AE 616/236: Mid-sem. 18/9/2024. Total=20. Duration=2 hrs.

Answers without detailed explanation will be heavily penalized.
Answer Q1 OR Q2, and all of Q3 through Q8.

1. (4 points) In Rayleigh flow, it is known that

$$\frac{T}{T^*} = \frac{(1+\gamma)^2 M^2}{(1+\gamma M^2)^2}, \quad \text{and} \quad \frac{T_0}{T_0^*} = \frac{(1+\gamma)M^2\{2+(\gamma-1)M^2\}}{(1+\gamma M^2)^2}.$$

Using only these and the energy equation, quantitatively show that there is a range of Mach numbers where cooling causes the static temperature to rise. Characterize the maximum achievable temperature in such a flow when air is the medium.

OR

2. (4 points) At the beginning of a duct carrying *nitrogen*, the pressure is 1.5 bar, the temperature is 248.2 K, and the Mach number is 0.80. After some heat transfer, the pressure is 2.5 bar. Determine the direction and amount of heat transfer, and the downstream temperature and Mach number.

3. (2 points) The difference between the total and static pressure before a normal shock is 5 bar. What is the maximum static pressure that can exist at this point ahead of the shock? The gas is *oxygen*.

4. (3 points) Consider the flow of a perfect gas (of gas constant R and specific heat ratio γ) through a duct of area A. Show that, if the mass flow rate through the duct is $\dot{m}$ and the total temperature is T_0, then the pressure at the Fanno-sonic state is given by the relation

$$p^{\#} = \frac{\dot{m}}{A}\left[\frac{2RT_0}{\gamma(\gamma+1)}\right]^{1/2}.$$

5. At section 'A' in a constant-area duct carrying air, the stagnation pressure is 147 kPa and the Mach number is 0.80. At section 'B' along the same 'Fanno line', the pressure is 131.93 kPa and the temperature is 50°C. The diameter and length of the duct are 20 cm and 4 m, respectively.

   (a) (1 point) Compute the Mach number at section 'B'.

   (b) (½ point) Compute the temperature at section 'A'.

   (c) (½ point) Which way is the air flowing?

   (d) (1 point) What is the Fanning friction factor of the duct?

6. A simple wedge with a total included angle of 34° is used to measure the Mach number of supersonic flows. When inserted into a wind tunnel and aligned with the flow, oblique shocks are observed at 38° angles to the free stream.

   (a) (1 point) What is the Mach number in the wind tunnel?

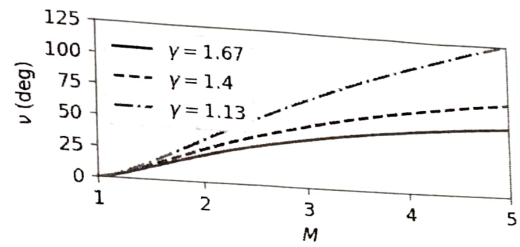
   (b) (1 point) Through what range of Mach numbers could this wedge be useful?

7. In the flow past a compression corner, the upstream Mach number and pressure are 3.5 and 1 atm., respectively. Downstream of the corner, the pressure is 5.48 atm..

   (a) (2 points) Calculate the deflection angle of the corner.

   (b) (1 point) What is the post-shock Mach number?

8. (3 points) The variation of the Prandtl-Meyer function $\nu(M)$ with Mach number M is shown in the graph below for three different specific heat ratios. Consider a supersonic *air* flow at a particular Mach number M_1 encountering a convex corner of a particular angle θ. Then comment on the changes to the downstream Mach number and the expansion fan angle, if the air stream is replaced by *helium* (keeping M_1 and θ fixed).



AE 616/236: End-sem. 19/11/2024. Total=40. Duration=3 hrs.

Answers without detailed explanation will be heavily penalized.
Answer Q1, and any 6 of the remaining questions.

1. (a) (1 point) Discuss the concept and utility of the 'diabatic-sonic' state in Rayleigh flow.

   (b) (1 point) Sketch the $h-s$ diagram for a Fanno flow, and discuss its implication.

   (c) (1 point) Discuss the phenomenon of Mach reflection?

   (d) (1 point) What is a free boundary, and how and why does an oblique shock reflect from it?

   (e) (2 points) Briefly describe the operation of a pressure-driven shock tube, along with plots of the typical variation of pressure, temperature and velocity vs. position in the tube, as well as a relevant $x-t$ diagram.

   (f) (1 point) Consider a cone with half angle β. What is its drag coefficient, as predicted by classical Newtonian theory in the context of hypersonic aerodynamics?

   (g) (1 point) Briefly describe the modelling of vibrational mode of excitation in a diatomic gas, and what it implies for temperature changes across a normal shock at high speeds.

   (h) (2 points) Briefly describe the modelling procedure that accounts for equilibrium chemical reactions in case of flow through a normal shock.

2. Air flows through a constant area frictionless duct with inlet temperature of 20°C.

   (a) (1 point) If the inlet Mach number is 0.5, what is the maximum possible exit stagnation temperature?

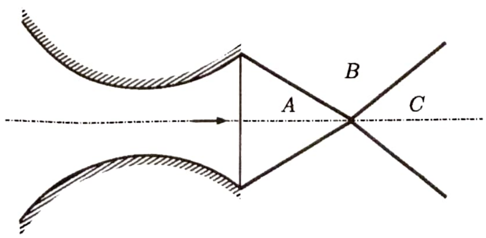
   (b) (4 points) It is desired to transfer heat such that the stagnation temperature at the duct's exit is 1180 K. For this condition what must be the limiting subsonic inlet Mach number?

3. Air is flowing into an insulated duct with a velocity of 153 m/s. The temperature and pressure at the inlet are 280°C and 28 bar respectively. The exit pressure is 15.7 bar.

   (a) (3½ points) Find the temperature at the exit of the duct.

   (b) (1½ points) If the duct diameter is 15 cm and the Fanning friction factor is 0.005, find its length.

4. (5 points) A supersonic flow leaves a two-dimensional nozzle as a parallel, one-dimensional flow (region A) with a Mach number of 2.6 and static pressure (in region A) of 50 kPa. The pressure of the atmosphere into which the jet discharges is 100 kPa. Find the pressures in regions B and C of the figure. Also describe qualitatively, quantitatively and diagrammatically the actual wave system that separates region B from A and region C from B.



5. (5 points) A reservoir containing air at 2 MPa is connected to ambient air at 100 kPa through a two-dimensional converging-diverging nozzle designed to produce one-dimensional flow at Mach 2.0 (see figure above; same as that for previous question). Find the pressures in regions A, B and C of the figure. Also describe qualitatively, quantitatively and diagrammatically the actual wave system that separates region B from A and region C from B.

6. Air maintained at 150 kPa and 40°C in a large reservoir flows through a converging-only nozzle having 40 mm diameter at exit.

   (a) (½ point) What is the back pressure that just chokes the nozzle?

   (b) (2½ points) What is the maximum possible mass flow rate?

   (c) (2 points) Find the mass flow rate when the back-pressure is 94.5 kPa.

   Do not use any formula for mass flow rate, other than its basic definition.

7. Air flows through a converging-diverging nozzle. At some section in the nozzle, the pressure, temperature, velocity and cross-sectional area are 2 bar, 200°C, 174.4 m/s and 1000 mm^2, respectively. Assuming isentropic flow conditions, determine:

   (a) (1 point) sonic velocity and Mach number at this section,

   (b) (1 point) stagnation temperature and pressure,

   (c) (1½ points) Mach number, velocity, and flow area at the exit section where the pressure is 1.1 bar, and

   (d) (1½ points) pressure, temperature, velocity and flow area at the throat section.

8. (5 points) Approximate knocking in an auto engine as the occurrence of a normal shock wave traveling at 1004 m/s downward, into the unburned mixture at 700 kPa and 500 K. Compared to this high speed of the shock, the piston's speed (say around 10 m/s upward) can be safely neglected. Determine the pressure acting on the piston face after the shock reflects from it. Assume the gas has the properties of air ($R = 287$ J/kg · K), and acts as a perfect gas with $\gamma = 1.4$. Use appropriate sketches to explain your approach.

9. (5 points) Starting from first principles, derive the relation between the static pressure ratio across a moving expansion wave in a one-dimensional duct, and the flow velocities (in the laboratory frame) ahead of and behind it (considering that both velocities may be non-zero). Use appropriate sketches.