NO :

Date :

Assignment 1

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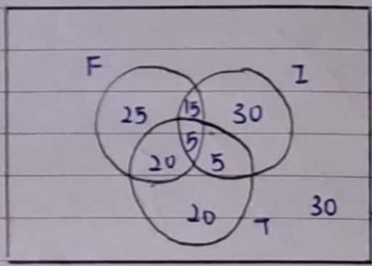
CHAU YING JIA A23CS0213

SABRINA HENG WEI QI A23CS0265

1. (a)(i) U



2

(a)(ii) $150 - 25 - 15 - 30 - 20 - 5 - 5 - 20 = 30$ 2

(iii) $15 + 20 + 5 = 40$ 2

(iv) $150 - 30 - 25 = 95$

(b)(i) $A = \{3, 5, 7, 9\}$ $|A| = 4$

$B = \{2, 3, 5, 7\}$ $|B| = 4$

$C = \{3, 6, 9\}$ $|C| = 3$

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(ii) Proper subset of A $= 2^n - 1$

$= 2^4 - 1$

$= 16 - 1$

$= 15$

3

(iii) $C \times B = \{3, 6, 9\} \times \{2, 3, 5, 7\}$

$= \{(3,2), (3,3), (3,5), (3,7), (6,2), (6,3), (6,5), (6,7),$

$(9,2), (9,3), (9,5), (9,7)\}$

2

2. (a) Truth Table

p	q	$\sim(p \vee q) \vee (\sim p \wedge q)$	$\sim p$
T	T	$\sim(T \vee T) \vee (\sim T \wedge T)$ $= F \vee F = F$	F
T	F	$\sim(T \vee F) \vee (\sim T \wedge F)$ $= F \vee F = F$	F
F	T	$\sim(F \vee T) \vee (\sim F \wedge T)$ $= F \vee T = T$	T
F	F	$\sim(F \vee F) \vee (\sim F \wedge F)$ $= T \vee F = T$	T

3

Logic Property Law

$\sim(p \vee q) \vee (\sim p \wedge q) \equiv (\sim p \wedge \sim q) \vee (\sim p \wedge q)$

$\equiv \sim p \wedge (\sim q \vee q)$

$\equiv \sim p$

proven

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Date

2 (b)(i) $(r \wedge q) \rightarrow p$

(ii) $(\neg r \vee \neg q) \rightarrow \neg p$

(iii) $\neg p \rightarrow (\neg r \vee \neg q)$

2

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(c) $\forall x\ (x^2 + 2x - 3 = 0)$

negation : $\exists x \sim (x^2 + 2x - 3 = 0)$

when $x = 5$, $5^2 + 2(5) - 3 = 0$

$32 \neq 0$

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$\therefore \exists x \sim (x^2 + 2x - 3 = 0)$ is TRUE

(d) (i) $\exists x\ (R(x) \wedge \sim P(x))$

(ii) $\forall x\ (R(x) \vee P(x))$

(iii) $\sim \exists x\ (R(x) \vee P(x))$

2

2

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R(x) : x can speak Russian

P(x) : x know C++

3 Case 1 : If a is odd and b is odd , $a^2 - 3b$ is odd.

$a^2 - 3b = (2x+1)^2 - 3(2x+1)$

$= 4x^2 + 4x + 1 - 6x - 3$

$= 4x^2 - 2x - 2$

$a^2 - 3b = 2(2x^2 - x - 1)$

$a^2 - 3b = 2t$.

$\therefore$ FALSE statement as $a^2 - 3b$ is even.

Case 2 : If a is odd and b is even, $a^2 - 3b$ is odd.

$a^2 - 3b = (2x+1)^2 - 3(2x)$

$= 4x^2 + 4x + 1 - 6x$

$= 4x^2 - 2x + 1$

$a^2 - 3b = 2(x^2 - x) + 1$

$a^2 - 3b = 2t + 1$.

$\therefore$ TRUE statement as $a^2 - 3b$ is odd.

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$\therefore$ Based on these cases, we could conclude that the theorem is false as there is a false statement in case 1.

Case 3 : If a is even and b is odd, $a^2 - 3b$ is odd.

$a^2 - 3b = (2x)^2 - 3(2x+1)$

$= 4x^2 - 6x - 3$

$a^2 - 3b = 2(2x^2 - 3x) - 3$

$a^2 - 3b = 2t - 3$

$\therefore$ TRUE statement as $a^2 - 3b$ is odd.

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