AE 616/236: Qz 1. 30/8/2024. Total=10. Duration=1 hr.

1. (2 points) Consider the following relation for a normal shock:

$$M_2^2 = \frac{1 + 0.5(\gamma - 1)M_1^2}{\gamma M_1^2 - 0.5(\gamma - 1)}.$$

Hence, give math and physical reasoning why M_2 must be less than 1.

Solution:

The pre-shock flow must be supersonic from second law of thermodynamics.

Let us assume that $M_2 > 1$. Then, we must have

$$\begin{aligned} &\gamma M_1^2 - 0.5(\gamma - 1) < 1 + 0.5(\gamma - 1)M_1^2 \\ \implies &\frac{\gamma + 1}{2} M_1^2 < \frac{\gamma + 1}{2} \\ \implies &M_1^2 < 1. \end{aligned}$$

But this is disallowed by the second law of thermodynamics, as stated above.

Thus, using this *reductio ad absurdum* procedure, we have proven the requisite, i.e., $M_2 \leq 1$.

2. (3 points) The pressures on either side of a normal shock wave are measured as 400 kPa and 100 kPa. Find the upstream and downstream Mach numbers and the loss in stagnation pressure, if air is the fluid.

Solution:

This is Example 2.2 of Yahya (2018).

The static pressure rises across the shock. Thus, we must have that $p_1 = 100$ kPa and $p_2 = 400$ kPa. Thus, the static pressure ratio across the shock is $p_2/p_1 = 4.0$. Looking up the normal shock tables for air, we have the nearest pressure ratio as 4.045 for $\boxed{M_1 = 1.9}$, $\boxed{M_2 = 0.5956}$ and $p_{02}/p_{01} = 0.7674$.

From the isentropic flow tables for air, for $M = 1.9$ we have for $p_0/p = 6.701$. Therefore, the total pressures before and after the shock are $p_{01} = 6.701 p_1 = 670.1$ kPa and $p_{02} = 0.7674 p_{01} = 514.2$ kPa. Thus, the loss in stagnation pressure across the shock is $\boxed{155.9 \text{ kPa}}$.

3. (1 point) Is the Rayleigh flow reversible? Justify.

Solution:

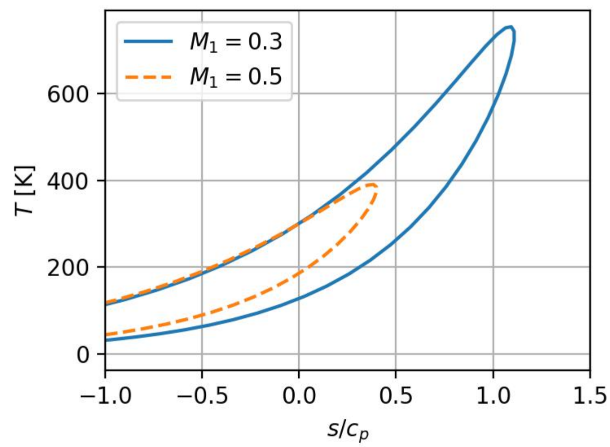
In the Rayleigh flow, the increase of entropy, noted for example in the Mollier diagram, is not necessarily due to irreversibility but due to heating. Indeed, cooling causes the entropy to decrease. The question is if some amount of heat, say q, is added in the Rayleigh flow between stations '1' & '2', and the same amount of heat is removed subsequently between stations '2' & '3', is the '3' state identical to the '1' state?

It is evident from a study of the governing integral equations that, in the energy equation, we will have $T_{03} = T_{02} - q/C_p = (T_{01} + q/C_p) - q/C_p = T_{01}$. Moreover, the heat addition/removal term does not appear explicitly in either of the two remaining equations – mass and momentum. Thus, there is nothing to distinguish between states '3' and '1'.

Aliter: One may take a more circuitous route to prove mathematically that the change in entropy is solely due to the heat addition and not due to any intrinsic irreversibility in the Rayleigh flow. For this, we go back to the original definition of entropy change in a process, viz. $\mathrm{d}s = T^{-1}\mathrm{d}q_{\text{rev}}$, and claim that $q \equiv q_{\text{rev}}$. Knowing the temperature variation, we can integrate this equation between states '1' and '2', and verify that the result is the same as obtained from the derived formula used, viz. $\Delta s = C_p \ln(T_2/T_1) - R\ln(p_2/p_1)$.

4. (4 points) Consider two Rayleigh flows – A & B – both with the same inlet temperature and pressure of 300 K and 1 bar, respectively. Flow A has inlet Mach number 0.3 whereas flow B has inlet Mach number 0.5. Assume that both flows have reference entropy of 0 at their respective inlet conditions. Then, on the same graph, sketch the $T - s$ curves for the two flows in absolute terms (i.e., without using any normalization or bias). Label the actual coordinates of enough relevant points on the curves so as to fix them. No marks if you do not justify your sketch.

Solution:



The two curves are shifted and stretched versions of the normalized $T/T^* - (s - s^*)/C_p$ curves for Rayleigh flow that were flashed in class. They must intersect at $(0, 300\text{ K})$.

On the normalized curve, flow A is represented by a point that is lower on the subsonic part ($(s - s^*)/C_p = \ln(T/T^*) - (\gamma-1)/\gamma \ln(p/p^*) = -1.110, T/T^* = 0.4089$) compared to flow B ($(s - s^*)/C_p = -0.4000, T/T^* = 0.7901$). Thus, the latter curve should be shifted leftwards relative to the former. Or, the first curve should be shifted rightwards by 1.110 whereas the second curve should be shifted rightwards by 0.4089 only. Coming to the temperature axis, we have $T^* = 733.7$ K for the first flow but only 379.7 K for the second one. Thus, these are temperatures at which the two curves should reach their respective peak entropies. Moreover, these also represent the multiplicative factors for scaling the ordinates of the original curve to arrive at the ordinates of the two new curves – the first curve will appear scaled about twice the second one. These considerations explain the curves plotted above.