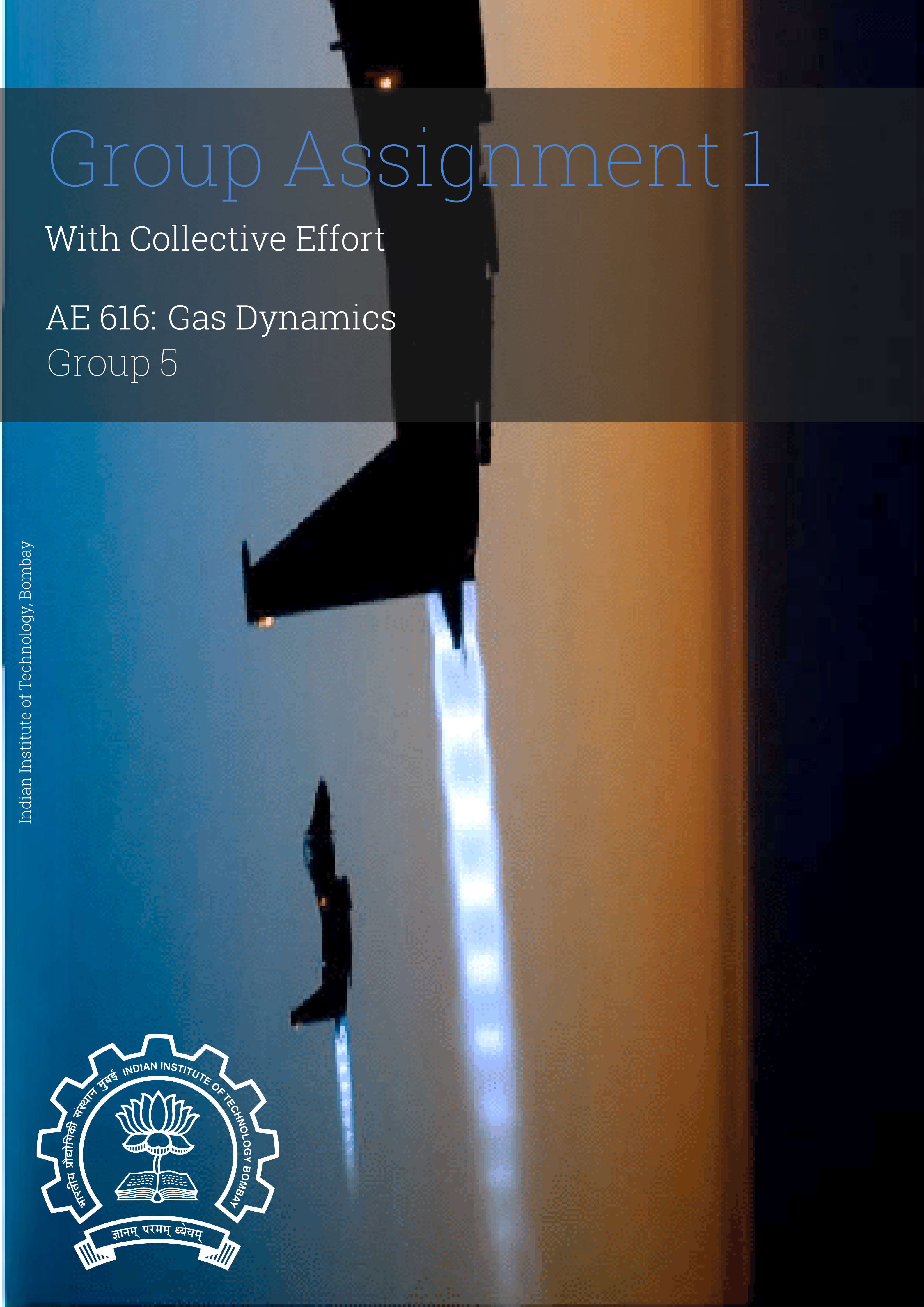

Group Assignment 1

With Collective Effort

AE 616: Gas Dynamics

Group 5

Indian Institute of Technology, Bombay

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With Collective Effort

by

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Student Name	Student ID
Piyush Hemantkumar Gautam	190010050
Sachin Kumar	24M0039
Manish Sunil Lakhode	24M0040
Kushal Shanbhag Narcinva	24M0058

Instructor: Prof. Aniruddha Sinha
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Nomenclature

Abbreviations

Symbols

Symbol	Definition	Unit
T_0	Stagnation Temperature	[K]
X^*	Parameter X in the Starred (Diabatic-Sonic) state	[unit(X)]
M	Mach Number	[ ]
c_p	Specific Heat at Constant Pressure	[J$\cdot$ Kg$^{-1}\cdot$ K^{-1}]
γ	Ratio of Specific Heats	[ ]

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Question 1

Derive a closed-form analytical expression for the Mach number in terms of the ratio of the total temperature to its diabatic sonic counterpart (i.e., T_0/T_0^*). Clearly identify the subsonic and supersonic solutions, with proper justification. Also, determine the ranges of T_0/T_0^* in which your equations are valid.

Solution:

As already derived in class,

$$\frac{T_{0,2}}{T_{0,1}} = \frac{\left(1+\gamma M_1^2\right)^2 M_2^2 \left(1-\frac{\gamma-1}{2}M_2^2\right)}{\left(1+\gamma M_2^2\right)^2 M_1^2 \left(1-\frac{\gamma-1}{2}M_1^2\right)} \quad \Longrightarrow \quad \frac{T_0}{T_0^*} = \frac{2(1+\gamma)M^2\left(1+\frac{\gamma-1}{2}M^2\right)}{(1+\gamma M^2)^2}$$

Let $a = \frac{T_0}{T_0^*},\ \ x = M^2$

Then $(1-\gamma^2(1-a))x^2 - (1+\gamma(1-a))x + a = 0$

This simplifies to

$$x = \tfrac{1+\gamma-a*\gamma\pm\sqrt{(1+\gamma-a*\gamma)^2-(1+a*\gamma^2-\gamma^2)}}{1+a*\gamma^2-\gamma^2}$$

$$x = \tfrac{1+\gamma-a*\gamma}{1+a*\gamma^2-\gamma^2} \pm \tfrac{\sqrt{\gamma^2*(1-a)+2*\gamma(1-a)+1-a}}{1-\gamma^2(1-a)}$$

Let $1-a=b$ $\tfrac{1+\gamma*b}{1-\gamma^2*b} = \tfrac{(\gamma+1)\sqrt{b}}{1-\gamma^2*b}$

$$\boxed{M^2 = \frac{1+\gamma(1-\frac{T_0}{T_0^*})}{1-\gamma^2(1-\frac{T_0}{T_0^*})} \pm \frac{1+\gamma}{1-\gamma^2(1-\frac{T_0}{T_0^*})}\sqrt{1-\frac{T_0}{T_0^*}}}$$

The + part of the solution would be the supersonic solution, and the - part would be the subsonic solution. This is because we know how a Rayleigh curve looks like, and that the only time we get a single Mach number here is at the starred state. Any other $\frac{T_0}{T_0^*}$ will give one subsonic and one supersonic solution. The above equations are only valid for the values of $\frac{T_0}{T_0^*}$ which give a positive M^2, since M physically cannot be non-real. That is,

$$\frac{1+\gamma(1-\frac{T_0}{T_0^*})}{1-\gamma^2(1-\frac{T_0}{T_0^*})} \pm \frac{1+\gamma}{1-\gamma^2(1-\frac{T_0}{T_0^*})}\sqrt{1-\frac{T_0}{T_0^*}} \geq 0$$

Since M^2 has to be real, $\frac{T_0}{T_0^*} \leq 1$.
Let us rewrite the inequality again in terms of $b = 1 - \frac{T_0}{T_0^*}$ (always positive):

$$\frac{1+\gamma b}{1-\gamma^2 b} \geq \mp \frac{1+\gamma}{1-\gamma^2 b}\sqrt{b}$$

For $b > \frac{1}{\gamma^2}$, the denominator is negative. Thus our condition gets divided into four parts:

1.1. Supersonic and $\mathbf{b} > \frac{1}{\gamma^2}$

$$1 + \gamma b \leq -(1+\gamma)\sqrt{b}$$

Clearly, this is impossible.

1.2. Supersonic and $b < \frac{1}{\gamma^2}$

$$1 + \gamma b \geq -(1+\gamma)\sqrt{b}$$

Clearly, this is always the case.

1.3. Subsonic and $b > \frac{1}{\gamma^2}$

$$\begin{aligned} 1 + \gamma b &\leq (1+\gamma)\sqrt{b} \\ (1+\gamma b)^2 &\leq (1+\gamma)^2 b \\ 1 + 2\gamma b + \gamma^2 b^2 &\leq b + 2\gamma b + \gamma^2 b \\ \gamma^2 b^2 - (1+\gamma^2) b + 1 &\leq 0 \\ (\gamma^2 b - 1)(b-1) &\leq 0 \\ &\boxed{b \in \left(\frac{1}{\gamma^2}, 1\right]} \end{aligned}$$

1.4. Subsonic and $b < \frac{1}{\gamma^2}$

$$\begin{aligned} 1 + \gamma b &\geq (1+\gamma)\sqrt{b} \\ (\gamma^2 b - 1)(b-1) &\geq 0 \\ &\boxed{b \in \left[0, \frac{1}{\gamma^2}\right)} \end{aligned}$$

1.5. Special case: $b = \frac{1}{\gamma^2}$

From our original quadratic equation, we get

$$\begin{aligned} -\left(1 + \frac{1}{\gamma}\right) x &= -\left(1 - \frac{1}{\gamma^2}\right) \\ x &= 1 - \frac{1}{\gamma} \\ M &= \sqrt{\frac{\gamma - 1}{\gamma}} \end{aligned}$$

Summary: In the subsonic regime, all values of $\frac{T_0}{T_0^*}$ except $1 - \frac{1}{\gamma^2}$ work, but for the supersonic regime, only those greater than $1 - \frac{1}{\gamma^2}$ work.

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Question 2

Write a code (and run it) to reproduce the graph flashed in class showing the ratios of thermodynamic properties to their respective diabatic sonic counterparts (e.g. $\frac{p}{p^*}$, $\frac{T}{T^*}$, $\frac{T_0}{T_0^*}$, etc.) versus Mach number. Also reproduce the T-s diagram flashed in class (with appropriate normalization).

Solution:

Code and grpahs for this Problem is given in Google colab. **Google Colab link**

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Question 3

Code up your solution of the first question and reuse parts from the second solution to design and implement a function with appropriate arguments and outputs, such that you can solve typical Rayleigh flow problems. Make this function in such a way that you can use it to solve examples 3.13 and 3.14 of Anderson's textbook (2003 edition) (or any two examples of your choice that exercise both the sub-sonic and supersonic parts of your code). Hence validate your code, with the given solutions to these example problems.

Solution:

Solution for this problem is given in Google Colab with code. **Google Colab link**