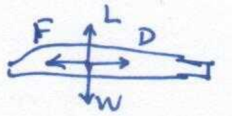
QUIZ-I

(1) Range of aircraft,



$F = D$ & $L = W$

$$F \cdot u_o = \eta_o \dot{m}_F \cdot Q = D \cdot u_o$$

$$\therefore \frac{\eta_o \dot{m}_F \cdot Q}{W} = \frac{D}{L} u_o$$

$$\Rightarrow -\frac{\eta_o Q}{g} \frac{dW}{W} = \frac{D}{L} u_o \cdot dt = \frac{dR}{(L/D)}$$

Using $\frac{dW}{dt} = -\dot{m}_F \cdot g$

Integrating, $R = \eta_o \cdot \frac{Q}{g} \frac{L}{D} \ln\left(\frac{W_i}{W_f}\right)$

Replacing. $\eta_o = \frac{F \cdot u_o}{\dot{m}_F \cdot Q} \Rightarrow \boxed{R = \left(M_o \cdot \frac{L}{D}\right) \frac{a_o/g}{TSFC} \ln\left(\frac{W_i}{W_f}\right)}$

Given, $M_o = 2.2$

$\frac{L}{D} = 3 \frac{M_o + 3}{M_o} = 7.09$

$\frac{W_i}{W_f} = 3$

$T_o = 210\,K \Rightarrow a_o = \sqrt{\gamma R T_o}$

where $\gamma = 1.4$, $R = 287\ J/kgK$

$a_o = 290.48\ m/s$

$TSFC = 0.2\ kg/N \cdot h = 5.55 \times 10^{-5} \frac{kg}{N \cdot s}$

$$\therefore R = 2.2 \times 7.09 \times \frac{290.48/9.81}{5.55 \times 10^{-5}} \ln(3) = 2.2 \times 7.09 \times 5.33 \times 10^5 \times \ln(3)$$

$$\therefore \boxed{R = 9133.57\ km}$$

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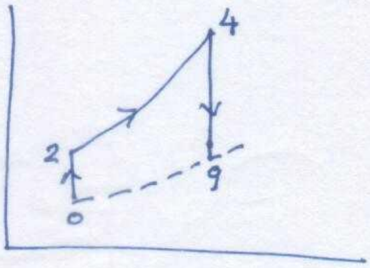
(2) Using energy balance across burner in Ramjet,

$$\dot{m}_o \cdot C_p T_{t2} + \dot{m}_F \cdot Q = (\dot{m}_o + \dot{m}_F) \cdot C_p \cdot T_{t4}$$



Simplifying. $f = \frac{C_p T_{t2}}{Q}\left(\frac{T_{t4}}{T_{t2}} - 1\right)$

$T_{t2} = T_o \cdot \tau_r$

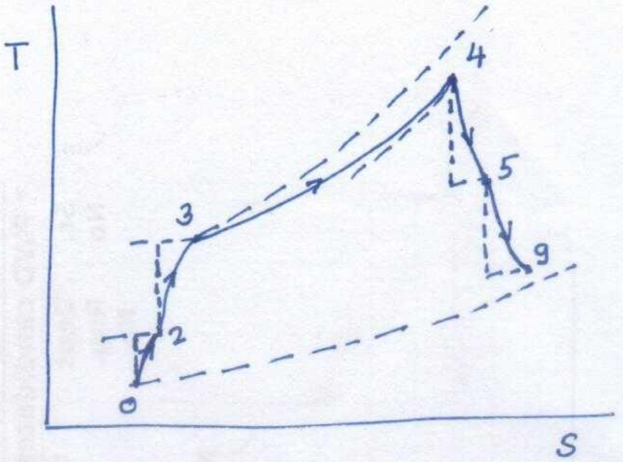
$T_{t4} = T_o \cdot \tau_\lambda \Rightarrow f = \frac{C_p \cdot T_o \cdot \tau_r}{Q}\left(\frac{\tau_\lambda}{\tau_r} - 1\right)$

From given data, $M_o = 3 \Rightarrow \tau_r = 1 + \frac{\gamma - 1}{2} M_o^2 = 2.8$

$T_o = 216K$, $T_{t4} = 2800\,K \Rightarrow \tau_\lambda = 12.963$

$\therefore$ Putting in above eq^n, $\boxed{f = 0.05}$

(3) Real turbojet engine,



Given data: $M_0 = 1.8$, $T_0 = 216\ K$, $T_{t4} = 1296\ K$

$$\therefore \quad \tau_r = 1 + \frac{\gamma - 1}{2} M_0^2 = 1.648$$

$$\tau_\lambda = \frac{T_{t4}}{T_0} = 6$$

Optimum compressor pressure ratio, $\pi_c = \left[\frac{\sqrt{\tau_\lambda}}{\tau_r} \right]^{\frac{\gamma}{\gamma - 1}} = 4$

Corresponding, $\tau_c = 1.486$

Thermal efficiency, $\eta_{th} = 1 - \frac{1}{\tau_r \tau_c} = \boxed{0.5916}$

$$\eta_{th} = \frac{\dot{m}_o (v_g^2 - v_0^2)}{2\, \dot{m}_f \cdot Q} = \frac{a_0^2 \left(\left(\frac{v_g}{a_0}\right)^2 - M_0^2\right)}{2 f Q}$$

$$f \cdot Q = c_p T_0 \tau_r \tau_c (\tau_b - 1) \quad \& \quad \tau_\lambda = \tau_r \tau_c \tau_b$$

$$\left(\frac{v_g}{a_0}\right)^2 = \frac{2}{\gamma - 1} (\tau_r \tau_c \tau_t - 1) \tau_b \qquad \& \quad \tau_t = 1 - \frac{\tau_r}{\tau_\lambda} (\tau_c - 1)$$

$$\eta_{th} = \frac{\gamma R \cancel{T_0} \left[\frac{2}{\gamma-1} (\tau_r \tau_c \tau_t - 1) \tau_b - M_0^2\right]}{2\, C_p \cancel{T_0}\, \tau_r \tau_c (\tau_b - 1)}$$

$$C_p = \frac{\gamma}{\gamma - 1} R \Rightarrow \frac{\gamma R}{C_p} = \gamma - 1$$

$$\tau_r = 1 + \frac{\gamma - 1}{2} M_0^2$$

$$M_0^2 = 2(\tau_r - 1)$$

$$\eta_{th} = \frac{2 (\tau_r \tau_c \tau_t - 1) \cdot \tau_b - (\gamma - 1) M_0^2}{2\, \tau_r \tau_c (\tau_b - 1)}$$

$$\eta_{th} = \frac{2 (\tau_r \tau_c \tau_t - 1) \tau_b - 2(\tau_r - 1)}{2\, \tau_r \tau_c (\tau_b - 1)} = \frac{(\tau_r \tau_c \tau_t - 1) \tau_b - (\tau_r - 1)}{\tau_r \tau_c (\tau_b - 1)}$$

$$\eta_{th} = \frac{\tau_\lambda \cdot \tau_t - \tau_b - \tau_r + 1}{\tau_r \tau_c (\tau_b - 1)} = \frac{\tau_\lambda \tau_t - \tau_r}{\tau_r \tau_c (\tau_b - 1)} - \frac{\cancel{(\tau_b - 1)}}{\tau_r \tau_c \cancel{(\tau_b - 1)}}$$

$$\eta_{th} = \frac{\tau_\lambda \tau_t - \tau_r}{\tau_\lambda - \tau_r \tau_c} - \frac{1}{\tau_r \tau_c}$$

$$\tau_\lambda \cdot \tau_t = \tau_\lambda - \tau_r (\tau_c - 1) = \tau_\lambda - \tau_r \tau_c + \tau_r$$

$$\therefore \eta_{th} = \frac{\tau_\lambda - \tau_r \tau_c + \cancel{\tau_r} - \cancel{\tau_r}}{\tau_\lambda - \tau_r \tau_c} - \frac{1}{\tau_r \tau_c}$$

$$\therefore \boxed{\eta_{th} = 1 - \frac{1}{\tau_r \tau_c}}$$