Statistics 108, Homework 6

Due: November 16th, 2018, In Class (turn in paper form)

*You need to show the steps to get the full credits.

This homework is to practice more on multiple linear regression. Attach the complete R codes for Problem 2 at the end of the homework. Total: 90 points.

1. (25 points) *Understanding the general linear regression model.* For each of the following models, indicate whether we can use the techniques of multiple linear regression model to estimate the coefficients β_i's or not. Explain. (Here, we assume that all X_i's are non-random, and ε_i's in each model are *i.i.d.* with $E(\varepsilon_i) = 0$ and $\text{Var}(\varepsilon_i) = \sigma^2$.)

   (a) $Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 \log X_{i2} + \beta_3 X_{i1}^2 + \varepsilon_i$.

   (b) $Y_i = \log(\beta_1 X_{i1}) + \beta_2 X_{i2} + \varepsilon_i$, where $\beta_1 > 0, X_{i1} > 0, \forall i$.

   (c) $Y_i = \log(\beta_1 + X_{i1}) + \beta_2 X_{i2} + \varepsilon_i$, where $\beta_1 > 0, X_{i1} > 0, \forall i$.

   (d) $Y_i = \exp(\beta_0 + \beta_1 X_{i1}) + \varepsilon_i$.

   (e) $Y_i = \exp(\beta_0 + \beta_1 X_{i1} + \beta_2 X_{i1} X_{i2} + \varepsilon_i)$.

2. (45 points) *Data analysis: general testing framework.* Consider the multiple linear regression model: $Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i3} + \beta_4 X_{i4} + \varepsilon_i,\ i = 1, \ldots, n, \varepsilon_i \overset{iid}{\sim} N(0, \sigma^2)$.

   Describe how you would test (at 0.05 significance level):

   (a) $H_0 : \beta_1 = \beta_2 = 0.$ vs. $H_a :$ either β_1 or β_2 not equal to 0.

   (b) $H_0 : \beta_1 = 1,\ \beta_2 = 2.$ vs. $H_a :$ not both equalities in H_0 holds.

   (c) $H_0 : \beta_2 = \beta_3.$ vs. $H_a : \beta_2 \neq \beta_3$.

   Perform the test for the dataset 'HW6Q2.txt".

3. (20 points) *Rigorous deviation.* Consider the multiple linear regression model in the matrix form $\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$ with $\text{E}(\boldsymbol{\varepsilon}) = \mathbf{0}$ and $\text{Var}(\boldsymbol{\varepsilon}) = \sigma^2 I_n$. Let $H = \mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T$ be the hat matrix. Show that

   (a) $H^T = H$, $H^2 = H$.

   (b) The diagonal elements of H are all between 0 and 1.