Department of Mathematics and Natural Sciences

PHY111 - Principles of Physics-I (Summer 2021)

Assignment-3

Total Marks: 20

Answer all questions.

1. Two 22.7 kg ice sleds initially at rest, are placed a short distance apart, one directly behind the other, as shown in Fig. 1. A 3.63 kg cat, standing on the left sled, jumps across to the right one and immediately comes back to the first. Both jumps are made horizontally at a speed of 3.05 m/s relative to the ice. Ignore the friction between the sled and ice.

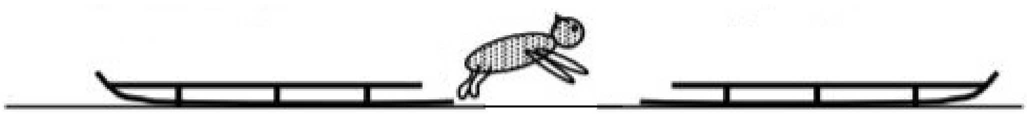


Fig. 1

(a) Find the final speeds of the two sleds. [6 marks]
(b) Calculate the impulse on the cat as it lands on the right sled. [2 marks]
(c) Find the average force on the right sled applied by the cat while landing. Consider that the cat takes 12 ms to finish the landing. [2 marks]

2. A 67 kg man stands at the front end of a uniform boat of mass 179 kg and of length, L = 2.5 m. Assume there is no friction or drag between the boat and water.

(a) What is the location of the center of mass of the system when the origin of our coordinate system (*i*) on the man's original location (*ii*) on the back end of the boat? [4 marks]
(b) If the man walks from the front end to the back end of the boat, by how much is the boat displaced? [3 marks]
(c) Now consider the man and his friend with identical mass of 67 kg are rowing the boat on a hot summer afternoon when they decide to go for a swim. The man jumps off the front of the boat at speed 3 m/s and his friend jumps off the back at speed 4 m/s. If the boat was moving forward at 1.5 m/s when they jumped, what is the speed of the boat after their jump? [3 marks]

(1) (a) Let, the x axis point to the right.

When the cat is on the left sled, both cat and sled are motionless.

$\therefore$ Initial momentum of the system, $P_{1i} = 0$

After the cat has made its first jump, the final momentum of the system,

$$P_{1f} = P_{\text{left sled}} + P_{\text{cat}}$$

Consider, $V_{L,x}$ = Velocity of the left sled

$$= (22.7 \times V_{L,x}) + (3.63 \times 3.05)$$

$$= 22.7 V_{L,x} + 11.072$$

According to momentum conservation law,

$$P_{1i} = P_{1f}$$

$$\Rightarrow 0 = 22.7 V_{L,x} + 11.072$$

$$\Rightarrow V_{L,x} = -0.488 \text{ ms}^{-1}$$

After landing on the right sled it moves with the same velocity as that sled.

Initial momentum of the right sled and cat system is,

$$P_{2i} = (3.63 \times 3.05) + 0$$

$$\Rightarrow P_{2i} = 11.072 \text{ kg.ms}^{-1}$$

Final momentum of the system,

$$P_{2f} = (22.7 + 3.63) V_{R,x}$$

Here $V_{R,x}$ = velocity of both cat and right sled

$$\Rightarrow P_{2f} = 26.33 V_{R,x}$$

According to momentum conservation law,

$$P_{2i} = P_{2f}$$

$$\Rightarrow 11.072 = 26.33 V_{R,x}$$

$$\Rightarrow V_{R,x} = 0.421 \text{ ms}^{-1}$$

Now, during the second jump, Initial momentum of cat and right sled system

$P_{3i} = P_{2f} = 26.33 \times V_{R,x}$

$\Rightarrow P_{3i} = 26.33 \times 0.421$

$\Rightarrow P_{3i} = 11.09 \ kg.ms^{-1}$

Final momentum of cat and right sled system,

$P_{3f} = P'_{cat} + P'_{right\ sled}$

$\Rightarrow P_{3f} = \{3.63 \times (-3.05)\} + (22.7 \times V'_{R,x})$

$V'_{R,x}$ = Velocity of right sled after 2nd jump

$\Rightarrow P_{3f} = -11.072 + 22.7 V'_{R,x}$

According to momentum conservation law,

$P_{3i} = P_{3f}$

$\Rightarrow 11.09 = -11.072 + 22.7 V'_{R,x}$

$\Rightarrow V'_{R,x} = 0.976 \ ms^{-1}$

Finally, for the cat's landing on the left sled, initial momentum of the system, $P_{4i} = P'_{cat} + P'_{left\ sled}$

$\Rightarrow P_{4i} = \{3.63 \times (-3.05)\} + \{(22.7) \times (-0.488)\}$

$\Rightarrow P_{4i} = -11.072 - 11.078$

$\Rightarrow P_{4i} = -22.15 \ kg.ms^{-1}$

After landing, final momentum, $P_{4f} = (22.7 + 3.63) V'_{L,x}$

$\Rightarrow P_{4f} = 26.33 V'_{L,x}$

$V'_{L,x}$ = final velocity of left sled

According to momentum conservation law,

$P_{4i} = P_{4f}$

$\Rightarrow -22.15 = 26.33 V'_{L,x}$

$\therefore V'_{L,x} = -0.841 \ ms^{-1}$

Left sled: $V'_{L,x} = -0.842 \ ms^{-1}$
speed $= 0.842 \ ms^{-1}$
Right sled: $V'_{R,x} = 0.976 \ ms^{-1}$
speed $= 0.976 \ ms^{-1}$

(b) Impulse on the cat as it lands on the right sled

$= (\text{Final} - \text{Initial})$ momentum of the cat

$= (3.63 \times 3.05) - (3.63 \times V_{R,x})$

$= 11.072 - (3.63 \times 0.421)$

$= 9.54 \ \text{kg.ms}^{-1}$

(c) We know,

Impulse = Average force × time

$\therefore F_{avg} = \dfrac{J_{\text{right sled}}}{\text{time}}$

$J_{\text{right sled}} = P_{final} - P_{initial}$

time = 12 ms
$= 1.2 \times 10^{-2}$ sec

$= (22.7 \times 0.421) - 0$

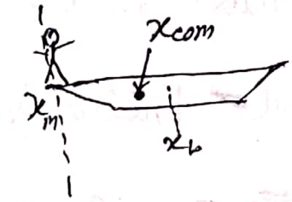
$= 9.56 \ \text{kg.ms}^{-1}$

$\therefore F_{avg} = \dfrac{9.56}{1.2 \times 10^{-2}}$

$= 7.97 \times 10^{2}$ N

2. (a)

(i) $$X_{com} = \frac{M_m x_m + M_b x_b}{M_m + M_b}$$

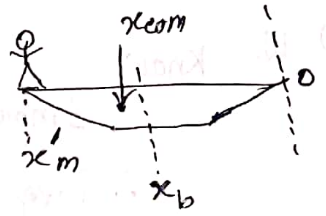


$$= \frac{(67 \times 0) + (179 \times 1.25)}{(67 + 179)}$$

$$= 0.91\ m$$

$L = 2.5m$
$M_m = 67\ Kg$
$M_b = 179\ kg$
$x_m = 0$
$x_b = L/2 = \frac{2.5}{2} = 1.25m$

(ii) $$X_{com} = \frac{M_m x_m' + M_b x_b}{M_m + M_b}$$



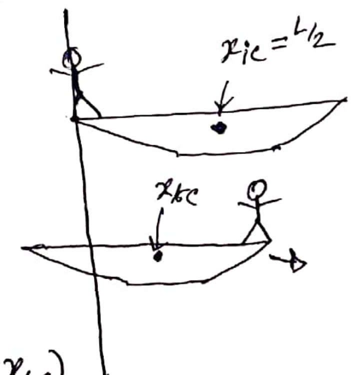
$$= \frac{(67 \times 2.5) + (179 \times 1.25)}{(67 + 179)}$$

$$= 1.59m$$

$x_m' = L = 2.5m$
$x_b = 1.25m$

(b) Initial position of the man is zero but after walking due to the displacement of the boat, let the position of the man is $(x_{fc} + L/2)$

Here, X_{fc} is the final position of the center of mass of the boat and $L/2$ is the initial position of the center of mass of the boat with respect to axis.



From (a)(i) Center of mass in 1st case, $= 0.91$

and in 2nd case, $$X_{com} = \frac{M_m X_m + M_b x_b}{M_m + M_b}$$

$$= \frac{\{67(x_{fc} + 1.25)\} + (179 \times x_{fc})}{(67 + 179)}$$

$$= \frac{246 x_{fc} + 80.4}{246}$$

We Know X_{com} does not move, $\therefore 0.91 = \frac{246 x_{fc} + 80.4}{246}$

$\Rightarrow x_{fc} = 0.583$

$\therefore$ Displacement of the boat $= x_{fc} - L/2 = 0.583 - 1.25 = -0.667m$

(c) We are considering, front/forward movement is positive and opposite is negative,

$m_1 = m_2 = 67\,kg$

$m_b = 179\,kg$

$\vec{V}_{1i} = \vec{V}_{2i} = \vec{V}_{bi} = 1.5\,ms^{-1}$

$\vec{V}_{1f} = 3\,ms^{-1}$

$\vec{V}_{2f} = -4\,ms^{-1}$

$\vec{V}_{bf} = ?$

According to momentum conservation law,

$$(m_1+m_2+m_b)\vec{V}_b = m_1\vec{V}_{1f} + m_2\vec{V}_{2f} + m_b\vec{V}_{bf}$$

$$\Rightarrow \vec{V}_{bf} = \frac{(m_1+m_2+m_b)\vec{V}_b - m_1\vec{V}_{1f} - m_2\vec{V}_{2f}}{m_b}$$

$$= \frac{\{(67+67+179)\times 1.5\} - (67\times 3) - \{67\times(-4)\}}{179}$$

$$= 2.997 \approx 3\,ms^{-1}$$

$\therefore$ Final speed of the boat is $3\,ms^{-1}$