Regression Metrics | MSE, MAE, RMSE | R^2 Score & Adjusted R^2 Score

➢ Evaluating Model Performance

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1. MAE (Mean Absolute Error)

$$MAE = \frac{1}{n}\sum_{i=1}^{n}|y_i - \hat{y}_i|$$

- **What it measures:**
  The **average absolute difference** between actual (yi) and predicted (y^i) values.
- **Characteristics:**
  1. Simple and easy to interpret.
  2. Treats all errors **equally**, ignoring their direction (positive/negative).
- **Limitation:**
  Less sensitive to **large errors** because it does not square differences.

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2. MSE (Mean Squared Error)

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$$

- **What it measures:**
  The **average squared difference** between actual and predicted values.
- **Characteristics:**
  1. Penalizes **larger errors** more heavily than smaller ones.
  2. Ensures errors are always positive because they're squared.
- **Limitation:**
  Units of error are **squared** (not in original units).

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3. RMSE (Root Mean Squared Error)

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}$$

- **What it measures:**
  Square root of MSE, giving errors in the **same unit as the target variable**.

- **Characteristics:**
    1. More interpretable than MSE because it's in the same scale as y.
    2. Still **penalizes large errors** more strongly than small ones.

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4. R^2 Score (Coefficient of Determination)

$$R^2 = 1 - \frac{\sum(y_i - \hat{y}_i)^2}{\sum(y_i - \bar{y})^2}$$

Where:

- y¯\bar = mean of actual values.
- **What it measures:**
  Proportion of **variance explained** by the regression model compared to a baseline model (mean prediction).
- **Interpretation:**
    1. R^2 = 1: Perfect prediction.
    2. R^2 = 0: Model predicts no better than mean.
    3. R^2 < 0: Model is worse than mean prediction.
- **Limitation:**
  R^2 **always increases** when more features are added, even if they are irrelevant.

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5. Adjusted R^2 Score

$$Adjusted\ R^2 = 1 - \left(1 - R^2\right)\frac{n-1}{n-p-1}$$

Where:

- n = number of data points
- p = number of independent variables (features)
- **What it measures:**
  Adjusts R^2 for the **number of predictors**, preventing overestimation when irrelevant features are added.
- **Interpretation:**
    1. Only increases when a new variable **improves model performance significantly**.
    2. More reliable than R^2 for **multiple regression models**.

➢ Key Takeaways

Metric	Measures	Good Value	Best Use
MAE	Avg. absolute error	Closer to 0	When all errors should be treated equally
MSE	Avg. squared error	Closer to 0	When large errors should be penalized more
RMSE	Square root of MSE	Closer to 0	Same units as target for easy interpretation
R^2	Variance explained	Closer to 1	Basic goodness of fit
Adj. R^2	Variance explained (adjusted for features)	Closer to 1	Multiple regression, avoids overfitting