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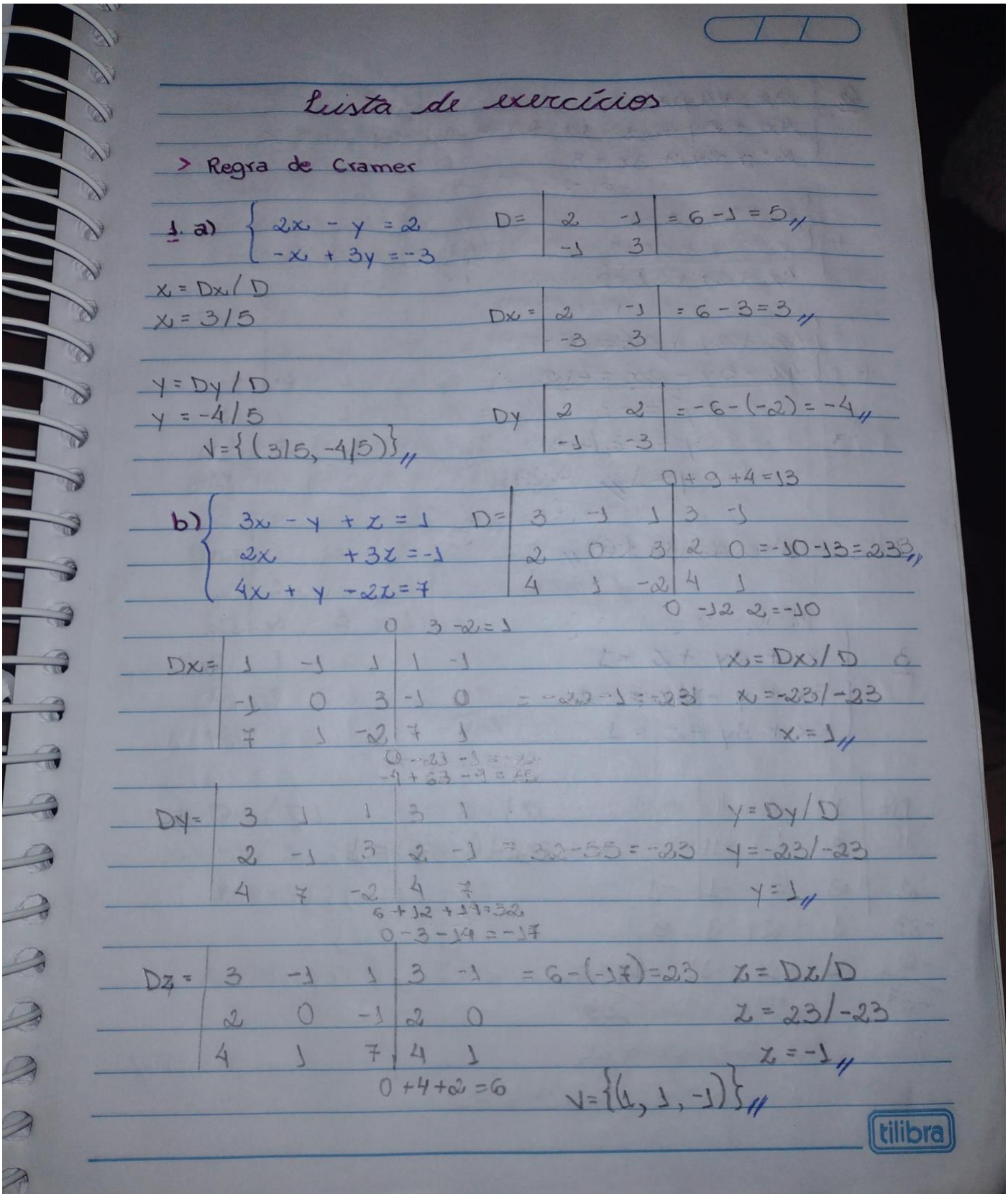

Lista de exercícios

> Regra de Cramer

1. a) $\begin{cases} 2x - y = 2 \\ -x + 3y = -3 \end{cases}$

$D = \begin{vmatrix} 2 & -1 \\ -1 & 3 \end{vmatrix} = 6 - 1 = 5$

$x = Dx / D$

$x = 3/5$

$Dx = \begin{vmatrix} 2 & -1 \\ -3 & 3 \end{vmatrix} = 6 - 3 = 3$

$y = Dy / D$

$y = -4/5$

$V = \{(3/5, -4/5)\}$

$Dy = \begin{vmatrix} 2 & 2 \\ -1 & -3 \end{vmatrix} = -6 - (-2) = -4$

b) $\begin{cases} 3x - y + z = 1 \\ 2x + 3z = -1 \\ 4x + y - 2z = 7 \end{cases}$

$D = \begin{vmatrix} 3 & -1 & 1 \\ 2 & 0 & 3 \\ 4 & 1 & -2 \end{vmatrix}$ 3 -1 / 2 0 / 4 1

0 + 9 + 4 = 13

0 - 12 - 2 = -10

0 - 3 - 20 = -23

$Dx = \begin{vmatrix} 1 & -1 & 1 \\ -1 & 0 & 3 \\ 7 & 1 & -2 \end{vmatrix}$ 1 -1 / -1 0 / 7 1

$x = Dx / D$

$x = -23 / -23$

$x = 1$

0 - 23 - 1 = -24

-7 + 63 - 4 = 52

$Dy = \begin{vmatrix} 3 & 1 & 1 \\ 2 & -1 & 3 \\ 4 & 7 & -2 \end{vmatrix}$ 3 1 / 2 -1 / 4 7

$y = Dy / D$

$y = -23 / -23$

$y = 1$

6 + 12 + 14 = 32

0 - 3 - 14 = -17

$Dz = \begin{vmatrix} 3 & -1 & 1 \\ 2 & 0 & -1 \\ 4 & 1 & 7 \end{vmatrix}$ 3 -1 / 2 0 / 4 1 $= 6 - (-17) = 23$

$z = Dz / D$

$z = 23 / -23$

$z = -1$

0 + 4 + 2 = 6

$V = \{(1, 1, -1)\}$

2. $\begin{cases} 3x + 4y - z = 1 \\ 4x + 5y + 2z = 12 \cdot (-1) \rightarrow -4x - 5y - 2z = -12 \\ x - 2y + 3z = 8 \end{cases}$

$$+ \begin{cases} 3x + 4y - z = 1 \\ x - 2y + 3z = 8 \end{cases}$$

$$4x + 2y + 2z = 9$$

$$+ \begin{cases} 4x + 2y + 2z = 9 \\ -4x - 5y - 2z = -12 \end{cases}$$

$$-3y = -3$$

$$y = -3/-3$$

$$y = 1 //$$

alternativa (a) //

3. $\begin{cases} x + 2y + z = 1 \\ 3x + y - 11z = -2 \\ 2x + 3y - z = 1 \end{cases}$

$$D = \begin{vmatrix} 1 & 2 & 1 \\ 3 & 1 & -11 \\ 2 & 3 & -1 \end{vmatrix} \quad \begin{matrix} \\ \\ -1 \end{matrix} = -36 - (-37) = 1$$

	1	2	1	
	3	1	-11	
2	2	3	-1	-1
-33	1	2	1	9
-6	3	1	-11	-44
-37				36

$$Dx = \begin{vmatrix} 1 & 2 & 1 \\ -2 & 1 & -11 \\ 1 & 1 & 3 & -1 \end{vmatrix} \quad -1$$

$$= -29 - (-28) = -1 //$$

$$\begin{array}{rrrrr} -33 & 1 & 2 & 1 & -6 \\ 4 & -2 & 1 & -11 & -22 \\ \hline -28 & & & & -29 \end{array}$$

$$Dy = \begin{vmatrix} 1 & 1 & 1 \\ 3 & -2 & -11 \\ -4 & 2 & 1 & -1 & 2 \end{vmatrix}$$

$$= -17 - (-18) = 1 //$$

$$\begin{array}{rrrrr} -11 & 1 & 1 & 1 & 3 \\ -3 & 3 & -2 & -11 & -22 \\ \hline -18 & & & & -17 \end{array}$$

$$Dz = \begin{vmatrix} 1 & 2 & 1 \\ 3 & 1 & -2 \\ 2 & 2 & 3 & 1 & 1 \end{vmatrix}$$

$$= 2 - 2 = 0 //$$

$$\begin{array}{rrrrr} -6 & 1 & 2 & 1 & 9 \\ 6 & 3 & 1 & -2 & -8 \\ \hline 2 & & & & 2 \end{array}$$

$x = Dx / D \qquad y = Dy / D \qquad z = Dz / D$

$x = -1 / 1 \qquad y = 1 / 1 \qquad z = 0 / 1$

$x = -1 \qquad y = 1 \qquad z = 0$

$x = a \quad y = b \quad z = c \qquad a + b + c$

$-1 + 1 + 0 = 0 //$

alternativa (c) //

4. $\begin{cases} x + 2y - 3z = 29 \\ x + 3y + 2z = 4 \\ x - y - 2z = 8 \end{cases}$

$D = \begin{vmatrix} 1 & 2 & -3 \\ 1 & 3 & 2 \\ 1 & -1 & -2 \end{vmatrix}$

$= 1 - (-15) = 16$ //

	1	2	-3	
-9	1	-1	-2	-6
-2	1	2	-3	+3
-4	1	3	2	4
-15				-1

$Dy = \begin{vmatrix} 1 & 29 & -3 \\ 1 & 4 & 2 \\ 1 & 8 & -2 \end{vmatrix}$

$= 26 - (54) = 80$ //

	1	29	-3	
-12	1	8	-2	-8
16	1	29	-3	-24
-58	1	4	2	58
-54				26

$y = Dy / D$

$y = 80 / 16$

$y = 5$ //

$Dz = \begin{vmatrix} 1 & 2 & 29 \\ 1 & 3 & 4 \\ 1 & -1 & 8 \end{vmatrix}$

$= 93 - 99 = -96$ //

	1	2	29	
-87	1	-1	8	29
-4	1	2	29	-29
16	1	3	4	8

$z = Dz / D$

$z = -96 / 16$

$z = -6$ //

$x + 2y - 3z = 29$

$x + 2.5 - 3.(-6) = 29$

$x + 10 + 18 = 29$

$x + 28 = 29$

$x = 29 - 28 \rightarrow x = 1$ //

$x + y + z$

$5 + (-6) + 1$

$5 - 6 + 1$

$-1 + 1 = 0$ //

alternativa (a) //

5) $\begin{cases} 2x + y = 5 \\ 2y + z = 3 \\ 3x + 2y + z = 7 \end{cases}$

$$D = \begin{vmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 3 & 2 & 1 \end{vmatrix} = 4 - 4 = 3$$

0 4 2 1 0 0
4 2 1 0 0
0 0 2 1 3/7

$$Dx = \begin{vmatrix} 5 & 1 & 0 \\ 3 & 2 & 1 \\ 7 & 2 & 1 \end{vmatrix} = 17 - 13 = 4$$

$x = Dx / D$
$x = 4/3$ //

0 10 5 1 0 0
10 5 1 0 0
3/13 3 2 1 7/17

$$Dy = \begin{vmatrix} 2 & 5 & 0 \\ 0 & 3 & 1 \\ 3 & 7 & 1 \end{vmatrix} = 21 - 14 = 7$$

$y = Dy / D$
$y = 7/3$ //

0 3 2 5 1 0 6
14 2 5 0 0
0 0 3 1 15/21

$$Dz = \begin{vmatrix} 2 & 1 & 5 \\ 0 & 2 & 3 \\ 3 & 2 & 7 \end{vmatrix} = 37 - 42 = -5$$

$z = Dz / D$
$z = -5/3$ //

30 2 1 5 28
12 2 1 5 0
0/42 0 2 3 9/37

alternativa (d) //

6. $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 2 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \\ -1 \end{bmatrix}$

$$D = \begin{vmatrix} 1 & 0 & 0 & 1 & 0 \\ 2 & 1 & 0 & 2 & 1 \\ -1 & 2 & 2 & -1 & 2 \end{vmatrix} = 2$$

$2 + 0 + 0 = 2$

$$Dx = \begin{vmatrix} 3 & 0 & 0 & 3 & 0 \\ 7 & 1 & 0 & 7 & 1 \\ -1 & 2 & 2 & 1 & 2 \end{vmatrix} = 6$$

$6 + 0 + 0$

$0 + 0 + 12$

$x = Dx/D$

$x = 6/2$

$x = 3$ //

$$Dy = \begin{vmatrix} 1 & 3 & 0 & 1 & 3 \\ 2 & 7 & 0 & 2 & 7 \\ -1 & -1 & 2 & -1 & -1 \end{vmatrix} = 14 - 12 = 2$$

$14 + 0 + 0$

$-3 + 14 + 0 = 11$

$y = Dy/D$

$y = 2/2$

$y = 1$ //

$$Dz = \begin{vmatrix} 1 & 0 & 3 & 1 & 0 \\ 2 & 1 & 7 & 2 & 1 \\ -1 & 2 & -1 & 1 & 2 \end{vmatrix} = 11 - 11 = 0$$

$-1 + 0 + 12 = 11$

$z = Dz/D$

$z = 0/2$

$z = 0$ //

alternativa (E) //

Lista de exercícios

1. $\begin{cases} 2x - y - 3z = -5 \\ x + 3y - z = 11 \\ x - 5z = 3 \end{cases}$

$$\begin{matrix} -0,5 \\ \\ \\ \end{matrix}\left(\begin{array}{ccc|c} 2 & -1 & -3 & -5 \\ 1 & 3 & -1 & 11 \\ 1 & 0 & -5 & 3 \end{array}\right) \sim \begin{matrix} \\ \\ -3,5 \end{matrix}\left(\begin{array}{ccc|c} & & & \\ 0 & 3,5 & 0,5 & 3,5 \\ 0 & 0,5 & -3,5 & 5,5 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} & & & \\ & & & \\ 0 & 0 & 12,5 & -12,5 \end{array}\right)$$

$12,5z = -12,5$

$z = -12,5 / 12,5$

$z = -1$ //

$x - 5z = 3$

$x - 5.(-1) = 3$

$x + 5 = 3$

$x = 3 - 5$

$x = -2$ //

$2x + 3y - z = 11$

$-2 + 3y - (-1) = 11$

$-1 + 3y = 11$

$3y = 12$

$y = 12/3$

$y = 4$ //

$V = \{(-2, 4, -1)\}$ //

2. $\begin{cases} x = 2y \\ 2y = 3z \rightarrow z = 2y/3 \\ x + y + z = 33 \end{cases}$

$x + y + z = 33$ (.3)

$2y + y + 2y/3 = 33$

$6y + 3y + 6y = 99$

$15y = 99$

$y = \frac{99}{15}$

$y = 3$

$x = 2y$

$x = 2 \cdot 3$

$x = 6$

$2y = 3z$

$2 \cdot 3 = 3z$

$6 = 3z$

$6/3 = z$

$z = 2$

$x + 2y + 3z$

$6 + 2 \cdot 3 + 3 \cdot 2$

$6 + 6 + 6 = 18$ //

alternativa (b) //

3. $\begin{cases} x + y + z = 0 \\ 2x - y - 2z = 1 \\ 6y + 3x = -12 \div 3 \rightarrow 2y + z = -4 \end{cases}$

$z = -2y - 4$

$x + y + z = 0$

$x + y - 2y - 4 = 0$

$x - y - 4 = 0$

$\downarrow$

$x = y + 4$

$\downarrow$

$x = -3 + 4$

$x = 1$

$2x - y - 2(-2y - 4) = 1$

$2x - y + 4y + 8 = 1$

$2x + 3y = -7$

$\downarrow$

$2(y + 4) + 3y = -7$

$2y + 8 + 3y = -7$

$5y = -7 - 8$

$5y = -15$

$y = -15/5$

$y = -3$

$x + y + z = 0$

$1 - 3 + z = 0$

$z = 2$ //

4. $a + b + c = 68$

caco $\rightarrow$ 20% = ali

$b + 0,2c = a$

$a \rightarrow$ 20% 3x mais que bia

$c + 0,2a = 3b$

$\begin{cases} a + b + c = 68 \rightarrow (b + 0,2c) + b + c = 68 \\ b + 0,2c = a \\ c + 0,2a = 3b \end{cases}$

$b + 0,2c + b + c = 68$

$2b + 1,2c = 68$

$\downarrow$

$2b = 68 - 1,2c$

$b = 34 - 0,6c$

$c + 0,2a = 3(34 - 0,6c)$

$c + 0,2a = 102 - 1,8c$

$c + 0,2a + 1,8c = 102 \rightarrow 2,8c + 0,2a = 102$

$2,8c + 0,2a = 102$

$2,8c + 0,2(b + 0,2c) = 102$

$2,8c + 0,2b + 0,04 = 102 \rightarrow 2,84c + 0,2b = 102$

$b = 34 - 0,6c$

$2,84c + 0,2(34 - 0,6c) = 102$

$2,84c + 6,8 - 0,12c = 102$

$2,72c = 102 - 6,8$

$2,72c = 95,2$

$c = 95,2 / 2,72$

$c = 35$

Caco tem 35,00 reais

$b = 34 - 0,6c$

$b = 34 - 0,6 \cdot 35$

$b = 34 - 21$

$b = 13$

Bia tem 13,00 reais

$a + b + c = 68$

$a + 13 + 35 = 68$

$a + 48 = 68$

$a = 68 - 48$

$a = 20$

Ali tem 20 reais

alternativa (a) R$ 15,00 a menos que Caco //

5. $\begin{cases} 3y + 4z = 134 \\ x + 5z = 115 \\ 2x + y = 48 \end{cases}$

$x = 510 / 34 = 15$

$y = 612 / 34 = 18$

$z = 680 / 34 = 20$

total = $x + y + z$

total = 15 + 18 + 20

total = 53

R$ 53,00

alternativa (a) //

$$D = \begin{vmatrix} 0 & 3 & 4 \\ 1 & 0 & 5 \\ 2 & 1 & 0 \end{vmatrix} = 34$$

0 + 0 + 3 + 4 + 4

0 + 1 + 0 + 5 + 30

34

$$Dx = \begin{vmatrix} 134 & 3 & 4 \\ 115 & 0 & 5 \\ 48 & 1 & 0 \end{vmatrix} = 1180 - 670 = 510$$

0 + 134 + 3 + 4 + 460

670 + 0 + 115 + 0 + 5 + 720 / 1180

$$Dy = \begin{vmatrix} 0 & 134 & 4 \\ 1 & 115 & 5 \\ 2 & 48 & 0 \end{vmatrix} = 1532 - 920 = 612$$

920 + 0 + 0 + 134 + 4 + 192

0 + 0 + 1 + 115 + 5 + 1340 / 1532

$$Dz = \begin{vmatrix} 0 & 3 & 134 \\ 1 & 0 & 115 \\ 2 & 1 & 48 \end{vmatrix} = 824 - 144 = 680$$

0 + 0 + 3 + 134 + 134

144 + 1 + 0 + 115 + 690 / 824