# Advanced image synthesis

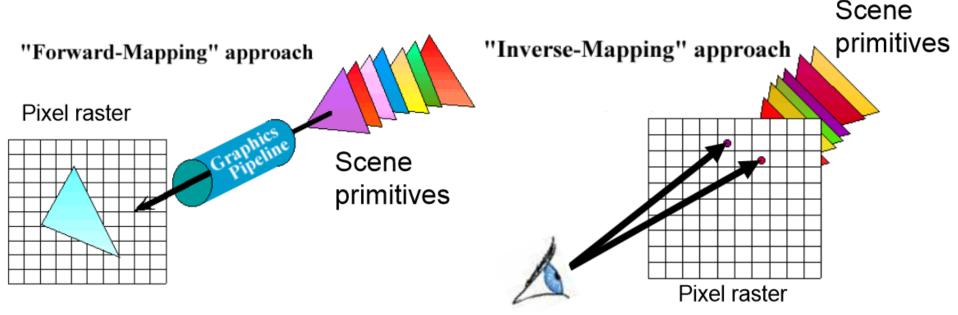
Romain Vergne – 2014/2015



#### Rasterization VS ray-casting

- For each triangle
  - Project triangle to image plane
  - For each pixel
    - Check pixel in triangle
    - Resolve visibility with z-buffer

- For each pixel
  - Compute pixel ray
  - For each triangle
    - Check ray-triangle intersection
    - Get closest intersection



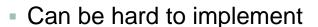


#### Ray-casting pros / cons

- Generality
  - Not limited to triangles: can render anything
  - Polygons, implicit, b-rep, etc...
- Shadows, reflection, refraction
  - Uniform handling
  - Directly obtained via recursion



Path tracing, photon mapping, etc...



- Entire scene in memory
- Can be slow with large scenes
  - But...







#### Ray-casting: summary

- For each pixel
  - Compute eye ray
  - For each object
    - Check ray-object intersection
    - Get closest intersection
    - Shade depending on light and normal vector

Finding intersection point and normal is the central part of ray-casting!



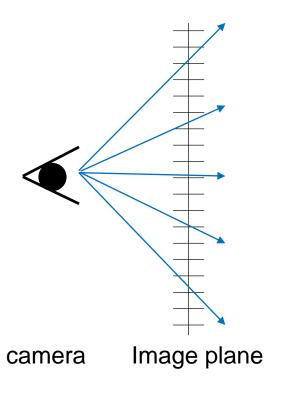
#### Eye ray and camera

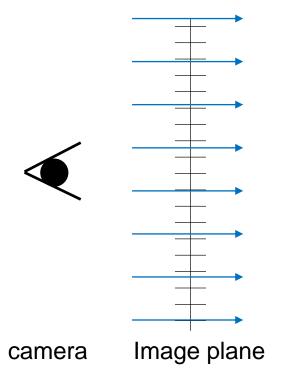
Persective

 $r = (x^*u,aspect^*y^*v,D^*w), normalized$  $P(t) = e + t^*r$  Orthographic

$$P(t) = o + t*w$$

$$o = e + x*size*u + y*size*v$$







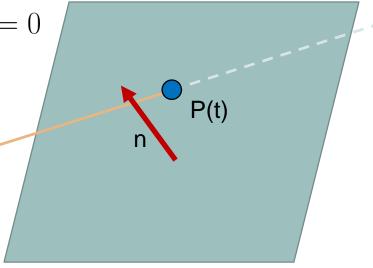
#### Ray-plane intersection

- Parametric ray equation:  $P(t) = r_o + r_d * t$
- Implicit plane equation: Ax + By + Cz + D = 0

$$n \cdot P + D = 0$$

- Signed distance to plane!
- Intersection:  $n \cdot (r_0 + r_d * t) + D = 0$

$$t = \frac{-(D + r_o \cdot n)}{r_d \cdot n}$$



rd rd

Normal: constant (n)



#### Ray-sphere intersection

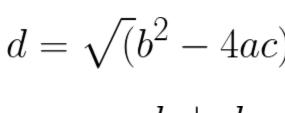
- Parametric ray equation:  $P(t) = r_o + r_d * t$
- Implicit sphere equation:  $||P O|| r^2 = 0$

$$||r_o + r_d * t - O|| - r^2 = 0$$

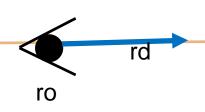
• 
$$(r_o + r_d * t - O) \cdot (r_o + r_d * t - O) - r^2 = 0$$

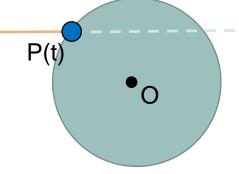
$$(r_d \cdot r_d)t^2 + (2r_o \cdot r_d - 2r_d \cdot O)t + (r_o \cdot r_o - 2r_o \cdot O + O \cdot O - r^2) = 0$$

$$- at^2 + bt + c = 0, \ a = 1$$



$$t = \frac{-b \pm d}{2a}$$





Normal: P-O (normalized)

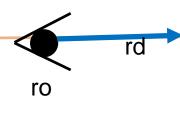
#### Ray-triangle intersection

- Ray-plane intersection
- Then test each edge...
- Better: parametric solution [Moller & Trumbore 97]

$$T(u,v) = (1-u-v)V_0 + uV_1 + vV_2, \quad u \ge 0, \ v \ge 0 \quad \text{and } u+v \le 1.$$

$$O + tD = (1 - u - v)V_0 + uV_1 + vV_2$$

See
http://www.cs.virginia.edu/~gfx/Courses/2003/ImageSynthesis/papers/Acceleration/Fast%20MinimumStorage%20RayTriangle%20Intersection.pdf



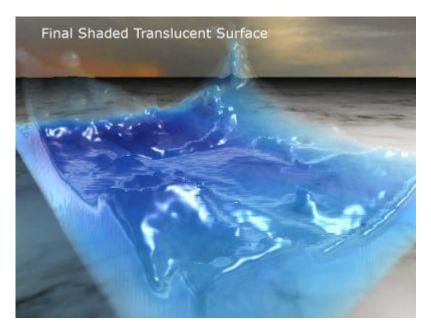




#### Ray-casting: summary

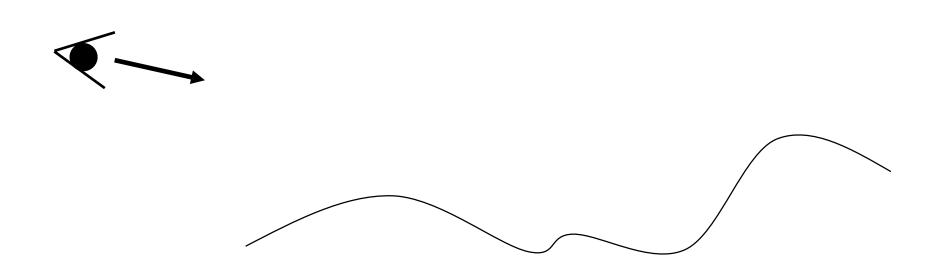
- For each pixel
  - Compute eye ray
  - For each object
    - Check ray-object intersection
    - Get closest intersection
    - Shade depending on light and normal vector

What if intersection cannot be computed analytically?





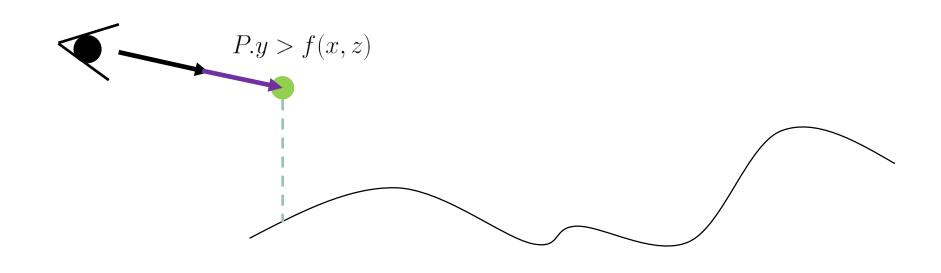
$$P(t) = r_0 + r_d * t$$



camera

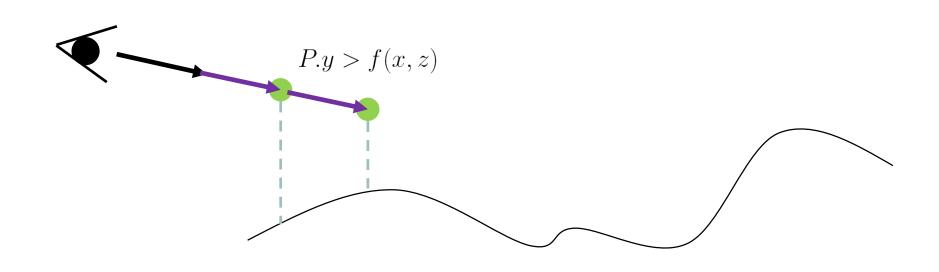


$$P(t) = r_0 + r_d * t$$

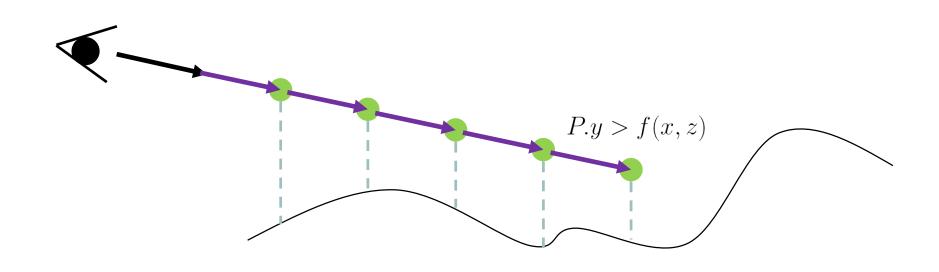




$$P(t) = r_0 + r_d * t$$



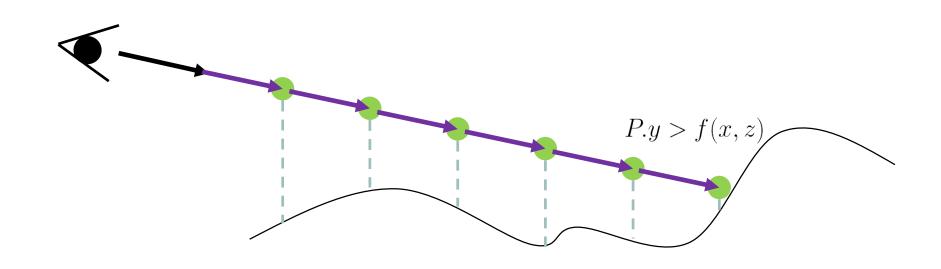
$$P(t) = r_0 + r_d * t$$







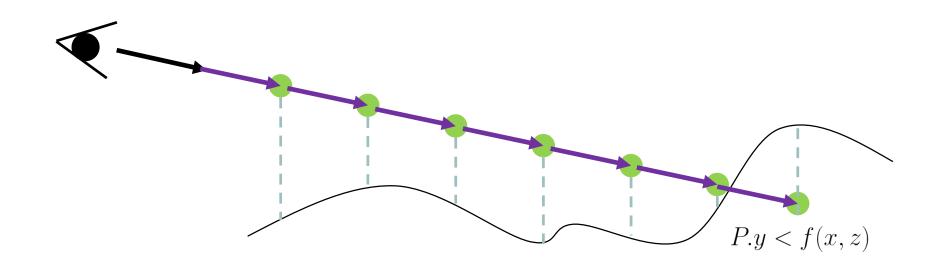
$$P(t) = r_0 + r_d * t$$





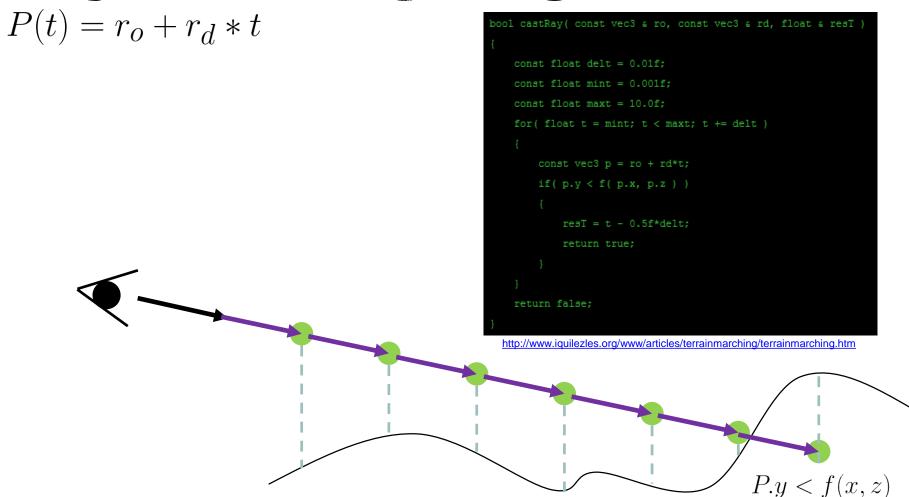


$$P(t) = r_0 + r_d * t$$









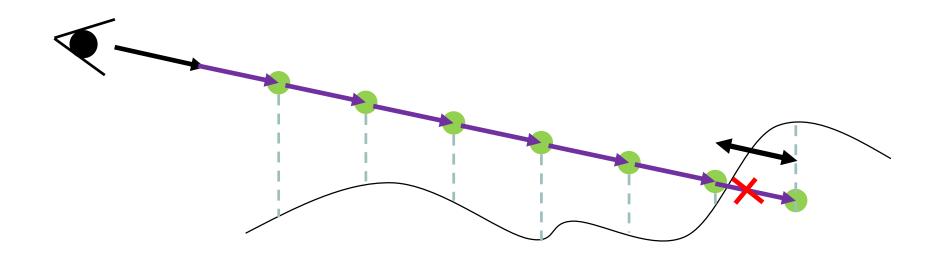
camera



$$P(t) = r_0 + r_d * t$$

#### **Optimizations:**

Interpolate between the 2 last positions

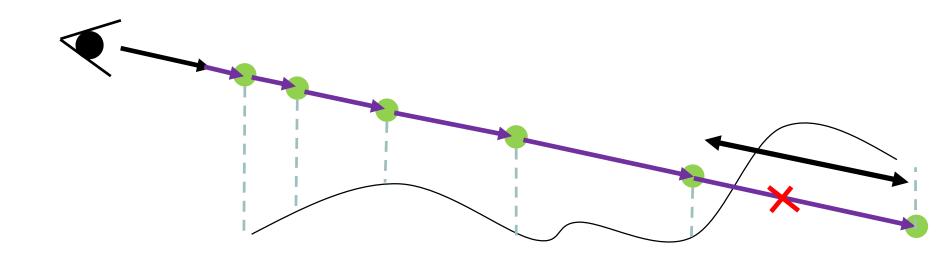




$$P(t) = r_0 + r_d * t$$

#### **Optimizations:**

- Interpolate between the 2 last positions
- Increase deltaT with distance from eye



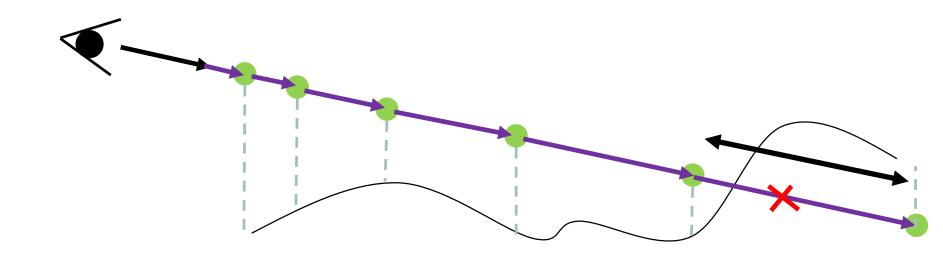




$$P(t) = r_0 + r_d * t$$

#### **Optimizations:**

- Interpolate between the 2 last positions
- Increase deltaT with distance from eye
- See: <a href="http://www.iquilezles.org">http://www.iquilezles.org</a>







 $P(t) = r_o + r_d * t$ 

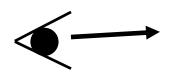
#### **Optimizations:**

- Interpolate between the 2 last positions
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- See: <a href="http://www.iquilezles.org">http://www.iquilezles.org</a>

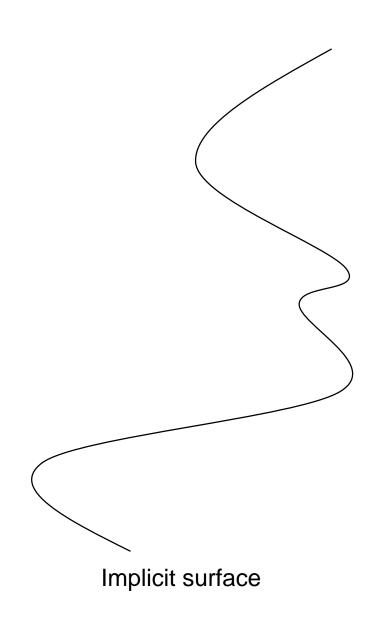
#### Normal computation:



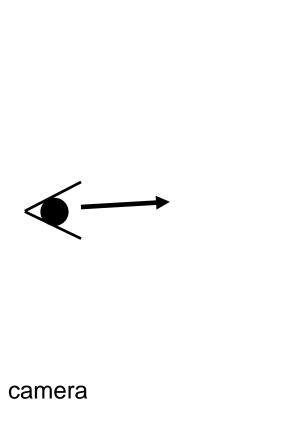
$$P(t) = r_o + r_d * t$$

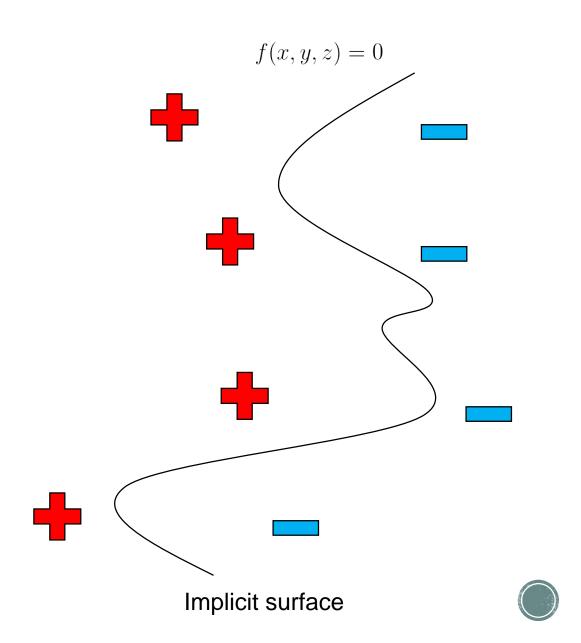


camera

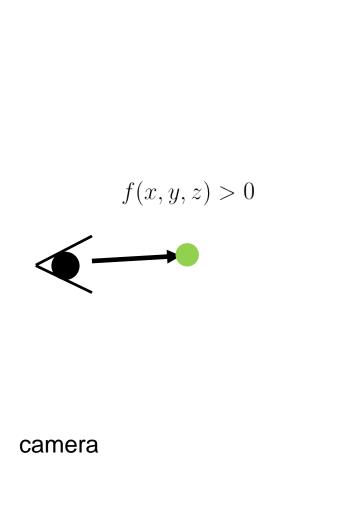


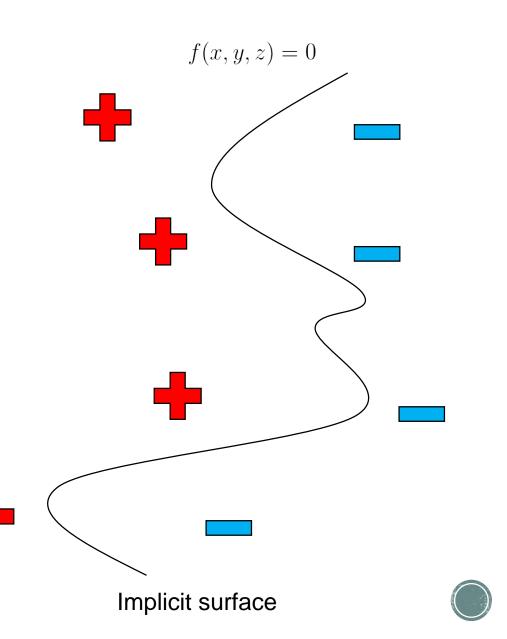
$$P(t) = r_o + r_d * t$$





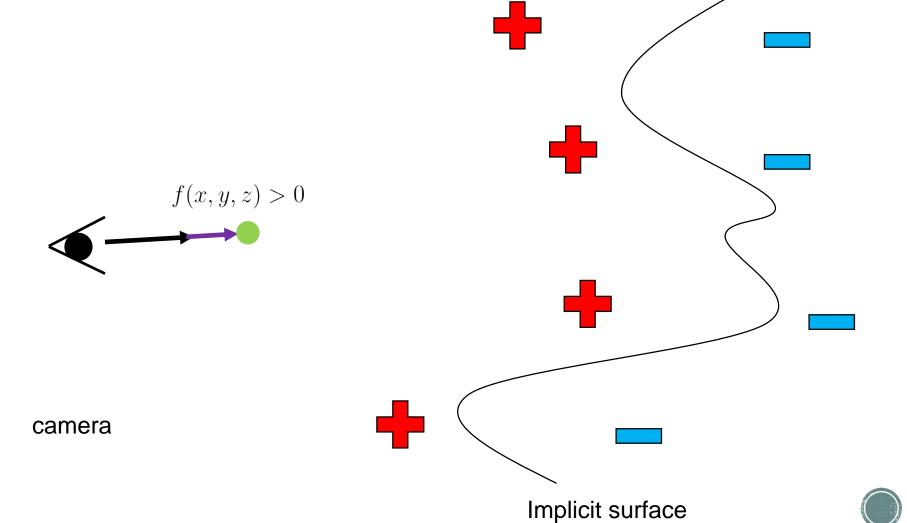
$$P(t) = r_o + r_d * t$$



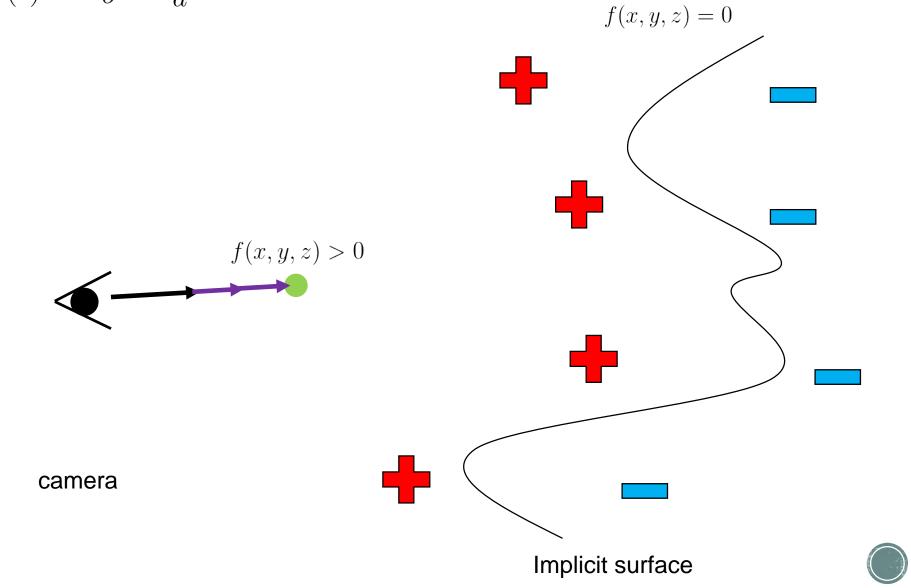


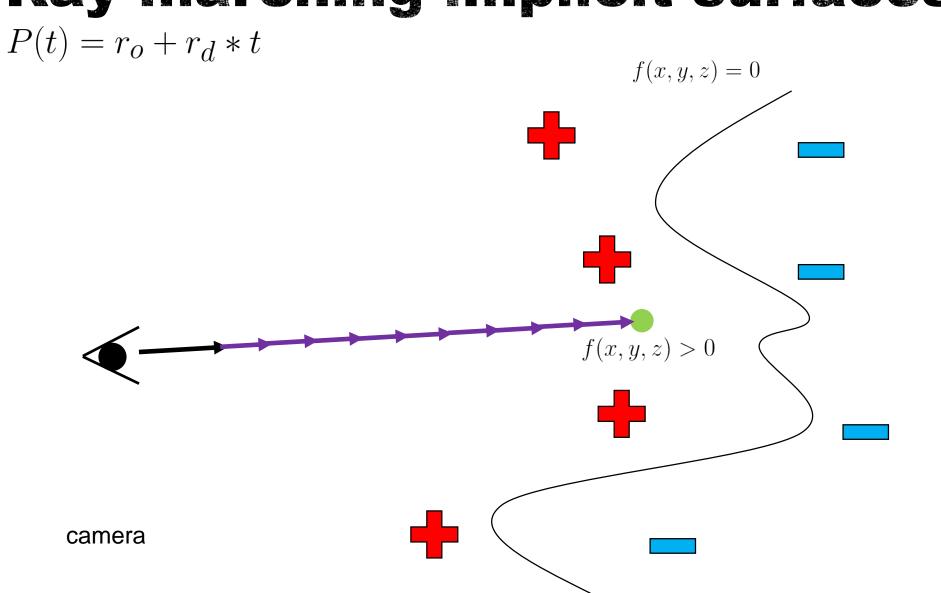
f(x, y, z) = 0

$$P(t) = r_0 + r_d * t$$

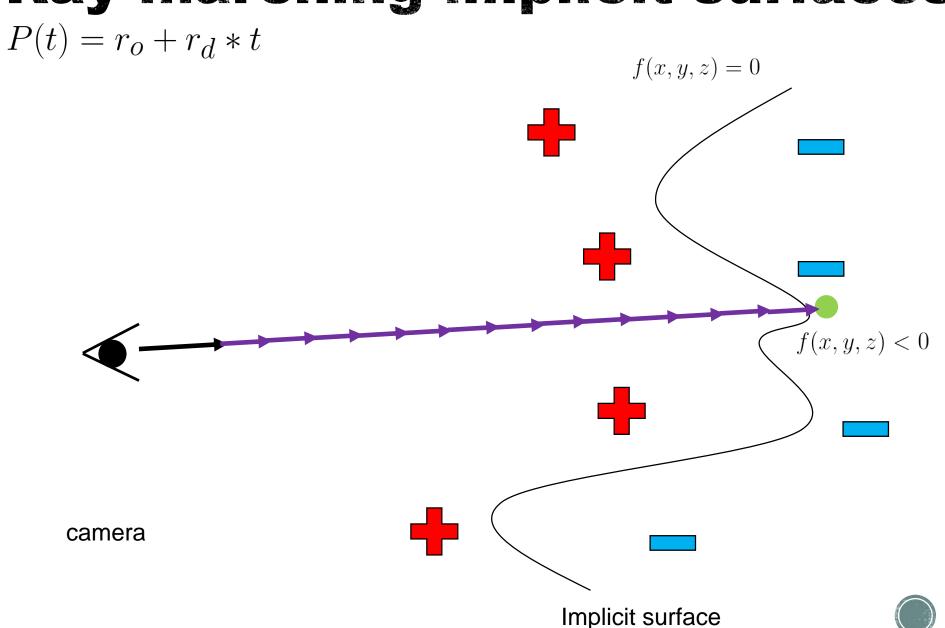


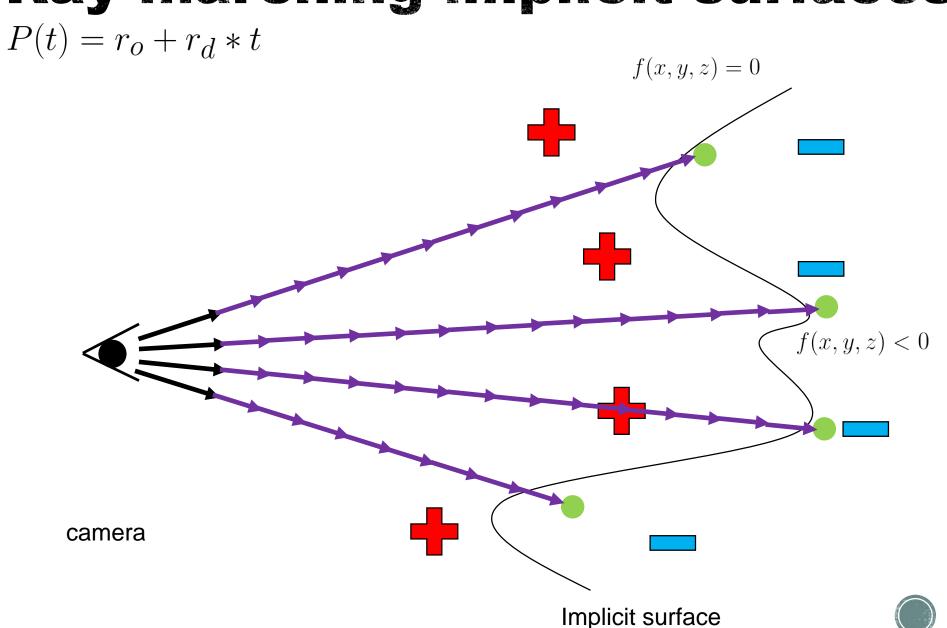
$$P(t) = r_0 + r_d * t$$

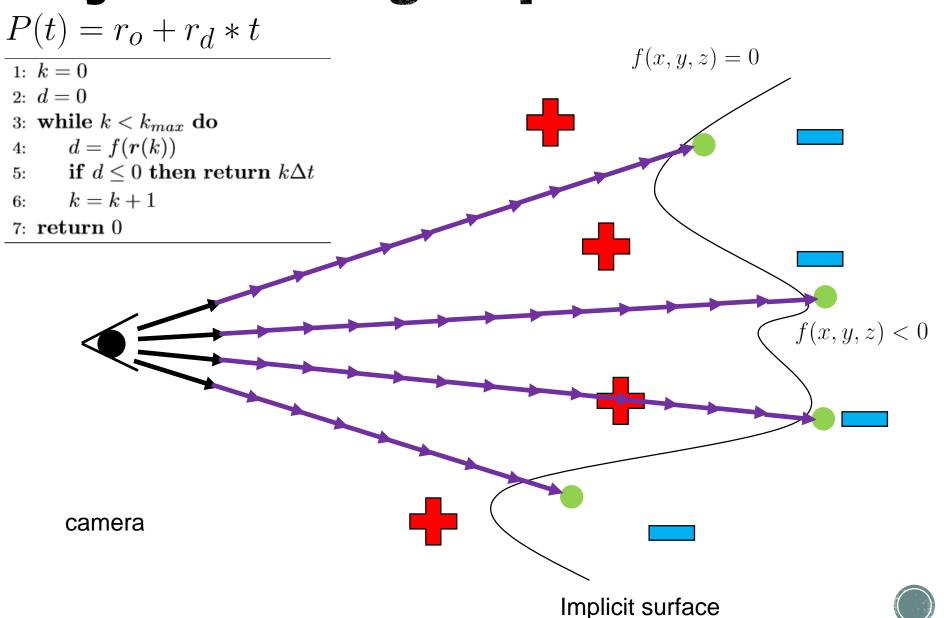




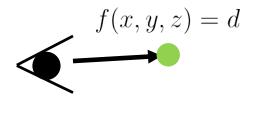
Implicit surface



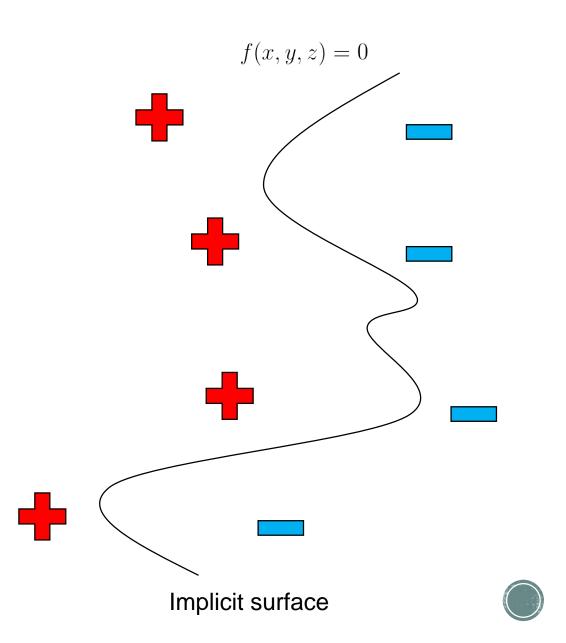


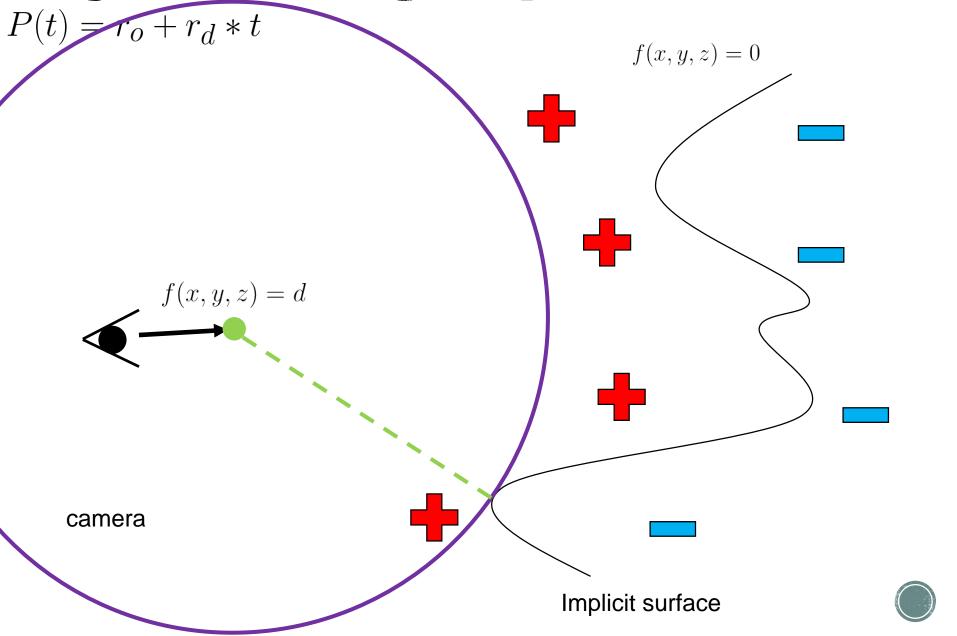


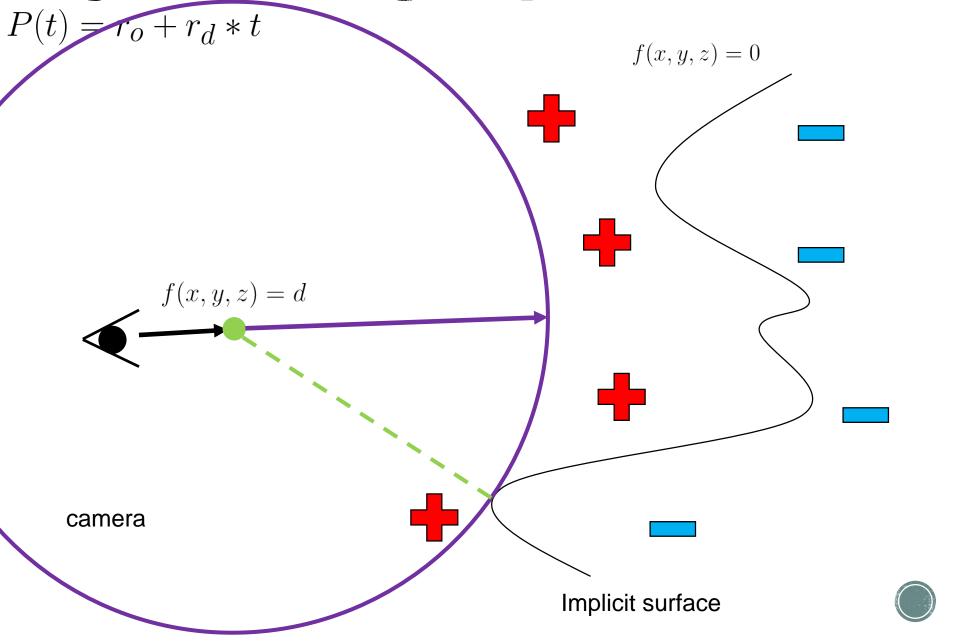
$$P(t) = r_o + r_d * t$$

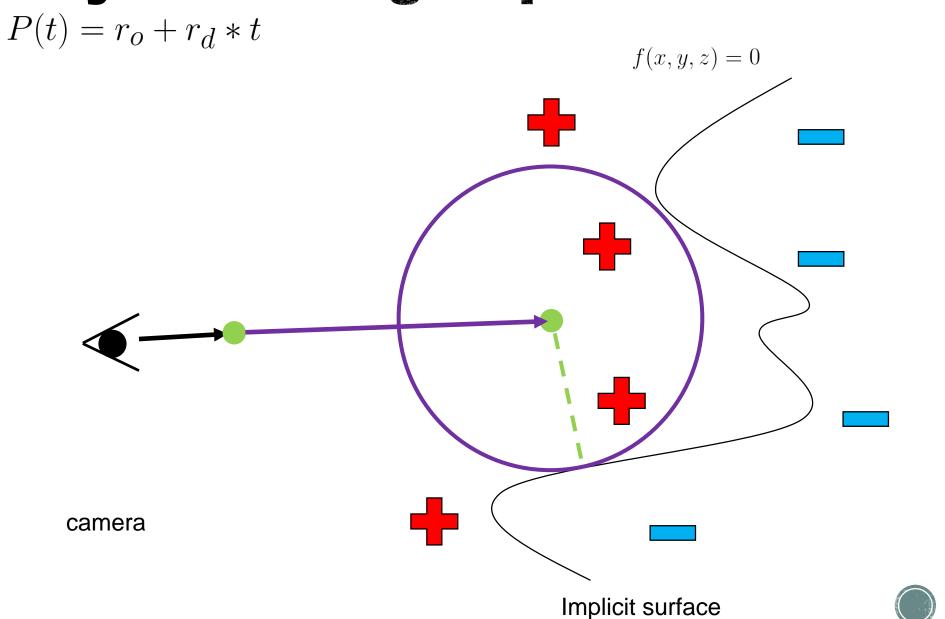


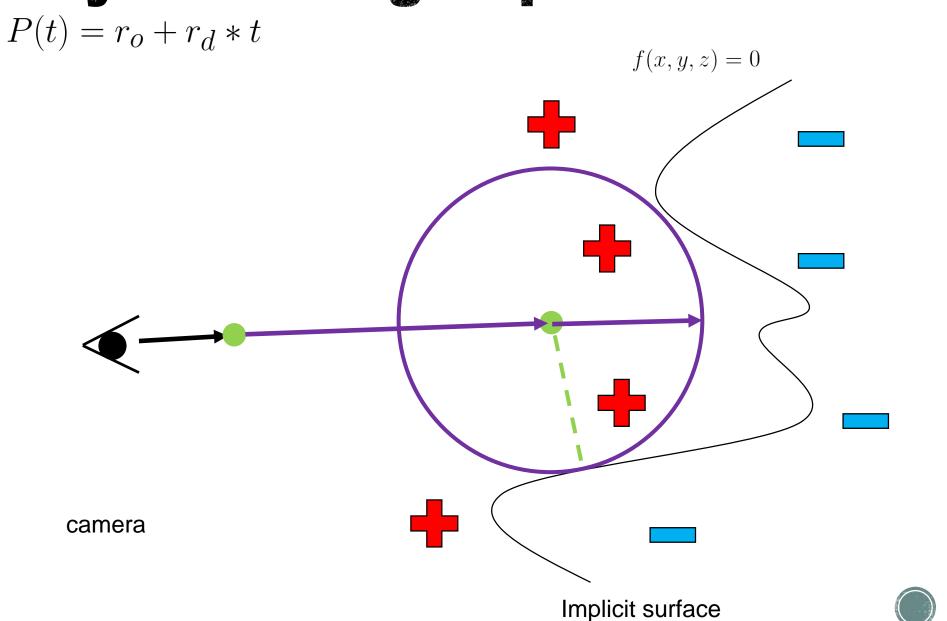
camera

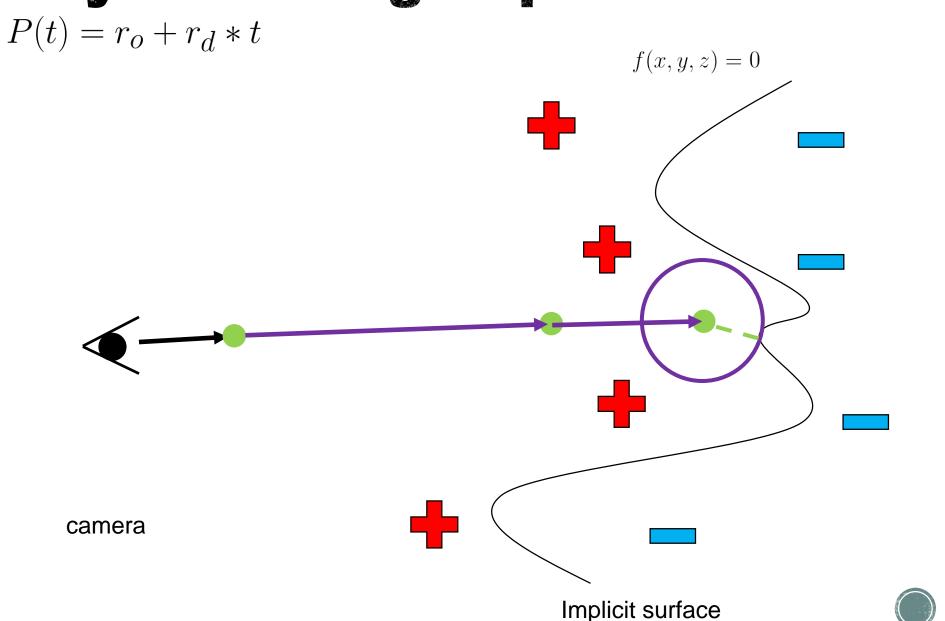


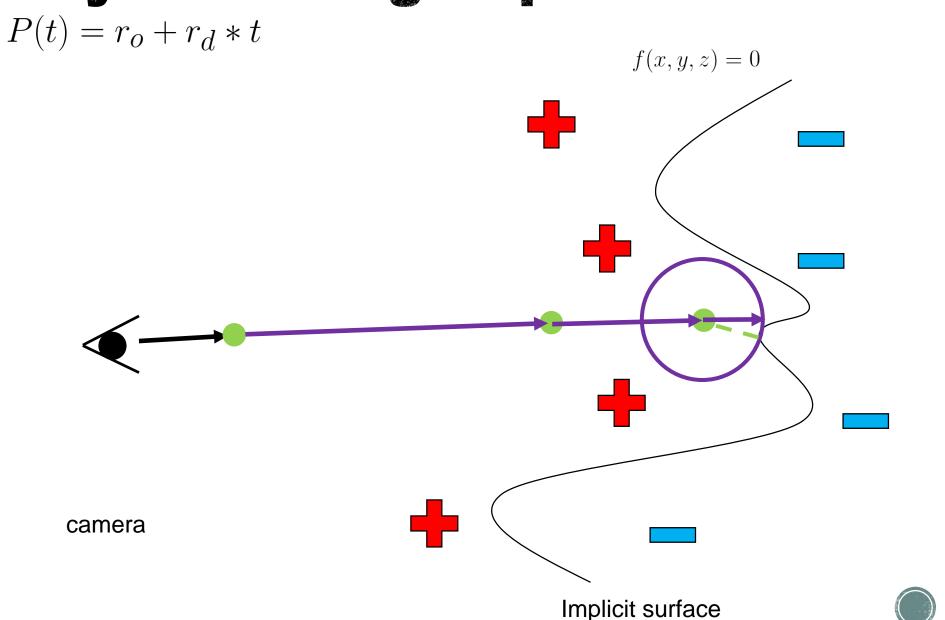


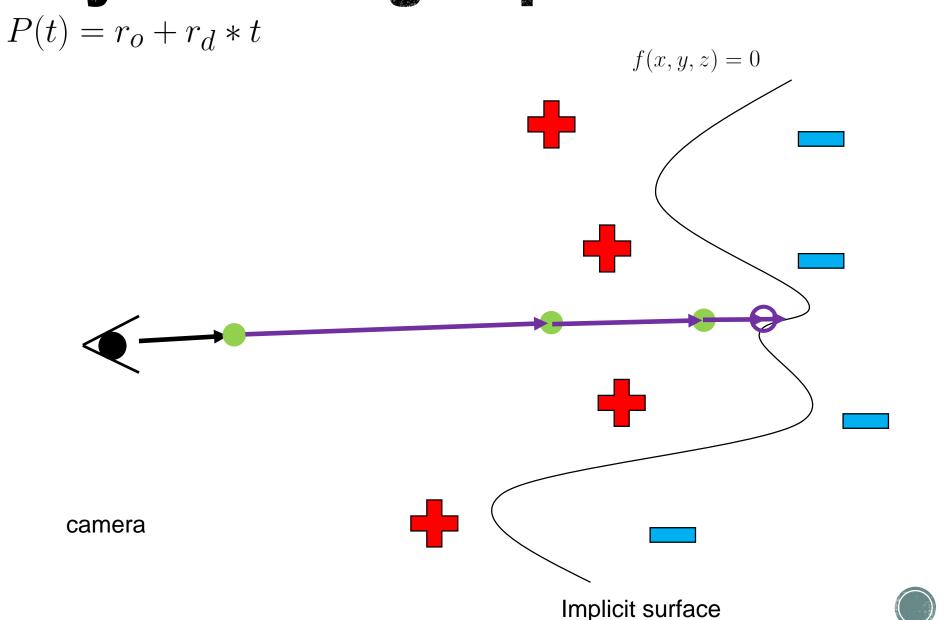


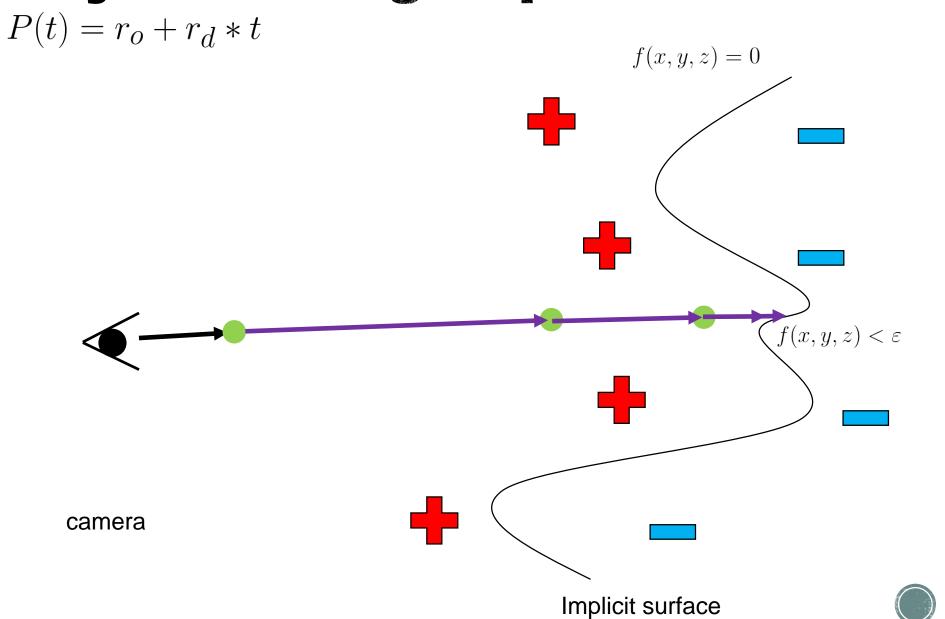






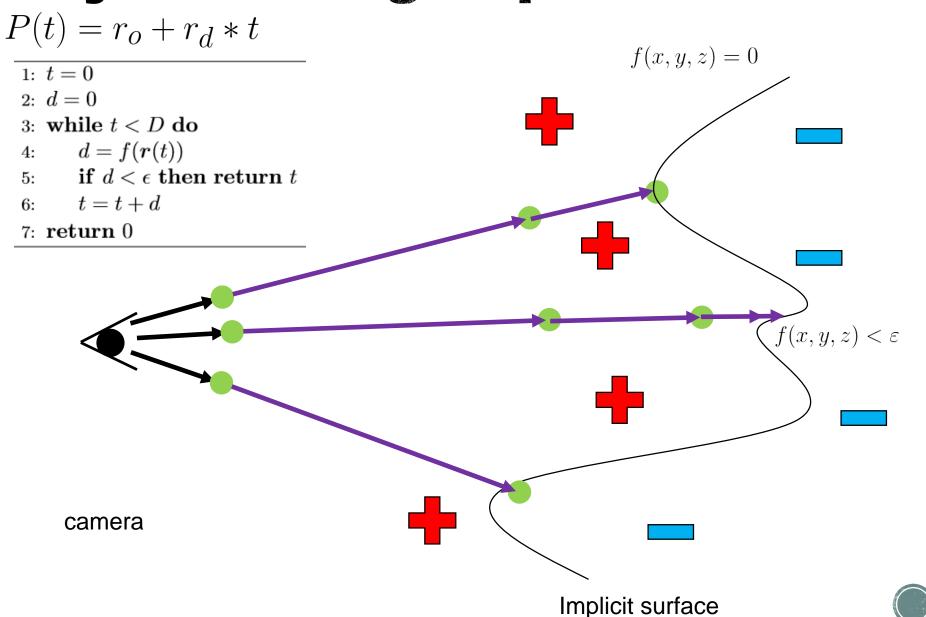






 $P(t) = r_0 + r_d * t$ f(x, y, z) = 0 $f(x, y, z) < \varepsilon$ camera

Implicit surface



## $P(t) = r_0 + r_d * t$

- 1: t = 0
- 2: d = 0
- 3: while t < D do
- $d = f(\mathbf{r}(t))$
- 5: if  $d < \epsilon$  then return t
- 6: t = t + d
- 7: return 0

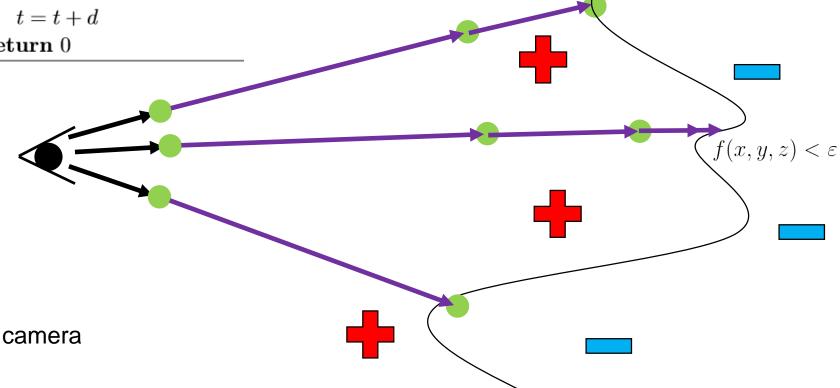
## Normal computation:

Implicit surface

$$n_x = f(x + \epsilon, y, z) - f(x - \epsilon, y, z)$$

$$n_y = f(x, y + \epsilon, z) - f(x, y - \epsilon, z)$$

$$n_z = f(x, y, z + \epsilon) - f(x, y, z - \epsilon)$$



## Sphere - signed

```
float sdSphere( vec3 p, float s )
{
  return length(p)-s;
}
```

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### Plane - signed

```
float sdPlane( vec3 p, vec4 n )
{
    // n must be normalized
    return dot(p,n.xyz) + n.w;
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## Box - signed

### Sphere - signed

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```

### Plane - signed

```
float sdPlane( vec3 p, vec4 n )
{
   // n must be normalized
  return dot(p,n.xyz) + n.w;
}
```

#### Box - signed

```
float sdBox( vec3 p, vec3 b )
{
  vec3 d = abs(p) - b;
  return min(max(d.x,max(d.y,d.z)),0.0) +
        length(max(d,0.0));
}
```

### Torus - signed

```
float sdTorus( vec3 p, vec2 t )
{
  vec2 q = vec2(length(p.xz)-t.x,p.y);
  return length(q)-t.y;
}
```

### Round Box - unsigned

```
float udRoundBox( vec3 p, vec3 b, float r )
{
  return length(max(abs(p)-b,0.0))-r;
}
```

### Cone - signed

```
float sdCone( vec3 p, vec2 c )
{
    // c must be normalized
    float q = length(p.xy);
    return dot(c,vec2(q,p.z));
}
```



### Sphere - signed

```
float sdSphere( vec3 p, float s )
{
  return length(p)-s;
}
```

#### Plane - signed

```
float sdPlane( vec3 p, vec4 n )
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   // n must be normalized
  return dot(p,n.xyz) + n.w;
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#### Box - signed

#### Torus - signed

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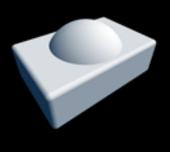
### Cone - signed

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float sdCone( vec3 p, vec2 c )
{
    // c must be normalized
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    return dot(c,vec2(q,p.z));
}
```

## Distance operations

### Union

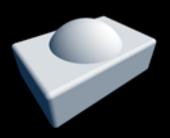
```
float opU( float d1, float d2 )
{
    return min(d1,d2);
}
```



## Distance operations

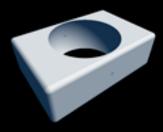
### Union

```
float opU( float d1, float d2 )
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    return min(d1,d2);
}
```



## Substraction

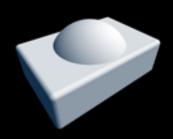
```
float opS( float d1, float d2 )
{
    return max(-d1,d2);
}
```



## Distance operations

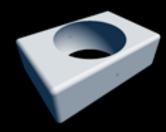
## Union

```
float opU( float d1, float d2 )
{
    return min(d1,d2);
}
```



## Substraction

```
float opS( float d1, float d2 )
{
    return max(-d1,d2);
}
```



## Intersection

```
float opI( float d1, float d2 )
{
    return max(d1,d2);
}
```



## Domain operations

## Repetition

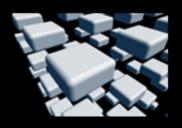
```
float opRep( vec3 p, vec3 c )
{
    vec3 q = mod(p,c)-0.5*c;
    return primitve( q );
}
```



# Domain operations

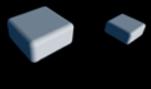
## Repetition

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#### Scale

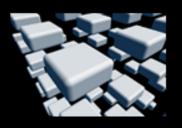
```
float opScale( vec3 p, float s )
{
    return primitive(p/s)*s;
}
```



# Domain operations

## Repetition

```
float opRep( vec3 p, vec3 c )
{
    vec3 q = mod(p,c)-0.5*c;
    return primitve( q );
}
```



#### Scale

```
float opScale( vec3 p, float s )
{
    return primitive(p/s)*s;
}
```



## Rotation/Translation

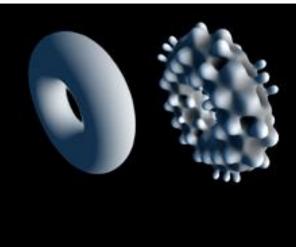
```
vec3 opTx( vec3 p, mat4 m )
{
    vec3 q = invert(m)*p;
    return primitive(q);
}
```



## Distance deformations

## Displacement

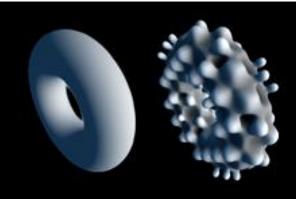
```
float opDisplace( vec3 p )
{
   float d1 = primitive(p);
   float d2 = displacement(p);
   return d1+d2;
}
```



## Distance deformations

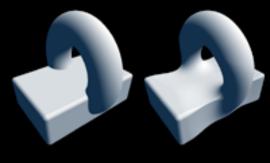
## Displacement

```
float opDisplace( vec3 p )
{
    float d1 = primitive(p);
    float d2 = displacement(p);
    return d1+d2;
}
```



## Blend

```
float opBlend( vec3 p )
{
    float d1 = primitiveA(p);
    float d2 = primitiveB(p);
    return smin( d1, d2 );
}
```

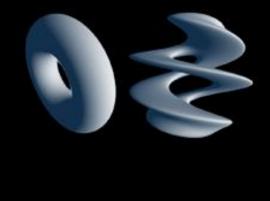


```
// polynomial smooth min (k = 0.1);
float smin( float a, float b, float k )
{
    float h = clamp( 0.5+0.5*(b-a)/k, 0.0, 1.0 );
    return mix( b, a, h ) - k*h*(1.0-h);
}
for instance...
```

## Domain deformations

### **Twist**

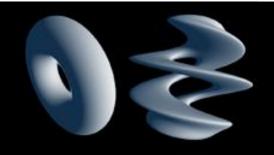
```
float opTwist( vec3 p )
{
    float c = cos(20.0*p.y);
    float s = sin(20.0*p.y);
    mat2 m = mat2(c,-s,s,c);
    vec3 q = vec3(m*p.xz,p.y);
    return primitive(q);
}
```



## Domain deformations

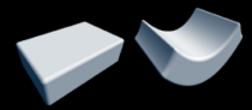
### **Twist**

```
float opTwist( vec3 p )
{
    float c = cos(20.0*p.y);
    float s = sin(20.0*p.y);
    mat2 m = mat2(c,-s,s,c);
    vec3 q = vec3(m*p.xz,p.y);
    return primitive(q);
}
```



## Cheap Bend

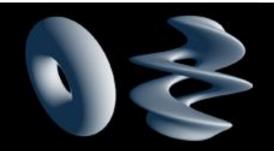
```
float opCheapBend( vec3 p )
{
    float c = cos(20.0*p.y);
    float s = sin(20.0*p.y);
    mat2 m = mat2(c,-s,s,c);
    vec3 q = vec3(m*p.xy,p.z);
    return primitive(q);
}
```



## Domain deformations

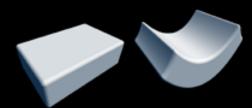
## **Twist**

```
float opTwist( vec3 p )
{
    float c = cos(20.0*p.y);
    float s = sin(20.0*p.y);
    mat2 m = mat2(c,-s,s,c);
    vec3 q = vec3(m*p.xz,p.y);
    return primitive(q);
}
```



## **Cheap Bend**

```
float opCheapBend( vec3 p )
{
    float c = cos(20.0*p.y);
    float s = sin(20.0*p.y);
    mat2 m = mat2(c,-s,s,c);
    vec3 q = vec3(m*p.xy,p.z);
    return primitive(q);
}
```



### Demos:

https://www.shadertoy.com/view/Xds3zNhttps://www.shadertoy.com/view/Mss3zM





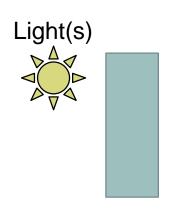
#include <stdlib.h> // card > aek.ppm #include <stdio.h> #include <math.h> typedef int i; typedef float f; struct v{ f x,y,z;v operator+(v r){return v(x+r.x ,y+r.y,z+r.z);}v operator\*(f r){ return v(x\*r,y\*r,z\*r);}f operator%(v r){return x\*r.x+y\*r.y+z\*r.z;}v(){}v operator^(v r ) {return v(y\*r.z-z\*r.y,z\*r.x-x\*r.z,x\*r. y-y\*r.x);} $v(f a, f b, f c){x=a; y=b; z=c;}v$ operator! () {return\*this\*(1/sqrt(\*this%\* this));}};i G[]={133022, 133266,133266, 133022, 254096, 131216, 131984, 131072, 258048, }; f R() {return(f) rand() / RAND MAX ;}i T(v o,v d,f&t,v&n) {t=1e9;i m=0;f p= -o.z/d.z; if (.01<p) t=p, n=v(0,0,1), m=1; for(i k=19;k--;)for(i j=9;j--;)if(G[j]&  $1 << k) \{v \ p=o+v(-k,0,-j-4); f \ b=p d, c=p p 1,q=b*b-c;if(q>0){f s=-b-sqrt(q);if(s<t}$ &&s>.01) t=s, n=! (p+d\*t), m=2;}}return m;} v S(v o, v d) {f t; v n; i m=T(o,d,t,n); if( !m) return v(.7,.6,1) \*pow(1-d.z,4); v h=o +d\*t, l=!(v(9+R(), 9+R(), 16)+h\*-1), r=d+n\* $(n^d-2)$ ; f b=1%n; if (b<0||T(h,1,t,n))b=0 ;f p=pow(1%r\*(b>0),99);if(m&1){h=h\*.2; return((i)(ceil(h.x)+ceil(h.y))&1?v(3,1),1):v(3,3,3))\*(b\*.2+.1);}return v(p,p,p )+S(h,r)\*.5;}i main(){printf("P6 512 " "512 255 "); v = v(-6, -16, 0), a = v(0, 0, 0)1)  $^{\circ}$ g) \*.002, b=! (g^a) \*.002, c=(a+b) \*-256+g ;for(i y=512; y--;) for(i x=512; x--;) {v p (9,9,9); for (i r=64;r--;) {v t=a\*(R()-.5) \*99+b\*(R()-.5)\*99;p=S(v(17,16,8)+t,!(t\*-1+(a\*(R()+x)+b\*(v+R())+c)\*16))\*3.5+p;printf("%c%c%c",(i)p.x,(i)p.v,(i)p.z);}}





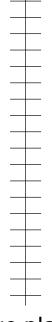
## **Basically 2 functions:**

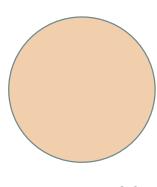
- trace
  - find intersection with an object
- directIllumination
  - Direct lighting at a given point







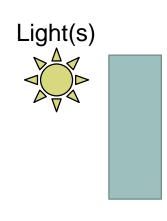




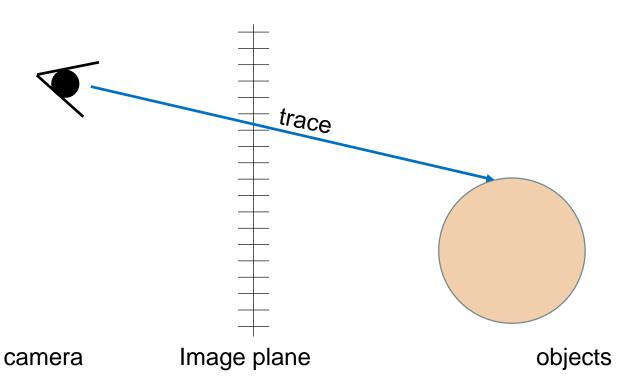
objects

## **Basically 2 functions:**

- trace
  - find intersection with an object
- directIllumination
  - Direct lighting at a given point



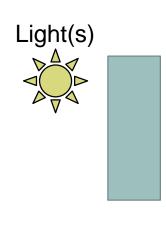


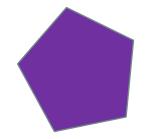


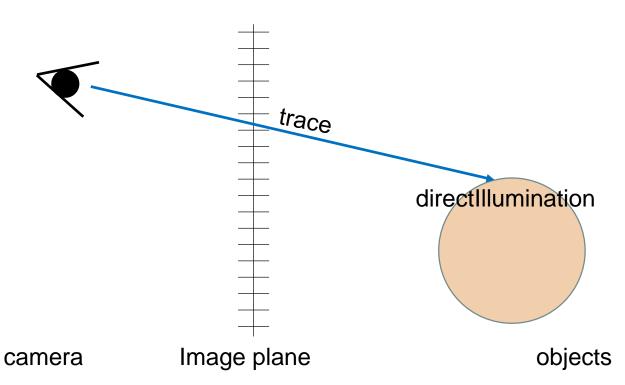


## **Basically 2 functions:**

- trace
  - find intersection with an object
- directIllumination
  - Direct lighting at a given point





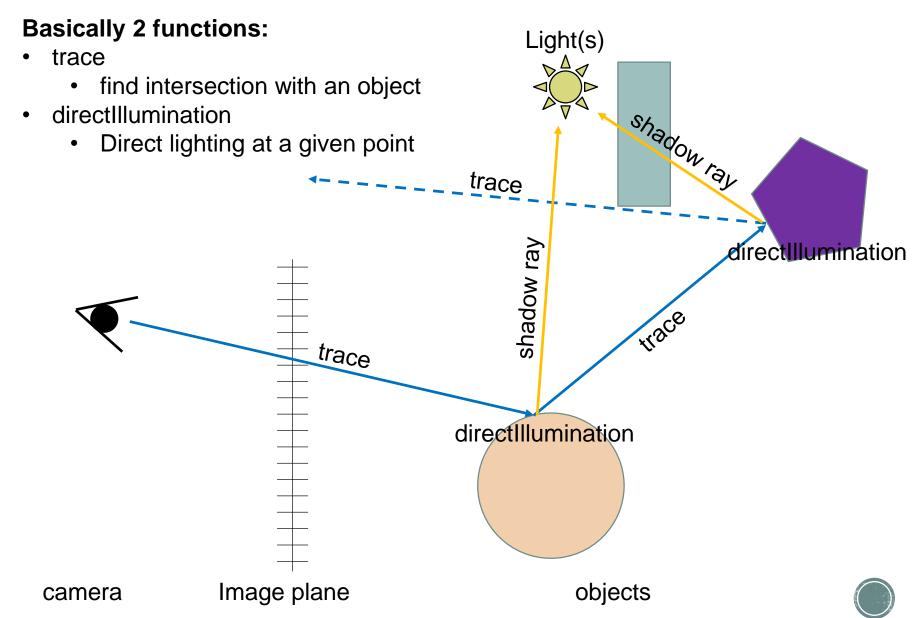




## **Basically 2 functions:** Light(s) trace find intersection with an object directIllumination Direct lighting at a given point shadow ray trace directIllumination Image plane objects camera

## **Basically 2 functions:** Light(s) trace find intersection with an object directIllumination Direct lighting at a given point shadow ray directIllumination trace directIllumination Image plane objects camera

## **Basically 2 functions:** Light(s) trace find intersection with an object shadow ray directIllumination Direct lighting at a given point shadow ray directIllumination trace directIllumination Image plane objects camera



```
color trace(ray) {
  • hit = intersectScene(ray)
  • if(hit) {
    color = directIllumination(hit)
    if hit is reflective
       color += c_refl * trace(reflected ray)
    if hit is transmissive
       color += c_trans * trace(refracted ray)
  } else
    color = background_color
  return color
```

```
color trace(ray) {
  hit = intersectScene(ray)
  • if(hit) {
    color = directIllumination(hit)
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       color += c_refl * trace(reflected ray)
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    • if hit is transmissive
       color += c_trans * trace(refracted ray)
  } else
    color = background_color
  return color
```

```
color directIllumination(hit) {
  - color = (0,0,0)
  for each light L {
    T = cast shadow ray to L
    if hit is not shadowed by L
      color += Ambient+diffuse+specular terms(L,hit)
  • }
  return color
```

```
color directIllumination(hit) {
  - color = (0,0,0)
  for each light L {
     T = cast shadow ray to L
    if hit is not shadowed by L
      color += Ambient+diffuse+specular terms(L,hit)
  • }
  return color
```

Difference between eye ray and shadow ray?



```
color directIllumination(hit) {
  - color = (0,0,0)
  for each light L {
    T = cast shadow ray to L
    if hit is not shadowed by L
      color += Ambient+diffuse+specular terms(L,hit)
  • }
  return color
```

Material properties (we will see it soon in details)



```
color trace(ray) {
  • hit = intersectScene(ray)
  if(hit) {
      color = directIllumination(hit)
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       color += c_refl * trace(reflected ray)
    • if hit is transmissive
       color += c_trans * trace(refracted ray)
  } else
    color = background_color
  return color
```

```
color trace(ray) {
  • hit = intersectScene(ray)
  • if(hit) {
                                                    Recursive!
    color = directIllumination(hit)
    if hit is reflective
       color += c_ref * trace(reflected ray)
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       color += c_trans * trace(refracted ray)
  else
    color = background_color
  return color
```

```
color trace(ray) {
  • hit = intersectScene(ray)
  • if(hit) {
                                                       Recursive!
    color = directIllumination(hit)
    if hit is reflective
       color += c_ref * trace(reflected ray)
    if hit is transmissive
       color += c_trans * trace(refracted ray)
  } else
                                       How to stop?
    color = background_color
                                         Recursion depth
  return color
                                             After a number of bounces
                                         Ray contribution
                                             Reflected/transmitted
                                             contribution becomes too small
```



```
color trace(ray) {
  • hit = intersectScene(ray)
  • if(hit) {
                                                     Recursive!
    color = directIllumination(hit)
    if hit is reflective
       color += c_ref * trace(reflected ray)
    if hit is transmissive
       color += c_trans * trace(refracted ray)
  else
                                      How to stop?
    color = background_color
  return color
```

# glossary

- Ray casting:
  - eye ray only
- Ray tracing:
  - all secondary rays
- Shadow ray:
  - surface to light ray (shadow test)
- Ray marching:
  - step by step surface intersection test



## References

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  - https://www.opengl.org/documentation/glsl/
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