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Lista de exercícios

1. $A = \begin{bmatrix} x & 1 \\ 5 & 2 \end{bmatrix}$ é inversa de $B = \begin{bmatrix} 3 & -1 \\ y & 2 \end{bmatrix}$

$$\begin{bmatrix} x & 1 \\ 5 & 3 \end{bmatrix} \cdot \begin{bmatrix} 3 & -1 \\ y & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3x + y & -x + 2 \\ 15 + 3y & -5 + 6 \end{bmatrix}$$

$$\begin{cases} 3x + y = 1 \\ 15 + 3y = 0 \end{cases}$$

$-x + 2 = 0 \rightarrow x = 2$

$-5 + 6 = 1$

$\rightarrow 3x + y = 1$

$3 \cdot 2 + y = 1$

$6 + y = 1$

$y = -5$

$x + y = ?$

$2 + (-5) = 2 - 5 = -3$ //

alternativa (c) //

2

p/ não admitir a inversão

$$\begin{array}{l} \\ A = \end{array} \begin{vmatrix} 1 & 0 & 1 \\ k & 1 & 3 \\ 1 & k & 3 \end{vmatrix} \begin{matrix} 1 & 0 \\ k & 1 \\ 1 & k \end{matrix}$$

$1 \quad 3k \quad 0$

$3 \quad 0 \quad k^2$

$\det A = 0$

$= k^2 + 3 - 1 - 3k$

$= k^2 + 2 - 3k$

$\Delta = (-3)^2 - 4.1.2$

$\Delta = 9 - 8$

$\Delta = 1$

$k = \frac{-(-3) \pm \sqrt{1}}{2.1}$

$k = \frac{3 \pm 1}{2}$

$k' = \frac{4}{2} = 2$ //

$k'' = \frac{2}{2} = 1$ //

alternativa (c) //

3 $A = \begin{vmatrix} 3 & 5 \\ 2 & 4 \end{vmatrix} \Rightarrow \det A = 12 - 10 = 2$

$B = A^{-1} = \begin{vmatrix} 4 & -5 \\ -2 & 3 \end{vmatrix} \div 2$

$B = \begin{vmatrix} 2 & -5/2 \\ -1 & 3/2 \end{vmatrix}$ //

alternativa (c) //

4

$$\begin{vmatrix} x & 1 & 2 \\ 3 & 1 & 2 \\ 10 & 1 & x \end{vmatrix} \begin{matrix} x & 1 \\ 3 & 1 \\ 10 & 1 \end{matrix}$$

$20 \quad 10 \quad 1 \quad x \quad x^2$

$2x \quad x \quad 1 \quad 2 \quad 6$

$3x \quad 3 \quad 1 \quad 2 \quad 20$

$\Rightarrow x^2 + 26 - 20 - 5x \Rightarrow x^2 - 5x + 6 \neq 0$

$x = \frac{-(-5) \pm \sqrt{1}}{2.1}$

$x = \frac{5 \pm 1}{2}$

$x' = \frac{6}{2} = 3$

$x'' = \frac{4}{2} = 2$

$x \neq 3$ //

$x \neq 2$ //

$\Delta = (-5)^2 - 4.1.6$

$\Delta = 25 - 24$

$\Delta = 1$

$\{x \neq 3 \text{ e } x \neq 2\}$ //

5. $A = \begin{bmatrix} -1 & -1 & 2 \\ 2 & 1 & -2 \\ 1 & 1 & -1 \end{bmatrix}$

$$2 + 2 + 2 = 6$$

$$\det A = \left|\begin{array}{ccc|cc} -1 & -1 & 2 & -1 & -1 \\ 2 & 1 & -2 & 2 & 1 \\ 1 & 1 & -1 & 1 & 1 \end{array}\right| = 7 - 6 = 1$$

$$1 + 2 + 4 = 7$$

$A' = \begin{bmatrix} 1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$ $\bar{A} = (A')^T = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix} \div 1$

$A = \begin{bmatrix} -1 & -1 & 2 \\ 2 & 1 & -2 \\ 1 & 1 & -1 \end{bmatrix}$ $A^{-1} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$

$A + A^{-1} = \begin{bmatrix} 0 & 0 & 2 \\ 2 & 0 & 0 \\ 2 & 1 & 0 \end{bmatrix}$ //

alternativa (D) //

6 $((xA)^t)^t = B^t$

$xAA^{-1} = B^tA^{-1}$

$x = B^tA^{-1}$

alternativa (B)

7. $A.B = C$ $\quad C = \begin{bmatrix} 4x + 5y \\ 5x + 6y \end{bmatrix}$ $\quad B = \begin{bmatrix} x \\ y \end{bmatrix}$

$A = C/B$

$\begin{bmatrix} 4x/x + 5y/y \\ 5x/x + 6y/y \end{bmatrix} \Rightarrow A = \begin{bmatrix} 4 & 5 \\ 5 & 6 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} -4 & 5 \\ 5 & -6 \end{bmatrix}$ //

alternativa (D) //

9. a) $(A+B).(A-B)$

$$A^2 - AB + BA - B^2 \; //$$

b) $(A+B)^2 \neq A^2 + 2.A.B + B^2$

$$(A+B)^2 = (A+B).(A+B) = A^2 + AB + BA + B^2$$

$$A^2 + AB + BA + B^2 = A^2 + 2AB + B^2$$

$$AB = BA \; //$$

c) $\det(-A) = (-1)^2 . \det A = \det A \neq 0$

$$\frac{\det(A)}{\det(-A)} = \frac{\det(A)}{\det(A)} = 1 \; //$$

d) $\det(AB) = 1$

$$\det(A).\det(B) = 1$$

$$\det B = \frac{1}{\det A} \; //$$