Advanced Monetary Economics

Lecture 7

Malte Rieth
Martin-Luther-Universität Halle-Wittenberg

What we have learned so far

- Monetary policy sets the short-term nominal interest rate or the money supply.
- Changes in the short-term nominal interest rate or money supply lead to changes in the mid- and long-term interest rate via the yield curve.
- Changes in interest rates (or money supply) effect the economy through different transmission mechanisms.
- One of these channels is the interest-rate channel.

Aim of this class

- In the following lectures we study theoretical models for money supply and the interest-rate channel.

- We want to use the model to answer the question: what is the effect of monetary policy on the economy?

- Starting point: How to model the household and firm sector under the assumption of *flexible prices*.

- Many predictions are at odds with the empirical evidence, but this is a useful starting point to introduce preferences, technology and notation.

- We assume that there is

- one representative household and
- one representative firm.

- Closed economy, no investment, no government consumption.

- No role for money except of unit of account.

- Later, we
  - assume that money generates utility (serves for transactions)
  - relax the strong assumption that prices are perfectly flexible.

- Then, money and monetary policy can have real effects.

1 Household

- The household maximizes its lifetime utility (she is infinitely lived).

- Future utility is discounted by a discount factor β.

- The household values consumption (C) and dislikes hours worked (N): $U(C_t, N_t)$, with $U_c > 0, U_{cc} \leq 0, U_N \leq 0, U_{NN} \leq 0$.

- Working yields nominal wage (W).

- The household receives dividends (D) from firm ownership, it can buy and sell bonds (B).

- Discount bonds have price Q, or pay interest rate $1/Q$.

- Households' problem: representative household solves

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, N_t\right)$$

subject to

$$P_t C_t + Q_t B_t \leq B_{t-1} + W_t N_t + D_t$$

further constraints include the non-negativity of C_t:

$$C_t > 0$$

and an No-Ponzi game condition.

- **Uncertainty and expectations:** In this model and in most macroeconomic models agents address uncertainty by forming expectations about the future.

- Agents attach to each possible outcome (payment) a probability.

- The expected outcome (payment) is then the sum over all outcomes where each outcome is weighted with its probability.

- Example: An asset a^J pays 50 Euro with probability 0.4 and 3 Euro with probability 0.6.

- The expected value of the payments is:

$$50 \cdot 0.4 + 3 \cdot 0.6 = 21.8. \tag{1}$$

- The expectation operator E performs the calculation in Equation (1).

- In this example, the expected value of payments by asset J are: $E[a^J] = 21.8$.

- When employing the expectation operator, it is important to specify the information set.

- E_t means: expectation conditional on information up to and including period t.

- The Lagrangian: Consider the utility function and the budget constraint only

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ U\left(C_t, N_t\right) - \lambda_t \left[P_t C_t + Q_t B_t - B_{t-1} - W_t N_t - D_t \right] \right\}.$$

- The non-negativity constraints are added using additional Lagrange multipliers.

- The Lagrangian written out: consider the utility function and the budget constraint only

$$\begin{aligned} \mathcal{L} \; = \; & E_0[\beta^0 \left\{ U\left(C_0, N_0\right) - \lambda_0 \left[P_0 C_0 + Q_0 B_0 - B_{-1} - W_0 N_0 - D_0 \right] \right\} \\ & + \beta^1 \left\{ U\left(C_1, N_1\right) - \lambda_1 \left[P_1 C_1 + Q_1 B_1 - B_0 - W_1 N_1 - D_1 \right] \right\} \\ & + \beta^2 \left\{ U\left(C_2, N_2\right) - \lambda_2 \left[P_2 C_2 + Q_2 B_2 - B_1 - W_2 N_2 - D_2 \right] \right\} \\ & + ...] \end{aligned}$$

- The agent chooses $C_0, C_1, C_2, \ldots$:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial C_0} &: \; E_0 \left\{ U_c\left(C_0, N_0\right) - \lambda_0 P_0 \right\} = 0 \\ \frac{\partial \mathcal{L}}{\partial C_1} &: \; E_0 \left\{ \beta U_c\left(C_1, N_1\right) - \beta \lambda_1 P_1 \right\} = 0 \\ \frac{\partial \mathcal{L}}{\partial C_2} &: \; E_0 \left\{ \beta^2 U_c\left(C_2, N_2\right) - \beta^2 \lambda_2 P_2 \right\} = 0. \end{aligned}$$

- In general, in period t, the first-order condition is given by:

$$\frac{\partial \mathcal{L}}{\partial C_t} : E_0 \left\{ U_c\left(C_t, N_t\right) - \lambda_t P_t \right\} = 0.$$

- We allow the household to form new expectations each period, i.e. to use the complete information set in period t:

$$\frac{\partial \mathcal{L}}{\partial C_t} : E_t \left\{ U_c\left(C_t, N_t\right) - \lambda_t P_t \right\} = 0.$$

- Variables in t are known to agent with certainty:

$$\frac{\partial \mathcal{L}}{\partial C_t} : U_c\left(C_t, N_t\right) - \lambda_t P_t = 0. \tag{2}$$

- The agent also chooses $N_0, N_1, N_2, \ldots$:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial N_0} &: \quad E_0\left\{U_N\left(C_0, N_0\right) + \lambda_0 W_0\right\} = 0 \\
\frac{\partial \mathcal{L}}{\partial N_1} &: \quad E_0\left\{\beta U_N\left(C_1, N_1\right) + \beta \lambda_1 W_1\right\} = 0 \\
\frac{\partial \mathcal{L}}{\partial N_2} &: \quad E_0\left\{\beta^2 U_N\left(C_2, N_2\right) + \beta^2 \lambda_2 W_2\right\} = 0.
\end{aligned}$$

- In general, in period t, the first-order condition is given by:

$$\frac{\partial \mathcal{L}}{\partial N_t} : U_N\left(C_t, N_t\right) + \lambda_t W_t = 0. \tag{3}$$

- The agent also chooses $B_0, B_1, B_2, \ldots$:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial B_0} &: \quad E_0 \left\{ -\lambda_0 Q_0 + \beta \lambda_1 \right\} = 0 \\
\frac{\partial \mathcal{L}}{\partial B_1} &: \quad E_0 \left\{ -\beta \lambda_1 Q_1 + \beta^2 \lambda_2 \right\} = 0 \\
\frac{\partial \mathcal{L}}{\partial B_2} &: \quad E_0 \left\{ -\beta^2 \lambda_2 Q_2 + \beta^3 \lambda_3 \right\} = 0.
\end{aligned}$$

- In general, in period t, the first-order condition is given by:

$$\frac{\partial \mathcal{L}}{\partial B_t} : -\lambda_t Q_t + E_t \left[\beta \lambda_{t+1} \right] = 0. \tag{4}$$

- Use conditions (2), (3), and (4) to obtain the optimality conditions of the representative household:

$$
\begin{aligned}
-\frac{U_N\left(C_t, N_t\right)}{U_C\left(C_t, N_t\right)} &= \frac{W_t}{P_t} \\
Q_t &= \beta E_t\left[\frac{U_C\left(C_{t+1}, N_{t+1}\right)}{U_C\left(C_t, N_t\right)} \frac{P_t}{P_{t+1}}\right].
\end{aligned}
$$

- Summary:
  - First-order conditions determine the behavior of the household – they are optimality conditions.
  - They determine how much to consume, to work and to save.
  - They have to be fulfilled – in case they are not, the household will change her behavior until they are fulfilled.

- Specification of the utility function of the household:

$$U\left(C_t, N_t\right) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}.$$

- Implied optimality conditions

$$\begin{aligned} C_t^{\sigma} N_t^{\varphi} &= \frac{W_t}{P_t} \\ Q_t &= \beta E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right]. \end{aligned}$$

- Taking the logarithm on both sides and defining $x_t \equiv \log(X_t)$ yields

$$\sigma c_t + \varphi n_t = w_t - p_t \tag{5}$$

$$c_t = E_t\left[c_{t+1}\right] - \frac{1}{\sigma}\left(i_t - E_t \pi_{t+1} + \rho\right), \tag{6}$$

where $i_t \equiv -\log Q_t$, $\rho \equiv -\log \beta$, and π_{t+1} is the inflation rate.

- (5) is the labor supply schedule depending on the real wage and marginal utility of consumption.

- (6) is the consumption Euler equation that balances consumption growth with the expected real interest rate.

2 Firms and markets

- A representative firm with technology

$$Y_t = A_t N_t^{1-\alpha}. \tag{7}$$

  - Firms produce output Y.
  - Input is labor N

  - A denotes technology, which is exogenously given.

- Profit maximization

$$\max P_t Y_t - W_t N_t$$

subject to equation (7), taking prices and wages as given.

- The optimality condition is

$$\frac{W_t}{P_t} = (1 - \alpha) A_t N_t^{-\alpha}.$$

- In log-linear terms

$$w_t - p_t = a_t - \alpha n_t + \log(1 - \alpha). \tag{8}$$

- This is a labor demand schedule: firms hire labor until its marginal product equals the real wage, taking technology a_t as given.

- Remember: closed economy, no exports/ imports, and no government purchases. So, goods market clearing implies

$$y_t = c_t \tag{9}$$

- Labor market clearing implies (5)=(8)

$$\sigma c_t + \varphi n_t = a_t - \alpha n_t + \log(1-\alpha) \tag{10}$$

- Asset market clearing: $B_t = 0$.

- Summary of the economy: employ (6), (9), (10) and a log-linear version of (7) to arrive at the following set of equation describing the economy

$$\begin{aligned}
\sigma y_t + \varphi n_t &= a_t - \alpha n_t + \log(1-\alpha) \\
y_t &= E_t[y_{t+1}] - \frac{1}{\sigma}(i_t - E_t \pi_{t+1} + \rho) \\
y_t &= a_t + (1-\alpha) n_t.
\end{aligned}$$

- Define the real interest rate as $r_t \equiv i_t - E_t\pi_{t+1}$ and instert this expression into the Euler equation to obtain

$$y_t = E_t\left[y_{t+1}\right] - \frac{1}{\sigma}\left(r_t + \rho\right).$$

- The real part of the economy contains 3 endogenous variable y_t, n_t, r_t and 3 equations

$$\begin{aligned} \sigma y_t + \varphi n_t &= a_t - \alpha n_t + \log\left(1-\alpha\right) && (11) \\ y_t &= E_t\left[y_{t+1}\right] - \frac{1}{\sigma}\left(r_t + \rho\right). && (12) \\ y_t &= a_t + \left(1-\alpha\right) n_t. && (13) \end{aligned}$$

- One exogenous variable: a_t.

- We can obtain a solution for the real variables independent of nominal variables! $\longrightarrow$ Classical dichotomy

- To see this, combine (11) and (13) to solve for y_t and n_t as functions of a_t. Then, use (12) to residually determine r.

- Because the equilibrium is independent of nominal variables, it is also independent of monetary policy: money is *neutral*.