(1) $P_0 = 10\,kPa$, $T_0 = 216\,K$, $M_0 = 0.9$, $\alpha = 0.3$, $\dot{m}_0 = 65\,kg/s$

$$\alpha = \frac{\dot{m}_F}{\dot{m}_C} \quad \& \quad \dot{m}_C + \dot{m}_F = \dot{m}_0 \Rightarrow \quad \dot{m}_C = 50\,kg/s, \quad \dot{m}_F = 15\,kg/s$$

	γ	C_p (J/kgK)	$R = \frac{\gamma-1}{\gamma} C_p$ (J/kgK)
Air	1.4	1004	286.85
Turbine	1.4	1004	286.85
Afterburner	1.33	1240	310

$$P_{t0} = P_0 \left[1 + \frac{\gamma-1}{2} M_0^2\right]^{\gamma/\gamma-1} = 16.913\,kPa$$

$$T_{t0} = T_0 \left[1 + \frac{\gamma-1}{2} M_0^2\right] = 251\,K$$

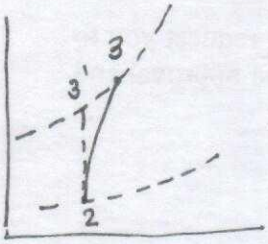
$$V_0 = \sqrt{\gamma R T_0} \cdot M_0$$

$$V_0 = 265.087\,m/s$$

Diffuser section: Adiabatic compression $\Rightarrow T_{t2} = T_{t0} = 251\,K$ (1)

Loss in total pressure = 5 % $\Rightarrow P_{t2} = 0.95\,P_{t0} = 16.067\,kPa$

Compressor section: $\pi_c = 24.5 \quad \therefore \frac{P_{t3}}{P_{t2}} = \frac{P_{t3'}}{P_{t2}} = 24.5$



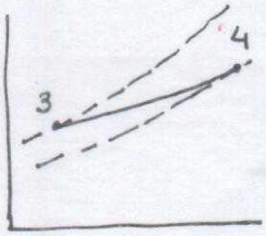
$$\Rightarrow P_{t3} = P_{t3'} = 393.65\,kPa$$

$$\frac{T_{t3'}}{T_{t2}} = \left(\frac{P_{t3'}}{P_{t2}}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_{t3'} = 626\,K$$

$$\eta_c = 0.9 = \frac{T_{t3'} - T_{t2}}{T_{t3} - T_{t2}} \Rightarrow T_{t3} = 667.67\,K$$ (2)

Combustor: Total pressure loss : 6 %. $\Rightarrow P_{t4} = 0.94\,P_{t3} = 370.031\,kPa$

$T_{t4} = 1850\,K$, $Q = 42000\,kJ/kg$, $\eta_b = 0.985$



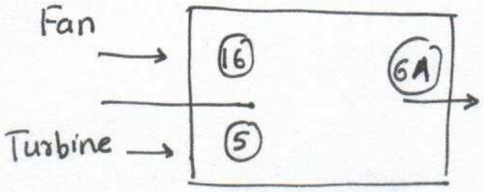
$$\dot{m}_c \cdot C_p \cdot T_{t3} + \eta_b \cdot \dot{m}_f \cdot Q = C_p \cdot T_{t4} \cdot (\dot{m}_c + \dot{m}_f)$$

$$\Rightarrow \dot{m}_f = \frac{\dot{m}_c \cdot C_p (T_{t4} - T_{t3})}{\eta_b \cdot Q - C_p T_{t4}} = 1.502\,kg/s$$

$$f = \frac{\dot{m}_f}{\dot{m}_c} = 0.03$$

$$\dot{m}_4 = (1+f)\,\dot{m}_c = 51.50\,kg/s$$ (2)

Fan and turbine section need to be analysed simultaneously by matching conditions at mixer inlet



Fan stream in mixer - 16

Turbine stream in mixer - 5

As per (vi) $\Rightarrow \quad P_{t5} = P_{t16}$

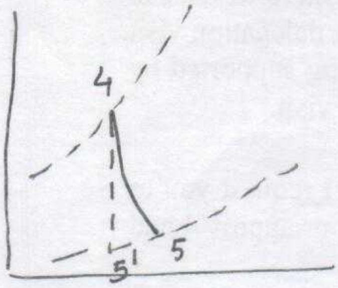
Let fan exit station is (13) & (13) to (16) is bypass duct.

As per (v) $\Rightarrow P_{t16} = 0.99\, P_{t13}$ & $T_{t16} = T_{t13}$

Turbine section: $\quad \dot{W}_C + \dot{W}_F = \eta_m \cdot \dot{W}_T$

$$\dot{m}_c \cdot C_p (T_{t3} - T_{t2}) + \alpha \cdot \dot{m}_c \cdot C_p (T_{t13} - T_{t2}) = \eta_m \cdot \dot{m}_c (1+f)\, C_p (T_{t4} - T_{t5})$$

$$\Rightarrow (T_{t3} - T_{t2}) + \alpha (T_{t13} - T_{t2}) = \eta_m \cdot (1+f)(T_{t4} - T_{t5})$$



$$\eta_T = \frac{T_{t4} - T_{t5}}{T_{t4} - T_{t5'}} \quad \text{and} \quad \frac{T_{t5'}}{T_{t4}} = \left(\frac{P_{t5}}{P_{t4}}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_{t4} - T_{t5} = \eta_T \cdot T_{t4}\left[1 - \left(\frac{P_{t5}}{P_{t4}}\right)^{\frac{\gamma-1}{\gamma}}\right] = \eta_T \cdot T_{t4}\left[1 - \left(\frac{0.99\, P_{t13}}{P_{t4}}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

Compression in fan is isentropic $\Rightarrow \quad \dfrac{T_{t13}}{T_{t2}} = \left(\dfrac{P_{t13}}{P_{t2}}\right)^{\frac{\gamma-1}{\gamma}}$

$$\Rightarrow (T_{t3} - T_{t2}) + \alpha \cdot T_{t2}\left[\left(\frac{P_{t13}}{P_{t2}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] = \eta_m \cdot (1+f)\, \eta_T \cdot T_{t4}\left[1 - \left(\frac{0.99\, P_{t13}}{P_{t4}}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

Solving for P_{t13}, $\Rightarrow \quad P_{t13} = 115.78$ kPa

$\therefore$ Fan pressure ratio, $\dfrac{P_{t13}}{P_{t2}} = 7.2$ & $T_{t13} = 441.3$ K

Also, $\quad P_{t16} = 114.622$ kPa & $T_{t16} = T_{t13} = 441.3$ K

$\therefore$ Turbine exit pressure, $\quad P_{t5} = 114.622$ kPa

8

$T_{t5} = \cancel{484.3}$ K

1365.69

Energy balance in mixer:

$$\dot{m}_c \cdot (1+f) \cdot C_p \cdot T_{t5} + \alpha \cdot \dot{m}_c \cdot C_p \cdot T_{t16} = \dot{m}_c (1+f+\alpha)\, C_p \cdot T_{t6A}$$

$$\therefore \quad T_{t6A} = \cancel{474.6} \text{ K}$$

1157.18

2

Pressure loss in mixer, = 5%.

$$\therefore \quad P_{t6A} = 0.95\, P_{t5} = 108.89 \text{ kPa}$$

Afterburner:- $\dot{m}_{6A} = \dot{m}_c (1+f) + \dot{m}_c \cdot \alpha = 66.5$ kg/s

$T_{t7} = 2000$ K (Given) ; $P_{t7} = 0.9\, P_{t6A} = 98.001$ kPa

$$\dot{m}_{6A} \cdot C_p \cdot T_{t6A} + \dot{m}_{f_{AB}} \cdot Q = (\dot{m}_{6A} + \dot{m}_{f,AB}) \cdot C_{p_{AB}} \cdot T_{t7}$$

2

$$\Rightarrow \dot{m}_{f,AB} = \cancel{3.37} \text{ kg/s}$$

2.218

Further $P_{t7} = 0.9\, P_{t6A} \Rightarrow P_{t7} = 98.001$ kPa

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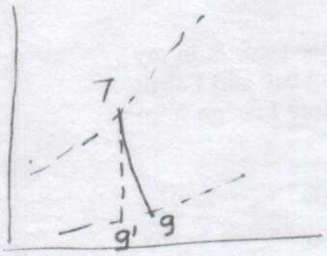
Nozzle: To check if nozzle is choked: $P_{t9} = P_{t7} = 98.001$ kPa



$$P_{t9} = P_{9^*} \cdot \left[1 + \frac{\gamma-1}{2} M_9^{*2}\right]^{\frac{\gamma}{\gamma-1}}$$

Let $M_{9^*} = 1 \Rightarrow P_{9^*} = 52.95$ kPa $> P_0$

$\therefore$ Nozzle will be choked.

$\therefore \quad M_9 = 1, \quad P_9 = 52.95$ kPa

$$\eta_N = \frac{T_{t7} - T_9}{T_{t7} - T_{9'}} \quad \& \quad T_{t9} = T_{9'} \left(\frac{P_{t9}}{P_9}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_{9'} = 1716.68 \text{ K}$$

$$\Rightarrow T_9 = 1730.84 \text{ K} \Rightarrow V_9 = a_9 = \sqrt{\gamma_9 R_9 T_9} = 844.76 \text{ m/s}$$

$$\rho_9 = \frac{P_9}{R_9 \cdot T_9} = 0.0986 \text{ kg/m}^3$$

3

$$\dot{m}_9 = \dot{m}_{6A} + \dot{m}_{f_{AB}} = \cancel{69.87} \text{ kg/s} = \rho_9 A_9 \cdot V_9$$

68.718

$$\Rightarrow A_9 = 0.8388 \text{ m}^2$$

Thrust, $F = (\dot{m}_g v_g - \dot{m}_o v_o) + A_g (p_g - p_o)$

$F = \cancel{39.88 + 36.02}\ 41.79 + 36.02$

$\boxed{\cancel{F = 75.9\ kN}}$ $\boxed{\cancel{F = 77.81\ kN}}$ $F = 76.84\ kN$

4

$$sfc = \frac{\dot{m}_f + \dot{m}_{f_{AB}}}{F} = \cancel{0.06025}\ 0.0478\ \frac{kg}{s \cdot kN}$$

Problem 2] Total compressor pressure ratio, $\pi_c = 8.9$

$$\pi_c = [\pi_{c_{LP}}]^{n_{LP}} \cdot [\pi_{c_{HP}}]^{n_{HP}} \quad \Big\} \quad \pi_{c_{LP}} = 1.2, \quad n_{LP} = 3, \quad n_{HP} = 5$$

$\pi_{c_{HP}} = 1.388$

$M_0 = 2, \quad P_0 = 10\ kPa, \quad T_0 = 210\ K$

$$T_{t0} = T_0\left[1 + \frac{\gamma - 1}{2} M_0^2\right] = 378\ K, \quad P_{t0} = P_0 \cdot \left(\frac{T_{t0}}{T_0}\right)^{\frac{\gamma}{\gamma-1}} = 78.244\ kPa$$

Intake : Adiabatic $\Rightarrow T_{t2} = T_{t0} = 378\ K$

$\pi_d = 0.88 \quad \Rightarrow \quad P_{t2} = 0.88\ P_{t0} = 68.855\ kPa$

$$\eta_d = \frac{\left(1 + \frac{\gamma-1}{2} M_0^2\right)(\pi_d)^{\frac{\gamma-1}{\gamma}} - 1}{\frac{\gamma-1}{2} M_0^2} = \frac{(1.8 \times 0.964) - 1}{0.8}$$

$\eta_d = 0.919$ or $91.9\ \%$.

Compressor inlet: $P_{t2} = 68.855\ kPa \quad T_{t2} = 378\ K$

Inlet mach no. for first stage, $M_2 = 0.5$

$$T_{t2} = T_2\left[1 + \frac{\gamma-1}{2} M_2^2\right] \Rightarrow T_2 = 360\ K$$

Befor IGV, air velocity $= M_2 \cdot \sqrt{\gamma R T_2}$

$\therefore \quad = 190.16\ m/s$

IGV, turn the flow by 20° w.r.t. z-direction without any loss.

$\gamma = 1.4$

$R = 287\ J/kg\,K$

$C_p = \frac{\gamma}{\gamma - 1} R$

$C_p = 1004.5\ J/kgK$

$\therefore \quad C_2 = 190.16\ m/s \quad \& \quad C_{Z_2} = C_2 \cdot \cos 20 = 178.69\ m/s$

$\alpha_2 = 20^\circ$

$$\frac{P_{t2}}{P_2} = \left(\frac{T_{t2}}{T_2}\right)^{\frac{\gamma}{\gamma-1}} \Rightarrow P_2 = 58.046 \text{ kPa}$$

$$\rho_2 = \frac{P_2}{RT_2} \Rightarrow \rho_2 = 0.5618 \text{ kg/m}^3$$

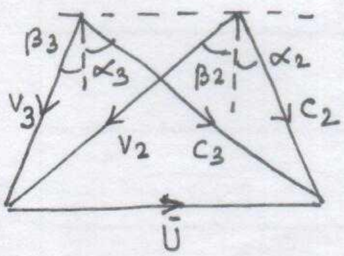
Hub-to-tip ratio, $= \frac{r_h}{r_t} = 0.2 \Rightarrow r_h = 0.2\, r_t$

$$\dot{m} = \rho_2 \cdot C_{z_2} \cdot \pi (r_t^2 - r_h^2) \Rightarrow r_t = 0.445 \text{ m}$$

$$\therefore r_h = 0.089 \text{ m}$$

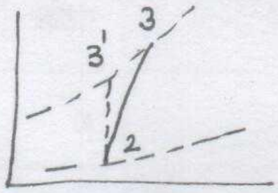
$$\bar{r} = \frac{r_h + r_t}{2} \Rightarrow \bar{r} = 0.267 \text{ m}$$

Degree of reaction $= \Lambda = 0.5 \Rightarrow$ symmetric velocity triangles



$\alpha_2 = \beta_3$ and $\alpha_3 = \beta_2$

Stage pressure ratio, $\pi_{cs} = 1.2$



$$T_{t3'} = T_{t2} (\pi_{cs})^{\frac{\gamma-1}{\gamma}}$$

$$T_{t3'} = 398.21 \text{ K}$$

$$\eta_c = \frac{T_{t3'} - T_{t2}}{T_{t3} - T_{t2}} = 0.92 \Rightarrow T_{t3} = 399.96 \text{ K}$$

Work done by stage, $W = C_p (T_{t3} - T_{t2}) = 22066.24$ J/kg

But $W = \bar{U}(C_{\theta 3} - C_{\theta 2})$

From velocity triangle,

$$C_{\theta 2} = C_{z_2} \cdot \tan\alpha_2 = 65.03 \text{ m/s}$$

$$C_{\theta 3} = \bar{U} - C_{z_2} \cdot \tan\beta_3$$

$$= \bar{U} - C_{z_2} \tan\alpha_2$$

$$C_{\theta 3} = \bar{U} - 65.03$$

$$\therefore 22066.24 = \bar{U}\left[\bar{U} - 65.03 - 65.03\right]$$

$$\Rightarrow \bar{U}^2 - 130.06\,\bar{U} - 22066.24 = 0$$

$$\bar{U} = 227.18 \text{ m/s}$$

(solving for $\bar{U}$ & taking +ve root)

$$\bar{U} = \frac{2\pi \bar{r} N}{60} \Rightarrow N = 8125.12 \text{ rpm}$$

$$C_{\theta 3} = \bar{U} - 65.03 = 162.15 \text{ m/s}$$

$$\tan\beta_2 = \frac{\bar{U} - C_{z_2}\tan\alpha_2}{C_{z_2}}$$

$$\beta_2 = 42.22^\circ$$

1st stage design parameters:

$r_h = 0.089$ m, $r_t = 0.445$ m, $\bar{r} = 0.267$ m, $N = 8125.12$ rpm

Blade angles, $\beta_2 = 42.22^\circ$, $\beta_3 = 20^\circ$

Design of LP turbine stage: LP turbine drives the LP compressor.

$\therefore$ rotational speed of LP turbine = 8125.15 rpm

Total pressure ratio of LP compressor = $(1.2)^3 = 1.728$

Overall. $\eta_c = 0.92$.

Let 2 & 3 represent inlet and exit of LP compr.

$$\eta_c = \frac{T_{t3}' - T_{t2}}{T_{t3} - T_{t2}} \quad \text{and} \quad \frac{T_{t3}'}{T_{t2}} = \left(\frac{P_{t3}}{P_{t2}}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_{t3}' = 441.938\ K$$

$$\therefore T_{t3} - T_{t2} = 69.498\ K$$

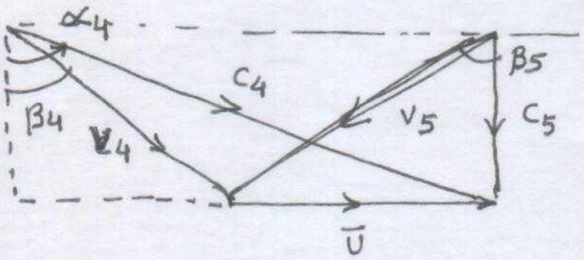
$$W_{LPC} = C_p (T_{t3} - T_{t2}) = 69810.741\ J/kg$$

$$W_{LPC} = W_{LPT} = 69810.741\ J/kg$$

Let 4 & 5 be inlet & outlet of LP turbine.

Impulse turbine with $\alpha_4 = 72^\circ$, $\eta_T = 0.95$, $T_{t4} = 900K$

Given C_5 = axial and $V_4 = V_5$



C_5 is purely axial $\Rightarrow C_{\theta 5} = 0$

$C_{\theta 4} = 2\bar{U}$ as $V_4 = V_5$

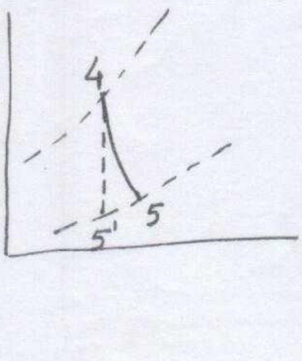
$$\therefore W_{LPT} = 2\bar{U}^2$$

$$\therefore \bar{U} = 186.83\ m/s$$

Mean turbine radius, $\bar{r} = \dfrac{\bar{U} \times 60}{2\pi N} \Rightarrow \bar{r} = 0.22\ m$

$$C_{z4} \cdot \tan\alpha_4 = C_{\theta 4} = 2\bar{U} \Rightarrow C_{z4} = 121.4\ m/s$$

$$\beta_4 = \beta_5 = \cancel{\tan^{-1}\left(\frac{C_{z4}}{-\bar{U}}\right)}\ \tan^{-1}\left(\frac{\bar{U}}{C_{z4}}\right) = 56.98^\circ$$



$$\dot{W}_{LPT} = C_p (T_{t4} - T_{t5}) \Rightarrow T_{t5} = 830.5\ K$$

$$\eta_T = 0.95 = \frac{T_{t4} - T_{t5}}{T_{t4} - T_{t5}'} \Rightarrow T_{t5}' = 826.84\ K$$

LPT pressure ratio, $\pi_t = \left(\dfrac{P_{t5}}{P_{t4}}\right) = \left(\dfrac{T_{t5}'}{T_{t4}}\right)^{\frac{\gamma}{\gamma-1}}$

$$\therefore \pi_t = 0.743$$