# Principal Component Analysis (PCA)
# Part 2: Problem Formulation & Step-by-Step Solution

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## ➢ MNIST Dataset Example

- Starts with a **real-world problem**: reducing dimensionality in **handwritten digit data (MNIST)**.
- Highlights PCA's role in simplifying data while retaining structure.

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## ➢ Problem Formulation

- Goal: Find a **lower-dimensional space** that retains as much **variance (information)** as possible.
- Key challenge: Identify the **optimal axes** (principal components) for projection.

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## ➢ Covariance and Covariance Matrix

- Covariance shows how features **vary together**.
- The **covariance matrix** captures pairwise covariances for all features — it's **symmetric** and helps understand the data's **spread and orientation**.

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## ➢ Eigenvectors and Eigenvalues

- **Eigenvectors**: Directions in space (axes) along which the data varies most.
- **Eigenvalues**: Amount of variance captured along each eigenvector.
- These are derived from the **covariance matrix**.
- Higher eigenvalue = more important direction.

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## ➢ Visualizing Linear Transformations

- Uses the Geogebra tool to **illustrate eigenvectors** as **invariant directions** under linear transformations.
- Great visual intuition: Eigenvectors **don't change direction** when transformed — they **stretch or shrink** (scaled by eigenvalues).

🔗 Visualization tool used

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## ➢ Eigendecomposition of Covariance Matrix

- Covariance matrix can be decomposed as:

$$\mathrm{Cov} = Q\Lambda Q^T$$

where:

1. Q: matrix of eigenvectors
2. Λ\Lambda: diagonal matrix of eigenvalues

- This breakdown is core to how PCA finds new axes for projection.

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## ➢ How to Solve PCA (Step-by-Step)

1. **Standardize the data** (mean = 0, variance = 1).
2. **Compute the covariance matrix**.
3. **Find eigenvectors & eigenvalues** of the covariance matrix.
4. **Sort eigenvectors** by descending eigenvalue.
5. **Select top-k** eigenvectors (k = target dimension).
6. **Project data** onto this reduced subspace.

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## ➢ How to Transform Points?

- New points can be projected as:

$$X_{\text{projected}} = X \cdot W$$

where W is the matrix of selected eigenvectors (principal components).

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## ➢ Key Takeaway:

PCA reduces high-dimensional data into a lower-dimensional space by projecting it onto **eigenvectors** of the covariance matrix — prioritizing directions that capture the most variance. This is achieved through **standardization**, **covariance analysis**, **eigendecomposition**, and **projection**.

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