Advanced Monetary Economics

Lecture 10

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What we have learned so far

- Monetary policy sets the short-run nominal interest rate or the money supply.

- Changes in interest rates or money supply effect the economy (among other channels) through the interest-rate channel.

- We want to study a dynamic model for the interest-rate channel: the New Keynesian model.

- We have studied two building blocks of the New Keynesian model:

  1. Household and firm sector

  2. Monetary policy authority

- Prices have been assumed to be flexible $\longrightarrow$ Monetary policy affects the real economy only if
  - money yields direct and non-separable utility (MIU)
  - money is need for transaction services (CIA)

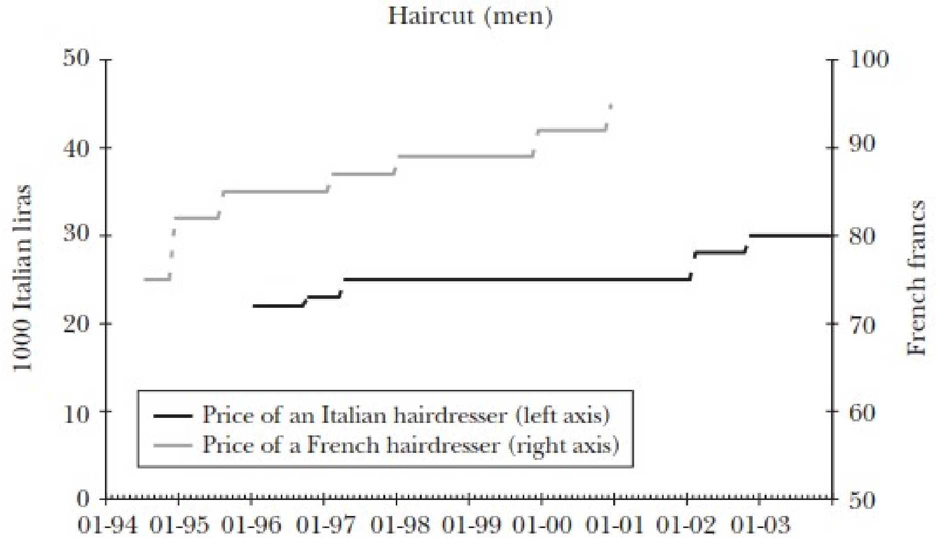
Outlook for the remaining lectures:

- Introduce sticky prices into the model.
- Solve the New-Keynesian model.
- Reinvestigate the effects of monetary policy on the real economy.

1 Sticky prices: evidence

- Modeling assumption from now on: firms adjust prices infrequently

- Some evidence on infrequent price adjustment



Note: Actual examples of trajectories, extracted from the French and Italian CPI databases.

Source: Dhyne et al 2006

- In order to model sticky prices, we need two ingredients:

  1. Firms need to have price setting power $\longrightarrow$ today we introduce different goods and monopolistic competition.

  2. A friction, which prevents firms to adjust prices frequently $\longrightarrow$ next week we introduce Calvo pricing.

2 Expenditure problem

- Main difference to the classical monetary model: household consumes a basket of consumption goods instead of one good only.

- Basket of consumption goods is denoted by: C.

- Basket consists of many consumption goods: $C(i)$.

- Consumption goods $C(i)$ are not perfect substitutes!

- The elasticity of substitution between these goods is ϵ with

$$C_t = \left[\int_0^1 C_t\,(i)^{1-\frac{1}{\epsilon}}\, di \right]^{\frac{\epsilon}{\epsilon-1}}. \tag{1}$$

- Expenditure problem of the household: The household has to choose between different goods $C(i)$ with prices $P(i)$.

- Maximize consumption basket C_t subject to an expenditure level.

- Expenditure level defined as:

$$X_t = \int_0^1 P_t(i)\, C_t(i)\, di.$$

- **2 goods example**
  - Goods: $C(1)$ and $C(2)$.
  - Prices: $P(1)$ and $P(2)$.

- Consumption basket:

$$C_t = \left[C_t(1)^{1-\frac{1}{\epsilon}} + C_t(2)^{1-\frac{1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}}.$$

- Expenditure level

$$X_t = P_t(1)\, C_t(1) + P_t(2)\, C_t(2).$$

- Maximization problem

$$\mathcal{L} = \left[C_t(1)^{1-\frac{1}{\epsilon}} + C_t(2)^{1-\frac{1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}} - \lambda \left[P_t(1)\, C_t(1) + P_t(2)\, C_t(2) - X_t \right]$$

- First-order conditions with $\left(1 - \frac{1}{\epsilon}\right)\left(\frac{\epsilon}{\epsilon-1}\right) = 1$

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial C_t(1)} &: \; C_t(1)^{-\frac{1}{\epsilon}} \left[C_t(1)^{1-\frac{1}{\epsilon}} + C_t(2)^{1-\frac{1}{\epsilon}} \right]^{\frac{1}{\epsilon-1}} - \lambda P_t(1) = 0 \\
\frac{\partial \mathcal{L}}{\partial C_t(2)} &: \; C_t(2)^{-\frac{1}{\epsilon}} \left[C_t(1)^{1-\frac{1}{\epsilon}} + C_t(2)^{1-\frac{1}{\epsilon}} \right]^{\frac{1}{\epsilon-1}} - \lambda P_t(2) = 0.
\end{aligned}$$

- Simplifying the first-order condition with respect to $C_t(1)$

$$\begin{aligned} C_t(1)^{-\frac{1}{\epsilon}}\left[C_t(1)^{1-\frac{1}{\epsilon}}+C_t(2)^{1-\frac{1}{\epsilon}}\right]^{\frac{1}{\epsilon-1}}-\lambda P_t(1) &= 0 \\ C_t(1)^{-\frac{1}{\epsilon}}\left[C_t(1)^{1-\frac{1}{\epsilon}}+C_t(2)^{1-\frac{1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}\frac{1}{\epsilon}}-\lambda P_t(1) &= 0 \\ C_t(1)^{-\frac{1}{\epsilon}}C_t^{\frac{1}{\epsilon}}-\lambda P_t(1) &= 0. \end{aligned}$$

- **In general**

$$C_t(i)^{-\frac{1}{\epsilon}}C_t^{\frac{1}{\epsilon}}-\lambda P_t(i)=0$$

- Expenditure problem with many goods

$$\mathcal{L}=\left[\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}}\,di\right]^{\frac{\epsilon}{\epsilon-1}}-\lambda\int_0^1 P_t(i)\,C_t(i)\,di$$

- First-order condition

$$\frac{\partial \mathcal{L}}{\partial C_t(i)} : C_t(i)^{-\frac{1}{\epsilon}} \left[\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} di \right]^{\frac{1}{\epsilon-1}} - \lambda P_t(i) = 0$$

- Simplifying

$$C_t(i)^{-\frac{1}{\epsilon}} C_t^{\frac{1}{\epsilon}} - \lambda P_t(i) = 0 \quad (2)$$

- **Demand function for good** i : From the first-order conditions, we can obtain the demand for good i.

- This demand is will be important once we consider the firm's problem.

- We want to derive the demand for good i as a function of:

1. the price of good i

2. the aggregate price level

3. aggregate output as measure for economic activity / income

- The aggregate price level is defined as:

$$P_t \equiv \left[\int_0^1 P_t(i)^{1-\epsilon}\, di\right]^{\frac{1}{1-\epsilon}}. \tag{3}$$

- In case of the two-good example P_t is given by:

$$P_t \equiv \left[P_t(1)^{1-\epsilon} + P_t(2)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}.$$

- To derive demand for good i with two goods only, we start with

$$\begin{aligned} C_t(1)^{-\frac{1}{\epsilon}} C_t^{\frac{1}{\epsilon}} - \lambda P_t(1) &= 0 \\ C_t(1) &= \lambda^{-\epsilon} P_t(1)^{-\epsilon} C_t. \end{aligned} \tag{4}$$

- In general

$$C_t(i) = \lambda^{-\epsilon} P_t(i)^{-\epsilon} C_t.$$

- Insert the expression for $C_t(i)$ into the definition of the consumption basket:

$$
\begin{aligned}
C_t &= \left[\left(\lambda^{-\epsilon} P_t(1)^{-\epsilon} C_t\right)^{1-\frac{1}{\epsilon}} + \left(\lambda^{-\epsilon} P_t(2)^{-\epsilon} C_t\right)^{1-\frac{1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}} \\
&= \left[\left[\left(P_t(1)^{-\epsilon}\right)^{1-\frac{1}{\epsilon}} + \left(P_t(2)^{-\epsilon}\right)^{1-\frac{1}{\epsilon}}\right]\left(\lambda^{-\epsilon} C_t\right)^{1-\frac{1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}} \\
&= \left[\left(P_t(1)^{-\epsilon}\right)^{1-\frac{1}{\epsilon}} + \left(P_t(2)^{-\epsilon}\right)^{1-\frac{1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}} \lambda^{-\epsilon} C_t \\
1 &= \left[P_t(1)^{-\epsilon+1} + P_t(2)^{-\epsilon+1}\right]^{\frac{\epsilon}{\epsilon-1}} \lambda^{-\epsilon}.
\end{aligned}
$$

- Simplifying and solving for λ yields

$$
\lambda^{-1} = \left[P_t(1)^{1-\epsilon} + P_t(2)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} = P_t \tag{5}
$$

- Insert (5) into (4)

$$C_t(1) = \left(\frac{P_t(1)}{P_t}\right)^{-\epsilon} C_t$$

- In general

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} C_t$$

- In case of many goods, the steps to derive the demand function are similar:

  1. Solve the first-order condition (2) for $C(i)$.
  2. Insert the solution into the defintion of the consumption index (1).
  3. λ_t and C_t can be taken out of the integral
  4. Use the definition of the aggregate price level (3).

5. Solve for λ_t, insert into the f.o.c for $C_t(i)$ (2).

6. Obtain

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} C_t.$$

- The demand for good i depends
  - negatively on the price of good $P_t(i)$
  - positively on aggregate consumption C_t
  - positively on the aggregate price level P_t

3 Household problem

- The household budget constraint is

$$\int_0^1 P_t(i)\,C_t(i)\,di + Q_tB_t \leq B_{t-1} + W_tN_t + D_t$$

- Before we continue, we find an expression for $\int_0^1 P_t(i)\,C_t(i)\,di$.

- Two goods example again

$$\begin{aligned}\sum_{i=1}^{2} P_t(i)\,C_t(i) &= P_t(1)\left(\frac{P_t(1)}{P_t}\right)^{-\epsilon} C_t + P_t(2)\left(\frac{P_t(2)}{P_t}\right)^{-\epsilon} C_t \\ &= P_tC_t\left[\left(\frac{P_t(1)}{P_t}\right)^{1-\epsilon} + \left(\frac{P_t(2)}{P_t}\right)^{1-\epsilon}\right]\end{aligned}$$

- Definition of the price index

$$
\begin{aligned}
P_t &= \left[P_t(1)^{1-\epsilon} + P_t(2)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \\
P_t^{1-\epsilon} &= P_t(1)^{1-\epsilon} + P_t(2)^{1-\epsilon} \\
1 &= \left(\frac{P_t(1)}{P_t}\right)^{1-\epsilon} + \left(\frac{P_t(2)}{P_t}\right)^{1-\epsilon}
\end{aligned}
$$

- Which implies that

$$\sum_{i=1}^{2} P_t(i)\, C_t(i) = P_t C_t$$

- Using this simplification in the housheold's budget constraint gives

$$P_t C_t + Q_t B_t \leq B_{t-1} + W_t N_t + D_t$$

- The maximization problem of the household then is

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t)$$

subject to

$$P_t C_t + Q_t B_t \leq B_{t-1} + W_t N_t + D_t$$

and the non-negativity of c_t : $c_t > 0$ and a No-Ponzi game condition.

- The first-order conditions to the problem can be summarized as:

$$\begin{aligned} -\frac{U_N(C_t, N_t)}{U_C(C_t, N_t)} &= \frac{W_t}{P_t} \\ Q_t &= \beta E_t \left[\frac{U_C(C_{t+1}, N_{t+1})}{U_C(C_t, N_t)} \frac{P_t}{P_{t+1}} \right]. \end{aligned}$$

- Specification of utility as before

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}.$$

- The implied log-linear optimality conditions for aggregate variables are as before

$$\begin{aligned} \sigma c_t + \varphi n_t &= w_t - p_t \\ c_t &= E_t[c_{t+1}] - \frac{1}{\sigma}(i_t - E_t \pi_{t+1} + \rho), \end{aligned}$$

where $i_t \equiv -\log Q_t$, $\rho \equiv -\log \beta$, and π_{t+1} is the inflation rate.

- Summary and outlook: In order to model sticky prices, we need two ingredients

1. Firms need to have price setting power $\longrightarrow$ today we introduced different goods and monopolistic competition.

2. A friction, which prevents firms to adjust prices frequently $\longrightarrow$ next week we introduce Calvo pricing.