Dr. Juliane Hennecke

Chair of Empirical Microeconomics

MARTIN-LUTHER-UNIVERSITÄT
HALLE-WITTENBERG

ADVANCED MICROECONOMICS

Consumer Theory I - Preferences and Utility

Winter Term 2025/2026

Consumer Theory

- Consumer theory model decisions of individual / households (HH).
- Producer theory models decisions of firms.
- Consumer theory more important/central.
    - In microeconomic theory, firms are ultimately owned by households.
        - Households provide the inputs (labor, capital, land)...
        - and they receive income in return (wages, interest, rent, profits).
    - Ultimate decision-making units in the economy are households, not firms.
- Decisions on household level more complex:
    - budget constraint(s) (time, income)
    - goals less straightforward than in case of firms (intertemporal, interpersonal)

Standard Approach - Rational Choice Analysis

- Rational Choice: HH have **rational preferences** which can be represented as **utility functions** and act according to them...
- HH take their decisions ...
  - ... so as to **maximize** their **utility** (captured by the utility function)
  - ... under a budget (and possibly other) constraint(s).
- Conditions: Consumers know which alternatives are available (complete information)
- Rational choice does not assume that people actually sit down and mathematically optimize their utility function, it just aims at describing the outcomes of human decisions as if they would mathematically optimize.

Outline

1) **Consumer Theory**
   1.1) Preferences and Utility (L1)
   1.2) Utility Maximization (L2)
   1.3) Expenditure Minimization (L3)
   1.4) Demand Functions and Comparative Statics (L4)
   1.5) Duality, Slutsky Equation and Types of Goods (L5)
2) **General Equilibrium**
   2.1) Exchange Economy (L6)
   2.2) Welfare Economics and General Equilibrium (L7)
3) **Decisions under Risk and Uncertainty**
   3.1) Expected Utility (L8)
   3.2) Risk Preferences (L9)
   3.3) Insurances (L10)
   3.4) Safety Regulations and the Value of a Statistical Life (11)
4) **Behavioral Economics** (L12)

Overview

Literature

- Varian (2022): Chapter 3+4 *(for refresher of basic concepts)*
- Autor (2016): Lecture Notes 3

Overview

Preferences

- Preferences refer to consumers' ability to compare different bundles of goods.
- Mathematically this is expressed with **binary relations** - also called **preference relations**.
- Assume there are two bundles of goods (A, B) (where each bundle may contain multiple goods).
- Then there are different possible preference relations over these bundles:
  - A $\succsim$ B – consumer **prefers** A **weakly** over B (or "A is at least as good as B")
  - A $\succ$ B – consumer **prefers** A **strictly** over B (or "A is better than B", in LN $A^P B$)
  - A $\sim$ B – consumer is **indifferent** between A and B (or "A is equal to B" , in LN $A^I B$)

Axioms on preference relations

Overview

- Axioms are **basic assumptions** about preference relations
- Axioms provide the logical foundation on which we can build utility theory and demand analysis:
  - To allow for mathematical representation of utility functions
  - To portray rational (optimizing) behavior
  - To derive “well-behaved” demand curves
- **Everything** that follows (rational choice theory) rests entirely on these assumptions (That’s why its important to understand them!)

Axioms on Preference Relations

Core axioms (ensure utility representation)

1. Complete
2. Transitive
3. Continuous

→ If a preference relation fulfills these three axioms, we can always find a utility function that represents the preference relation.

Additional assumptions (make analysis easier)

4. Monotone (implies non-satiation)
5. Convex (implies diminishing marginal rate of substitution)

→ Axioms not necessary for finding utility functions but often intuitive (capture fundamental features of observed behavior) and furthermore useful because they make the maths easier.

Axiom 1 - Completeness

Definition

The preference relation $\succsim$ is complete if $\forall\ A, B \in X$ (where X is the set of all bundles of goods), $A \succsim B$ or $B \succsim A$ or both.

- The consumer can establish a preference ordering for any two bundles of goods A and B.
- For any comparison of bundles, the consumer will choose one and only one of the three options:
  1. $A \succ B$
  2. $B \succ A$
  3. $B \sim A$

Axiom 2 - Transitivity

Definition

The preference relation $\succsim$ is **transitive**, if $A \succsim B$ and $B \succsim C$ implies $A \succsim C \ \forall \ A, B, C \in X$.

- Consumers are consistent in their preferences i.e., no circular preferences.
- **Example:**
    - Rolling a dice is transitive (6 beats 5, 5 beats 4, ... ,6 beats 1)
    - Rock–Paper–Scissors is not transitive (P beats R, R beats S but P does not S)
- Preferences that are both complete and transitive are called rational preferences.

Axiom 3 - Continuity

Definition

The preference relation $\succsim$ is **continuous** if $A \succsim B$ and C lies in ϵ radius around B, then $A \succsim C \; \forall \; A, B, C \in X$.

- If you prefer bundle A to bundle B, then you won't suddenly flip your preference when we make small changes to B. $\rightarrow$ Preferences change "smoothly," not in abrupt jumps.
- Not necessarily an intuitive property, but a property that is necessary to derive well-behaved demand curves.

Axiom 4 - Monotonicity (Non-satiation)

Definition

A and B each consist of two goods X and Y, where X_A is the quantity of X in bundle A etc.
If we assume that $Y_A = Y_B$ and $X_A > X_B$, a non-satiated preference relation implies that $A \succ B$ (strictly non-satiated) or $A \succsim B$ (weakly non-satiated).

- This axiom implies that **more is better** (strictly non-satiated) or **more cannot be worse** (weakly non-satiated).
- Consumer always places positive value on **more** consumption.

Axiom 5 - Convexity

Definition

The preference relation $\succsim$ is **convex** if for any two bundles of goods $A, B \in X$ where $B \succsim A$ and C a bundle on the line segment joining bundles A and B then $C \succsim A$ (weak convexity) or $C \succ A$ (strong convexity).

- In words: When B is at least as good as A, then every combination of A and B is at least as good as A (weak convexity) or better than A (strong convexity).
- Example: If I like to have 100 cups of coffee at least as much as I like 100 pieces of cake, I will like 50 cups of coffee with 50 pieces of cake more (or at least as much as) having only coffee or cake.

Overview

Utility functions

- Preference relations are difficult to work with.
- If we are considering preferences that fulfill axioms 1-3 (completeness, transitivity and continuity), we can always find a **utility function** that **represents** the preference relation.
- Utility functions are mathematical functions which assign values (utility levels) to bundles of goods (e.g., $u(x, y) = x + y$, i.e., $u(A) = X_A + Y_A$).
- What does that mean?
  - A utility function $u()$ represents a preference relation $\succsim$ if $\forall$ $A, B \in X$, $A \succsim B \Leftrightarrow u(A) \geq u(B)$.
  - Analogously also for other preference relations, i.e., $A \succ B \Leftrightarrow u(A) > u(B)$ and $A \sim B \Leftrightarrow u(A) = u(B)$.
  - Thus, the utility function preserves the **ranking of alternatives** of the preference relation.

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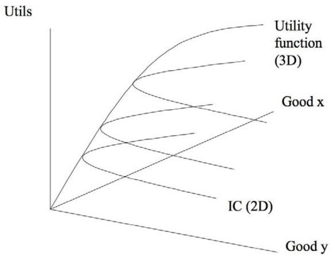
Indifference Curves

- Utility functions and preferences can be visualized using indifference curves (IDC).
- The map of all possible indifference curves represents a consumer's preferences.

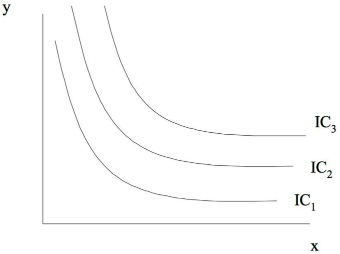
Indifference Curve

An indifference curve consists of all bundles of goods that give the consumer the same utility level $u(x) = \overline{u}$.

Indifference curves



Source: Autor (2016), Lecture Notes 3, p. 5



Source: Autor (2016), Lecture Notes 3, p. 5

Indifference curves and preferences

- With axioms 1–3, we can always find utility functions and indifference curves that represent the preferences.
  - With **Axiom 1 (completeness)**, every pair of consumption bundles can be compared, so every bundle lies on some indifference curve.
  - With **Axiom 2 (transitivity)**, indifference curves do not cross (preferences are consistent).
  - With **Axiom 3 (continuity)**, indifference curves are unbroken and connected (no jumps or gaps).
- With **Axiom 4 (non-satiation / monotonicity)**, there are infinitely many indifference curves and higher curves correspond to strictly preferred bundles.
- With **Axiom 5 (convexity)**, indifference curves are convex to the origin, which means a preference for variety (and implies a diminishing marginal rate of substitution).

Overview

Marginal rate of substitution

Definition

The **marginal rate of substitution (MRS)** tells us how many units of good y the consumer is willing to give up to gain one additional unit of x.

- Mathematically this is the **slope of the indifference curve**.
- This slope may change along the indifference curve.

Marginal rate of substitution - Example

- Assumptions
  - 2 goods: hours of sleep (X) and points on the exam (Y)
  - 2 bundles: A=($X = 5$, $Y = 60$) and B=($X = 6$, $Y = 50$)
- A and B are on one indifference curve (for student S) $\rightarrow$ S is indifference between A and B
- S is willing to give up 10 points in situation A to gain one additional hour of sleep.

$\rightarrow$ MRS (sleep for points) $= |\frac{-10}{1}| = 10$

- S may be willing to give up only 5 points instead of 10 for one additional hour of sleep if the initial bundle had 7 instead of 5 hours of sleep $\rightarrow$ MRS depends on the starting point

Marginal rate of substitution

and marginal utility

- Every point on the indifference curve has the same utility level, $u(x, y) = \overline{u}$
- When moving along the indifference curve, we know that $du = 0$ (no change in utility)
- We can take the total derivative of $u(x, y) = \overline{u}$:

$$0 = du = \frac{\partial u}{\partial x} dx + \frac{\partial u}{\partial y} dy$$

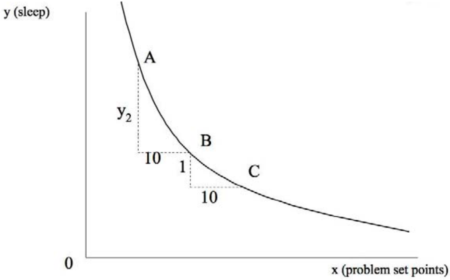
and rearrange

$$-\frac{dy}{dx} = \frac{\frac{\partial u}{\partial x}}{\frac{\partial u}{\partial y}} = \frac{MU_x}{MU_y} = MRS$$

- The MRS (of x for y) is thus equal to the ratio of the marginal utilities

Diminishing marginal rate of substitution

- If MRS decreases when moving along the indifference curve, we call this a diminishing marginal rate of substitution
- I.e. if x is higher (and y lower, i.e., point B below instead of A) the consumer is willing to give up less y per additional unit of x:



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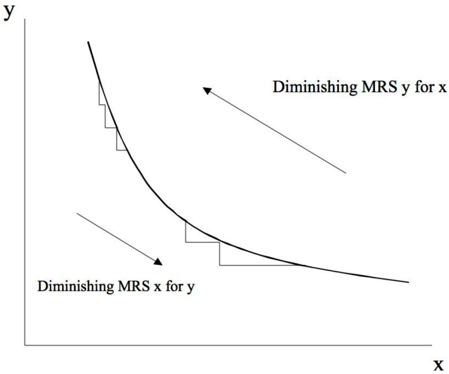
Diminishing marginal rate of substitution



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- MRS defined of x for y or the other way around
- Diminishing MRS implies that consumers prefer combinations of goods over extremes (only x or only y)

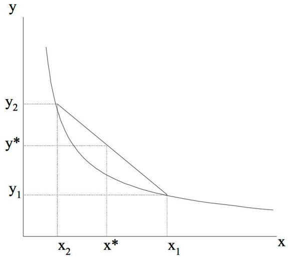
Overview

Remember: Indifference curves and preferences

- With **Axiom 1 (completeness)**, every pair of consumption bundles can be compared, so every bundle lies on some indifference curve.
- With **Axiom 2 (transitivity)**, indifference curves do not cross (preferences are consistent).
- With **Axiom 3 (continuity)**, indifference curves are unbroken and connected (no jumps or gaps).
- With **Axiom 4 (non-satiation / monotonicity)**, there are infinitely many indifference curves and higher curves correspond to strictly preferred bundles.
- With **Axiom 5 (convexity)**, indifference curves are convex to the origin, which means a preference for variety (and implies a diminishing marginal rate of substitution).

Convexity

- A utility function has a diminishing marginal rate of substitution iff (if and only if) the IDC is convex.
- What is a convex function?
  - A function is convex if the line segment of any two points of the graph of the function lies above the graph of the function.



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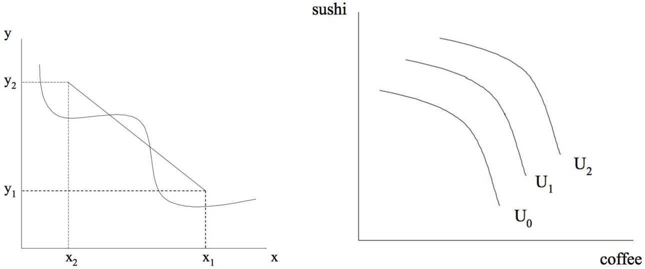
Convexity

- Examples of non-convex curves - MRS is not (always) diminishing

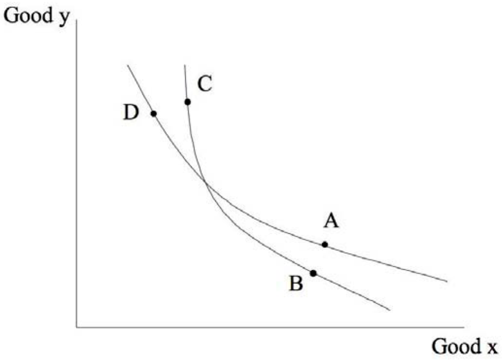


Source: Autor (2016), Lecture Notes 3, p. 10

- Remember: Convexity not a necessary condition for well-behaved demand curves → utility representation of these preferences is possible

Non-crossing indifference curves

- Transitivity (+ monotonicity) $\rightarrow$ non-crossing indifference curves



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Overview

Cardinal vs. Ordinal utility

If a utility function $u()$ is to be interpreted cardinally, $u(A)$ has to represent a cardinal number (e.g. 'utils'), which implies that distances between different values of the function are meaningful.

- Example: If $u(A) = 12$ and $u(B) = 6$ an cardinal utility function, then A gives twice as much utility as B.
- Interpersonal comparisons possible

If a utility function is to be interpreted ordinally, it just keeps the ranking of the underlying preferences (the preference ordering) but the values are not themselves meaningful.

- Example: If $u(A) = 12$ and $u(B) = 6$, with an ordinal utility function A is preferred to B, but we cannot infer by how much A is better than B.
- No interpersonal comparisons possible

Ordinal utility in consumer theory

- For a cardinal interpretation, we'd have to **assume** that there are absolute levels of utility.
- Strong assumption not needed for most of consumer theory.
- To model decisions over bundles of goods an ordinal utility function(focus on the ranking/ordering of values) is sufficient.
- Important is only that the utility function keeps the ordering of alternatives and that the rate of substitution (MRS) is well-defined.

Monontonic Transformation

- **Problem:** How do we preserve existence of the MRS properties without imposing cardinality?
  - $\rightarrow$ Even if different utility functions represent the same preferences, the MRS has to be the same for these different utility functions
- **Solution:** Weakened definition of utility functions $\rightarrow$ Utility function is only defined up to a monotonic transformation

Monotonic Transformation

A function $g(x)$ is a monotone transformation if it is a **strictly monotone increasing** function, i.e. it is a rank-preserving transformation.
Formally: If $g(x)$ is differentiable then $g'(x) > 0 \ \forall x$.

- Examples: $g(x) = x + 1$; $g(x) = 2x$; $g(x) = ln(x)$ for $y > 0$

Monotonic Transformation of Utility Functions

If $U^2(x,y)$ is a monotone transformation of $U(x,y)$, i.e. $U^2(x,y) = f(U(x,y))$ where $f()$ is monotone in $U(x,y)$, then:

1. U^1 and U^2 exhibit identical preference rankings
2. MRS of $U^1(\overline{U})$ and $U^2(\overline{U})$ are the same
3. U^1 and U^2 are equivalent for consumer theory

- A Monotonic transformation of a utility function is a utility function that represents the same preferences as the original function

Monotonic Transformation of Utility Functions

- Example:
  1. Cobb-Douglas utility function $U(x,y) = x^{\alpha}y^{\beta}$

$$
\begin{aligned}
dU &= \alpha x^{\alpha-1}y^{\beta}dx + x^{\alpha}\beta y^{\beta-1}dy = 0 \\
-\frac{dy}{dx} &= \frac{\alpha x^{\alpha-1}y^{\beta}}{x^{\alpha}\beta y^{\beta-1}} = \frac{\alpha}{\beta}\frac{y}{x} = MRS
\end{aligned}
$$

  2. Monotone transformation of $U(x,y)$, e.g. $U^2(x,y) = ln(U(x,y)) = \alpha lnx + \beta lny$

$$
\begin{aligned}
dU^2 &= \frac{\alpha}{x}dx + \frac{\beta}{y}dy = 0 \\
-\frac{dy}{dx} &= \frac{\frac{\alpha}{x}}{\frac{\beta}{y}} = \frac{\alpha}{\beta}\frac{y}{x} = MRS
\end{aligned}
$$

Monotonic Transformation of Utility Functions

- General demonstration
  - Assume $U^2(x,y)$ is a monotone transformation of $U^1(x,y) = f(x,y)$, i.e., $U^2(x,y) = g(f(x,y))$.
  - Then MRS of $U^2(x,y)$:

$$
\begin{aligned}
0 &= g'\left(f(x,y)\right)\frac{\partial f}{\partial x}dx + g'\left(f(x,y)\right)\frac{\partial f}{\partial y}dy \\
-\frac{dy}{dx} &= \frac{g'(f(x,y))\frac{\partial f}{\partial x}}{g'(f(x,y))\frac{\partial f}{\partial y}} \\
&= \frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}} = \frac{MU_x^1}{MU_y^1}
\end{aligned}
$$

Overview

Summary

1. Preferences are the foundation of decisions taken by consumers.
2. Preferences can be expressed in form of preference relations.
3. If preferences are rational (i.e., complete and transitive) and continuous, one can always find a utility function that represents the preferences (i.e., keeps the ordering of alternatives).
4. Utility functions can be represented as indifference curves.
5. Additional properties of preferences (axioms 4 and 5) buy additional (nice) properties of the utility functions and indifference curves.
6. For consumer theory usually an ordinal interpretation of utility functions suffices.

Main Keywords

- Rational Choice
- Preference Relations
- Rationality
- Completeness
- Transitivity
- Continuity
- Monotonicity (Non-satiation)
- Convexity (Preference for Variety)
- Utility Function
- Indifference Curves
- Marginal Rate of Substitution (MRS)
- Ordinal vs. Cardinal Utility
- Monotonic Transformation

Questions?