MA106 Tutorial 2

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Tutorial Problems

Note: These aren't complete solutions. These are just hints, solutions have been discussed during the tutorial sessions.

Q1

The system of linear equations $Ax = b$, where $A \in \mathbb{R}^{3\times 5}$ where column C_i is the coefficients of x_i in all the equations. $b \in {}^{3\times 1}$ is a column vector made up of the constant terms in the right hand side of the equation. To solve the system, construct augmented matrix $A^+ = [A|b]$ and apply Elementary Row operations until you arrive at a very simple equivalent linear equation system.(**Gauss Elimination Method**)

Q2

Apply ERO's on augmented matrix $[H_3|I_3]$, until H_3 transforms into its RREF(Reduced Row Echlon Form) or RCF(Row Canonical Form). If H_3 reduces to identity, then H_3 is invertible and the matrix at the right partition is the inverse. **How does this work?**

$$E_n \dots E_2 E_1 A = I$$
$$A = E_1^{-1} E_2^{-1} \dots E_n^{-1}$$
$$A^{-1} = E_n \dots E_2 E_1 = E_n \dots E_2 E_1 I$$

Answer: $\begin{bmatrix} 9 & -36 & 30 \\ -36 & 192 & -180 \\ 30 & -180 & 180 \end{bmatrix}$

Q3

JEE Problem, write equation of line, and generalise a point on this line in terms of a parameter say t, and now let this point lie on the plane, find value of t and hence the point.

Answer:$(1/10, -4/10, 3/10)$

Q5

All $x \in \mathbb{R}^2$ such that $Ax = 0$.

Answer: $x + 2y = 0$

Q6

Left to Right:
If invertible, Ax = linear span of Column space = 0 =¿ x = 0. This implies Column vectors are independent.
Right to Left:
If independent column vectors, A^T has independent row vectors, ERO's reduce it to I. **Why?** If in RCF of A, there had been a zero row, then that row in the original A can be written as a linear combination of other rows, but our rows are independent, hence contradiction. Therefore, all rows are non zero, hence RCF = I. Therefore, A^T is invertible which further implies A is invertible.

Practice Problems

Q11

It is obvious that RCF is either I or it has atleast one non zero row. If it has a non zero row, the last row must definitely be zero. **Why?**
Case I: A is invertible
RCF of A(say B) must also be invertible, since ERM's are all invertible and product of invertible matrices is also invertible. If B has a non zero row, since B is invertible consider inverse as C. $BC = I$, but BC has last row zero, whereas I doesn't have such. **Contradiction!**
Alternate argument: B is invertible, then by results proved from Q6 of Tutorial Problems, B^T is invertible and hence columns of B^T are linearly independent. Columns of B^T are same as rows of B. But one non zero row, makes the set of rows linearly dependent. **Contradiction!**

Case II: A is not invertible
Again from result proved in Q6, A has linearly dependent row matrices. Therefore atleast one row is definitely zero, hence the last row of RCF of A is zero.
Alternate argument: Ax = 0 has atleast one non zero solution, let that be w, then $Aw = 0$, also $Bw = 0$, where B is RCF of A. Assume B has no non zero row, then it must be identity, $Iw = w = 0$, but w is non zero. **Contradiction!** Therefore, B has a non zero row, therefore the last row is zero.