Thoughts About TARN

April 22, 2015

1 Thoughts

- As Alex observed, about 94% of paths stay inside $(90, 110)$ and lead to a payoff of -100. Only about 6% of paths exit these barriers.
- However, what happens is that when a path exits the barrier, there is a big discontinuity in the payoff because of the discontinuity of f.
- So what happens is that the paths that leave the barriers have a payoff that is much different from -100.
- Can we 'push' some particles towards leaving the barriers? Tried this today (01/04/15) with some weighting fuunctions and managed to do somewhat.
- But it is not clear how this will help us.
- The payoffs (of the M paths) are not continuous and this is inherent to the problem.
- But so what?
- The estimated payoff is the mean of a (large) bunch of negative values and a (small) bunch of positive values. Can we basically use importance sampling to estimate these negative values and positive values acccurately?
- It is not immediately obvious how to do it.

Further thoughts:

1. $$\begin{aligned}
\mathbb{E}\left[\sum_{i=1}^{\tau_m} f(S_i)\right] &= \mathbb{E}\left[\sum_{i=1}^{\tau_m} f(S_i)\middle|\text{exits by time 5}\right]\mathbb{P}(\text{exits by time 5}) \\
&\quad +\mathbb{E}\left[\sum_{i=1}^{\tau_m} f(S_i)\middle|\text{doesn't exit by time 5}\right]\mathbb{P}(\text{doesn't exit by time 5}) \\
&= \mathbb{E}\left[\sum_{i=1}^{\tau_m} f(S_i)\middle|\text{exits by time 5}\right]\mathbb{P}(\text{exits by time 5}) \\
&\quad -100\times\mathbb{P}(\text{doesn't exit by time 5})
\end{aligned}$$

2. Is the problem with naive MC that it is not approximating $\mathbb{P}(\text{exits by time 5})$ well?
3. $\mathbb{P}(\text{does not exit by time 5})$ could potentially be calculated by SMC.
4. Estimating $[\sum_{i=1}^{\tau_m} f(S_i)|\text{doesn't exit by time 5}]$ is the challenge. Don't know how to do it.

2 Transforming The Problem To A More Familiar Setting

- We observe that

$$f(s) = \begin{cases} 2(s-110)+20 & \text{if } s > 110 \\ 2(80-s)+20 & \text{if } s < 90 \\ -20 & \text{if } 90 \le s \le 110 \end{cases}$$

- Define $g_m(s_{1:m}) = 100 + \sum_{i=1}^{\tau} f(s_{1:m})$ to be the new payoff function. Then $g(s_{1:m}) \ge 0$.

- The sample space is $\mathcal{S} = \mathbb{R}^m$. We can divide the sample space into two parts: $\mathcal{S} = \mathcal{S}_1 \cup \mathcal{S}_2$, where

$$\mathcal{S}_1 = \{s_{1:m} \in \mathbb{R}^m : L_5 = 100\}$$
$$\mathcal{S}_2 = \{s_{1:m} \in \mathbb{R}^m : L_5 < 100\}$$

  Particles in $\mathcal{S}_1$ lead to a payoff of 0 and particles in $\mathcal{S}_2$ lead to a positive payoff.

- This is similar to the barrier option case, wherein particles in one part of the sample space lead to a 0 payoff and particles in the other part lead to a positive payoff. The idea is to explore $\mathcal{S}_2$ more.

- For this, define

$$h_n(s_{1:n}) := \left[100 + \sum_{i=1}^{n} f(s_i)\right] \mathbb{1}\{L_n < 100\}$$

  for $n = 1, 2, \ldots, 5$.

  Then we can write

$$g_m(s_{1:m}) = h_5(s_{1:5}) \times \frac{g_m(s_{1:m})}{h_5(s_{1:5})}$$

- We say that a particle is 'dead' if it is still inside the pit by time 5, i.e, when $L_5 = 100$.

- If a particle is dead, then it leads to a 0 payoff and is rejected and resampled among particles that are still alive. This is similar to the barrier option case.