Department of Mathematics and Natural Sciences

PHY111 - Principles of Physics-I (Summer 2021)

Assignment-4

Total Marks: 20

Answer all questions.

1. Two blocks, of masses M = 2.3 kg and 2M are connected to a spring of spring constant k = 180 N/m that has one end fixed, as shown in the Figure-1. The coefficient of kinetic friction between the horizontal surface and the block is 0.12. The pulley is frictionless and has a negligible mass. The blocks are released from rest with the spring relaxed.

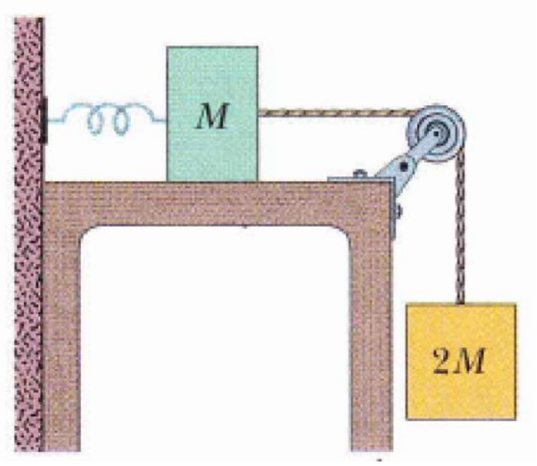

Figure-1

(a) What is the work done by the friction on the block of mass M? [2 marks]
(b) What is the combined kinetic energy of the two blocks when the hanging block has fallen 8 cm? [4 marks]
(c) What maximum distance does the hanging block fall before momentarily stopping? [4 marks]

2. A block of mass m rests on a plane inclined at θ with the horizontal. The block is attached to a spring of constant k as shown in Figure-2. The coefficients of static and kinetic friction between the block and plane are μ_s and μ_k respectively. Very slowly, the spring is pulled upward along the plane until the block starts to move.

(a) Obtain an expression for the extension d of the spring the instant the block moves.

[4 marks]

(b) Determine the value of μ_k such that the block comes to rest just as the spring is in its unstressed condition, that is, neither extended nor compressed.

[6 marks]

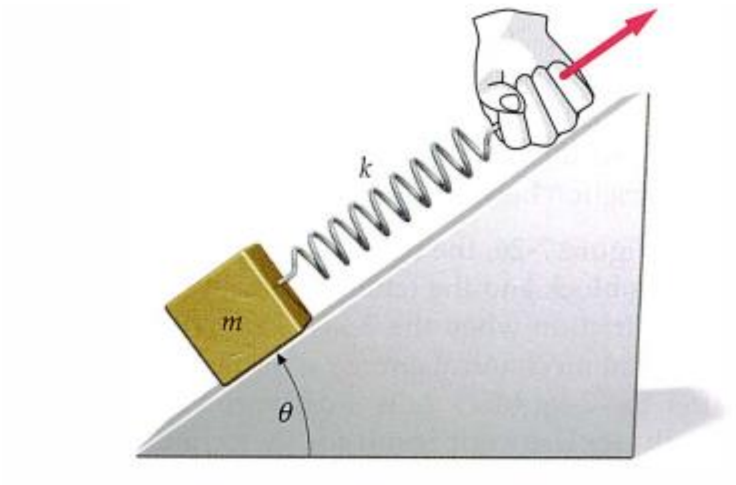

Figure-2

Assignment - 4

Question-1 solution:—

(a) Frictional force on mass, M is $f_K = \mu_K Mg$

$= 0.12 \times 2.3 \times 9.8$

$= 2.7\, N$

Work done by frictional force, $W_f = f_K \times d = 2.7 \times 0.08 = 0.216\, J$

(b) Using the principle of conservation of energy for the system

$\Delta E + \Delta E_{th} = 0$

$\Rightarrow E_f - E_i + \Delta E_{th} = 0$

$\Rightarrow K_f + U_f + \Delta E_{th} = 0$

$\Rightarrow K_f = -U_f - \Delta E_{th}$

Here

$W_f = \Delta E_{th} = 0.216\, J$

$E_i = K_i + U_i = 0$

$= -\left\{ 2Mg \times (-0.08) + \frac{1}{2} \times k \times (0.08)^2 \right\} - \Delta E_{th}$

$= -\left\{ 2 \times 2.3 \times 9.8 \times (-0.08) + \frac{1}{2} \times 180 \times (0.08)^2 \right\} - 0.216$

$\boxed{K_f = 2.81\, J}$

(c) $E_f - E_i + \Delta E_{th} = 0$ (for the system)

$\Rightarrow K_f + U_f - 0 + f_K d = 0$

$\Rightarrow 0 + 2 \times 2.3 \times 9.8 \times (-0.08) + \frac{1}{2} \times 180 \times d^2 + 2.7 \times d = 0$

$E_i = 0$

$W_f = \Delta E_{th} = f_K d$

$K_f = 0$

$\Rightarrow 90d^2 - 45.08d + 2.7d = 0$

$\Rightarrow 90d^2 - 42.38d = 0$

$\Rightarrow d(90d - 42.38) = 0$

$d = 0$ not applicable

$\boxed{d = 0.47\, m}$

Solution of Assignment-4

Wednesday, September 1, 2021 9:32 AM

Principles of Physics-I, PHY111

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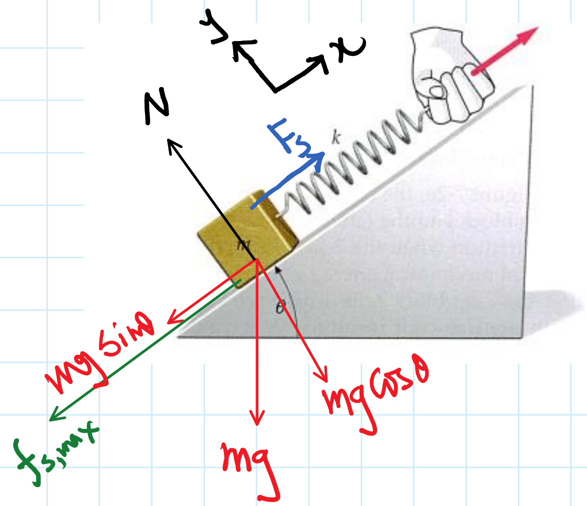
1. A block of mass m rests on a plane inclined at θ with the horizontal. The block is attached to a spring of constant k as shown in Figure-2. The coefficients of static and kinetic friction between the block and plane are μ_s and μ_k respectively. Very slowly, the spring is pulled upward along the plane until the block starts to move.

   (a) Obtain an expression for the extension d of the spring the instant the block moves. [4 marks]

   (b) Determine the value of μ_k such that the block comes to rest just as the spring is in its unstressed condition, that is, neither extended nor compressed. [6 marks]



Sol^n:- (a) Here, $\sum F_y = 0$

$$\Rightarrow N - mg\cos\theta = 0$$

$$\Rightarrow \boxed{N = mg\cos\theta}$$

Now, for the condition,

$$F_s = mg\sin\theta + f_{s,max}$$

$$\Rightarrow kd = mg\sin\theta + \mu_s N$$

$$\Rightarrow kd = mg\sin\theta + \mu_s mg\cos\theta$$

$$\Rightarrow \boxed{d = \frac{mg}{k}(\sin\theta + \mu_s\cos\theta)}$$

4m

(b) For this case we have to find the value of μ_k so that block come to rest. so

$$\Delta E_{sys} = 0$$

$$\Rightarrow \Delta K + \Delta U + W_f = 0$$

Work done by friction

$$\Rightarrow K_f - K_i + U_f - U_i + W_f = 0$$

$$\Rightarrow K_i + U_i = K_f + U_f + W_f$$

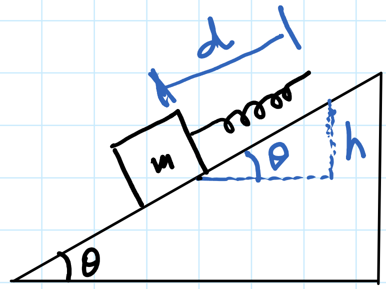
b $\rightarrow$ block, s $\rightarrow$ spring

U_{is} = Initial P.E of spring

K_{ib} = Initial K.E of block

$$\Rightarrow K_{ib} + K_{is} + U_{ib} + U_{is} = K_{fb} + K_{fs} + U_{fb} + U_{fs} + W_f$$

$$\Rightarrow \underbrace{K_{ib}}_{0} + \underbrace{K_{is}}_{0} + \underbrace{U_{ib}}_{0} + U_{is} = \underbrace{K_{fb}}_{0} + \underbrace{K_{fs}}_{0} + U_{fb} + \underbrace{U_{fs}}_{0} + W_f$$



$$\Rightarrow \quad U_{is} = U_{fb} + W_f$$

$$\Rightarrow \frac{1}{2} k d^2 = mgd\sin\theta - f_k d$$

$$\Rightarrow \frac{1}{2} k d = mg\sin\theta - f_k$$

$$\Rightarrow \quad \frac{1}{2}\cancel{k}\,\frac{\cancel{mg}}{\cancel{k}}(\sin\theta + \mu_s\cos\theta) = \cancel{mg}\sin\theta - \mu_k \cancel{mg\cos\theta}$$

$$\Rightarrow \quad \frac{1}{2}\sin\theta + \frac{1}{2}\mu_s\cos\theta = \sin\theta - \mu_k\cos\theta$$

$$\Rightarrow \quad \frac{1}{2}\tan\theta + \frac{1}{2}\mu_s = \tan\theta - \mu_k \quad [\text{Dividing by } \cos\theta \text{ on both side}]$$

$$\Rightarrow \quad \mu_k = \tan\theta - \frac{1}{2}\tan\theta - \frac{1}{2}\mu_s$$

$$\Rightarrow \quad \boxed{\mu_k = \frac{1}{2}(\tan\theta - \mu_s)}$$

Ans