

DIGITAL IMAGE PROCESSING SYSTEMS

Ω UNIT - I

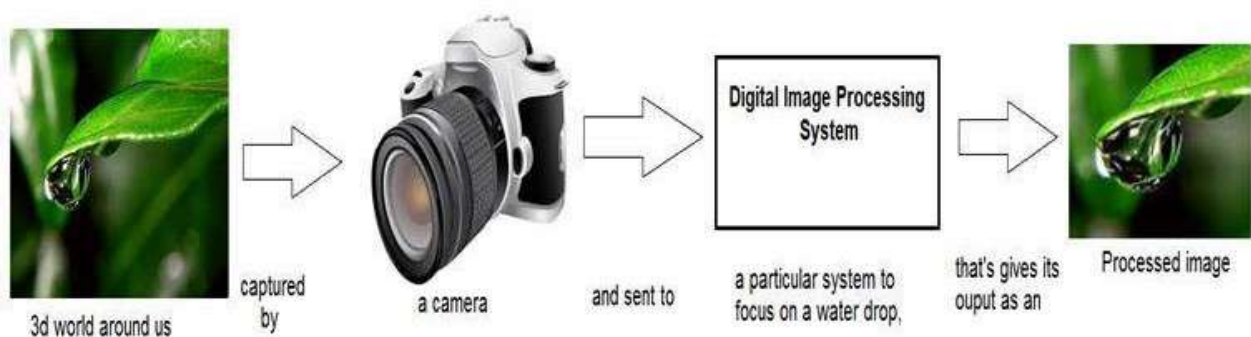
DIP: Introduction Fundamental Steps in DIP – Elementary of Visual Perceptions – Image Sensing and Image Acquisition – Image Sampling and Quantization – Image Geometry. Spatial Domain: Relationship between Pixels Basic Gray Level Transformations Histogram Processing Smoothing Spatial Filters Sharpening Spatial Filters

❖ Introduction to Digital image processing

What is DPI?

- ✦ Digital image processing deals with manipulation of digital images through a digital computer.
- ✦ It is a subfield of signals and systems but focus particularly on images. DIP focuses on developing a computer system that is able to perform processing on an image.
- ✦ The input of that system is a digital image and the system process that image using efficient algorithms, and gives an image as an output.
- ✦ The most common example is Adobe Photoshop. It is one of the widely used application for processing digital images

How it works



In the above figure, an image has been captured by a camera and has been sent to a digital system to remove all the other details, and just focus on the water drop by zooming it in such a way that the quality of the image remains the same.

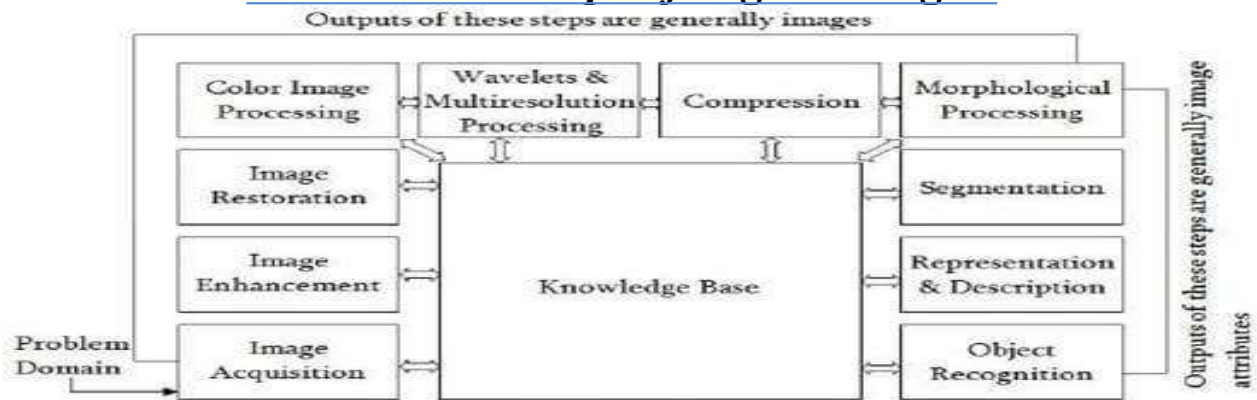
- ✦ In computer science, digital image processing uses algorithms to perform image processing on digital images to extract some useful information.
- ✦ Digital image processing has many advantages as compared to analog image processing.
- ✦ Wide range of algorithms can be applied to input data which can avoid problems such as noise and signal distortion during processing. As we know, images are defined in two dimensions, so DIP can be modeled in multidimensional systems.

❖ Purpose of Image processing

The main purpose of the DIP is divided into following 5 groups:

1. **Visualization:** The objects which are not visible, they are observed.
2. **Image sharpening and restoration:** It is used for better image resolution.
3. **Image retrieval:** An image of interest can be seen
4. **Measurement of pattern:** In an image, all the objects are measured.
5. **Image Recognition:** Each object in an image can be distinguished.

Fundamental Steps of Digital Image :



1. Image Acquisition

Image acquisition is the first step of the fundamental steps of DIP. In this stage, an image is given in the digital form. Generally, in this stage, pre-processing such as scaling is done.

2. Image Enhancement

Image enhancement is the simplest and most attractive area of DIP. In this stage details which are not known, or we can say that interesting features of an image is highlighted. Such as brightness, contrast, etc...

3. Image Restoration

Image restoration is the stage in which the appearance of an image is improved.

4. Colour Image Processing

Colour image processing is a famous area because it has increased the use of digital images on the internet. This includes colour modelling, processing in a digital domain, etc....

5. Wavelets and Multi-Resolution Processing

In this stage, an image is represented in various degrees of resolution. Image is divided into smaller regions for data compression and for the pyramidal representation.

6. Compression

Compression is a technique which is used for reducing the requirement of storing an image. It is a very important stage because it is very necessary to compress data for internet use.

7. Morphological Processing

This stage deals with tools which are used for extracting the components of the image, which is useful in the representation and description of shape.

8. Segmentation

In this stage, an image is partitioned into its objects. Segmentation is the most difficult task in DIP. It is a process which takes a lot of time for the successful solution of imaging problems which requires objects to be identified individually.

9. Representation and Description

Representation and description follow the output of the segmentation stage. The output is a raw pixel data which has all points of the region itself. To transform the raw data, representation is the only solution. Whereas description is used for extracting information to differentiate one class of objects from another.

10. Object recognition

In this stage, the label is assigned to the object, which is based on descriptors.

11. Knowledge Base

Knowledge is the last stage in DIP. In this stage, important information of the image is located, which limits the searching processes. The knowledge base is very complex when the image database has a high-resolution satellite.

➤ Elementary of Visual Perception

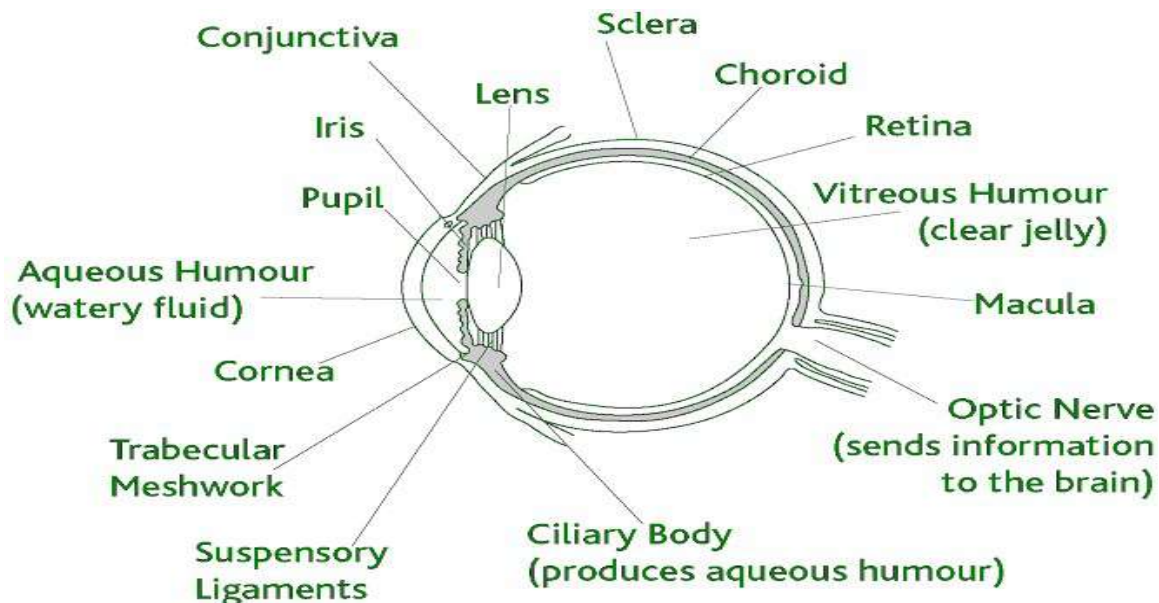
- ✦ The field of digital image processing is built on the foundation of mathematical and probabilistic formulation, but human intuition and analysis play the main role to make the selection between various techniques, and the choice or selection is basically made on subjective, visual judgements.

In human visual perception, the eyes act as the sensor or camera, neurons act as the connecting cable and the brain acts as the processor.

The basic elements of visual perceptions are:

1. Structure of Eye
2. Image Formation in the Eye
3. Brightness Adaptation and Discrimination

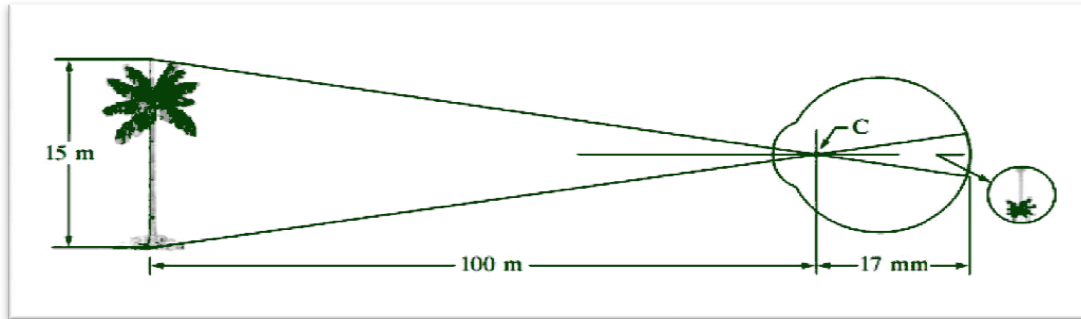
✦ **Structure of Eye:**



- ❖ The human eye is a slightly asymmetrical sphere with an average diameter of the length of 20mm to 25mm. It has a volume of about 6.5cc. The eye is just like a camera
- ❖ . The external object is seen as the camera take the picture of any object. Light enters the eye through a small hole called the pupil, a black looking aperture having the quality of contraction of eye when exposed to bright light and is focused on the retina which is like a camera film.
- ✦ The lens, iris, and cornea are nourished by clear fluid, know as anterior chamber.
- ✦ The fluid flows from ciliary body to the pupil and is absorbed through the channels in the angle of the anterior chamber.
- ✦ The delicate balance of aqueous production and absorption controls pressure within the eye.
- ✦ Cones in eye number between 6 to 7 million which are highly sensitive to colours.
- ✦ Human visualizes the coloured image in daylight due to these cones. The cone vision is also called as photopic or bright-light vision.
- ✦ Rods in the eye are much larger between 75 to 150 million and are distributed over the retinal surface
- ✦ Rods are not involved in the colour vision and are sensitive to low levels of illumination.

❖ Image Formation in the Eye:

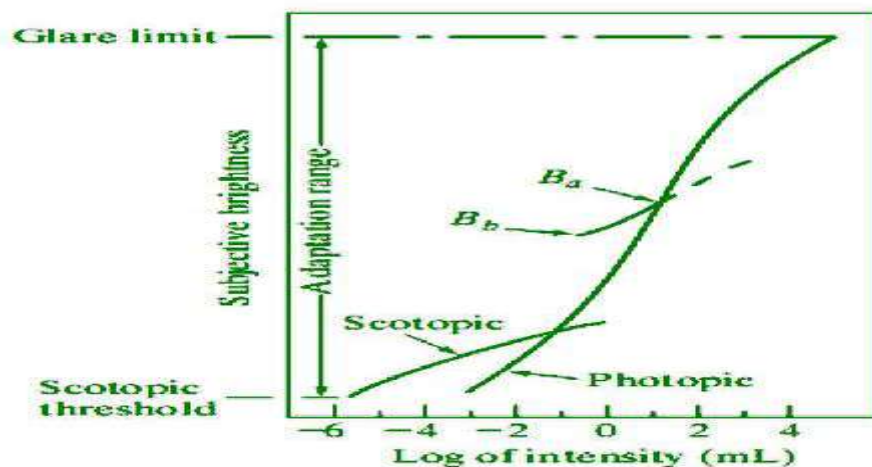
- When the lens of the eye focus an image of the outside world onto a light - sensitive membrane in the back of the eye is called retina and the image is formed.
- The lens of the eye focuses light on the photoreceptive cells of the retina which detects the photons of light and responds by producing neural impulses.



- ✦ The distance between the lens and the retina is about 17mm and the focal length is approximately 14mm to 17mm.

❖ Brightness Adaptation and Discrimination:

- ❖ Digital images are displayed as a discrete set of intensities.
- ❖ The eyes ability to discriminate black and white at different intensity levels is an important consideration in presenting image processing result.

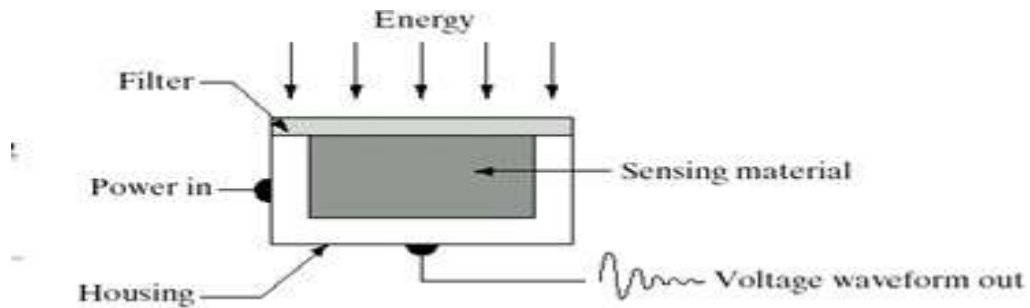


- ✦ The range of light intensity levels to which the human visual system can adapt is of the order of 10^{10} from the scotopic threshold to the glare limit. In a photopic vision, the range is about 10^6

❖ Image Sensing and Acquisition

There are 3 principal sensor arrangements (produce an electrical output proportional to light intensity).

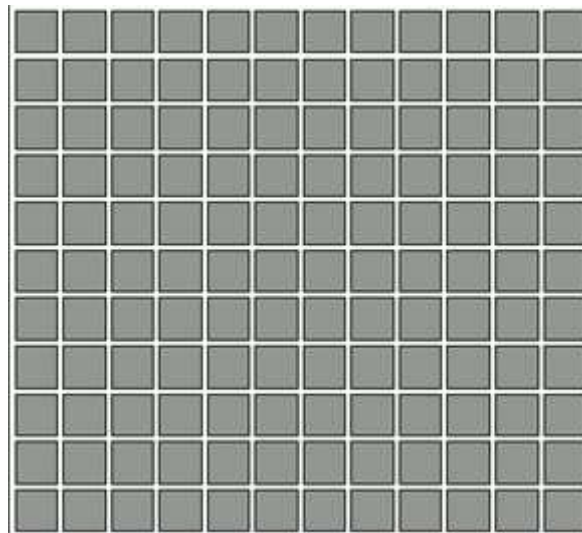
(I) Single imaging Sensor (II) Line sensor (III) Array sensor



(I) Single image



(ii) Sensor line sensor



(iii) Array sensor

❖ Image Acquisition using a single sensor

- ❖ The most common sensor of this type is the photodiode, which is constructed of silicon materials and whose output voltage waveform is proportional to light.
- ❖ The use of a filter in front of a sensor improves selectivity. For example, a green (pass) filter in front of a light sensor favors light in the green band of the color spectrum.
- ❖ As a consequence, the sensor output will be stronger for green light than for other components in the visible

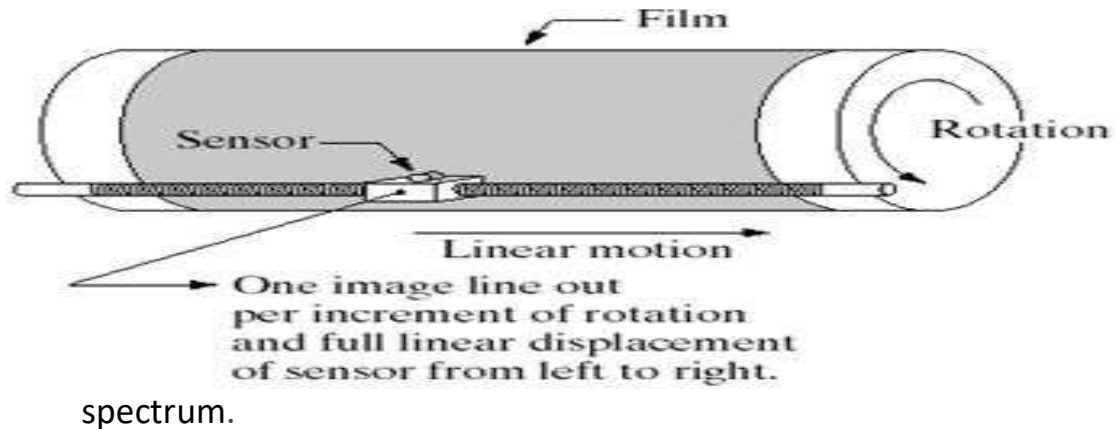
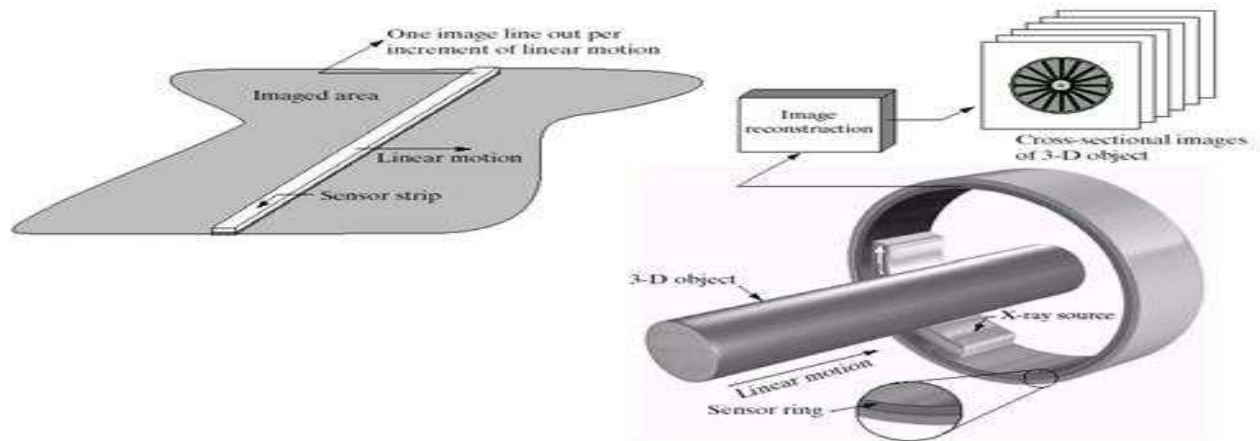


Fig: Combining a single sensor with motion to generate a 2-D image

- ✓ In order to generate a 2-D image using a single sensor, there have to be relative displacements in both the x- and y-directions between the sensor and the area to be imaged.
- ✓ An arrangement used in high precision scanning, where a film negative is mounted onto a drum whose mechanical rotation provides displacement in one dimension.

- ✓ The single sensor is mounted on a lead screw that provides motion in the perpendicular direction. Since mechanical motion can be controlled with high precision, this method is an inexpensive (but slow) way to obtain high-resolution image



❖ Image Acquisition using Sensor Strips

Fig: (a) Image acquisition using linear sensor strip (b) Image acquisition using circular sensor strip.

- ❖ The strip provides imaging elements in one direction. Motion perpendicular to the strip provides imaging in the other direction.
- ❖ This is the type of arrangement used in most flatbed scanners. Sensing devices with 4000 or more in-line sensors are possible.
- ❖ In-line sensors are used routinely in airborne imaging

applications, in which the imaging system is mounted on an aircraft that flies at a constant altitude and speed over the geographical area to be imaged.

- ❖ One-dimensional imaging sensor strips that respond to various bands of the electromagnetic spectrum are mounted perpendicular to the direction of flight.
- ❖ The imaging strip gives one line of an image at a time and the motion of the strip completes the other dimension of a two-dimensional image.
- ❖ Sensor strips mounted in a ring configuration are used in medical and industrial imaging to obtain cross-sectional (“slice”) images of 3-d objects.
- ❖ A rotating X-ray source provides illumination and the portion of the sensors opposite the source collect the X-ray energy that pass through the object.
- ❖ The sensors obviously have to be sensitive to X-ray energy). This is the basis for medical and industrial computerized axial tomography (CAT) imaging.

❖ Image Acquisition using Sensor Arrays

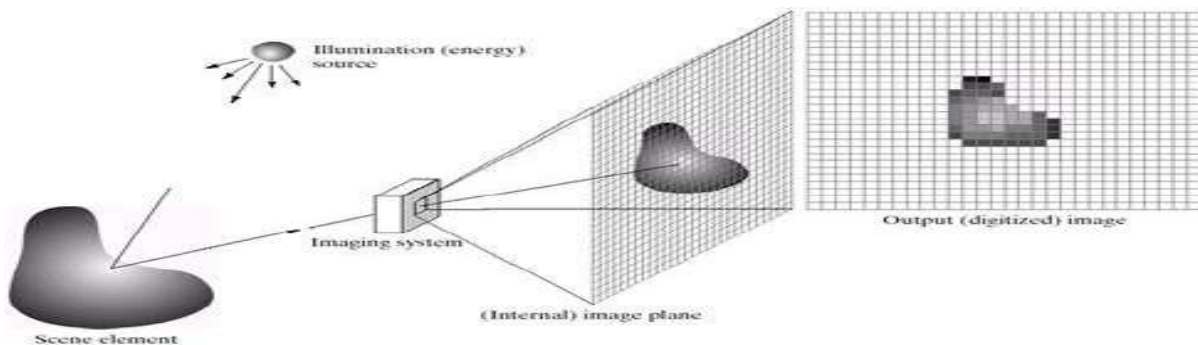


Fig: An example of the digital image acquisition process (a) energy source (b) An element of a scene (d) Projection of the scene into the image (e) digitized image

- ✦ This type of arrangement is found in digital cameras. A typical sensor for these cameras is a CCD array, which can be manufactured with a broad range of sensing properties and can be packaged in rugged arrays of $4000 * 4000$ elements or more.
- ✦ CCD sensors are used widely in digital cameras and other light sensing instruments.
- ✦ The response of each sensor is proportional to the integral of the light energy projected onto the surface of the sensor, a property that is used in astronomical and other applications requiring low noise images.
- ✦ The first function performed by the imaging system is to collect the incoming energy and focus it onto an image plane.

- ✦ If the illumination is light, the front end of the imaging system is a lens, which projects the viewed scene onto the lens focal plane.
- ✦ The sensor array, which is coincident with the focal plane, produces outputs proportional to the integral of the light received at each sensor.
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❖ Image Sampling and Quantization

Introduction

- ⤴ Images have become an indispensable part of our lives.
- ⤴ We tend to take photographs of every occasion to remember them.

- ▲ But the pictures that we take are mostly analog images. We cannot process or store these analog images on our computers.
- ▲ Digital images are more useful than analog images. We can store them on computers, apply digital image processing, make hundreds and thousands of copies, and share them over the internet.
- Before going forward with the article **Image Sampling and Quantization**, it is essential to know how a digital image is represented on our computer.

What is an analog image?

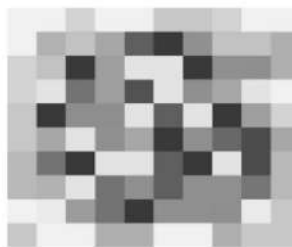
- ✦ When we capture the image of an object, we use image sensors to sense the incoming light and form the image.
- ✦ Image sensors convert the incoming light from an object into electrical signals that can be stored and viewed later. These analog signals are continuous.
- ✦ The images are stored in an analog form. Thus, the image formed has continuous variation in the tone.
- ✦ We cannot process analog images by a computer. Analog signals contain infinite points, and we need infinite memory to store them. We need to convert the analog images into digital images to store and process by a computer.

❖ Analog image to digital image conversion

- ✦ An analog image is converted to a digital image by digitizing the analog signals. We apply sampling and quantization to the analog signals to convert them into digital form.
- ✦ A digital image is formed by arranging pixels in rows and columns. Each pixel has a particular integral value. The computer process that integral value and show us that pixel, the arrangement of the pixels form the digital image.
- ✦ We use sampling and quantization to change the continuous analog image into quantized integral values that will represent each pixel and ultimately form the digital image.



**ANALOG
IMAGE**



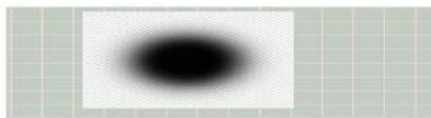
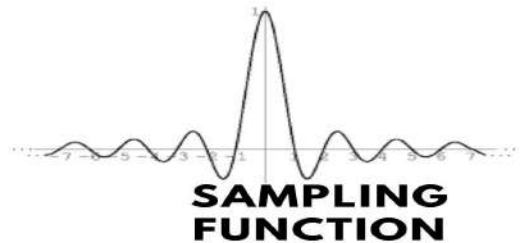
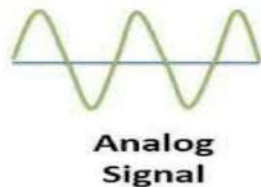
**DIGITAL
SAMPLING**

35	58	-51	-15	-12
18	27	1	-5	3
-35	-50	19	7	-18
-20	2	34	36	12
-5	-32	-15	45	37
7	-8	11	4	7
-26	-2	7	-44	-21
-44	35	48	-37	-3

**PIXEL
QUANTIZATION**

❖ What is Image Sampling

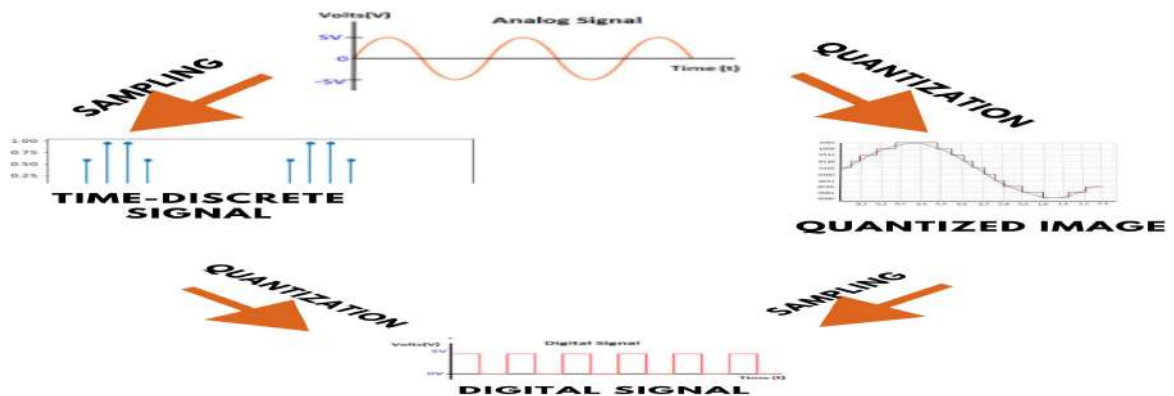
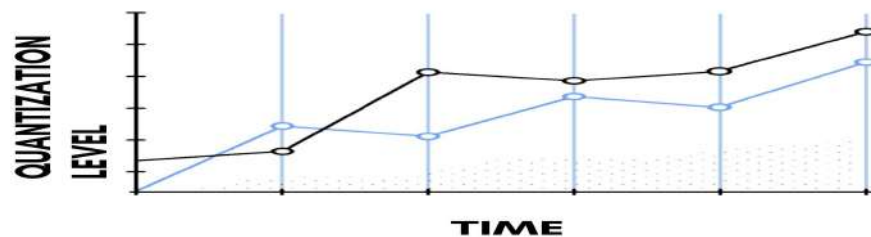
- Sampling is the process of converting an analog signal into discrete values.
- In layman's terms, we can say that sampling is the process of recording an analog signal at regular intervals of time.
- A sampling function is applied to the analog signal that results in the sampled signal.
- We get a finite number of samples of an analog signal. The number of samples gives us the number of pixels.
- More samples will result in higher image quality of the digital image because of more pixels.



The sampled signal is then quantized to get the value of each pixel.
Let us look at how quantization is achieved..

❖ What is Image quantization

- ✦ After sampling the analog signal, we will apply quantization. Quantization digitizes the amplitude of the sampled signal.
- ✦ Quantization is done by rounding off the amplitude of each sample and then assigning a different value according to its amplitude.
- ✦ Each value will represent a different color tone.



94	178	124	90	131	0
23	94	135	147	94	138
153	120	140	73	162	6
72	64	10	124	56	64
3	60	75	82	86	129
116	92	165	106	170	89

- Each pixel is assigned an integer value after quantization. Each number represents a different shade of grey.
- The collection of these pixels will form the image. In the above example, there are 256 quantization level.
- In Digital Image Processing, signals captured from the physical world need to be translated into digital form by “Digitization” Process.
- In order to become suitable for digital processing, an image function $f(x,y)$ must be digitized both spatially and in amplitude.

This digitization process involves two main processes called

1. Sampling:

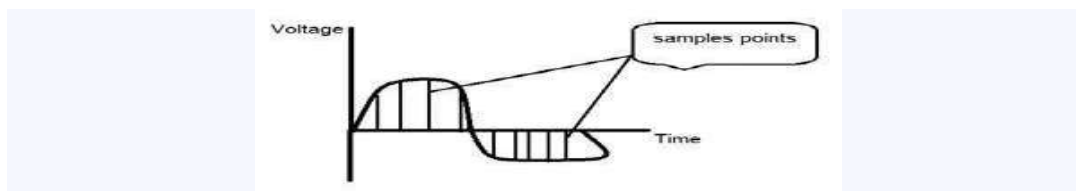
- ❖ Digitizing the co-ordinate value is called sampling.

2. Quantization:

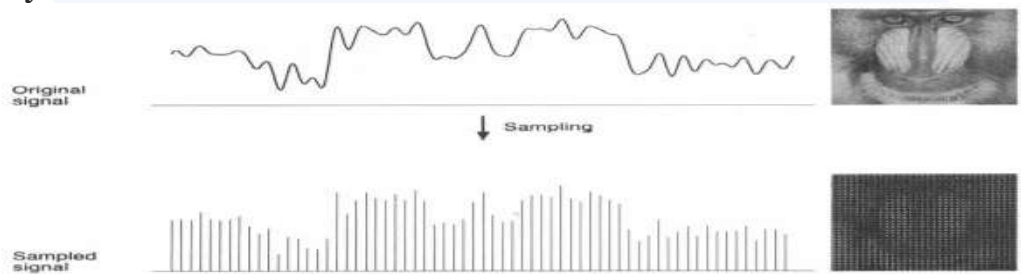
- ❖ Digitizing the amplitude value is called quantization.

❖ Sampling

- Since an analogue image is continuous not just in its co-ordinates (x axis), but also in its amplitude (y axis), so the part that deals with the digitizing of co-ordinates is known as sampling.
- In digitizing sampling is done on independent variable. In case of equation $y = \sin(x)$, it is done on x variable.



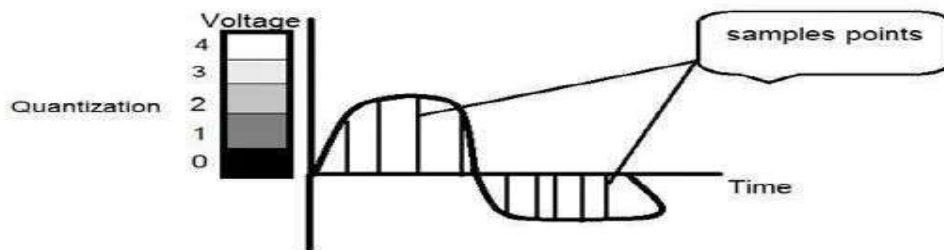
- ✦ There are some random variations in the signal caused by noise. In sampling we reduce this noise by taking samples.
- ✦ It is obvious that more samples we take, the quality of the image would be more better , the noise would be more removed and same happens vice versa.
- ✦ if you take sampling on the x axis, the signal is not converted to digital format, unless you take sampling of the y-axis too which is known as quantization.
- ✦ Sampling has a relationship with image pixels.
- ✦ The total number of pixels in an image can be calculated as
- ✦ $\text{Pixels} = \text{total no of rows} * \text{total no of columns}.$
- ✦ we have total of 36 pixels, that means we have a square image of 6X 6.
- ✦ That refers to 36 pixels of this image.
- ✦ The number sample is directly equal to the number of sensors on CCD array.



❖ Quantization

- ✦ Quantization is a process of transforming a real valued sampled image to one taking only a finite number of distinct values. Under quantization process the amplitude values of the image are digitized.

- ✦ Quantization is opposite to sampling because it is done on “y axis” while sampling is done on “x axis”
- ✦ when you are quantizing an image, you are actually dividing a signal into quanta(partitions).



- ✦ Now let's see how quantization is done. Here we assign levels to the values generated by sampling process.
- ✦ In the image showed in sampling explanation, although the samples has been taken, but they were still spanning vertically to a continuous range of gray level values.
- ✦ In the image shown below, these vertically ranging values have been quantized into 5 different levels or partitions
- ✦ Ranging from 0 black to 4 white. This level could vary according to the type of image you want.
- ✦ There is a relationship between Quantization with gray level resolution.
- ✦ The above quantized image represents 5 different levels of gray and that means the image formed from this signal, would only have 5 different colors. It would be a black and white image more or less with some colors of gray.
- ✦ When we want to improve the quality of image, we can increase the levels assign to the sampled image.

- ✦ If we increase this level to 256, it means we have a gray scale image.
- ✦ Most digital IP devices uses quantization into k equal intervals. If b-bits per pixel are used,

$$\text{No. of quantization levels} = k = 2^b$$

- The number of quantization levels should be high enough for human perception of fine shading details in the image.
- The occurrence of false contours is the main problem in image which has been quantized with insufficient brightness levels.

Here is an example for image quantization process.



❖ Difference between Sampling vs Quantization

Sampling	Quantization
Sampling establishes the number of pixels in a digital image.	Quantization establishes the color of each pixel.
Sampling digitalizes an analog signal's x-axis.	Quantization digitalizes its y-axis.
An analog signal's amplitude value is noted at predetermined intervals during sampling.	In quantization, the amplitude values are rounded off, and the values are given.
Sampling is carried out before quantization.	Quantization is carried out following sampling.

❖ *Geometric Transformation of Images*

- ❖ Geometric transformation is one of the critical techniques used in image processing. It is the process of modifying the geometric properties of an image, such as its position, size, rotation, or shape. **That perform other operations that require changing the spatial arrangement of pixels in an image.**
- ❖ A set of image transformations where the geometry of image is changed without altering its actual pixel values are commonly referred to as “Geometric” transformation. In general, you can apply multiple operations on it, but, the actual pixel values will remain unchanged.
- ❖ pixel values are not changed the positions of pixel values are changed.

What is Image Processing

- Image processing is a subfield of computer science and signal processing that involves manipulating and analyzing digital images to improve their quality, extract useful information, or perform specific operations.
- It employs various algorithms and techniques to perform tasks such as noise reduction, edge detection, and image enhancement.

How are Image Processing and Geometric Transformation interrelated?

- Image processing and geometric transformation are interrelated as geometric transformations play a vital role in manipulating and modifying digital images.
- In the context of image processing, geometric transformations are used to alter the spatial relationships within an image without affecting its geometric properties.
- The interrelation between image processing and geometric transformation is evident in the various applications of geometric transformations in image processing.
- By applying transformation geometry, geometric transformations can be applied to geometric images to achieve the desired results in image processing tasks.

❖ Image processing and geometric transformation are interrelated through their shared goal of modifying and enhancing digital images. **Geometric transformations play a crucial role in achieving this by allowing the manipulation of geometric images and the application of transformation geometry in various image-processing tasks.**

❖ Why is it used?



❖ Correct images for avoiding lens distortion.

Perfect lens shows straight lines and the distortion in the lens is when the lines have deviated inwards or outwards. So to avoid this, we need to have correct images.

❖ *Images according to camera orientation.*

The camera is oriented at a specific angle so the image will be at a specific angle. Now we need the image at some other angle, so with the help of geometric transformations, we can easily do this.

❖ Different Types of Geometric Transformation in Image Processing

1. Projective Transformation

- ✦ Also known as **homography**, this geometric transformation is the most general form and includes all other transformations.
- ✦ Projective transformations can change the perspective of an image, making it suitable for tasks like panorama stitching and rectifying images.

2. Euclidean Transformation

Also known as **rigid transformation**, it combines translation and rotation operations. Euclidean transformation in digital image processing preserves the distances and angles between points in the image.

3. Perspective Transformation

- ✦ Perspective transformation, also known as **homography**, is a projective transformation that maps points in one plane to another while preserving straight lines.
- ✦ It can account for changes in perspective and orientation, making it suitable for applications like image stitching, rectifying images, and

augmented reality. In computer vision, perspective transformation is often represented as a 3×3 matrix called the homography matrix.

- ✦ When applied to an image, this matrix can correct perspective distortions, align images from different viewpoints, and create panoramas.

4. Matrix Multiplication and Transformations

- ✦ Matrix multiplication is a fundamental operation in linear algebra that allows the combination of multiple linear transformations into a single transformation matrix.
- ✦ transformation matrices are used to represent various operations like rotation, scaling, translation, shearing, and perspective transformation.

By multiplying transformation matrices, multiple transformations can be combined and applied simultaneously to an image or a set of points. This enables efficient and compact representation of complex geometric transformations.

5. Affine Transformations

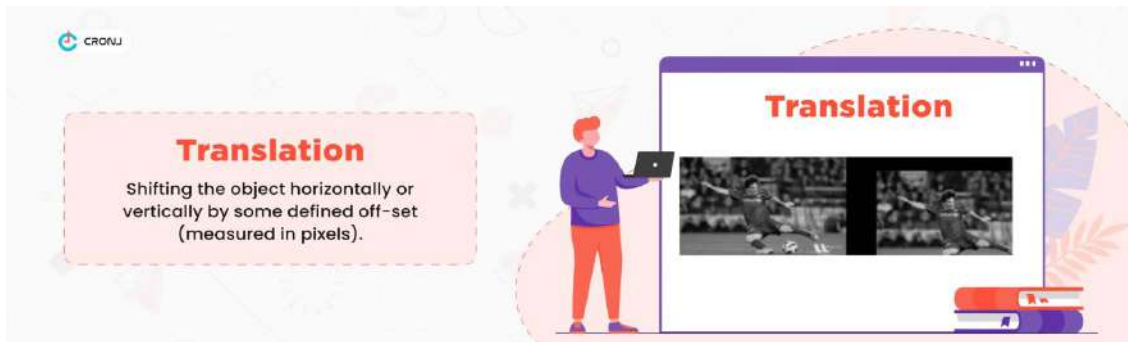
An affine transformation is a transformation that preserves collinearity and the ratio of distances (for example – the midpoint of a line segment is still the midpoint even after the transformation))

an affine transformation is a composition of translations, rotations, shears, and magnifications.

❖ Reflection

This geometric transformation involves flipping an image along a specified axis (horizontal, vertical, or diagonal), creating a mirror image.

❖ Translation



$$x' = x + A \quad (\text{Eq. 1})$$

$$y' = y + B \quad (\text{Eq. 2})$$

Here, let's say, $A = 100$, $B = 80$ (Translation in x and y-direction respectively)

(x, y) – point in input image

(x', y') – point in output image

It means that each pixel is shifted 100 pixels in the x-direction and each pixel is shifted 80 pixels in the y-direction.

You can get a clearer picture of this concept by this example given below.



Fig: Original image

Fig: scaling image

Scaling



- We can resize an image in terms of scaling factor or we can provide the size of the image we want.

❖ *An example.*

If we have an image of size (300 x 200) and we want to transform it into an image of shape (600 x 100).

The scaling in x- direction will be : $600/300 = 2$. (we denote it as $S1 = 2$)

Similarly $S2 = 100/200 = 1/2$.

Basically it means that the pixel in the input image at position (x, y) = (100,100) will be mapped to a new position (x', y') in the output image,

$$(x', y') = (100 * 2, 100 * 1/2) = (200, 50)$$

$$X' = x * S1 \text{ (Eq. 3)}$$

$$Y' = y * S2 \text{ (Eq. 4)}$$



Fig : original image



Fig: Scaling image

❖ Rotation



It performs a geometric transform which maps the position of a point in current image to the output image by rotating it by the user-defined angle through the specified axis. **Rotation** is basically used for improvised visual appearance.

Implementation ?

It's a little different from other transforms.

$$X' = x \cdot \cos(\Theta) - y \cdot \sin(\Theta) \text{ (Eq. 5)}$$

$$Y' = y \cdot \sin(\Theta) + x \cdot \cos(\Theta) \text{ (Eq. 6)}$$

Here, (x, y) is the position of pixel in input image.

(x', y') is the position of pixel in output image.

Θ is the angle by which an image to be rotated.

❖ Shearing

Shearing an image means shifting the pixel values either horizontally or vertically. You must be thinking that it is sounding like Translation, but the difference is that shifting is not done by the same amount.



Basically, this shifts some part of an image to a direction and other parts to some other direction. *For instance*, horizontal shearing will shift the upper part to the right and lower part to the left. You can better understand this concept by visualization.

✦ Relationships between Pixels

we will look at some of the most essential relationships between pixels in a digital image. As previously stated, an image is denoted by f . (x, y) . This tutorial uses lowercase letters like p and q to refer to specific pixels.

Neighbours of a Pixel

A pixel p at coordinates (x, y) has four horizontal and vertical neighbors whose coordinates are given by

$$(x+1, y), (x-1, y), (x, y+1), (x, y-1)$$

This set of pixels, called the 4-neighbors of p , is denoted by $N_4(p)$. The four diagonal neighbors of p have coordinates

$$(x+1, y+1), (x+1, y-1), (x-1, y+1), (x-1, y-1)$$

and are denoted by $N_D(p)$.

Diagonal neighbors together with the 4-neighbors are called the 8-neighbors of p , denoted by $N_8(p)$.

$$N_8(p) = N_4(p) + N_D(p)$$

The set of image locations of the neighbors of a point p is called the neighborhood of p . The neighborhood is said to be closed if it contains p . Otherwise, the neighborhood is said to be open.

$(x - 1, y + 1)$	$(x, y + 1)$	$(x + 1, y + 1)$
$(x - 1, y)$	(x, y)	$(x + 1, y)$
$(x - 1, y - 1)$	$(x, y - 1)$	$(x + 1, y - 1)$

❖ Adjacency between pixels

Let V be the set of intensity values used to define adjacency. In a binary image, $V = \{1\}$ if we are referring to the adjacency of pixels with the value 1. In a gray-scale image, the idea is the same but set V typically contains more elements.

For example, in the adjacency of pixels with intensity values ranging from 0 to 255, set V might be any subset of these 256 values.

There are three types of adjacency:

1. **4-adjacency:**

Two pixels p and q with values from V are 4-adjacent if q is in the set $N_4(p)$.

2. **8-adjacency:**

Two pixels p and q with values from V are 8-adjacent if q is in the set $N_8(p)$.

3. **m-adjacency (Mixed Adjacency):**

Two pixels p and q with values from V are m-adjacent if

1. q is in $N_4(p)$, or
2. q is in $N_D(p)$ and the set $N_4(p) \cap N_4(q)$ has no pixels whose values are from V .

Mixed adjacency is a modification of 8-adjacency, and is introduced to eliminate the ambiguities that may result from using 8-adjacency.

❖ Connectivity between pixels

➤ It is a vital topic in **digital image processing**.

It is used to define the boundaries of objects and region components in an image.

Let S represent a subset of pixels in an image. Two pixels p and q are said to be connected in S if there exists a path between them consisting entirely of pixels in S .

For any pixel p in S , the set of pixels that are connected to it in S is called a **connected component** of S . If it only has one component, and that component is connected, then S is called a **connected set**.

On the basis of adjacency, there are three forms of connectivity. They are as follows:

1. **4-connectivity:** If two or more pixels are 4-adjacent to each other, they are said to be 4-connected.
2. **8-connectivity:** If two or more pixels are 8-adjacent to each other, they are said to be 8-connected.
3. **M-connectivity:** If two or more pixels are m-adjacent to each other, they are said to be m-connected.

```

0  1  1
0  1  0
0  0  1
  
```

Fig: An arrangement of pixels

```

0  1—1
   |
0  1  0
   |
0  0  1
  
```

Fig: 4-connectivity of pixels

```

0  1—1
   | \
0  1  0
   | \
0  0  1
  
```

Fig: 8-connectivity of pixels

```

0  1—1
   | \
0  1  0
   | \
0  0  1
  
```

Fig: m-connectivity of pixels

❖ Region

Let R represent a subset of pixels in an image. We call R a **region** of the image if R is a connected set.

Two regions, R_i and R_j , are said to be **adjacent** if their union forms a connected set.

Regions that are not adjacent are said to be **disjoint**. We consider 4 and 8-adjacency when referring to regions.

❖ Boundary

The boundary is also known as the **border** or **contour**.

The **boundary** of a region R is the set of pixels in R that are adjacent to pixels in the complement of R .

What is adjacency in Image Processing

- ❖ Two pixels are connected if they are neighbours and their gray levels satisfy some specified criterion of similarity.

Gray Level Transformation

We have discussed some of the basic transformations in our tutorial of Basic transformation. In this tutorial we will look at some of the basic gray level transformations.

Image enhancement

Enhancing an image provides better contrast and a more detailed image as compare to non enhanced image. Image enhancement has very applications. It is used to enhance medical images, images captured in remote sensing, images from satellite etc,...

The transformation function has been given below $s = T(r)$

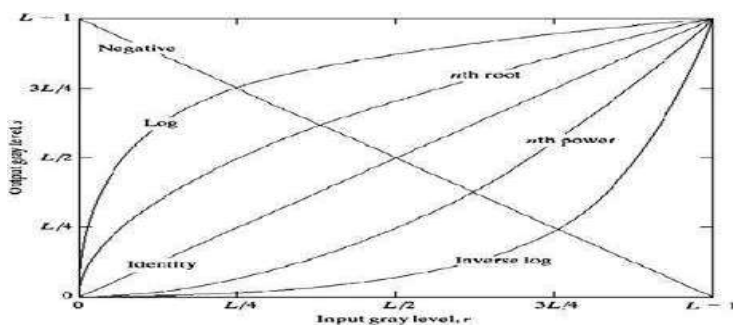
where r is the pixels of the input image and s is the pixels of the output image. T is a transformation function that maps each value of r to each value of s . Image enhancement can be done through gray level transformations which are discussed below.

Gray level transformation

There are three basic gray level transformation.

- Linear
- Logarithmic
- Power – law

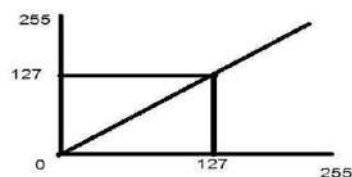
The overall graph of these transitions has been shown below



Linear transformation

- First we will look at the linear transformation. Linear transformation includes simple identity and negative transformation.
- Identity transformation has been discussed in our tutorial of image transformation, but a brief description of this transformation has been given here.
- Identity transition is shown by a straight line. In this transition, each value of the input image is directly mapped to each other value of output image.

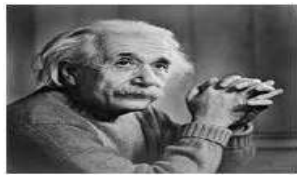
That results in the same input image and output image. And hence is called identity transformation. It has been shown below:



❖ Negative transformation

The second linear transformation is negative transformation, which is invert of identity transformation. In negative transformation, each value of the input image is subtracted from the L-1 and mapped onto the output image.

The result is somewhat like this.



Input Image



Output Image

In this case the following

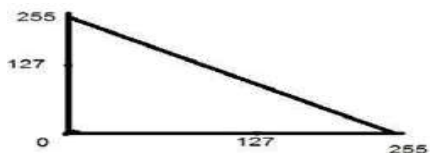
transition has been done.

$$s = (L - 1) - r$$

since the input image of Einstein is an 8 bpp image, so the number of levels in this image are 256.

Putting 256 in the equation, we get this $s = 255 - r$

So each value is subtracted by 255 and the result image has been shown above. So what happens is that, the lighter pixels become dark and the darker picture becomes light. And it results in image negative. It has been shown in the graph below.



❖ Logarithmic transformations

Logarithmic transformation further contains two type of transformation. Log transformation and inverse log transformation.

❖ Log transformation

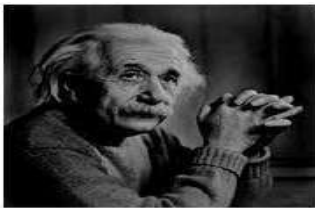
The log transformations can be defined by this formula $s = c \log(r + 1)$.

Where s and r are the pixel values of the output and the input image and c is a constant. The value 1 is added to each of the pixel value of the input image because if there is a pixel intensity of 0 in the image, then $\log(0)$ is equal to infinity. So 1 is added, to make the minimum value at least 1.

During log transformation, the dark pixels in an image are expanded as compare to the higher pixel values. The higher pixel values are kind of compressed in log transformation. This result in following image enhancement.

The value of c in the log transform adjust the kind of enhancement you are looking for.

Input Image



output Image



Log Transform Image

The inverse log transform is opposite to log transform.

Power – Law transformations

There are further two transformation is power law transformations, that include n th power and n throot transformation. These transformations can be given by the expression:

$$s = cr^{\gamma}$$

This symbol γ is called gamma, due to which this transformation is also known as gamma transformation.

Variation in the value of γ varies the enhancement of the images. Different display

devices / monitors have their own gamma correction, that's why they display their image at different intensity.

This type of transformation is used for enhancing images for different type of display devices. The gamma of different display devices is different. For example Gamma of CRT lies in between of 1.8 to 2.5, that means the image displayed on CRT is dark.

Correcting gamma.

$$s = cr^\gamma \quad s = cr^{(1/2.5)}$$

The same image but with different gamma values has been shown here.

For example



Gamma=6



Gamma =8



Gamma =10

❖ Filtering in the spatial domain (Spatial Filtering)

refers to image operators that change the gray value at any pixel (x,y) depending on the pixel values in a square neighborhood centered at (x,y) using a fixed integer matrix of the same size. **The integer matrix is called a *filter, mask, kernel or a window*.**

The mechanism of spatial filtering, shown below, consists simply of moving the filter mask from pixel to pixel in an image. At each pixel (x,y), the response of the filter at that pixel is calculated using a predefined relationship (linear or nonlinear).

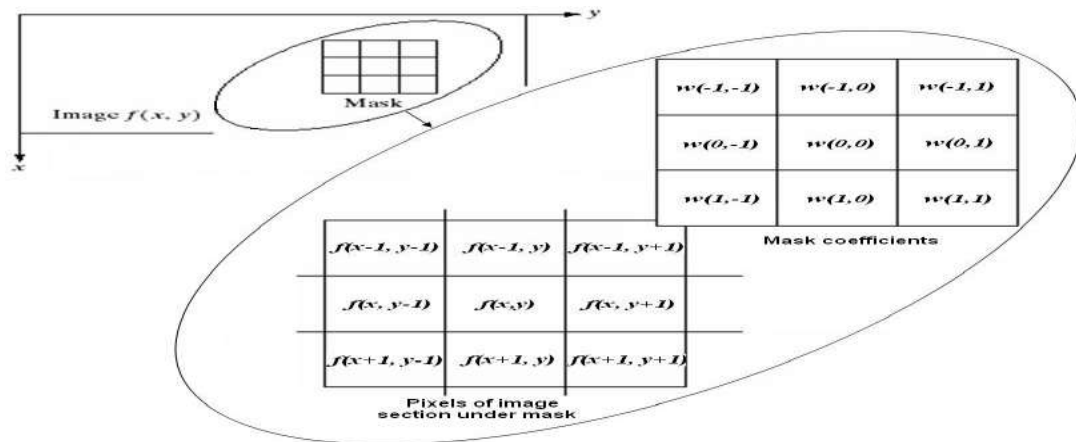


Figure 6.1 Spatial filtering

Note:

The size of mask must be odd (i.e. 3×3, 5×5, etc.) to ensure it has a center.

The smallest meaningful size is 3×3.

Linear Spatial Filtering (Convolution)

The process consists of moving the filter mask from pixel to pixel in an image.

At each pixel (x, y) , the response is given by a sum of products of the filter coefficients and the corresponding image pixels in the area spanned by the filter mask.

For the 3×3 mask shown in the previous figure, the result (or response), R , of linear filtering is:

$$R = w(-1, -1)f(x - 1, y - 1) + w(-1, 0)f(x - 1, y) + \dots \\ + w(0, 0)f(x, y) + \dots + w(1, 0)f(x + 1, y) + w(1, 1)f(x + 1, y + 1)$$

In general, linear filtering of an image f of size $M \times N$ with a filter mask of size $m \times n$ is given by the expression:

$$(x, y) = \sum_{s=-a}^a \sum_{t=-b}^b w(s, t)f(x + s, y + t)$$

where $a = (m - 1)/2$ and $b = (n - 1)/2$. To generate a complete filtered image this equation must be applied for $x = 0, 1, 2, \dots, M-1$ and $y = 0, 1, 2, \dots, N-1$.

❖ Nonlinear Spatial Filtering

- ✦ **The operation also consists of moving the filter mask from pixel to pixel in an image.**
- ✦ **The filtering operation is based conditionally on the values of the pixels in the neighborhood, and they do not explicitly use coefficients in the sum-of-products manner.**

For example, noise reduction can be achieved effectively with a nonlinear filter whose basic function is to compute the median gray-level value in the neighborhood in which the filter is located.

computation of the median is a nonlinear operation.

Example:

Use the following 3×3 mask to perform the convolution process on the shaded pixels in the 5×5 image below. Write the filtered image.

0	1/6	0
1/6	1/3	1/6
0	1/6	0

3×3 mask

30	40	50	70	90
40	50	80	60	100
35	255	70	0	120
30	45	80	100	130
40	50	90	125	140

5×5 image

Solution:

$$0 \times 30 + \frac{1}{6} \times 40 + 0 \times 50 + \frac{1}{6} \times 40 + \frac{1}{3} \times 50 + \frac{1}{6} \times 80 + 0 \times 35 + \frac{1}{6} \times 255 + 0 \times 70 = 85$$

$$0 \times 40 + \frac{1}{6} \times 50 + 0 \times 70 + \frac{1}{6} \times 50 + \frac{1}{3} \times 80 + \frac{1}{6} \times 60 + 0 \times 255 + \frac{1}{6} \times 70 + 0 \times 0 = 65$$

$$0 \times 50 + \frac{1}{6} \times 70 + 0 \times 90 + \frac{1}{6} \times 80 + \frac{1}{3} \times 60 + \frac{1}{6} \times 100 + 0 \times 70 + \frac{1}{6} \times 0 + 0 \times 120 =$$

$$0 \times 40 + \frac{1}{6} \times 50 + 0 \times 80 + \frac{1}{6} \times 35 + \frac{1}{3} \times 255 + \frac{1}{6} \times 70 + 0 \times 30 + \frac{1}{6} \times 45 + 0 \times 80 = 118$$

Filtered image =

30	40	50	70	90
40	85	65	61	100
35	118	92	58	120
30	84	77	89	130
40	50	90	125	140

❖ Spatial Filters

Spatial filters can be classified by effect into:

1. Smoothing Spatial Filters: also called lowpass filters. They include:

1.1 Averaging linear filters

1.2 Order-statistics nonlinear filters.

❖ Sharpening Spatial Filters: also called highpass filters. For example, the Laplacian linear filter.

❖ Smoothing Spatial Filters

are used for blurring and for noise reduction. Blurring is used in preprocessing steps to:

- remove small details from an image prior to (large) object extraction
- bridge small gaps in lines or curves.

Noise reduction can be accomplished by blurring with a linear filter and also by nonlinear filtering.

❖ Averaging linear filters

- The response of averaging filter is simply the average of the pixels contained in the neighborhood of the filter mask.
- The output of averaging filters is a smoothed image with reduced "sharp" transitions in gray levels.
- Noise and edges consist of sharp transitions in gray levels. Thus smoothing filters are used for noise reduction; however, they have the undesirable side effect that they blur edges.

The figure below shows two 3×3 averaging filters.

$$\frac{1}{9} \times \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Standard average filter

$$\frac{1}{16} \times \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

Weighted average filter

Note:

Weighted average filter has different coefficients to give more importance (weight) to some pixels at the expense of others. The idea behind that is to reduce blurring in the smoothing process.

Averaging linear filtering of an image f of size $M \times N$ with a filter mask of size $m \times n$ is given by the expression:

$$g(x, y) = \frac{\sum_{s=-a}^a \sum_{t=-b}^b w(s, t) f(x + s, y + t)}{\sum_{s=-a}^a \sum_{t=-b}^b w(s, t)}$$

To generate a complete filtered image this equation must be applied for $x = 0, 1, 2, \dots, M-1$ and $y = 0, 1, 2, \dots, N-1$.

Figure below shows an example of applying the standard averaging filter.



(a)



(b)



(c)



(d)



(e)



(f)

Figure 6.2 Effect of averaging filter.

(a) Original image. (b)-(f) Results of smoothing with square averaging filter masks of sizes $n = 3, 5, 9, 15$, and 35 , respectively.

As shown in the figure, the effects of averaging linear filter are:

1. Blurring which is increased whenever the mask size increases.
2. Blending (removing) small objects with the background. The size of the mask establishes the relative size of the blended objects.
3. Black border because of padding the borders of the original image.
4. Reduced image quality.

✦ Order-statistics filters

are nonlinear spatial filters whose response is based on ordering (ranking) the pixels contained in the neighborhood, and then replacing the value of the center pixel with the value determined by the ranking result.

Examples : include **Max, Min, and Median filters.**

❖ Median filter

- It replaces the value at the center by the median pixel value in the neighborhood, (i.e. the middle element after they are sorted). Median filters are particularly useful in removing impulse noise (also known as salt-and-pepper noise). Salt = 255, pepper = 0 gray levels.

In a 3×3 neighborhood the median is the 5th largest value, in a 5×5 neighborhood the 13th largest value, and so on.

For example, suppose that a 3×3 neighborhood has gray levels (10, 20, 0, 20, 255, 20, 20, 25, 15). These values are sorted as (0,10,15,20,20,20,20,25,255), which results in a median of 20 that replaces the original pixel value 255 (salt noise).

Example:

Consider the following 5×5 image:

20	30	50	80	100
30	20	80	100	110
25	255	70	0	120
30	30	80	100	130
40	50	90	125	140

Apply a 3×3 median filter on the shaded pixels, and write the filtered image.

Solution

20	30	50	80	100
30	20	80	100	110
25	255	70	0	120
30	30	80	100	130
40	50	90	125	140

20	30	50	80	100
30	20	80	100	110
25	255	70	0	120
30	30	80	100	130
40	50	90	125	140

Sort:

20, 25, 30, 30, 30, 70, 80, 80, 255

20	30	50	80	100
30	20	80	100	110
25	255	70	0	120
30	30	80	100	130
40	50	90	125	140

Sort

0, 70, 80, 80, 100, 100, 110, 120, 130

Filtered Image =

20	30	50	80	100
30	20	80	100	110
25	30	80	100	120
30	30	80	100	130
40	50	90	125	140

Sort

0, 20, 30, 70, 80, 80, 100, 100, 255

Figure below shows an example of applying the median filter on an image corrupted with salt-and-pepper noise.

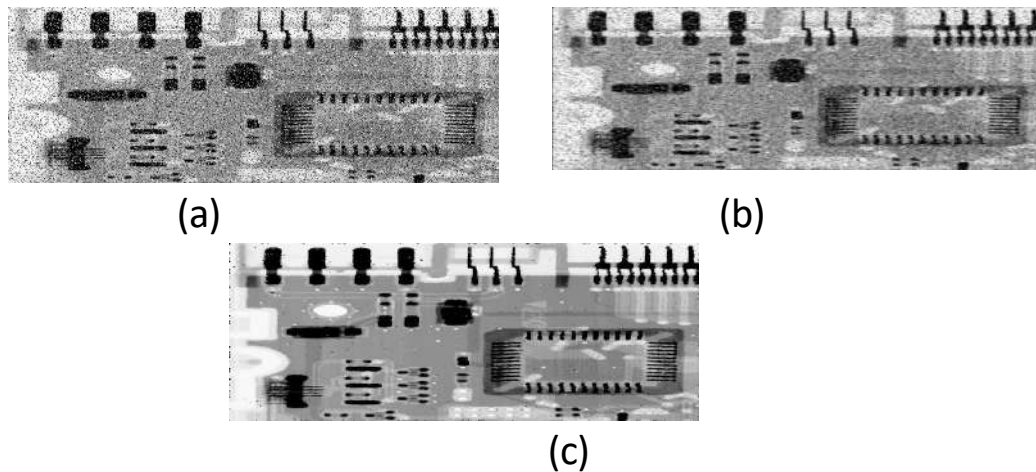


Figure 6.3 Effect of median filter. (a) Image corrupted by salt & pepper noise. (b) Result of applying 3×3 standard averaging filter on (a). (c) Result of applying 3×3 median filter on (a).

As shown in the figure, the effects of median filter are:

1. Noise reduction
2. Less blurring than averaging linear filter

✦ Sharpening Spatial Filters

- ✦ Sharpening aims to highlight fine details (e.g. edges) in an image, or enhance detail that has been blurred through errors or imperfect capturing devices.
- ✦ Image blurring can be achieved using averaging filters, and hence sharpening can be achieved by operators that invert averaging operators. In mathematics, averaging is equivalent to the concept of integration, and differentiation inverts integration. Thus, sharpening spatial filters can be represented by partial derivatives.

✦ Partial derivatives of digital functions

The first order partial derivatives of the digital image $f(x,y)$ are:

The first derivative must be:

$$\frac{\partial f}{\partial x} = (x+1, y) - f(x, y) \quad \text{and}$$

$$\frac{\partial f}{\partial y} = (x, y+1) - f(x, y)$$

zero along flat segments (i.e. constant gray values).

1) non-zero at the outset of gray level step or ramp (edges or noise)

2) non-zero along segments of continuing changes (i.e. ramps).

The second order partial derivatives of the digital image $f(x,y)$ are:

$$\frac{\partial^2 f}{\partial x^2} = (x+1, y) + f(x-1, y) - 2f(x, y)$$

$$\frac{\partial^2 f}{\partial y^2} = (x, y+1) + f(x, y-1) - 2f(x, y)$$

The second derivative must be:

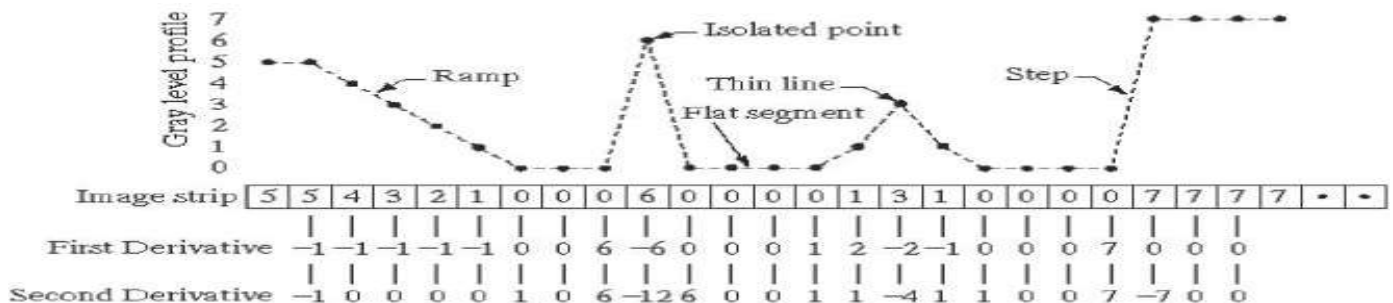
1) zero along flat segments.

2) nonzero at the outset and end of a gray-level step or ramp; zero along ramps

Consider the example below:



partial derivatives



We conclude that:

- 1st derivative detects thick edges while 2nd derivative detects thin edges.
- 2nd derivative has much stronger response at gray-level step than 1st derivative.

Thus, we can expect a second-order derivative to enhance fine detail (thin lines, edges, including noise) much more than a first-order derivative.

❖ The Laplacian Filter

The Laplacian operator of an image $f(x,y)$ is:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

This equation can be implemented using the 3×3 mask:

$$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

Since the Laplacian filter is a linear spatial filter, we can apply it using the same mechanism of the convolution process. **This will produce a laplacian image that has grayish edge lines and other discontinuities, all superimposed on a dark, featureless background.**

Background features can be "recovered" while still preserving the sharpening effect of the Laplacian operation simply by adding the original and Laplacian images

$$g(x, y) = f(x, y) + A^2 f(x, y)$$

The figure below shows an example of using Laplacian filter to sharpen an image.

Example of applying Laplacian filter



(A) Original Image



(B) Laplacian image



(C) Changed image