes,

$$x_i = \frac{-1}{\lambda} \ln(R_i).$$

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Example:

Give the exponential random variate x_i with mean $\lambda=1$, then generate the values of exponential variates x_i with mean 1, given random numbers R_i are follows:

i	1	2	3	4	5
R_i	0.1306	0.0422	0.6597	0.7965	0.7696

Sol:

Given random numbers, the random variate x_i values can be generated by using the formula $x_i = F^{-1}(R_i)$

$$= \frac{-1}{\lambda} \ln(1-R_i)$$

$$x_1 = \frac{-1}{\lambda} \ln(1-0.1306)$$

$$= \frac{-1}{\lambda} \ln(0.8694)$$

$$= -1(-0.1399)$$

$$= 0.1399$$

$$x_2 = \frac{-1}{\lambda} \ln(1-0.0422)$$

$$= -\frac{1}{1} \ln(0.9578)$$

$$= -1(-0.0431)$$

$$= 0.0431$$

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$$x_3 = \frac{-1}{\lambda} \ln(1-0.6597)$$

$$= \frac{-1}{1} \ln(0.3403)$$

$$= -1(-1.07792)$$

$$= 1.0779$$

$$x_4 = \frac{-1}{\lambda} \ln(1-0.7965)$$

$$= \frac{-1}{1} \ln(0.2035)$$

$$= -1(-1.5920)$$

$$= 1.5920$$

$$x_5 = \frac{-1}{\lambda} \ln(1-0.7696)$$

$$= \frac{-1}{1} \ln(0.2304)$$

$$= -1(-1.4679)$$

$$= 1.4679$$

i	1	2	3	4	5
R_i	0.1306	0.0422	0.6597	0.7965	0.7696
x_i	0.1399	0.0431	1.0779	1.5920	1.4679

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ITT for Uniform distribution:

The random variable x having uniform distribution in the interval $[a,b]$ has pdf:

$$f(x) = \begin{cases} \frac{1}{b-a}, & \text{if } a \le x \le b \\ 0, & \text{otherwise} \end{cases}$$

and cdf is:

$$F(x) = \begin{cases} 0, & \text{if } x < a \\ \frac{x-a}{b-a}, & \text{if } a \le x \le b \\ 1, & \text{if } x > b. \end{cases}$$

To apply ITT to generate random variable which follows uniform distribution

Step 1: Compute the cdf of desired random variable x

i.e. $F(x) = \frac{x-a}{b-a}$

Step 2: Set $F(x) = R$

i.e. $\frac{x-a}{b-a} = R$

Step 3: Solve the equation $F(x) = R$ for x in term of R.

i.e. $x - a = R(b-a)$

$\Rightarrow x = R(b-a) + a$

Step 4: Compute x_i using the formula

$x_i = R_i(b-a) + a$

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Example:

Generate random variates that follows uniform distribution in the interval [3,5]. Given the random numbers are: 0.779, 0.921, 0.186, 0.284, 0.653.

Sol:

Uniform distribution is

For the given set of random numbers, the random variates are generated using the formula: $x_i = R_i(b-a)+a$

$x_1 = R_1(5-3)+3$
$= 0.779 * 2 + 3$
$= 4.558$

$x_2 = R_2(5-3)+3$
$= 0.921 * 2 + 3$
$= 4.842$

$x_3 = R_3(5-3)+3$
$= 0.186 * 2 + 3$
$= 3.372$

$x_4 = R_4(5-3)+3$
$= 0.284 * 2 + 3$
$= 3.568$

$x_5 = R_5(5-3)+3$
$= 0.653 * 2 + 3$
$= 4.306$

i	1	2	3	4	5
R_i	0.779	0.921	0.186	0.284	0.653
x_i	4.558	4.842	3.372	3.568	4.306

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Acceptance-Rejection Technique:

It is another random variate generation technique. It is useful when we need to sample from a distribution whose inverse function is difficult to solve.

Suppose that an analyst need to devise a method for generating random variates x, uniformly distributed between $1/4$ and 1. One way to proceed would be to follow these steps:

Step 1 : Generate random number R.

Step 2 : If $R \geq 1/4$, accept $x = R$, then go to step 3.
If $R < 1/4$, reject R and return to step 1.

Step 3 : If another uniform random variate on $[1/4, 1]$ is needed, repeat the procedure beginning at step 1, if not stop.

- Each time step 1 is executed, a new random number must be generated.
- Step 2 is an acceptance or rejection technique.
- The efficiency of an acceptance-rejection technique depends heavily on being able to minimize the number of rejection.

Acceptance-Rejection Technique for Poisson distribution:

- In Probability theory and statistics, the Poisson distribution name after French mathematician Simeon Denis Poisson.
- A poisson random variable N with mean $\alpha > 0$ has the probability mass function (pmf),

$$P(n) = P(N=n) = \frac{e^{-\alpha} \cdot \alpha^n}{n!}, \quad n = 0, 1, 2, \ldots$$

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Here, N can also be thought of number of arrivals from Poisson arrival process in one unit of time.

The procedure for generating a Poisson random variate N is given by the following steps:

Step 1: Set $n=0$, $P=1$

Step 2: Generate random number R_{n+1} & replace P by $P \cdot R_{n+1}$
i.e. $P = P \cdot R_{n+1}$

Step 3: If $P < e^{-\alpha}$ then accept $N=n$. Otherwise, reject the current value n and increase n by 1 and return to step 2.

Example:

Using acceptance and rejection technique, generate 3 Poisson variates with mean $\alpha = 0.2$ with given random numbers are 0.4357, 0.4146, 0.8353, 0.9952, 0.8004

Sol:

Given,

$\alpha = 0.2$, the procedure is as follows:

Step 1: set $n=0$, $p=1$.

Step 2: $R_{n+1} = R_1 = 0.4357$.

and, $P = P \cdot R_{n+1} = 1 * 0.4357 = 0.4357$

Step 3: Check whether $P < e^{-\alpha}$

$0.4357 < e^{-0.2}$

$0.4357 < 0.8187$ (i.e. True)

So, we accept $N=n$.

Since, we require 3 random variates, we restart with step1 again.

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Step 1 : set $n=0$, $P=1$

Step 2 : $R_{n+1} = R_1 = 0.4146$

and, $P = P \cdot R_{n+1} = 1 * 0.4146 = 0.4146$

Step 3 : Check whether $P < e^{-\alpha}$

$0.4146 < e^{-0.2}$

$0.4146 < 0.8187$ (i.e. True)

So, we accept $N = n$

For 3rd random variate,

Step 1 : set $n=0$, $P=1$

Step 2 : $R_{n+1} = R_1 = 0.8353$

and, $P = P \cdot R_{n+1} = 1 * 0.8353 = 0.8353$

Step 3 : Check whether $P < e^{-\alpha}$

$0.8353 < 0.8187$ (i.e. False)

Hence, $N = 0$ is rejected.

Now,

$n = n+1$ and go to step 2.

$R_{n+1} = R_2 = 0.9953$

$P = P \cdot R_{n+1} = (0.8353) \cdot (0.9952)$

$= 0.8313$

Step 3 : Check whether $P < e^{-0.2}$

$0.8313 < 0.8187$ which is false

Hence, $N = 1$ is rejected.

Again,

$n = n+1 = 1+1 = 2$ and go to step 2.

Step 2 : $R_{n+1} = R_3 = 0.8004$

$P = P \cdot R_{n+1} = 0.8313 * 0.8004$

$= 0.6653$

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Step 3 : Check whether $P < e^{-\lambda}$.

i.e. $0.6653 < e^{-0.2}$

$0.6653 < 0.8187$ (i.e. True)

So, we accept $N = n$

i.e. $N = 2$ is accepted.

Thus, the required variates are 0, 0 and 2.

Calculation of above problem can be summarized as the following table:

n	R_{n+1}	P	Accept/Reject ($P < e^{-\lambda}$)	Result
0	0.4357	0.4357	accept	$N = 0$
0	0.4146	0.4146	accept	$N = 0$
0	0.8353	0.8353	reject	-
1	0.9952	0.8313	reject	-
2	0.8004	0.6653	accept	$N = 2$.

Testing for Randomness:-

The desirable properties of random numbers are uniformity and independence. To ensure that these desirable properties are achieved, a number of test can be performed. The test can be placed into two categories according to the properties of interest. The first one is testing for uniformity and second one is testing for independence.

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1. Testing for uniformity:-

The testing for uniformity can be achieved through different frequency test. These test use the Kolmogorov-Smirnov test and the Chi-Square test to compare the distribution of the set of numbers generated to the uniform distribution. Hence in this category, we will discuss two types of test.

a) Kolmogorov-Smirnov test
b) Chi-Square test.

2. Testing for independence:-

Test for independence includes three types of test as given below:

a) Auto-correlation test: Test the correlation between numbers and compares the sample correlation to the expected correlation of zero.

b) Gap test: Counts the number of digits that appear between repetition of particular digit and then uses the Kolmogorov-Smirnov test to compare with the expected size of gap.

c) Poker test: Treats numbers group together as a Poker hand. then the hands obtained are compared to what is expected using Chi-square test.

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Testing hypothesis for Randomness:

In testing for uniformity, the hypothesis are as follows:

$H_0 : R_i \sim U[0,1]$

$H_1 : R_i \nsim U[0,1]$

The null hypothesis H_0 reads that the numbers are distributed uniformally on the interval $[0,1]$. Failure to reject the null hypothesis means that no evidence of non-uniformity has been detected on the basis of this test. This does not imply that further testing of the generator for uniformity is unnecessary.

In testing for independence, the hypothesis are as follows:

$H_0 : R_i \sim$ independently

$H_1 : R_i \nsim$ independently

The null hypothesis H_0 reads the number are independent. Failure to reject the null hypothesis means that evidence of dependence has not been detected by this test. This doesn't imply that further testing of generator for independence is unnecessary.

For each test, a level of significance 'α' must be stated. The level 'α' is the probability of rejecting the null hypothesis given that the null hypothesis is true.

i.e. $\alpha = P$ (reject H_0, H_0 is true)

The decision maker set the value of 'α' for any test. Frequently α is set to 0.01 or 0.05.

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Kolmogorov Smirnov (K-S) Test:

- This test compares the continuous cdf, $F(x)$ of the uniform distribution with the empirical cdf, $S_N(x)$ of the sample of N observations.

By definition,

$$F(x) = x, \quad 0 \le x \le 1$$

- If the sample from the random-number generator is $R_1, R_2, \ldots, R_N$ then the empirical cdf, $S_N(x)$ is defined as:

$$S_N(x) = \frac{\text{number of } R_1, R_2, \ldots, R_N \text{ which are less than } x}{N}$$

- As N becomes larger $S_N(x)$ should become a better approximation to $F(x)$, provided that the null hypothesis is true.
- The K-S test is based on the largest absolute deviation between $F(x)$ and $S_N(x)$ over the range of the random variable that is based on the statistics.

$$D = \text{Max} \, |F(x) - S_N(x)|.$$

- The sampling distribution of D is known; it is tabulated as a function of N.
- For testing against a uniform cdf, the test procedure follows these steps:

Step 1: Rank the data from smallest to largest. Let R_i denote the ith smallest observation so that,

$$R_1 \le R_2 \le \ldots \le R_N$$

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Step 2: Compute:

$$D^+ = \max_{1 \le i \le N} \left\{ \frac{i}{N} - R_i \right\}$$

$$D^- = \max_{1 \le i \le N} \left\{ R_i - \frac{i-1}{N} \right\}$$

Step 3: Compute:

$$D = \max \{ D^+, D^- \}$$

Step 4: Determine the critical value D_α from the table for a specified α and for given sample size N.

Step 5: If the sample statistics D is greater than the critical value D_α, the null hypothesis that the data are sample from the uniform distribution is rejected. If $D < D_\alpha$, then we conclude that no difference has been detected between the true distribution of $R_1, R_2, \ldots R_N$ and the uniform distribution.

Example:

Suppose that 5 random numbers 0.44, 0.81, 0.14, 0.05 and 0.93 were generated and it is desired to perform test for uniformity using the K-S test with level of significance (α) 0.05.

Sol:

Step 1: Rank the numbers from smallest to largest i.e. 0.05, 0.14, 0.44, 0.81, 0.93

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Step 2: Compute D^+ and D^-.

i	R_i	i/N	$i/N - R_i\ (D^+)$	$R_i - \frac{i-1}{N}\ (D^-)$
1	0.05	$1/5 = 0.2$	0.15	0.05
2	0.14	$2/5 = 0.4$	0.26	–
3	0.44	0.6	0.16	0.04
4	0.81	0.8	–	0.21
5	0.93	1	0.07	0.13

$$D^+ = \max_{1 \le i \le N} \left\{ \frac{i}{N} - R_i \right\} = 0.26$$

$$D^- = \max_{1 \le i \le N} \left\{ R_i - \frac{i-1}{N} \right\} = 0.21$$

Step 3: Compute:

$$D = \max\{D^+, D^-\} = \max\{0.26, 0.21\} = 0.26$$

Step 4: The critical value of D_α obtained from the table for $\alpha = 0.05$ and $N = 5$ i.e. $(D_\alpha, N) = 0.565$

Step 5: Since the computed value $D = 0.26$ is less than the tabulated value $(D_\alpha, N) = 0.565$, the hypothesis of no difference between the distribution of the generated numbers and the uniform distribution is accepted.

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Chi-Square Test :

The Chi-square test uses the sample statistics,

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

where,

O_i is the observed number in the ith class.
E_i is the expected number in the ith class.
n is the number of classes.

- For the uniform distribution E_i, the expected number in each class is given by,

$$E_i = \frac{N}{n}$$

- For equally spaced classes, N is the total number of observations.
- It can be shown that χ_0^2 follow Chi-square distribution with n-1 degree of freedom. If χ_0^2 (computed) $\leq$ table value of χ for a given α then null hypothesis is accepted.

Example:

1) If the Chi-square test with $\alpha = 0.05$ to test for whether the data shown next are uniformly distributed.

0.34	0.90	0.25	0.89	0.87	0.44	0.12	0.21	0.46	0.67
0.83	0.76	0.79	0.64	0.70	0.81	0.94	0.74	0.22	0.74
0.46	0.99	0.77	0.67	0.54	0.41	0.52	0.73	0.99	0.02
0.47	0.30	0.17	0.82	0.56	0.05	0.45	0.31	0.78	0.05
0.79	0.71	0.26	0.19	0.82	0.93	0.65	0.37	0.39	0.42
0.99	0.17	0.99	0.46	0.05	0.66	0.10	0.42	0.18	0.49

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0.37	0.51	0.54	0.01	0.81	0.28	0.69	0.34	0.75	0.49
0.72	0.43	0.56	0.97	0.30	0.94	0.96	0.58	0.73	0.05
0.06	0.39	0.84	0.24	0.40	0.64	0.40	0.19	0.79	0.62
0.18	0.26	0.97	0.88	0.64	0.47	0.60	0.11	0.29	0.78

Soln:

There are 100 number in a given list in the interval [0,1] i.e. $N = 100$. Let us take $n = 10$ intervals of equal length like $[0, 0.1)$, $[0.1, 0.2)$, $[0.2, 0.3)$ ..... $[0.9, 1.0)$.
Now we will calculate the test statistics χ^2 using the formula,

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

$$= \sum_{i=1}^{10} \frac{(O_i - E_i)^2}{E_i}$$

where,

$$E_i = \frac{100}{10} = 10 \text{ i.e. } \frac{N}{n}$$

O_i indicates total number of elements actually observed in the interval i i.e. O_1 indicates no. of elements in the range $[0, 0.1)$, O_2 indicates no. of elements in the range $[0.1, 0.2)$ .... and so on.

Now, The table for Chi-square statistic is:

Interval (i)	O_i	E_i	$O_i - E_i$	$(O_i - E_i)^2$	$(O_i - E_i)^2 / E_i$
1	8	10	-2	4	0.4
2	8	10	-2	4	0.4
3	10	10	0	0	0
4	9	10	-1	1	0.1
5	12	10	2	4	0.4
6	8	10	-2	4	0.4
7	10	10	0	0	0
8	14	10	4	16	0.16
9	10	10	0	0	0
10	11	10	1	1	0.1
	100				$\chi^2_{computed} = 1.96$

From the above table, the essential computation for Chi-square test is shown:

Here, the degree of freedom $= (n-1) = (10-1) = 9$

$\alpha = 0.05$, then tabulated value is,

$$\chi^2_{0.05,9} = 16.9$$

Since χ^2 is much less than tabulated value of χ^2, then null hypothesis of uniform distribution is not rejected.

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2) The following is the set of the single-digit numbers from a random number generator.

6	7	0	6	9	9	0	6	4	6
4	0	8	2	6	6	1	2	6	8
5	6	0	4	7	1	3	5	0	7
1	4	9	8	6	0	9	6	6	7
1	0	4	7	9	2	0	1	4	8
6	9	7	7	5	4	2	3	3	3
6	0	5	8	2	5	8	8	3	1
4	0	8	1	7	0	0	6	2	8
5	6	0	8	0	6	9	7	0	0
3	1	5	4	3	8	3	3	2	4

Using appropriate test, check whether the numbers are uniformly distributed.

Soln:

The test uses $n = 10$ intervals of equal length, namely $[0,1), [1,2), [2,3), [3,4), [4,5), [5,6), [6,7), [7,8), [8,9), [9,10)$.

level of significance $(\alpha) = 0.05$

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The test for Chi-square statistic is:

Class-interval(i)	O_i	E_i	$O_i - E_i$	$(O_i - E_i)^2$	$(O_i - E_i)^2 / E_i$
1	16	10	6	36	3.6
2	8	10	-2	4	0.4
3	7	10	-3	9	0.9
4	9	10	-1	1	0.1
5	10	10	0	0	0
6	7	10	-3	9	0.9
7	16	10	6	36	3.6
8	9	10	-1	1	0.1
9	11	10	1	1	0.1
10	7	10	-3	9	0.9
					10.6

Then,

$$\chi_0^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

$$= 10.6$$

Tabulated value, $\chi^2_{\alpha, n-1} = \chi^2_{0.05, 9} = 16.9$

Since, $\chi_0^2 = 10.6 < \chi^2_{\alpha, n-1} = 16.9$

Hence, null hypothesis of a uniform distribution is not rejected.

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Test for auto correlation:-

- Auto correlation test is a statistical test that determine whether a random number generator is independent random number in a sequence.
- The test for the auto correlation is concerned with the dependence between number in a sequence.
- The test compute the auto correlation between every 'm' numbers (m is known as lag) starting in index.

The variables involved in this test are:

$m \rightarrow$ is the lag, the space between the number being tested.

$i \rightarrow$ is the index from which we start

$N \rightarrow$ is the number of random number generated.

$M \rightarrow$ is the largest integer such that $i+(M+1)m \leq N$.

- Now the autocorrelation estimator ρ_{im} between R_i, R_{i+m}, R_{i+2m}, ----, $R_{i+(M+1)m}$ and its standard deviation is computed as follows:

$$\hat{\rho}_{im} = \frac{1}{M+1}\left[\sum_{k=0}^{M} R_{i+km} \cdot R_{i+(k+1)m}\right] - 0.25$$

and,

$$\sigma_{\hat{\rho}_{im}} = \frac{\sqrt{13M+7}}{12(M+1)}$$

Then the test statistic can be formed as follows:

$$Z_0 = \frac{\hat{\rho}_{im}}{\sigma_{\hat{\rho}_{im}}}.$$

- After computing Z_0, do not reject the null hypothesis of independence if $-Z_{\alpha/2} \leq Z_0 \leq Z_{\alpha/2}$.

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Example:

Test for whether the 3rd, 8th, 13th and so on numbers in the sequence with $\alpha = 0.05$ are independent.

0.12	0.01	0.23	0.28	0.89	0.31	0.64	0.26	0.83	0.93
0.99	0.15	0.33	0.35	0.91	0.41	0.60	0.27	0.75	0.88
0.68	0.49	0.05	0.43	0.95	0.58	0.19	0.36	0.69	0.87.

Sol:

Here, we have to start with the third number. Hence,

$i = 3$ (beginning with the third number)

$m = 5$ (gaps between numbers)

$N = 30$ (total numbers)

Now,

we calculate value of M using the condition;

$$i + (M+1)m \leq N$$

or, $3 + (M+1)5 \leq 30$

or, $3 + 5M + 5 \leq 30$

or, $5M \leq 22$

or, $M = 22/5$

$\simeq 4$

$\boxed{\therefore M = 4}$ (M is the largest integer)

Then,

$$\hat{\rho}_{im} = \frac{1}{M+1}\left[\sum_{k=0}^{M} R_{i+km} \cdot R_{i+(k+1)m}\right] - 0.25$$

$$\hat{\rho}_{3,5} = \frac{1}{4+1}\left[R_3 * R_8 + R_8 * R_{13} + R_{13} * R_{18} + R_{18} * R_{23} + R_{23} * R_{28}\right] - 0.25$$

$$= \frac{1}{5}\left[(0.23 * 0.26) + (0.26 * 0.33) + (0.33 * 0.27) + (0.27 * 0.05) + (0.05 * 0.36)\right] - 0.25$$

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$$= \frac{1}{5}\left[0.0598 + 0.0858 + 0.0891 + 0.0135 + 0.018\right] - 0.25$$

$$= \frac{1}{5}\left[0.2662\right] - 0.25$$

$$= 0.05324 - 0.25$$

$$\hat{\rho}_{3.5} = -0.1945$$

And,

$$\sigma_{\hat{\rho}_{im}} = \frac{\sqrt{13M+7}}{12(M+1)} = \frac{\sqrt{13*4+7}}{12(4+1)} = \frac{\sqrt{59}}{60} = 0.1280$$

Then the test statistics become,

$$Z_0 = \frac{\hat{\rho}_{im}}{\sigma_{\hat{\rho}_{im}}}$$

$$= \frac{-0.1945}{0.1280}$$

$$= -1.5195$$

Now, the critical value is $Z_{\alpha/2} = Z_{0.025} = 1.96$ [from table]

Since, $-Z_{\alpha/2} \leq Z_0 \leq Z_{\alpha/2}$ i.e. $-1.96 \leq -1.5195 \leq 1.96$

$\therefore$ The hypothesis of independence cannot be rejected on the basis of this test.

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to check independence of random number

Gap Test:

- The gap test is used to determine the significance of the interval between the recurrence of the same digit.
- A gap of length x occurs between the recurrence of same digit.
- If we are only concerned with digits between 0-9 then the probability of a particular gap length can be determined by;

$$P(\text{gap of } x) = 0.9^x * 0.1 \quad \text{where } x = 0, 1, 2, 3, \ldots$$

- Then the theoretical frequency distribution for randomly ordered digit is given by,

$$P(\text{gap} \le x) = F(x) = 0.1 \sum_{n=0}^{x} (0.9)^x = 1 - 0.9^{x+1}$$

Gap Test Algorithm:

Step 1: Define the hypothesis testing for independence as

H_0 : $R_i \sim$ independently

H_1 : $R_i \not\sim$ independently.

Step 2: Determine the number of gaps and length of each gap associated with each digit (0, 1, 2, 3, 4, 5, 6, 7, 8, 9)

Step 3: Select the interval width based on no. of gaps and generate the frequency distribution table for sample of gaps and apply K-S test.

Gap length	Frequency	Relative frequency	Cumulative frequency $S_N(x)$	Theoretical frequency $(F(x))$	$\lvert F(x) - S_N(x) \rvert$

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Step 4 : Compute test statistic D which is maximum of
$$D = \text{Max.} \{ | F(x) - S_N(x) \}$$

Step 5 : Determine the critical value, D_α for sample of size N for α from table.

Step 6 : If $D < D_\alpha$ then H_0 is accepted.

Example :

Analyze the 110 digit below to test whether they are independent, use $\alpha = 0.05$.

4, 1, 3, 5, 1, 7, 2, 8, 2, 0, 7, 9, 1, 3, 5, 2, 7, 9, 4, 1, 6, 3, 3, 9, 6,
3, 4, 8, 2, 3, 1, 9, 4, 4, 6, 8, 4, 1, 3, 8, 9, 5, 5, 7, 3, 9, 5, 9, 8,
5, 3, 2, 2, 3, 7, 4, 7, 0, 3, 6, 3, 5, 9, 9, 5, 5, 5, 0, 4, 6, 8,
0, 4, 7, 0, 3, 3, 0, 9, 5, 7, 9, 5, 1, 6, 6, 3, 8, 8, 8, 9, 2, 9, 1, 8,
5, 4, 4, 5, 0, 2, 3, 9, 7, 1, 2, 0, 3, 6, 3.

Sol^n :

The number of gaps is given by the number of data values minus the number of distinct digits i.e. 110 − 10 = 100.
Then the number of gaps associated with various digits are tabulated below:

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Digits	Length of each gaps	No. of gaps
0	47, 9, 3, 2, 2, 21, 6	7
1	2, 7, 6, 10, 6, 45, 9, 10	8
2	1, 6, 12, 22, 0, 38, 8, 4	8
3	10, 7, 0, 2, 3, 8, 5, 5, 3, 4, 1, 14, 0, 9, 14, 5, 1	17
4	17, 7, 5, 0, 2, 18, 12, 3, 23, 0	10
5	10, 26, 0, 3, 2, 11, 2, 0, 0, 12, 2, 12, 2	13
6	3, 9, 24, 9, 15, 0, 22	7
7	4, 5, 26, 10, 1, 16, 6, 22	8
8	19, 7, 3, 8, 21, 16, 0, 0, 4	9
9	5, 5, 7, 8, 4, 1, 14, 0, 15, 2, 8, 1, 9.	13

- There are 18 threes in the list thus 17 gaps can occur
- Similarly, the gap associated with other digits can be calculated.
- Probability of other gap can be calculated, the calculation for gap test is shown in following table:

$1-(0.9)^{17+1} = 1-0.9^{3+1}$
$= 1-0.9^4$
$= 0.3439$

Gap length	Frequency	Relative frequency	Cumulative frequency $S_N(x)$	Theoretical frequency $F(x)$	$\lvert F(x) - S_N(x) \rvert$
0-3	35	0.35	0.35	0.3439	0.0061
4-7	22	0.22	0.57	0.5695	0.0005
8-11	17	0.17	0.74	0.7175	0.0225
12-15	9	0.09	0.83	0.8146	0.0154
16-19	5	0.05	0.88	0.8784	0.0016
20-23	6	0.06	0.94	0.9202	0.0198
24-27	3	0.03	0.97	0.9476	0.0224
28-31	0	0.00	0.97	0.9656	0.0044
32-35	0	0.00	0.97	0.9774	0.0074
36-39	1	0.01	0.98	0.9852	0.0052
40-43	0	0.00	0.98	0.9903	0.0103
44-47	2	0.02	1.00	0.9936	0.0064

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Note:- For value of $N > 35$, critical value is given by $\frac{1.36}{\sqrt{N}}$

Theoretical frequency for interval (0-3) is calculated as:

$$P(gap \le 3) = 1 - 0.9^{3+1}$$
$$= 0.3439$$

and other so on....

The critical value for D is given by $D_{0.05} = \frac{1.36}{\sqrt{100}} = 0.136$

Since $D = Max\,|F(x) - S_N(x)|$

$= 0.0224 < D_{0.05}$ i.e. $0.0224 < 0.136$.

So, we do not reject hypothesis of independence on the basis of this test i.e. accepted.

Poker Test:

- The poker test for independence is based on the frequency in which certain digits are repeated in a series of numbers.
- It basically consider three digits numbers.
- Random numbers are rounded upto 3 digits and classified as:
  a) All digits different
  b) All digits identical
  c) Exactly one pair of identical digit.

- The probability associated with each of these possibilities is given by the following:

a) P (three different digits) = P (second digit different from first) * P (third digit different from first and second)

$$= \frac{9}{10} \times \frac{8}{10} = 0.72$$

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b) P (three identical digits) = P (second digit same as first) × P (third digit same as first)

$$= \frac{1}{10} \times \frac{1}{10} = 0.01$$

c) P (exactly one pair) = 1 − 0.72 − 0.01
= 0.27.

- Observed value compared with expected value using chi-square test.

Poker Test Algorithm:

Step 1: Define hypothesis testing for independence as
$H_0 : R_i \sim$ independently
$H_1 : R_i \not\sim$ independently

Step 2: Generate frequency distribution table for 3 combinations and apply chi-square test.

Step 3: Compute test statistic as:

$$\chi_0^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

Step 4: Determine the critical value at α and with degree of freedom $(n-1)$.

Step 5: $\chi_0^2 > \chi^2_{\alpha,(n-1)}$, then H_0 is rejected.

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Example:

A sequence of 1000 three digits number has been generated and analysis indicate that 680 have three different digits, 289 contain exactly one pair of like digits and 31 contains like digit. Based on the poker test, are these numbers independent? Let $\alpha = 0.05$.

Sol:

All the necessary computation for Poker test is summarized in the following table:

Combination (i)	Observed frequency (O_i)	Expected frequency (E_i)	$\frac{(O_i - E_i)^2}{E_i}$
Three diff. digit	680	$0.72 \times 1000 = 720$	2.22
Three like digit	31	$0.01 \times 1000 = 10$	44.10
Exactly one pair	289	$0.27 \times 1000 = 270$	1.33
			47.65

The appropriate degree of freedom is one less than the number of class interval.

$(n-1)$ i.e. $3 - 1 = 2$.

$(n=3)$ cause 3 different varexa ni fekiho ni

Since, $\chi_0^2 > \chi^2_{0.05,2} = 5.99$

The independence of the number is rejected on the basis of this test.

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UNIT 4:
VERIFICATION & VALIDATION OF SIMULATION MODELS

Verification:-

In simple word, verification is verifying the document. verification activities involve, here are inspection and reviews. It is a low level activities. (निरीक्षण और (evaluation))

Validation:-

Validation is a validate, the actual and expected output of software/system. Validation activities involved, in this is testing the software application. It is a high level activities.

Verification and Validation of Simulation Models:

One of the most important and difficult task facing a model developer is the verification and validation of simulation model. It is the job of model developer to work closely with the end users throughout the period of development and validation to reduce the doubt and to increase the models credibility.

The goal of the validation process is two fold.

1) To produce a model that represents true system behaviour closely enough for the model to be used as a substitute for the actual system for the purpose of experimenting with system, analyzing system behaviour and predicting system performance.

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ii) To increase credibility of the model to an acceptable level so that the model will be used by managers and other decision makers.

Validation should not be seen as an isolated set of procedure that follows model development, but rather as an integral part of model development. Conceptually, the verification and validation process consists of following components:

i) Verification is concerned with building the model correctly. It proceeds by the comparison of conceptual model to the computer representation that implements the conceptions. It ask the question: Is the model implemented correctly in the simulation software?, are the input parameters and logical structure of the model correctly represented.

ii) Validation is concerned with building the correct model. It attempts to confirm that a model is an accurate representation of the real system. In otherword, validation is a overall process of comparing the model and its behaviour to the real system.

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Model Building:-

The model building is the process of constructing a model of a real world system. It consists of following steps:

Step 1: The first step in model building consists of observing the real system and interactions among their various components and of collection of data on their behaviour. But observation alone seldom yield sufficient understanding of the system behaviour. People familiar with the system or any sub-system should take advantage of their special knowledge. Operators, technicians, repair and maintainance personnel, engineer, supervisor and managers understand certain aspect of the system that might be unfamiliar to others. As model development proceeds, new questions may arise, and the model developers will return to this step of learning true system structure and behaviour.

Step 2:

The second step in model building is the construction of a conceptual model i.e. a collection of assumptions on the components and the structure of the system, plus on the hypothesis on the values of model input parameters as illustrated in figure below.

Step 3:

The third step is the translation of the operational model into a computer recognizable form i.e. computerized model.

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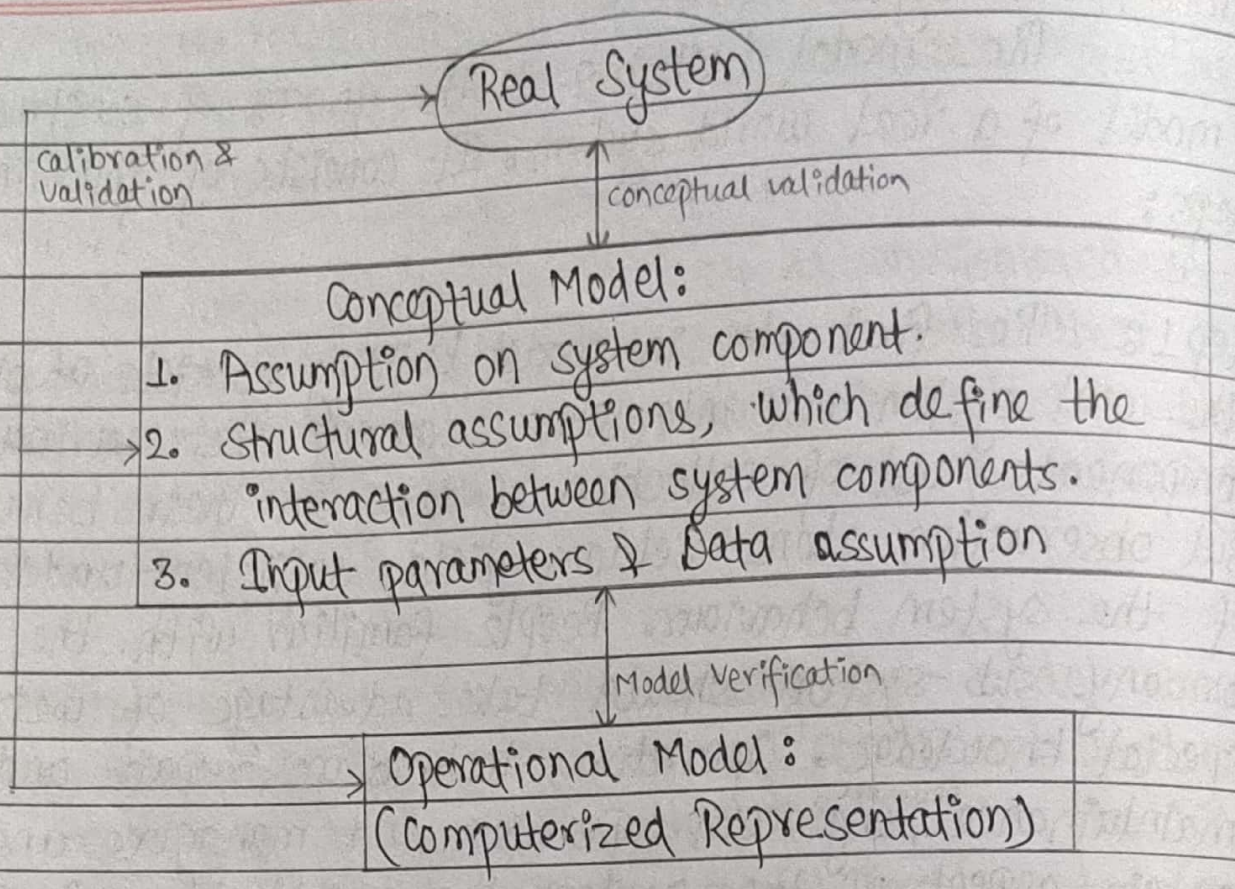

[Fig:- Model Building, Verification & Validation]

Verification of Simulation Model:

Verification is concerned with building the model correctly. It proceeds by the comparison of the conceptual model to the computerized representation that implements that conception. The purpose of model verification is to ensure that the conceptual model is reflected accurately in the operational model. The conceptual model quite often involves some degree of abstraction about system operation or some amount of simplification of actual operations. Verification asks the following question; Is the conceptual model accurately represented by the operational model?

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Many common sense suggestion can be given for use in the verification process:

i) Have the operational model checked by someone other than its developer, preferrably an expert in the simulation software being used.

ii) Make a flow diagram that include each logically possible action a system can take when an event occurs and follow the model logic for each action for each event type.

iii) Closely examine the model output for reasonableness under the variety of settings of the input parameter.

iv) Have the operational model print the input parameters at the end of simulation, to be sure that these parameters values have not been change without intention or accidently.

v) Make the operational model as self documenting as possible.

vi) If the operational model is animated verify that what is seen in the animation imitates the actual system.

vii) The Interactive Run Controller (IRC) or debugger is an essential components of successful simulation model building. Even the best simulation analyst makes mistakes or commits logical errors when building a model. The IRC assist in finding and correcting those errors in the following ways:-

a) The simulation can be monitored as it progresses.

b) Attention can be focused on a particular entity, line of code or procedure.

c) Values of selected model component can be observed.

d) The simulation can be temporarily suspended or paused not only to view information but also to reassign values or redirect entities.

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viii) Graphical interfaces are recommended for accomplishing verification and validation.

Calibration & Validation of models:

Calibration and validation although are conceptually distinct but they usually conducted simultaneously. Validation

- Validation is the overall process of comparing the model and its behaviour to the real system and its behaviour.
- Calibration is the iterative process of comparing the model to the real system making adjustment to the model, comparing the revised model to the reality and so on.

Following figure shows the relationship of model calibration to the overall validation process.

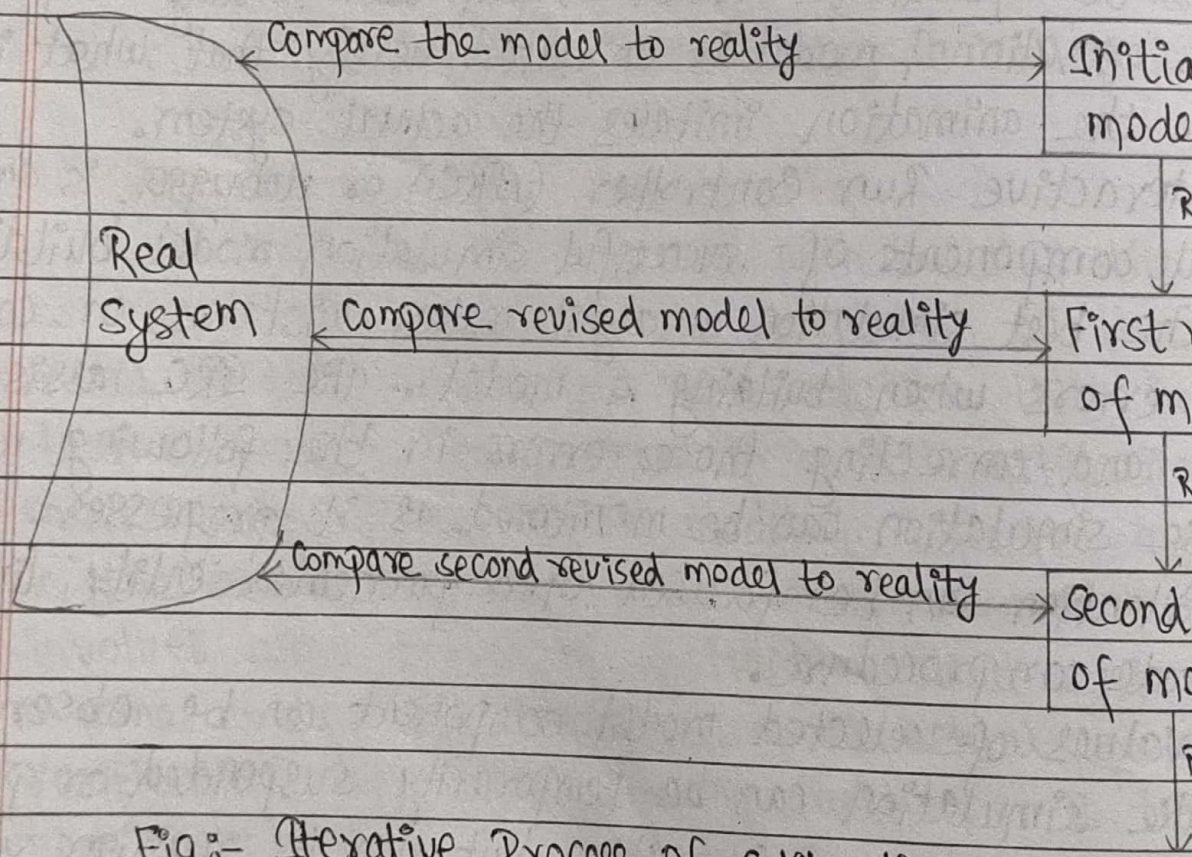

Fig:- Iterative Process of Calibrating a model

The comparison of model to the reality is carried out by a variety of test. Test can be of subjective or objective. Subjective test usually involve people, who are knowledgable about one or more aspect of the system, making judgement about the model and its output. Objective test always require data on systems behaviour plus the corresponding data produced by the model. Then one or more statistical test are performed to compare some aspect of system data set with the same aspect of the model data set. This iterative process of comparing model with system and then revising both the conceptual and operational models to accomodate any perceived model deficiencies is continued until the model is judged to be sufficiently accurate.

Validation Process:

As an aid (help) in the validation process, Naylor & Finger [1967] formulated a 3-step approach that has been widely followed.

Step 1 : Build a model that has high face validity.
Step 2 : Validate model assumption
Step 3 : Compare the model input-output transformation to corresponding input-output transformation for the real system.

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1. Face validity:-

- The first goal of the simulation modular is to construct a model that appears reasonable on its face to model users and others who are knowledgeable about the real system being simulated.
- The potential user of a model should be involved in the model construction from its conceptualization to its implementation to ensure that a high degree of realism (attitude) is build into the model through reasonable assumption regarding system structure and through reliable data.
- Sensitively, analysis can also be used to check model face validity.

2. Validation of model assumption:

Model assumption categorized into two general classes:

a) Structural assumption
b) Data assumption

a) Structural assumption:-

- It involves the questions of how the system operates and usually involve simplification and abstraction of reality. For example: consider the customer queuing and service facility in bank, customer can form one line or there can be an individual line for each teller.
- If there are many line, customer could be served strictly or First-Come-First-Serve basis or some customer could change line if one line is moving faster.
- Structure assumption should be verified by actual observation

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during appropriate time period and by discussion with manager and teller regarding bank policies and actual implementation of these policies.

b) Data assumption :-

It should be based on the collection of reliable data and correct statistical analysis of data. For example :- In a bank study as discussed previously data were collected on :

*. inter-arrival time of customer during periods of peak loading.
*. inter_arrival time during slack period (loose period).
*. service time for commercial account
*. service time for personal account.

The procedure for analyzing input data consists of three steps :

*. Identify an appropriate probability distribution.
*. Estimate the parameters for hypothesized distribution
*. Validate the assumped statistical model by goodness of fit such as Chi-square test, K-S test or by graphical methods.

3. Validating Input-Output Transformation :-

- In this phase of validation process, the model is viewed as input-output transformation i.e. the model accepts values of input parameter and transform these input into output measure of performance.
- It is this correspondance that is being validated.
- A necessary condition for validation of input-output transformation is that some version of the system under study exist, so that

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system data under at least one set of input condition can be collected to compare the model prediction.

- If the system is in planning stage and no system operating data can be collected complete input-output validation is not possible.
- Validation increases modeller's confidence that the model of existing system is accurate.
- To get accurate model, some changes in operational model is done.
- Changes in operational model ranging from relatively minor to relatively major includes the following:

* Minor changes of single numerical parameters such as speed of the machine, arrival rate of customer, etc.
* Minor changes of the form of a statistical distribution such as the distribution of a service time or a time to failure of a machine.

* Major changes in the logical structure of a sub-system such as change in queue discipline for a waiting line model or a change in the scheduling rule for job-shop model.
* Major changes involves difference design for the new system such as computerized inventory control system replacing an older non-computerized design.

- If the changes to the computerized representation of system is minor such as in item 1 & 2, these changes can be carefully verified and output from new model can be accepted with considerable confidence.

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- Partial validation of substantial changes in item 3 & 4 might be possible.

Techniques for input-output validation :-

After designing the model, we need to check whether the input-output of model is valid with respect to the system. This validation can be done in two ways:

i) Input-output validation using historical input data.
ii) Input-output validation using a Turing test.

i) Input-output validation using historical input data :-

- When using artificial generated data as input data, the modeler expects the model to produce event pattern that are compatible with but not identical to the event pattern that occurred in the real system during the period of data collection.
- For eg: in the bank model, artificial input data $\{X_{1n}, X_{2n}, \ldots ; n = 1, 2, \ldots\}$ for inter-arrival and service time are generated and replicates of the output data 'Y' is compared to what was observed in the real system by means of hypothesis test.

- An alternative to generating input data is to use the actual historical record $\{A_n, S_n ; n = 1, 2, \ldots\}$ to derive the simulation model and then to compare model output with system data to implement this technique for the bank model.
- The data $A_1, A_2, \ldots$ & $S_1, S_2, \ldots$ would have to be entered into the model into array or stored in a file to be read as the need arise.

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- To conduct a validation test using historical input data it is important that all the input data (A_n, S_n) and all the system response data such as average delay (z) be collected during the same time period. Otherwise, the comparison of model response to system response, such as comparison of average delay in the model (Y) to that in the system (z) could be misleading. The response depends both on the input and on the structure of the system (or model).

- Implementation of this technique could be difficult in large system because of the need for simultaneous data collection of all input variables (to) and those response variable of primary interest. In some system, electronic counters and devices are used to ease the data-collection task by automatically recording certain types of data.

ii) Input-output validation using a Turing test:-

- In addition to statistical tests or when no statistical test is readily applicable, persons knowledgeable about system behaviour can be used to compare model output to system output.
- For example, suppose that five reports of system performance over five different days are prepared, and simulation outputs are used to produce five 'fake' reports.
- These 10 reports should all be in exactly in the same format and should contain information of the type that manage and engineer have previously seen on the system.
- These ten reports are randomly shuffled and give to the engineers who is asked to decide which report are fake and which are real.

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- If engineer identifies substantial number of fake reports the model builder questions the engineer and use the information gained to improve the model.
- If the engineer cannot distinguish between fake and real reports with any consistency, the model will conclude that this test provides no evidence of model inadequacy.
- This type of validation test is called as Turing test.

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UNIT 5:
ANALYSIS OF SIMULATION OUTPUT

Nature of the problem :-

Once a stochastic variable has been introduced into a simulation model, almost all the system variables describing the system behaviour also become stochastic because of the way endogeneous events make one variable depends upon another. Hence, it needs some stochastic method to analyze the simulation output.

A large body of statistical method has been developed over the years to analyze the result in science engineering and other fields where experimental observations are made. Because of experimental nature of the system simulation, it seems natural to attempt applying these method to simulation results. But most of the experiment pre-suppose that the observation being made are mutually independent, however simulation results are not mutually independent. For example; customer waiting time in a queuing system are not IID (Identically & Independently Distributed). Thus, it is difficult to apply classical statistical technique to analysis of simulation output. This need has also lead to the development of new statistical methods, and it is the subject of current research work.

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Types of Simulation on the basis of output:-

To analyze the simulation output, the simulation can be categorized into two broad categories:

i) Terminating or transient simulation
ii) Non-terminating or steady-state simulation.

i) Termination or finite simulation:-

It takes place at a specified time or is caused by some specified events. For example; Banks open at 8:30 am with no customer present and closed at 4:30 pm. The terminating simulation runs for some specified duration of time T_E where E is a specified event that stops the simulation. It starts at time 0 under well specified initial condition and ends at the stopping time T_E.

ii) Non-terminating or steady state Simulation:-

It is a system that runs continuously or at least over a very long period of time. The purpose of steady state simulation is the study of long run behaviour of system. For example, continuously operating communication system where the objective of computation of mean delay of packet in the long run.

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Some Definitions:

a) Independently and Identically Distribution (IID) random variables :-

Usually, a random variable is drawn from an infinite population that has a stationary probability distribution with a finite mean μ, and finite variance σ^2. This means that the population distribution is not affected by the number of samples already made, nor does it change with time. Further the value of one sample is not affected in any way by the value of any other sample i.e. the random variables are mutually independent. Random variables that meet all these conditions are said to be independently and identically distributed, usually abbreviated to IID.

b) Central Limit Theorem :-

It states that the sum of n IID variables, drawn from a population has a mean of μ and a variance of σ^2 is approximately distributed as a normal variable with a mean of $n\mu$ and a variance of $n\sigma^2$.

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Estimation Methods :-

Any normal distribution can be transformed into a standard normal distribution, that has a mean of 0 and a variance of 1. Let us consider the estimation of a performance parameter θ (or ϕ) of a simulated system. It is desired to have a point estimate and an internal estimate of θ (or ϕ). The length of the internal estimate is a measure of the error in the point estimate. The simulation output data are of the form $\{Y_1, Y_2, \ldots Y_n\}$ for estimating θ, we refer to such output data as discrete-time data, because the index n is discrete valued. The simulation output data are of the form $\{Y(t), 0 \le t \le T_E\}$ for estimating ϕ, we refer to such output data as continuous-time data, because the index i is continuous valued, where $Y(t)$ represents estimation of data based on different time, t is estimate time, T_E is time event.

a) Point Estimation :-

The point estimator of θ based on the data $\{Y_1, \ldots, Y_n\}$ is defined by

$$\hat{\theta} = \frac{1}{n} \sum_{i=1}^{n} Y_i$$

where,

$\hat{\theta}$ is a sample mean based on a sample of size n. Computer simulation languages may refer to this as a discrete-time statistic.

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The point estimator $\hat{\theta}$ is said to be unbiased for θ if its expected value is θ i.e. if

$$E(\hat{\theta}) = \theta$$

However,

$$E(\hat{\theta}) \neq \theta$$

and the difference $E(\hat{\theta}) - \theta$ is called bias of $\hat{\theta}$.

For the continuous data $\{Y(t), 0 \leq t \leq T_E\}$ with mean ϕ then the point estimator is given by

$$\hat{\phi} = \frac{1}{T_E} \int_0^{T_E} Y(t)\, dt$$

b) Interval Estimation:-

- Suppose that model is the normal distribution with mean θ and variance σ^2 both unknown. Let $\bar{Y}_i$ be the average cycle time for the ith replication and σ is day-to-day variation of average cycle time.

- Suppose our goal is to estimat θ. If we are planning to be in business for a long time, then θ is a relevant parameter, because it is the long-run mean daily cycle time.

- The natural estimator for θ is the overall sample mean of R independent replications,

$$\bar{Y}_{..} = \sum_{i=1}^{R} \bar{Y}_i / R$$

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- But $\bar{Y}_{..}$ is not θ, it is an estimate, based on a sample, and it has error.
- A confidence interval is a measure of that error.
- Let,

$$S^2 = \frac{1}{R-1} \sum_{i=1}^{R} (Y_i - \bar{Y}_{..})^2$$

be the sample variance across the R replications.

- The usual confidence interval which assume the Y_i is normally distributed and it is:

$$\bar{Y}_{..} \pm t_{\alpha/2, R-1} \frac{S}{\sqrt{R}}$$

where,

$t_{\alpha/2, R-1}$ is the quantile of t-distribution with R-1 degree of freedom that cuts off $\alpha/2$ of the area of each tail.

Simulation Run Statistics:-

It is method to handle problems that arise in measuring statistics from simulation run. In the estimation method, it is assumed that the observations are mutually independent and the distribution from which they are drawn is stationary. Unfortunately, many statistics of interest in simulation do not meet these conditions.

For example;

Consider a single server system in which the arrival occurs with Poisson distribution and service time has an exponential and queue discipline is FIFO with no

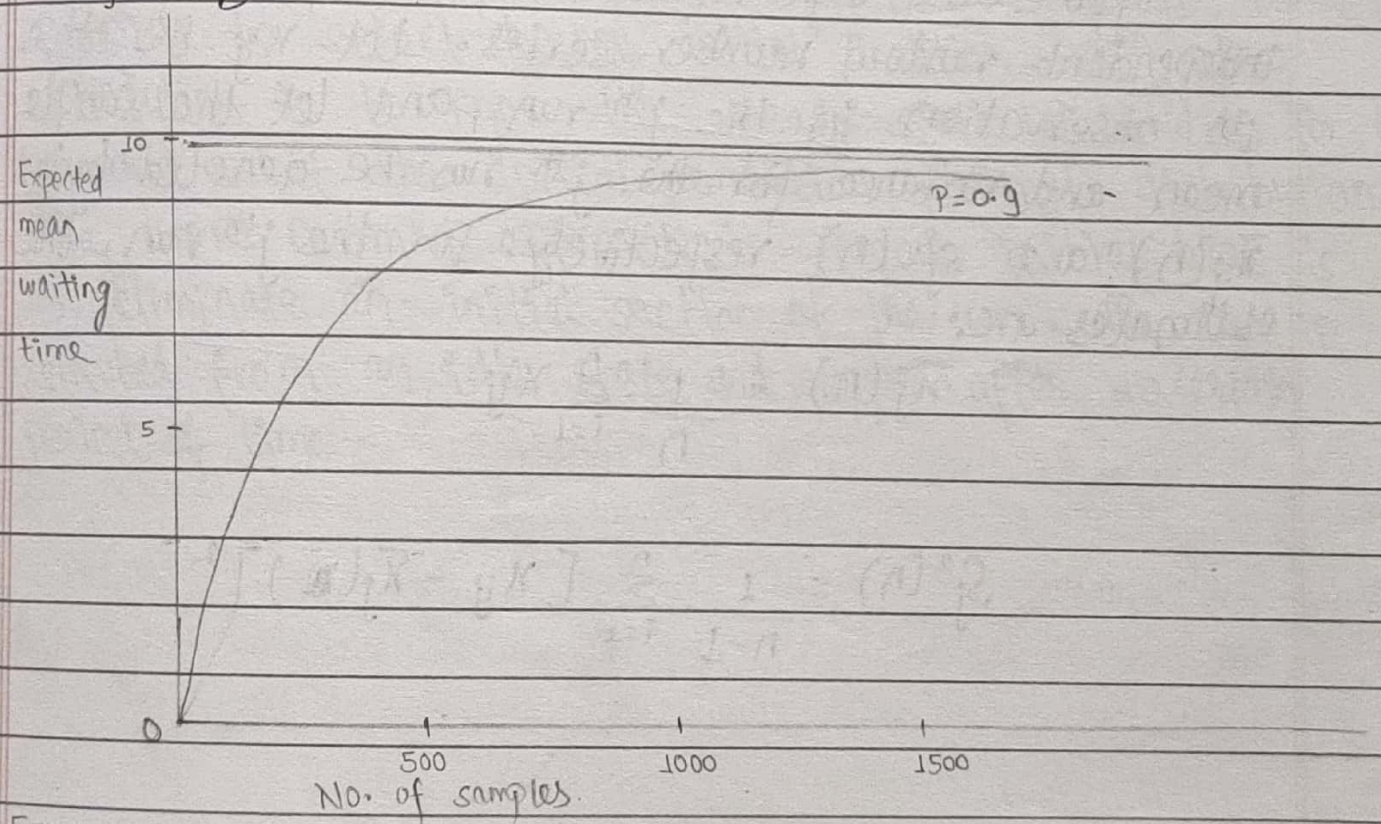
priority. Suppose the study objective is to measure the mean waiting time. In simulation run, the simplest approach is to estimate the mean waiting time by accumulating the waiting time of n successive entities and dividing by n. This sample mean is denoted by $\bar{x}(n)$ or $\bar{x}$.

$$\bar{x}(n) = \frac{\sum_{i=1}^{n} x_i}{n}$$

where, x_i is the individual waiting time. $(i = 1, 2, 3, \ldots)$

Waiting time measured in this way are not independent. Whenever a waiting line forms, the waiting time of each entity on the line clearly dependent upon the waiting time of its predecessors. Such data are called auto-correlated.

The following figures shows mean waiting time for different sample size.



[Fig:- Mean waiting time in M/M/1 system for different sample sizes]

where,

first M = inter-arrival time is distributed with Poison distribution

second M = service time is distributed exponentially.

1 = single server.

(P = 0.9) is starting from an initial empty state with a server utilization of 0.9.

Replication of Runs:

One way of obtaining independent results is to repeat the simulation. Repeating the experiment with different random numbers for the same sample size n gives a set of independent determinations of the sample mean $\bar{x}(n)$.

Suppose the experiment is repeated p times with independent random number series. Let x_{ij} be the ith observation in the jth run, and let the sample mean and variance for the jth run be denoted by $\bar{x}_j(n)$ and $s_j^2(n)$ respectively. For the jth run, the estimates are:

$$\bar{x}_j(n) = \frac{1}{n} \sum_{i=1}^{n} x_{ij}$$

$$s_j^2(n) = \frac{1}{n-1} \sum_{i=1}^{n} [x_{ij} - \bar{x}_j(n)]^2$$

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Combining the results of p independent measurement gives the following estimates for the mean waiting time $\bar{x}$ and the variance s^2 of the population

$$\bar{x} = \frac{1}{p} \sum_{j=1}^{p} \bar{x}_j(n)$$

$$s^2 = \frac{1}{p} \sum_{j=1}^{p} s_j^2(n)$$

Elimination of Initial Bias :-

The system can be started in a more representative state than the empty state or the first part of simulation run can be ignored. In some simulation studies, particularly of existing system, there may be information available on the expected condition that makes it feasible to select better initial condition. The ideal solution is to know the steady state distribution for the system and select the initial condition for that distribution. The more common approach to remove initial bias is to eliminate an initial section of the run. The run is started from an idle state and stopped after a certain period of time.

(Imp) sir

Elimination of Initial Bias :-

The exponential results in simulation may contain initial bias, which is needed to be removed. There are two general approaches that can be taken to remove initial bias:

a) The system can be started in a more representative than an empty state.

b) The first part of the simulation can be removed.

(Tmsh sir)

In some simulation studies, where the expected value is available, that makes it is feasible to select better initial condition. The ideal solution is to know the steady state distribution for the system and select the initial conditions from that distribution.

The more common approach to remove initial bias is to eliminate an initial section of the run. The run is started from an idle state and stopped after a certain period of time. The entities existing in that system at that time are left as they are. The run is then restarted with the statistics being gathered from the point of re-start. The main disadvantage of this approach is that there is no simple rules to decide how long an interval should be eliminated. Another disadvantage of eliminating the first part of a simulation run is that the estimate of the variance must be based on less information.

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UNIT 6 :

SIMULATION LANGUAGE

Basic Concepts of Simulation Software:-

Simulation software is a program that allows the user to observe an operation through simulation without actually performing that operation. Simulation software is used widely to design equipment so that the final product will be as close to design as possible without expensive in process modification.

There are essentially three types of languages one can use for simulation:

a) General Purpose Programming Language (GPPL)
b) Simulation Programming Languages (SPL)
c) Simulation tools or environment or software

a) General Purpose Programming Language:-

General Purpose Programming language such as java, C++, C, etc can be used to develop the simulation model of real system as a computer program and can be used for the experimental purpose. These languages are used to develop many types of applications. There are a number of advantages to implement simulations in general languages. They tend to be very fast, there are few limitations on what can be done, programmers are easily found, and simulation can be done on a number of different computers. The disadvantage of this approach

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is cost. Simulation based on general languages take knowledge of simulation implementation and are generally very large program. The time needed to design, code and verify such a system may be very overwhelming.

b) Simulation programming Languages :-

These languages are simulation specific and they include GPSS, SIMAM, etc. These languages take most of the work out of creating a simulation. These systems can do all the queue manipulation needed in a single line. The main disadvantage of this language are in their limited domain. It is difficult, but not impossible to create a financial portfolio planner software in GPSS. It is just not designed for it. On the other hand, another specialized simulation language, a 123 spreasheet, can easily handle it.

c) Simulation Environment / tools :-

These includes simulation environment or IDE such as Arena, AutoMod, etc. These products includes common characteristics such as graphic user interface (GUI) and an environment that supports all aspects of a simulation study.

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Specific Simulation Programming Languages :-

The overall simulation language can be categorized into two broad classes:

a) Discrete system simulation language
b) Continuous system simulation language

a) Discrete System Simulation Language :-

A number of programming languages have been developed to simplify the task of writing discrete system simulation program. These programs include a language with which to describe the system and a programming system that will establish a system image and execute a simulation algorithm. Each language is based on set of concept used for describing system. The word world-view has come to be used to describe this aspect of simulation program. The user of the program must learn the world view of particular language he/she is using and be able to describe the system in those terms. One of the language that is used for DSSL is GPSS.

b) Continuous System Simulation Language :-

These languages use the familiar statement type of input for digital computer allowing a program to be programmed directly from the equation of a mathematical model, rather than requiring the equations to be broken into a functional element. It includes a variety of algebraic and logical expression to describe the relation between variables. Several implementations of CSSL has been published.

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One particular CSSL that illustrate the nature of these language is the Continuous system Modeling Program (CSMP, version 3) i.e. CSMP III.

General Purpose Simulation System (GPSS) Language:

Introduction:-

GPSS is one of the earliest discrete simulation language, it was developed by Geaffrey Gordon and presented in two papers in 1961 & 1962. GPSS was designed specially for analysis who were not necessarily computer programmer. It is particularly suitable for modeling traffic and queuing systems. The GPSS language has been developed over many years by the IBM corporation. It has been implemented on several different manufacturer's machines and there are variations in the different implementations. The version GPSS/360 and GPS V are most popular.

General Descriptions:-

- The system to be simulated in GPSS is described as a block diagram in which the blocks represent the activities and lines joining the blocks indicates the sequence in which the activities can be executed.
- When there is choice of activities, more than one line leaves a block and the condition for the choice is stated at the block.

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- The use of block is to describe the system being simulated, make easy and familiar to understand what kind of system we are going to simulate and what are the essential or important part of the system are included in our simulation model.
- The GPSS defines a set of 48 specific block types, each of which represents a characteristic action of a system.
- The program user must draw a block diagram of the system using only these blocks and then write the corresponding code for each block.

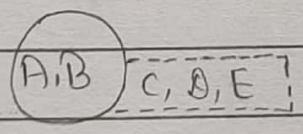
GPSS Blocks:-

Each block type in GPSS is given a name that is descriptive of the block action and is represented by a particular symbol. Each block type has a number of data fields. As the blocks are described, the fields will be referred to as field A, B, C and so on. Some of the common blocks that are used in GPSS are:

1) ADVANCE Block:- It is concerned with representing the expenditure of time.

A, B

II) GENERATE Block :- It controls the interval between successive arrivals of transactions.



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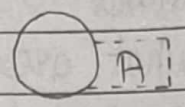
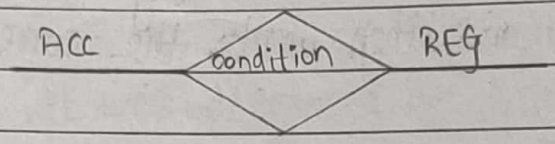
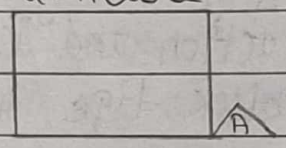
III) TERMINATE Block:- It destroys the active transaction.



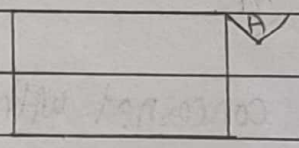
IV) TRANSFER Blocks:- It allows some location other than the next sequential location to be selected. The choice is normally between two blocks referred to as next blocks A and B.



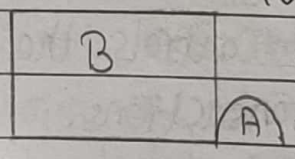
V) SEIZE Block:- It allows a transaction to engage a facility if it is available.



VI) RELEASE Block:- It allows the transaction to disengage the facility.



VII) ENTER Block:- It allows a transaction to occupy space in a storage, if it is available.



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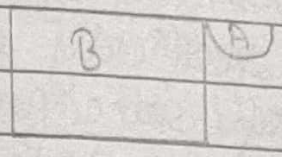
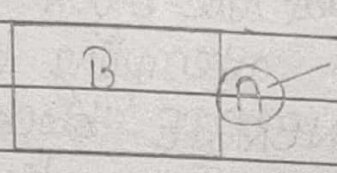
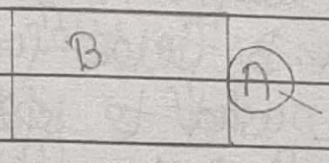
viii) LEAVE Block:- It allows a transaction to give up the space.



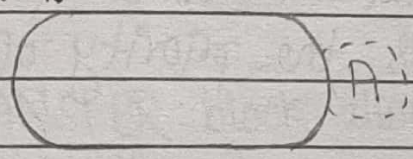
ix) QUEUE Block:- It increases the queue numbered in field A.



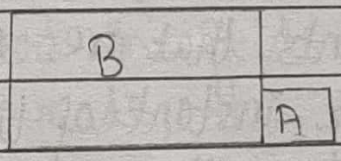
x) DEPART Block:- It decreases the queue numbered in field A.



xi) MARK Block :- It simply notes the time of arrival on the transaction.



xii) TABULATE Block:- It subtracts the time noted by a MARK block from the time of arrival at the TABULATE block.



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Action Times:-

Clock time is represented by an integer number. The unit of time is not specifically started but all time are expressed in terms of same unit. The program computes an interval of time called an action time for each transaction as it enters an ADVANCE block, and transaction remains at the block for this interval of simulated time before attempting to proceed. The action time at the GENERATE block controls the interval between successive intervals of transactions.

An action time is defined by giving a mean and modifier as the A and B fields for the block. The C field however can be used to specify an offset time as the time when the first transaction will arrive. The field D is used to specify a limit to the total no. of transaction that will come from the block. The E field determines the priority of the transactions.

Succession of Events:-

The programs maintain records of when each transaction in the system is due to move. It proceeds by completing all movements that are scheduled for execution at a particular instant of time and that can logically be performed. When there is more than one transaction, the program processes transactions in the order of their priority class.

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Normally, a transaction spends no time at a block other than at an ADVANCE block. Once the program has begun moving a transaction, it continues to move the transaction through the block diagram until one of following conditions arises:

i) The transaction may enter an ADVANCE block with a non-zero action time.
ii) The conditions in the system may be such that the action the transaction is attempting to execute by entering a block cannot be performed at the current time.
iii) A third possibility is that the transaction enters a TERMINATE block, in which case, it is removed from the simulation.
iv) A fourth possibility is that a transaction may be put on a chain.

Choice of Paths:-

The TRANSFER block allows some location other than the next sequential location to be selected. The choice is normally between two blocks referred to as next blocks A and B. The method used for choosing is indicated by a selection factor in field A of the TRANSFER block. Next blocks A and B are placed in fields B and C respectively. If no choice is to be made, the selection factor is left blank. An unconditional transfer is then made to next block A.

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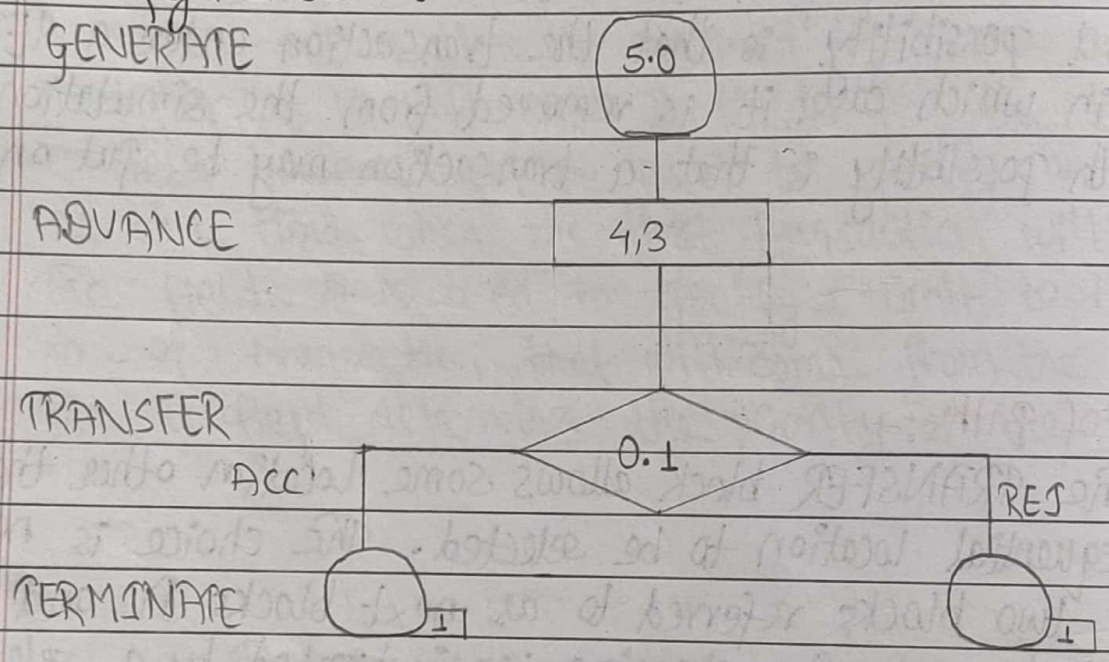
Example :- Simulation of a Manufacturing Shop

Let us consider a machine tool in a manufacturing shop is turning out parts at the rate of one every 5 minutes. As they are finished, the parts go to an inspector, who takes 4±3 minutes to examine each one and rejects about 10% of the parts. Each part will be represented by on transaction, and the time unit selected for problem will be 1 minute.

The block diagram representing the system is as shown in figure below:



[Fig:- Manufacturing Shop]

- Here a GENERATE block is used to represent the output of the machine by creating one transaction.
- An ADVANCE block with a mean of 4 and modifier of 3 is used to represent inspector. The time spent on

inspection will be anyone of the values 1, 2, 3, 4, 5, 6, or 7, with equal probability given to each value.

- Upon completion of the inspection, transactions go to TRANSFER block with selection factor of 0.1 so that 90% of the part of to next location called ACC, to represent acceptance part and 10% go to another location called REJ to represent rejected parts.
- Since there is no further interest in the history of parts in this simulation, both location reached from the TRANSFER blocks to TERMINATE blocks.

The equivalent program code (GPSS source code)

```
        GENERATE 5                ; create parts
        ADVANCE 4,3               ; inspects
        TRANSFER 0.1,ACC,REJ      ; selects or rejects
ACC     TERMINATE 1               ; Accepted
REJ     TERMINATE 1               ; Rejected
        START 1000                ; Run for 1000 parts
```

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Facilities and storages :-

There are many permanent entities associated with the system being simulated such as items of equipment which operates on the transactions. Two types of permanent entities are defined in GPSS to represent system equipment: facility & storage.

A facility is defined as an entity that can be engaged by a single transaction at a time. A storage is defined as an entity that can be occupied by many transactions at a time, upto some predetermined limit.

Four types of blocks SEIZE, RELEASE, ENTER & LEAVE are concerned with using facilities and storages. Field A in each case indicates which facility or storage is indicated and the choice is usually marked in the flag attached to the symbols of the blocks. If the field B of the ENTER and LEAVE blocks are blank, the storage contents are changed by one. If there is a number (≥ 1) then the contents changed by that value.

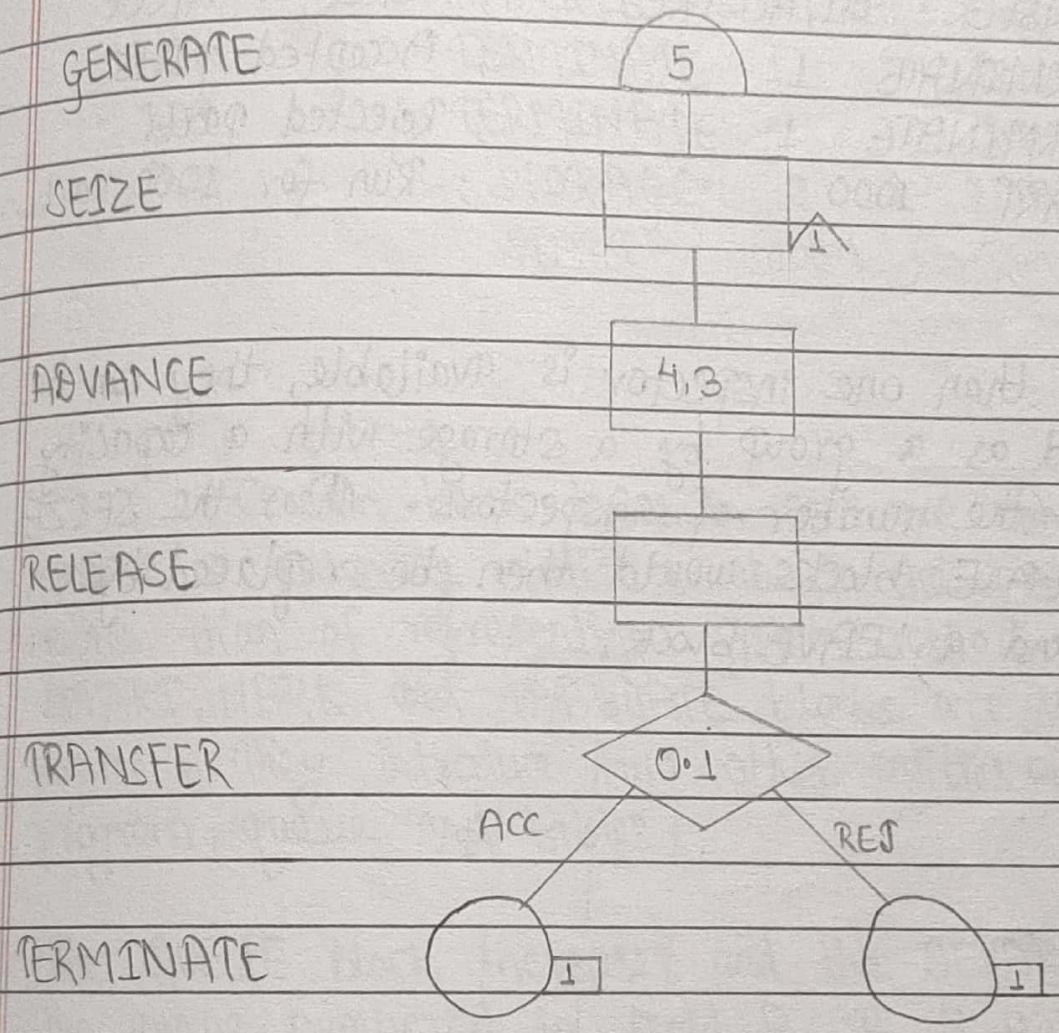
Example :-

Manufacturing shop - model with storage and facility

Let us consider again the manufacturing shop. Since the average inspection time is 4 minutes and the average generation rate of parts is one every 5 minutes, there will normally be only one part inspected at a time. Ocassionally however a new part can arrive before the previous part has completed its inspection. In this situation, in the

previous example, the ADVANCE block will result more than one transaction at a time.

Assuming that there is only one inspector, it is necessary to represent the inspector by a facility to simulate the fact that only one part at a time can be inspected. Then the block diagram above situation can be represented as:



[Fig:- Manufacturing Shop]

The equivalent program code (GPSS source code)

```
        GENERATE   5                  ; create parts
        SEIZE      1                  ; Get inspector
        ADVANCE    4,3                ; inspect
        RELEASE    1                  ; free inspector
        TRANSFER   0.1,ACC,REJ        ; select or reject
ACC     TERMINATE  1                  ; Accepted parts.
REJ     TERMINATE  1                  ; Rejected parts
        START      1000               ; Run for 1000 parts.
```

If more than one inspector is available, they can be represented as a group by a storage with a capacity equal to the number of inspectors. Then the SEIZE and RELEASE blocks would then be replaced by an ENTER and a LEAVE block.

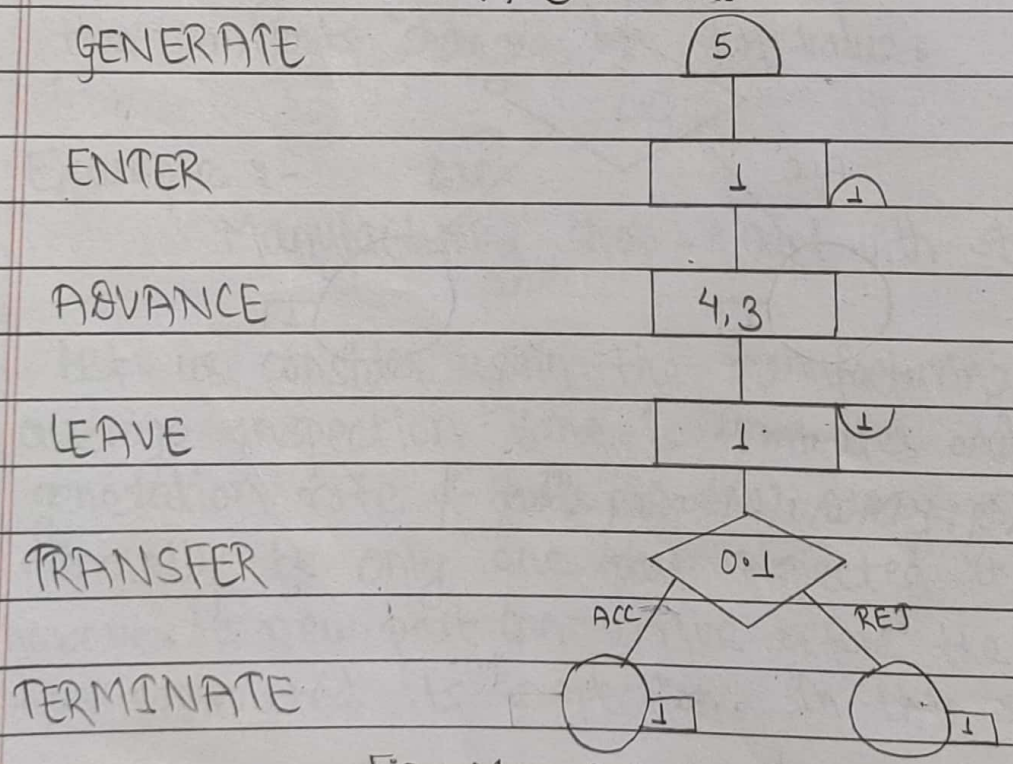


Fig:- Manufacturing Shop

The equivalent program code (Source code)

```
        GENERATE   5          ; create parts.
        ENTER      insp,1     ; Get inspector
        ADVANCE    4,3        ; inspect
        LEAVE      insp,1     ; free inspector
        TRANSFER   0.1,ACC,RES ; select or reject
ACC     TERMINATE  1          ; Accepted parts
RES     TERMINATE  1          ; Rejected parts
insp    STORAGE    3          ; Number of storage
        START      1000       ; Run 1000 parts.
```

Gathering Statistics :-

Certain block types in GPSS are constructed for the purpose of gathering statistics about the system performance, rather than of representing system actions. The QUEUE, DEPART, MARK and TABULATE blocks are used for this purpose. They introduce two other entities of the GPSS program, queue and tables.

The QUEUE block increases and the DEPART block decreases the queue numbered in field A. If B field is blank, then the change is a unit change, otherwise the value of field B≥1 is used.

It is also desirable to measure the length of time taken by transaction to move through the system or parts of

the system, and this can be done with the MARK and TABULATE blocks.

Example :-

Manufacturing shop model with statistics

Let us again consider the manufacturing shop model. In the previous example, the properties of GENERATE block is such that, if a transaction is unable to leave the block at the time it is created, no further creation are made until the block is cleared. Therefore, it assumes that if one part arrives when all inspectors are busy, the machining of further parts stops until the machine is cleared.

A more realistic situation is that parts will accumulate on the inspector work bench if inspection does not finish quickly enough. In that case, it would be of interest to measure the size of the queue of work that occurs. This can be done as shown in figure below:

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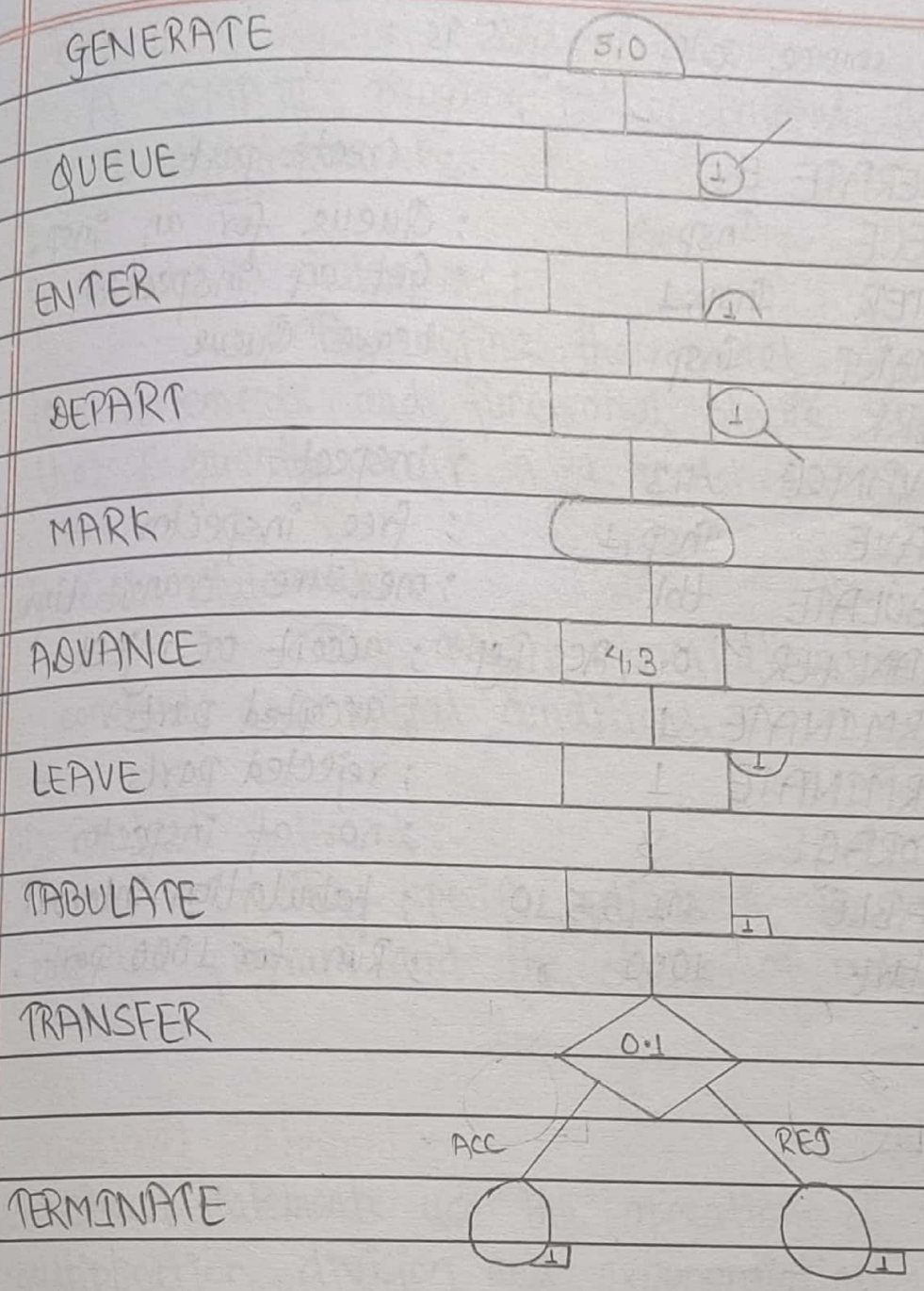

[Fig :- Manufacturing Shop]

The equivalent source code in GPSS is :

```
        GENERATE 5              ; create parts
        QUEUE    Insp1          ; Queue for an insp.
        ENTER    Insp,1         ; Get an inspection
        DEPART   Insp1          ; Leave Queue
        MARK
        ADVANCE  4,3            ; inspect
        LEAVE    Insp,1         ; free inspector
        TABULATE tbl            ; measure transit time
        TRANSFER 0.1,ACC,Rej    ; accept or reject
ACC     TERMINATE 1             ; accepted parts
Rej     TERMINATE 1             ; rejected parts
Insp    STORAGE  3              ; no. of inspector
tbl     TABLE    M1,5,5,10      ; tabulation interval
        Start    1000           ; Run for 1000 parts.
```

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Continuous System Modeling Program Version III (CSMP III):

A CSMP III program is constructed from three general types of statements.

i) Structural Statements :-

They define the model. They consist of FORTRAN-like statements, and functional blocks designed for operations that frequently occur in a model definition.

ii) Data Statements :-

They assign numerical values to parameters, constants and initial conditions.

iii) Control Statements :-

They specify options in the assembly and execution of the program and the choice of output.

Structural Statements :-

- Structural statements use the operations of addition, subtraction, multiplication, division and exponentiation using the same notation and rules as are used in FORTRAN.
- For example, if the model includes the equation,

$$X = \frac{6Y}{W} + (Z-2)^2$$

then the equivalent expression in CSMP III is :

```
X = 6.0 *Y / W + (Z-2.0)** 2.0
```

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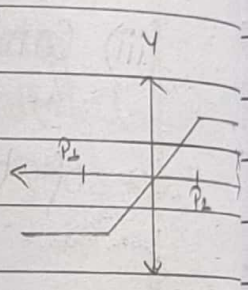
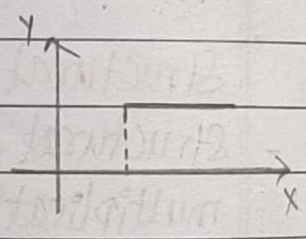
- There are many other functional blocks in addition to provide simulation specific operations.
- Some of them are exponential function, trigonometric function and function for taking maximum values.

Functional Block in CSMP III:

List of functional block that are used in CSMP III are as follows:

	General form	Function
1.	Y = INTGRL (IC, X) Y(0) = IC (Name: Integrator)	$Y = \int_0^t X\,dt + IC$
2.	Y = LIMIT (P1, P2, X) (Name: Limiter)	Y = P1 X < P1 Y = P2 X > P2 Y = X P1 ≤ X ≤ P2 
3.	Y = STEP (P) (Name: Step function)	Y = 0 t < P Y = 1 t ≥ P 
4.	Y = EXP (X) (Name: Exponential)	$Y = e^x$
5.	Y = ALOG (X) (Name: Natural logrithm)	$Y = \ln(X)$

	General form	Function
6.	Y = SIN (X) (Name : Trigonometric Sine)	$Y = \sin(X)$
7.	Y = COS (X) (Name : Trigonometric cosine)	$Y = \cos(X)$
8.	Y = SQRT (X) (Name : Square root)	$Y = \sqrt{X} = X^{1/2}$
9.	Y = ABS (X) (Name : Absolute value)	$Y = \lvert X \rvert$
10.	Y = AMAX1 ($X_1, X_2, \ldots, X_n$) (Name : Largest value)	$Y = \max(X_1, X_2, \ldots, X_n)$
11.	Y = AMIN1 ($X_1, X_2, \ldots, X_n$) (Name : Smallest value)	$Y = \min(X_1, X_2, \ldots, X_n)$

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Data Statement :-

Data statements are used to set the initial values to the model parameter. Some of the data statements are as follows :

i) INCON :- It is used to set the initial values of the integration - function block.

ii) CONST :- It is used to set the parameter values.

iii) PARAM :- It is control statement but it can also be used to assign values to individual parameter. Generally it is used to specify a series of values for one parameter.

Example :

CONST A = 0.5 , XDOT = 1.25 , YDOT = 6.22
PARAM D = (0.25, 0.50 , 0.75, 1.0)

Control Statements :-

They specify options in the assembly and execution of the program and the choice of output. Control statements that are commonly used in CSMP III are as follows :

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i) TIMER :- It is used to specify certain time interval.

Example:-

TIMER DELT = 0.005, FINTIM = 1.5, PRDEL = 0.1, OUTDEL = 0.1

where,

DELT = Integration interval

FINTIM = Finish time

PRDEL = Interval at which to print results

OUTDEL = Interval at which to print-plot

ii) PRINT and PRTPLT :-

They are used to provide printed and print-plotted output respectively.

iii) TITLE and LABEL :-

They are used to put headings on the printed and print-plotted outputs respectively.

iv) END and ENDJOB :-

The END statement is used to specify the end of run of a model. The ENDJOB statement is used to specify the end of simulation.

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Example:-

A CSMP III program for the automobile wheel suspension problem. It has been coded for the case where M = 2.0, F = 1, and K = 400, as specified in the CONST statement. A PARAM statement has been used to produce runs with different values of D to give different values to the dumping ratior. The program integrates with an interval of 0.005, and runs for a time of 1.5.

Sol^n:

```
TITLE AUTOMOBILE SUSPENSION SYSTEN
*
PARAM D = (5.65 , 16.968 , 39.592 , 56.56, 113.12)
*
X2DOT = (1.0/m) * (K*F - K*X - D*XDOT)
XDOT = INTGRL (0.0, X2DOT)
X    = INTGRL (0.0, XDOT)
*
CONST M=20, F=1.0 , K=400.0
TIMER DELT = 0.005 , FINTIM = 1.5 , PRDEL = 0.05
OUTDEL = 0.05
PRINT X , XDOT , X2DOT
PRTPLT X
LABEL DISPLACEMENT VERSUS TIME
END
ENDJOB
```

Hybrid Simulation:-

For the most studies the model is clearly either continuous or discrete nature, and that is the determining factor in deciding whether to use an analog or digital computer for system simulation. However, in some situation we have to combine digital and analog computer to provide hybrid simulation. The form taken by hybrid simulation depends upon the application. It is possible that the system being simulated is an interconnection of continuous and discrete sub-system, which can be best modelled by an analog and digital computer being linked together.

The hybrid simulation requires some technological developments, for its operation. High-speed converters are needed to transform signals from one form of representation to the other. For the practical use mini-computers are generally used for hybrid simulation to lower the cost.

The term "hybrid simulation" is generally reserved for the case in which functionally distinct analog and digital computers are linked together for the purpose of simulation.

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Feedback System:-

A significant factor in the performance of many system is that a coupling occurs between the input and output of the system. The term feedback is used to describe the phenomenum. A home heating system controlled by a thermostat is a simple example of a feedback system. The system has a furnace whose purpose is to heat a room. and the output of the system can be measured as room temperature. Depending upon whether the temperature is below or above the thermostat setting, the furnace will be turned off or on, so that information is fed back from the output to the input. In this case, there are only two states, either the furnace is on or off.

Another example of a feedback system in which there is continuous control is the aircraft system. Here the input is desired aircraft heading and the output is the actual heading. The gyroscope of the autopilot is able to detect the difference between the two headings. A feedback is established by using the difference to operate the control surface, since change of heading will then affect the signal being used to control the heading. The difference between desired heading and actual heading is called the error. It is denoted by ϵ.

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