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MN2 - Tarefa 6

Gauss-Hermite, Gauss-Laguerre e Gauss-Chebyshev para $n = 4$.

(Hermite) $I_H = \int_{-\infty}^{+\infty} e^{-x^2} f(x)\,dx \approx \sum_{K=1}^{4} f(x)\,w_K$

(Laguerre) $I_L = \int_{0}^{+\infty} e^{-x} f(x)\,dx \approx \sum_{K=1}^{4} f(x)\,w_K$

(Chebyshev) $I_C = \int_{-1}^{1} \frac{1}{\sqrt{1-x^2}} f(x)\,dx \approx \sum_{K=1}^{4} f(x)\,w_K$

Polinômios de Hermite, Laguerre e Chebyshev de grau 4:

$H_4(x) = 16x^4 - 48x^2 + 12$

$L_4(x) = \frac{1}{24}(x^4 - 16x^3 + 72x^2 - 96x + 24)$

$T_4(x) = 8x^4 - 8x^2 + 1$

Raízes x_K de Hermite ($H_4(x)$)

$x_1^{(H)} = -\sqrt{\frac{3+\sqrt{6}}{2}}$ $\quad$ $x_3^{(H)} = \sqrt{\frac{3-\sqrt{6}}{2}}$

$x_2^{(H)} = -\sqrt{\frac{3-\sqrt{6}}{2}}$ $\quad$ $x_4^{(H)} = \sqrt{\frac{3+\sqrt{6}}{2}}$

S/L | T/M | Q/M | Q/J | S/V | S/S | D/D

Raízes x_K de Laguerre

$x_1^{(L)} = 0,32254$ $\quad x_3^{(L)} = 4,53662$

$x_2^{(L)} = 1,74576$ $\quad x_4^{(L)} = 9,39507$

Raízes x_K de Chebyshev

$$x_1^{(C)} = -\frac{\sqrt{2+\sqrt{2}}}{2} \qquad x_3^{(C)} = \sqrt{\frac{2-\sqrt{2}}{2}}$$

$$x_2^{(C)} = -\sqrt{\frac{2-\sqrt{2}}{2}} \qquad x_4^{(C)} = \sqrt{\frac{2+\sqrt{2}}{2}}$$

Calculando-se agora w_K, $K = 1, \ldots, n$:

(Hermite)

$$w_K^{(H)} = \frac{2^{n-1} n! \sqrt{\pi}}{n^2 [H_{n-1}(x_K^{(H)})]^2}$$

$$w_1^{(H)} = \frac{2^3 4! \sqrt{\pi}}{4^2 [H_3(x_1^{(H)})]^2} = 0,08131 = w_4^{(H)}$$

$$w_2^{(H)} = \frac{2^3 4! \sqrt{\pi}}{4^2 [H_3(x_2^{(H)})]^2} = 0,80491 = w_3^{(H)}$$

(Laguerre)

$$w_K^{(L)} = \frac{x_K^{(L)}}{(n+1)^2 [L_{n+1}(x_K^{(L)})]^2}$$

$$w_1^{(L)} = \frac{x_1^{(L)}}{5^2 [L_5(x_1^{(L)})]^2} = 0,60315$$

$$w_2^{(L)} = \frac{x_2^{(L)}}{5^2 [L_5(x_2^{(L)})]^2} = 0,35742$$

$$w_3^{(L)} = \frac{x_3^{(L)}}{S^2[L_5(x_3^{(L)})]^2} = 0,03888$$

$$w_4^{(L)} = \frac{x_4^{(L)}}{S^2[L_5(x_4^{(L)})]^2} = 0,00053$$

(Chebyshev)

$$w_k = \frac{\pi}{n}$$

$$w_1 = w_2 = w_3 = w_4 = \frac{\pi}{4} = 0,78539$$