TP TOMOGRAPHY parallel geometry

Formula $\mu(x) = \int_0^{\pi} g_F(\varphi, \vec{x}\cdot\vec{\theta}(\varphi))\, d\varphi$

with $g_F(\varphi, s) = \int_{\mathbb{R}} \hat{g}_\varphi(\sigma)|\sigma|\, e^{+2i\pi\sigma s}\, d\sigma$

and the data are $g(\varphi, s)$

In Practice g is sampled

sinogram(k,l) $= g_{k,l} = g(\varphi_k, s_l)$ $\quad \begin{cases} \varphi_k = \frac{k\pi}{P} & k = 0, \dots, P-1 \\ s_l = -1 + l\frac{2}{Q-1} & l = 0, \dots, Q-1 \end{cases}$

$g_{F,k}(l) = g_{F,\varphi_k}(s_l)$ $\quad l = 0 \dots Q-1$ is computed by Fast Fourier Transform Discrete

let $G_k = FFT(g_k)$ where $g_k(l) = g(k, l)$

then $G_k(n)$ estimate (up to fftshift) $c_n(g_k) = \frac{1}{2}\int_{-1}^{1} g_{\varphi_k}(s)\, e^{-\frac{2i\pi s n}{2}}\, ds$

$= \frac{1}{2}\int_{\mathbb{R}} g_{\varphi_k}(s)\, e^{-2i\pi s(\frac{n}{2})}\, ds$

$= \frac{1}{2}\hat{g}_{\varphi_k}(\sigma_n)$ with $\sigma_n = \frac{n}{2}$

thus $2G_k(n) \cdot \sigma_n$ is an estimate of $\hat{g}_{\varphi_k}(\sigma_n)|\sigma_n|$

$$\| $$

$$2G_k(n)\left|\frac{n}{2}\right| = G_k(n)|n|$$

thus Filtering $\begin{cases} \text{filter} = -\frac{Q}{2} : 1 : \frac{Q}{2} - 1 \\ \text{filter} = \text{abs}(\text{filter}) \\ \text{filter} = \text{fftshift}(\text{filter}) \end{cases}$

For all $k = 1 .. P$

$$G_k = FFT(g_k)$$

$$FG_k = filter .* G_k$$

$$gF_k = ifft(FG_k)$$

etc.

Backprojection:

$$\mu(x) = \int_0^{\pi} g_F(\varphi, \vec{x} \cdot \theta_{(\varphi)})\, d\varphi$$

$$\simeq \frac{\pi}{P} \sum_{k=1}^{P} g_F(\varphi_k, \vec{x} \cdot \bar{\theta}_{\varphi})$$

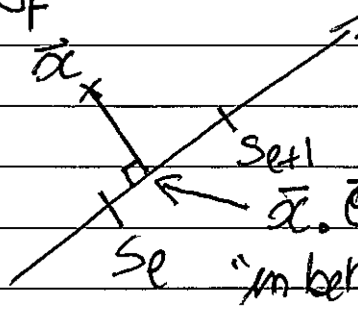
$$\mu(\vec{x}) = \frac{\pi}{P} \sum_{k=0}^{P} g_F(k, \vec{x} \cdot \bar{\theta}_{(\varphi)})$$

Rk 1 in Practice μ computed in an image $[-1\ 1] \times [-1\ 1]$
at center of pixel $x_{i,j}$
thus image$(i,j) = \mu(x_{i,j})$

$$x_{i,j} = \begin{pmatrix} -1 + (i - \frac{1}{2}) h \\ -1 + (j - \frac{1}{2}) h \end{pmatrix} \quad \text{where } h = \frac{2}{N}$$

$i = 1, \dots, N$
$j = 1, \dots, N$

Rk 2 g_F has been computed at (φ_k, s_ℓ)



$$\bar{\theta}(\varphi_k) = \begin{pmatrix} \cos\varphi_k \\ \sin\varphi_k \end{pmatrix}$$

$\bar{x} \cdot \bar{\theta}(\varphi_k) = x_1 \cos\varphi_k + x_2 \sin\varphi_k$

"in between" s_ℓ and $s_{\ell+1}$ $\Rightarrow$ linear interpolation