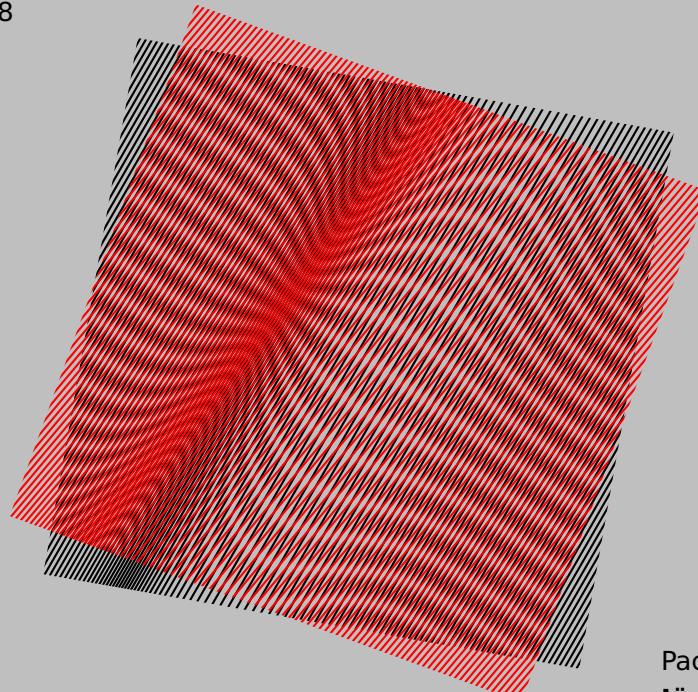


PSTricks

pst-moire v. 2.1

A PSTricks package to draw moiré patterns

November 22, 2018



Package author(s):
Jürgen Gilg
Manuel Luque
Jean-Michel Sarlat

Contents

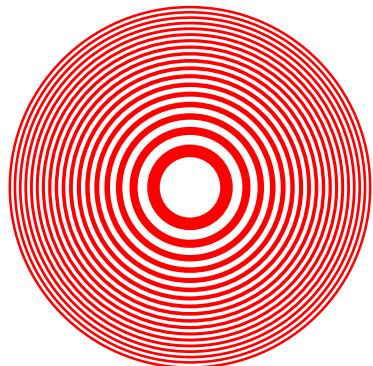
1. The provided patterns	4
2. The command \psmoire	6
3. The command \addtomoirelisttype	7
3.1. How to make a custom moiré types	7
4. Opacity and Blendmodes	12
4.1. Opacity	12
4.2. Blendmodes	13
5. The moiré types	14
5.1. type=Fresnel	14
5.2. type=linear	15
5.3. type=radial	16
5.4. type=Bouasse	17
5.5. type=circle	18
5.6. type=Gauss	20
5.7. type=square	21
5.8. type=Newton	22
5.9. type=dot	23
5.10. type=chess	24
6. Examples of combined moiré patterns	25
7. Random moirés	28
7.1. The command \psRandomDotPatterns	29
7.2. The command \psRandomDot	34
8. Glass-patterns	36
8.1. The command \psGlassPattern	36
9. Animations	39
10. Theory—for the interested user	44
10.1. The contribution of “éditions Kangourou”	44
10.2. The contribution of Henri Bouasse	45
10.3. The humble contributions of our group	46
10.4. The construction of a Gauss network	47
10.5. Some moiré figures	49
11. List of all optional arguments for pst-moire	54
A. PostScript Font Names	55

The `pst-moire` package makes it possible to very simply create a variety of patterns obtained either by dragging one pattern on another, or by rotating one on the other. Moiré effects sometimes look very interesting. This document provides the necessary commands and divers examples.

For the interested user, we present a section **Theory** (see pages 44-53) for the mathematical background of moiré patterns.

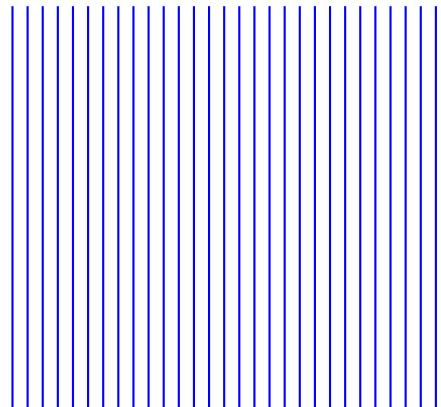
1. The provided patterns

Bands of Fresnel



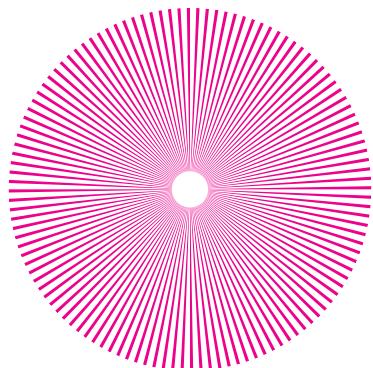
type=Fresnel

Equidistant lines



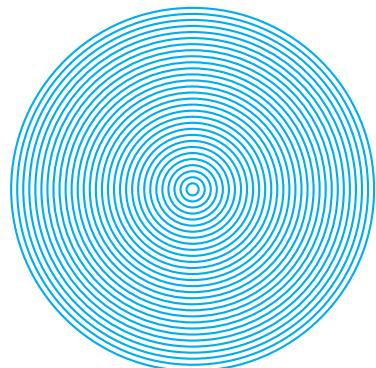
type=linear

Radii with 3°



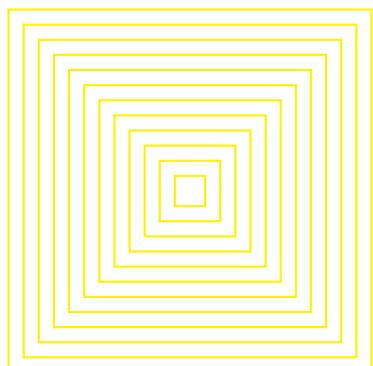
type=radial

Concentric circles



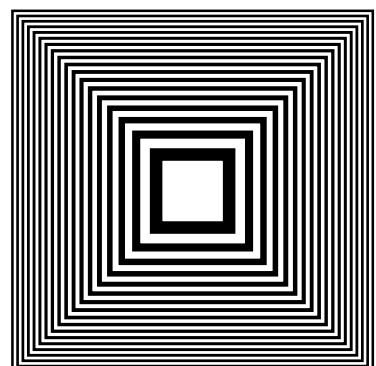
type=circle

Squares



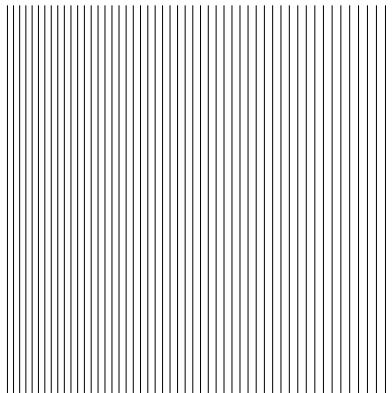
type=square

Squares of Newton



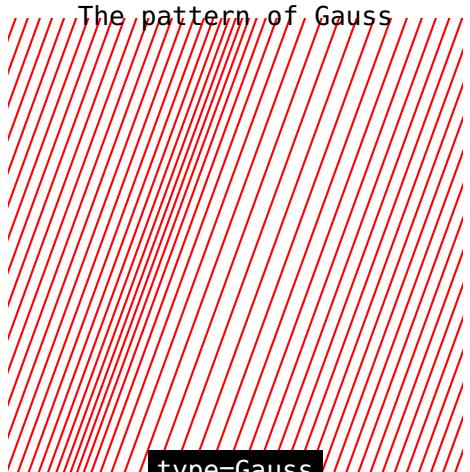
type=Newton

The pattern of H. Bouasse



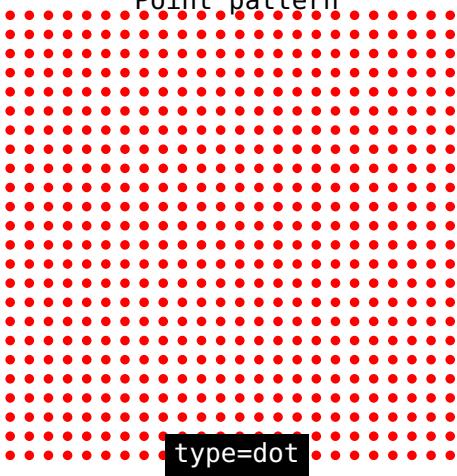
type=Bouasse

The pattern of Gauss



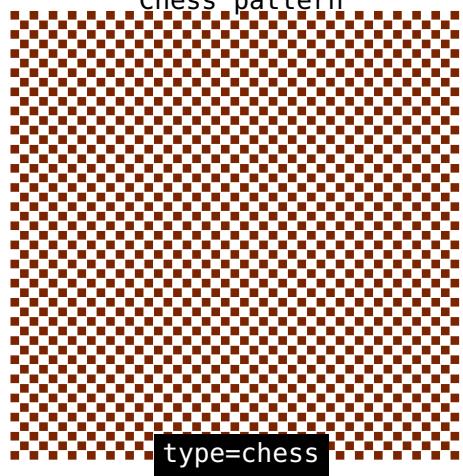
type=Gauss

Point pattern



type=dot

Chess pattern



type=chess

2. The command \psmoire

```
\psmoire [Options] (x , y)
```

The command \psmoire contains the options `type=`, `Rmax=`, `scale=`, `Alpha=`, `rotate=`, `E=`, `n=` and `T=`. The optional argument `(x , y)` gives the x and y center of the image. If not chosen `(0,0)` is taken by default.

Name	Default	Meaning
<code>type</code>	<code>Fresnel</code>	The type of pattern
<code>rotate</code>	<code>0</code>	Rotation angle of the pattern (in degrees).
<code>Rmax</code>	<code>6</code>	For <code>type=Gauss</code> – image dimensions: $(-Rmax, -Rmax) * (Rmax, Rmax)$ For <code>type=linear</code> – $2*Rmax$ is the length of the lines For <code>type=radial</code> – maximal radii length all (in cm)
<code>scale</code>	<code>1</code>	Scaling factor for the image available for the types: <code>type=Fresnel/Gauss/Newton/dot/chess</code>
<code>Alpha</code>	<code>70</code>	Slope of the lines (in degrees): for <code>type=Gauss</code> only
<code>E</code>	<code>0.25</code>	x-Distance between two points on the Gaussian curve (in cm): for <code>type=Gauss</code> only
<code>n</code>	<code>30</code>	Number of circles/lines: for <code>type=circle</code> , <code>type=linear</code> only.
<code>T</code>	<code>2</code>	Distance between two adjacent circles/lines (in mm): for <code>type=circle</code> , <code>type=linear</code> only.

Note:

- Thickness of the lines/circles: done with the usual PStricks key `linewidth=1mm` (for example).
- Color of the lines/circles: done with the usual PStricks key `linecolor=red` (for example).

We set the type of pattern like: `type=Gauss` (for example).

```
\psmoire[options,type=Gauss](x,y)
```

For the types that are not affected by `scale`, simply use the PStricks key `unit=`. If no position coordinate is specified, the center of the image is placed at `(0,0)`. The PStricks thickness key `linewidth=` does not affect the following types: `Fresnel`, `Newton` and `radial`

3. The command `\addtomoirelisttype`

```
\addtomoirelisttype{name}
```

The command `\addtomoirelisttype` only comes with one mandatory argument—the name the customer chooses.

If we like to generate our own pattern we can achieve this by using the command `\addtomoirelisttype{name}` within the preamble which then references to a file named `pst-name.pro`

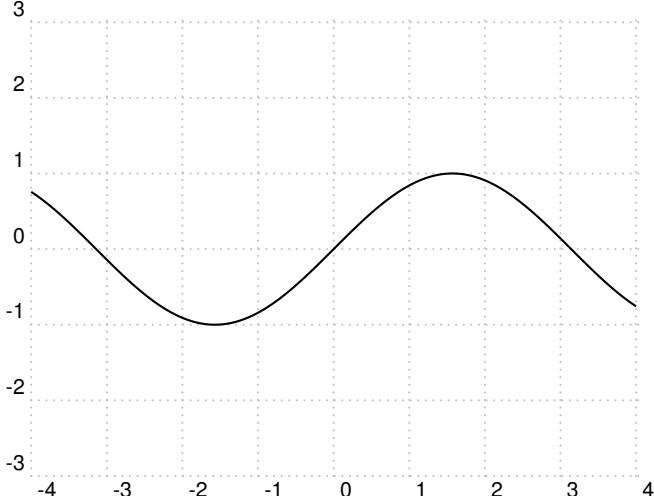
where the prefix `pst-` is automatically generated. This file we need to code ourselves in PostScript language, save within the working folder and then load it within the preamble like

```
\pstheader{pst-name.pro}
\addtomoirelisttype{name}
```

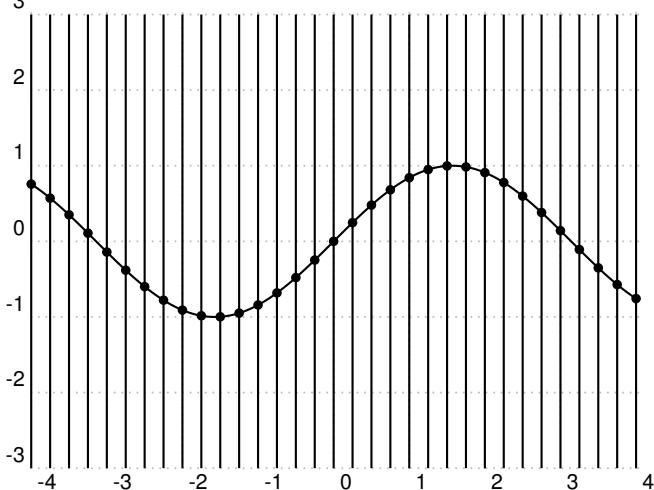
3.1. How to make a custom moiré types

Sinusoidal pattern

$$y = a \sin\left(2\pi \frac{x}{T}\right)$$



For the grating of equidistant vertical lines we have: $x = ne$, e displacement and n an integer. The ordinates of the intersection points are: $y_n = a \sin(2\pi \frac{ne}{T})$. Within the following figure, the displacement is set to 0.25.



We determine the equations of the straight lines passing through these points and which are inclined by an angle α with respect to the horizontal. This is to setup with the key `Alpha=` (in degrees). The general equation of such a line is given by: $y = x \tan(\alpha) + b$, we determine b to go through one of the previous points.

$$ne \tan(\alpha) + b = a \sin(2\pi \frac{ne}{T})$$

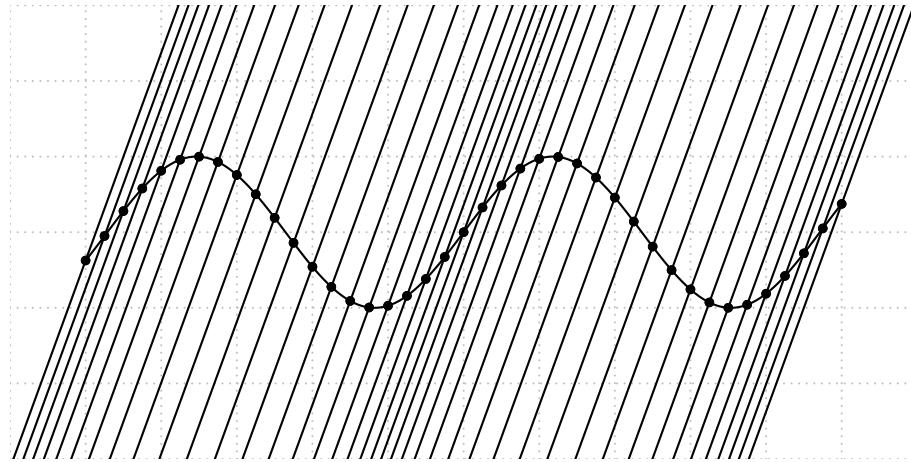
and thus we get b .

$$b = a \sin(2\pi \frac{ne}{T}) - ne \tan(\alpha)$$

For every value of n we receive one line.

$$y = x \tan(\alpha) + a \sin(2\pi \frac{ne}{T}) - ne \tan(\alpha)$$

We draw some lines, setting $a = 1$, $T = 2\pi$, $-20 < n < +20$, $e = 0.5$ and $\alpha = 70^\circ$



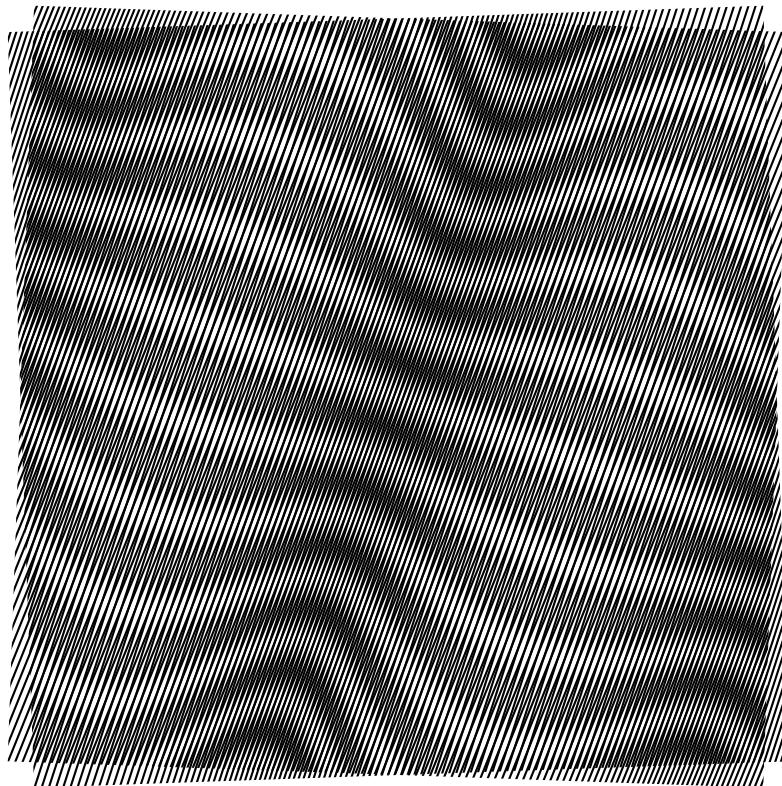
Now we need to code that in PostScript and name that file `pst-sin.pro` and save it within the working folder.

```

moireDict begin
/pst-sin {
0 0 translate
2 dict begin
/A1 0.5 def % amplitude
/TRAME {
-50 E1 2 mul 50 {/n exch def
gsave
    n E1 mul unit % x
    A1 unit 360 Tr div n E1 mul mul sin mul % y
    translate
    linecolor
    linewidth
    -6 unit -6 m mul unit moveto
    6 unit 6 m mul unit lineto
    stroke
grestore
} for
} def
Runit neg dup
Runit 2 mul dup
rectclip
    TRAME
end
} def
end
}

```

This results within the following custom generated pattern.



```
\begin{pspicture}(-5,-6)(5,6)
\psset{Rmax=5,T=2,E=0.2}
\psmoire[type=sin,rotate=2]
\psmoire[type=sin,rotate=-2]
\end{pspicture}
```

Note: Now we can use our custom generated type=sin with all the other options available.

Cosine pattern

Here another PostScript code saved with the file name `pst-cosine.pro` within the working folder.

```

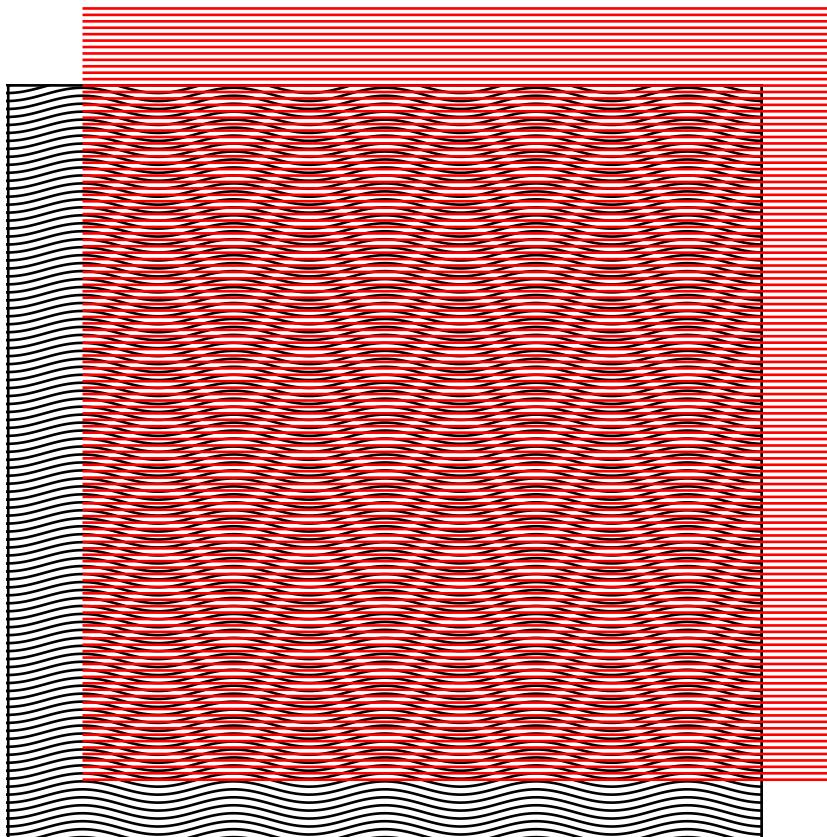
moireDict begin
/pst-cosine {
/amplitud 2.5 def
/period 2 def
/cos1 [
-8 0.05 8 {/x exch def % 320 pts
x unit
360 x mul period div cos amplitud mul
} for
] def
%
/drawcos {
newpath
cos1 0 get cos1 1 get moveto
0 2 cos1 length 2 sub {/i exch def
cos1 i get cos1 i 1 add get lineto
} for
stroke
} def
gsave
Runit neg dup
Runit 2 mul dup
rectclip
0 -8 unit translate
nr {
0 E1 translate
drawcos
} repeat
grestore
} def
end

```

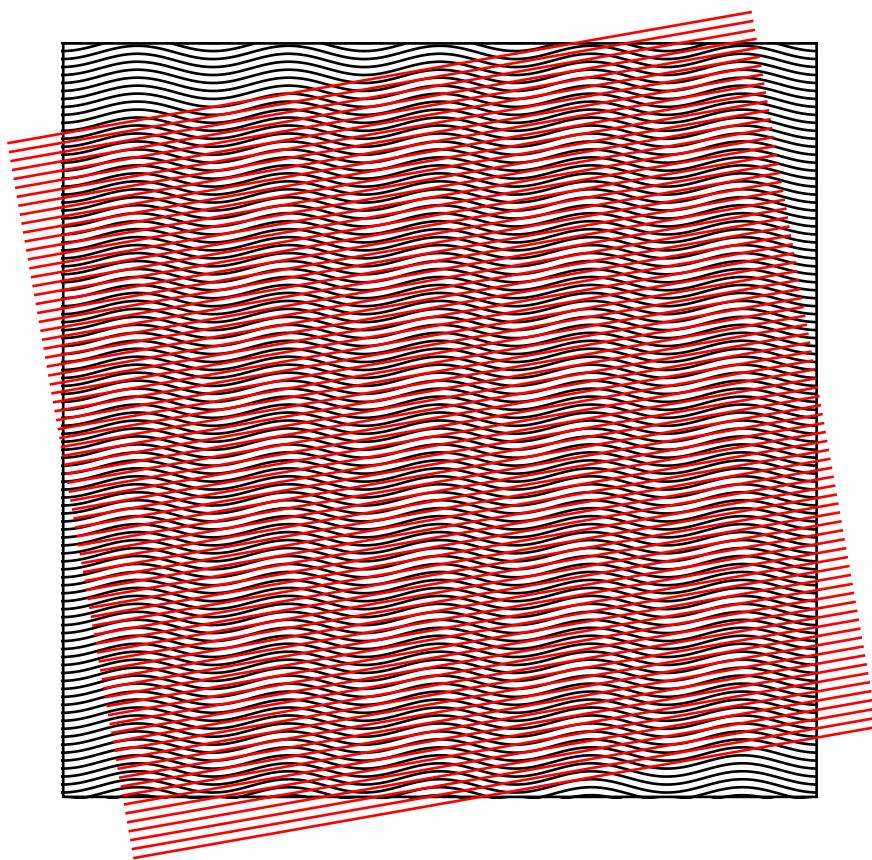
Setting into the preamble:

```
\pstheader{pst-cosine.pro}
\addtomoirelisttype{cosine}
```

Example 1:



```
\begin{pspicture}(-6,-6)(6,6)
\psset{Rmax=5,linewidth=1pt}
\psframe(-5,-5)(5,5)
\psmoire[% type=cosine, E=3, n=400, rotate=0]
\psmoire[% type=linear, T=0.85, rotate=-90, n=120, linecolor=red](1,1)
\end{pspicture}
```

Example 2:

```
\begin{pspicture}(-6,-6)(6,6)
\psset{Rmax=5,linewidth=1pt}
\psframe(-5,-5)(5,5)
\psmoire[%  

type=cosine,  

E=3,  

n=400,  

rotate=0]
\psmoire[%  

type=linear,  

T=1.2,  

rotate=100,  

n=80,  

linecolor=red]
\end{pspicture}
```

4. Opacity and Blendmodes

If we want to highlight the color of the intersecting area of the lines of two or more overlapping moiré patterns differently, we can either use *opacity* or *blendmodes*.

4.1. Opacity

In case we want to add some opacity to the lines of the moiré patterns, we just set, i. e.

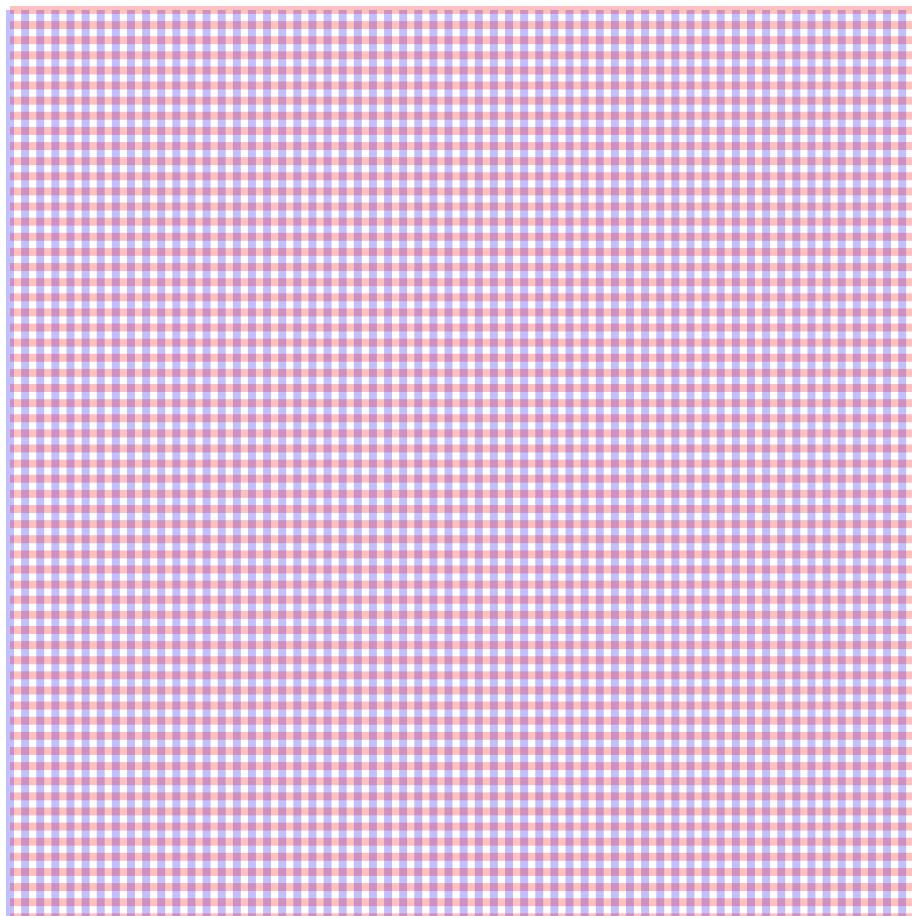
```
\pstVerb{%
0.25 .setopacityalpha
}
```

within the `\pspicture` environment.

Distiller users set instead:

```
\pstVerb{%
[ /ca 0.25 /CA 0.25 /SetTransparency pdfmark
}
```

Note: The value for the opacity needs to be between 0 and 1.



```
\begin{pspicture}(-6,-6)(6,6)
\pstVerb{%
0.25 .setopacityalpha
}
\psmoire[type=linear, linecolor=blue, linewidth=3pt, n=60]
\psmoire[type=linear, linecolor=red, linewidth=3pt, rotate=90, n=60]
\end{pspicture}
```

4.2. Blendmodes

In case we want to overlap various moiré patterns we can use the following blendmodes:

/Lighten, /Darken, /Normal, /Multiply, /Screen, /Overlay, /ColorDodge,
 /ColorBurn, /HardLight, /SoftLight, /Difference, /Exclusion, /Saturation,
 /Color, /Luminosity.

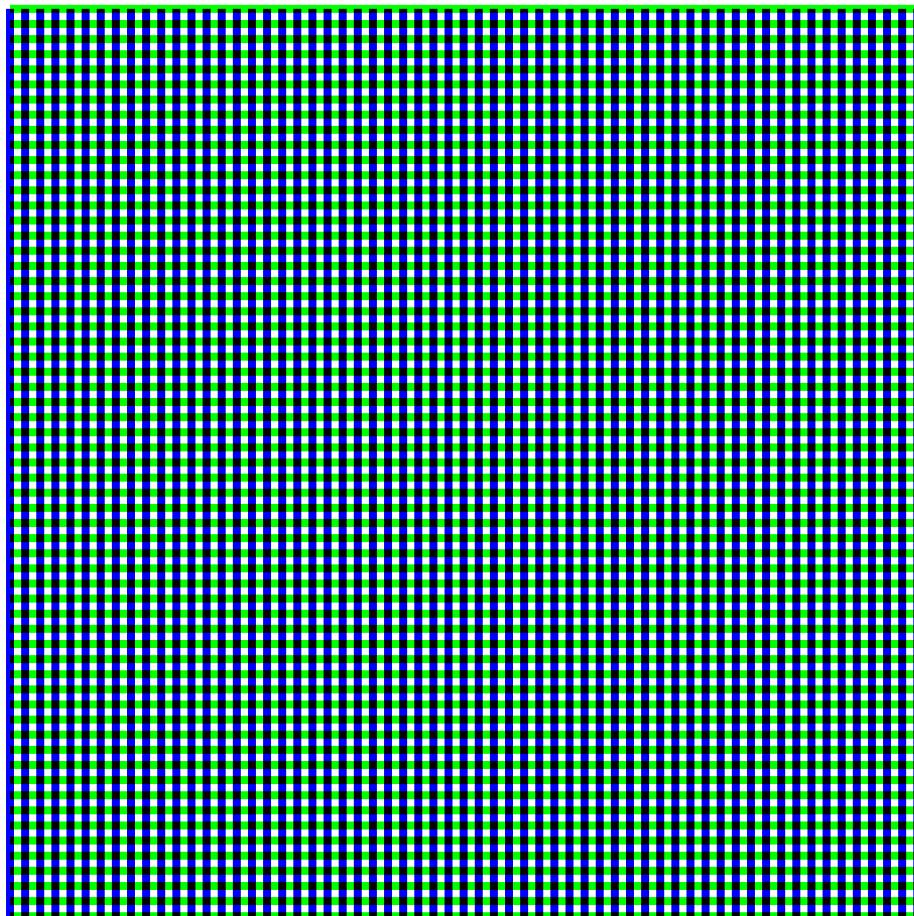
We just set, i. e.

```
\pstVerb{%
/Darken .setblendmode
}
```

within the `\pspicture` environment.

Distiller users set instead:

```
\pstVerb{%
[ /BM /Darken /ca 1 /CA 1 /SetTransparency pdfmark
}
```



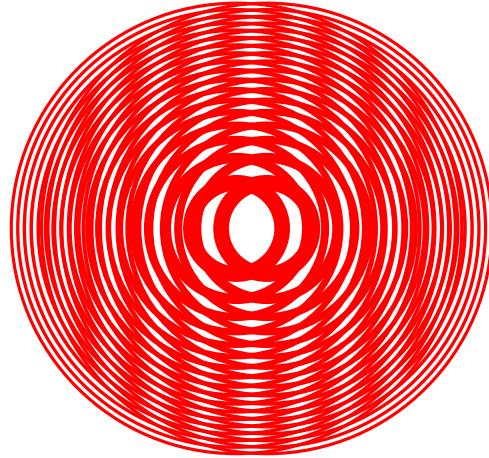
```
\begin{pspicture}(-6,-6)(6,6)
\pstVerb{%
/Darken .setblendmode
}
\psmoire[type=linear, linecolor=blue, linewidth=3pt, n=60]
\psmoire[type=linear, linecolor=green, linewidth=3pt, rotate=90, n=60]
\end{pspicture}
```

5. The moiré types

5.1. type=Fresnel

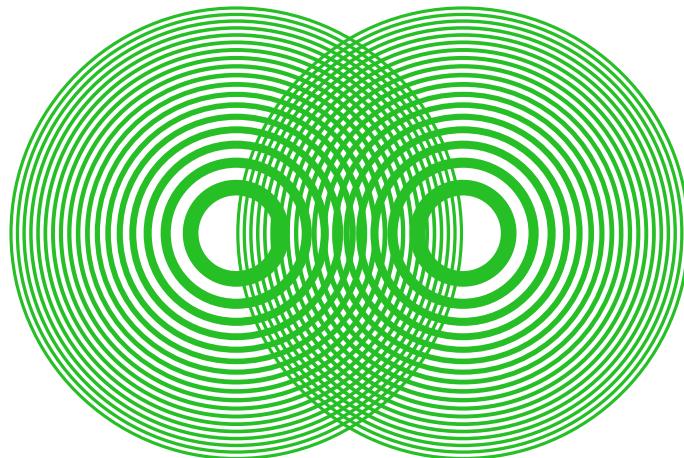
The type=Fresnel consists of concentric circles with incrementing radii of \sqrt{n} . The maximum radius is given by Rmax. The linewidth is fixed (cannot be changed by linewidth=), due to the thickness of the circle varies.

Example 1: Overlapping pattern with centers close to each other



```
\begin{pspicture}(-3,-3)(3,3)
\psmoire[linecolor=red,scale=0.5](-0.2,0)
\psmoire[linecolor=red,scale=0.5](0.2,0)
\end{pspicture}
```

Example 2: Overlapping pattern with centers far from each other



```
\begin{pspicture}(-5,-3)(5,3)
\psmoire[linecolor={[rgb]{0.15 0.75 0.15}},scale=0.5](-1.5,0)
\psmoire[linecolor={[rgb]{0.15 0.75 0.15}},scale=0.5](1.5,0)
\end{pspicture}
```

5.2. type=linear

The type=linear offers two more keys to be more flexible:

n	Number of lines -1	Default: 30
T	Distance between the middle of two lines in mm	Default: 2

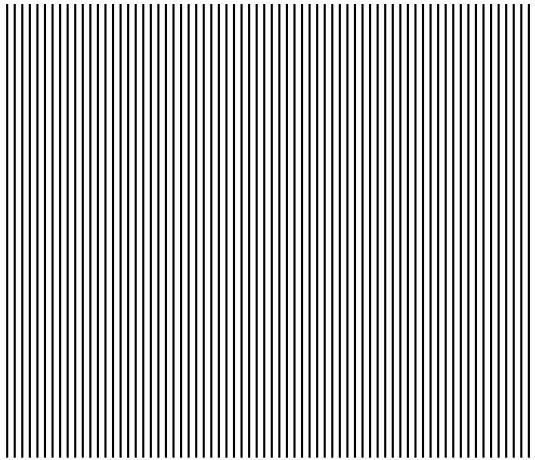
The height of the lines is given by: $2*R_{max}$, the width of the image is $n*T$

If we like a distance that is equal to the thickness of the line, we set:

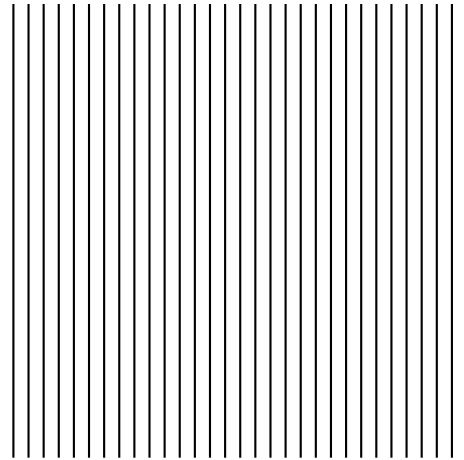
`linewidth=T/2`

If $T=2$, we set `linewidth=0.1`.

Example 1: Basic pattern with the keys n= and T=

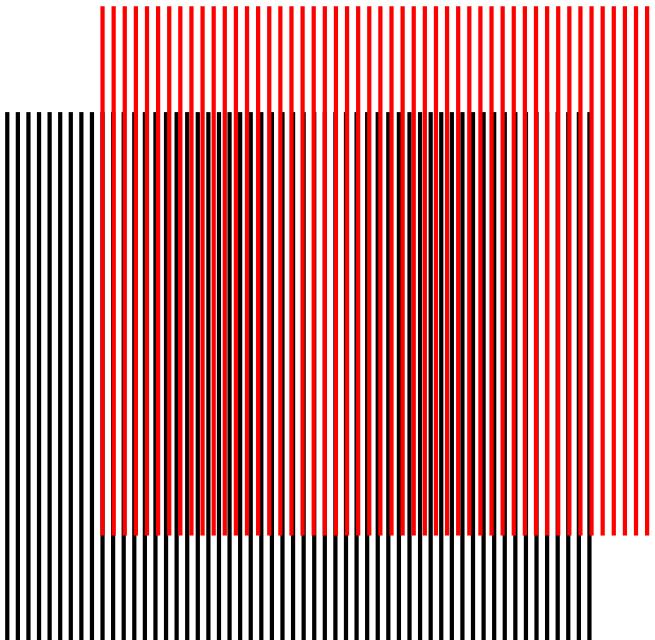


```
\begin{pspicture}(-3,-3)(3,3)
\psmoire[type=linear,n=70,T=1,Rmax=3]
\end{pspicture}
```



```
\begin{pspicture}(-3,-3)(3,3)
\psmoire[type=linear,n=30,T=2,Rmax=3]
\end{pspicture}
```

Example 2: Overlapping patterns

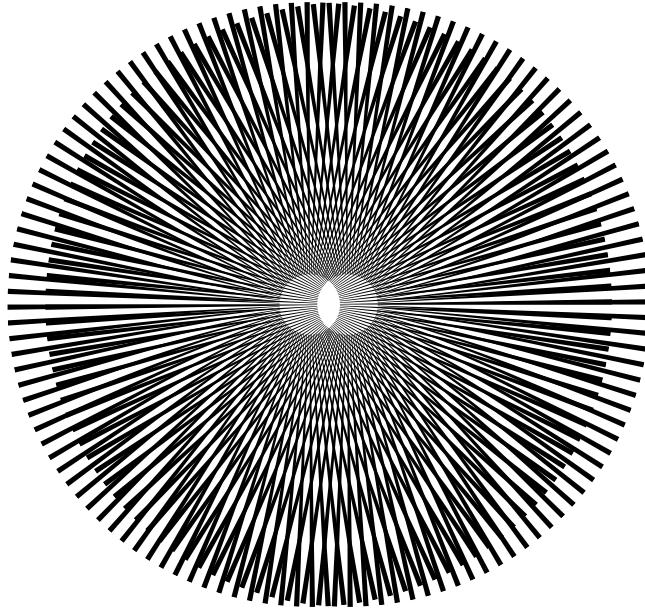


```
\begin{pspicture}(-5,-5)(5,7)
\psset{linewidth=0.08,type=linear,n=50,Rmax=5}
\psmoire[T=2,n=55]
\psmoire[T=2.1,linecolor=red](1.8,2)
\end{pspicture}
```

5.3. type=radial

The type=radial consists of radial rays—to be more precise, it consists of 120 sectors. The maximal radius of the sectors is given by Rmax=.

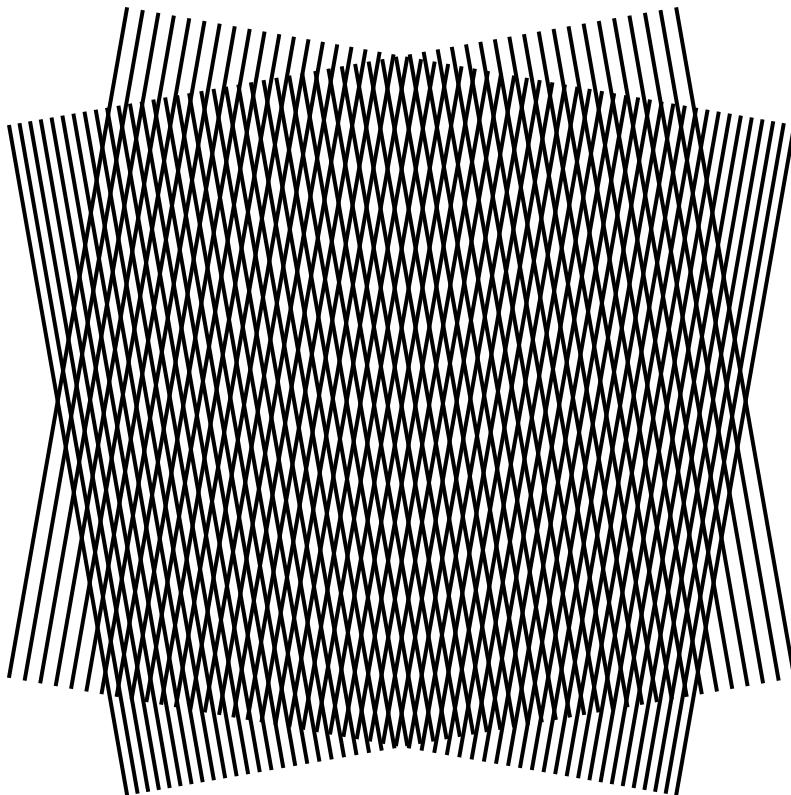
Example 1: Overlapping patterns with centers close to each other



```
\begin{pspicture}(-4,-4)(4,4)
\psmoire[Rmax=4,type=radial](-0.25,0)
\psmoire[Rmax=4,type=radial](0.25,0)
\end{pspicture}
```

5.4. type=Bouasse

For some detailed information about the type=Bouasse as pattern, see page 45.



```
\begin{pspicture}(-6,-6)(6,6)
\psmoire[type=Bouasse,rotate=10]
\psmoire[type=Bouasse,rotate=170]
\end{pspicture}
```

5.5. type=circle

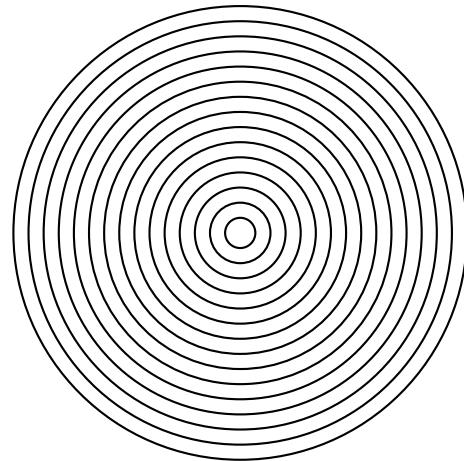
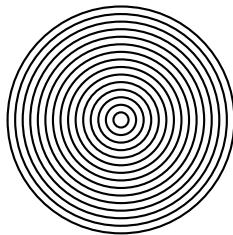
The type=circle offers two more keys to be more flexible:

n	Number of circles	Default: 30
T	Distance between two adjacent circles in mm	Default: 2

The key Rmax has no effect. The maximal width/height of the image is calculated by: n*T, by default: $30 \cdot 2 = 60$ mm = 6 cm. Another way could be to use the default values and play with the usual PSTricks key unit=.

The idea to make that type more flexible came from example 11.8 page 373 of the book “The Theory of the Moiré Phenomenon” of Isaac Amidror.

Example 1: Basic pattern with the keys n= and T=

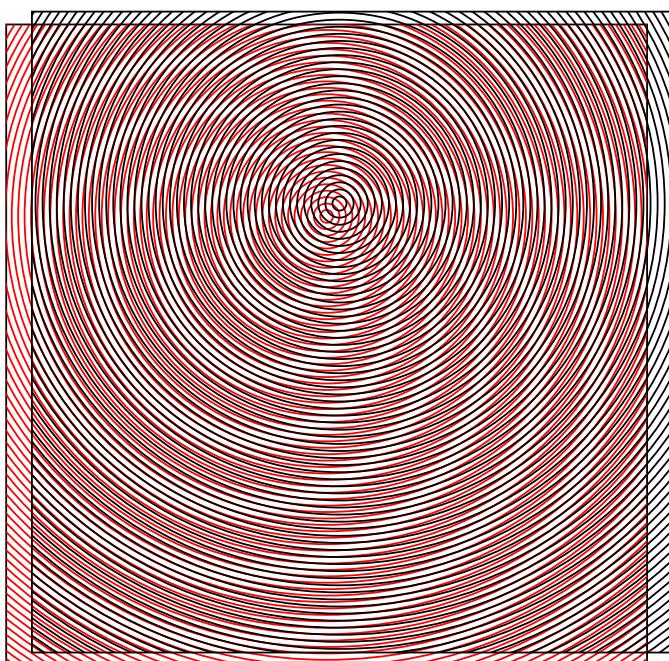


```
\begin{pspicture}(-3,-3)(3,3)
\psmoire[type=circle,n=15,T=1]
\end{pspicture}
```

```
\begin{pspicture}(-3,-3)(3,3)
\psmoire[type=circle,n=15,T=2]
\end{pspicture}
```

Example 2: Overlapping patterns

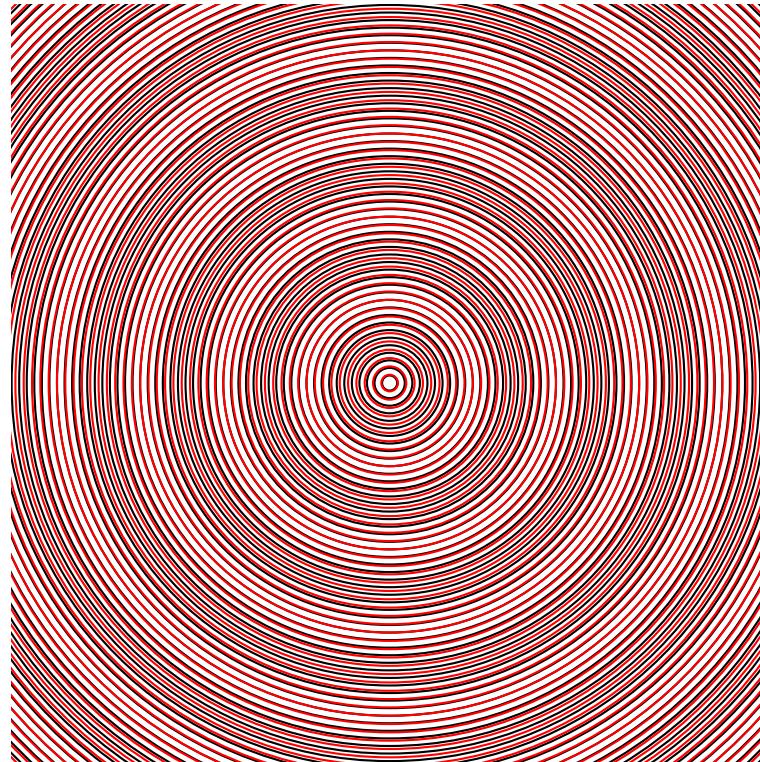
The following moiré is given by a superposition of two circular gratings, however their line spacings T1 and T2 are slightly different and they are shifted by $+/- (x_0, y_0)$ from its origin.



```
% With two circular patterns.
% T1 and T2 are slightly different.
% r1=m*T1 : T1=1 mm
% r2=n*T2 : T2=1.1 mm
% The moirés are: Ovals of Descartes
% demonstration of the figure 11.8 page 373 of
% "The Theory of the Moiré Phenomenon" de Isaac Amidror
% Volume 1 : Periodic Layers
% The image is found on the front cover of the book
\begin{pspicture}(-5,-5)(4,4.5)
\psclip{\psframe(-5,-6)(5,4)}
\psmoire[linecolor=red,type=circle,T=1,n=100](0,1)%
\endpsclip%
\rput(0.2,1.2){%
\psclip{\psframe(-4.8,-7)(5.2,3)}
\psmoire[type=circle,T=1.1,n=100]%
\endpsclip}%
\end{pspicture}
```

Example 3: Overlapping patterns

The following moiré is given by the superposition of two circular gratings when both gratings are centered at their origins, however their line spacings T_1 and T_2 are slightly different.



```
\begin{pspicture*}(-5,-5)(5,5)
\psmoire[type=circle,T=1,n=100]%
\psmoire[type=circle,T=1.1,n=100,linecolor=red]%
\end{pspicture*}
```

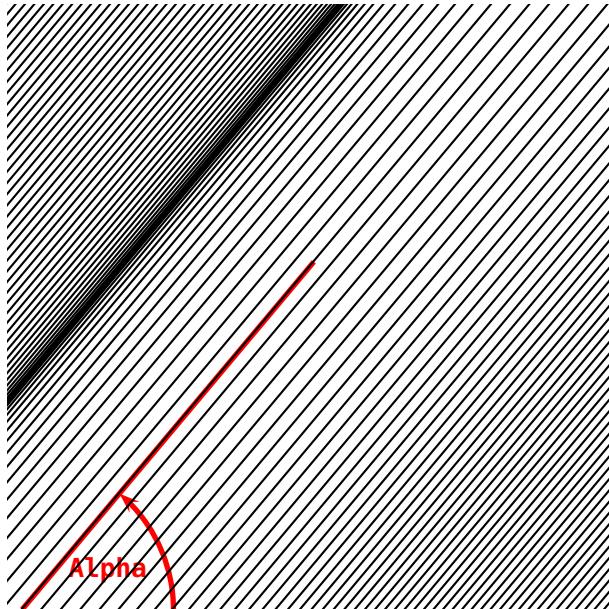
5.6. type=Gauss

The type=Gauss offers two keys to be more flexible:

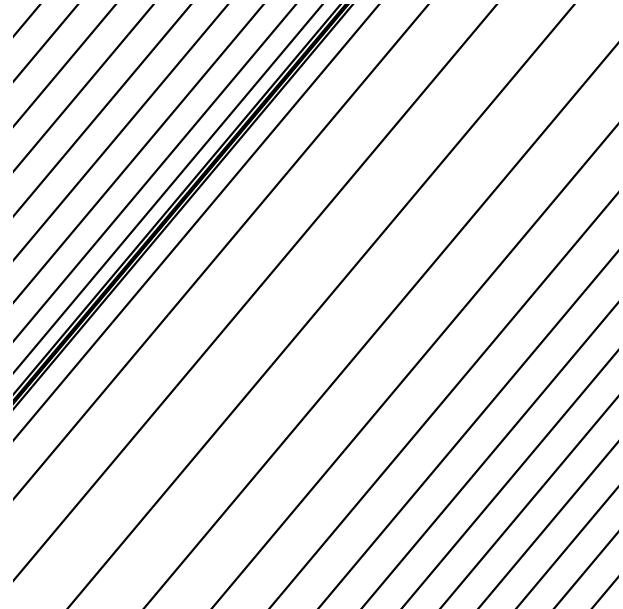
Alpha	Slope of the lines	Default: 70
E	x-distance between two points on the Gaussian curve	Default: 0.25

The key Alpha= is self-explanatory—see the following example. For the key E=, see the sketch of the Gaussian curve on page 47.

Example 1: Basic pattern

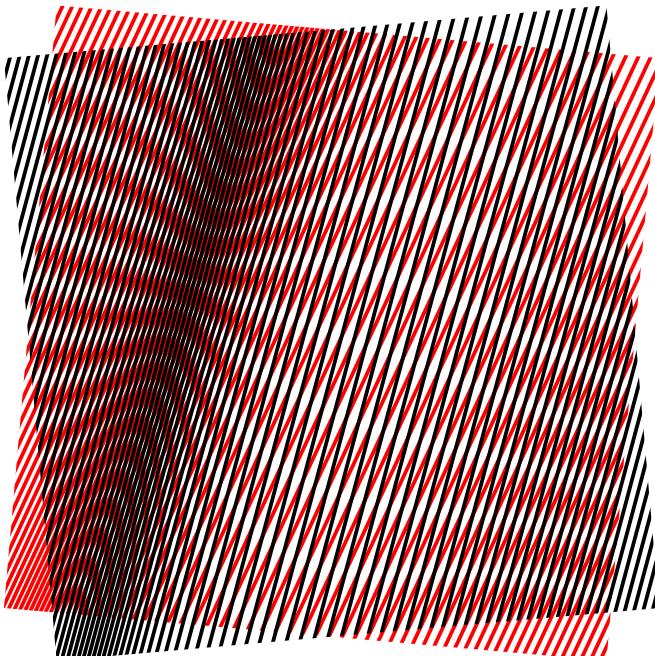


```
\begin{pspicture}(-4,-4)(4,4)
\psmoire[Rmax=4,type=Gauss,Alpha=50]
\end{pspicture}
```



```
\begin{pspicture}(-4,-4)(4,4)
\psmoire[Rmax=4,type=Gauss,Alpha=50,E=0.5]
\end{pspicture}
```

Example 2: Overlapping patterns



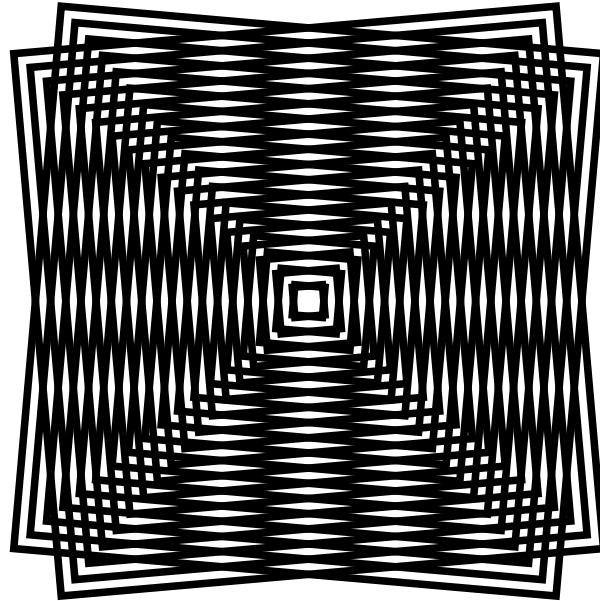
```
\psset{linewidth=0.5mm,Rmax=4}
\begin{pspicture}(-4,-4)(4,4)
\psmoire[type=Gauss,rotate=-5,linecolor=red]
\psmoire[type=Gauss,rotate=5]
\end{pspicture}
```

5.7. type=square

The type=square consists of equidistant squares with a distance 2 mm each.

Example 1:

The following moiré is given by a superposition of two square gratings when both gratings are rotated by $+/- 5^\circ$ around its origin.



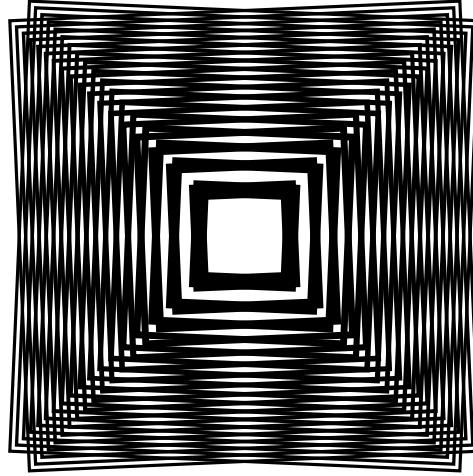
```
\psset{linewidth=1mm,scale=0.6}
\begin{pspicture}(-4,-4)(4,4)
\psmoire[type=square,rotate=-5]
\psmoire[type=square,rotate=5]
\end{pspicture}
```

5.8. type=Newton

The `type=Newton` consists of concentric squares with incrementing side length of \sqrt{n} . The maximum side length is given by `Rmax`. The linewidth is fixed (cannot be changed by `linewidth=`), due to the thickness of the squares varies.

Example 1:

The following moiré is given by a superposition of two Newton gratings when both gratings are rotated by $+/- 2.5^\circ$ around its origin.



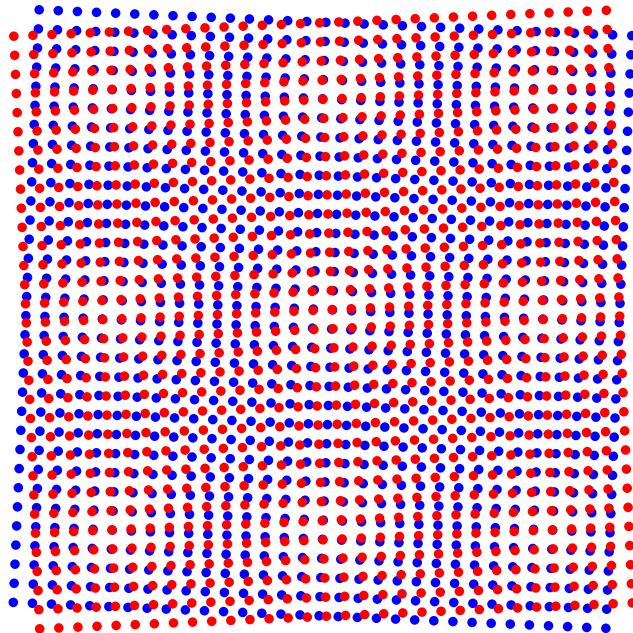
```
\psset{linewidth=0.5mm,scale=0.5}
\begin{pspicture}(-4,-4)(4,4)
\psmoire[type=Newton,rotate=-2.5]
\psmoire[type=Newton,rotate=2.5]
\end{pspicture}
```

5.9. type=dot

The `type=dot` consists of dots bordered within a square of side lengths `Rmax*Rmax`. The `dotstyle=` and `dotsize=` can be individually setup. Its colors can be chosen by the usual PStricks key `linecolor`.

Example 1:

The following moiré is given by a superposition of two dot gratings when both gratings are rotated by $+/- 2.5^\circ$ around its origin.



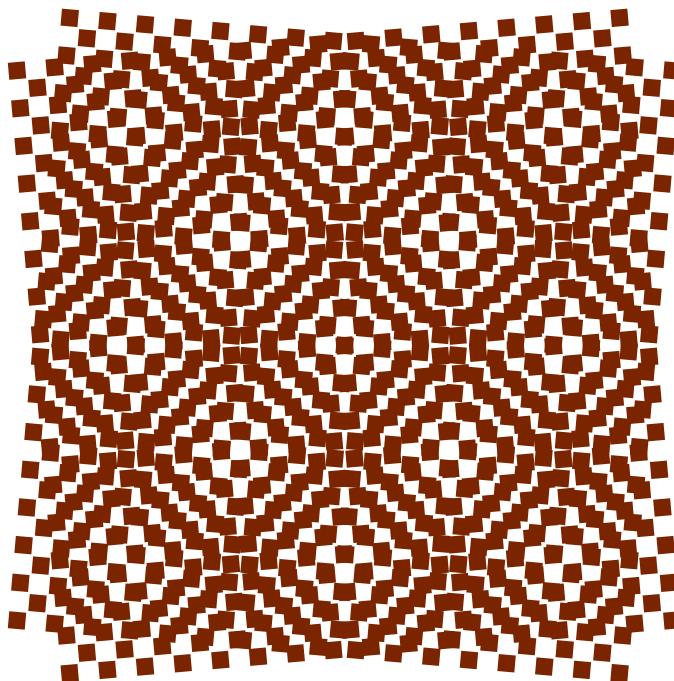
```
\psset{Rmax=4}
\begin{pspicture}(-5,-5)(5,5)
\psmoire[type=dot,linecolor=blue,rotate=-2.5]
\psmoire[type=dot,rotate=2.5,linecolor=red]
\end{pspicture}
```

5.10. type=chess

The `type=chess` consists of squares bordered within a square of side lengths `Rmax*Rmax`. The `dotstyle=` and `dotsize=` can be individually setup. Its colors can be chosen by the usual PSTricks key `linecolor`.

Example 1:

The following moiré is given by a superposition of two chess gratings when both gratings are rotated by $+/- 5^\circ$ around its origin.

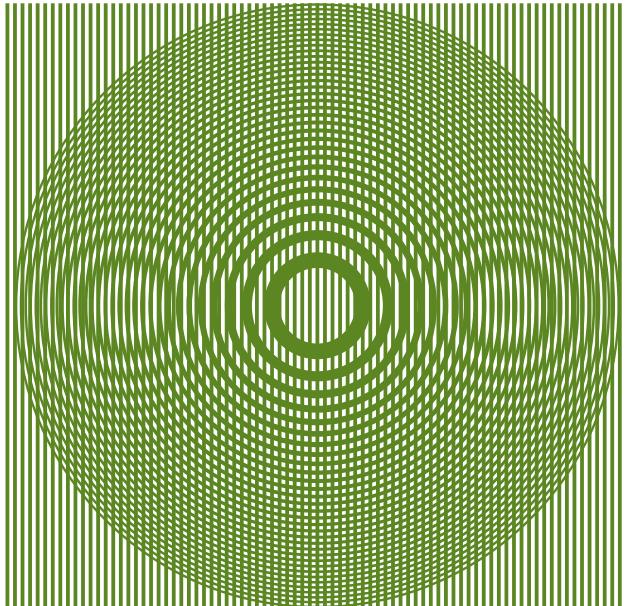


```
\psset{Rmax=4,dotstyle=square*,dotsize=0.25cm}
\begin{pspicture}(-4,-4)(4,5)
\psmoire[type=chess,rotate=-5]
\psmoire[type=chess,rotate=5]
\end{pspicture}
```

6. Examples of combined moiré patterns

Example 1:

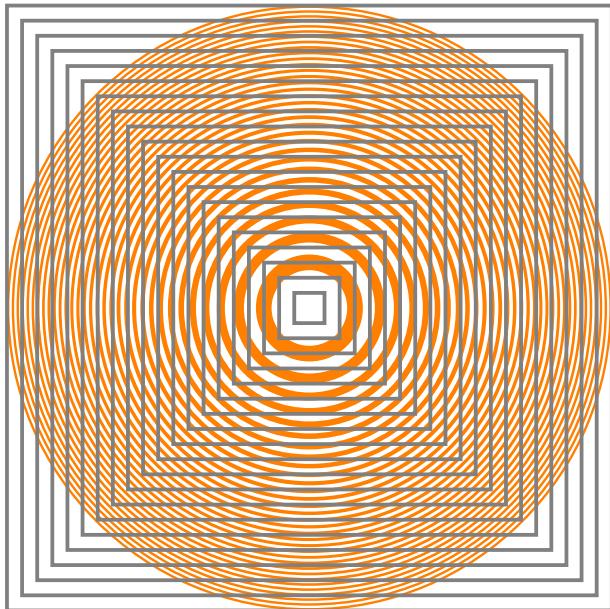
type=Fresnel + type=linear



```
\psset{Rmax=8,linewidth=0.5mm,scale=0.5,
linecolor={[rgb]{0.357 0.525 0.13}}}
\begin{pspicture}(-5,-5)(5,5)
\rput(0,4.75){\texttt{type=Fresnel + type=linear}}
\psmoire[type=Fresnel]
\psmoire[type=linear,n=40]
\psmoire[type=linear,n=41](-0.1,0)
\end{pspicture}
```

Example 2:

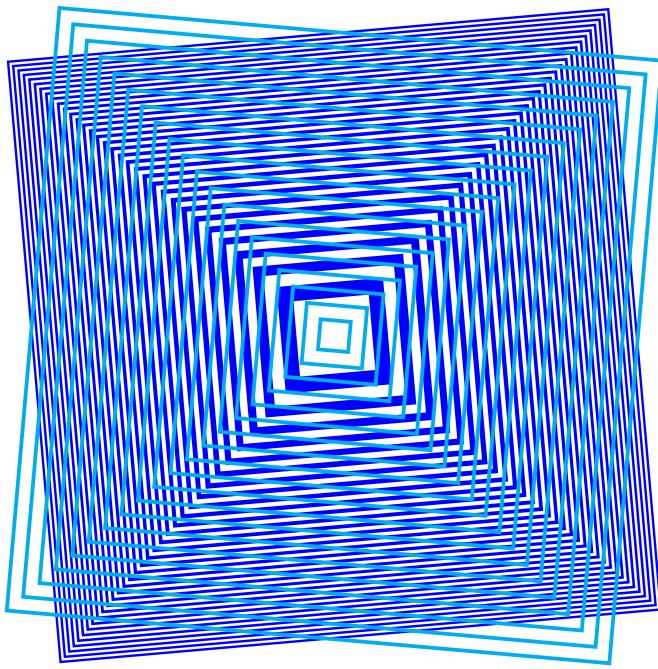
type=Fresnel + type=square



```
\psset{Rmax=8,linewidth=0.5mm,scale=0.5}
\begin{pspicture}(-5,-5)(5,5)
\rput(0,4.75){\texttt{type=Fresnel + type=square}}
\psmoire[type=Fresnel,linecolor=orange]
\psmoire[type=square,linecolor=gray]
\end{pspicture}
```

Example 3:

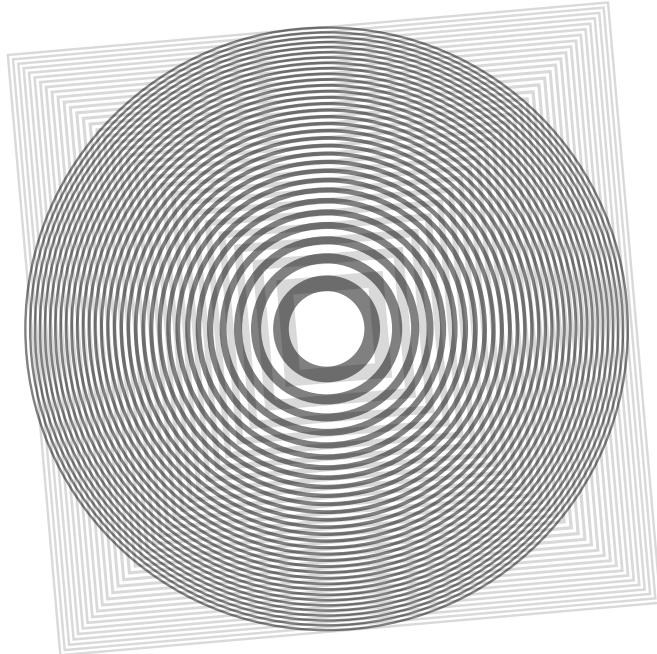
type=Newton + type=square



```
\psset{Rmax=8,linewidth=0.5mm,scale=0.5}
\begin{pspicture}(-5,-5)(5,5)
\rput(0,4.75){\texttt{type=Newton + type=square}}
\psmoire[type=Newton,linecolor=blue,rotate=5]
\psmoire[type=square,linecolor=cyan,rotate=-5]
\end{pspicture}
```

Example 4:

type=Fresnel + type=Newton



```
\psset{Rmax=8,scale=0.5}
\begin{pspicture}(-5,-5)(5,5)
\pstVerb{%
/Multiply .setblendmode
}
\rput(0,4.75){\texttt{type=Fresnel + type=Newton}}
\psmoire[% type=Fresnel,
linecolor=gray
](-0.05,0)
\psmoire[% type=Newton,
linecolor=gray!30,
rotate=5
](0.05,0)
\end{pspicture}
```

These following rotating moirés were obtained with the use of the `pst-lens` package. It is the reproduction, with the tools of PSTricks, of the image 6, page 137 of the book “*Les phénomènes naturels*” of the Library *Pour la Science*, Berlin (1978). This image is accompanied by the following comment:

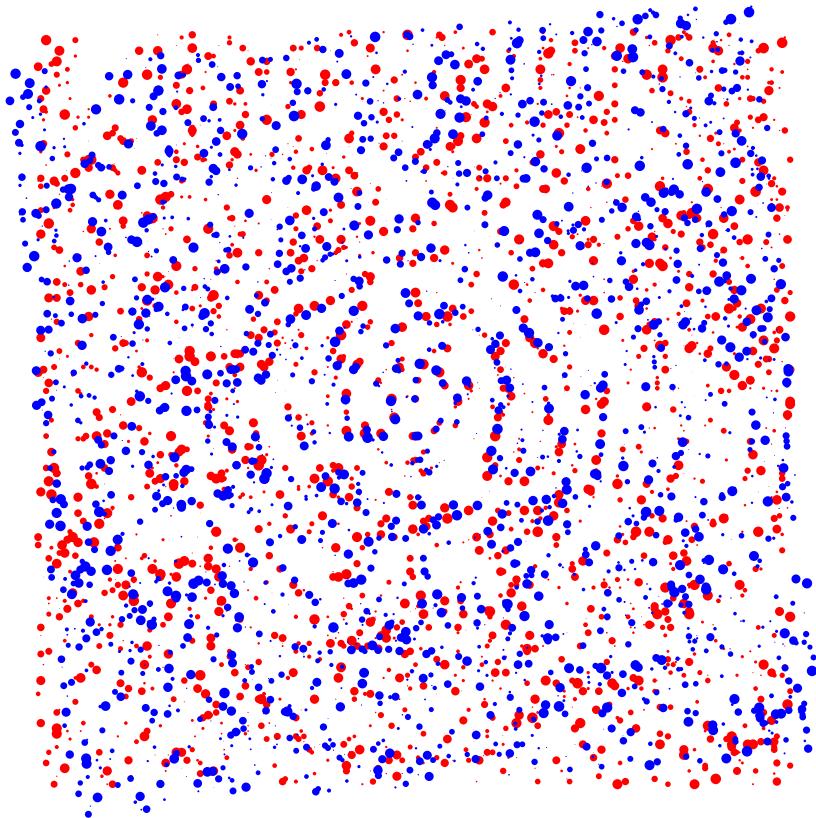
« Ces moirés tournants apparaissent quand les lentilles placées sur une trame et observées avec une autre trame identique à la première. La grosse lentille (convergente) réduit la trame inférieure, tandis que la petite lentille (divergente) la grossit. En conséquence, les moirés obtenus ont des sens de rotation opposés. Une figure de moiré ondulée traduit la présence d’aberrations optiques dans la lentille. »

Animation: Big lens: magnification of 1.2, small lens: magnification of 0.8

```
\begin{animateinline}[controls,palindrome,
  begin={\begin{pspicture}(-7,-7)(7,7)},
  end={\end{pspicture}}]{10}% 10 image/s
\multiframe{20}{i=-10+1}%
\psset{LensHandle=false,LensShadow=false}
\psset{linecolor=red,linewidth=0.1,type=linear,n=60}
\psmoire%
\PstLens[LensMagnification=1.2,LensSize=2](1,1.5){\psmoire}
\PstLens[LensMagnification=0.8,LensSize=1.5](-2,-2){\psmoire}
\psmoire[linecolor=black,rotate=i]
\end{animateinline}
```

7. Random moirés

Figure 1: Circular: $a=1$, $b=1$ $\theta = 5$



"The moiré patterns are formed from the superimposition of two random dot patterns: an original and a second pattern generated following a linear or nonlinear transformation of the original. Though each set is random, a variety of different spatial patterns such as circles, spirals, hyperbolae, can be generated by introducing correlations between the two sets of dots. In this image 1, for each dot there is a corresponding "partner" dot that lies along the circumference of a circle centered at the point of rotation of the two images. The visual system is able to decode these correlations, thereby perceiving the underlying global transformation."

http://www.scholarpedia.org/article/Glass_patterns

The article mentioned above contains a history of the physiological interpretations of the perception of moiré patterns and we reproduced the beginning of the paragraph.

The authors Leon Glass and Matthew A. Smith propose the following transformations:

$$x' = ax \cos \theta - by \sin \theta$$

$$y' = ax \sin \theta + by \cos \theta$$

which give the following Eigenvalues:

$$\lambda_{\pm} = \frac{(a+b) \cos \theta \pm \sqrt{(a-b)^2 - (a+b)^2 \sin^2 \theta}}{2}$$

Another article of Leon Glass:

http://www.medicine.mcgill.ca/physio/glasslab/pub_pdf/dots_mi_2002.pdf

We added the terms (x_0, y_0) to be able to also illustrate translations:

$$x' = ax \cos \theta - by \sin \theta + x_0$$

$$y' = ax \sin \theta + by \cos \theta + y_0$$

θ is the angle of rotation between the two layers. The values of a and b are scaling factors respectively for x and y .

7.1. The command `\psRandomDotPatterns`

`\psRandomDotPatterns [Options]`

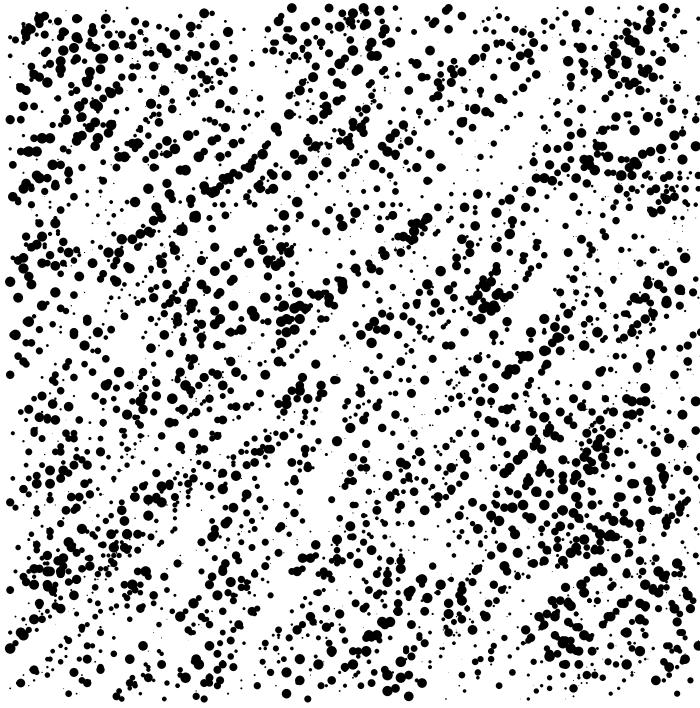
The command `\psRandomDotPatterns` contains the options `NumberDots=`, `coefficients=`, `srand=`, `layer=` and `revlayer=`.

Name	Default	Meaning
<code>NumberDots</code>	2000	Number of randomly arranged dots.
<code>coefficients</code>	1.05 1.05 0 0 0	$a \ b \ \theta \ x_0 \ y_0$. If the two last values are omitted (they are responsible for an eventual translation), they are automatically set to 0.
<code>srand</code>	10	Random seed of PostScript for the randomly arranged dots.
<code>layer</code>	true	If set to <code>false</code> , the first layer is hidden.
<code>revlayer</code>	true	If set to <code>false</code> , the second layer is hidden.

Note: We can draw superposed layers within a given square of 10 cm length. We can scale the image with the PStricks key `unit=`.

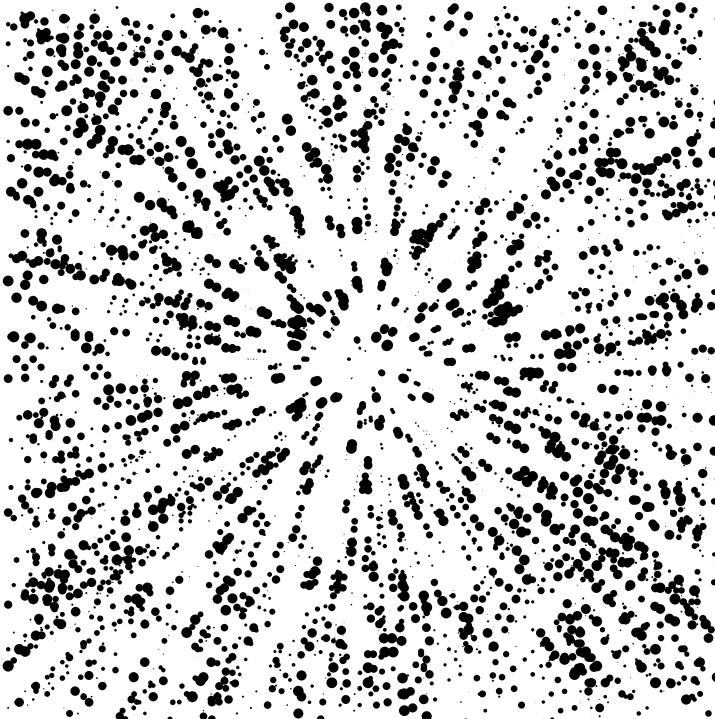
The radius of the dots is random, it is between 0 and 2 pt. The color of the points of the first layer is fixed with the PStricks key `linecolor=` and the color of the second layer with the PStricks key `fillcolor=`.

Example 1: Translation ($a = 1$, $b = 1$, $\theta = 0$, $x_0 = 5$, $y_0 = 5$)

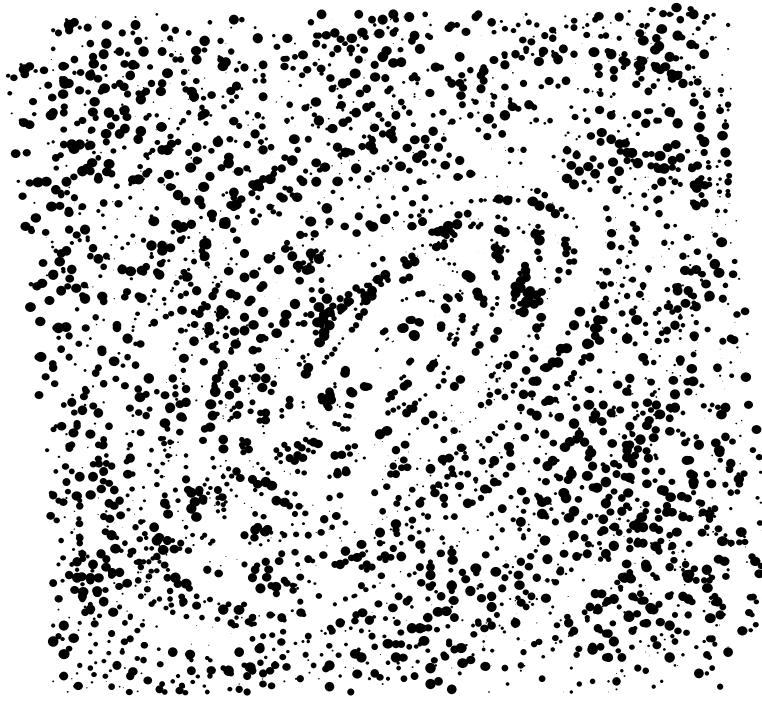


```
\begin{pspicture}(-5.5,-5.5)(5.5,5.5)
\psRandomDotPatterns[%  
coefficients=1 1 0 5 5  
]
\end{pspicture}
```

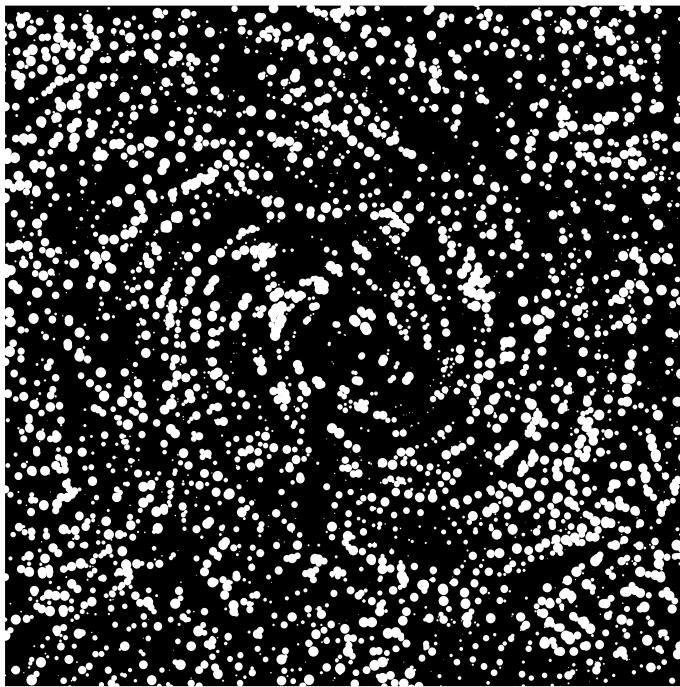
Example 2: Radial ($a = 1.05$, $b = 1.05$, $\theta = 0$)



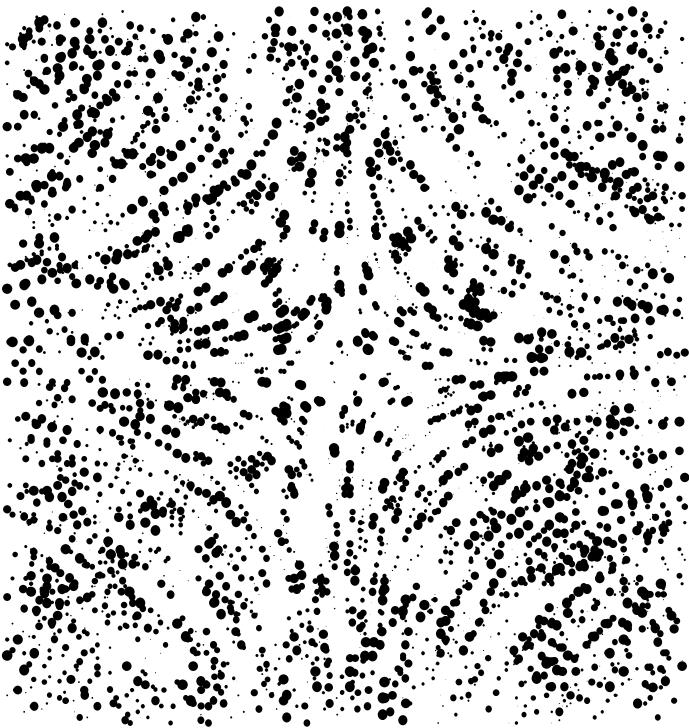
```
\begin{pspicture}(-5.5,-5.5)(5.5,5.5)
\psRandomDotPatterns
\end{pspicture}
```

Example 3: Elliptic ($a = 1.05$, $b = 0.95$, $\theta = 5$)

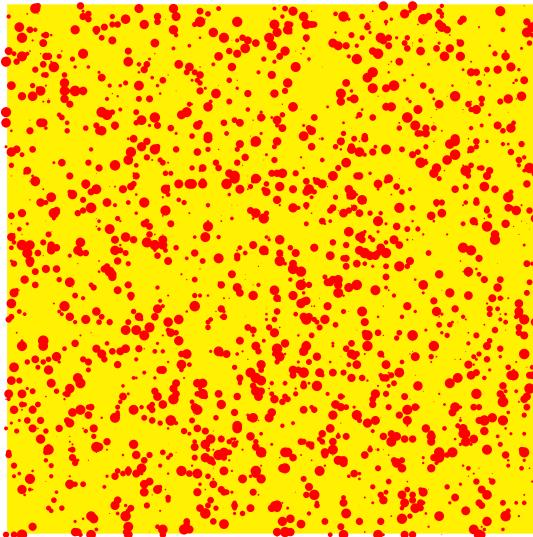
```
\begin{pspicture}(-5.5,-5.5)(5.5,5.5)
\psRandomDotPatterns[coefficients=1.05 0.95 5]
\end{pspicture}
```

Example 4: Spiral ($a = 1$, $b = 1.05$, $\theta = 5$)

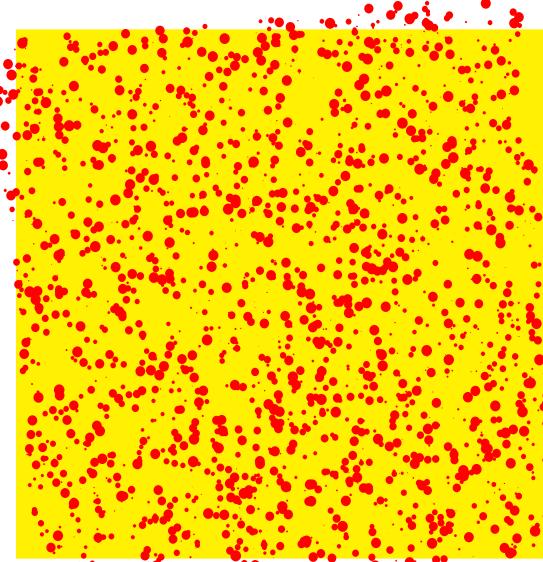
```
\begin{pspicture}(-5.5,-5.5)(5.5,5.5)
\psframe*(-5.5,-5.5)(5.5,5.5)
\psRandomDotPatterns[% 
coefficients=1 1.05 5,
linecolor=white,
fillcolor=white
]
\end{pspicture}
```

Example 5: Hyperbolic ($a = 0.95, b = 1.05, \theta = 0$)

```
\begin{pspicture}(-5.5,-5.5)(5.5,5.5)
\psRandomDotPatterns[coefficients=0.95 1.05 0]
\end{pspicture}
```

The keys layer and revlayer

`coefficients=1 1.05 5, revlayer=false`

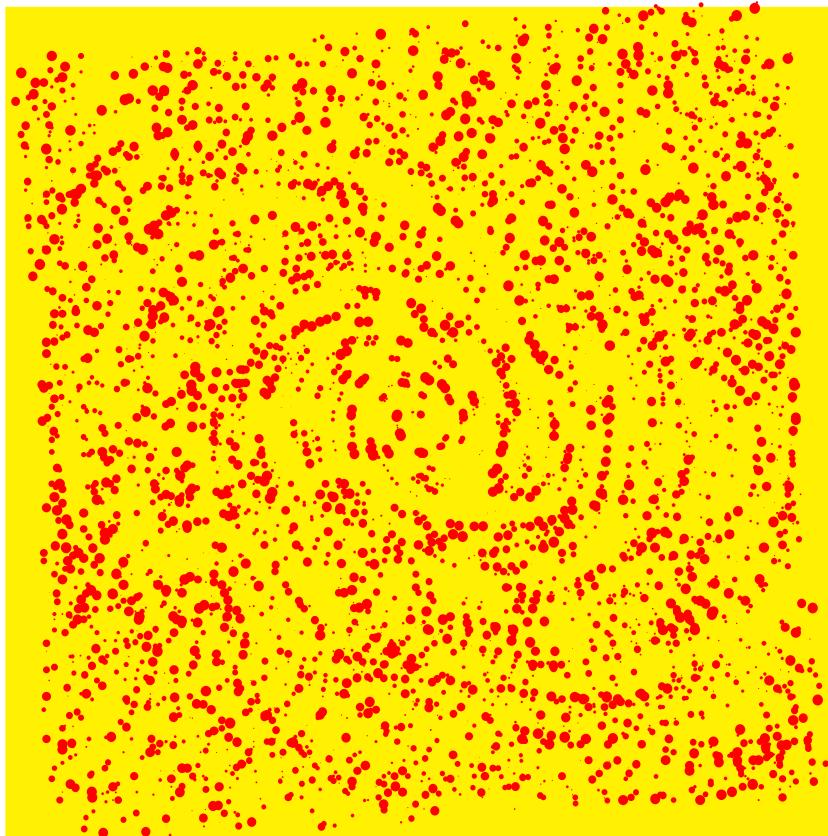


`coefficients=1 1.05 5, layer=false`

```
\begin{pspicture}(-3.5,-4.5)(3.5,3.5)
\psframe*[linecolor=yellow](-3.5,-3.5)(3.5,3.5)
\psRandomDotPatterns[% 
unit=0.7,
revlayer=false,
coefficients=1 1.05 5,
linecolor=red]
\rput(0,-4){%
coefficients=1 1.05 5,
\textbf{revlayer=false}}
\end{pspicture}
```

```
\begin{pspicture}(-3.5,-4.5)(3.5,3.5)
\psframe*[linecolor=yellow](-3.5,-3.5)(3.5,3.5)
\psRandomDotPatterns[% 
unit=0.7,
layer=false,
coefficients=1 1.05 5,
fillcolor=red]
\rput(0,-4){%
coefficients=1 1.05 5,
\textbf{layer=false}}
\end{pspicture}
```

Both layers superposed.



coefficients=1 1.05 5

```
\begin{pspicture}(-5.5,-5.5)(5.5,6)
\psframe*[linecolor=yellow](-5.5,-5.5)(5.5,5.5)
\psRandomDotPatterns[%
coefficients=1 1.05 5
linecolor=red,
fillcolor=red]
\rput(0,-6){coefficients=1 1.05 5}
\end{pspicture}
```

7.2. The command \psRandomDot

Emin Gabrielyan presents on

<https://docs.switzenet.com/people/emin-gabrielyan/070212-random-moire/>

another amazing moiré pattern (his second example).

A plate perforated with holes (the *revealing layer*), is placed on top of a fixed layer (the *base layer*) filled with the digit “2”, and then rotated.

These holes of the revealing layer are arranged randomly, and the digits “2” are arranged on the base layer at exactly the same places (just larger than the holes and rotated by -90 degrees). Both layers are therefore correlated—this is the necessary condition for the existence of this random moiré phenomenon.

Why do we see the digit “2” as a *halo* with a magnifying effect as a function of the angle—upright or upside down according to the sign of the angle? The holes of the revealing layer cover the digits “2” of the base layer and these superposed parts reconstruct an enlarged digit “2”; what is the explanation of this phenomenon?

\psRandomDot [Options] (x , y)

The command \psRandomDot contains the options `hole=`, `r=`, `p=`, `k=`, `symbole=`, `rotate=`, `PSfont=`, `fontsize=`, `vadjust=` and `hadjust=`.

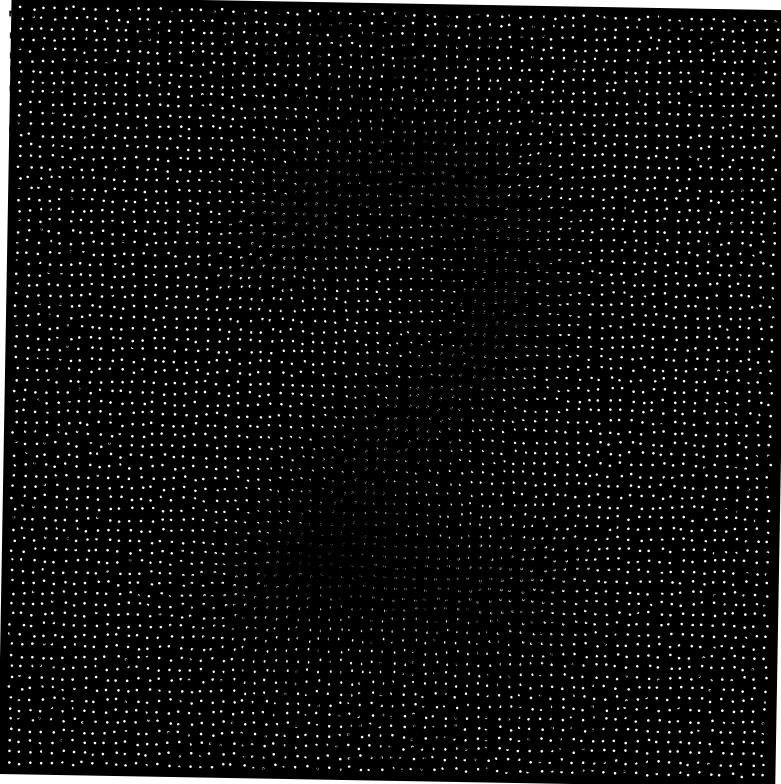
The optional argument `(x , y)` sets up the dimensions `x` and `y` of the image. If not chosen (10,10) is taken by default.

Name	Default	Meaning
hole	round	Types of holes: hole=round (circle holes), hole=square (square holes)
r	0.5	Radius for circle/side length of square of the holes (in pt)
p	4	Distance between the holes (in pt).
rotate	0	Angle of rotation between base layer and revealing layer (in degrees) Typical values: -2<rotate<2
k	0.2	Factor for the dispersion of the holes
symbole	2	Digit or letter for the base layer
PSfont	Helvetica-Bold	Font family of PS fonts
fontsize	4.7	Font size (in pt)
vadjust	-1.5	Vertical adjustment of the superposed image
hadjust	0	Horizontal adjustment of the superposed image

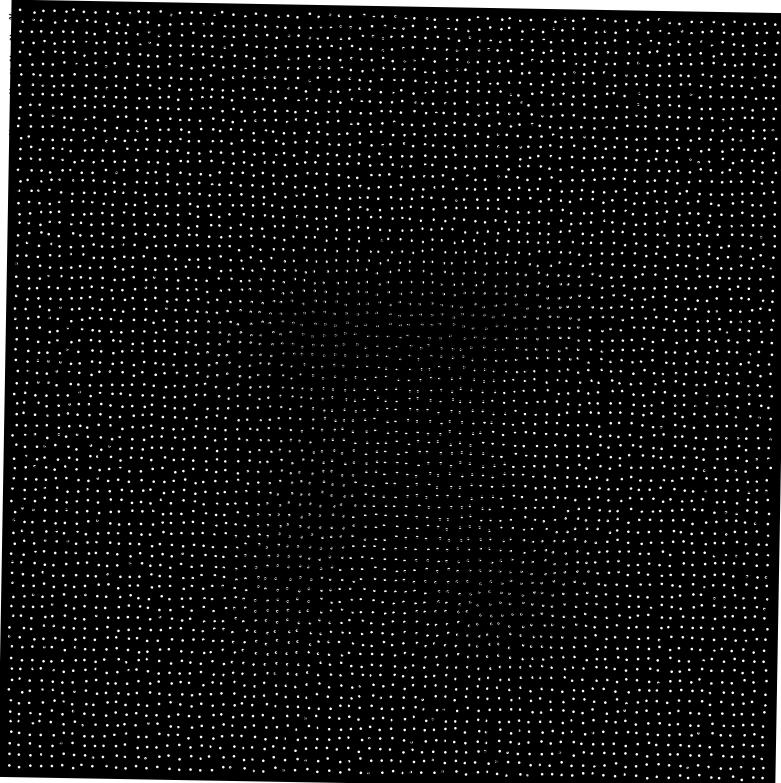
Principle:

1. We draw a path (`newpath`) that starts along a square (or a rectangle) anti-clockwise.
2. On a 2D grid, where points are setup with a constant distance of `p`, the centers of the circle/square shaped holes (with dimensions `r`) are then randomly distributed around each of the points on the grid within a circle with the *distribution radius* $\rho = k \cdot p$. (The bigger `k`, the more disorder. The bigger `p`, the less points inside the given grid of a square/rectangle base layer.)

Additional Remark: For a list of some more *PostScript font names*, look up the appendix on page 55.

Example 1:

```
\begin{pspicture}(-5,-5.5)(5,5)
\psRandomDot[
r=0.5,
p=4,
k=0.2,
symbole=2,
rotate=-1,
PSfont=Helvetica-Bold,
fontsize=4.7,
vadjust=-1.5,
hadjust=0,
hole=round](10,10)
\end{pspicture}
```

Example 2:

```
\begin{pspicture}(-5,-5.5)(5,5)
\psRandomDot[
r=0.5,
p=4,
k=0.2,
symbole=p,
rotate=-1,
PSfont=Symbol,
fontsize=4.7,
vadjust=-1.5,
hadjust=0,
hole=round](10,10)
\end{pspicture}
```

8. Glass-patterns

Glass-patterns result from a superposition of two random dotted layers—a first original layer and a second layer which is a transformed layer of the original.

These patterns received their names from Leon Glass.

8.1. The command \psGlassPattern

```
\psGlassPattern [Options]
```

The command `\psGlassPattern` contains the options `function=` and `layers=true/false`.

Name	Default	Meaning
<code>function</code>	<code>5 r mul t 5 mul sin neg 0.5 mul 1 add mul 2.5 sub</code>	The equation of the function <code>r</code> and <code>t</code> are the variables of the function in polar coordinates $r = \sqrt{x^2 + y^2}$, $t = \theta$
<code>layers</code>	<code>true</code>	Both layers are shown. If set to <code>false</code> , only the revealing layer with the hidden shape of the function is shown.

Note: The dimensions of the layers are 15 cm x 15 cm and can be modified with `unit=`.

Remarks:

- The key `function=` is the equation of a function in polar coordinates with the variables `r` (radius coordinate) and `t`, which is an angle in degrees.

The star-like functions are of the type: $z=r*(1-0.5*\cos(\theta))$, with each value of z there corresponds a star-like function with the equation (in polar coordinates):

$$r=z/(1-0.5*\cos(\theta))$$

The equation of a function can either be entered with RVN (Reverse Polish Notation = PostScript notation) or when the PStricks key `algebraic=true` is set, it is possible to enter the equation in algebraic notation (therefore we need to transform the variable `t` from degrees into radians which can be done with the substitution `t -> t*Pi/180`.

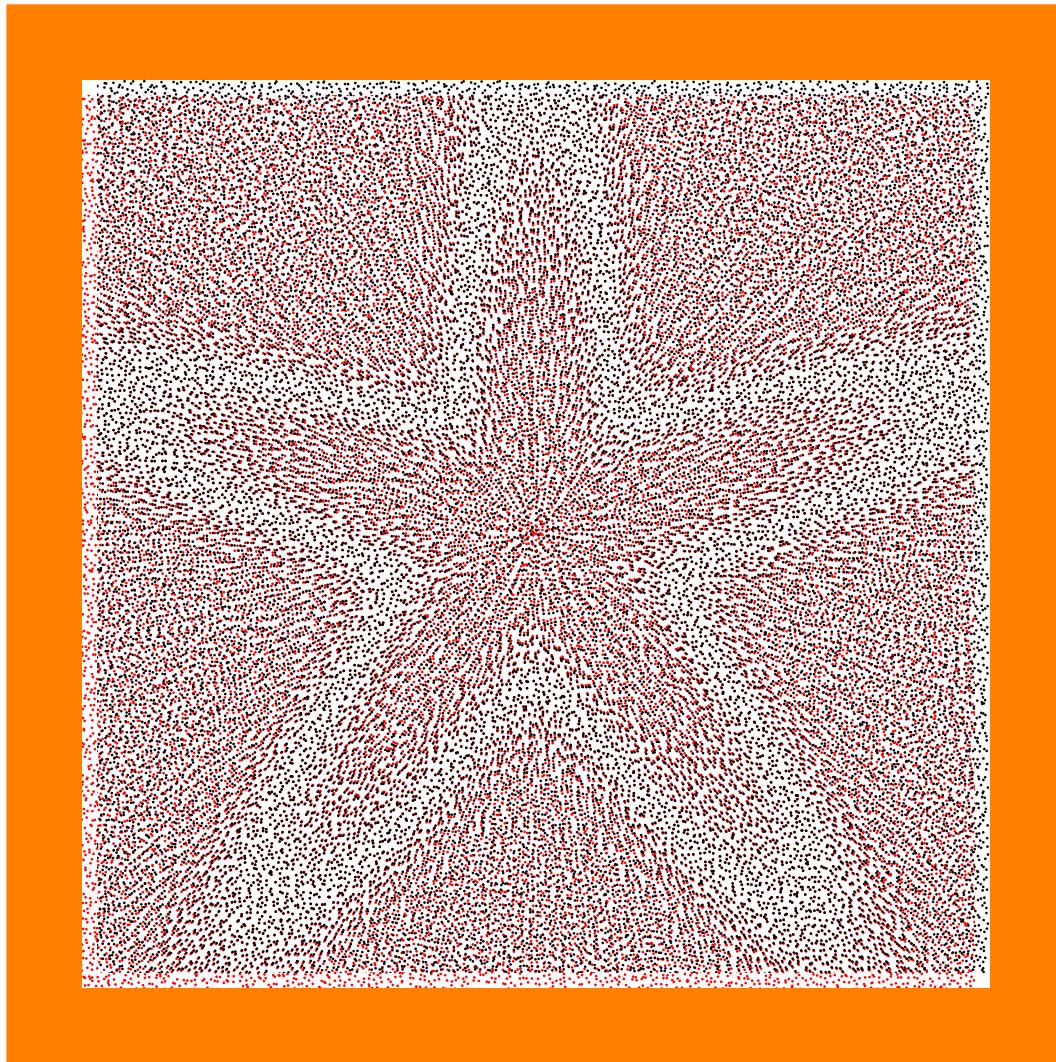
- If the key `layers=false` is set to false, only the layer with the hidden shape of the function is shown.
- The colors of the randomly arranged dots can be chosen with the PStricks keys `linecolor=` (for the first layer) and `fillcolor=` (for the second layer).
- The size and shape of the dots can be setup with the PStricks keys `dotsize=` and `dotstyle=`.

Example 1:

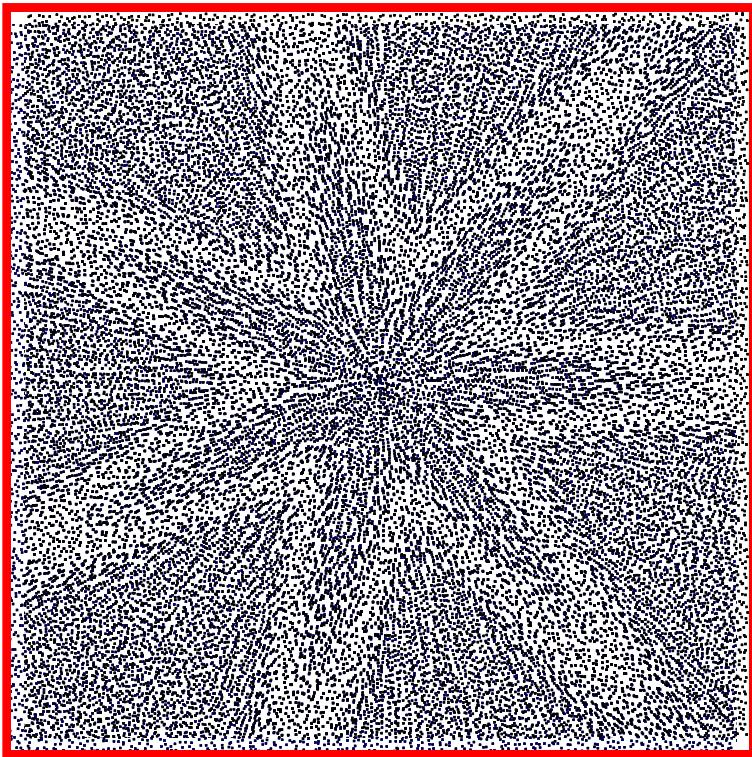
Here a citation from Isaac Amidror from his book:

"The Theory of the Moiré Phenomenum", Volume II: Aperiodic Layers, **3-18: Synthesis of a layer superposition having a predefined fixed locus.**

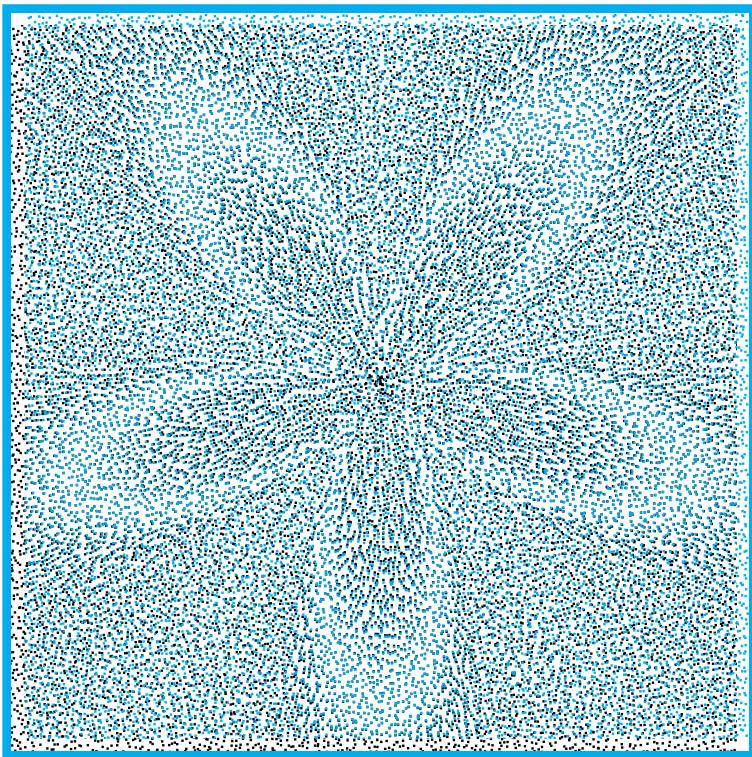
"Design layer transformations $g_1(x, y)$ and $g_2(x, y)$ that will produce in the superposition of two initially identical random screens a fixed locus consisting of a star-like curve that surrounds the origin as shown in the figure on the front cover of this book. Hint: In this case, you may consider a top-opened conic surface having star-like level lines, such as $z = r(1 + 0.5 \cos 5\theta)$, or, possibly, $z = r/(1 + 0.5 \cos 5\theta)$, which gives a slightly different star. You may adjust the orientation of the star by replacing \cos by \sin or by $-\sin$, as seems suitable. In order to have this surface intersect the x, y plane along a star, you need to lower it by some constant z_0 : $z = r(1 + 0.5 \cos 5\theta) - z_0$. But if you wish to obtain a more complex surface that intersects the x, y plane on a family of concentric stars, you may consider a surface such as: $z = \sin(r(1 + 0.5 \cos 5\theta))$."



```
\begin{pspicture}(-7,-7)(7,7)
\psframe*[linecolor=orange](-7,-7)(7,7)
% z=5*r*(1-0.5*sin(5*t*Pi/180))-2.5
\psGlassPattern[unit=0.8,linecolor=red]
\end{pspicture}
```

Example 2:

```
\begin{pspicture}(-5,-5)(5,5)
\psframe*[linecolor=red](-5,-5)(5,5)
% in algebraic notation
% t in degrees;
% arguments of sin and cos in radians
% convert t -> t*Pi/180
\psGlassPattern[% 
unit=0.65,
dotsize=1pt,
dotstyle=square,
linecolor={[rgb]{0 0 0.5}},
algebraic,
function=5*r*(1-0.5*cos(7*t*Pi/180))-2.5]
\end{pspicture}
```

Example 3:

```
\begin{pspicture}(-5,-5)(5,5)
\psframe*[linecolor=cyan](-5,-5)(5,5)
% in algebraic notation
% t in degrees;
% arguments of sin and cos in radians
% convert t -> t*Pi/180
\psGlassPattern[% 
unit=0.65,
dotsize=1pt,
dotstyle=square*,
linecolor=black,
fillcolor=cyan,
algebraic,
function=5*r/(1-0.75*sin(5*t*Pi/180))-2.5]
\end{pspicture}
```

9. Animations

Some interactive moiré JavaScript based applications can be found on:

<https://melusine.eu.org/syracuse/G/pstricks/pst-moire/moirej/>

The following animations are all generated with the `animate` package of Alexander Grahn.

<https://ctan.org/pkg/animate>

Animation 1:

```
\begin{animateinline}[%  
  controls, palindrome,  
  begin={\begin{pspicture}(-6,-6)(6,6)},  
  end={\end{pspicture}}}  
  ]{5}% 5 image/s  
\multiframe{36}{i=0+1}{%  
  \psmoire[type=Newton,rotate=-\i]%;  
  \psmoire[type=Newton,rotate=\i]%;  
}  
\end{animateinline}
```

Animation 2:

```
\begin{animateinline}[%  
  controls, palindrome,  
  begin={\begin{pspicture}(-6,-6)(6,6)},  
  end={\end{pspicture}}  
 ]{10}{ 10 image/s  
\multiframe{36}{r=0+0.1}{%  
 \psframe*[linecolor=black](-6,-6)(6,6)  
 \psmoire[type=linear,rotate=-\r,linewidth=0.05,linecolor=yellow,n=60](0,0)%  
 \psmoire[type=linear,rotate=\r,linewidth=0.15,linecolor=black,n=60](0,0)%  
 \psframe[linecolor=yellow,linewidth=5pt](-6,-6)(6,6)  
 }  
\end{animateinline}
```

Animation 3:

```
\definecolor{moire1}{rgb}{0.98,0.89,0.56}
\definecolor{moire2}{rgb}{0.357,0.525,0.13}
\definecolor{moire3}{rgb}{0.2,0.05,0.015}
\definecolor{moire4}{rgb}{0.070.41 0.255}
\begin{animateinline}[%  
    controls,palindrome,  
    begin={\begin{pspicture}(-6,-6)(6,6)},  
    end={\end{pspicture}}  
    ]{5}% 5 image/s  
\multiframe{20}{r=0+0.025}{%  
\psset{linewidth=2.5pt}  
\psmoire[type=circle, linecolor=moire1](0,\r)  
\psmoire[type=circle, linecolor=moire2](0,-\r)  
\psmoire[type=circle, linecolor=moire3](\r,0)  
\psmoire[type=circle, linecolor=moire4](-\r,0)  
}  
\end{animateinline}
```

Animation 4:

```
\begin{animateinline}[%  
  controls, palindrome,  
  begin={\begin{pspicture}(-6,-6)(6,6)},  
  end={\end{pspicture}}  
 ]{5}% 5 image/s  
\multiframe{30}{r=0+0.025}{%  
\psset{linewidth=1pt}  
\psmoire[type=radial, linecolor=red](\r,0)  
\psmoire[type=radial, linecolor=green](-\r,0)  
\psmoire[type=radial, linecolor=blue](0,-\r)  
}  
\end{animateinline}
```

Animation 5:

The reason to finally setup this PSTricks package came from a post card “*turn the top part*”—bought years ago in a boutique of the *Centre Beaubourg* in Paris—showing up this quite spectacular phenomenon of the *moiré effect* and the following code was quickly ready made to redesign it within PSTricks.

```
\def\myMoiré{%
\psset{dimen=inner,linewidth=0pt}
\def\carre{%
\pnodes{AL}{0,0}(-1.5,1.5)(-1.5,1.2)(-1.5,0.9)(-1.5,0.7)(-1.5,0.4)(-1.5,0.2)(-1.5,0)%
(-1.5,-0.2)(-1.5,-0.4)(-1.5,-0.6)(-1.5,-0.75)(-1.5,-0.9)(-1.5,-1.05)(-1.5,-1.15)%
(-1.5,-1.25)(-1.5,-1.3)(-1.5,-1.4)(-1.5,-1.45)
\pnodes{AR}{0,0}(1.5,-1.5)(1.5,-1.2)(1.5,-0.9)(1.5,-0.7)(1.5,-0.4)(1.5,-0.2)(1.5,0)%
(1.5,0.2)(1.5,0.4)(1.5,0.6)(1.5,0.75)(1.5,0.9)(1.5,1.05)(1.5,1.15)(1.5,1.25)%
(1.5,1.3)(1.5,1.4)(1.5,1.45)
\multido{\iA=1+2,\iB=2+2}{9}{\pspolygon*(AL\iA)(AR\iA)(AR\iB)(AL\iB)}%
\pnodes{BL}{0,0}(-1.2,1.5)(-0.9,1.5)(-0.7,1.5)(-0.4,1.5)(-0.2,1.5)(0.0,1.5)(0.2,1.5)%
(0.4,1.5)(0.6,1.5)(0.75,1.5)(0.9,1.5)(1.05,1.5)(1.15,1.5)(1.25,1.5)(1.3,1.5)%
(1.4,1.5)(1.45,1.5)(1.5,1.5)
\pnodes{BR}{0,0}(1.2,-1.5)(0.9,-1.5)(0.7,-1.5)(0.4,-1.5)(0.2,-1.5)(0,-1.5)(-0.2,-1.5)%
(-0.4,-1.5)(-0.6,-1.5)(-0.75,-1.5)(-0.9,-1.5)(-1.05,-1.5)(-1.15,-1.5)(-1.25,-1.5)%
(-1.3,-1.5)(-1.4,-1.5)(-1.45,-1.5)(-1.5,-1.5)
\multido{\iA=1+2,\iB=2+2}{9}{\pspolygon*(BL\iA)(BR\iA)(BR\iB)(BL\iB)}%
\def\half{%
\rput(0,0){\carre}
\rput(-3,0){\psscalebox{-1 1}{\carre}}
}
\def\pattern{%
\rput(0,0){\half}
\rput(0,-3){\psscalebox{1 -1}{\half}}
}
\multido{\iA=0+6}{2}{\multido{\iB=0+-6}{3}{\rput(\iA,\iB){\pattern}}}

\psset{unit=0.75}
\begin{animateinline}[controls,loop,
  begin={\begin{pspicture}(-7,-19)(10,5)},
  end={\end{pspicture}}]{10}% 10 frames/s (velocity of the animation)
\multiframe{11}{i=0+2}{% number of frames
\rput(0,0){\myMoiré}
\psrotate{1.5,-7}{\i}{\myMoiré}
}
\multiframe{21}{i=20+-2}{%
\rput(0,0){\myMoiré}
\psrotate{1.5,-7}{\i}{\myMoiré}
}
\multiframe{10}{i=-20+2}{%
\rput(0,0){\myMoiré}
\psrotate{1.5,-7}{\i}{\myMoiré}
}
\end{animateinline}
```

10. Theory—for the interested user

10.1. The contribution of “éditions Kangourou”

“Le Kangourou des mathématiques”: <http://www.mathkang.org/> published a revue in 2002, “Les malices du Kangourou” that contains a magnificent article from pages 18 to 26 titled “Mirifices et mirobolants moirés” and on the back cover “La règle à moirer” (“The ruler”). The article and the ruler are available at the following addresses:

<http://www.mathkang.org/cite/moires9p.pdf>
<http://www.mathkang.org/cite/moirer.html>

The ruler can be purchased at the following address:

<http://www.mathkang.org/catalogue/prodmoir.html>

In the article the sketches are very beautiful and the part “Mathématisation du phénomène” is remarkable! It contains the following moirés:

- a network of parallel straight lines superposed each with a rotation in different direction;
- shifted superposition of two networks consisting of radial rays (or rather sectors);
- shifted superposition of two frames of Fresnel rings, well known as Newton’s rings observed in optics.

10.2. The contribution of Henri Bouasse

This is the chapter of his book “*Vision et reproduction des formes et des couleurs*” published at Librairie Delagrave in Paris in 1917. His demonstration and the diagram within his book have been reproduced here:

Parallel straight lines

1° — Consider two straight lines respectively parallel:

$$x \cos \theta + y \sin \theta = bt + ct^2 \quad x \cos \theta - y \sin \theta = b\tau - c\tau^2 \quad (1)$$

For $t = \tau = 0$, we get the two lines OS_2 and OS_1 ; they obviously have the same angle θ with the axis Oy .

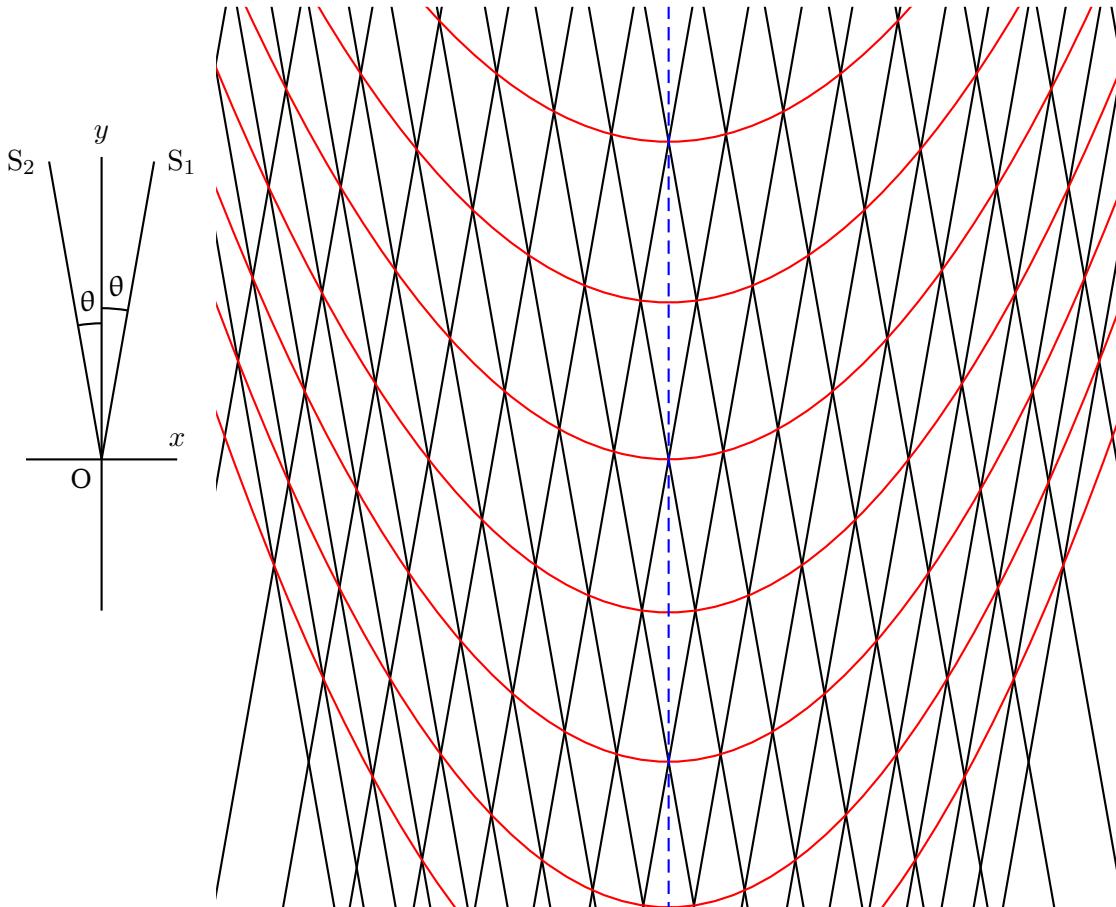


Figure 2: Moiré: parallel lines

The curves of the intersection points, which correspond to the small diagonals of the parallelograms, satisfy the following condition:

$$t - \tau = \mu = \text{constant}$$

Adding and reordering the equations (1):

$$\begin{aligned} 2x \cos \theta &= (b + c\mu)(t + \tau) \\ 2y \sin \theta &= b\mu + c(t^2 + \tau^2) = b\mu + c(\mu^2 + 2t\tau) \end{aligned}$$

To complete the elimination, we will use the following relation:

$$(t + \tau)^2 - 4t\tau = \mu^2$$

Thus:

$$\frac{4x^2 \cos^2 \theta}{(b + c\mu)^2} - \frac{4y \sin \theta}{c} + \frac{2b\mu}{c} + \mu^2 = 0 \quad (2)$$

The wanted curves are parabolas which have the Oy axis in common.

Its vertices are given by:

$$y = \frac{\mu(2b + c\mu)}{4 \sin \theta} \quad (3)$$

2° — The parameter c is small compared to the parameter b , so the equations simplify.

The equation (2) becomes:

$$\frac{4x^2 \cos^2 \theta}{b^2} - 4y \sin \theta + 2b\mu = 0$$

This is the same parabola for all the values of μ sliding parallelly to Oy . The vertices are given by:

$$y = \mu \frac{b}{2 \sin \theta} \quad (4)$$

The radius of curvature at the vertex of the parabola is:

$$R = \frac{b^2}{2c} \frac{\sin \theta}{\cos^2 \theta}$$

If the parallel straight lines are equidistant ($c = 0$), the parabolas degenerate to straight lines (4); in other words, the radius of curvature becomes infinite.

3° — To make an experiment, we trace with “*China ink*” on a paper 51 parallel lines with a length of i. e. 20 cm, where the distance between two adjacent lines increases from 2 mm (between the first two lines) to 3 mm (between the last two lines), following the formula:

$$s = 2t + 0.01t^2$$

We take a photo by reducing to the half or a quarter. We generate two diapositives¹. We realize the phenomenon when placing one over the other by rotating one of them.

We think that if you had followed the given instructions, you might be as well convinced—as we are—it would have been a pity to have left this beautiful demonstration “of that time” in oblivion!

10.3. The humble contributions of our group

These demonstrations contain:

1. the moirés of Newton. In fact it is a construction similar to that of the Fresnel rings. Here the progression of the squares is such that the areas between two consecutive squares are equal to the area of the central square. One out of every two intervals is made opaque. The resulting moiré figures are equilateral hyperbolas.
2. the moirés obtained by the superposition of Fresnel rings and a network of parallel lines result as well in Fresnel rings.

The source files (L^AT_EX) and pdf are found within the repository:

<http://melusine.eu.org/syracuse/G/pstricks/pst-moire/moiredoc/>

¹ spelling of the time.

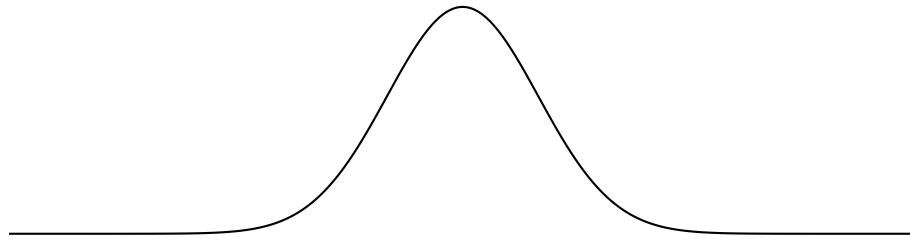
10.4. The construction of a Gauss network

This method is discussed on page 136 of the book "Les phénomènes naturels" edited in 1978 by the revue "Pour la Science" and distributed by the Berlin editions.

« The Gauss network is obtained by drawing a series of equidistant vertical lines on a Gaussian curve, then by drawing parallel oblique lines passing through the points of intersection between the vertical lines and the Gaussian curve. »

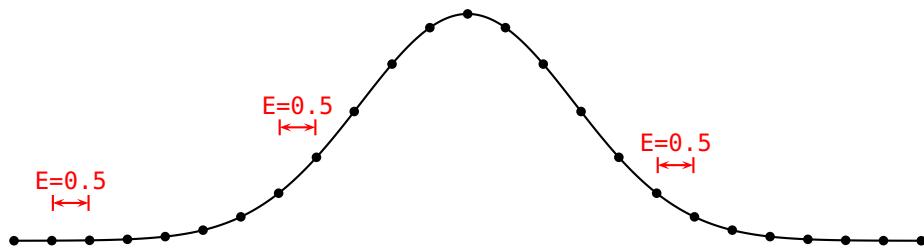
Gaussian curve

$$y = ae^{-(kx)^2}$$



Determination of the points of intersection

The equidistant vertical line network has for equation: $x = ne$, e is the spacing and n is an integer. The ordinates of the intersection points are: $y_n = ae^{-(kne)^2}$. Within the following figure, we set the spacing between the points on the Gaussian curve with the key $E=0.5$ (in cm).



Drawing the network of the straight lines

We determine the equations of the straight lines passing through these points and which are inclined by an angle α with respect to the horizontal. This is to setup with the key Alpha= (in degrees).

The general equation of such a line is given by: $y = x \tan(\alpha) + b$, we determine b to go through one of the previous points.

$$ne \tan(\alpha) + b = ae^{-(kne)^2}$$

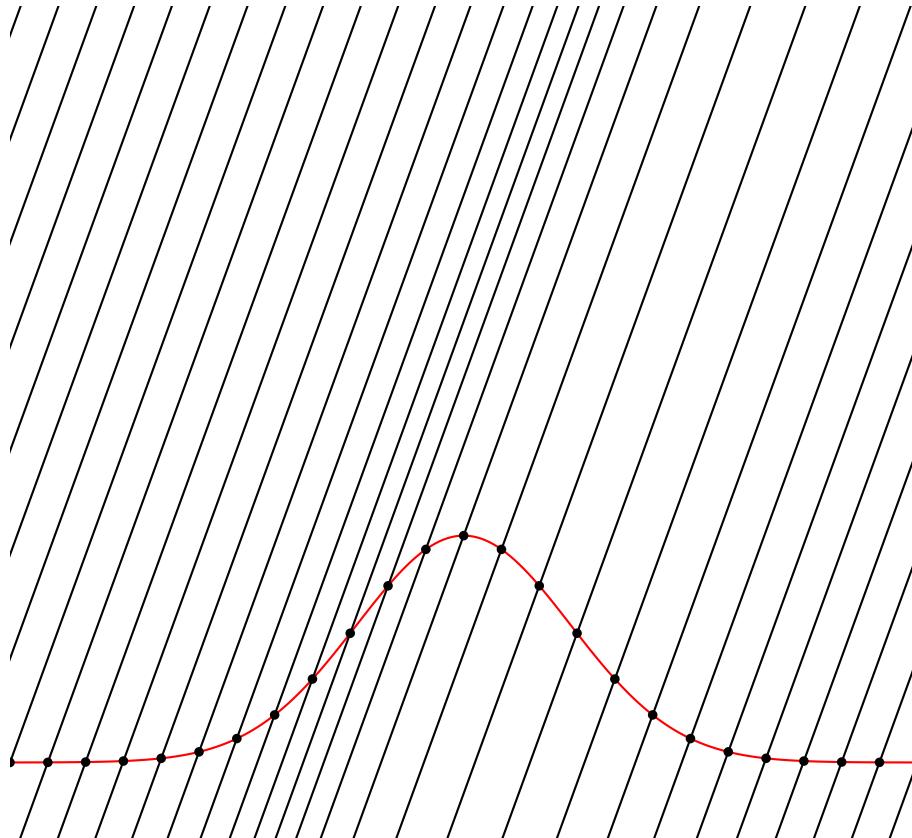
so we get b .

$$b = ae^{-(kne)^2} - ne \tan(\alpha)$$

For every value of n we get a straight line.

$$y = x \tan(\alpha) + ae^{-(kne)^2} - ne \tan(\alpha)$$

Let's draw some of these straight lines. Setting $a = 3$, $k = 0.5$, $-20 < n < +20$, $e = 0.5$ and $\alpha = 70^\circ$

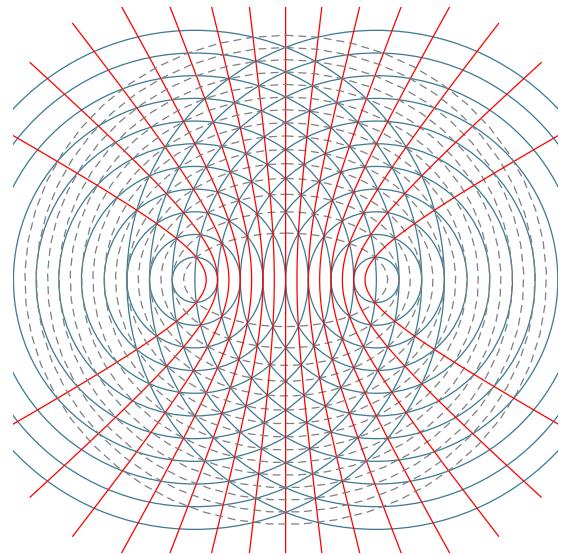
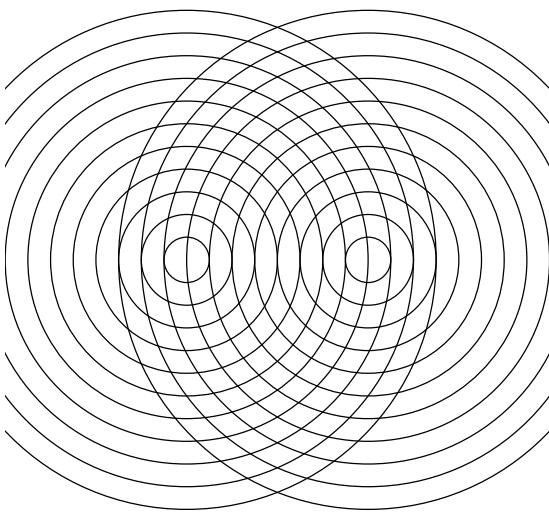


```
\begin{pspicture}(-6,-1)(6,10)
\parametricplot[plotpoints=1000, linecolor=red]{-6}{6}{%
    t
    3 2.71828 0.5 t mul dup mul neg exp mul}
\pstVerb{/A 3 def
/K 0.5 def
/E 0.5 def
/Alpha 70 def
/m {Alpha dup sin exch cos div} bind def % pente de la droite
}%
\multido{\n=-20+1}{41}{%
\pnode(! \n\space E mul \% x
    A 2.71828 K \n\space E mul mul dup mul neg exp mul){A}
\psdot(A)
\rput(A){\psline(! -4 -4 m mul)(! 4 4 m mul)}
}
\end{pspicture}
```

The last step is to translate these lines into PostScript code.

10.5. Some moiré figures

type=circle + type=circle



Mathematization

Equidistant radii that increase like: $r_n = n \cdot a$, with $a > 0$.

We have

$$(x - c)^2 + y^2 = k^2 p^2$$

$$(x + c)^2 + y^2 = k^2 q^2$$

It is necessarily: $p - q = m \in \mathbb{Z}$, thus

$$p = \frac{1}{k} \sqrt{(x - c)^2 + y^2}$$

$$q = \frac{1}{k} \sqrt{(x + c)^2 + y^2}$$

and $p - q = m$

$$\sqrt{(x - c)^2 + y^2} - \sqrt{(x + c)^2 + y^2} = km$$

$$r_p = p \cdot k \quad r_q = q \cdot k$$

$$r_p - r_q = (p - q) \cdot k = m \cdot k$$

The points of the moiré curves are such that the difference in distances to the two centers is constant. The moiré curves are hyperbolas focussing the centers of circles.

We pose: $a = \frac{km}{2}$ and $b^2 = c^2 - a^2$. The equations of this family of hyperbolas are written like:

$$x = a \cosh(t)$$

$$y = b \sinh(t)$$

or:

$$x = \frac{a}{\cos(t)}$$

$$y = b \tan(t)$$

If we go from a point of intersection (p, q) to a point $(p + 1, q - 1)$, the sum of the distances remains constant.

As a result, we say:

$$r_p + r_q = (p + q) \cdot k = n \cdot k$$

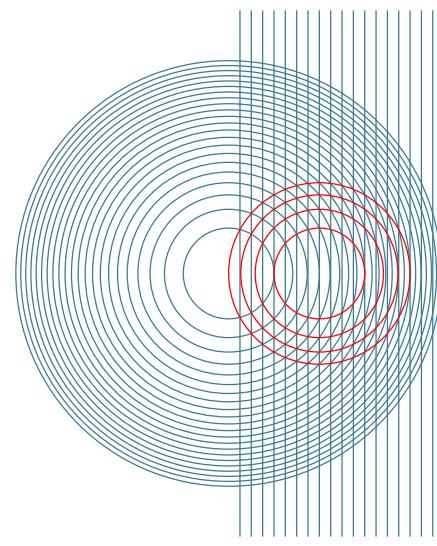
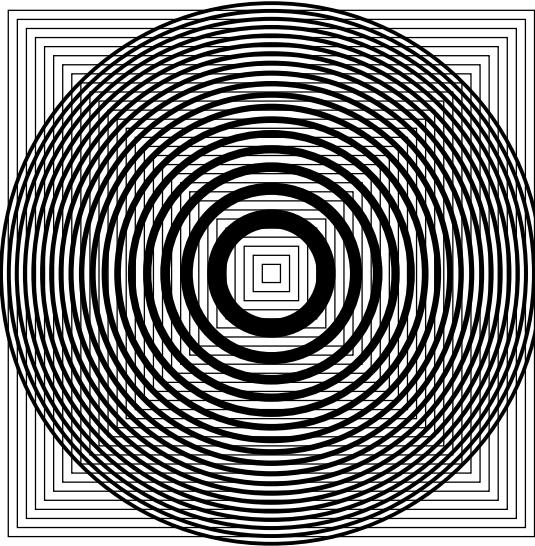
This family of moiré curves are ellipses with equations like:

$$x = a \cos(t) \quad y = b \sin(t)$$

with:

$$b^2 = a^2 - c^2$$

type=square + type=Fresnel



Mathematization

The abscissa of the edges of the square with $x > 0$ increase with: $x_n = a \cdot n$, with $a > 0$

The radii increase with: $r_n = \sqrt{n}$.

We have

$$\begin{aligned} x &= ap \\ x^2 + y^2 &= q \end{aligned}$$

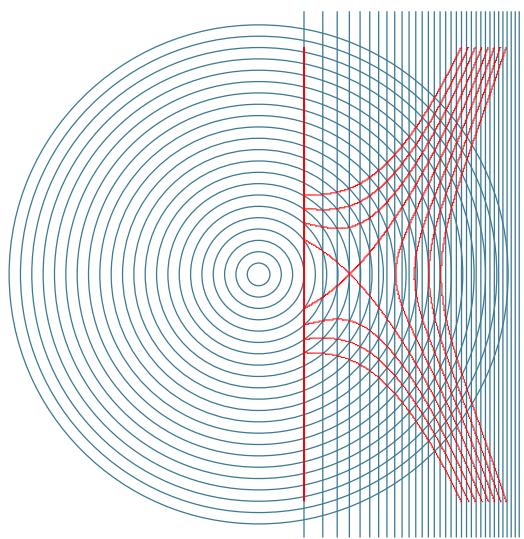
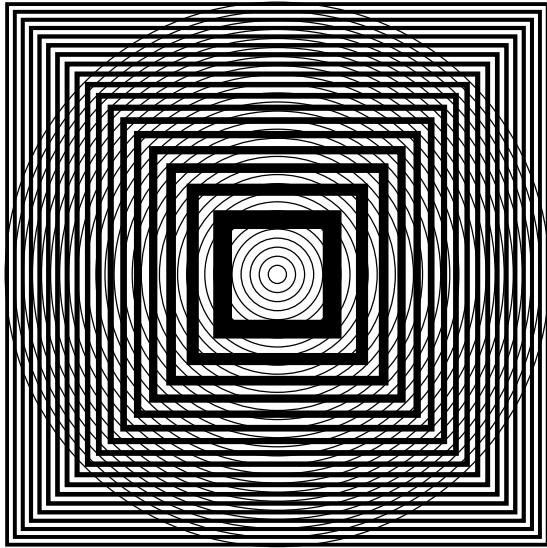
$$\begin{aligned} p &= \frac{x}{a} \\ q &= x^2 + y^2 \end{aligned}$$

On a curve of moiré, we verify: $p - q = m \in \mathbb{Z}$:

$$\left(x - \frac{1}{2a} \right)^2 + y^2 = m + \frac{1}{4a^2}$$

This family of moiré curves are circles with the center at $(\frac{1}{2a}, 0)$ and with a radius of $r_m = \sqrt{m + \frac{1}{4a^2}}$

type=circle + type=Newton



Mathematization

The abscissa of the edges of the square with $x > 0$ increase with: $x_n = \sqrt{n}$.

The radii increase with: $r_n = n \cdot a$, with $a > 0$.

We have:

$$\begin{aligned} x &= \sqrt{p} \\ x^2 + y^2 &= a^2 q^2 \end{aligned}$$

It is necessarily: $p - q = m \in \mathbb{Z}$,
thus

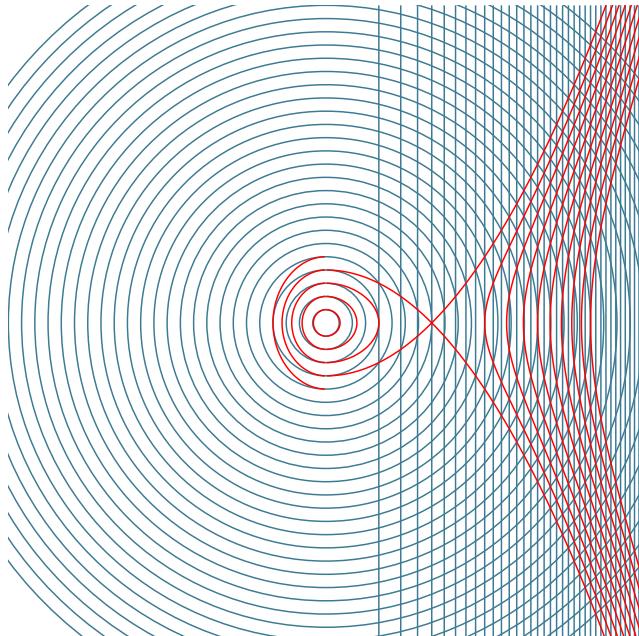
$$\begin{aligned} p &= x^2 \\ q &= \frac{1}{a} \sqrt{x^2 + y^2} \end{aligned}$$

and $p - q = m$

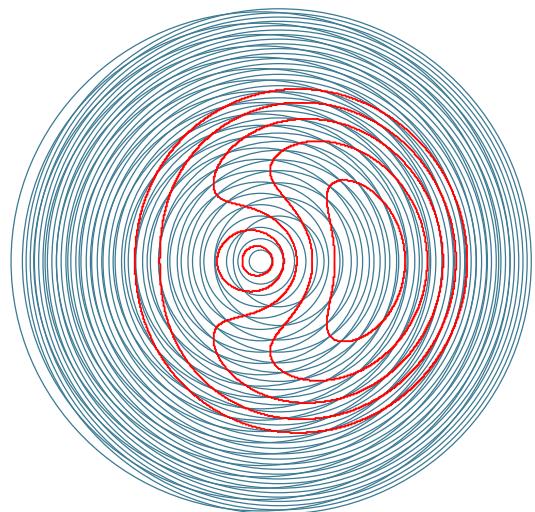
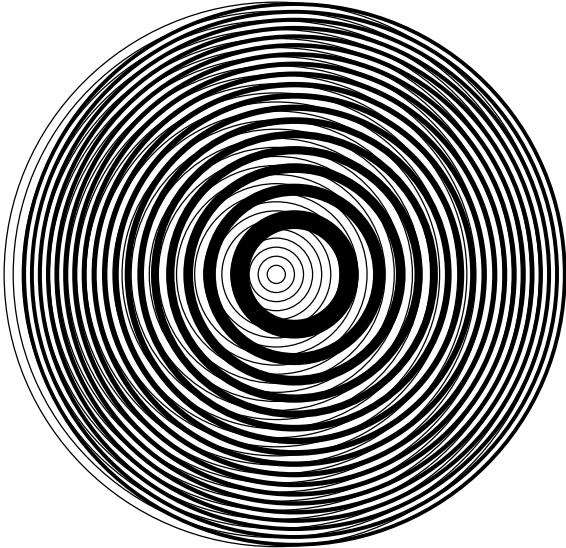
$$x^2 - \frac{1}{a} \sqrt{x^2 + y^2} = m$$

In polar coordinates:

$$\begin{aligned} \rho^2 (\cos \theta)^2 - \frac{\rho}{a} - m &= 0 \\ \Delta &= \left(\frac{-1}{a}\right)^2 + 4m(\cos \theta)^2 \\ \rho &= \frac{\frac{1}{a} \pm \sqrt{\Delta}}{2(\cos \theta)^2} \end{aligned}$$



type=circle + type=Fresnel



Mathematization

The radii of the Fresnel circles increase: $r_n = \sqrt{n}$.

The radii of the equidistant circles increase: $r_n = n \cdot a$, with $a > 0$.

The centers of the circles are placed at $(x_M, 0)$ and $(-x_M, 0)$, we have p and q as integers:

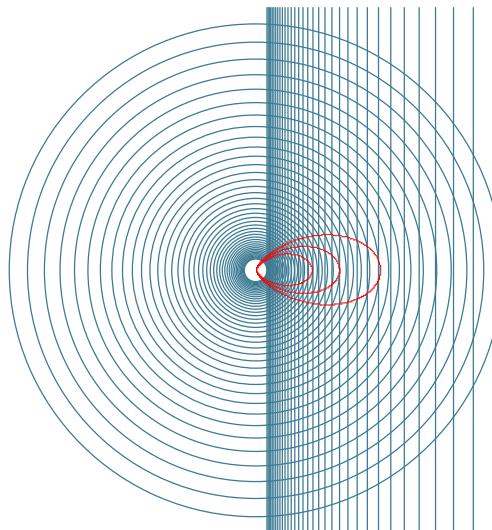
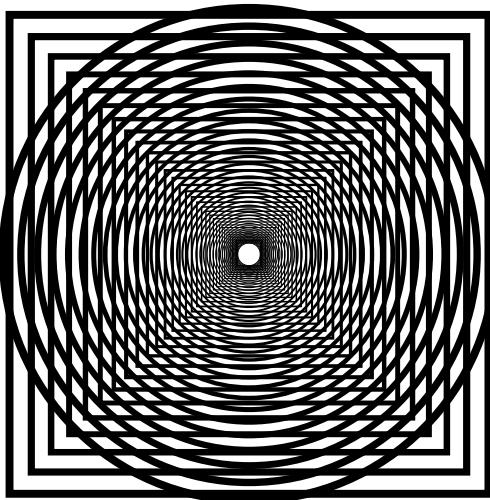
$$\begin{aligned} (x - x_M)^2 + y^2 &= p \\ (x + x_M)^2 + y^2 &= a^2 q^2 \\ p &= (x - x_M)^2 + y^2 \\ q &= \frac{1}{a} \sqrt{(x + x_M)^2 + y^2} \end{aligned}$$

One moiré curve line is determined by: $p - q = m \in \mathbb{Z}$, thus:

$$(x - x_M)^2 + y^2 - \frac{1}{a} \sqrt{(x + x_M)^2 + y^2} = m$$

which is the implicit equation of a moiré curve line with m .

Circles and Squares (both of increasing thickness)



Mathematization

The abscissa of the edges of the square with $x > 0$ increase with: $x_n = \frac{1}{4}a^n$, with $a > 1$.

The radii increase with: $r_n = \frac{1}{4}b^n$, with $b > 1$.

We have

$$\begin{aligned}x_p &= \frac{1}{4}a^p \\r_q^2 &= x^2 + y^2 \\x^2 + y^2 &= \frac{1}{16}b^{2q}\end{aligned}$$

If we consider the point determined by the intersection $p \cap q$, the next point will be $(p+1) \cap (q+1)$, the next one at $(p+2) \cap (q+2)$, etc., so that the difference between the indices remains constant. As a result, the moiré lines are characterized by the relation $p - q = m \in \mathbb{Z}$, m determines a moiré curve.

$$\begin{aligned}p &= \frac{\ln(4x)}{\ln a} \\q &= \frac{\ln[16(x^2 + y^2)]}{2 \ln b}\end{aligned}$$

and $p - q = m$

$$\ln(4x)^{\frac{1}{\ln a}} - \ln[16(x^2 + y^2)]^{\frac{1}{2 \ln b}} = m$$

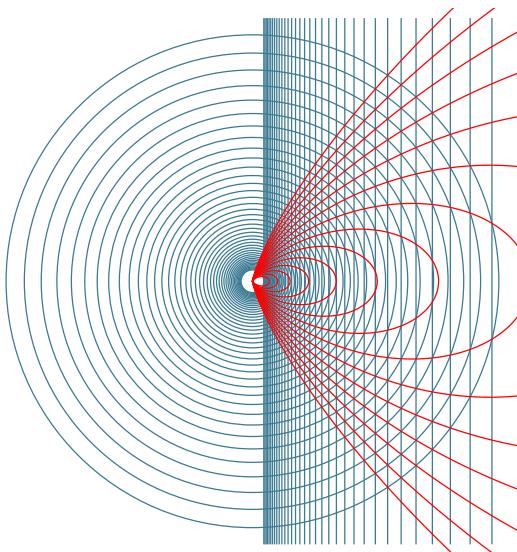
finally gives

$$\frac{(4x)^{\frac{1}{\ln a}}}{[16(x^2 + y^2)]^{\frac{1}{2 \ln b}}} = e^m$$

We transform this implicit equation into a polar equation by setting $\rho^2 = x^2 + y^2$ and $x = \rho \cos \theta$. Setting $\alpha = \frac{1}{\ln a}$ and $\beta = \frac{1}{2 \ln b}$. The equation becomes:

$$\begin{aligned}\frac{\rho^\alpha (\cos \theta)^\alpha \cdot 4^\alpha}{\rho^{2\beta} \cdot 4^{2\beta}} &= e^m \\\rho^{\alpha-2\beta} (\cos \theta)^\alpha &= e^m \cdot 4^{2\beta-\alpha} \\\rho &= \frac{1}{4} \left(\frac{e^m}{(\cos \theta)^\alpha} \right)^{\frac{1}{\alpha-2\beta}}\end{aligned}$$

We can trace some elements of this family of curves:



11. List of all optional arguments for *pst-moire*

Key	Type	Default
Rmax	ordinary	[none]
scale	ordinary	[none]
Alpha	ordinary	[none]
rotate	ordinary	[none]
E	ordinary	[none]
n	ordinary	[none]
T	ordinary	[none]
r	ordinary	[none]
p	ordinary	[none]
k	ordinary	[none]
symbole	ordinary	[none]
PSfont	ordinary	[none]
fontsize	ordinary	[none]
vadjust	ordinary	[none]
hadjust	ordinary	[none]
hole	ordinary	[none]
type	ordinary	[none]
function	ordinary	[none]
layers	boolean	true
NumberDots	ordinary	[none]
coefficients	ordinary	[none]
srand	ordinary	[none]
revlayer	boolean	true
layer	boolean	true

A. PostScript Font Names

Ghostscript

uagd8a	URWGothicL-Demi
uagdo8a	URWGothicL-DemiObli
uagk8a	URWGothicL-Book
uagko8a	URWGothicL-BookObli
ubkd8a	URWBookmanL-DemiBold
ubkdi8a	URWBookmanL-DemiBoldItal
ubkl8a	URWBookmanL-Ligh
ubkli8a	URWBookmanL-LighItal
ucrb8a	NimbusMonL-Bold
ucrbo8a	NimbusMonL-BoldObli
ucrr8a	NimbusMonL-Regu
ucrro8a	NimbusMonL-ReguObli
uhvb8a	NimbusSanL-Bold
uhvb8ac	NimbusSanL-BoldCond
uhvbo8a	NimbusSanL-BoldItalic
uhvbo8ac	NimbusSanL-BoldCondItalic
uhvr8a	NimbusSanL-Regu
uhvr8ac	NimbusSanL-ReguCond
uhvro8a	NimbusSanL-ReguItalic
uhvro8ac	NimbusSanL-ReguCondItalic
uncb8a	CenturySchL-Bold
uncbi8a	CenturySchL-BoldItalic
uncr8a	CenturySchL-Roma
uncri8a	CenturySchL-Ital
uplb8a	URWPalladioL-Bold
uplbi8a	URWPalladioL-BoldItalic
uplr8a	URWPalladioL-Roma
upli8a	URWPalladioL-Ital
usyr	StandardSymL
utmb8a	NimbusRomNo9L-Medi
utmbi8a	NimbusRomNo9L-MediItalic
utmri8a	NimbusRomNo9L-Regu
utmri8a	NimbusRomNo9L-ReguItalic
uzcmi8a	URWChanceryL-MediItalic
uzdr	Dingbats

Adobe Basic 14

Times	Times-Roman
	Times-Italic
	Times-Bold
	Times-BoldItalic
Helvetica	Helvetica
	Helvetica-Oblique
	Helvetica-Bold
	Helvetica-BoldOblique
Courier	Courier
	Courier-Oblique
	Courier-Bold
	Courier-BoldOblique
ZapfDingbats	ZapfDingbats
Symbol	Symbol