Design of a Control System for an Autonomous Vehicle Based on PID

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Abstract

The autonomous vehicle is a mobile robot affiliating multi-sensor navigation and positioning, intelligent decision making, and control technology. This report presents the control system for the autonomous vehicle and discusses the path tracking and stability of motion to navigate in unknown environments effectively. In this approach, we develop a two-degree-of-freedom dynamic model to represent the path-tracking problem in a state-space format. For controlling the instantaneous path error, traditional controllers have difficulty guaranteeing performance and stability. We will use the adaptive-PID controller method to overcome these drawbacks. With this approach, the flexibility of the vehicle control system will be increased and achieve significant advantages.

Problem Statement

"We need to design a PID controller for autonomous vehicle that allows path-tracking of autonomous vehicle. The vehicle control can be separated into lateral and longitudinal controls. We will focus on the lateral control to follow a given trajectory with a minimum of track error."

Introduction

The path-tracking control of an autonomous vehicle is one of the most challenging automation challenges because of constraints on mobility, speed of motion, high-speed operation, complex interaction with the environment, and typically a lack of prior information. We can separate the vehicle control into lateral and longitudinal controls. Here we focus on the lateral control to follow a given trajectory with a minimum of track error. In past studies, various theories and methods have been evaluated. However, the PID control method has gained much attention in recent times. Its simple structure has many advantages, such as good control effect and robust and easy implementation. Unfortunately, this method does not automatically adapt to the environment caused of the complexity of vehicle dynamics, the uncertainty of the external environments, and the non-holonomic constraint of the vehicle. An adaptive control algorithm has received attention and has been studied to overcome these difficulties.
Recently, adaptive control has been a great success in the industrial field. The adaptive control system can adapt and undertake corrective control action because of environment changes, achieving optimal or suboptimal control effects. The adaptive control method includes adaptive PID control based on neural networks, fuzzy adaptive PID control, model reference adaptive PID control, adaptive PID control based on genetic algorithms, and many more. However, to solve the trajectory tracking problem of unmanned vehicles, not all the adaptive PID control will give the best result. We required the fluctuation of output in the control procedure caused by the

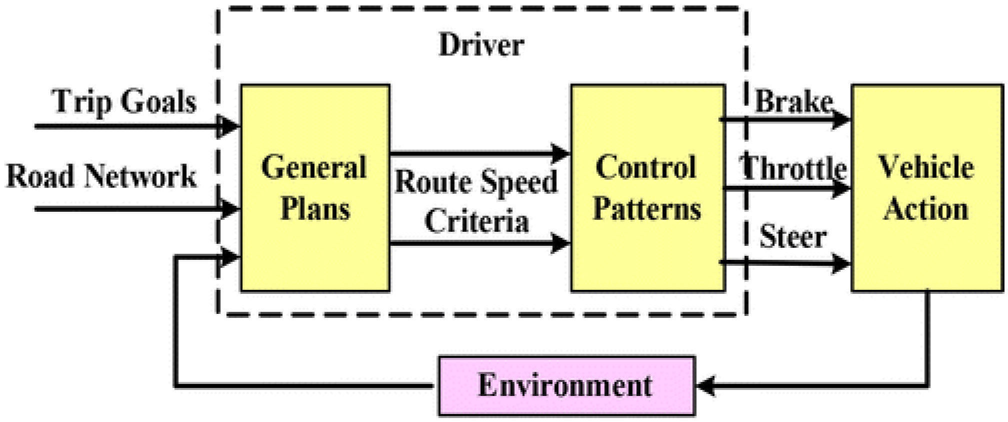


Figure 1. Model of the driver-vehicle-road closed loop system.

disturbance to be as small as possible; namely, the production of the steady-state equation is as tiny as possible. For that reason, we designed the adaptive controller based on the generalized minimum variance method. The standard of this adaptive controller is simple, and it is easy to understand. It can correct parameters constantly online and set the controller's parameters constantly, so we can gradually obtain the fundamental dynamic behavior of the process.

Mathematic Model Analysis of the Vehicle Control System

The external forces and torques acting on the vehicle are two main types: tire contact forces and aerodynamic forces. But the vehicle motion dealt with in this paper is mainly generated by the tire forces produced by the vehicle motion itself. Three forces act upon the tire, namely longitudinal force, lateral force and vertical force. The effect of longitudinal force will cause vehicle traction and braking. Driver controls the magnitude of the vehicle's driving force by the acceleration pedal and shift gear and controls the magnitude of braking force by the braking system. The effect of lateral force is to make the vehicle turn. The driver makes the tires generate a steering angle using the steering system to control the lateral force of the tires. The effect of vertical force is good adhesion of the vehicle to the road. For a general vehicle travelling on a city road, the effect of aerodynamics is little, therefore, we can ignore aerodynamic force in this problem of designing the vehicle's controller as mentioned below. Several different models have been used to simulate the dynamics of the vehicle. One common approach is to treat a four-wheeled vehicle as a two-wheeled system, also called the “bicycle model”, which makes the analysis of vehicle motion simpler. Under the following assumption, we can build a two degree-of-freedom dynamic model for describing the motion of the vehicle.

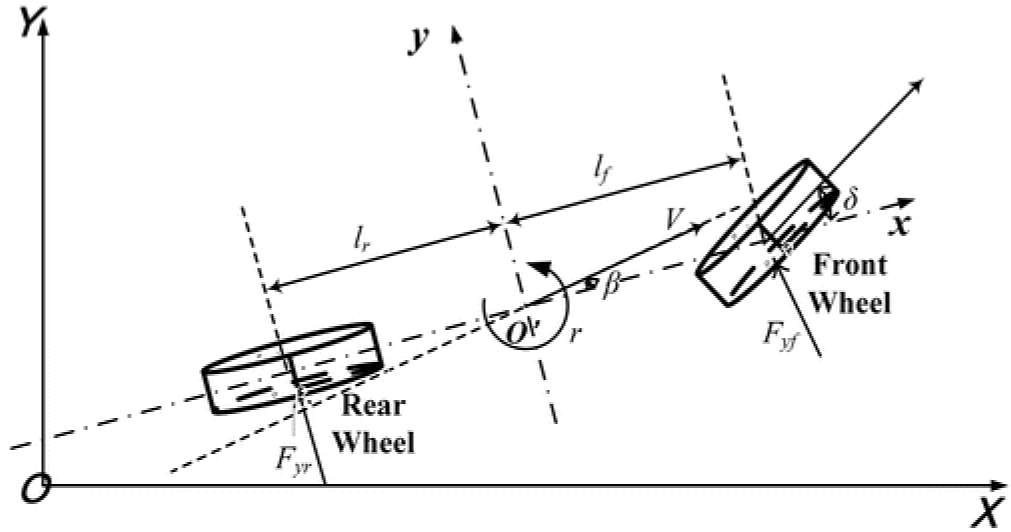


Figure 2. Equivalent bicycle model.

1. Supposing that the vehicle travels on a flat and level road, and that there is no input of vertical angle caused by road unevenness, we can ignore the vertical force and its coupling effects related with vehicle dynamics.
2. The structure of the vehicle is rigid including the suspension system.
3. Putting the input on the tire directly ignoring the steering system; or supposing the steering system is rigid, which puts the input imposed on the turning tires through the steering wheel with a fixed transmission ratio.
4. Ignoring aerodynamic force.
5. The vehicle is disturbed merely by the small perturbation in the equilibrium point, this means the input angle of the front wheel is small enough to ensure the linearity of equations of the vehicle motion.

As the left and right tire side-slip angles are equal, the steer angle is small and there is negligible roll motion. This is suitable for the left and right tires of the front and rear wheels to be concentrated at the intersecting point of the vehicle x-axis with the front and rear axles as shown in Figure 2. In this model, we set up a vehicle-centered coordinate system, O'-xyz. The rigid body vehicle has a velocity component of u in the longitudinal, x direction, and v in the lateral, y direction. The vehicle also has an angular velocity of r around the center of gravity. The net force components in x and y direction are ΣFx and ΣFx, and the external torque around z axis is ΣMz. The lateral motion of the vehicle is described below:

$$\mathrm{m}(\dot{u} - vr) = \sum F_x \qquad (1)$$

$$m(\dot{v} + ur) = \sum F_y \qquad (2)$$

$$I\dot{r} = \sum \mathrm{M}_z \qquad (3)$$

here, m is the vehicle inertia mass. I is vehicle yaw moment inertia.
Usually, the velocity component of u in the longitudinal is larger than the velocity component of u in the lateral. Therefore, we can represent u as:

$$u = u_c + \Delta u \qquad (4)$$

Here, uc is the velocity in heading direction and Δu is a disturbance of the velocity. We consider the vehicle is driving at uniform velocity, therefore, ΣFx=0, and with this small disturbance, Δur and vr can be ignored as negligible. Then the lateral motion of the vehicle can be described using the two degree-of-freedom model by the decoupling equations as below:

$$m(\dot{v} + u_c r) = \sum F_y \qquad (5)$$

$$I\dot{r} = \sum \mathrm{M}_z \qquad (6)$$

Usually, if there is no difference in the characteristics in the left and right tires, the lateral forces of the left and right tires will be equal. Taking the front and rear lateral forces as Fyf and Fyr, and the distances of the front and rear wheel axles from the center of gravity are a and b, then the equations are expressed as:

$$m(\dot{v} + u_c r) = \mathrm{F_{yf}} + \mathrm{F_{yr}} \qquad (7)$$

$$I\dot{r} = \mathrm{aF_{yf}} - \mathrm{bF_{yr}} \qquad (8)$$

If the tyre side-slip stiffness C is known, Fy is proportional to the side-slip angle α. When a side-slip angle is positive, Fy acts in the negative y-direction and can be written as below:

$$F_y = -C\alpha \qquad (9)$$

Because the rear wheel is not the steering wheel, the side-slip angle of the rear wheel can be approximated as:

In response to an arbitrary front wheel steer angle, δf. the side-slip angle of the rear wheel can be approximated as:

$$\alpha_r \approx \frac{v - br}{u_c} \qquad (10)$$

Substituting into the previous Eqns (4) and (5), the equations now become the fundamental equations of motion describing the vehicle plane motion as below:

$$m(\dot{v}+u_c r)=\mathrm{C_f}\delta_f-\frac{(\mathrm{C_f}+\mathrm{C_r})}{u_c}v-\frac{(aC_f-bC_r)}{u_c}r \quad (12)$$

$$\square\dot{r}=\mathrm{aC_f}\delta_f-\frac{(\mathrm{aC_f}+b\mathrm{C_r})}{u_c}v-\frac{(a^2C_f-b^2C_r)}{u_c}r \quad (13)$$

Rearranging equations (12) and (13) yields the state space system described as below:

$$\dot{\mathbf{X}}=\mathbf{AX}+\mathbf{BU} \qquad (14)$$

Where:

$$\mathbf{A}=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \quad \mathbf{B}=\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}, \quad \mathbf{X}=\begin{bmatrix} v \\ r \end{bmatrix}, \quad \mathbf{U}=\delta_f(t)$$

With:

$$a_{11}=-\frac{C_f+C_r}{u_c m}, \quad a_{12}=-u_c-\frac{aC_f-bC_r}{u_c m}$$

$$a_{21}=-\frac{aC_f-bC_r}{u_c I}, \quad a_{22}=-\frac{a^2C_f-b^2C_r}{u_c I}$$

$$b_1=\frac{C_f}{m}, \quad b_2=\frac{aC_f}{I}$$

Vehicle Parameter	Value
vehicle mass/*kg*	2325
vehicle yaw moment inertia $I/kg\cdot m^2$	4132
wheelbase/m	3.025
longitudinal position of front wheel from vehicle centre of gravity *a/m*	1.430
height of vehicle centre of gravity h_g/m	0.5
cornering stiffness of front tyre $C_f/N\cdot rad^{-1}$	80000
cornering stiffness of rear tyre $C_r/N\cdot rad^{-1}$	96000

Table 1. Pertinent vehicle parameters of Intelligent Pioneer.

Assume that the vehicle is driving at a constant velocity with 20m/s (uc=20m/s) and referring to the table of pertinent vehicle parameters given in Table 1, substituting these into Equation 14, we have:

$$\begin{bmatrix} \dot{v} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} -3.785 & -19.167 \\ 0.469 & 0.976 \end{bmatrix} \begin{bmatrix} v \\ r \end{bmatrix} + \begin{bmatrix} 34.409 \\ 27.686 \end{bmatrix} \delta_f(t) \quad (15)$$

In the dynamic equation above, the two state variables are yaw rate and lateral velocity. In this paper a lateral controller is designed to reduce lateral error E of trajectory tracking. The lateral path error E is a function of the lateral velocity V, the heading θ, and the longitudinal velocity V. This relation is shown in Equations 16 and 17.

$$\dot{\mathrm{E}} = \mathrm{v} + \mathrm{u}_c \theta \qquad (16)$$

$$\dot{\theta} = \mathrm{r} \qquad (17)$$

The augmented state space model is shown in Equation 18.

$$\begin{bmatrix} \dot{v} \\ \dot{r} \\ \dot{\theta} \\ \dot{E} \end{bmatrix} = \begin{bmatrix} -3.785 & -19.167 & 0 & 0 \\ 0.469 & 0.976 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 20 & 0 \end{bmatrix} \begin{bmatrix} v \\ r \\ \theta \\ E \end{bmatrix} + \begin{bmatrix} 34.409 \\ 27.686 \\ 0 \\ 0 \end{bmatrix} \delta_f(t) \quad (18)$$

It is assumed that the lateral path error E is the measurable output of the system. Consequently, the equation can be described as:

$$\mathbf{Y} = \mathbf{CX} + \mathbf{DU} \qquad (19)$$

Here the C matrix is described as:

$$\mathbf{C} = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}, \mathbf{D} = 0$$

The lateral path error E is also the quantity which must be controlled. This system's open-loop control transfer function of interest is thus the transfer function from steering angle input to path error output. This may be determined using Equation 20.

$$\begin{aligned} G(s) &= \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D} \\ &= \frac{34.41\, s^2 - 10.52\, s + 2419}{s^4 + 2.809\, s^3 + 5.295\, s^2} \end{aligned} \qquad (20)$$

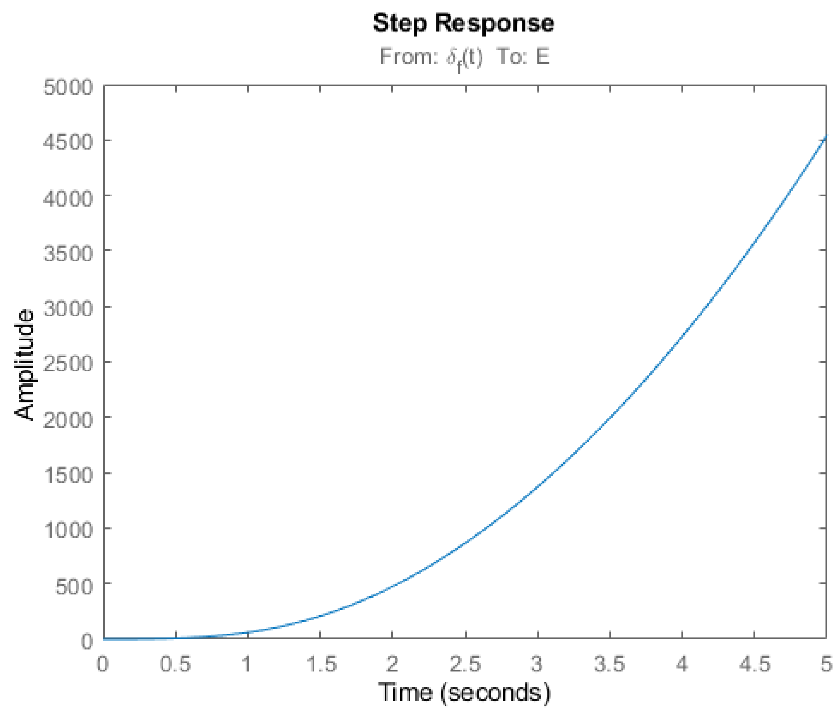
Note that the relative degree is 2, the numerator is Hurwitz, the denominator has a double root at the origin and the sign of the high frequency gain is known (positive). Now that the structure of the plant is known, the next section describes the design of a model reference controller for controlling the lateral path error.

Controller Design Based on PID

Equation 20 can be written as below:

$$G(s) = \frac{1}{s^2}\frac{34.41s^2 - 10.52s + 2419}{s^2 + 2.809s + 5.295} = \frac{1}{s^2}C(s) \quad (21)$$

Because 0 is the double pole of the system, the system will be unstable. We must use velocity feedback, see as Figure 5.



This is the step response of the above open loop transfer function. We can see that the above system is not stable.

We must use feedback control.

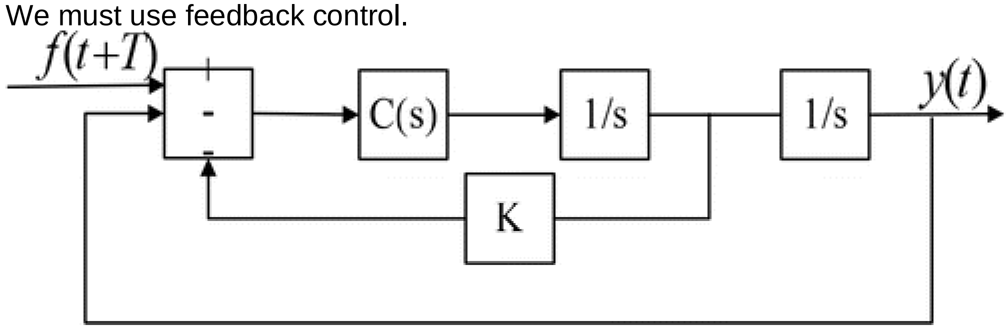


Figure 5. Closed-loop control strategy

The coefficient is chosen at *K*=10 to make the additional zero point nearby the origin, therefore, *H*(*s*) = (1 + 10s). The system's open-loop transfer function becomes:

$$\left(\frac{1}{s^2}\right)C(s)H(s) = \frac{(1+10s)(34.41s^2 - 10.52s + 2419)}{s^2(s^2 + 2.809s + 5.295)} \quad (22)$$

Now we also need to further design a PID controller to improve the performance. The PID control law is given by:

$$G_c(s) = K_p + \frac{K_I}{s} + K_D s \qquad (23)$$

We chose K_P=1, K_I=0.5, K_D=10. The block diagram of closed-loop PID control is shown in Figure 8.

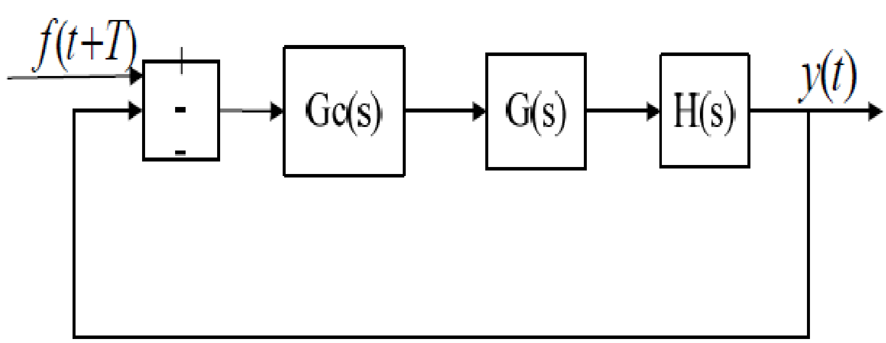
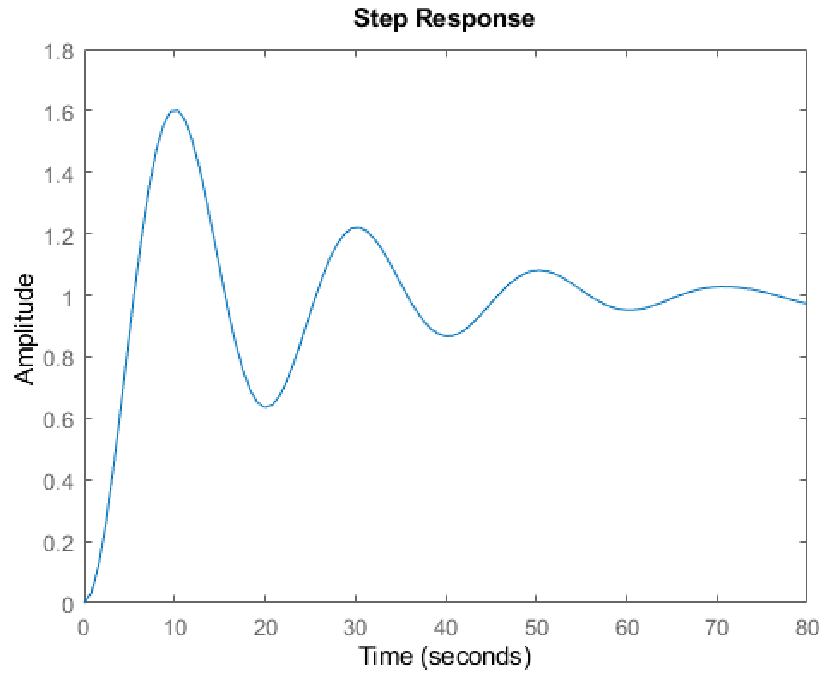
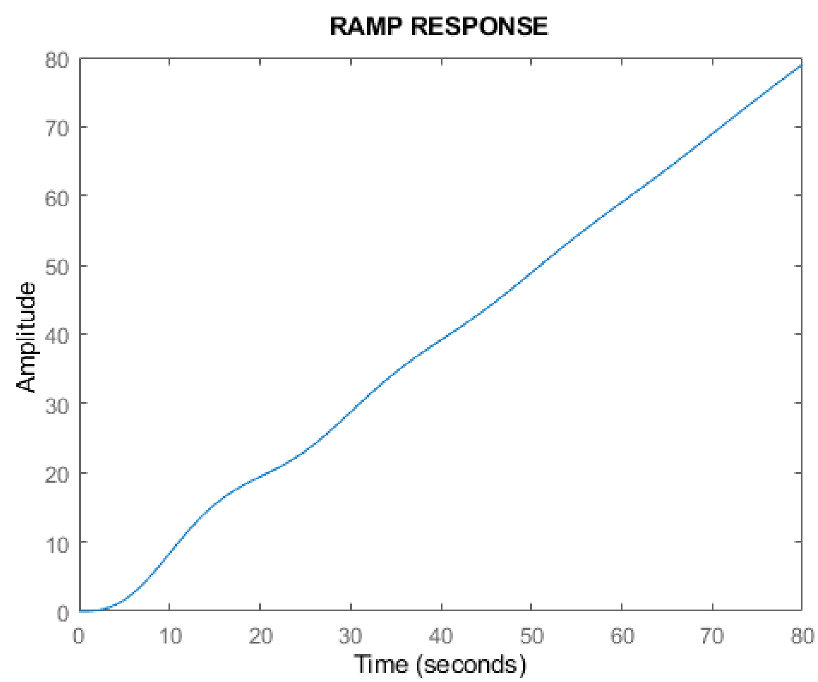


Figure 8. Block diagram of PID control.

The above designed system has the following response for step and ramp input.



Step Response
Amplitude
Time (seconds)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
0
10
20
30
40
50
60
70
80



RAMP RESPONSE
Amplitude
Time (seconds)
0
10
20
30
40
50
60
70
80
0
10
20
30
40
50
60
70
80

CONCLUSIONS

In this paper a simplified bicycle model of an automobile was used to model the vehicle's lateral dynamics. Based on the vehicle parameters for Intelligent Pioneer, a suitable reference model was developed. Full control on the state variable E was achieved for fix parameters available As, can be seen in the graphs. Analysis is done using MATLAB, step and ramp response is potted. But above analysis is just a preliminary approach under ideal dynamics models, we have ignored many factors which make automatic lateral control of vehicles difficult. These include changing vehicle parameters with time, changing road conditions, as well as disturbances caused by GPS signal attenuation and other factors. Traditional controllers have difficulty in guaranteeing performance and stability over a wide range of parameter changes. In order to solve these problems and make the system automatically adapt to changes of the environment and parameters, for which we needed to design an adaptive PID controller.

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