Advanced Monetary Economics

Lecture 12

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What we have learned so far

- strong evidence in micro data for price stickiness
- Calvo pricing: probability to reset price $1 - \theta$
- firms take into account that they may not be able to reset price and that they have market power
- New Keynesian Phillips curve says inflation today depends on:
  - inflation tomorrow
  - future marginal costs/ outupt
- nominal interest rate affect real rates (because prices are sticky), which affects output (Euler equation), which affects inflation today

Outlook

- collect model equations
- solve the New-Keynesian model
- reinvestigate the effects of monetary policy on the real economy
- analyze the effect of technology shocks

1 The New-Keynesian model

- market clear in the goods market requires

$$Y_t(i) = C_t(i)$$

- define aggregate output as

$$
\begin{aligned}
Y_t &\equiv \left[\int_0^1 Y_t(i)^{1-\frac{1}{\epsilon}}\,di\right]^{\frac{\epsilon}{\epsilon-1}} \\
&= \left[\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}}\,di\right]^{\frac{\epsilon}{\epsilon-1}} \\
&= C_t
\end{aligned}
$$

- remember market clearing in the labor market implies

$$N_t = \int_0^1 N_t(i)\,di$$

- Specification of utility as before

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}.$$

- The implied log-linear optimality conditions for aggregate variables are as before

$$
\begin{aligned}
\sigma c_t + \varphi n_t &= w_t - p_t \\
c_t &= E_t\left[c_{t+1}\right] - \frac{1}{\sigma}\left(i_t - E_t \pi_{t+1} + \rho\right)
\end{aligned}
$$

or in terms of output

$$
\begin{aligned}
\sigma y_t + \varphi n_t &= w_t - p_t \\
y_t &= E_t\left[y_{t+1}\right] - \frac{1}{\sigma}\left(i_t - E_t \pi_{t+1} + \rho\right)
\end{aligned}
$$

- The New Keynesian Phillips curve is

$$
\pi_t = \beta E_t \pi_{t+1} + \chi \widehat{mc}_t
$$

with

$$
\chi = \frac{(1-\theta)(1-\beta\theta)}{\theta} \frac{(1-\alpha)}{(1-\alpha+\alpha\varepsilon)}.
$$

- log-linear marginal costs are

$$mc_t = w_t - p_t - \frac{1}{1-\alpha}(a_t - \alpha y_t) - \log(1-\alpha)$$

- monetary policy sets the short-run interest rate

$$i_t = \rho + \phi \pi_t + \nu_t$$

- putting things together

$$\begin{aligned}
\sigma y_t + \varphi n_t &= w_t - p_t \\
y_t &= E_t[y_{t+1}] - \frac{1}{\sigma}(i_t - E_t \pi_{t+1} - \rho) \\
\pi_t &= \beta E_t \pi_{t+1} + \chi \widehat{mc}_t \\
mc_t &= w_t - p_t - (a_t - \alpha n_t) - \log(1-\alpha) \\
i_t &= \rho + \phi \pi_t + \nu_t \\
y_t &= a_t + (1-\alpha) n_t
\end{aligned}$$

and exogenous processes for a_t and ν_t

- compute log deviations from the steady state of a generic variable as

$$\widehat{x}_t \equiv \log X_t - \log X = x_t - x$$

- For example

$$\begin{aligned}
MC_t &= \frac{W_t/P_t}{(1-\alpha)\,A_t N_t^{-\alpha}} \\
\log MC_t &= \log\left(W_t/P_t\right) - \log\left((1-\alpha)\,A_t N_t^{-\alpha}\right) \\
mc_t &= w_t - p_t - (a_t - \alpha n_t) - \log(1-\alpha) \\
mc_t - mc &= w_t - p_t - (a_t - \alpha n_t) - \log(1-\alpha) \\
&\quad - [w - p - (a - \alpha n) - \log(1-\alpha)] \\
\widehat{mc}_t &= \hat{w}_t - \hat{p}_t - (\hat{a}_t - \alpha \hat{n}_t)
\end{aligned}$$

- doing this with all equations gives

$$\begin{aligned}
\sigma \hat{y}_t + \varphi \hat{n}_t &= \hat{w}_t - \hat{p}_t \\
\hat{y}_t &= E_t \left[\hat{y}_{t+1}\right] - \frac{1}{\sigma} \left(\hat{\imath}_t - E_t \hat{\pi}_{t+1}\right) \\
\hat{\pi}_t &= \beta E_t \hat{\pi}_{t+1} + \chi \widehat{mc}_t \\
\widehat{mc}_t &= \hat{w}_t - \hat{p}_t - \left(\hat{a}_t - \alpha \hat{n}_t\right) \\
\hat{\imath}_t &= \phi \hat{\pi}_t + \hat{\nu}_t \\
\hat{y}_t &= \hat{a}_t + (1 - \alpha)\, \hat{n}_t
\end{aligned}$$

- simplyifing $\widehat{mc}_t$ yields

$$\begin{aligned}
\widehat{mc}_t &= \hat{w}_t - \hat{p}_t - \left(\hat{a}_t - \alpha \hat{n}_t\right) \\
&= \sigma \hat{y}_t + \varphi \hat{n}_t - \left(\hat{a}_t - \alpha \hat{n}_t\right) \\
&= \sigma \hat{y}_t + (\varphi + \alpha)\, \hat{n}_t - \hat{a}_t
\end{aligned}$$

- using the aggregate production function gives

$$\begin{aligned}\widehat{mc}_t &= \sigma\hat{y}_t + (\varphi+\alpha)\,\hat{n}_t - \hat{a}_t \\ &= \sigma\hat{y}_t - \hat{a}_t + \frac{(\varphi+\alpha)}{(1-\alpha)}\,(\hat{y}_t - \hat{a}_t) \\ &= \left[\sigma + \frac{\varphi+\alpha}{1-\alpha}\right]\hat{y}_t - \frac{1+\varphi}{1-\alpha}\hat{a}_t\end{aligned}$$

- replacing $\widehat{mc}_t$ obtains the canonical New Keynesian model with three endogenous variables output, inflation and the nominal interest rate:

$$\begin{aligned}\hat{y}_t &= E_t\left[\hat{y}_{t+1}\right] - \frac{1}{\sigma}\left(\hat{\imath}_t - E_t\hat{\pi}_{t+1}\right) \\ \hat{\pi}_t &= \beta E_t\hat{\pi}_{t+1} + \chi\left[\sigma + \frac{\varphi+\alpha}{1-\alpha}\right]\hat{y}_t - \frac{\chi\,(1+\varphi)}{1-\alpha}\hat{a}_t \\ \hat{\imath}_t &= \phi\hat{\pi}_t + \hat{\nu}_t,\end{aligned}$$

- The competitive equilibrium are sequences $\{\hat{\imath}_t, \hat{y}_t, \hat{\pi}_t\}_{t=0}^{\infty}$ for given exogenous processes $\hat{a}_t$ and $\hat{\nu}_t$.

- For example, technology follows an AR(1) process $\hat{a}_t = \rho \hat{a}_{t-1} + \zeta_t$, with $\zeta_t \sim \mathcal{N}\left(0, 0.1^2\right)$, $\rho \in [0, 1]$ and the monetary policy shock is iid.

- Note that in steady state for the linearized system it holds that $0 = 0$ for all three equations. So, when implementing this system, we do not need to solve for the steady state.

2 Calibration

- Reminder: parameters in the model determine the behavior of households, firms and monetary policy, for example
  - how important are future periods,

  - how sensitive is the labor supply to a change in wages,
  - how much does the nominal interest rate change after a change in inflation

- We have to pick numbers to quantify the behavior. Here, we follow the text book of Galí (2008).

- discount factor $\beta = 0.99$, which implies steady state real annual interest rate of about 4 percent

- production elasticity of labor $1 - \alpha = 1 - 1/3 = 2/3$

- log utility as befor $\sigma = 1$

- Frisch elasticity of labor supply $\varphi = 1$

- demand elasticity $\varepsilon = 6$

- Calvo parameter $\theta = 2/3$ implies average price duration of $1/\left(1-\theta\right) = 3$ quarters

- from estimates and the Taylor principle: $\phi = 1.5$.

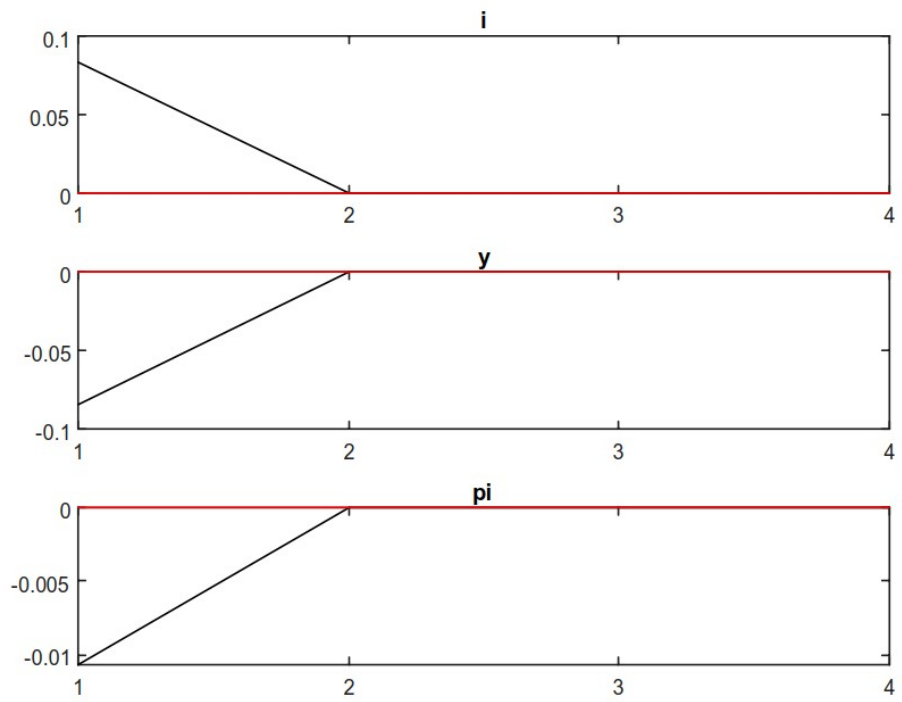
- autocorrelation of the technology shock: $\rho = 0.9$

3 Results

3.1 Monetary policy shock

- analysis of a monetary policy shock of 10%: $\hat{\nu}_t = +0.1$

- the impulse responses of the interest rate, output, inflation for 4 quarters are:



- because inflation falls and monetary policy responds positively to inflation, the actual increase in the interest rate is smaller than the 10% shock

- prices are sticky and the real rate ($\hat{r}_t = \hat{\imath}_t - E_t \hat{\pi}_{t+1}$) increases as the nominal rate rises

- households consume less today and more tomorrow: $\frac{1}{\sigma}(\hat{r}_t) = E_t[\hat{y}_{t+1}] - \hat{y}_t$

- output growth is positive, savings increase

- as current output falls, current inflation declines as well: $\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \chi \left[\sigma + \frac{\varphi+\alpha}{1-\alpha}\right] \hat{y}_t$, (with $\hat{a}_t = 0$)

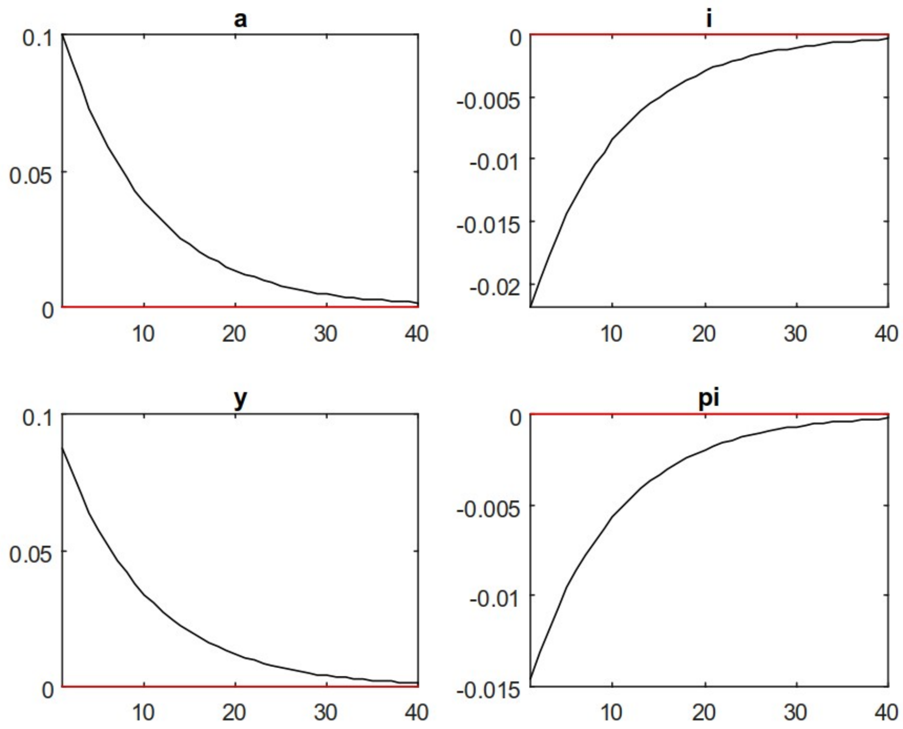
- thereafter, prices adjust so that all variables return to their initial level

- In the long run monetary policy is neutral and only changes the price level.

3.2 Technology shock

- let us analyze a technology shock of one standard deviation, that is, 10%: $\zeta_t = 0.1$

- impulse responses



- now, a_t increases actually by 10% because it is purely exogenous

- the increase in technology and the responses of the other variables are much more persistent because the shock is highly autocorrelated

- inflation falls as higher productivy lowers marginal costs: $\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \chi \left[\sigma + \frac{\varphi+\alpha}{1-\alpha}\right] \hat{y}_t - \frac{\chi(1+\varphi)}{1-\alpha} \hat{a}_t$

- because monetary policy cares only about inflation in our simple model (the systematic component of monetary policy is $\phi \hat{\pi}_t$), the nominal interest rate falls by more than one-for-one with inflation

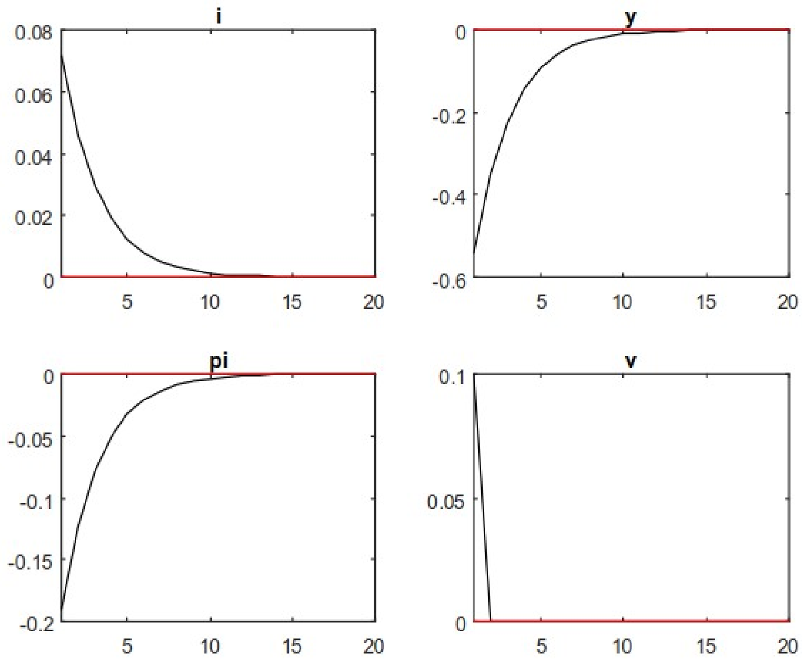
- the real interest rate falls, as well

- housheolds pull consumption forward and output increases

3.3 Extensions

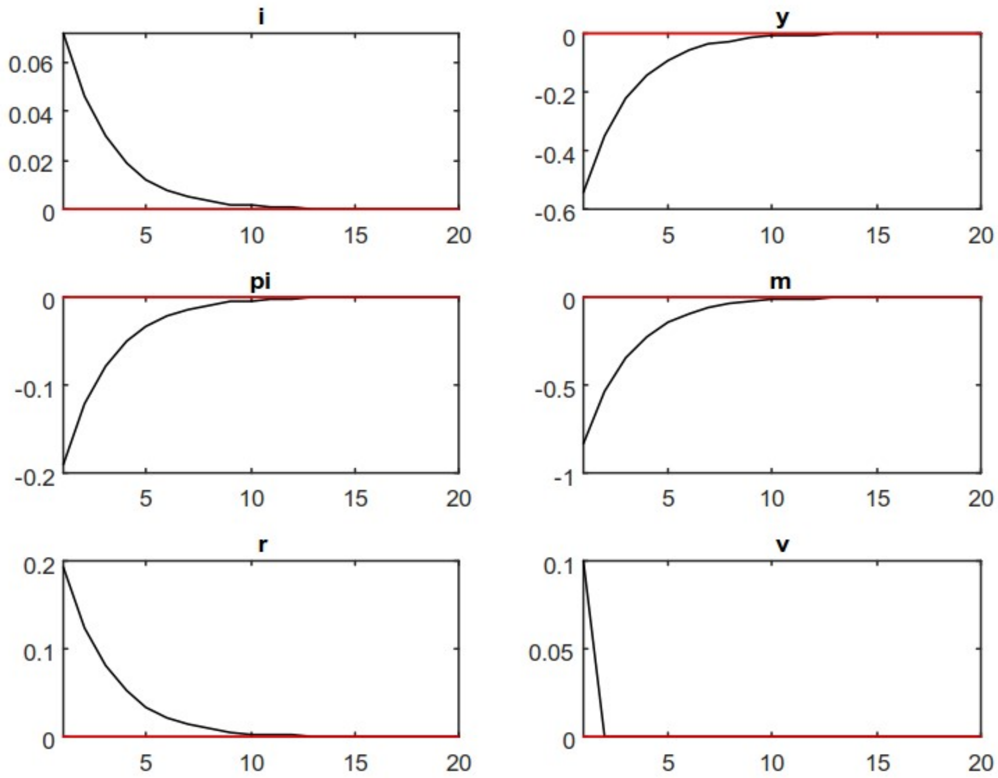
- A modified Taylor rule is $\hat{\imath}_t = \rho_\pi \hat{\imath}_{t-1} + (1 - \rho_\pi)\,\phi\hat{\pi}_t + \hat{\nu}_t$, with $\rho_\pi = 0.9$. This captures the aim of central banks to smooth interest rates and avoid erratic movements in $\hat{\imath}_t$.

- impulse responses to monetary shock



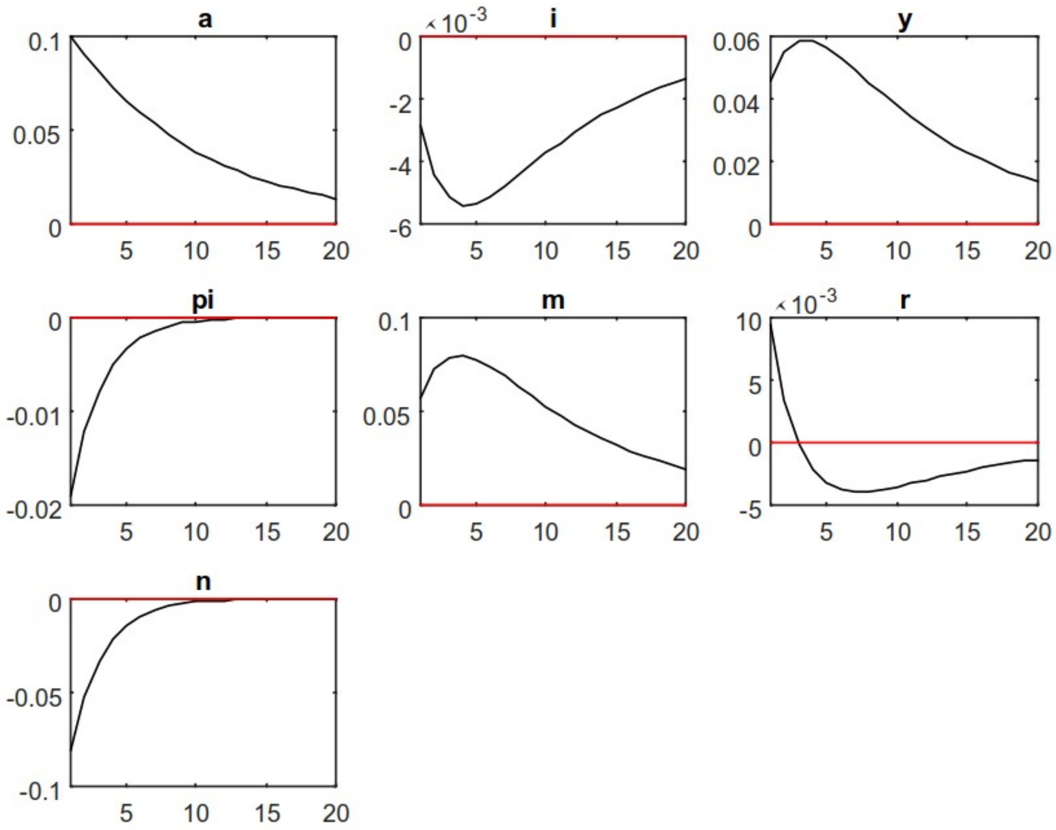
- add the real rate $\hat{r}_t = \hat{\imath}_t - E_t \hat{\pi}_{t+1}$ and real money demand $\hat{m}_t = \hat{y}_t - \eta \hat{\imath}_t$ to the model, with $\eta = 4$

- impulse responses to monetary policy shock



- re-introduce labor $\hat{y}_t = \hat{a}_t + (1-\alpha)\,\hat{n}_t$

- impulse responses to a technology shock



4 Summary

- the canonical New Keynesian model consists of three equations for output, inflation and the nominal interest rate

- the linearized version is simple and intuitive:

  - the New Keynesian Phillips curve determines inflation, given the output gap (and technology)

  - the Euler equation determines demand today and tomorrow, given nominal and real interest rates

  - monetary policy sets the short-term interest rate

- contractionary monetary policy shocks lead to a decline in inflation and output, consistent with empirical evidence

- We have built a model for the interest rate channel.

- favorable technology shocks lead to an increase in output and a fall of prices

- monetary policy shocks are sometimes called demand shocks because prices and output respond in the same direction

- technology shocks are sometimes called supply shocks, because output and prices respond in opposite directions

- we can modify and extend the model to include other channels of monetary policy, for example, the balance sheet channel or the exchange rate channel