

Lista de exercícios

1. a) $\begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$
 $\begin{matrix} 3 & 10 \end{matrix}$

$$\det = 10 - 3 = 7 //$$

b) $\begin{bmatrix} -2 & -4 \\ 3 & 6 \end{bmatrix}$
 $\begin{matrix} -12 \\ -12 \end{matrix}$

$$\det = -12 - (-12) = -12 + 12 = 0 //$$

c) $\begin{bmatrix} 3 & -1 & 1 \\ 2 & 1 & -1 \\ 1 & 4 & -2 \end{bmatrix}$
 $\begin{matrix} 1 & -12 & 4 \\ 3 & -1 \\ 2 & 1 \\ 1 & 4 \end{matrix}$

$$\det = 3 - (-7) = 10 //$$

$$\begin{matrix} -6 & 1 & 8 \end{matrix}$$

$$1 + (-12) + 4 = 1 - 12 + 4 = -7$$

$$-6 + 1 + 8 = 3$$

d) $\begin{bmatrix} 3 & 2 & -1 \\ 2 & 3 & 1 \\ 1 & 1 & 4 \end{bmatrix}$
 $\begin{matrix} -3 & 36 \end{matrix}$

$$\det = 36 - 16 = 20 //$$

$$\begin{matrix} 3 & 3 & 2 & -1 & -2 \end{matrix}$$

$$36 + (-2) + 2 = 36 - 2 + 2 = 36$$

$$\begin{matrix} 16 & 2 & 3 & 1 & 2 \end{matrix}$$

$$-3 + 3 + 16 = 16$$

2. $A = (a_{ij})_{3 \times 3}$

$$a_{ij} = \begin{cases} -3 & \text{se } i=j \\ 0 & \text{se } i \neq j \end{cases}$$

$$\begin{vmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{vmatrix} = -27$$

$$\det A = -27 - 0 = -27 //$$

3.

$$\begin{vmatrix} x & 1 & x \\ 3 & x & 4 \\ 1 & 3 & 3 \end{vmatrix} = -3$$

$$(3x^2 + 9x + 4) - x^2 + 12x + 9$$

$$2x^2 - 3x - 5 + 3$$

$$2x^2 - 3x - 2 = 0$$

$$12x \quad x \quad 1 \quad x \quad 9x$$

$$2x^2 + x - 4x - 2 = 0$$

$$9 \quad 3 \quad x \quad 4 \quad 4$$

$$x(2x + 1) - 4x - 2 = 0$$

$$(2x + 1)(x - 2) = 0$$

$$3x^2 + 9x + 4$$

$$2x + 1 = 0$$

$$x' = -1/2 //$$

$$x^2 + 12x + 9$$

$$x - 2 = 0$$

$$x'' = 2 //$$

alternativa (E) //

4.

$$\begin{vmatrix} x-1 & -1 & 0 \\ 0 & x+1 & -1 \\ 2 & -1 & x+1 \end{vmatrix}$$

$$x-1 \quad -1 \quad 0$$

$$0 \quad x+1 \quad -1$$

$$[(x-1)(x+1) - (-1)(x+1)] - 2(x-1) = 2$$

$$[x^2 - x - 1 + 2] - (2x - 2) = 2$$

$$x^2 - x - 1 + 2 - 2x + 2 = 2$$

$$x^2 - 3x + 3 = 2$$

5. $A = (a_{ij})_{3 \times 2}$

$a_{ij} = 2i - 3j$

$B = (b_{jk})_{2 \times 3}$

$b_{jk} = k - j$

$B = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & -1 \end{bmatrix}$

$b_{11} = 1 - 1 = 0$

$b_{12} = 2 - 1 = 1$

$b_{13} = 3 - 1 = 2$

$b_{21} = 1 - 2 = -1$

$b_{22} = 2 - 2 = 0$

$b_{23} = 3 - 2 = 1$

$A = \begin{bmatrix} -1 & -4 \\ 1 & -2 \\ 3 & 0 \end{bmatrix}$

$a_{11} = 2 \cdot 1 - 3 \cdot 1 = 2 - 3 = -1$

$a_{12} = 2 \cdot 1 - 3 \cdot 2 = 2 - 6 = -4$

$a_{21} = 2 \cdot 2 - 3 \cdot 1 = 4 - 3 = 1$

$a_{22} = 2 \cdot 2 - 3 \cdot 2 = 4 - 6 = -2$

$a_{31} = 2 \cdot 3 - 3 \cdot 1 = 6 - 3 = 3$

$a_{32} = 2 \cdot 3 - 3 \cdot 2 = 6 - 6 = 0$

$\begin{matrix} (-1 \cdot 0)(-4 \cdot -1) & (-1 \cdot 1)(-4 \cdot 0) & (-1 \cdot 2)(-4 \cdot -1) \\ (1 \cdot 0)(2 \cdot -1) & (1 \cdot 1)(2 \cdot 0) & (1 \cdot 2)(2 \cdot -1) \\ (3 \cdot 0)(0 \cdot -1) & (3 \cdot 1)(0 \cdot 0) & (3 \cdot 2)(0 \cdot -1) \end{matrix}$

$\begin{bmatrix} -0+4 & -1-0 & -2-4 \\ 0+2 & 1-0 & 2-2 \\ 0+0 & 3+0 & 6-0 \end{bmatrix}$

$\exists A_{3 \times 2} \cdot B_{2 \times 3}$

$= \begin{bmatrix} 4 & -1 & -6 \\ 2 & 1 & 0 \\ 0 & 3 & 6 \end{bmatrix}$

$\begin{bmatrix} 4 & -1 & -6 \\ 2 & 1 & 0 \\ 0 & 3 & 6 \end{bmatrix}$

$\begin{bmatrix} 0 & 4 & -1 & -6 & -36 \\ -12 & 2 & 1 & 0 & 0 \end{bmatrix}$

$\det A \cdot B = (24 + 0 - 36) - (0 - 12 + 0)$

$\det A \cdot B = -12 + 12$

$\det A \cdot B = 0 //$

alternativa (c) //

$$6. A = \begin{bmatrix} 2 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}_{2 \times 3}$$

$$B = \begin{bmatrix} 1 & -1 \\ -1 & 1 \\ 0 & 2 \end{bmatrix}_{3 \times 2}$$

$$\begin{bmatrix} (2 \cdot 1)(0 \cdot -1)(-1 \cdot 0) & (2 \cdot -1)(0 \cdot 1)(-1 \cdot 2) \\ (-1 \cdot 1)(1 \cdot -1)(0 \cdot 0) & (-1 \cdot -1)(1 \cdot 1)(0 \cdot 2) \end{bmatrix}$$

$$\begin{bmatrix} 2+0-0 & -2+0-2 \\ -1-1-0 & 1+1-0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -4 \\ -2 & 2 \end{bmatrix}$$

$$A_{2 \times 3} \cdot B_{3 \times 2} = \begin{bmatrix} 2 & -4 \\ -2 & 2 \end{bmatrix}$$

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4

$$\text{det } A \cdot B = 4 - 8 = -4 //$$

alternativa (D) //