Department of Mathematics and Natural Sciences

PHY111 - Principles of Physics-I (Summer 2021)

Assignment-1

Total Marks: 20

Answer all questions.

1. Three vectors are given by: $\vec{a} = 2\hat{\imath} - 3\hat{\jmath} + 5\hat{k}$, $\vec{b} = -2\hat{\imath} + 7\hat{\jmath} - \hat{k}$ and $\vec{c} = \lambda\hat{\imath} + 2\hat{\jmath} - 4\hat{k}$.
   a. (3 marks): Find λ, so that $\left[\frac{\vec{a}\times\vec{b}}{8}\right] \cdot [2\vec{c}] = -4$.
   b. (2 marks): Calculate the angle between $\vec{b}$ and a position vector with coordinate (0, 0,-3).

2. Two vectors **A** and **B** with the magnitude 750 N and 900 N respectively, which create some angles with the Cartesian coordinate as shown in the Figure-1. Consider the unit vectors $\hat{e}_1$, $\hat{e}_2$ and $\hat{e}_3$ for the +*x*, +*y* and +*z* axis, respectively.

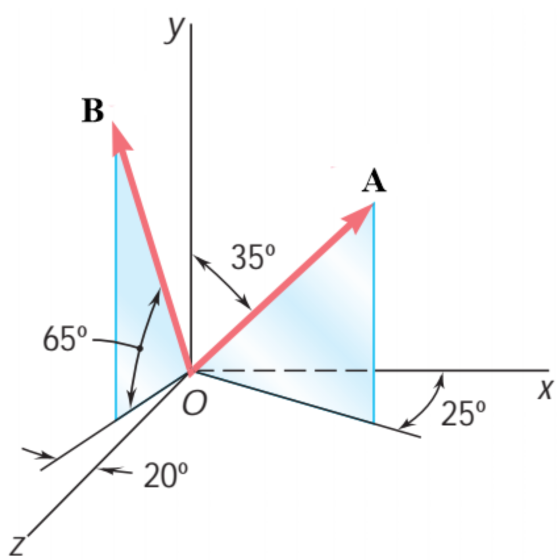


Figure-1

   a. (4 marks): Express the vectors **A** and **B** in terms of unit vectors $\hat{e}_1$, $\hat{e}_2$ and $\hat{e}_3$.
   b. (1 mark): Calculate the resultant vector **R** = **A**+**B**.

3. (5 marks): Pin ***P*** at the end of the telescoping rod as shown in Figure-2 slides along the fixed parabolic path, $y^2 = 40x$, where x and y are measured in millimeters. The y coordinate of ***P*** varies with time t (measured in seconds) according to $y = 4t^2 + 6t$ mm. When $y = 30$ mm, compute the velocity and the acceleration vector of ***P***.

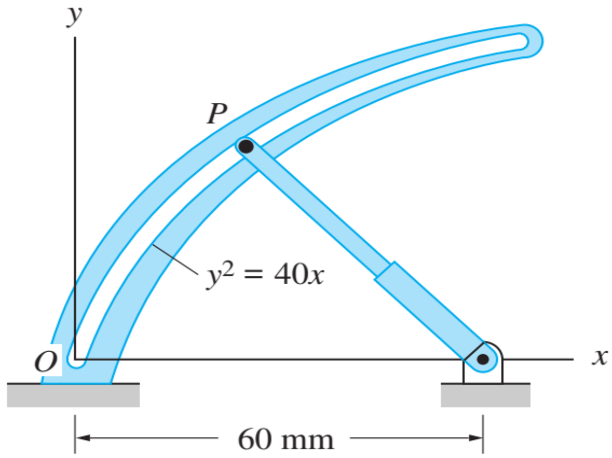


Figure-2

4. A team of engineering students designs a medium size catapult which launches 8 kg steel spheres. The launch speed is v_0 = 45 m/sec, the launch angle is = 35° above the horizontal, and the launch position is 6 m above ground level as shown in Figure-3. The students use an athletic field with an adjoining slope topped by an 8 m fence as shown in the Figure, which is 130 m away from the releasing point of steel spheres. Neglect the air resistance for the following calculations.

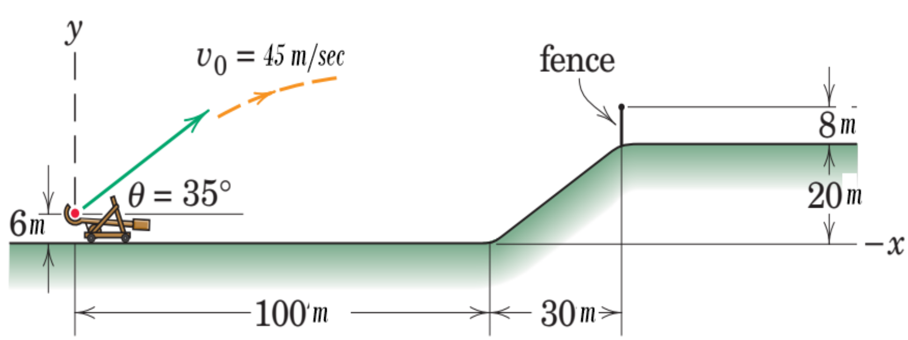


Figure-3

a. (4 marks): Determine steel sphere's time duration t_f of the flight.
b. (1 mark): Calculate the x-y coordinates of the point of first impact of the steel sphere.

① (a) Here, $\vec{a} = 2\hat{i} - 3\hat{j} + 5\hat{k}$, $\vec{b} = -2\hat{i} + 7\hat{j} - \hat{k}$, $\vec{c} = \lambda\hat{i} + 2\hat{j} - 4\hat{k}$

Now, $$[\vec{a} \times \vec{b}] = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 5 \\ -2 & 7 & -1 \end{vmatrix}$$

$$= \hat{i}(3-35) - \hat{j}(-2+10) + \hat{k}(14-6)$$

$$= -32\hat{i} - 8\hat{j} + 8\hat{k}$$

$$\therefore \left[\frac{\vec{a} \times \vec{b}}{8}\right] = \frac{-32\hat{i} - 8\hat{j} + 8\hat{k}}{8}$$

$$= -4\hat{i} - \hat{j} + \hat{k}$$

Again, $2\vec{c} = 2\lambda\hat{i} + 4\hat{j} - 8\hat{k}$

$$\therefore \left[\frac{\vec{a} \times \vec{b}}{8}\right] \cdot (2\vec{c}) = -4$$

$$\Rightarrow (-4\hat{i} - \hat{j} + \hat{k}) \cdot (2\lambda\hat{i} + 4\hat{j} - 8\hat{k}) = -4$$

$$\Rightarrow -8\lambda - 4 - 8 = -4$$

$$\Rightarrow \lambda = \frac{-8}{8}$$

$$\therefore \lambda = -1 \text{ (Ans)}$$

(b) Given position vector, $\vec{R} = -3\hat{k}$ and $\vec{b} = -2\hat{i} + 7\hat{j} - \hat{k}$

Now, $\vec{b} \cdot \vec{R} = bR\cos\theta$

$\therefore \theta = \cos^{-1} \dfrac{(\vec{b} \cdot \vec{R})}{bR}$

$= \cos^{-1} \dfrac{3}{\sqrt{54} \times 3}$

$= 82.18^\circ$

(Ans)

Here,

$b = \sqrt{(-2)^2 + 7^2 + (-1)^2}$

$= \sqrt{54}$

$R = \sqrt{(-3)^2}$

$= 3$

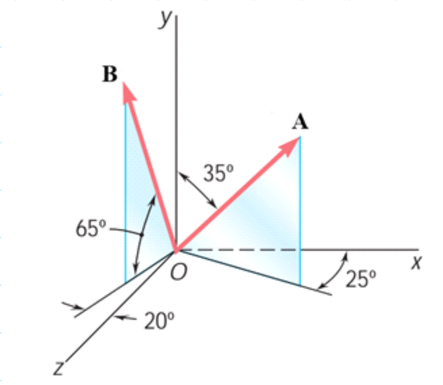
$\vec{b} \cdot \vec{R} = (-2\hat{i} + 7\hat{j} - \hat{k}) \cdot (-3\hat{k})$

$= 3$

Principles of Physics-I, PHY111

1. Two vectors **A** and **B** with the magnitude 750 N and 900 N respectively are acting on a point O, which are creates some angles with the Cartesian coordinate as shown in the Figure-1. Consider the unit vectors $\widehat{e_1}$, $\widehat{e_2}$ and $\widehat{e_3}$ for the +x, +y and +z respectively.

a) (4 marks): Express the vectors **A** and **B** in terms of unit vectors $\widehat{e_1}$, $\widehat{e_2}$ and $\widehat{e_3}$.
b) (1 mark): Calculate the resultant vector **A**+**B**.



Sol^n: (a)

$A_{xz} = A\sin 35° = 750 \sin 35°$

or $\boxed{A_{xz} = 430.18\,N}$

Now, $A_x = A_{xz} \cos 25°$

or $A_x = A \sin 35° \cos 25°$

or $\boxed{A_x = 389.87\,N}$

and $A_z = A_{xz} \sin 25° = A \sin 35° \sin 25°$

$\Rightarrow \boxed{A_z = 181.80\,N}$

And $A_y = A \cos 35° = 750 \cos 35° = 614.36\,N$

or $\boxed{A_y = 614.36\,N}$

So, $\vec{A} = A_x \hat{e}_1 + A_y \hat{e}_2 + A_z \hat{e}_3$

or $\boxed{\vec{A} = 389.87\,\hat{e}_1 + 614.36\,\hat{e}_2 + 181.80\,\hat{e}_3}$ Ans

Now, $B_{zx} = B\cos 65° = 900 \cos 65° = 380.35\,N$

So. $B_z = B_{zx} \cos 20° = 900 \cos 65° \cos 20° = 357.41\,N$

or $\boxed{B_z = 357.41\,N}$

And $B_y = B \sin 65° = 900 \sin 65°$

$\Rightarrow \boxed{B_y = 815.67\,N}$

And $B_x = B_{zx} \sin 20^\circ = 900 \cos 65^\circ \sin 20^\circ$

$$\Rightarrow \boxed{B_x = 130.08\,N}$$

So.
$$\vec{B} = -B_x \hat{e}_1 + B_y \hat{e}_2 + B_z \hat{e}_3$$

$$\Rightarrow \boxed{\vec{B} = -130.08\hat{e}_1 + 815.67\hat{e}_2 + 357.41\hat{e}_3}$$

Ans

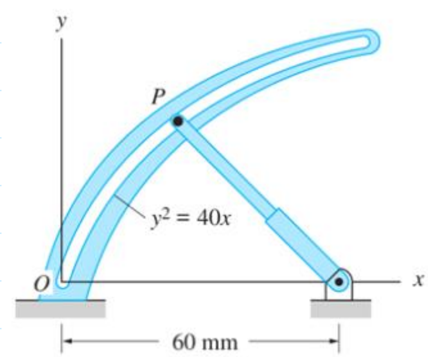
(6) The resultant vector,

$$\vec{R} = \vec{A} + \vec{B}$$

$$\text{or } \boxed{\vec{R} = 259.78\hat{e}_1 + 1430.03\hat{e}_2 + 539.21\hat{e}_3}$$

Ans

1. (5 marks): Pin P at the end of the telescoping rod as shown in Figure-2 slides along the fixed parabolic path, $y^2 = 40x$, where x and y are measured in millimeters. The y coordinate of P varies with time t (measured in seconds) according to $y = 4t^2 + 6t$ mm. When $y = 30$ mm, compute the velocity and the acceleration vector of P.



Soln:- Given that, $y = 4t^2 + 6t$ — (1)

and $y^2 = 40x$

$$\Rightarrow x = \frac{y^2}{40} = \frac{(4t^2+6t)^2}{40}$$

$$\Rightarrow x = 0.4t^4 + 1.2t^3 + 0.9t^2 \quad \text{— (2)}$$

Now, velocity component,

$$v_x = \frac{dx}{dt} = 1.6t^3 + 3.6t^2 + 1.8t \quad \text{— (3)}$$

and $v_y = \frac{dy}{dt} = 8t + 6$ — (4)

Now, when $y = 30$ mm, then eqn (1) $\Rightarrow$

$$\boxed{30 = 4t^2 + 6t}$$

which implies $t = 2{\cdot}09$ sec

So, $v_x = 34{\cdot}1$ mm/s and $v_y = 22{\cdot}7$ mm/s

So $\boxed{\vec{v} = v_x\hat{i} + v_y\hat{j} = 34{\cdot}1\,\hat{i} + 22{\cdot}7\,\hat{j} \text{ mm/s}}$ Ans

Now acceleration, $\vec{a} = a_x\hat{i} + a_y\hat{j}$

$a_x = \frac{dv_x}{dt} = 4{\cdot}8t^2 + 7{\cdot}2t + 1{\cdot}8$ mm/s²

and $a_y = \frac{dv_y}{dt} = 8$ mm/s²

Now, at $t = 2{\cdot}09$ sec

$a_x = 37{\cdot}8$ mm/s² and $a_y = 8$ mm/s²

So, $\boxed{\vec{a} = 37{\cdot}8\,\hat{i} + 8\,\hat{j} \text{ mm/s}^2}$ Ans

1. A team of engineering students designs a medium size catapult which launches 8 kg steel spheres. The launch speed is v_0 = 45 m/sec, the launch angle is = 35° above the horizontal, and the launch position is 6 m above ground level as shown inFigure-3. The students use an athletic field with an adjoining slope topped by an 8 m fence as shown Figure, which is 130 m away from the releasing point of steel spheres. Neglect the air resistance for the following calculations.

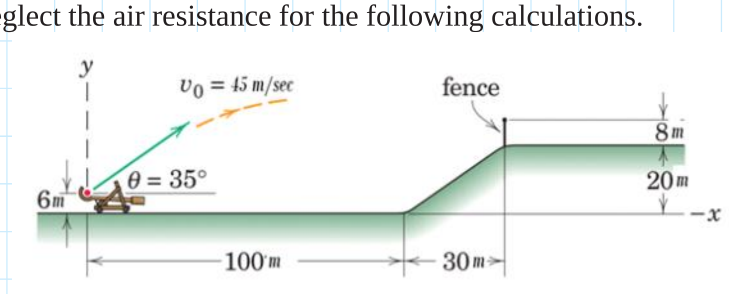


Figure-3

a) (4 marks): Determine the time of flight t_f of the steel sphere.

b) (1 mark): Calculate the *x*-*y* coordinates of the point of first impact of the steel sphere.

Sol^n:- (a) We need to check first, whether steel sphere clear the fence or not.

Here, $x = v_{0x} t$

$\Rightarrow 130 = 45 \cos 35^\circ \times t$

$\Rightarrow \boxed{t = 3.53 \text{ sec}}$

Now, $y = y_0 + v_{0y} t + \frac{1}{2} a_y t^2$

$\Rightarrow y = 6 + 45 \sin 35^\circ \times 3.53 - 4.9 \times (3.53)^2$

$\Rightarrow \boxed{y = 36.04 \text{ m}}$

Since the y-coordinate of the top of the fence is 28 m, so the projectile will clear the fence.

Now, the flight of time t_f is

$$y = y_0 + v_{0y} t_f + \frac{1}{2} a_y t_f^2$$

$\Rightarrow 20 = 6 + 45 \sin 35^\circ \times t_f - 4.9 t_f^2$

$\Rightarrow 4.9 t_f^2 - 25.81 t_f + 14 = 0$

$\therefore t_f = 4.65 \text{ sec}$ and $t_f = 0.65 \text{ sec}$

But $t_f = 0.65$ sec is not acceptable, since in this time projectile does not clear the fence.

So. $\boxed{t_f = 4.65 \text{ sec}}$

Ans

(b) x-coordinate, $x = 45 \cos 35^\circ \times 4.65 = 171.41$ m

and $y = 20$ m

So. $\boxed{(x, y) \text{ coordinate of first impact} = (171.41, 20)}$

Ans