❖ DIGITAL IMAGE PROCESSING SYSTEMS

Ω UNIT-II

Frequency Domain: Smoothing Frequency Domain Filters – Sharpening Frequency Domain Filters Homomorphic Filtering. Image Restoration: Image Restoration Model – Degradation Model Noise Models Restoration in the Presence of Noise Estimating the Degradation Function – Inverse Filtering – Wiener Filter.

❖ Smoothing frequency domain filters

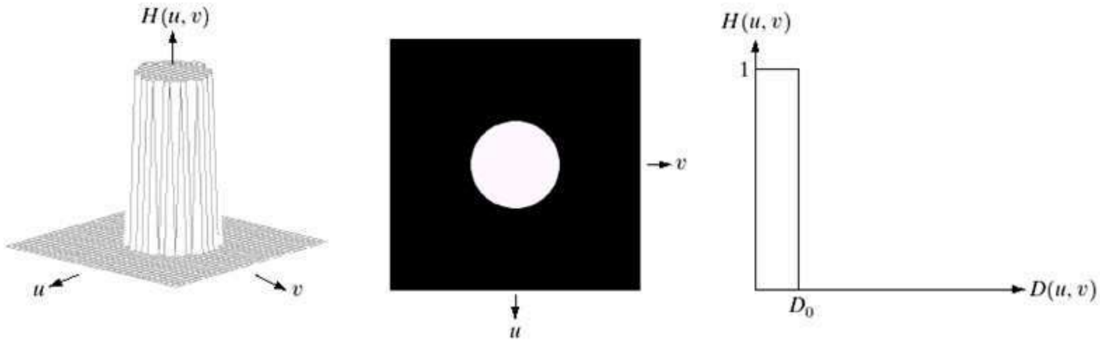
Ideal Lowpass Filter (ILPF)

ILPF is the simplest lowpass filter that "cuts off" all high frequency components of the DFT that are at a distance greater than a specified distance D_0 from the origin of the (centered) transform.

The transfer function of this filter is:

$$H(u,v) = \begin{cases} 1 & if \;\; D(u,v) \le D_0 \\ 0 & if \;\; D(u,v) > D_0 \end{cases}$$

where D_0 is the cutoff frequency, and $D(u,v) = \sqrt{(u - M/2)^2 + (v - N/2)^2}$



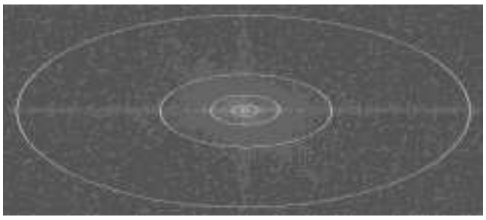
(a) (b) (c)

Figure 7.1 (a) Ideal lowpass filter. (b) ILPF as an image. (c) ILPF radial cross section

The ILPF indicates that all frequencies inside a circle of radius D_0 are passed with no attenuation, whereas all frequencies outside this circle are completely attenuated.

The next figure shows a gray image with its Fourier spectrum. The circles superimposed on the spectrum represent cutoff frequencies 5, 15, 30, 80 and 230.

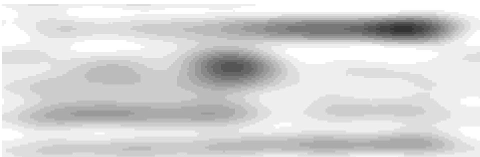
(a) Original image.



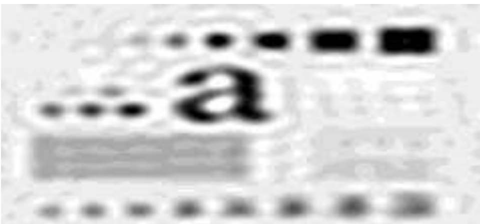
(b) its Fourier spectrum

The figure below shows the results of applying ILPF with the previous cutoff frequencies.

(a)



(b)



(c)

(d)

(e)

(f)

Figure 7.3 (a) Original image. (b) - (f) Results of ILPF with cutoff frequencies 5, 15, 30, 80,and 230 respectively.

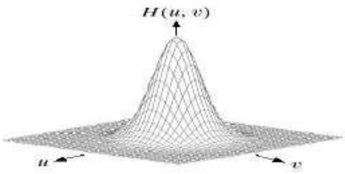
We can see the following effects of ILPF:

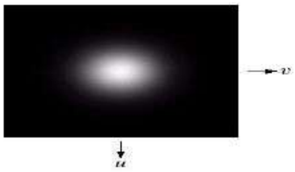
1. Blurring effect which decreases as the cutoff frequency increases.
2. Ringing effect which becomes finer (i.e. decreases) as the cutoff frequency increases.

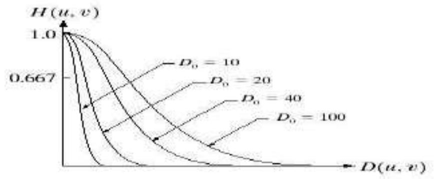
❖ Gaussian Lowpass Filter (GLPF)

The GLPF with cutoff frequency D_0 is defined as:

$$H(u,v) = e^{-D^2(u,v)/2D_0{}^2}$$







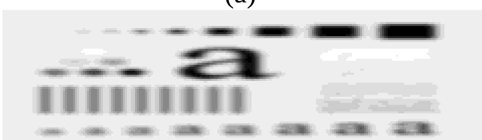
(a) Gaussian lowpass filter. (b) GLPF as an image. (c) GLPF radial cross section

Unlike ILPF, the GLPF transfer function does not have a sharp transitionthat establishes a clear cutoff between passed and filtered frequencies. Instead, GLPF has a smooth transition between low and high frequencies.

results of applying GLPF on the image with the same previous cutoff frequencies.

(a)

(b)



(c)

(d)

(e)

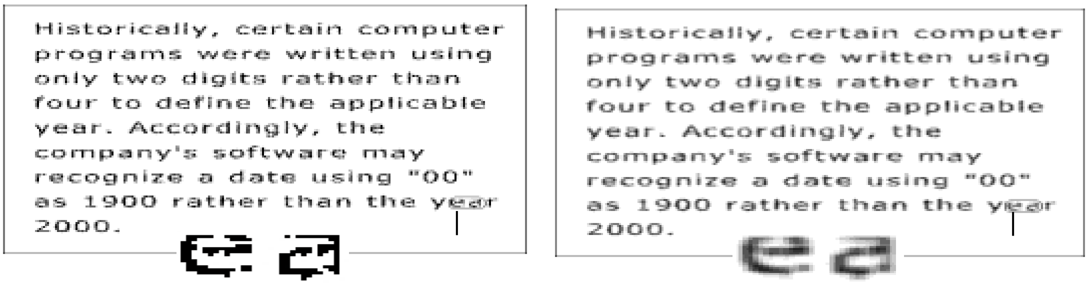
(f)

Figure 7.5 (a) Original image. (b) - (f) Results of GLPF with cutoff frequencies 5, 15, 30, 80,and 230 respectively.

We can see the following effects of GLPF compared to ILPF:

1. Smooth transition in blurring as a function of increasing cutoff frequency.
2. No ringing effect.

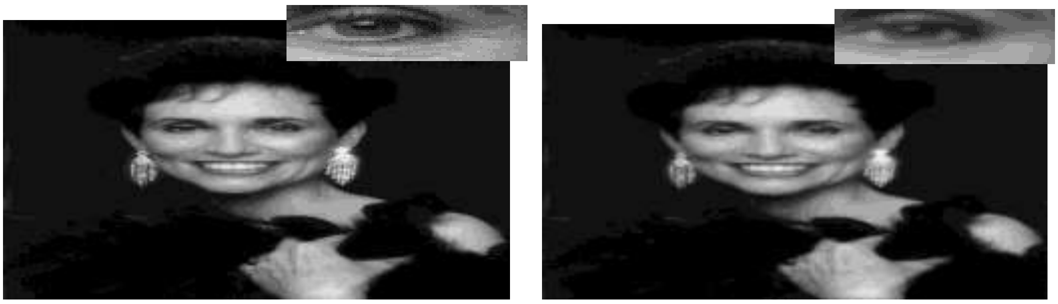
Smoothing (lowpass) filtering is useful in many applications. For example, GLPF can be used to bridge small gaps in broken characters byblurring it as shown below. This is useful for character recognition.



(a) (b)

Figure 7.6 (a) Text of poor resolution. (b) Result of applying GLPF with cutoff=80 on (a)

GLPF can also be used for cosmetic processing prior to printing and publishing as shown below.



(a) Original image. (b) Result of filtering with GLPF with cutoff=80

❖ Sharpening frequency domain filters

Edges and sudden changes in gray levels are associated with high frequencies. Thus to enhance and sharpen significant details we need to use highpass filters in the frequency domain

For any lowpass filter there is a highpass filter:

$$H_{hp}(u, v) = 1 - H_{lp}(u, v)$$

❖ Ideal Highpass Filter (IHPF)

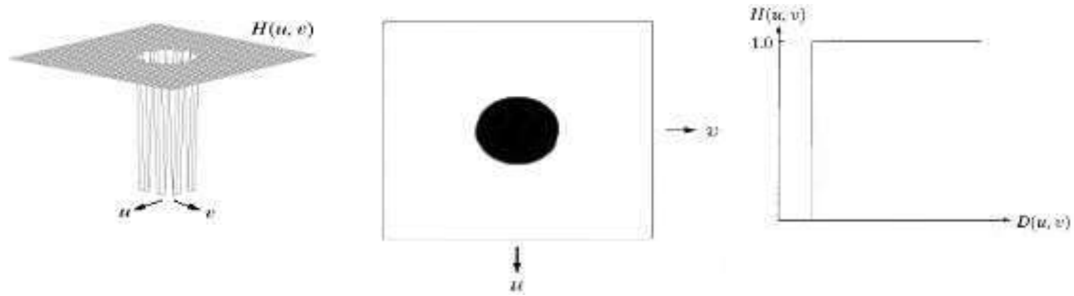
The IHPF cuts off all low frequencies of the DFT but maintain the highones

$$H(u,v) = \begin{cases} 1 & if \;\; D(u,v) > D_0 \\ 0 & if \;\; D(u,v) \le D_0 \end{cases}$$

that are within a certain distance from the center of the DFT.

$$D(u,v) = \sqrt{(u - M/2)^2 + (v - N/2)^2}$$

where D_0 is the cutoff frequency, and



(a) Ideal highpass filter. (b) IHPF as an image. (c) IHPF radial cross section

The IHPF sets to zero all frequencies inside a circle of radius D_0 while passing, without attenuation, all frequencies outside the circle.

The figure below shows the results of applying IHPF with cutoff frequencies 15, 30, and 80.

(a)

(b)

(c)

(d)

(a) Original image. (b) - (d) Results of IHPF with cutoff frequencies 15, 30, and 80respectively.

We can see the following effects of IHPF:

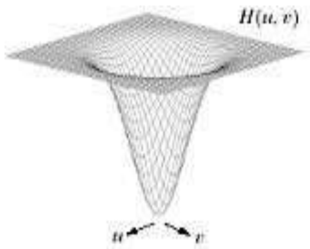
1. Ringing effect.
2. Edge distortion (i.e. distorted, thickened object boundaries).

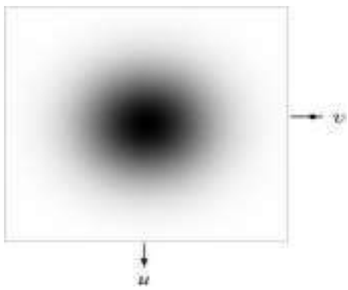
Both effects are decreased as the cutoff frequency increases.

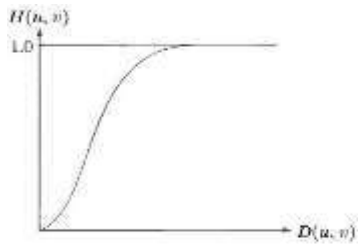
❖ <u>Gaussian Highpass Filter (GHPF)</u>

The Gaussian Highpass Filter (GHPF) with cutoff frequency at distance D_0 is defined as:

$$H(u,v) = 1 - e^{-D^2(u,v)/2{D_0}^2}$$







(a) Gaussian highpass filter. (b) GHPF as an image. (c) GHPF radial cross section

The figure below shows the results of applying GHPF with cutoff frequencies 15, 30 and 80.

(a)

(b)

(c)

(d)

(a) Original image. (b) - (d) Results of GHPF with cutoff frequencies 15, 30, and80 respectively.

The effects of GHPF in comparison with IHPF are:

1. No ringing effect.
2. Less edge distortion.
3. The results are smoother than those obtained by IHPF.

❖ Homomorphic filtering:

The illumination-reflectance model can be used to develop a frequency domain procedure for improving the appearance of an image by simultaneous gray-level range compression and contrast enhancement. An image f(x, y) can be expressed as the product of illumination and reflectance components:

$$f(x, y) = i(x, y)r(x, y).$$

Equation above cannot be used directly to operate separately on the frequency components of illumination and reflectance because the Fourier transform of the product of two functions is not separable; in other words,

where F_i (u, v) and F_r (u, v) are the Fourier transformation

$$\Im\{f(x, y)\} \neq \Im\{i(x, y)\}\Im\{r(x, y)\}.$$

Suppose, however, that we define

$$\begin{aligned} z(x, y) &= \ln f(x, y) \\ &= \ln i(x, y) + \ln r(x, y). \end{aligned}$$

Then

$$\begin{aligned} \Im\{z(x, y)\} &= \Im\{\ln f(x, y)\} \\ &= \Im\{\ln i(x, y)\} + \Im\{\ln r(x, y)\} \end{aligned}$$

or

$$Z(u, v) = F_i(u, v) + F_r(u, v)$$

sforms of ln i(x, y) and ln r(x, y), respectively. If we process Z (u, v) by means of a filter function H (u, v) then, from

$$\begin{aligned} S(u, v) &= H(u, v)Z(u, v) \\ &= H(u, v)F_i(u, v) + H(u, v)F_r(u, v) \end{aligned}$$

where S (u, v) is the Fourier transform of the result. In the spatial domain,

$$s(x, y) = \Im^{-1}\{S(u, v)\}$$
$$= \Im^{-1}\{H(u, v)F_i(u, v)\} + \Im^{-1}\{H(u, v)F_r(u, v)\}.$$

By letting

$$i'(x, y) = \Im^{-1}\{H(u, v)F_i(u, v)\}$$

and

$$r'(x, y) = \Im^{-1}\{H(u, v)F_r(u, v)\},$$

Now we have $$s(x, y) = i'(x, y) + r'(x, y).$$

Finally, as z (x, y) was formed by taking the logarithm of the original image f (x, y), the inverse (exponential) operation yields the desired enhanced image, denoted by g(x, y); that is,

$$g(x, y) = e^{s(x, y)}$$
$$= e^{i'(x, y)} \cdot e^{r'(x, y)}$$
$$= i_0(x, y)r_0(x, y)$$

where

$$i_0(x, y) = e^{i'(x, y)}$$

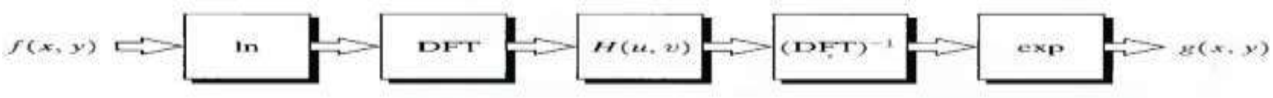


Fig.9.1 Homomorphic filtering approach for image enhancement

and

$$r_0(x, y) = e^{r'(x, y)}$$

- Are the illumination and reflectance components of the output image. The enhancement approach using the foregoing concepts is summarized in Fig. 9.1.
- **This method is based on a special case of a class of systems known as homomorphic systems.** In this particular application, the key to the approach is the separation of the illumination and reflectance components achieved. The homomorphic filter **function H (u, v)** can then operate on these components separately.
- The illumination component of an image generally is characterized by slow spatial variations, while the reflectance component tends to vary abruptly, particularly at the junctions of dissimilar objects.
- **These characteristics lead to associating the low frequencies of the Fourier transform of the logarithm of an image with illumination and the high frequencies with reflectance.**
- These associations are rough approximations, they can be used to advantage in image enhancement.
- A good deal of control can be gained over the illumination and reflectance components with a homomorphic filter.

- **This control requires specification of a filter function H (u, v) that affects the low- and high-frequency components of the Fourier transform in different ways.** Figure 9.2 shows a cross section of such a filter. If the parameters γ_L and γ_H are chosen so that $\gamma_L < 1$ and $\gamma_H > 1$, the filter function shown in Fig. 9.2 tends to decrease the contribution made by the low frequencies (illumination) and amplify the contribution made by high frequencies (reflectance). The net result is simultaneous dynamic range compression and contrast enhancement.

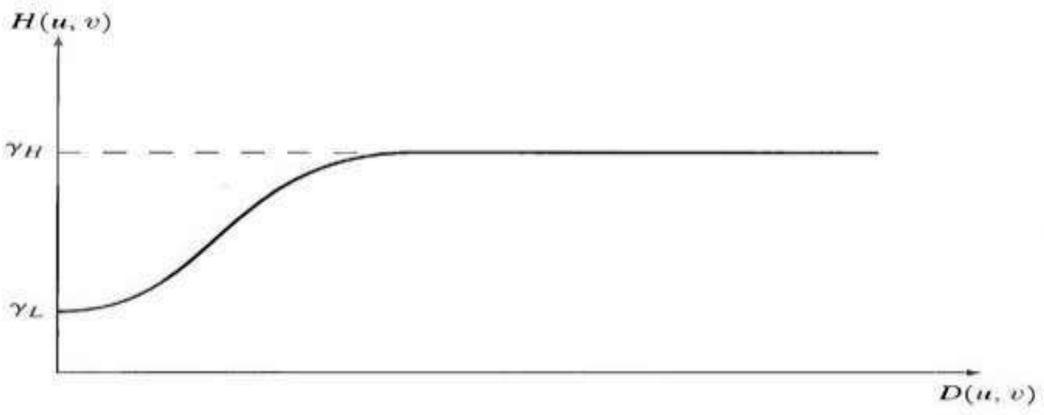


Fig.9.2 Cross section of a circularly symmetric filter function D (u. v) is the distance from the origin of the centered transform.

❖ Explain a Model of the Image Degradation/Restoration Process.

❖ Digital Image Processing

The degradation process is modeled as a degradation function that, together with an additive noise term, operates on an input image f(x, y) to produce a degraded image g(x, y). Given g(x, y), some knowledge about the degradation function H, and some knowledge about the additive noise term η(x, y),

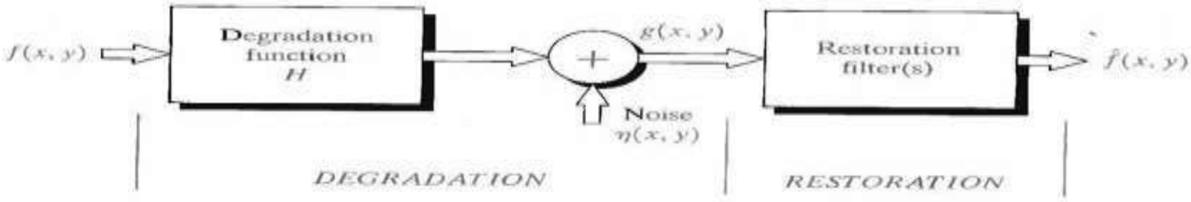
The objective of restoration is to obtain an estimate *f*(x, y) of the original image. the estimate should be as close as possible to the original input image and, in general, the more we know about H and η, the closer *f*(x, y) will be to f(x, y).The degraded image is given in the spatial domain by

$$g(x, y) = h(x, y) * f(x, y) + \eta(x, y)$$

where h (x, y) is the spatial representation of the degradation function and, the symbol * indicates convolution. Convolution in the spatial domain is equal to multiplication in the frequency domain, hence

G (u, v) = H (u, v) F (u, v) + N (u, v)

where the terms in capital letters are the Fourier transforms of the corresponding terms in above equation.



model of the image degradation/restoration process.

❖ Noise Models in Digital Image Processing

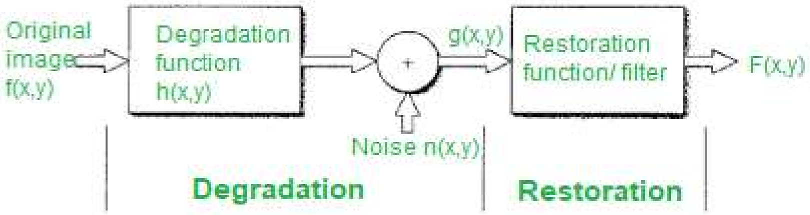
The principal source of noise in digital images arises during image acquisition and transmission. The performance of imaging sensors is affected by a variety of environmental and mechanical factors of the instrument, resulting in the addition of undesirable noise in the image. Images are also corrupted during the transmission process due to non-ideal channel characteristics.

Generally, a mathematical model of image degradation and its restoration is used for processing. The figure below shows the presence of a degradation function *h(x,y)* and an external noise *n(x,y)* component coming into the original image signal *f(x,y)* thereby producing a final degraded image *g(x,y)*. This part composes the degradation model. Mathematically we can write the following :

$$g(x, y) = h(x, y) * f(x, y) + n(x, y)$$

Where * indicates convolution in the spatial domain.

The goal of the restoration function or the restoration filter is to obtain a close replica *F(x,y)* of the original image.



The external noise is probabilistic in nature and there are several noise models used frequently in the field of digital image processing. We have several probability density functions of the noise.

❖ Noise Models

Gaussian Noise

Because of its mathematical simplicity, **The Gaussian noise model is often used in practice and even in situations where they are marginally applicable at best. Here, m is the mean and σ^2 is the variance.**

Gaussian noise arises in an image due to factors such as electronic circuit noise and sensor noise due to poor illumination or high temperature.

$$p(z) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(z-m)^2}{2\sigma^2}}$$

Rayleigh Noise

$$p(z) = \frac{2}{b}(z-a)e^{\frac{-(z-a)^2}{b}} for z \geq a, and p(z) = 0 otherwise.$$

Here mean m and variance σ^2 are the following:

$$m = a + \sqrt{\pi b/4}$$

$$\sigma^2 = \frac{b(4-\pi)}{4}$$

Rayleigh noise is usually used to characterize noise phenomena in range imaging.

Erlang (or gamma) Noise

$$p(z) = \frac{a^b z^{b-1}}{(b-1)!} e^{-az} for z \geq 0 and p(z) = 0 otherwise.$$

Here ! indicates factorial. The mean and variance are given below.

$$m = b/a, 2 = b/a2$$

Gamma noise density finds application in laser imaging.

Exponential Noise

$$p(z) = ae^{-az} for z \geq 0 and p(z) = 0 otherwise.$$

Here a > 0. The mean and variance of this noise pdf are:

$$m = 1/a$$
$$2 = 1/\mathrm{a}2$$

This density function is a special case of b = 1.

Exponential noise is also commonly present in cases of laser imaging.

Uniform Noise

$$p(z) = \frac{1}{b-a} ifa \leq z \leq b, andp(z) = 0otherwise.$$

The mean and variance are given below.

$$m = \frac{a+b}{2}$$
$$\sigma^2 = \frac{(b-a)^2}{12}$$

Uniform noise is not practically present but is often used in numerical simulations to analyze systems.

Impulse Noise

$$p(z) = Paforz = a, p(z) = Pbforz = b, p(z) = 0otherwise.$$

If b > a, intensity b will appear as a light dot in the image. Conversely, level a will appear like a black dot in the image. Hence, this presence of white and black dots in the image resembles to salt-and-pepper granules, hence also called salt-and-pepper noise. When either P_a or P_b is zero, it is called unipolar noise. The origin of impulse noise is quick transients such as faulty switching in cameras or other such cases.

❖ Restoration in the presence of Noise only- Spatial filtering:

When the only degradation present in an image is noise, i.e.

$g(x,y)=f(x,y)+\eta(x,y)$ or

$G(u,v)= F(u,v)+ N(u,v)$

The noise terms are unknown so subtracting them from g(x,y) or G(u,v) is not a realistic approach. In the case of periodic noise it is possible to estimate N(u,v) from the spectrumG(u,v).

So N(u,v) can be subtracted from G(u,v) to obtain an estimate of original image. Spatial filtering can be done when only additive noise is present. The following techniques can be used to reduce the noise effect:

i) Mean Filter:

ii) (a)Arithmetic Mean filter:

It is the simplest mean filter. Let Sxy represents the set of coordinates in the sub image of size m*n centered at point (x,y). The arithmetic mean filter computes the average value of the corrupted image g(x,y) in the area defined by Sxy. The value of the restored image f at any point (x,y) is the arithmetic mean computed

using the pixels in the region defined by Sxy.

This operation can be using a convolution mask in which all coefficients have value 1/mn. A mean filter smoothes local variations in image Noise is reduced as a result of blurring. For every pixel in the image, the pixel value is replaced by the mean value of its neighboring pixels with a weight .This will resulted in a smoothing effect in the image.

(b) Geometric Mean filter:

An image restored using a geometric mean filter is given by the expression

$$\hat{f}(x,y)=\left(\prod_{(s,t)\in Sxy} g(s,t)\right)^{1/mn}$$

This filter is useful for flinging the darkest point in image. Also, it reduces salt noise of the min operation.

(c) Midpoint filter:

The midpoint filter simply computes the midpoint between the maximum and minimum values in the area encompassed by

$$\hat{f}(x,y)=\left(\max_{(s,t)\in Sxy}\{g(s,t)\}+\min_{(s,t)\in Sxy}\{g(s,t)\}\right)/2$$

It comeliness the order statistics and averaging .This filter works best for randomly distributed noise like Gaussian or uniform noise.

(d) Harmonic Mean filter:

The harmonic mean filtering operation is given by the expression

$$\hat{f}(x,y)=\sum_{(s,t)\in Sxy} g(s,t)^{Q+1} \Big/ \sum_{(s,t)\in Sxy} g(s,t)^{Q}$$

The harmonic mean filter works well for salt noise but fails for pepper noise. It does well with Gaussian noise also.

(c) Order statistics filter:

Order statistics filters are spatial filters whose response is based on ordering the pixel contained in the image area encompassed by the filter. The response of the filter at any point is determined by the ranking result.

Median filter:

It is the best order statistic filter; it replaces the value of a pixel by the median of gray levels in the Neighborhood of the pixel.

$$\hat{f}(x,y) = \underset{(s,t)\in Sxy}{\text{median}}\{g(s,t)\} \qquad \hat{f}(x,y) = \left(\prod_{(s,t)\in Sxy} g(s,t)\right)^{1/mn}$$

The original of the pixel is included in the computation of the median of the filter are quite possible because for certain types of random noise, the provide excellent noise reduction capabilities with considerably less blurring then smoothing filters of similar size. These are effective for bipolar and unipolor impulse noise.

Max and Min filter:

Using the l00th percentile of ranked set of numbers is called the max filter and is given by the equation

$$\hat{f}(x,y) = \left(\max_{(s,t)\in Sxy}\{g(s,t)\} + \min_{(s,t)\in Sxy}\{g(s,t)\}\right)\Big/2$$

It is used for finding the brightest point in an image. Pepper noise in the image has very low values, it is reduced by max filter using the max selection process in the sublimated area skys. The 0th percentile filter is min filter.

$$\hat{f}(x,y) = \sum_{(s,t)\in Sxy} g(s,t)^{Q+1} \Big/ \sum_{(s,t)\in Sxy} g(s,t)^{Q}$$

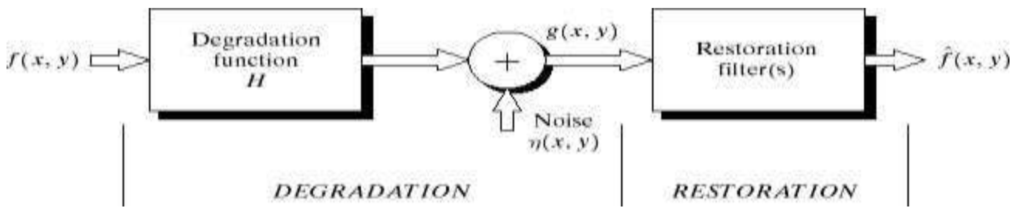
IMAGE RESTORATION AND RECONSTRUCTION

Introduction

Image Restoration is the process to recover an image that has been degraded by using a priori knowledge of the degradation phenomenon. These techniques are oriented toward modeling the degradation and applying the inverse process to recover the original image. Restoration improves image in some predefined sense. Image enhancement techniques are subjective process, where as image restoration techniques are objective process.

3.1 A Model of Image Degradation/Restoration Process

Image Degradation process operates on a degradation function that operates on an input image with an additive noise term to produce degraded image. The image degradation model is shown below.



A Model of Image Degradation/Restoration Process

Let $f(x, y)$ is an input image and $g(x, y)$ is the degraded image with some knowledge about the degradation function H and some knowledge about the additive noise term $\eta(x, y)$. The objective of the restoration is to obtain an estimate $f^{\wedge}(x, y)$ of the original image. If H is a linear, position-invariant process, then the degraded image is given in the spatial domain by

$$g(x,y)=f(x,y)*h(x,y)+\eta(x,y)$$

Where $h(x, y)$ is the spatial representation of the degraded function. The degrade image in frequency domain is represented as

$$G(u,v)=F(u,v)H(u,v)+N(u,v)$$

The terms in the capital letters are the Fourier Transform of the corresponding termsin the spatial domain.

3.2 Noise Models

The principle sources of noise in digital image are due to image acquisition and transmission.

- During image acquisition, the performance of image sensors gets affected by a varietyof factors such as environmental conditions and the quality of sensing elements.
- During image transmission, the images are corrupted due to the interference introduced in the channel used for transmission.

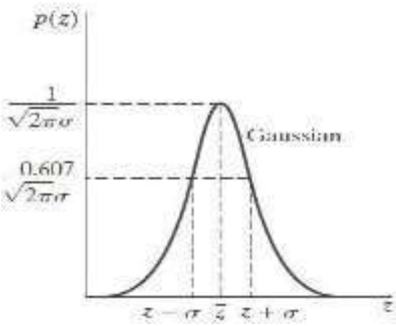
The Noise components are considered as random variables, characterized by a probability density function. The most common PDFs found in digital image processing applications are given below.

Gaussian Noise

Gaussian noise is also known as „normal" noise. The Probability density function of aGaussian random variable z is given by

$$p(z) = \frac{1}{\sqrt{2\pi\sigma}} e^{-(z-\bar{z})^2/2\sigma^2}$$

Where z represents intensity, $\bar{z}$ is the mean and σ is its standard deviation and its square (σ^2) is called the variance of z. The values of Gaussian noise is approximately 70% will be in the range $[(\bar{z} - \sigma), (\bar{z} + \sigma)]$ and 95% will be in the range $[(\bar{z} - 2\sigma), (\bar{z} + 2\sigma)]$.

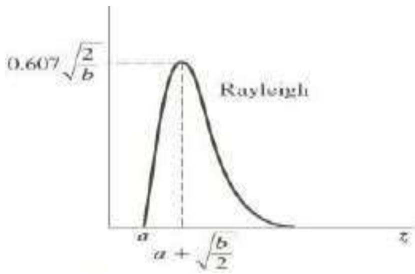


Rayleigh Noise

The PDF of Rayleigh Noise is given by

$$p(z) = \begin{cases} \frac{2}{b}(z-a)e^{(z-a)^2/b} & \text{for } z \geq a \\ 0 & \text{for } z < a \end{cases}$$

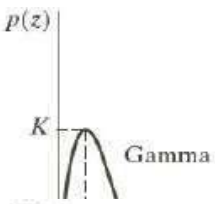
$$\text{Mean:} \quad \bar{z} = a + \sqrt{\pi b/4} \qquad \text{Variance:} \quad \sigma^2 = \frac{b(4-\pi)}{4}$$



Applications:

- It is used for characterizing noise phenomenon in range imaging.
- It describes the error in the measurement instrument.
- It describes the noise affected in radar.
- It determines the noise occurred when the signal is passed through the band passfilter.

Erlang (gamma) Noise



The probability der is given by

$$p(z) = \begin{cases} \frac{a^b z^{b-1}}{(b-1)!}e^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases} \qquad \begin{array}{l} a > 0 \\ \text{b positive integer} \end{array}$$

$$\text{Mean:} \quad \bar{z} = \frac{b}{a} \qquad \text{Variance:} \quad \sigma^2 = \frac{b}{a^2}$$

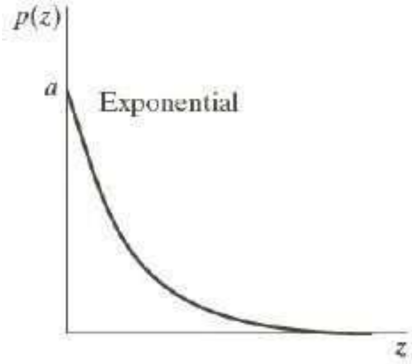
Exponential Noise

The probability density function of Exponential noise is given by

$$p(z) = \begin{cases} ae^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases}$$

p(z) is maximum at z=0

Mean: $z = \frac{1}{a}$ Variance: $\sigma^2 = \frac{1}{a^2}$



Applications:

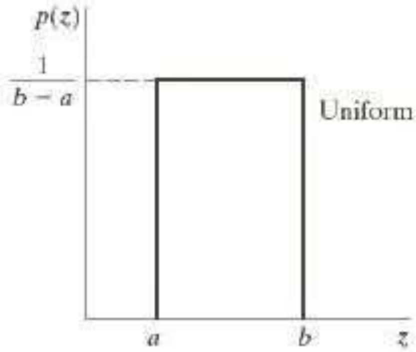
- It is used to describe the size of the raindrop.
- It is used to describe the fluctuations in received power reflected from certain targets
- It finds application in Laser imaging.

Uniform Noise

The probability density function of Uniform noise is given by

$$p(z) = \begin{cases} \frac{1}{b-a} & \text{if } a \leq z \leq b \\ 0 & \text{otherwise} \end{cases}$$

Mean: $\bar{z} =$ $a + b$ $\frac{(b-a)^2}{12}$

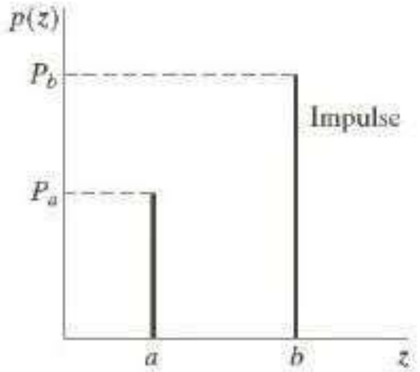


Salt and Pepper Noise (Impulse Noise)

The probability density function of Salt and Impulse noise is given by

$$p(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & \text{for } z = b \\ 0 & \text{otherwise} \end{cases}$$

$$P_a = P_b \Rightarrow \textit{unipolar} \text{ noise}$$



If b>a, gray level b will appear as a light dot in image. Level a will appear like a dark dot. The salt and pepper noise is also called as bi-polar impulse noise or Data-drop-out and spike noise.

Periodic Noise

Periodic noise in an image occurred from electrical or elecrtomechnaical interference during image acquisition. This is the only type of spatially dependent noise and the parameters are estimated by the Fourier spectrum of the image. Periodic noise tends to produce frequency spikes that often can be detected even by visual analysis. The mean and variance are defined as

$$\bar{z} = \sum_{i=0}^{L-1} z_i p_S(z_i)$$

$$\sigma^2 = \sum_{i=0}^{L-1} (z_i - \bar{z})^2 p_S(z_i)$$

❖ *Types of Restoration Filters*

Restoration Filters are the type of filters that are used for operation of noisy image and estimating the clean and original image. It may consists of processes that are used for blurring or the reverse processes that are used for inverse of blur. Filter used in restoration is different from the filter used in enhancement process.

- **Types of Restoration Filters:** There are three types of Restoration Inverse Filter, Pseudo Inverse Filter, Wiener Filter.

1. Inverse Filter: Inverse Filtering is the process of receiving the input of a system from its output. It is the simplest approach to restore the original image once the degradation function is known.

H'(u, v) = 1 / H(u, v)

Let,

F'(u, v) -> Fourier transform of the restored image

G(u, v) -> Fourier transform of the degraded image

H(u, v) -> Estimated or derived or known degradation function

then **F'(u, v) = G(u, v)/H(u, v)**

where, G(u, v) = F(u, v).H(u, v) + N(u, v)

and F'(u, v) = f(u, v) - N(u, v)/H(u, v)

Note: Inverse filtering is not regularly used in its original form.

2. Pseudo Inverse Filter:

Pseudo inverse filter is the modified version of the inverse filter and stabilized inverse filter. Pseudo inverse filtering gives more better result than inverse filtering but both inverse and pseudo inverse are sensitive to noise. Pseudo inverse filtering is defined as:

H'(u, v) = 1/H(u, v), H(u, v)!=0

H'(u, v) = 0, otherwise

3. Wiener Filter:

(Minimum Mean Square Error Filter). Wiener filter executes and optimal trade off between filtering and noise smoothing. IT removes the addition noise and inputs in the blurring simultaneously. Weiner filter is real and even. It minimizes the overall mean square error by:

$e^2 = F\{(f-f')^2\}$

where, f -> original image

f' -> restored image

E{.} -> mean value of arguments

$H(u, v) = H'(u, v)/(|H(u, v)|^2 + (Sn(u, v)/Sf(u, v))$

where H(u, v) -> Transform of degradation function

Sn(u, v) -> Power spectrum of the noise

Sf(u, v) -> Power spectrum of the undergraded original image

No blur only additive noise:

$H(u, v)=1$

$W(u, v) = 1 / (1 + Sn(u, v)/Sf(u, v))$

$W(u, v) = SNR/(1 + SNR)$

where, $SNR = Sf(u, v)/Sn(u, v)$

No noise only blur:

$Sn(u, v)=0$

$W(u, v) = 1/H(u, v)$

❖ Drawbacks of Restoration Filters:

- Not effective when images are restored for the human eye.
- Cannot handle the common cause of non-stationary signals and noise.
- Cannot handle spatially variant blurring point spread function.

3.3 Restoration in the Presence of Noise only-Spatial Filtering

When the only degradation present in an image is noise,

$$g(x, y) = f(x,y)+ \eta(x,y)$$

and

$$G (u, v) = F (u, v) + N (u, v)$$

The noise terms are unknown so subtracting them from g(x, y) or G (u, v) is not a realistic approach. In the case of periodic noise it is possible to estimate N (u, v) from the spectrum G (u, v). So N (u, v) can be subtracted from G (u, v) to obtain an estimate of original image. Spatial filtering can be done when only additive noise is present.

<u>Mean Filters</u>

Arithmetic Mean Filter:

It is the simplest mean filter. Let S_{xy} represents the set of coordinates in the sub image of size m*n centered at point (x, y). The arithmetic mean filter computes the average value of the corrupted image g(x, y) in the area defined by S_{xy}. The value of the restored image f at any point (x, y) is the arithmetic mean computed using the pixels in the region defined by S_{xy}.

$$\hat{f}(x, y) = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} g(s, t)$$

This operation can be using a convolution mask in which all coefficients have value 1/mn. A mean filter smoothes local variations in image Noise is reduced as a result of blurring.

Geometric Mean Filter:

An image restored using a geometric mean filter is given by the expression

$$\hat{f}(x, y) = \left[\prod_{(s,t)\in S_{xy}} g(s, t) \right]^{\frac{1}{mn}}.$$

Here, each restored pixel is given by the product of the pixel in the sub-image window, raised to the power 1/mn. A Geometric means filter achieves smoothing comparable to the arithmetic mean filter, but it tends to lose image details in the process.

Harmonic Mean Filter:

The harmonic mean filtering operation is given by the expression

$$\hat{f}(x, y) = \frac{mn}{\displaystyle\sum_{(s,t)\in S_{xy}} \frac{1}{g(s, t)}}.$$

The harmonic mean filter works well for salt noise but fails for pepper noise. It doeswell also with other types of noise.

Contra harmonic Mean Filter:

The contra harmonic mean filter yields a restored image based on the expression

$$\hat{f}(x, y) = \frac{\displaystyle\sum_{(s,t)\in S_{xy}} g(s, t)^{Q+1}}{\displaystyle\sum_{(s,t)\in S_{xy}} g(s, t)^{Q}}$$

Where Q is called the order of the filter and this filter is well suited for reducing the effects of salt and pepper noise. For positive values of Q the filter eliminates pepper noise. For negative values of Q it eliminates salt noise. It cannot do both simultaneously. The contra harmonic filter reduces to arithmetic mean filter if Q=0 and to the harmonic filter if Q= -1.

Order-Static Filters

Order statistics filters are spatial filters whose response is based on ordering the pixel contained in the image area encompassed by the filter. The response of the filter at any point is determined by the ranking result.

Median Filter:

It is the best known order statistic filter. It replaces the value of a pixel by the median of gray levels in the Neighborhood of the pixel.

$$\hat{f}(x, y) = \underset{(s,t)\in S_{xy}}{\text{median}}\{g(s, t)\}$$

The value of the pixel at *(x, y)* is included in the computation of the median. Median filters are quite popular because for certain types of random noise, they provide excellent noise reduction capabilities with considerably less blurring than smoothing filters of similar size. These are effective for bipolar and unipolar impulse noise.

❖ Max and Min Filters:

The median filter represents the 50^{th} percentile of a ranked set of numbers. If using the 100^{th} percentile results is called "Max Filter". It can be defined as

$$\hat{f}(x, y) = \max_{(s,t)\in S_{xy}} \{g(s, t)\}$$

This filter is used for finding the brightest point in an image. Pepper noise in theimage has very low values, it is reduced by max filter using the max selection process in the sublimated area S_{XY}.

The 0^{th} percentile filter is min filter

$$\hat{f}(x, y) = \min_{(s,t)\in S_{xy}} \{g(s, t)\}$$

This filter is useful for flinging the darkest point in image. Also, it reduces salt noiseas a result of the min operation.

Midpoint Filter:

The midpoint filter simply computes the midpoint between the maximum andminimum values in the area encompassed by the filter.

$$\hat{f}(x, y) = \frac{1}{2}\left[\max_{(s,t)\in S_{xy}} \{g(s, t)\} + \min_{(s,t)\in S_{xy}} \{g(s, t)\}\right].$$

It combines the order statistics and averaging .This filter works best for randomlydistributed noise like Gaussian or uniform noise.

Alpha-trimmed mean Filter:

If we delete the d/2 lowest and the d/2 highest intensity values of *g(s, t)* in the neighborhood S_{XY}. Let $g_r(s, t)$ represents the remaining *mn-d* pixels. A filter formed by averaging the reaming pixels is called alpha-trimmed mean filter.

$$\hat{f}(x, y) = \frac{1}{mn - d} \sum_{(s,t)\in S_{xy}} g_r(s, t)$$

The value of d can range from 0 to *mn*-1. If d=0 this filter reduces to arithmetic mean filter. If d=*mn*-1, the filter becomes a median filter. For the other values of d the alpha- trimmed median filter is useful for multiple types of noise, such as combination of salt-and- pepper and Gaussian noise.

3.4 Adaptive Filters

daptive filter whose behavior changes based on the statistical characteristics of the image inside the filter region S_{xy}.

Adaptive, local noise reduction filter

The simplest statistical measures of a random variable are its mean and variance. The mean gives a measure of average intensity in the region over which the mean is computed and the variance gives a measure of contrast in that region.

Let the filter is operate on a local region S_{XY}. The response of the filter at any point *(x, y)* is based on four quantities: (a) *g(x, y)*, the value of noisy image at *(x, y)*; (b) σ^2_η the variance of the noise corrupting *f(x, y)* to form *g(x, y)*; (c) m_L, the local mean of the pixels in S_{XY}; and (d) σ^2_L, the local variance of the pixels in S_{XY}. Hence the behavior of the filter is,

- If σ^2_η is zero, the filter should return simply the value of *g(x, y)*.
- If the local variance is high relative to σ^2_η that means ($\sigma^2_L > \sigma^2_\eta$), the filter should return a value close to *g(x, y)*.
- If the two variances are equal, the filter returns the arithmetic mean value of the pixelin S_{XY}.

An adaptive filter for obtaining the restored image is

$$\hat{f}(x, y) = g(x, y) - \frac{\sigma_\eta^2}{\sigma_L^2}\left[g(x, y) - m_L\right]$$

The only quantity that needs to be known or estimated is the variance of the overall noise is σ_η^2. The other parameters are computed from the pixels in S_{XY}.

Adaptive median filter

Adaptive median filters are used to preserve the details while smoothing non impulse. However it changes the size of S_{XY} during the filtering operation, depending on certain conditions. The output of the filter is a single value used to replace the value of pixel at (x, y).Let us consider the following parameters,

z_{min} = minimum intensity value in S_{XY}

z_{max} = maximum intensity value in S_{XY}

z_{med} = median of intensity values in S_{XY}

z_{xy} = intensity value at co-ordinates (x, y)

S_{max} = maximum allowed size of S_{XY}

The adaptive median filtering algorithm works in two stages, denoted as stage A and stage B as follows:

Stage A: $A1 = z_{med} - z_{min}$

$A2 = z_{med} - z_{max}$

If $A1>0$ AND $A2<0$, go to stage B Else increase the window size

If window size $\leq S_{max}$ repeat stage A

Else output z_{med}

Stage B: $B1 = z_{xy} - z_{min}$

$B2 = z_{xy} - z_{max}$

If $B1>0$ AND $B2<0$, output z_{xy}.Else output z_{med}

3.5 Periodic Noise Reduction by Frequency Domain Filtering

Periodic noise in images are appears as concentrated bursts of energy in the Fourier transform at locations corresponding to the frequencies of the periodic interference. This can be removed by using selective filters.

Band Reject Filter:

The Band Reject Filter transfer function is defined as

Ideal	Butterworth	Gaussian
$H(u,v) = \begin{cases} 0 & \text{if } D_0 - \frac{W}{2} \leq D \leq D_0 + \frac{W}{2} \\ 1 & \text{otherwise} \end{cases}$	$H(u,v) = \dfrac{1}{1 + \left[\dfrac{DW}{D^2 - D_0^2}\right]^{2n}}$	$H(u,v) = 1 - e^{-\left[\frac{D^2 - D_0^2}{DW}\right]^2}$

Where „W" is the width of the band, D is the distance *D (u, v)* from the centre of the filter, D_o is the cutoff frequency and n is the order of the Butterworth filter. The band reject filters are very effective in removing periodic noise and the ringing effect normally small. The perspective plots of these filters are

(a) Ideal (b) Butterworth and (c) Gaussian Band Reject Filters

Band Pass Filter:

A *band pass* filter performs the opposite operation of a band reject filter. ABand pass filter is obtained from the band reject filter as

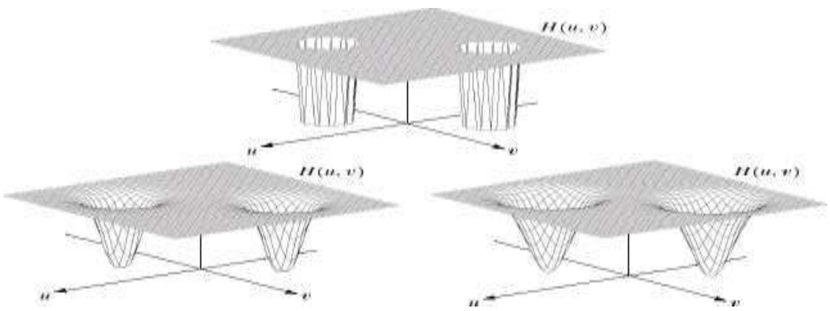
$$H_{BP}(u,v) = 1 - H_{BR}(u,v)$$

Notch Filters:

A Notch filter Reject (or pass) frequencies in a predefined neighborhood about the centre of the frequency rectangle. It is constructed as products of high pass filters whose centers have been translated to the centers of the notches. The general form is defin

$$H_{NR}(u,v) = \prod_{k=1}^{Q} \underbrace{H_k(u,v)}_{\substack{\text{centre at} \\ (u_k, v_k)}} \underbrace{H_{-k}(u,v)}_{\substack{\text{centre at} \\ (-u_k, -v_k)}}$$

Where $H_k(u,v)$ and $H_{-k}(u,v)$ are high pass filters whose centers are at (u_k, v_k) and $(-u_k, -v_k)$ respectively. These centers are specified with respect to the center of the frequency rectangle (M/2, N/2).



(a) Ideal (b) Butterworth and (c) Gaussian Notch Reject FiltersA *Notch Pass filter* (NP) is obtained from a *Notch Reject filter* (NR) using:

$$H_{NP}(u,v) = 1 - H_{NR}(u,v)$$

3.6 Linear, Position-Invariant Degradations

The input –output relation relationship before the restoration stage is expressed as

$$g(x,y)=H[f(x,y)]+\eta(x,y)$$

Let us assume that $\eta(x, y)=0$ then

$$g(x, y)=H[f(x,y)]$$

If H is linear

$$H[af_1(x,y)+bf_2(x,y)]=aH[f_1(x,y)]+bH[f_2(x,y)]$$

Where a and b are scalars. $f_1(x,y)$ and $f_2(x,y)$ are any two input images. If a=b=1 $H[f_1(x,y)+f_2(x,y)]=H[f_1(x,y)]+H[f_2(x,y)]$

It is called the property of additivity. This property says that, if H is a linear operator,the response to a sum of two inputs is equal to the sum of the two responses.

An operator having the input –output relation relationship $g(x, y)=H[f(x,y)]$ is said tobe position invariant if

$$H[f(x-\alpha, y-\beta)] = g(x-\alpha, y-\beta)$$

It indicates that the response at any point in the image depends only on the value ofthe input at that point not on its position. If the impulse signal can be considered

$$f(x,y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)\boxed{\delta(x-\alpha,y-\beta)}\,d\alpha\, d\beta$$

impulse

$$g(x,y)=H[f(x,y)]=H\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)\delta(x-\alpha,y-\beta)\,d\alpha\, d\beta\right]$$

linear

$$g(x,y)=H[f(x,y)]=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} H\left[f(\alpha,\beta)\delta(x-\alpha,y-\beta)\right]d\alpha\, d\beta$$

$$g(x,y)=H[f(x,y)]=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)\boxed{H\left[\delta(x-\alpha,y-\beta)\right]}\,d\alpha\, d\beta$$

$$h(x,\alpha,y,\beta)=H\left[\delta(x-\alpha,y-\beta)\right]$$

Impulse response (point spread function)

$$h(x,y)=H\left[\delta(x,y)\right]$$

If position-invariant

$$H\left[\delta(x-\alpha,y-\beta)\right]=h(x-\alpha,y-\beta)$$

$$g(x,y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)h(x-\alpha,y-\beta)\,d\alpha\, d\beta$$

$\eta(x,y)\neq 0$

$$g(x,y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)h(x-\alpha,y-\beta)\,d\alpha\, d\beta+\eta(x,y)$$

3.7 Estimating the Degradation Function

There are three principle ways to estimate the degradation function for use in image restoration: (1) Observation, (2) Experimentation and (3) mathematical modeling.

Estimation by Image observation:

In this process the degradation function is estimated by observing the Image. Select the sub image whose signal content is strong. Let the observed sub image be denoted by $g_s(x, y)$ and the processed sub image is $f^\wedge(x, y)$. The estimated degradation function can be expressed as

$$H_s(u, v) = \frac{G_s(u, v)}{\hat{F}_s(u, v)}$$

From the characteristics of this equation, we then deduce the complete degradation function *H (u, v)* based on the consideration of position invariance.

Estimation by Experimentation:

The degrade image can be estimated accurately when the equipment is identical to the one used to obtain the degraded image. Images similar to the degraded image can be acquired with various system settings until they are degraded as closely as possible to the image we wish to restore. Now obtain the impulse response of the degradation by imaging an impulse using the same system settings.

An impulse is simulated by a bright dot of light, as bright as possible to reduce the effect of noise. Then the degradation image can be expressed as

$$H(u, v) = \frac{G(u, v)}{A}$$

Where $G(u, v)$ is the Fourier transform of observed image and A is a constant describing the strength of the impulse.

Estimation by Modeling:

Image degradation function can be estimated by modeling includes the environmentalconditions that cause degradations. It can be expressed as

$$H(u, v) = e^{-k(u^2+v^2)^{5/6}}$$

Where k is a constant that depends on the nature of the turbulence. Let *f(x, y)* is an image that undergoes planar motion and that x_o (t) and y_o (t) are the time varying components in the direction of x and y. The total blurring image *g*(*x*, *y*) is expressed as

$$g(x, y) = \int_0^T f[x - x_0(t), y - y_0(t)]\, dt$$

The Fourier Transform of *g*(*x, y*) is *G(u, v)*

$$G(u, v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x, y)e^{-j2\pi(ux+vy)}\, dx\, dy$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[\int_0^T f[x - x_0(t), y - y_0(t)]\, dt\right]e^{-j2\pi(ux+vy)}\, dx\, dy$$

Reversing the order of the integration then

$$G(u, v) = \int_0^T\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f[x - x_0(t), y - y_0(t)]e^{-j2\pi(ux+vy)}\, dx\, dy\right] dt$$

$$G(u, v) = \int_0^T F(u, v)e^{-j2\pi[ux_0(t)+vy_0(t)]}\, dt$$

$$= F(u, v)\int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]}\, dt$$

Let us define *H (u, v)*

$$H(u, v) = \int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]}\, dt$$

Then the above expression can be expressed as

$$G(u, v) = H(u, v)F(u, v)$$

If the motion variables x_o(t) and y_o(t) are known then the degradation function $H(u, v)$

can becomes

$$H(u,v) = \frac{T}{\pi(ua+vb)} \sin[\pi(ua+vb)] e^{-j\pi(ua+vb)}$$

3.8 Inverse Filtering

The simplest approach to restoration is direct inverse filtering, where we can estimate

$\hat{F}$ (u, v) of the transform of the original image simply by dividing the transform of the degraded image G (u,v) by the degraded function

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)}$$

We know that $G(u,v) = H(u,v)F(u,v) + N(u,v)$

Then

$$\hat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

From the above expression we can observe that if we know the degrade function we cannot recover the un-degraded image exactly because N (u, v) is not known. If the degrade function has zero or very small values, then the ratio N (u, v)/ H(u, v) could easily dominate the $\hat{F}$(u, v). To avoid this disadvantage we limit the filter frequency values near the origin because H(u, v) values are maximum at the origin.

3.9 Minimum Mean Square Error Filtering (Wiener Filtering)

In this filtering process, it incorporates both the degrade function and statistical characteristics of noise .The images and noise in this method are considered as random variables. The objective is to find an estimate f^ of the uncorrupted image f such that the mean square error between them is minimized. This error is measured by

$$e^2 = E\{(f - \hat{f})^2\}$$

Where E { } is the expected value of the argument. It is assumed that noise and image are uncorrelated; one or the other has zero mean; the intensity levels in the estimate are a linear function of the levels in the degraded image. Based on these conditions the minimum of the error function in frequency domain is given by

$$\begin{aligned}\hat{F}(u,v) &= \left[\frac{H^*(u,v)S_f(u,v)}{S_f(u,v)|H(u,v)|^2 + S_\eta(u,v)}\right]G(u,v)\\ &= \left[\frac{H^*(u,v)}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)}\right]G(u,v)\\ &= \left[\frac{1}{H(u,v)}\frac{|H(u,v)|^2}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)}\right]G(u,v)\end{aligned}$$

This result is known as "*Wiener filter*". The terms inside the bracket is commonly referred as the minimum mean square error filter or the least square error filter. It does not have the same problem as the inverse filter with zeros in the degraded function, unless the entire denominator is zero for the same values of *u* and *v*.

H(u, v) = degraded function

*H**(u, v) = complex conjugate of H(u, v)

$$S_\eta(u,v) = |N(u,v)|^2 = \text{power spectrum of the noise}$$

$$S_f(u,v) = |F(u,v)|^2 = \text{power spectrum of the undegraded image}$$

The signal to noise ratio in frequency domain

$$\text{SNR} = \frac{\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}|F(u,v)|^2}{\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}|N(u,v)|^2}$$

The signal to noise ratio in spatial domain

The mean square xpression

$$\text{SNR} = \frac{\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\hat{f}(x,y)^2}{\sum^{M-1}\sum^{N-1}}$$

$$\text{MSE} = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}[f(x,y) - \hat{f}(x,y)]^2$$

The modified expression to estimate $f^{\wedge}$ by using minimum mean square error filtering

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + K}\right] G(u,v)$$

3.10 Constrained Least Square Filtering

The problem with the wiener filter is that it is necessary to know the power spectrumof noise and image. The degraded image is given in the spatial domain by

$$g(x,y)=f(x,y)*h(x,y)+\eta(x,y)$$

In vector-matrix form

$$\mathbf{g} = \mathbf{Hf} + \boldsymbol{\eta}$$

Where $\boldsymbol{g}$, $\boldsymbol{f}$, $\boldsymbol{\eta}$ vectors of dimension MNx1.$\boldsymbol{H}$ matrix of dimension MN x MN is very large and is highly sensitive to noise. Optimality of restoration based on a measure of smoothness: using Laplacian operator. The restoration

$$C = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [\nabla^2 f(x,y)]^2$$

must be constrained by the parameters is to find the minimum criterion function c is defined as

subject to the constraint

$$\|\mathbf{g} - \mathbf{H\hat{f}}\|^2 = \|\boldsymbol{\eta}\|^2$$

The frequency domain solution to this optimization problem is given by theexpression

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \gamma|P(u,v)|^2}\right] G(u,v)$$

Where γ is a parameter that must be adjusted so that constraint is satisfied and $P(u, v)$is the Fourier transform of the function

$$p(x,y) = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

It is possible to ad... tively until acceptable results areachieved. A p... y iteration is as follows.

Define a residual vector r as $\mathbf{r} = \mathbf{g} - \mathbf{H\hat{f}}$

Hence $\hat{F}$ (u, v) is a function of γ, then r also a function of this parameter is a monotonically increasing function of γ .

$$\phi(\gamma) = \mathbf{r}^T\mathbf{r}$$
$$= \|\mathbf{r}\|^2$$

We want to adjust γ so that

$$\|\mathbf{r}\|^2 = \|\boldsymbol{\eta}\|^2 \pm a$$

Where a is an accuracy factor. If $\|\mathbf{r}\|^2 = \|\boldsymbol{\eta}\|^2$ the constarint is satisfied. Becauseφ(γ) is monotonic , finding the desired value of γ is not

1. Specify an initial value of γ.
2. Compute $\|\mathbf{r}\|^2$.
3. Stop if Eq. (5.9-8) is satisfied; otherwise return to Step 2 after increasing γ if $\|\mathbf{r}\|^2 < \|\boldsymbol{\eta}\|^2 - a$ or decreasing γ if $\|\mathbf{r}\|^2 > \|\boldsymbol{\eta}\|^2 + a$. Use the new value of γ in Eq. (5.9-4) to recompute the optimum estimate $\hat{F}(u, v)$.

The variance and the mean of the entire image is

$$\sigma_\eta^2 = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\left[\eta(x, y) - m_\eta\right]^2$$

$$m_\eta = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\eta(x, y)$$

ence the noise is

$$\|\boldsymbol{\eta}\|^2 = MN\left[\sigma_\eta^2 + m_\eta^2\right]$$

3.11 Geometric Mean Filter

Geometric mean filter is slightly generalized wiener filter in the form

$$\hat{F}(u, v) = \left[\frac{H^*(u, v)}{|H(u, v)|^2}\right]^\alpha \left[\frac{H^*(u, v)}{|H(u, v)|^2 + \beta\left[\dfrac{S_\eta(u, v)}{S_f(u, v)}\right]}\right]^{1-\alpha} G(u, v)$$

α and β being positive real constants. Based on the values of α and β, geometric meanfilter performs the different actions

$\alpha = 1$ => inverse filter

$\alpha = 0$ => *parametric Wiener filter* (standard Wiener filter when $\beta =$

$1)\alpha = 1/2$ => actual geometric mean

$\alpha = 1/2$ and $\beta = 1$ => *spectrum equalization filter*

PREVIOUS QUESTIONS

1. What is meant by image restoration? Explain the image degradation model
2. Discuss about the noise models
3. Explain the concept of algebraic image restoration
4. Discuss the advantages and disadvantages of wiener filter with regard to image restoration.
5. Explain about noise modeling based on distribution function
6. Explain about wiener filter in noise removal
7. What is geometric mean filter? Explain
8. Explain the following. a) Minimum Mean square error filtering. b) Inverse filtering.
9. Discuss about Constrained Least Square restoration of a digital image in detail.
10. Explain in detail about different types of order statistics filters for Restoration.
11. Name different types of estimating the degradation function for use in image restoration and explain in detail estimation by modeling.
12. Explain periodic noise reduction by frequency domain filtering
13. Explain adaptive filter and also what the two levels of adaptive median filteringalgorithms are

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