Differentiation

formula

1) $\frac{d}{dx}(x^n) = nx^{n-1}$

2) $\frac{d}{dx} e^x = e^x$

3 $\frac{d}{dx} a^x = a^x \log a$

4) $\frac{d}{dx} \sin x = \cos x$

5) $\frac{d}{dx} \cos x = -\sin x$

6) $\frac{d}{dx} \tan x = \sec^2 x$

7 $\frac{d}{dx} \cot x = -\operatorname{cosec}^2 x$

8) $\frac{d}{dx} \sec x = \sec x \cdot \tan x$

9) $\frac{d}{dx} \operatorname{cosec} x = -\operatorname{cosec} x \cot x$

10 $\frac{d}{dx} \log x = \frac{1}{x}$

Ques $\frac{d}{dx} 2 \sin x = 2 \frac{d}{dx} \sin x = 2 \cos x$

Sum rule

$$\frac{d}{dx}\left(f(x)+g(x)\right)$$

$$\frac{d}{dx} f(x) + \frac{d}{dx} g(x)$$

Ques Ex $\frac{d}{dx}(\sin x + \cos x)$

$$\frac{d}{dx}\sin x + \frac{d}{dx}\cos x$$

$$\cos x - \sin x$$

Ques $\frac{d}{dx}\left(2\cos x - 3\sin x + 4\tan x + 7e^x + 3\sqrt{x}\right)$

$$2\frac{d}{dx}\cos x - 3\frac{d}{dx}\sin x + 4\frac{d}{dx}\tan x + 7\frac{d}{dx}e^x + 3\frac{d}{dx}\sqrt{x}$$

$$-2\sin x - 3\cos x + 4\sec^2 x + 7e^x + \frac{3}{2\sqrt{x}}$$

$$\frac{d}{dx}\sqrt{x} = x^{1/2} = \frac{1}{2}x^{1/2-1} = \frac{1}{2}x^{-1/2} = \frac{1}{2x^{1/2}}$$

$$\sqrt{x} = \frac{1}{2\sqrt{x}}$$

$$\frac{d}{dx}\frac{1}{x} = \frac{-1}{x^2}$$

Product rule

$$\frac{d}{dx}\left[f(x)\cdot g(x)\right]$$

$$f(x)\frac{d}{dx}g(x) + g(x)\frac{d}{dx}f(x)$$

Ques $\frac{d}{dx} \sqrt{x} \sin x$

$\sqrt{x} \frac{d}{dx} \sin x + \sin x \frac{d}{dx} \sqrt{x}$

$\sqrt{x} \cos x + \frac{\sin x}{2\sqrt{x}}$

Ques $\frac{d}{dx} 5 e^x \tan x$

$5 \frac{d}{dx} e^x \tan x$

$5 \left[e^x \frac{d}{dx} \tan x + \tan x \frac{d}{dx} e^x \right]$

$5 \left[e^x \sec^2 x + \tan x \, e^x \right]$

$5 e^x \left[\sec^2 x + \tan x \right]$

Ques $\frac{d}{dx} e^x \sin x \log x$

$\sin x \log x \frac{d}{dx} e^x + e^x \log x \frac{d}{dx} \sin x + e^x \sin x \frac{d}{dx} \log x$

$\sin x \log x \, e^x + e^x \log x \cos x + e^x \sin x \frac{1}{x}$

Quotient rule

$\frac{d}{dx} \frac{f(x)}{g(x)}$

$\frac{g(x) \frac{d}{dx} f(x) - f(x) \frac{d}{dx} g(x)}{[g(x)]^2}$

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Ques $\frac{d}{dx} \frac{(x + \cos x)}{\tan x}$

$$\frac{\tan x \frac{d}{dx}(x + \cos x) - (x + \cos x) \frac{d}{dx} \tan x}{(\tan x)^2}$$

$$\frac{\tan x (1 - \sin x) - (x + \cos x) \sec^2 x}{\tan^2 x}$$

Function of Function
chain rule

Ques $\frac{d}{dx} \sin(x+5)$

$\cos(x+5) \frac{d}{dx}(x+5)$

$\cos(x+5)\ (1+0)$

$\cos(x+5)$

Ques $\frac{d}{dx} \tan(x^2 + 2x + 6)$

$\sec^2(x^2+2x+6) \frac{d}{dx}(x^2+2x+6)$

$\sec^2 x (x^2+2x+6)\ (2x+5)$

Ques e^{3x}

$e^{3x} \frac{d}{dx} 3x = 3e^{3x}$

Ques $\frac{d}{dx} \sin^5 x$

$5 \sin^4 x \frac{d}{dx} \sin x = 5 \sin^4 x \cos x$

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Ques $\frac{d}{dx} \tan^7 x$

$7\tan^6 x \frac{d}{dx} \tan x = 7\tan^6 x \sec^2 x$

Ques $(3x+5)^6$

$6(3x+5)^5 \frac{d}{dx}(3x+5)$

$= 6(3x+5)^5 (3+0)$

$= 18(3x+5)$

Ques $\log 5x$

$\frac{1}{5x} \frac{d}{dx} 5x = \cancel{5} \cdot \frac{1}{\cancel{5}x} = \frac{1}{x}$

Ques $\frac{d}{dx}\sqrt{\sin x}$

$\frac{1}{2\sqrt{\sin x}} \frac{d}{dx} \sin x = \frac{1}{2\sqrt{\sin x}} \cos x + a \frac{d}{dx}(x+a)$

Ques $\sec(\tan(\sqrt{x}))$

$\sec(\tan(\sqrt{5})) \cdot \tan(\tan(\sqrt{x})) \frac{d}{dx}(\tan(\sqrt{x}))$

" " $(\sec^2(\sqrt{x})) \frac{d}{dx}\sqrt{x}$

" " " $\frac{1}{2\sqrt{x}}$

$\sec(\tan(\sqrt{5})) \cdot \tan(\tan(\sqrt{5})) \frac{(\sec^2(\sqrt{x}))}{2\sqrt{x}}$

Ques $\cos x^3 \cdot \sin^2(x^3)$

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$\frac{d}{dx} \cos x^3 \cdot \sin^2(x^5)$

$\cos x^3 \frac{d}{dx} \sin^2(x^5) + \sin^2(x^5) \frac{d}{dx} \cos x^3$

$\cos x^3 \; 2\sin(x^5) \frac{d}{dx} \sin(x^5) - \sin^2(x^5) \sin x^3 \frac{d}{dx} x^3$

$\cos x^3 \; 2\sin(x^5) \cos(x^5) \frac{d}{dx}(x^5) - \sin^2(x^5) \sin x^3 \; 3x^2$

$\cos x^3 \; 2\sin(x^5) \cos(x^5) \; 5x^4 - \sin^2(x^5) \sin x^3 \; 3x^2$

Implicit function

with respect to (x, y)

Ques $x^2 + e^{x-y} + 2y = 3$

$\frac{d}{dx} x^2 + \frac{d}{dx} e^{x-y} + \frac{d}{dx} 2y = \frac{d}{dx} 3$

$2x + e^{x-y} \frac{d}{dx}(x-y) + 2 \frac{d}{dx} y = 0$

$2x + e^{x-y}\left(1 - \frac{dy}{dx}\right) + 2\frac{dy}{dx}$

$2x + e^{x-y} - e^{x-y}\frac{dy}{dx} + 2\frac{dy}{dx}$

$2x + e^{x-y} = e^{x-y}\frac{dy}{dx} + 2\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{2x + e^{x-y}}{e^{x-y} + 2}$

Ques $x^2 + 2xy + y^3 = 42$

$$\frac{d}{dx} x^2 + 2\frac{d}{dx}(x \cdot y) + \frac{d}{dx} y^3 = \frac{d}{dx} 42$$

$$2x + 2\left(x \frac{d}{dx} y + y \frac{d}{dx} x\right) + 3y^2 \frac{d}{dx} y = 0$$

$$2x + 2\left(x \frac{dy}{dx} + y\right) + 3y^2 \frac{dy}{dx} = 0$$

$$2x + 2x\frac{dy}{dx} + 2y + 3y^2 \frac{dy}{dx}$$

$$\frac{dy}{dx}(2x + 3y^2) = -2x - 2y$$

$$\frac{dy}{dx} = \frac{-2(x+y)}{(2x+3y^2)}$$

Ques $\sin xy + \cos(x+y) = 1$

$$\frac{d}{dx} \sin xy + \frac{d}{dx} \cos(x+y) = \frac{d}{dx} 1$$

$$\cos xy \frac{d}{dx}(x \cdot y) - \sin(x+y) \frac{d}{dx}(x+y)$$

$$\cos xy \left(x \frac{d}{dx} y + y \frac{d}{dx} x\right) - \sin(x+y)\left(1 + \frac{dy}{dx}\right)$$

$$\cos xy \left(x \frac{dy}{dx} + y\right) - \sin(x+y)\left(1 + \frac{dy}{dx}\right)$$

$$x \cos xy \frac{dy}{dx} + y \cos xy - \sin(x+y) - \sin(x+y)\frac{dy}{dx}$$

$$\frac{dy}{dx}\left(x \cos xy - \sin(x+y)\right) = \sin(x+y) - y\cos xy$$

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$$\frac{dy}{dx} = \frac{\sin(x+y) - y\cos xy}{[x\cos xy - \sin(x+y)]}$$

Q4 $x\sqrt{1+y} + y\sqrt{1+x} = 0$ prove $\frac{dy}{dx} = \frac{-1}{(1+x)^2}$

$$x\sqrt{1+y} = -y\sqrt{1+x}$$

Sq^2 both sides

$$x^2(1+y) = y^2(1+x)$$

$$x^2 - y^2 = xy^2 - x^2y$$

$$(x-y)(x+y) = -xy(y-x)$$

$$(x+y) = -xy$$

$$y + xy = -x$$

$$y = \frac{-x}{1+x}$$

$$\frac{dy}{dx} = \frac{(1+x)\frac{d}{dx}(-x) - (-x)\frac{d}{dx}(1+x)}{(1+x)^2}$$

$$\frac{dy}{dx} = \frac{-1-x+x}{(1+x)^2} = \frac{-1}{(1+x)^2}$$

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Trigonometric Substitution

$\sin 2\theta = 2\sin\theta\cos\theta = \dfrac{2\tan\theta}{1+\tan^2\theta}$

$\cos 2\theta = 2\cos^2\theta - 1$

$= 1 - 2\sin^2\theta$

$= \cos^2\theta - \sin^2\theta$

$= \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$

$\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

$\sin 4\theta = 2\sin 2\theta\cos 2\theta$

$\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

Inverse formula

$\dfrac{d}{dx}\sin^{-1}x = \dfrac{1}{\sqrt{1-x^2}}$

$\dfrac{d}{dx}\cos^{-1}x = \dfrac{-1}{\sqrt{1-x^2}}$

$\dfrac{d}{dx}\tan^{-1}x = \dfrac{1}{1+x^2}$

$\dfrac{d}{dx}\cot^{-1}x = \dfrac{-1}{1+x^2}$

$\dfrac{d}{dx}\sec^{-1}x = \dfrac{1}{|x|\sqrt{x^2-1}}$

$\dfrac{d}{dx}\operatorname{cosec}^{-1}x = \dfrac{-1}{|x|\sqrt{x^2-1}}$

Ques $y = \cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)$

$y = \cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)$

Let $x = \tan\theta$

$\tan^{-1}x = \theta$

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$$y = \cos^{-1}\left[\frac{1-\tan^2\theta}{1+\tan^2\theta}\right]$$

$$y = \cos^{-1}(\cos 2\theta)$$

$$y = 2\theta$$

diff. with respect to x

$$\frac{dy}{dx} = 2\frac{d}{dx}\tan^{-1}x$$

$$\frac{dy}{dx} = \frac{2}{1+x^2}$$

2) $\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

Let $x = \tan\theta$, $\tan^{-1}x = \theta$

$$y = \sin^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$$

$$y = \sin^{-1}(\cos 2\theta) \rightarrow \cos\theta = \sin(90-\theta)$$

$$y = \sin^{-1}\left(\sin\frac{\pi}{2} - 2\theta\right)$$

$$y = \frac{\pi}{2} - 2\theta$$

differenciate with respect to x

$$\frac{d}{dx}y = \frac{d}{dx}\frac{\pi}{2} - 2\cdot\frac{d}{dx}\tan^{-1}x$$

$$\frac{dy}{dx} = 0 - 2\frac{1}{1+x^2}$$

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$$\frac{dy}{dx} = \frac{-2}{1+x^2}$$

3 $\cos^{-1}\left[2x\sqrt{1-x^2}\right]$

Let $x = \sin\theta$ or $\sin^{-1} x = \theta$

$y = \cos^{-1}\left[2\sin\theta\sqrt{1-\sin^2\theta}\right]$

$y = \cos^{-1}\left[2\sin\theta\cos\theta\right]$

$y = \cos^{-1}\left[\sin 2\theta\right]$

$y = \cos^{-1}\left(\cos\left(\frac{\pi}{2} - 2\theta\right)\right)$

$y = \frac{\pi}{2} - 2\theta$

$y = \frac{\pi}{2} - 2\sin^{-1} x$

diff. with respect to x

$$\frac{dy}{dx} = 0 - \frac{2 \cdot 1}{\sqrt{1-x^2}}$$

$$\frac{dy}{dx} = \frac{-2}{\sqrt{1-x^2}}$$

4 $\tan^{-1}\left[\frac{\sqrt{1+x^2}-1}{x}\right]$

$x = \tan\theta$

$\tan^{-1} x = \theta$

$\tan^{-1}\left[\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right]$

$\tan^{-1}\left[\frac{\sec-1}{\tan\theta}\right]$

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$$\tan^{-1}\left[\frac{\frac{1}{\cos\theta}-1}{\frac{\sin\theta}{\cos\theta}}\right]$$

$$\tan^{-1}\left[\frac{\frac{1-\cos\theta}{\cancel{\cos\theta}}}{\frac{\sin\theta}{\cancel{\cos\theta}}}\right]$$

$$\tan^{-1}\left[\frac{1-\cos\theta}{\sin\theta}\right]$$

$$\tan^{-1}\left[\frac{\cancel{2}\sin^2\theta/2}{\cancel{2}\sin\theta/2\ \cos\theta/2}\right]$$

$$\tan^{-1}\left[\frac{\sin\theta/2}{\cos\theta/2}\right]$$

$$\tan^{-1}\left[\tan\theta/2\right]$$

$$y = \theta/2$$

$$y = \frac{1}{2}\tan^{-1}x$$

$$\frac{dy}{dx} = \frac{1}{2}\cdot\frac{1}{(1+x^2)}$$

Log Differenciation

$$\log m + \log n = \log mn$$
$$\log m - \log n = \log\frac{m}{n}$$

$\log m^n = n \log m$

(1) $x^{\sin x}$

Taking log on both the sides

$y = x^{\sin x}$

$\log y = \log x^{\sin x}$

$\log y = \sin x \log x$

differenciating with respect to x

$\frac{1}{y} \frac{dy}{dx} = \sin x \frac{1}{x} + \log x \cos x$

$\frac{1}{y} \frac{dy}{dx} = \left[\frac{\sin x}{x} + \cos x \cdot \log x\right]$

$\frac{dy}{dx} = y\left[\frac{\sin x}{x} + \cos x \log x\right]$

$\frac{dy}{dx} = x^{\sin x}\left[\frac{\sin x}{x} + \cos x \log x\right]$

$(\sin x)^{\log x}$

$y = (\sin x)^{\log x}$

Taking log on both the Sides

$\log y = \log (\sin x)^{\log x}$

$\log y = \log x \cdot \log(\sin x)$

diff: with respect to x

$\frac{1}{y} \cdot \frac{dy}{dx} = \log \cdot \frac{1}{\sin x} \cdot \cos x + \log(\sin x) \frac{1}{x}$

$$\frac{1}{y}\frac{dy}{dx} = \log\frac{\cos x}{\sin x} + \frac{\log(\sin x)}{x}$$

$$\frac{dy}{dx} = y\left[\log \cot x + \frac{\log(\sin x)}{x}\right]$$

$$\frac{dy}{dx} = (\sin x)^{\log x}\left[\log \cdot \cot x + \frac{\log(\sin x)}{x}\right]$$

<u>Ques</u> $\cos x \cdot \cos 2x \cdot \cos 3x$

$$y = \cos x \cos 2x \cdot \cos 3x$$

Taking log both the Sides

$$\log y = \log(\cos x \cdot \cos 2x \cdot \cos 3x)$$

$$\log y = \log(\cos x) + \log(\cos 2x) + \log(\cos 3x)$$

diff. w. r. t. x.

$$\frac{1}{y}\frac{dy}{dx} = \left[\frac{1}{\cos x} \cdot -\sin x - \frac{2\sin 2x}{\cos 2x} - \frac{3\sin 3x}{\cos 3x}\right]$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = -\left[\tan x + 2\tan 2x + 3\tan 3x\right]$$

$$\frac{dy}{dx} = -y\left[\tan x + 2\tan 2x + 3\tan 3x\right]$$

$$\frac{dy}{dx} = -(\cos x \cdot \cos 2x \cdot \cos 3x)\left[\tan x + 2\tan 2x + 3\tan 3x\right]$$

Ques $\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$ 1)

$$y = \left[\frac{(x-1)\ (x-2)}{(x-3)\ (x-4)\ (x-5)}\right]^{1/2}$$

Taking log on both the Sides

$$\log y = \log\left[\frac{(x-1)\ (x-2)}{(x-3)\ (x-4)\ (x-5)}\right]^{1/2}$$

$$\log y = \frac{1}{2}\log\left[\frac{(x-1)\ (x-2)}{(x-3)\ (x-4)\ (x-5)}\right]$$

$$\log y = \frac{1}{2}\ \log(x-1)(x-2) - \log(x-3)(x-4)(x-5)$$

$$\log y = \frac{1}{2}\ \log(x-1) + \log(x-2) - \log(x-3) + \log(x-4) + \log(x-5)$$

$$\log y = \frac{1}{2}\ \log(x-1) + \log(x-2) - \log(x-3) - \log(x-4) - \log(x-5)$$

diff w. respect to x

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2}\left[\frac{1}{x-1} + \frac{\cancel{\log}1}{x-2} - \frac{1}{x-3} - \frac{1}{x-4} - \frac{1}{x-5}\right)$$

$$\frac{dy}{dx} = \frac{1}{2}\left(\sqrt{\frac{(x-1)(x-2)}{(x-3)\ (x-4)}}\right)\left(\frac{1}{x-1} + \frac{1}{x-2} - \frac{1}{x-3} - \frac{1}{x-4} - \frac{1}{x-5}\right]$$

$$y \quad (\sin x)^{\tan x} + (\cos x)^{\sec x}$$

$$(\sin x)^{\tan x} = u \quad , \quad (\cos x)^{\sec x} = v$$

$$y = u + v$$

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diff. w. r. t. x

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$u = (\sin x)^{\tan x}$$

Taking log both the Sides

$$\log u \cancel{\frac{1}{u}\frac{du}{dx}} = \tan x \cancel{(\sin x} \tan x \log(\sin x)$$

diff. w. r. t. x

$$\frac{1}{u}\frac{du}{dx} = \left[\tan x \cdot \frac{\cos x}{\sin x} + \log \sin x \cdot \sec^2 x\right]$$

$$\frac{1}{u}\frac{du}{dx} = \left[\tan x \cdot \cot x + \log \sin x \sec^2 x\right]$$

$$\frac{du}{dx} = u\left[1 + \log \sin x \sec^2 x\right]$$

$$\frac{du}{dx} = (\sin x)^{\tan x}\left[1 + \log \sin x \sec^2 x\right]$$

$$v = (\cos x)^{\sec x}$$

Taking log both the Sides

$$\log v = \log(\cos x)^{\sec x}$$

$$\log v = \sec x \log(\cos x)$$

differenciating with respect to x

$$\frac{1}{v}\frac{dv}{dx} = \sec x \cdot \frac{(-\sin x)}{\cos x} + \log(\cos x)(\sec x \cdot \tan x$$

$\frac{1}{v} \cdot \frac{dv}{dx} \Big[-\tan x \sec x + \log \cos x (\sec x \cdot \tan x) \Big)$

$\frac{dv}{dx} = v \tan x \sec x \Big[\log \cos x - 1 \Big]$

$\frac{dv}{dx} = (\cos x)^{\sec x} \tan x \sec x \Big[\log \cos x - 1 \Big]$

$\frac{dy}{dx} = u + v$

putting the value of u & v

$\frac{dy}{dx} = (\sin x)^{\tan x} \Big[1 + \log \sin x \sec^2 x \Big] + (\cos x)^{\sec x} \tan x \sec x \Big[\log \cos x - 1 \Big]$

Paramatric form

$x = a \sec^3 \theta$ & $y = a \tan^2 \theta$ find $\frac{dy}{dx}$ at $\theta = \pi/3$

$y = a \tan^3 \theta$

diff with respect to x

$\frac{dy}{d\theta} = a\, 3 \tan^2 \theta \cdot \sec^2 \theta$

$\frac{dy}{d\theta} = 3a \tan^2 \theta \sec^2 \theta$

$x = a \sec^3 \theta$

$\frac{dx}{d\theta} = a \cdot 3 \sec^2 \theta \cdot (\sec \theta \cdot \tan \theta)$

$\frac{dx}{d\theta} = 3a \sec^3 \theta \cdot \tan \theta$

$\frac{dy}{dx} = \frac{3a \tan^2 \theta \sec^2 \theta}{3a \sec^3 \theta \tan \theta}$

$\frac{dy}{dx} = \frac{\tan \theta}{\sec \theta}$

$\frac{dy}{dx} = \frac{\sin \theta}{\cos \theta \times \sec \theta} = \frac{dy}{dx} = \sin \theta$

at $\theta = \pi/3$

$\sin \frac{\pi}{3} = \sqrt{3}/2$

$\frac{dy}{dx} = \sqrt{3}/2$