

1. A 15 kg box frame is sliding down a 20° incline plane with a 10 kg steel sphere suspended inside the box as shown in Fig. 1. Consider the coefficient of kinetic friction between the box and the incline is 0.15.

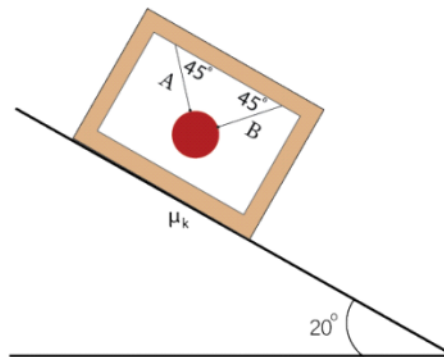
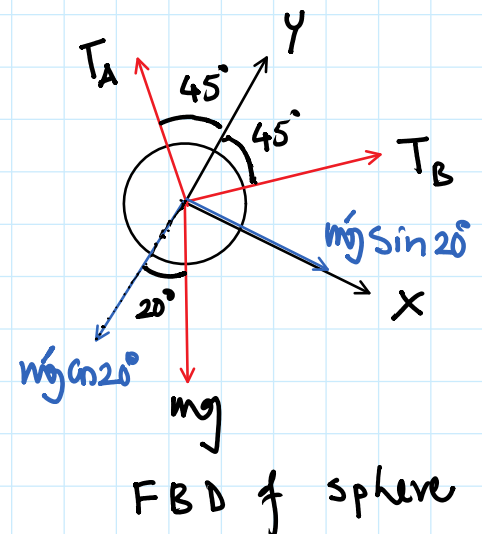
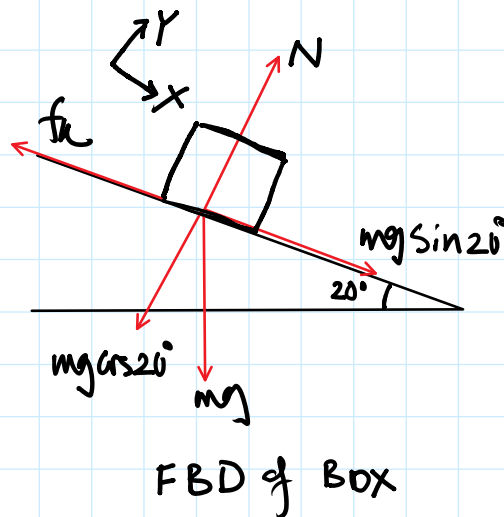


Fig. 1

- (2 marks) Draw a free body diagram of the forces on the box and on the sphere.
- (3 marks) What is the acceleration of the box?
- (5 marks) Calculate the tension in each of the supporting wires A and B.

Soln: (a)



(b) Box and sphere as a unit

$$\begin{aligned} \sum F_y &= 0 \\ \Rightarrow N - mg \sin 20^\circ - mg \sin 20^\circ &= 0 \\ \Rightarrow N &= (m + m')g \sin 20^\circ = 25g \sin 20^\circ \\ \Rightarrow \boxed{N = 230.22 \text{ N}} \end{aligned}$$

Ans. $\Sigma F_x = ma$

$$\Rightarrow mg \sin 20^\circ + m'g \sin 20^\circ - f_k = (m + m')a$$

$$\Rightarrow (m + m')g \sin 20^\circ - \mu_k N = (m + m')a$$

$$\Rightarrow a = \frac{25g \sin 20^\circ - 0.15 \times 230.22}{25}$$

$$\Rightarrow \boxed{a = 1.97 \text{ m/s}^2} \quad \underline{\text{Ans}}$$

(c) Sphere alone

$$\Sigma F_y = 0$$

$$\Rightarrow T_A \cos 45^\circ + T_B \cos 45^\circ - mg \cos 20^\circ = 0$$

$$\Rightarrow \boxed{T_A + T_B = 130.23 \text{ N}}$$

Ans. $\Sigma F_x = ma$

$$\Rightarrow T_B \sin 45^\circ - T_A \sin 45^\circ + mg \sin 20^\circ = m'a$$

$$\Rightarrow \boxed{T_B - T_A = -17.54}$$

So, $\boxed{T_A = 74.88 \text{ N}}$

and $\boxed{T_B = 55.34 \text{ N}}$

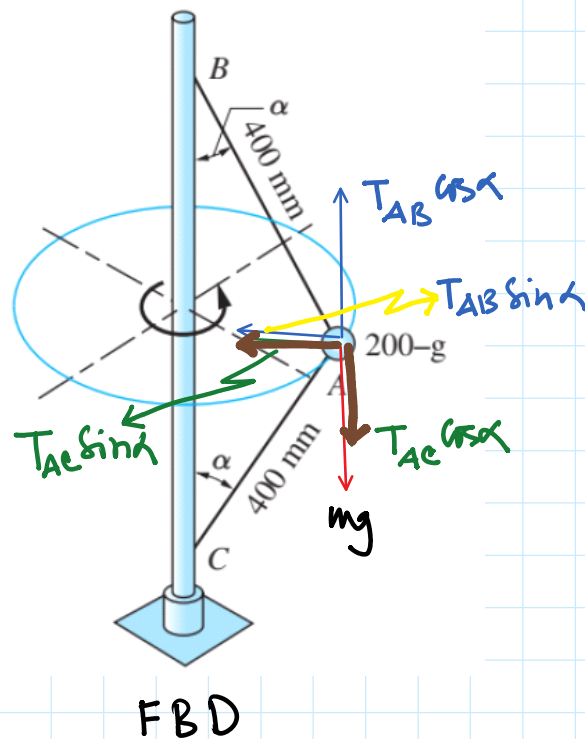
Ans

2. The strings AB and AC connect the 200 g ball to the vertical shaft as shown in Fig. 2. The shaft rotates at a constant angular speed ω and the ball travels in a horizontal circle with the strings inclined at $\alpha = 30^\circ$ to the shaft. Given that the tension in string AC is 4 N.

- (2 marks) Draw a free body diagram of the forces on the ball.
- (3 marks) Calculate the radial acceleration of the ball.
- (4 marks) Find the value of the angular speed ω .
- (1 marks) What is the tangential acceleration of the ball?

Soln:

(a)



(b) Radial acceleration is a_c .

Here $\sum F_y = 0$

$$T_{AB} \cos \alpha - T_{AC} \cos \alpha - mg = 0$$

$$\Rightarrow T_{AB} \cos 30^\circ = mg + T_{AC} \cos 30^\circ$$

$$\Rightarrow T_{AB} = \frac{0.2 \times 9.8 + 4 \cos 30^\circ}{\cos 30^\circ}$$

$$\Rightarrow \boxed{T_{AB} = 6.26 \text{ N}}$$

$$\text{Now, } \sum F_n = m a_c$$

$$\Rightarrow T_{AB} \sin 30^\circ + T_{AC} \sin 30^\circ = m a_c$$

$$\Rightarrow a_c = \frac{T_{AB} \sin 30^\circ + T_{AC} \sin 30^\circ}{m} = 25.65 \text{ m/s}^2$$

$$\Rightarrow \boxed{a_c = 25.65 \text{ m/s}^2} \quad \underline{\text{Ans}}$$

© Now, $a_c = \omega^2 r$

$$\Rightarrow \omega = \sqrt{\frac{a_c}{r}}$$

$$\Rightarrow \omega = \sqrt{\frac{25.65}{0.2}}$$

$$\Rightarrow \boxed{\omega = 11.32 \text{ rad/s}} \quad \underline{\text{Ans}}$$



Here,
 $r = 0.4 \sin 30^\circ$
 $r = 0.2 \text{ m}$

(d) Tangential acceleration is zero, since it rotate at a constant angular speed. Ans