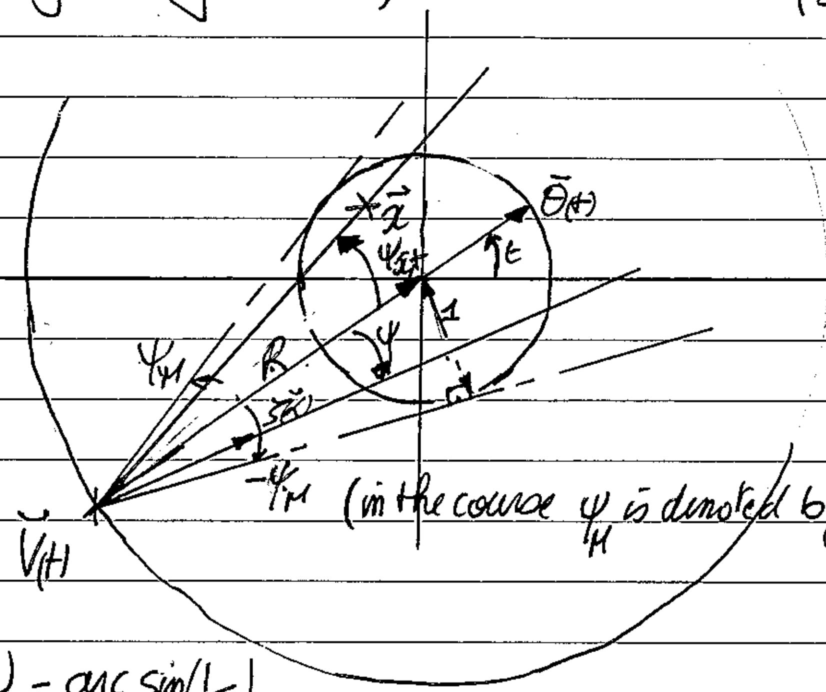
Algorithm F.BP for F.B. data. circular trajectory.

The algorithm is nothing but the discretization of the analytical formula in three steps: data Weighting; Filtering; Weighted Back projection

Acquisition Geometry (circular). $\vec{v}(t) = -R\,\vec{\theta}(t) = -R\begin{pmatrix}\cos t\\ \sin t\end{pmatrix}$ (here $R=3$)



Rq that $\psi_{\vec{x},t}$ is s.th. $\frac{\vec{x}-\vec{v}(t)}{\|x-v(t)\|}\cdot\vec{\theta}(t) = \cos(\psi_{\vec{x},t})$

(in the course ψ_M is denoted by α_M)

$\psi_M = \arcsin\left(\frac{1}{R}\right)$. let α such that the line of integration for $d(t,\alpha)$ is $\vec{v}(t)+\mathbb{R}\,\vec{\zeta}(\alpha)$

then $\alpha = t - \frac{\pi}{2} + \psi$

(here) note that α depends on t ~~but not~~ ψ, $\psi\in[-\psi_M\ \psi_M]$

the Jacobian (weight) is $|\vec{v}'(t)\cdot\vec{\theta}(\alpha)| = |R\sin(t-\alpha)|$ indep of t

$\left(s = \vec{v}(t)\cdot\vec{\theta}(\alpha) \to J = \frac{\partial s}{\partial t}\right)$ $= |R\sin(\alpha-t)| = |R\sin(\psi-\frac{\pi}{2})|$

$= -R(\cos t\cos\alpha + \sin t\sin\alpha) = -R\cos(t-\alpha)$ $= |R\cos\psi| = R\cos\psi$

because $\psi\in[-\arcsin(\frac{1}{R})\ \arcsin(\frac{1}{R})]$

Thus Remarking that $\alpha_\ell = t - \frac{\pi}{2} + \psi_\ell$ $\ell = 1\ldots ndet$

with $\psi = -\psi_M : h_\psi : \psi_M$

where $h_\psi\ (= h_\alpha) = \frac{2\psi_M}{(ndet-1)}$ i.e. $\psi_\ell = -\psi_M + (\ell-1)h_\psi$

we have the <u>WEIGHTING</u>:

weight $= R\cos(\psi)$

For all t Wdata$(t,:)$ = data$(t,:)$.* weight

(for all projections)

Filtering

the filter is given in the direct space but ready for us with convolution with fft and zeropadding (filter is of length $2ndet+1$, filter(1) is filter at 0)

filter=fbSheppLoganfilter(ndet, h_ψ, cutoff); ← play around $\frac{(1+R)*nproj}{2}$

ftfilter = fft(filter);

% data zero padding

zpwdata = zeros(nproj, 2ndet+1); zpwdata(:, 1:ndet) = wdata;

for all proj t (can be more efficiently done)

- ftzpwdata = fft(zpwdata(t, :))
- filteredftzpwdata = ftzpwdata .* ftfilter
- filteredzpwdata(t, :) = real(ifft(filteredftzpwdata)).

filteredwdata = filteredzpwdata(:, 1:ndet);

Backprojection at $\vec{x}_{i,j} = (x(i), x(j))$ where $x = -1+\frac{h}{2} : h : 1$

where $h = \frac{2}{N_{image}}$ $N_{image} \, 2ndet$

if $(x(i)^2 + x(j)^2) < 1$ only

we have to compute $\alpha_{\vec{x},t}$ such that $\vec{v}(t) + R\vec{\zeta}(\alpha_{\vec{x},t})$ pass through $\vec{x}$

this is equivalent to compute $\psi_{\vec{x},t}$

We remark in the figure that

$$\cos\psi_{\vec{x},t} = \frac{\vec{x}-\vec{v}(t)}{\|\vec{x}-\vec{v}(t)\|} \cdot \vec{\theta}(t)$$

(Rq $\vec{\theta}(t) = \frac{-\vec{v}(t)}{\|\vec{v}(t)\|}$)

$$\psi_{x,t} = \pm \operatorname{acos}\left(\frac{\vec{x}-\vec{v}(t)}{\|\vec{x}-\vec{v}(t)\|} \cdot \vec{\theta}(t)\right)$$

the sign is given by $\operatorname{sign}(\vec{x} \cdot \vec{\zeta}(t))$ here.

Backprojection:

For all pixel $(x(i), x(j))$

- if $(x(i)^2 + x(j)^2) < 1$
  - for all t (for all proj) compute $\psi_{x_{i,j},t}$
    - $l = 1 + \operatorname{floor}\left(\frac{\psi_{x_{i,j},t} + \psi_M}{h_\psi}\right)$
    - $mu(i,j) = mu(i,j) + \frac{\text{linearinterpo}(\text{filteredwdata}, l, l+1, \psi_{x_{i,j},t})}{\|\vec{x}-\vec{v}(t)\|^2}$

$\psi_l < \psi_x < \psi_{l+1}$

$-\psi_M + (l-1)h_\psi \simeq \psi_x$

$l = 1 + \operatorname{floor}\left(\frac{\psi_x + \psi_M}{h_\psi}\right)$

mu = mu/a..