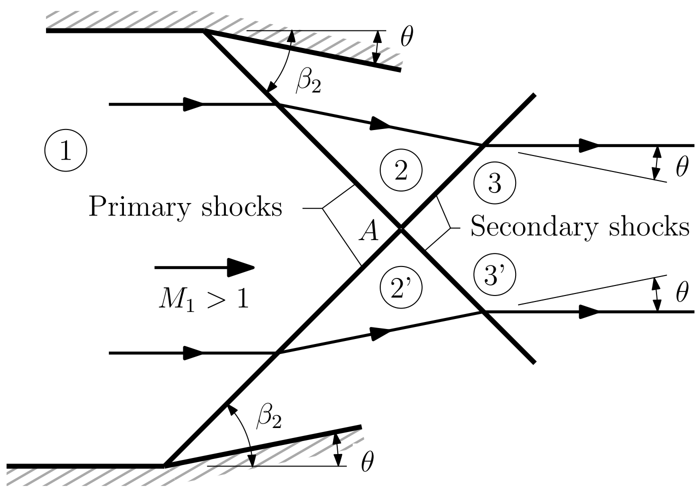
AE 616/236: Qz 2. 23/10/2024. Total=10. Duration=1 hr.

1. (4 points) A 2d duct carries air at Mach 3 and 1 atm pressure. At a point along the duct, the floor of the duct is wedged in by 5°. At a point slightly further along, the ceiling is also wedged in by the same 5°. The oblique shocks thus formed must interact with each other. (a) Draw a sketch of the flow, and determine the (b) flow angle, (c) Mach numbers, and (d) pressure, prevailing immediately downstream of this first interaction.

Solution:



Since the two wedges are at the same angle, the primary oblique shocks are of the same strength and angle. Even though the floor and ceiling wedges are not encountered at the same streamwise location, the shock system is still going to be symmetric, albeit about a plane that is not centered in the duct. Due to this symmetry, the problem will be akin to reflection of an oblique shock at an impermeable planar wall. In other words, the flow downstream of the first shock interaction point is going to be parallel to the original flow direction; i.e., the flow angle $\boxed{\Phi = 0°}$. This also means that there is no slipline that is formed in this case, so that a single Mach number and a single entropy prevails downstream of the first shock-shock interaction.

From the $\theta - \beta - M_1$ oblique shock chart, when an $M_1 = 3$ flow is turned into itself by $\theta = 5°$ then the oblique shock angle is $\beta \approx 23.1°$. Thus, the pre-shock shock-normal Mach number is $M_{n,1} = M_1 \sin\beta = 3 \times \sin 23.1° = 1.177$. In the normal shock table, the nearest entry for pre-shock Mach number is 1.18, for which $M_{n,2/1} = 0.8549$ and $p_2/p_1 = 1.458$. Then, for the primary shock, the post-shock Mach number is $M_2 = M_{n,2/1}/\sin(\beta - \theta) = 0.8549/\sin(23.1° - 5°) = 2.752$.

Due to the symmetry of the flow, we know that the flow must turn back by the original angle of 5° through the 'refracted' shock. From the $\theta - \beta - M_1$ oblique shock chart, when an $M = 2.8$ (nearest entry to $M_2 = 2.752$) flow is turned into itself by $\theta_2 = 5°$ then the oblique shock angle is $\beta_2 \approx 24.6°$. Thus, the pre-shock shock-normal Mach number is $M_{n,2/3} = M_2 \sin\beta_2 = 2.752 \times \sin 24.6° = 1.145$. In the normal shock table, the nearest entry for pre-shock Mach number is 1.14, for which $M_{n,3/2} = 0.8820$ and $p_3/p_2 = 1.350$. Then, for the primary shock, the common post-shock Mach number is $M_3 = M_{n,3/2}/\sin(\beta_2 - \theta_2) = 0.8820/\sin(24.6° - 5°) = \boxed{2.629}$.

The common pressure behind the first shock interaction is $p_3 = (p_3/p_2)(p_2/p_1)p_1 = 1.350 \times 1.458 \times 1 = \boxed{1.968 \text{ atm}}$.

2. (4 points) A nozzle's efficiency may be defined as the ratio of the actual change in kinetic energy that it produces to its ideal counterpart (for a given pressure difference). Air at 450 K and 1.6 atm feeds a converging-only nozzle having an efficiency of 94%. The receiver pressure is 1 atm. What is the actual nozzle outlet temperature? Assume that both the actual and ideal cases are adiabatic.

Solution:

We first recall the stagnation-to-back-pressure for a converging nozzle operating at its critical condition (viz. sonic exit). It is found from the isentropic flow table for sonic condition: $(p_0/p_b)_{crit} = 1.893$. For the given reservoir pressure of 1.6 atm, the critical back-pressure is $(p_b)_{crit} = p_0/(p_0/p_b)_{crit} = 1.6/1.893 = 0.8452$ atm, which is lower than the prevailing back-pressure of 1 atm. Thus, we are guaranteed that the nozzle is not choked.

The reservoir-to-exit pressure ratio is $p_0/p_e = 1.6/1 = 1.6$. From the isentropic flow tables, the nearest entry for the pressure ratio is 1.587 corresponding to $M = 0.84$ and $T_0/T = 1.141$. Thus, the ideal (isentropic) nozzle outlet temperature is $T_{es} = (T_{es}/T_0)T_0 = 450/1.141 = 394.4$ K.

From the adiabatic energy equation, we have

$$T_1 - T_2 = \frac{1}{2C_p}(V_2^2 - V_1^2) = \frac{\Delta KE}{C_p}.$$

Thus, the efficiency of the nozzle given by $\eta = \Delta KE/\Delta KE_s$, may be expressed as

$$\eta = \frac{\Delta KE}{\Delta KE_s} = \frac{T_0 - T_e}{T_0 - T_{es}}.$$

This means that the non-isentropy (or inefficiency) of the nozzle manifests as a decrease in the temperature drop through the nozzle (as well as a decrease in entropy).

From this, we can find the actual exit temperature as

$$T_e = T_0 - \eta(T_0 - T_{es}) = 450 - 0.94(450 - 394.4) = \boxed{397.7 \text{ K}}.$$

3. (2 points) Discuss the starting problem of a supersonic intake.

Solution:

The ideal supersonic intake is a C-D duct, where the deceleration is potentially shockless at the design diffuser-inlet Mach number M_{design}. Let the corresponding area ratio be A_{inlet}/A_t. However, the aircraft has to be accelerated from rest, so that the freestream Mach number will initially be less than M_{design}. Indeed, at subsonic flight speeds, the C-D duct will act like a nozzle, accelerating the flow through it. At a particular subsonic Mach number, we will have the corresponding $A/A^* = A_{inlet}/A_t$, so that sonic flow will exist at the throat. Further increase in flight Mach number will lead to *spillage*, as the throat is choked. In this condition, the intake is unable to handle all the mass flow that is coming at its inlet, so that the excess mass must flow outside.

When the flight Mach number just becomes supersonic, this spillage will result in a bow shock being formed – the intake is behaving as a bluff body. If one considers the flow just behind the normal portion of the bow shock, then it is subsonic but close to unity too, so that the choking will continue. As the Mach number increases further, the bow shock will strengthen and move closer to the inlet. However, the post-shock subsonic Mach number behind the normal portion of the bow shock will remain too high for isentropic acceleration to the sonic throat.

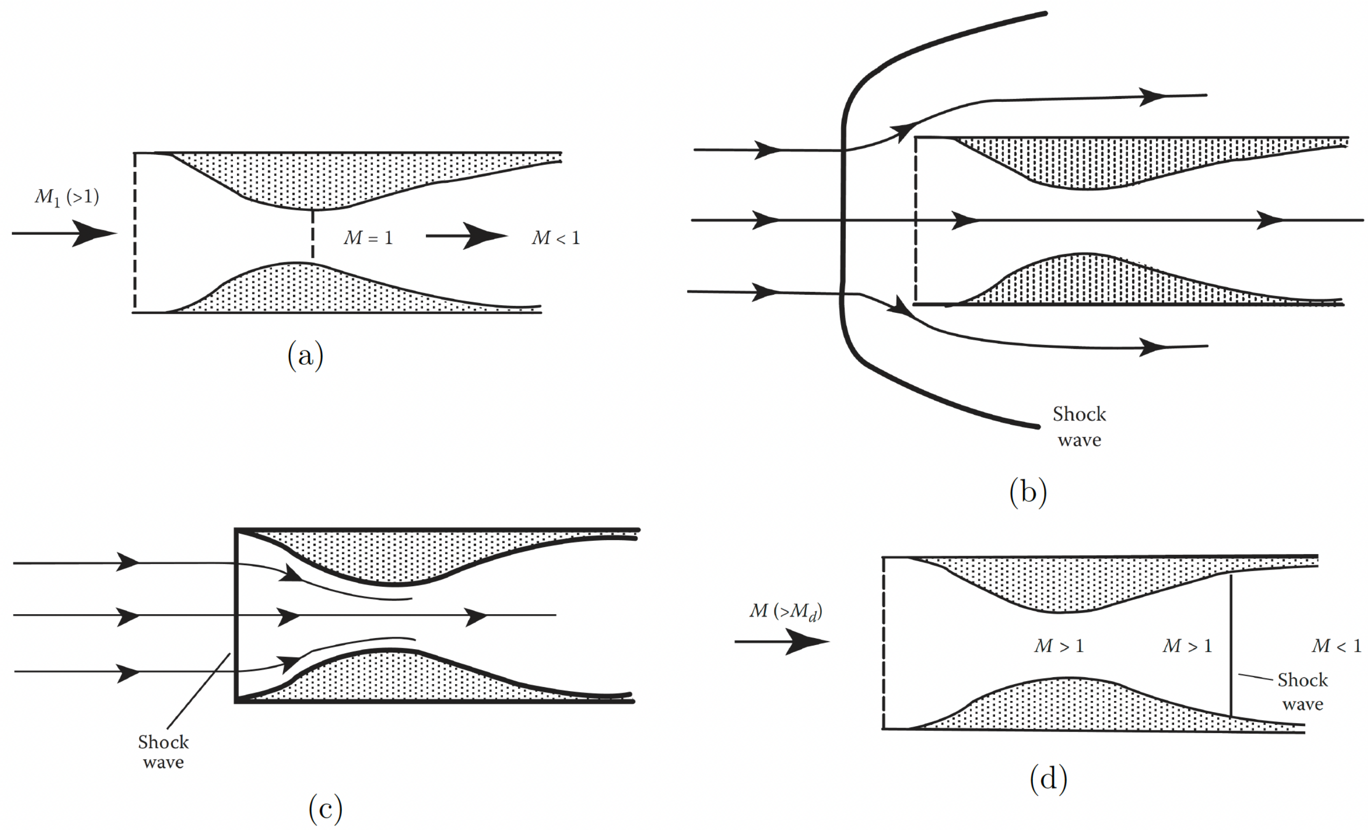
It may be observed from the isentropic flow tables in conjunction with the normal shock tables that A/A^* for a supersonic (pre-shock) Mach number M_1 is always greater than that for the corresponding post-shock Mach number M_2. Thus, even at the design speed, the A/A^* corresponding to the subsonic post-shock flow will still be lower than A_{inlet}/A_t, so that the spillage, and associated bow shock, will continue to occur.

Indeed, one has to '*overspeed*' in order for M_2 behind the corresponding normal shock to correspond to $A/A^* = A_{inlet}/A_t$. At this speed, the bow shock will become a normal shock standing right at the inlet.

A slight increase of the speed will cause the shock to enter into the converging section. However, the normal shock is unstable in the converging section, and will be '*swallowed*'; it will come to settle in the diverging section. Then, reduction of the flow speed will cause the shock wave to move back towards the throat. As the flight speed approaches the design speed *from above*, the shock will be just downstream of the throat and infinitesimally weak (since the upstream flow is just supersonic).

However, if the aircraft were to slow down ever so slightly, then the shock will need to move to the converging section where it is unstable, thereby resulting in it being '*disgorged*'. Then, one has to overspeed to the requisite Mach number for the intake to be restarted.

This is clearly an issue – the so-called 'starting problem'!



Starting of a fixed-geometry converging-diverging diffuser. (a) Ideal operation. (b) Flow near intake when $M < M_{\text{design}}$. (c) Flow near intake before shock is swallowed. (d) Flow in intake after shock wave is swallowed. Reproduced from Oosthuizen and Carscallen (2014).