Operation Research

→ History
→ Meaning & Definition
→ features / characteristics
→ Model of OR (Phases of OR)
→ Decision Making in OR
→ Limitation of OR

History of OR

British Army (World war II) → US Army
↓ lack of limited resources ↓ L.P.P
↓
Group of Scientist
↓
Scientific Approach
↓
Win the particular battle

1970-80 → Business, house, Agriculture

Meaning & Defination

Limited Resources $\xrightarrow[\text{way}]{\text{scientific}}$ Man made System best design & operation

operation of a System (with optimum solution → Scientific tools & techniques

Research in Operation

Art of wining war without fighting.

Operation research (OR) is an analytical method of problem solving & decision-making that useful in the management of organizations. In operation research, we broke problem into basic Component & then Solve it (M-A)

features / characteristics

Use of Interdisciplinary team

Complete System orientation

Scientific Methods

Quality in Decision

Scope of OR

Military | Planning | Agriculture | Corporation | Transportation

Limitations of OR

Lack of Qualitative factors

Limited factor Consideration

Specific Category of problems

Resistance of Employees

classmate
Date
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Model

Definition - A model is an ideal representation of a real life System. System can be a problem, process, operation, object or event.

Examples - photograph, Roadmap, accounting statement like profit & loss account & balance sheet are all models since they partially represents the reality

Types of model

1 Classifications based on function

i) Normative Models :- These models provide the best Solution to problems subjects to certain limitations. These models are also called optimizing models or prescriptive models because they prescribe what have to be done.

Ex :- Linear programming, X-ray of healthy man, C.P.M & PERT planning model.

ii) Predictive Model :- These models predict the outcomes regarding certain events due to a given set of alternatives for the problem. They can answer " what is type of questions"

Ex :- Television network predict this election outcome before counting the votes based on the survey result.

Descriptive model :- These models describe the system under study based on observation, survey, questionnaire results.

Ex:- Organization chart, Plant layout diagram, Scale models etc.

2 Classification of Structure

i) Iconic Models ii) Analogue Models iii) Symbolic Mode

i) Iconic Model :- Iconic Model is a physical or pictorial or visual representation of the real system They are scaled up or Scaled down versions of the particular system they represent.

Examples :- Model / Blue prints of proposed building, models of Sun & planets are scaled down & model of atom, models of Cells in human body are scaled up.

ii) Analogue Models :- These models represent a System by a set of properties which is different from the original System & the does not resemble it physically.

Ex. A barometer that indicates change in atmospheric pressure through movement of a needle. Graphs, flow diagrams, Charts etc.

iii) Symbolic model (Mathematical Model) - In these model the various components of the system and their inter - relationship are denoted by letters, numbers & other types of Symbols.

Ex:- Queuing model, Inventory model, Linear programming Models etc.

3. Classification based on Nature of Environment

(i) Deterministic Model :- In these model all parameters & functional relationship are assumed to be Known with certainty when decision is to be made.

Ex:- Linear programming, Transportation, Assignment Models.

(ii) Probabilistic Model / Stochastic model - These type of model usually handle such situations in which outcome of managerial actions can not be predicted with certainty.

Ex:- Insurance companies are willing to ensure aganist risk of fire, accidents, sickness.

(i) Principle of Modelling

choose your model well
the Choice of model profoundly impacts the analysis of the problem & the design of the solution.

2 Every model may be expressed at different levels of precision
→ The same model can be scaled up (or down) to different granularities.

③ the best models are connected to reality

Simplify the model, but don't hide important details.

Linear Programming Problem

linear programming is a method to achieve the best outcome in a mathematical model whose requirement are represented by linear relationships. linear programming is special case of mathematical programming.

linear programming is a process of optimising the problem which are subjected to certain constraints. It means that it is the process of maximising or minimizing the linear functions under linear inequality constraints. The problem of solving linear programs is considered as the easiest one.

Main Component L.P.P

Decision Variable

Objective function

Subjective Condition

Convex Set

2-D

3-D

$C_1 x_1 + C_2 x_2 = K$

Value of x_1 & x_2 = ~~Convex~~ Hyperplane

$C_1 x_1 + C_2 x_2 + C_3 x_3 = K$

Value of x_1, x_2 & x_3 = Hyperplane

Hyperplane

In R^n (i.e n-dimensional space) the set of points $x = (x_1, x_2, \dots x_n)$ Satisfying the equ^n $C_1 x_1 + C_2 x_2 + \dots + C_n x_n = k$ (not all $C_i = 0$,) is called a hyper plane for given value of C's

If $K = 0$ then the hyper plane is passes through the origin

In matrix notation the eq^n of the hyperplane can be written as $CX = K$

where C is row vector

$$C = [C_1, C_2, \dots, C_n]$$

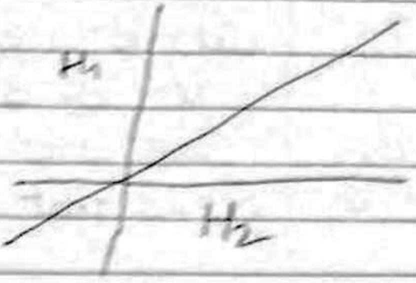
X is column vector $= \begin{bmatrix} x_1 \\ x_2 \\ x_n \end{bmatrix}$

and = $K = (K)$

If a hyperplane divides R^n into two half space which we denoted by

$H_1 = \{ x \mid cx \geq k \}$

$H_2 = \{ x \mid cx \leq k \}$



Defination of Convex Set

Let $S \in R^n$, If for every two point $x_2 \in S$ the line Segment joining x_1 & x_2 is connected in the set S, then S is called convex set.

we can express x_1 & x_2 as

$x_1 \{ x_1^1, x_2^1, x_3^1, \ldots, x_n^1 \}$ & $x_2 =$
$\{ x_1^{(2)}, x_2^2, x_3^2, x_n^2 \}$

Here each of the points x_1 & x_2 has n co-ordinate

$S \in R^2$

S has a two points P_1 & P_2

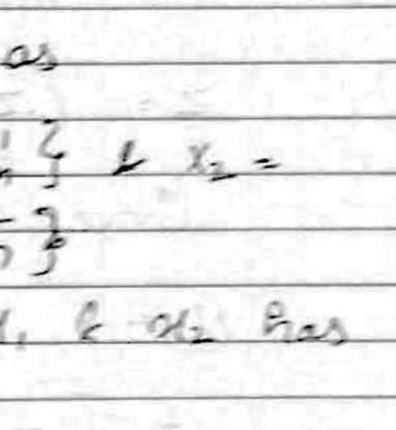
for point

$V = \lambda p_2 (1-\lambda) P_1$

Here,

$V \in S$

S is a Convex set



the line Segment $v = \lambda x_2 + (1-\lambda) x_1$ where $0 \le \lambda \le 1$ lies in S that is $v \in S$

Q Show that the given Set

① $X = \{(x_1, x_2) : x_1 \le 5, x_2 \ge 3\}$ is a Convex set.

Let $x = (x_1, x_2)$ & $y = (y_1, y_2) \in X$
i.e $x, y \in X$

$\therefore x_1 \le 5, x_2 \ge 3$ & $y_1 \le 5$ $y_2 \ge 3$

$Z = (Z_1, Z_2) = \lambda x + (1-\lambda) y$ is a Convex combination of x & y

Now

$Z_1, Z_2 = \lambda (x_1, x_2) + (1-\lambda) y_1, y_2$

$Z_1 = \lambda x_1 + (1-\lambda) y_1$

and $Z_2 = \lambda x_2 + (1-\lambda) y_2$

Now

$Z_1 = \lambda x_1 + (1-\lambda) y_1$
$\le \lambda \cancel{5} + 5 - \cancel{\lambda 5}$

$Z_1 \le 5$

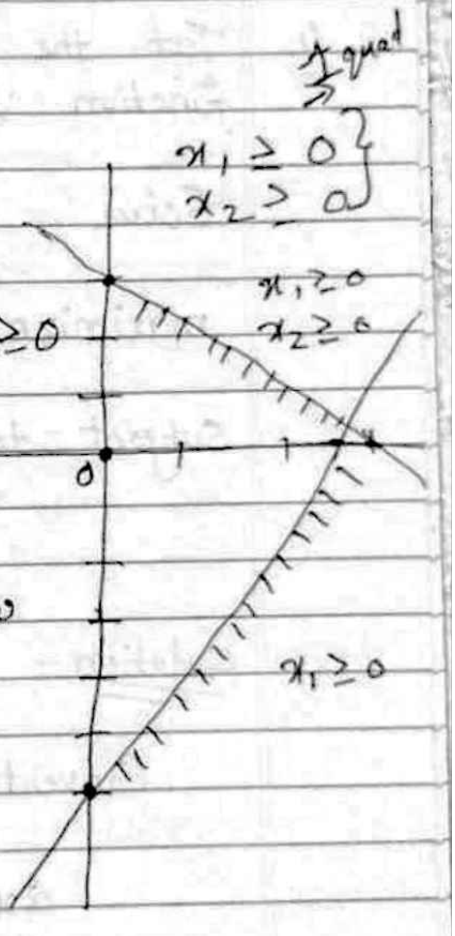
$Z_2 = \lambda x_2 + (1-\lambda) y_2$

$\ge \lambda 3 + 3 - 3\lambda$

$Z_2 \ge 3$

$Z_1 \le 5, \quad Z_2 \ge 3$ is a Convex set

Graphical Solution of L.P.P



$x + y \le 3$

x	0	3
y	3	0

$0 \le 3$

(0,3) (3,0)

it always use to show the reigon
if Condition is true points show the origin

$x_1 - x_2 \le 5$
$2x_1 + 5x_2 \ge 2$
$2x_1 - x_2 \ge 5$
$2x_1 - x_2 = 5$

$x_1 = 0$		$5/2$
$x_2 = -5$		0

if we equate by 0 we get $0 \ge 5$ is false means the reigon shows opposite to the origin working rule of G.M

① Convert all Constraints into equations.

2) plot the graph of equation & apply the condition given in constraint.

3 Identify feasible reigon (that satisfy all Constraints) find its Corners. (extreme points)

4 Test the maximum / minimum value of objective function in there corners.

Solve L.P.P by Graphical method

maximize $Z = 50x_1 + 18x_2$

Subject to constraints : $2x_1 + x_2 \le 100$
$x_1 + x_2 \le 80$

and $x_1, x_2 \ge 0$

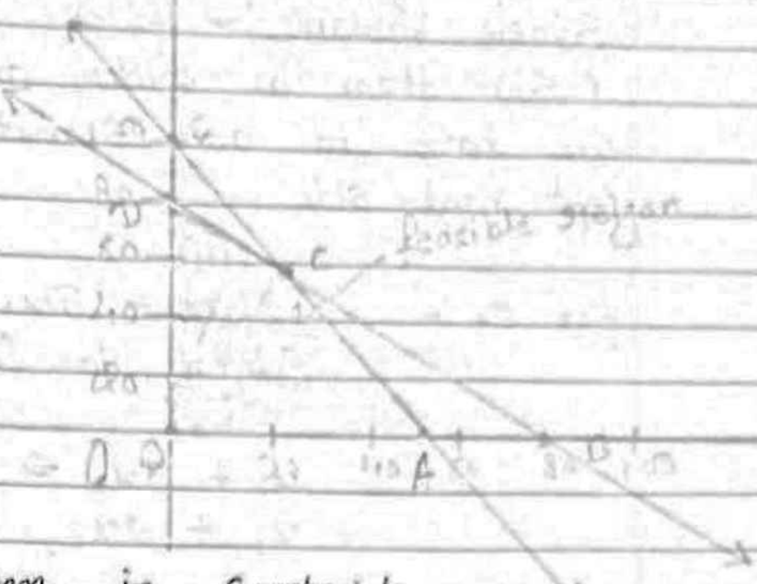
Solution

Convert all Constraints into equation

$2x_1 + x_2 = 100$ —— ①
$x_1 + x_2 = 80$ —— ⑪

points of line ① (by putting 0 to both variable)
$(0, 100)$, $(50, 0)$

points on line ② $(0, 80)$, $(80, 0)$



$0 \leq 100$
$0 \leq 80$
points shows to the origin

Apply Condition given in Constraints we get feasible reigon

O A C D O

getting point of C

$$
\begin{array}{r}
2x_1 + x_2 = 100 \\
-x_1 \mp x_2 = -80 \\
\hline
x_1 = 20 \\
x_2 = 60
\end{array}
\quad (20, 60)
$$

Test of optimality

Corners	$Z = 50x_1 + 18x_2$
O (0,0)	$Z = 0$
A (50,0)	$Z = 50 \times 50 + 0 = 2500$
C (20,60)	$Z = 50 \times 20 + 18 \times 60 = 2080$
D (0,80)	$Z = 50 \times 0 + 18 \times 80 = 1440$

Maximize value of $Z = 2500$ optimum Solⁿ. $x_1 = 50,\ x_2 = 0$

Slack Variable :- if a Constraints has a sign ($\leq$) then in order to make it an equality we have to add Something positive to the left hand side.

for Ex :- $Max z = 5x_1 + 8x_2$

Subject to Constraints

$$x_1 + 3x_2 \leq 5$$
$$x_1 + 2x_2 \leq 2$$
$$\text{and} \quad x_1 , x_2 \geq 0$$

The positive variables which are added to the left hand sides to the constraints to convert them into equalities are called slack variables.

Surplus Variable :- if a constraints has a sign ($\geq$) then in order to make it a equality we have to Subtract some thing positive to the left hand side.

for ex :- $Max z = x_1 + x_2$

Subject to Constraints

$$2x_1 + x_2 \geq 4$$
$$x_1 + 7x_2 \geq 7$$
$$\& \quad x_1 , x_2 \geq 0$$

The positive variable which are substracted to the left hand side to the constraints to Convert them into equalities are called Surplus variable.

Use of Slack variable $\longrightarrow$ also called basic Variables

Now, using S.V

$$s_1 \geq 0 \ , \ s_2 \geq 0$$

$$Max z = 5x_1 + 8x_2 + 0s_1 + 0s_2$$

subject to constraint

$$x_1 + 3x_2 + s_1 + 0s_2 = 5$$
$$x_1 + 2x_2 + 0s_1 + s_2 = 2$$
$$\& \ x_1, x_2, s_1, s_2 \geq 0$$

Use of Surplus variable

Now, using Surplus variable

$$s_1 \geq 0 \ , \ s_2 \geq 0$$

$$Max z = x_1 + x_2 + 0s_1 + 0s_2 - Ma_1 - Ma_2$$

subject to Constraints

$$2x_1 + x_2 - s_1 + 0s_2 + a_1 + 0a_2 = 4$$
$$x_1 + 7x_2 + 0s_1 - s_2 + 0a_1 + a_2 = 7$$

$$\& \ x_1, x_2, s_1, s_2, a_1, a_2 \geq 0$$

Matrix form of L.P.P

Find all basic Sol^n of the following system

$x_1 + 2x_2 + x_3 = 4$
$2x_1 + x_2 + 5x_3 = 5$

Sol^n $\rightarrow$ Coefficient Matrix Matrix

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$
$\alpha_1 \quad \alpha_2 \quad \alpha_3$

$$A \quad X = B$$

n = no. of variable = 3
m = " " eq^n = 2

$$Sol^n = {}^nC_m = {}^3C_2 = \frac{\lfloor 3}{\lfloor 2 \, \lfloor 3-2} = 3$$

So, here is three basic solution

$$B_1 = [\alpha_1 \ \alpha_2] = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

$$B_2 = [\alpha_1 \ \alpha_3] = \begin{bmatrix} 1 & 1 \\ 2 & 5 \end{bmatrix}$$

$$\beta_3 = [\alpha_2 \ \alpha_3] = \begin{bmatrix} 2 & 1 \\ 1 & 5 \end{bmatrix}$$

$$|B_1| = \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} = 1 - 4 = -3 \neq 0$$

$$|B_2| = \begin{vmatrix} 1 & 1 \\ 2 & 5 \end{vmatrix} = 5 - 2 = 3 \neq 0$$

$$|B_3| = \begin{vmatrix} 2 & 1 \\ 1 & 5 \end{vmatrix} = 10 - 1 = 9 \neq 0$$

Hence all the three sets of two variables of A is L. I.

Hence all the three basic solution exist.

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x_{B_1} = B_1^{-1} \cdot b \quad \text{——— (1)}$$

$$B_1^{-1} = \frac{1}{|B_1|} \text{ adj } B_1$$

$$\text{Coefficient of } B_1 = \begin{bmatrix} +1 & -2 \\ -2 & +1 \end{bmatrix}$$

adj B_1 = Transpose of C

$$\text{adj } B_1 = \begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix}$$ Put in eqn (1)

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \frac{1}{-3} \begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$$= \frac{1}{-3} \begin{bmatrix} 4 - 10 \\ -8 + 5 \end{bmatrix} = \frac{1}{-3} \begin{bmatrix} -6 \\ -3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_3 \end{bmatrix} = x_{B_2} = B_2^{-1} \cdot b = \frac{1}{3} \begin{bmatrix} 5 & -1 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$$\frac{1}{3} \begin{bmatrix} 20 - 5 \\ -8 + 5 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 15 \\ -3 \end{bmatrix} = \begin{bmatrix} 5 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} x_2 \\ x_3 \end{bmatrix} = x B_3 = B_3^{-1} \cdot b = \frac{1}{|B_3|} \operatorname{adj}(B_3) \cdot b$$

$$= \frac{1}{9} \begin{bmatrix} 5 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$$= \frac{1}{9} \begin{bmatrix} 20 - 5 \\ -4 + 10 \end{bmatrix}$$

$$= \frac{1}{9} \begin{bmatrix} 15 \\ 6 \end{bmatrix} = \begin{bmatrix} 5/3 \\ 2/3 \end{bmatrix}$$

$$\begin{bmatrix} x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5/3 \\ 2/3 \end{bmatrix}$$

Basic solutions are $(2, 1, 0)$ $(5, 0, -1)$ & $(0, 5/3, 2/3)$