

Lista de exercícios

$$1. \binom{8}{3} = \frac{n!}{k!(n-k)!} \rightarrow \frac{8!}{3!(8-3)!} = \frac{8!}{3! 5!} = \frac{8 \cdot 7 \cdot 6 \cdot 5!}{3! 5!}$$

$$= \frac{336}{3 \cdot 2 \cdot 1} = \frac{336}{6} = 56 \rightarrow \text{alternativa B,}$$

$$2. \binom{200}{198} = \frac{200!}{198!(200-198)!} = \frac{200 \cdot 199 \cdot 198!}{198! 2!} = \frac{200 \cdot 199}{2 \cdot 1} = 19900 \rightarrow \text{alternativa A,}$$

$$3. \binom{n-1}{2} = \binom{n+1}{4}$$

$$\frac{(n-1)!(n+1)}{2!(n-1-2)!} = \frac{(n+1)!}{4!(n+1-4)!}$$

$$\frac{n-1!}{2!(n-3)!} = \frac{(n+1)!}{4!(n-3)!}$$

$$\frac{(n-1) \cdot (n-2) \cdot (n-3)!}{2! (n-3)!} = \frac{(n+1) \cdot n \cdot (n-1) \cdot (n-2) \cdot (n-3)!}{4! (n-3)!}$$

$$\frac{(n-1) \cdot (n-2)}{2} - \frac{(n+1) \cdot n \cdot (n-1) \cdot (n-2)}{4 \cdot 3 \cdot 2 \cdot 1} = 0$$

$$\frac{(n-1) \cdot (n-2)}{2} - \frac{(n+1) \cdot n \cdot (n-1) \cdot (n-2)}{24} = 0$$

$$12(n-1) \cdot (n-2) - n \cdot (n+1) \cdot (n-1) \cdot (n-2) = 0$$

\Leftrightarrow

$$12(n-1) \cdot (n-2) - n \cdot (n+1) \cdot (n-1) \cdot (n-2) = 0$$

$$(n-1) \cdot (n-2) \cdot (12 - n \cdot (n+1)) = 0$$

$$(n-1) \cdot (n-2) \cdot (-n^2 - n + 12) = 0$$

$$n-1 = 0 \quad n-2 = 0$$

$$n = 1 //$$

$$n = 2 //$$

$$-n^2 - n + 12$$

$$n = \frac{-(-1) \pm \sqrt{49}}{2}$$

$$\Delta = (-1)^2 - 4 \cdot (-1) \cdot 12$$

$$\Delta = 1 + 48$$

$$n = \frac{1 + 7}{-2} = \frac{8}{-2} = -4$$

$$\Delta = 1 + 48$$

$$n = \frac{1 - 7}{-2} = \frac{-6}{-2} = 3 //$$

$$\Delta = 49$$

$$n = 3 //$$

$$n = 1$$

$$n = 2$$

$$n = 3$$

$n = -4 \rightarrow$ não convém

$$\downarrow \{1, 2, 3\}$$

$$4. \quad \binom{20}{13} + \binom{20}{14} = \binom{21}{24} \quad \binom{k}{n} = \binom{n}{n-k} \rightarrow \binom{21}{24} = \binom{21}{21-24}$$

soma na linha 20

$$\binom{21}{21-24}$$

alternativa C //

$$5. \quad \binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \dots + \binom{n}{n} = \underbrace{2^n}_{(9)}$$

soma na linha

6. a) $\sum_{p=0}^{10} \binom{10}{p} = \binom{10}{0} + \binom{10}{1} + \binom{10}{2} + \binom{10}{3} + \dots + \binom{10}{10}$
 $2^{10} = \underbrace{1024}_1$

b) $\sum_{p=0}^9 \binom{10}{p} = \binom{10}{0} + \binom{10}{1} + \binom{10}{2} + \binom{10}{3} + \dots + \binom{10}{9}$
 linha 10 - $\binom{10}{10}$

$$2^{10} - 1 \rightarrow 1024 - 1 = \underbrace{1023}_1$$

c) $\sum_{p=0}^9 \binom{9}{p} = \binom{9}{0} + \binom{9}{1} + \binom{9}{2} + \dots + \binom{9}{9}$
 $2^9 - \binom{9}{0} - \binom{9}{1} -$

$$2^9 - 1 - 9$$

$$512 - 10 = \underbrace{502}_1$$

d) $\sum_{p=4}^{10} \binom{p}{4} = \sum_{p=4}^{10} \frac{p!}{4!(p-4)!} = \frac{4!}{4!(4-4)!} + \frac{5!}{4!(5-4)!} + \frac{6!}{4!(6-4)!} + \frac{7!}{4!(7-4)!} + \frac{8!}{4!(8-4)!} + \frac{9!}{4!(9-4)!} + \frac{10!}{4!(10-4)!}$

$$\begin{aligned} & \frac{1}{1} + \frac{5}{1} + \frac{6 \cdot 5}{2!} + \frac{7 \cdot 6 \cdot 5}{3!} + \frac{8 \cdot 7 \cdot 6 \cdot 5}{4!} + \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5}{5!} \\ & + \frac{10 \cdot 9 \cdot 8 \cdot 7}{4!} \end{aligned}$$

$$\frac{(2! \cdot 3 \cdot 4 \cdot 5) \cdot 7 + 2520 + 6720 + 15120 + 30240}{2! \cdot 3 \cdot 4 \cdot 5}$$

$$\frac{5! \cdot 7 + 2520 + 6720 + 15120 + 30240}{5!}$$

$$\frac{7 \cdot 120 + 54600}{120}$$

$$\frac{840 + 54600}{120} = \underline{\underline{462}}$$

$$7. \sum_{k=0}^m \binom{m}{k} = 512$$

$$\binom{m}{0} + \binom{m}{1} + \binom{m}{2} + \dots + \binom{m}{m}$$

$$2^m = 512$$

$$2^9 = 512$$

$$\underline{\underline{m = 9}}$$

alternativa E //