

Lista de exercícios

$$1. \binom{8}{3} = \frac{n!}{k!(n-k)!} \rightarrow \frac{8!}{3!(8-3)!} = \frac{8!}{3! \cdot 5!} = \frac{8 \cdot 7 \cdot 6 \cdot 5!}{3! \cdot 5!} = \frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1} = \frac{336}{6} = 56 \rightarrow \text{alternativa B}$$

$$2. \binom{200}{198} = \frac{200!}{198!(200-198)!} = \frac{200 \cdot 199 \cdot 198!}{198! \cdot 2!} = \frac{39800}{2} = 19900 \rightarrow \text{alternativa A}$$

$$3. \binom{n-1}{2} = \binom{n+1}{4}$$

$$\frac{(n-1)! \cdot n!}{2! (n-1-2)!} = \frac{(n+1)!}{4! (n+1-4)!}$$

$$\frac{n-1!}{2! (n-3)!} = \frac{(n+1)!}{4! (n-3)!}$$

$$\frac{(n-1) \cdot (n-2) \cdot \cancel{(n-3)!}}{2! \cdot \cancel{(n-3)!}} = \frac{(n+1) \cdot n \cdot (n-1) \cdot (n-2) \cdot \cancel{(n-3)!}}{4! \cdot \cancel{(n-3)!}}$$

$$\frac{(n-1) \cdot (n-2)}{2} - \frac{(n+1) \cdot n \cdot (n-1) \cdot (n-2)}{4 \cdot 3 \cdot 2 \cdot 1} = 0$$

$$\frac{(n-1) \cdot (n-2)}{2} - \frac{(n+1) \cdot n \cdot (n-1) \cdot (n-2)}{24} = 0$$

$$12(n-1) \cdot (n-2) - n \cdot (n+1) \cdot (n-1) \cdot (n-2) = 0$$

$$12(n-1) \cdot (n-2) - n \cdot (n+1) \cdot (n-1) \cdot (n-2) = 0$$

$$(n-1) \cdot (n-2) \cdot (12 - n \cdot (n+1)) = 0$$

$$(n-1) \cdot (n-2) \cdot (-n^2 - n + 12) = 0$$

$$n-1=0 \quad n-2=0$$

$$n=1 // \quad n=2 //$$

$$-n^2 - n + 12$$

$$n = -(-1) \pm \sqrt{49}$$

$$\Delta = (-1)^2 - 4 \cdot (-1) \cdot 12$$

$$2 \cdot (-1)$$

$$\Delta = 1 + 48$$

$$\Delta = 49$$

$$n = \frac{1 \pm 7}{-2}$$

$$n' = \frac{1+7}{-2} = \frac{8}{-2} = -4$$

$$n'' = \frac{1-7}{-2} = \frac{-6}{-2} = 3 //$$

$$n=1$$

$$n=2$$

$$n=3$$

$$n=-4 \rightarrow \text{n\~{a}o conv\~{e}m}$$

$$V = \{1, 2, 3\}$$

$$4. \begin{pmatrix} 20 \\ 13 \end{pmatrix} + \begin{pmatrix} 20 \\ 14 \end{pmatrix} = \begin{pmatrix} 21 \\ 14 \end{pmatrix} \quad \begin{pmatrix} k \\ n \end{pmatrix} = \begin{pmatrix} n \\ n-k \end{pmatrix} \rightarrow \begin{pmatrix} 21 \\ 14 \end{pmatrix} = \begin{pmatrix} 21 \\ 21-14 \end{pmatrix}$$

soma na linha 20

$$\begin{pmatrix} 21 \\ 7 \end{pmatrix}$$

alternativa C //

$$5. \binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \dots + \binom{n}{n} = 2^n$$

soma na linha

6. a) $\sum_{p=0}^{10} \binom{10}{p} = \binom{10}{0} + \binom{10}{1} + \binom{10}{2} + \binom{10}{3} + \dots + \binom{10}{10}$
 $2^{10} = 1024$

b) $\sum_{p=0}^9 \binom{10}{p} = \binom{10}{0} + \binom{10}{1} + \binom{10}{2} + \binom{10}{3} + \dots + \binom{10}{9}$
 linha 10 - $\binom{10}{10}$
 $2^{10} - 1 \rightarrow 1024 - 1 = 1023$

c) $\sum_{p=2}^9 \binom{9}{p} = \binom{9}{2} + \binom{9}{3} + \binom{9}{4} + \dots + \binom{9}{9}$
 $2^9 - \binom{9}{0} - \binom{9}{1}$
 $2^9 - 1 - 9$
 $512 - 10 = 502$

d) $\sum_{p=4}^{10} \binom{p}{4} = \sum_{p=4}^{10} \frac{p!}{4!(p-4)!} = \frac{4!}{4!(4-4)!} + \frac{5!}{4!(5-4)!} + \frac{6!}{4!(6-4)!}$

$+ \frac{7!}{4!(7-4)!} + \frac{8!}{4!(8-4)!} + \frac{9!}{4!(9-4)!} + \frac{10!}{4!(10-4)!}$

$\frac{1}{1} + \frac{5}{2!} + \frac{6 \cdot 5}{3!} + \frac{7 \cdot 6 \cdot 5}{4!} + \frac{8 \cdot 7 \cdot 6 \cdot 5}{4!} + \frac{9 \cdot 8 \cdot 7 \cdot 6}{4!}$
 $+ \frac{10 \cdot 9 \cdot 8 \cdot 7}{4!}$

$$\frac{(2! \cdot 3 \cdot 4 \cdot 5) \cdot 7 + 2520 + 6720 + 15120 + 30240}{2! \cdot 3 \cdot 4 \cdot 5}$$

$$\frac{5! \cdot 7 + 2520 + 6720 + 15120 + 30240}{5!}$$

$$\frac{7 \cdot 120 + 54600}{120}$$

$$\frac{840 + 54600}{120} = \underline{462}$$

$$\sum_{k=0}^m \binom{m}{k} = 512$$

$$\binom{m}{0} + \binom{m}{1} + \binom{m}{2} + \dots + \binom{m}{m}$$

$$2^m = 512$$

$$2^9 = 512$$

$$\underline{m=9} \quad \text{alternativa E}$$