

MN 2 - Tarefa 2

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Desenvolvimento de Newton-Cotes para um polinômio de substituição de grau 4.

$$I_i = \int_{x_I}^{x_F} f(x) dx \approx \int_{x_I}^{x_F} p(x) dx$$

grau 4  $\Rightarrow$  5 pontos

Polinômio de interpolação de Newton

$$q(\Delta) = \sum_{k=0}^n \binom{\Delta}{k} \Delta^k f_0$$

$$k=0 \Rightarrow \binom{\Delta}{0} = \frac{\Delta!}{0!(\Delta-0)!} = 1; \Delta^0 f_0 = f_0$$

$$k=1 \Rightarrow \binom{\Delta}{1} = \frac{\Delta(\Delta-1)!}{1!(\Delta-1)!} = \Delta; \Delta^1 f_0 = f_1 - f_0$$

$$k=2 \Rightarrow \binom{\Delta}{2} = \frac{\Delta(\Delta-1)(\Delta-2)!}{2!(\Delta-2)!} = \frac{1}{2} \Delta(\Delta-1); \Delta^2 f_0 = f_2 - 2f_1 + f_0$$

$$k=3 \Rightarrow \binom{\Delta}{3} = \frac{\Delta(\Delta-1)(\Delta-2)(\Delta-3)!}{3!(\Delta-3)!} = \frac{1}{6} \Delta(\Delta-1)(\Delta-2); \Delta^3 f_0 = f_3 - 3f_2 + 3f_1 - f_0$$

$$k=4 \Rightarrow \binom{\Delta}{4} = \frac{\Delta(\Delta-1)(\Delta-2)(\Delta-3)(\Delta-4)!}{4!(\Delta-4)!} = \frac{1}{24} \Delta(\Delta-1)(\Delta-2)(\Delta-3); \Delta^4 f_0 = f_4 - 4f_3 + 6f_2 - 4f_1 + f_0$$

$$\begin{aligned} q(\Delta) = & f_0 \frac{1}{24} (24 + \Delta^4 - 10\Delta^3 + 35\Delta^2 - 50\Delta) \\ & + f_1 \frac{1}{6} (-\Delta^4 + 9\Delta^3 - 26\Delta^2 + 24\Delta) \\ & + f_2 \frac{1}{4} (\Delta^4 - 8\Delta^3 + 19\Delta^2 - 12\Delta) \\ & + f_3 \frac{1}{6} (-\Delta^4 + 7\Delta^3 - 14\Delta^2 + 8\Delta) \\ & + f_4 \frac{1}{24} (\Delta^4 - 6\Delta^3 + 11\Delta^2 - 6\Delta) \end{aligned}$$

Fechada

O polinômio de interpolação deve passar por  $f(x_i)$ ,  $f(x_r)$  e por 3 pontos intermediários de maneira que os 5 pontos sejam igualmente espaçados por uma distância  $h = \frac{\Delta x}{4}$ .

$$\int_{x_i}^{x_r} f(x) dx \approx \int_{x_i}^{x_r} p(x) dx = \int_{\eta_i}^{\eta_r} p(x(\eta)) \frac{dx(\eta)}{d\eta} d\eta = h \int_0^4 g(\eta) d\eta$$

$$x(\eta) = x_i + \eta h$$

$$\begin{aligned} = & h \left[ F_0 \frac{1}{24} \int_0^4 (24 + \eta^4 - 10\eta^3 + 35\eta^2 - 50\eta) d\eta \right. \\ & + F_1 \frac{1}{6} \int_0^4 (-\eta^4 + 9\eta^3 - 26\eta^2 + 24\eta) d\eta \\ & + F_2 \frac{1}{4} \int_0^4 (\eta^4 - 8\eta^3 + 19\eta^2 - 12\eta) d\eta \\ & + F_3 \frac{1}{6} \int_0^4 (-\eta^4 + 7\eta^3 - 14\eta^2 + 8\eta) d\eta \\ & \left. + F_4 \frac{1}{24} \int_0^4 (\eta^4 - 6\eta^3 + 11\eta^2 - 6\eta) d\eta \right] \end{aligned}$$

$$= h \left( F_0 \frac{14}{45} + F_1 \frac{64}{45} + F_2 \frac{24}{45} + F_3 \frac{64}{45} + F_4 \frac{14}{45} \right)$$

$$= \frac{2h}{45} (7F_0 + 32F_1 + 12F_2 + 32F_3 + 7F_4)$$

Alberta

O polinômio de interpolação deve passar por 6 pontos entre  $f(x_s)$  e  $f(x_r)$ , exclusivamente, de forma que sejam igualmente espaçados por uma distância  $h = \frac{\Delta x}{6}$ .

$$\int_{x_r}^{x_r} f(x) dx \approx \int_{x_r}^{x_r} p(x) dx = \int_{x_s}^{x_r} p(x(s)) \frac{dx(s)}{ds} ds = h \int_{-1}^5 q(s) ds$$

$x(s) = x_i + h + sh$

$$= h \left[ F_0 \frac{1}{24} \int_{-1}^5 (24 + s^4 - 10s^3 + 35s^2 - 50s) ds \right. \\
+ F_1 \frac{1}{6} \int_{-1}^5 (-s^4 + 9s^3 - 26s^2 + 24s) ds \\
+ F_2 \frac{1}{4} \int_{-1}^5 (s^4 - 8s^3 + 19s^2 - 12s) ds \\
+ F_3 \frac{1}{6} \int_{-1}^5 (-s^4 + 7s^3 - 14s^2 + 8s) ds \\
\left. + F_4 \frac{1}{24} \int_{-1}^5 (s^4 - 6s^3 + 11s^2 - 6s) ds \right]$$

$$= h \left( F_0 \frac{33}{10} - F_1 \frac{42}{10} + F_2 \frac{78}{10} - F_3 \frac{42}{10} + F_4 \frac{33}{10} \right)$$

$$= \frac{3h}{10} (11F_0 - 14F_1 + 26F_2 - 14F_3 + 11F_4)$$