S/L | T/M | Q/M | Q/J | S/V | S/S | D/D

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Gauss-Legendre com 4 pontos de interpolação

Dessa forma:

$$I = \int_{x_I}^{x_F} f(x)dx \approx \frac{x_F - x_I}{2}\left[\sum_{k=1}^{4} f(x(\alpha_k))\omega_k\right] =$$

$$= \frac{x_F - x_I}{2}\left[f(x(\alpha_1))\omega_1 + f(x(\alpha_2))\omega_2 + f(x(\alpha_3))\omega_3 + f(x(\alpha_4))\omega_4\right]$$

Precisamos encontrar $\alpha_1, \alpha_2, \alpha_3$ e α_4, vide raízes do polinômio de Legendre de grau 4,

$$P_4(\alpha) = \frac{1}{2^4 4!}\frac{d^4}{d\alpha^4}\left[(\alpha^2-1)^4\right] = \frac{1}{384}\frac{d^4}{d\alpha^4}\left[\alpha^8 - 4\alpha^6 + 6\alpha^4 - 4\alpha^2 + 1\right]$$

$$= \frac{1}{384}(1680\alpha^4 - 1440\alpha^2 + 144) = \frac{1}{8}(35\alpha^4 - 30\alpha^2 + 3)$$

Calculando-se as raízes:

$$\alpha_1 = -\sqrt{\frac{15-2\sqrt{30}}{35}}$$

$$\alpha_2 = -\sqrt{\frac{15+2\sqrt{30}}{35}}$$

$$\alpha_3 = \sqrt{\frac{15-2\sqrt{30}}{35}}$$

$$\alpha_4 = \sqrt{\frac{15+2\sqrt{30}}{35}}$$

Calculando $x(\alpha_K)$, $K=1,\dots,4$

$$x(\alpha_K) = \frac{x_I + x_F}{2} + \frac{x_F - x_I}{2}\alpha_K$$

$$x(\alpha_1) = x\left(-\sqrt{\frac{15-2\sqrt{30}}{35}}\right) = \frac{x_I + x_F}{2} - \frac{x_F - x_I}{2}\sqrt{\frac{15-2\sqrt{30}}{35}}$$

$$x(\alpha_2) = x\left(-\sqrt{\frac{15+2\sqrt{30}}{35}}\right) = \frac{x_I + x_F}{2} - \frac{x_F - x_I}{2}\sqrt{\frac{15+2\sqrt{30}}{35}}$$

$$x(\alpha_3) = x\left(\sqrt{\frac{15-2\sqrt{30}}{35}}\right) = \frac{x_I + x_F}{2} + \frac{x_F - x_I}{2}\sqrt{\frac{15-2\sqrt{30}}{35}}$$

$$x(\alpha_4) = x\left(\sqrt{\frac{15+2\sqrt{30}}{35}}\right) = \frac{x_I + x_F}{2} + \frac{x_F - x_I}{2}\sqrt{\frac{15+2\sqrt{30}}{35}}$$

Calculando w_K, $K=1,\dots,4$

$$w_K = \int_{-1}^{1} L_K(\alpha)\,d\alpha$$

Precisamos calcular $L_K(\alpha)$

$$L_1(\alpha) = \frac{\alpha-\alpha_2}{\alpha_1-\alpha_2}\,\frac{\alpha-\alpha_3}{\alpha_1-\alpha_3}\,\frac{\alpha-\alpha_4}{\alpha-\alpha_4} = \frac{35\sqrt{7}\,\alpha^3}{8\sqrt{90-12\sqrt{30}}} - \frac{\sqrt{3675-490\sqrt{30}}\,\alpha^2}{8\sqrt{90-12\sqrt{30}}}$$

$$- \frac{15\sqrt{7}\,\alpha}{8\sqrt{90-12\sqrt{30}}} - \frac{\sqrt{105}\,\alpha}{2\sqrt{180-24\sqrt{30}}} + \frac{\sqrt{315+42\sqrt{30}}}{8\sqrt{90-12\sqrt{30}}}$$

$$L_2(\alpha) = -\Big(35\sqrt{35}\,\alpha^3 - 35\sqrt{15+2\sqrt{30}}\,\alpha^2 - 15\sqrt{35}\,\alpha + 10\sqrt{42}\,\alpha +$$

$$15\sqrt{15+2\sqrt{30}} - 2\sqrt{450+60\sqrt{3}}\Big)\cdot\frac{1}{70\sqrt{35}\left(\frac{15+2\sqrt{30}}{35}\right)^{3/2}} -$$

$$\overline{30\sqrt{15+2\sqrt{30}} + 4\sqrt{450+60\sqrt{30}}}$$

Não é preciso calcular $L_3(\alpha)$ e nem $L_4(\alpha)$, pois $w_1 = w_4$ e $w_2 = w_3$.

Portanto,

$$w_1 = w_4 = \int_{-1}^{1} L_1(\alpha)\,d\alpha = \frac{18+\sqrt{30}}{36}$$

$$w_2 = w_3 = \int_{-1}^{1} L_2(\alpha)\,d\alpha = \frac{18-\sqrt{30}}{36}$$

$$I \approx \frac{x_F - x_I}{2}\left[f\left(\frac{x_I + x_F}{2} - \frac{x_F - x_I}{2}\sqrt{\frac{15-2\sqrt{30}}{35}}\right)\frac{18+\sqrt{30}}{36} \right.$$

$$+ f\left(\frac{x_I + x_F}{2} - \frac{x_F - x_I}{2}\sqrt{\frac{15+2\sqrt{30}}{35}}\right)\frac{18-\sqrt{30}}{36}$$

$$+ f\left(\frac{x_I + x_F}{2} + \frac{x_F - x_I}{2}\sqrt{\frac{15-2\sqrt{30}}{35}}\right)\frac{18-\sqrt{30}}{36}$$

$$\left. + f\left(\frac{x_I + x_F}{2} + \frac{x_F - x_I}{2}\sqrt{\frac{15+2\sqrt{30}}{35}}\right)\frac{18+\sqrt{30}}{36} \right]$$

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