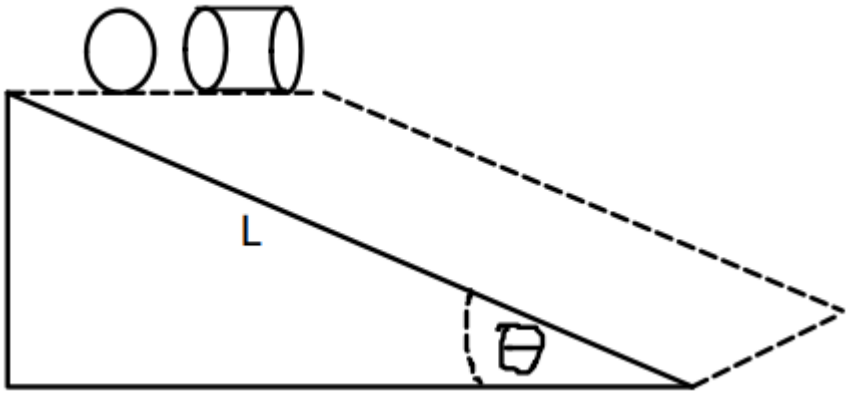
Department of Mathematics and Natural Sciences
PHY 111 - Principles of Physics-I (Summer-2021)
Assignment 5
Total Marks : 20
Answer all the questions

1. Consider a solid cylinder of mass m and radius r and a solid sphere of mass M and radius R are placed at rest on the top of an inclined plane that makes an angle θ with the ground. The length of the inclined plane is L. Now solve the following problems.

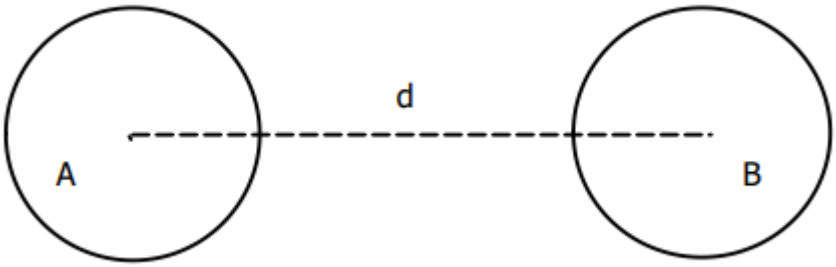


   (a) Find the moment of inertia of the cylinder and sphere. [1 mark]

   (b) Find the potential energy of the cylinder and the sphere when they are at the top of the inclined plane. [1 mark]

   (c) Now we let go of the cylinder and the sphere at the same time so that the roll without slipping along the incline plane. Find out which one of the solid bodies have greater linear velocity than the other when they come down from the inclined plane. [3 marks]

2. Two spherical celestial objects, call them A and B have mass M_1 and M_2 with radius R_1 and R_2 respectively. Initially they are at a distance d from each other. Now solve the following problems.



   (a) What would be the gravitational acceleration on the surface of object A ? You should ignore B's presence in this case. [2 marks]

   (b) Find the escape velocity of object B. Again ignore A's presence. [2 marks]

   (c) Now lets say these two objects, A & B start to move towards each other dude to mutual gravitational attraction. Find the time when they will collide with each other. Express you answers in terms of the given parameters and any universal constant. [6 marks]

3. A $65\,kg$ person jumps from a window to a fire net $18\,m$ below, which stretches the net $1.1\,m$. Assuming the net behaves like a simple spring, solve the following problems.

   (a) How much it would stretch if the same person was lying in it ? [3 Marks]

   (b) How much it would stretch if the same person jumped from a height of $35\,m$? [2 marks]

Question 1

(a) $I_S = \frac{2}{5}MR^2$, $I_C = \frac{1}{2}MR^2$ $\frac{1}{2}mr^2$

(b)

$E_S = Mgh$, $E_C = mgh$

here $\boxed{h = L\sin\theta}$ so, $E_S = MgL\sin\theta$

$E_C = mgL\sin\theta$

(c) Sphere:

$Mgh = \frac{1}{2}Mv_1^2 + \frac{1}{2}I_1\omega_1^2$ $\left[v_1 = \omega R,\ I_1 = \frac{2}{5}MR^2\right]$

$\Rightarrow Mgh = \frac{1}{2}Mv_1^2 + \frac{1}{5}Mv_1^2$

$\Rightarrow gh = \frac{7}{10}v_1^2 \Rightarrow \boxed{v_1^2 = \frac{10}{7}gh}$ → (1)

Cylinder

$mgh = \frac{1}{2}mv_2^2 + \frac{1}{2}I_2\omega_2^2$ $\left|\begin{array}{l} v_2 = \omega_2 r \\ I_2 = \frac{1}{2}mr^2 \end{array}\right.$

$\Rightarrow mgh = \frac{1}{2}mv_2^2 + \frac{1}{4}mv_2^2$

$\Rightarrow gh = \frac{3}{4}v_2^2 \Rightarrow \boxed{v_2^2 = \frac{4}{3}gh}$ → (2)

(1) ÷ (2) $\left(\frac{v_1}{v_2}\right)^2 = \left(\frac{10}{7} \times \frac{3}{4}\right) = \frac{15}{14} < 1$

$\therefore v_1 < v_2$ (always.)

Question 2°

(a) $g_1 = \frac{GM_1}{R_1^2}$ → (Ans.)

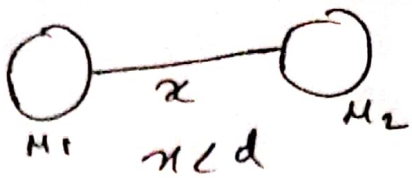
(b) $\frac{1}{2} m v_e^2 = \frac{GM_2 m}{R_2}$

m = test particle mass

$\Rightarrow$ $v_e = \sqrt{\frac{2GM_2}{R_2}}$ → (Ans.)

(c)

(c) Energy conservation:



$$-\frac{GM_1M_2}{d} = \frac{1}{2}M_1v^2 - \frac{GM_1M_2}{x}$$

$$\Rightarrow v_1^2 = 2GM_2\left(\frac{1}{x} - \frac{1}{d}\right)$$

$$\Rightarrow v_1 = \sqrt{2GM_2}\sqrt{\frac{d-x}{xd}} = \sqrt{\frac{2GM_2}{d}}\sqrt{\frac{d-x}{x}}$$

$$\Rightarrow -2\frac{dx}{dt} = \sqrt{\frac{2GM_2}{d}}\sqrt{\frac{d-x}{x}}$$

Here we get a factor 2 because we are watching from a frame of M_2.

→ P.T.O

$$\Rightarrow -\int_{d}^{0} \frac{dx\sqrt{x}}{\sqrt{d-x}} = \gamma \int_{0}^{T} dt$$

$$\boxed{\gamma = \frac{1}{2}\sqrt{\frac{2qM}{d}}}$$

$$\Rightarrow \int_{0}^{d} \frac{\sqrt{x}\,dx}{\sqrt{d-x}} = \gamma T \quad \rightarrow (1)$$

let $x = d\sin^2\theta \quad \therefore dx = 2d\sin\theta\cos\theta\, d\theta$

so, $x=0 \Rightarrow \theta = 0$, $x = d \Rightarrow \theta = \frac{\pi}{2}$

$$\int_{0}^{\frac{\pi}{2}} \frac{2\sqrt{d}\sin^2\theta\cdot\cos\theta\, d\theta\cdot(d)}{\sqrt{d}\cos\theta}$$

$$\Rightarrow \int_{0}^{\pi/2} 2d\sin^2\theta\, d\theta = \left(\int_{0}^{\pi/2}(1-\cos 2\theta)\,d\theta\right).d$$

$$= \left(\frac{\pi}{2} - \frac{1}{2}\sin 2\theta\Big|_{0}^{\pi/2}\right).d$$

$$= \frac{\pi d}{2}$$

putting this in (1) $\Rightarrow$

$$\frac{\pi d}{2} = \frac{1}{2}\sqrt{\frac{2qM}{d}}\,T \Rightarrow \boxed{T = \frac{d^{3/2}\pi}{\sqrt{2qM}}}$$

$\rightarrow$ (Ans)

$\Rightarrow$ [illegible]

Solution of Assignment-5 **Principles of Physics-I, PHY111**

Wednesday, September 1, 2021 9:32 AM

3. A 65 kg person jumps from a window to a fire net 18 m below, which stretches the net 1.1 m. Assuming the net behaves like a simple spring, solve the following problems.

(a) How much it would stretch if the same person was lying in it ? [3 Marks]
(b) How much it would stretch if the same person jumped from a height of 35 m ? [2 marks]

Solⁿ (a) If the person were to lie on the fire net, they would stretch the net an amount such that the upward force of the net would be equal to their weight.

i.e. $F_s = F_g$

$\Rightarrow kx = mg$

$\Rightarrow \boxed{x = \frac{mg}{k}}$ ——— (1)

Now, from energy conservation.

$E_t = E_b$

$\Rightarrow mgh_t = mgh_b + \frac{1}{2}kx^2$ [$h_t \rightarrow$ height at top, $h_b \rightarrow$ height at bottom]

$\Rightarrow 2mgh_t = 2mgh_b + kx^2$

$\Rightarrow k = \frac{2mg(h_t - h_b)}{x^2}$

$\Rightarrow k = \frac{2 \times 65 \times 9.8 \times [18 - (-1.1)]}{(1.1)^2}$

$\Rightarrow \boxed{k = 2.011 \times 10^4 \text{ N/m}}$

Now, from eqⁿ (1) $\Rightarrow$ $\boxed{x = \frac{65 \times 9.8}{2.011 \times 10^4} = 0.0316 \text{ m}}$ Ans

(b) From energy conservation.

$E_t = E_b$

$\Rightarrow mgh_t = mgh_b + \frac{1}{2}kx^2$

$$\Rightarrow \quad mgh_t = -mgx + \frac{1}{2}kx^2$$

$$\Rightarrow \quad \frac{2mg}{k} h_t = -\frac{2mg}{k} x + x^2$$

$$\Rightarrow \quad x^2 - \frac{2mg}{k} x - \frac{2mg}{k} h_t = 0$$

$$\Rightarrow \quad x^2 - \frac{2\times 65 \times 9.8}{2.011 \times 10^4} x - \frac{2 \times 65 \times 9.8}{2.011 \times 10^4} \times 35 = 0$$

$$\Rightarrow \quad x^2 - 0.0633x - 2.2173 = 0$$

$$\therefore \quad x = 1.52 \text{ m} \quad \text{or} \quad x = -1.45 \text{ m (Not acceptable)}$$

Sw. $\boxed{x = 1.52 \text{ m}}$ Ans