Date / /

A set of (m x n) numbers arranged in the form of an ordered set of m rows & n columns is called m x n matrix.

$$A = [a_{ij}]_{m\times n} \qquad A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ a_{31} & a_{32} & \cdots & a_{3n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mxn} \end{bmatrix}_{m\times n}$$

Types of Matrix

Sq^2 matrix = A matrix in which the number of rows is equal to the number of columns

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 5 \end{bmatrix}_{2\times 2} \qquad B = \begin{bmatrix} 1 & 3 & 6 \\ 2 & 4 & 4 \\ 0 & 5 & 2 \end{bmatrix}_{3\times 3}$$

Horizontal Matrix – Matrix in which number of columns are more than number of ~~columns~~ rows

$$A = \begin{array}{cc} & C \\ R & \begin{bmatrix} 4 & -1 & 6 \\ 2 & 0 & 8 \end{bmatrix} \end{array}$$

Vertical Matrix – Matrix in which no. of rows are more than no. of columns.

$$B = \begin{bmatrix} 4 & 3 \\ 3 & 2 \\ 2 & 1 \end{bmatrix}$$

Row matrix :- A matrix having only one row.

$$C = [0 \quad 1 \quad 0]$$

Column matrix :- A matrix having only one column

$$A = \begin{bmatrix} 5 \\ 7 \\ 9 \end{bmatrix}$$

Date / /

Diagonal Matrix :- A sq^2 matrix $A = [a_{ij}]_{m \times n}$ is called a diagonal matrix if $a_{ij} = 0$ for all $i \neq j$

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

diagonal elements denoted by diag $[d_1, d_2, \ldots, d_n]$

Scaler matrix :- A diagonal matrix in which all the diagonal elements are equal.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Identity or unit Matrix

A diagonal Matrix in which all diagonal elements are equal to one.

The identity matrix of order n is denoted by I_n.

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Null matrix :- all elements are zero :-

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Upper Δ matrix :- where $a_{ij} = 0$ $[a_{ij}]$ for all $i > j$

Lower Δ matrix :- where $[a_{ij}]$ for all $i < j$

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 4 \end{bmatrix} \qquad A_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 8 & 3 & 2 \end{bmatrix}$$

Date

Operation of matrix :

Equality :- Two matrix of same order are equal if their corresponding elements are equal.

Ques :- $\begin{bmatrix} 2x+4 & 3y-x \\ z+2 & x+z \end{bmatrix} = \begin{bmatrix} 0 & 5 \\ 3 & -1 \end{bmatrix}$

find $x+y+z$

$$2x + 4 = 0 \quad \text{--- } 1$$
$$x = -2$$
$$3y - (-2) = 5 \quad \text{--- } 2$$
$$y = 1$$
$$z + 2 = 3 \quad \text{--- } 3$$
$$z = 1$$
$$x + z = -1$$
$$-1 = -1$$
$$x + y + z = -2 + 1 + 1 = 0$$

Addition :- Addition of two matrix is possible only if they are of same order.

Subtraction :- ,,

Multiplication of Matrix with a Scaler

The scaler gets multiplied in every element of the matrix.

if $A = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}_{2\times 2}$

Then $2A = \begin{bmatrix} 2 & 8 \\ 4 & 2 \end{bmatrix}$

Multiplication of two matrix

Matrix multiplication is done Row x Column.

for two matrix $A_{m\times n}$ & $B_{p\times q}$, the multiplication $(A \times B)$ is possible only if number of Columns of A is equal to the number of rows of B i.e.; $n = p$

resultant of matrix C will be order $m \times q$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & -4 \\ 5 & 6 \end{bmatrix}_{3\times 2} \quad \& \quad B = \begin{bmatrix} 4 & 5 & 6 \\ 7 & -8 & 2 \end{bmatrix}_{2\times 3}$$

$A \cdot B = B \cdot A$

$$AB = \begin{bmatrix} 4+14 & 5-16 & 6+4 \\ 12-28 & 15+32 & 18-8 \\ 20+42 & 25-48 & 30+12 \end{bmatrix}$$

$$AB \begin{bmatrix} 18 & -11 & 10 \\ -16 & 47 & 10 \\ 62 & -23 & 42 \end{bmatrix}_{3\times 3}$$

$$B = \begin{bmatrix} 4 & 5 & 6 \\ 7 & -8 & 2 \end{bmatrix}_{2\times 3} \quad A = \begin{bmatrix} 1 & 2 \\ 3 & -4 \\ 5 & 6 \end{bmatrix}_{3\times 2}$$

$$B \cdot A \begin{bmatrix} 4+15+30 & 8-20+36 \\ 7-24+10 & 14+32+12 \end{bmatrix}$$

$$B \cdot A \begin{bmatrix} 49 & 24 \\ -7 & 58 \end{bmatrix}_{2\times 2}$$

Page No.

Date / /

$$AI_n = I_n A = A$$

$$\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$$

Ques Find value of x, if $[1 \; x \; 1] \underset{3\times3}{\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 2 & 5 \end{bmatrix}} \underset{3\times1}{\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}} = 0$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 2 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 - 4 + 9 \\ 4 - 10 + 18 \\ 3 - 4 + 15 \end{bmatrix} = \begin{bmatrix} 6 \\ 12 \\ 14 \end{bmatrix}$$

$$\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 12 \\ 14 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 12x & 14 \end{bmatrix}$$

$$6 + 12x + 14 = 0$$

$$6 + 12x = -14$$

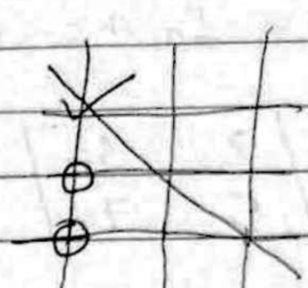
$$12x = -14 - 6$$

$$12x = -20$$

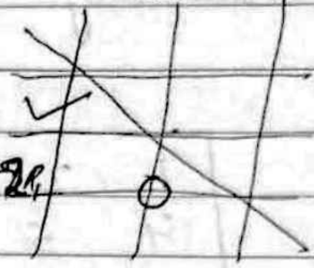
$$x = \frac{-20}{12} = \frac{5}{3}$$

$$x = -\frac{5}{3}$$

① Gauss elimination method :-
Rank of Matrix :-



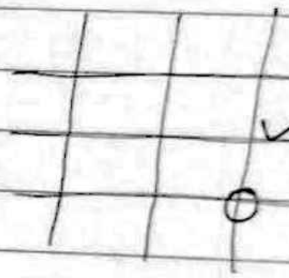
Ques 1 $A = \begin{bmatrix} 1 & 4 & 3 & 2 \\ 1 & 2 & 3 & 4 \\ 2 & 6 & 7 & 5 \end{bmatrix}$ $R_2 \to R_2 - R_1$



$\begin{bmatrix} 1 & 4 & 3 & 2 \\ 0 & -2 & 0 & 2 \\ 2 & 6 & 7 & 5 \end{bmatrix}$ $R_3 \to R_3 - 2R_1$

$\begin{bmatrix} 1 & 4 & 3 & 2 \\ 0 & -2 & 0 & 2 \\ 0 & -2 & -1 & 1 \end{bmatrix}$ $R_3 \to R_3 - R_2$

$\begin{bmatrix} 1 & 4 & 3 & 2 \\ 0 & -2 & 0 & 2 \\ 0 & 0 & -1 & -1 \end{bmatrix}$



Rank of matrix = 3

Ques 2 $\begin{bmatrix} 2 & 3 & 6 \\ 4 & 2 & 8 \\ 4 & 6 & 10 \end{bmatrix}$ $R_2 \to R_2 - 2R_1$

$\begin{bmatrix} 2 & 3 & 6 \\ 0 & -4 & -4 \\ 4 & 6 & 10 \end{bmatrix}$ $R_3 \to R_3 - 2R_1$

$\begin{bmatrix} 2 & 3 & 6 \\ 0 & -4 & -4 \\ 0 & 0 & -2 \end{bmatrix}$ $R_3 \to 2R_3 - R_2$

Page No.

Date / /

$$\begin{bmatrix} 2 & 3 & 6 \\ 0 & -4 & -4 \\ 0 & 0 & 0 \end{bmatrix}$$

Rank of matrix = 2