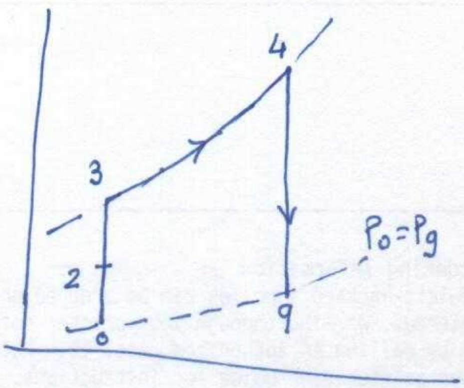
Q.1] Ideal Ramjet cycle with modifications



0-2 → Compression by Ram or diffuser

2-3 → Compressor

$$F = (\dot m_g v_g - \dot m_o v_o) + A_g (P_g - P_o)$$

For ideal case, $\dot m_g = \dot m_o$ & $P_g = P_o$

$$F = \dot m_o (v_g - v_o)$$

$$\therefore \frac{F}{\dot m_o} = a_o \left[\frac{v_g}{a_o} - M_o \right] \quad \text{-----} \quad (1)$$

$$\left(\frac{v_g}{a_o}\right)^2 = \left(\frac{a_g}{a_o}\right)^2 M_g^2 = \frac{T_g}{T_o} M_g^2$$

$$P_{tg} = P_o \cdot \frac{P_{to}}{P_o} \cdot \frac{P_{t2}}{P_{to}} \cdot \frac{P_{t3}}{P_{t2}} \cdot \frac{P_{t4}}{P_{t3}} \cdot \frac{P_{tg}}{P_{t4}} = P_o \cdot \pi_r \cdot \pi_d \cdot \pi_c \cdot \pi_b \cdot \pi_n$$

As $\pi_d = \pi_b = \pi_n = 1 \implies P_{tg} = P_o \cdot \pi_r \cdot \pi_c$

$$\frac{P_{tg}}{P_g} = \frac{P_{tg}}{P_o} \cdot \frac{P_o}{P_g} = \frac{P_{tg}}{P_o} = \pi_r \pi_c$$

Also $\frac{P_{tg}}{P_g} = \left[1 + \frac{\gamma - 1}{2} M_g^2\right]^{\frac{\gamma}{\gamma-1}} \implies M_g^2 = \frac{2}{\gamma - 1}(\tau_r \tau_c - 1)$

$$T_{tg} = T_o \cdot \frac{T_{to}}{T_o} \cdot \frac{T_{t2}}{T_{to}} \cdot \frac{T_{t3}}{T_{t2}} \cdot \frac{T_{t4}}{T_{t3}} \cdot \frac{T_{tg}}{T_{t4}} = T_o \cdot \tau_r \tau_d \tau_c \tau_b \tau_n$$

$$\Rightarrow T_{tg} = T_o \cdot \tau_r \tau_c \tau_b$$

$$\frac{T_g}{T_o} = \frac{T_{tg}/T_o}{T_{tg}/T_g} = \frac{T_{tg}/T_o}{(P_{tg}/P_g)^{\frac{\gamma-1}{\gamma}}} = \frac{\tau_r \tau_c \tau_b}{\tau_r \tau_c} = \tau_b$$

$$\therefore \left(\frac{v_g}{a_o}\right)^2 = \tau_b \cdot \frac{2}{\gamma - 1}(\tau_r \tau_c - 1)$$

$$\tau_b = \frac{T_{t4}}{T_{t3}} = \frac{T_{t4}/T_0}{T_{t3}/T_0} = \frac{\tau_\lambda}{\tau_r \tau_c} \Rightarrow \left(\frac{v_9}{a_0}\right)^2 = \frac{2}{\gamma-1}\cdot\frac{\tau_\lambda}{\tau_r\tau_c}\left(\tau_r\tau_c - 1\right)$$

$$\therefore \boxed{\frac{F}{\dot m_0} = a_0\left[\sqrt{\frac{2}{\gamma-1}\cdot\frac{\tau_\lambda}{\tau_r\tau_c}\left(\tau_r\tau_c-1\right)} - M_0\right]}$$

Electric power requirement, $= \dot W_c = \dot m_0 \cdot C_p \left(T_{t3} - T_{t2}\right)$

$$\dot W_c = \dot m_0 \cdot C_p \cdot T_{t2} \left(\tau_c - 1\right)$$

$$\boxed{\dot W_c = \dot m_0 C_p T_0 \tau_r \left(\tau_c - 1\right)}$$

$$T_{t2} = T_0 \cdot \frac{T_{t0}}{T_0} \cdot \frac{T_{t2}}{T_{t0}}$$

$$T_{t2} = T_0 \cdot \tau_r$$

$T_0 = 220 K$, $\pi_c = 4$, $T_{t4} = 1430$ K

$M_0 = 0$ for static thrust.

$\tau_\lambda = \frac{T_{t4}}{T_0} = 6.5$, $\tau_c = (\pi_c)^{\frac{\gamma-1}{\gamma}} = 1.486$

$\tau_r = 1 + \frac{\gamma-1}{2} M_0^2 = 1$ & $a_0 = \sqrt{\gamma R T_0} = 297.31$ m/s

Putting this in expression for specific thrust,

$$\boxed{\frac{F}{\dot m_0} = 969.3 \text{ m/s}}$$

Q.2] Diffuser section:

Given condition - Take-off (near static condition)

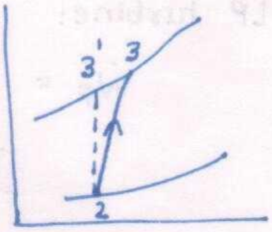
$\Rightarrow V_0 \approx 0$ & $M_0 \approx 0$

We can neglect the ram effect.

$P_{t2} = P_{t0} = P_0 = 1$ bar $= 100$ kPa

$T_{t2} = T_{t0} = T_0 = 288$ K

Compressor section: $\pi_c = 25$, $e_c = 0.9$

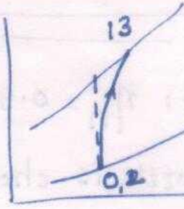


$$\pi_c = \frac{P_{t3}}{P_{t2}} = \frac{P_{t3'}}{P_{t2}} = 25$$

$$\frac{T_{t3}}{T_{t2}} = \left(\frac{P_{t3}}{P_{t2}}\right)^{\frac{\gamma-1}{\gamma \cdot e_c}} \Rightarrow T_{t3} = 800.17 \text{ K}$$

$\therefore P_{t3} = 25$ bar & $T_{t3} = 800.17$ K

Fan or bypass section: $\pi_F = 1.65$, $e_F = 0.9$



$$\therefore \frac{T_{t13}}{T_{t2}} = \left(\frac{P_{t13}}{P_{t2}}\right)^{\frac{\gamma-1}{\gamma \cdot e_F}} \Rightarrow T_{t13} = 337.6 \text{ K}$$

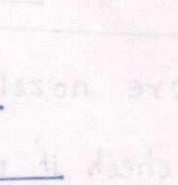
$\therefore P_{t13} = 1.65$ bar & $T_{t13} = 337.6$ K

Mass flow rates: Total $\dot{m}_0 = 215$ kg/s, $\alpha = 5 = \frac{\dot{m}_F}{\dot{m}_c}$

$\dot{m}_0 = \dot{m}_c + \dot{m}_F \Longrightarrow \underline{\dot{m}_c = 35.83 \text{ kg/s}}$

& $\underline{\dot{m}_F = 179.17 \text{ kg/s}}$

Burner section: $T_{t4} = 1550$ K,



$$\dot{m}_c \cdot C_{p_c} \cdot T_{t3} + \dot{m}_F \cdot Q = (\dot{m}_c + \dot{m}_F) \cdot C_{p_t} \cdot T_{t4}$$

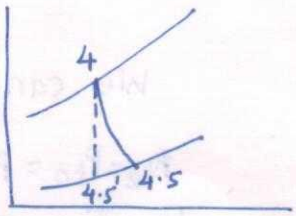
$$\frac{\dot{m}_F}{\dot{m}_c} = \frac{C_{p_t} \cdot T_{t4} - C_{p_c} \cdot T_{t3}}{Q - C_{p_t} \cdot T_{t4}} = \underline{f = 0.0237}$$

$\Delta P_t = 1.5$ bar $\Rightarrow P_{t4} = 25 - 1.5 = \underline{23.5 \text{ bar}}$

HP turbine: $e_t = 0.9$

$$\dot{W}_C = \dot{W}_T \Rightarrow \dot{m}_c \cdot C_{p_c} \cdot (T_{t3} - T_{t2}) = (\dot{m}_c + \dot{m}_f) \cdot C_{p_t} \cdot (T_{t4} - T_{t4.5})$$

$$\Rightarrow \underline{T_{t4.5} = 1112 \text{ K}}$$



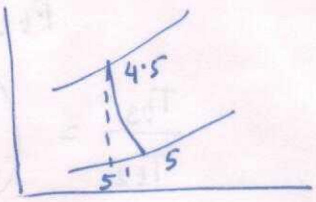
$$\frac{P_{t4.5}}{P_{t4}} = \left(\frac{T_{t4.5}}{T_{t4}}\right)^{\frac{\gamma}{e_t(\gamma-1)}} \quad \text{where } \underline{\gamma = 1.33}$$

$$\Rightarrow \underline{P_{t4.5} = 5.31 \text{ bar}}$$

LP turbine: $e_t = 0.9$

$$\dot{W}_F = \dot{W}_T \Rightarrow \dot{m}_c \cdot \alpha \cdot C_{p_c} (T_{t13} - T_{t2}) = (\dot{m}_c + \dot{m}_f) C_{p_t} \cdot (T_{t4.5} - T_{t5})$$

$$\underline{T_{t5} = 900 \text{ K}}$$



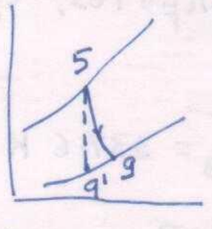
$$\frac{P_{t5}}{P_{t4.5}} = \left(\frac{T_{t5}}{T_{t4.5}}\right)^{\frac{\gamma}{e_t(\gamma-1)}} \quad \text{where } \gamma = 1.33$$

$$\Rightarrow \underline{P_{t5} = 2.074 \text{ bar}}$$

Core nozzle: $\eta_n = 0.95$

To check if nozzle is choked

Let $M_9 = 1$ & Use



$$\frac{P_{t9}}{P_9^*} = \frac{P_{t5}}{P_9^*} = \left[1 + \frac{\gamma-1}{2} M_9^2\right]^{\frac{\gamma}{\gamma-1}} \quad \text{where } \gamma = 1.33$$

$$\Rightarrow P_9^* = 1.126 \text{ bar} > P_0$$

$\therefore$ Nozzle is choked $\Rightarrow M_9 = 1$

$$\eta_n = \frac{T_{t5} - T_9}{T_{t5} - T_{9'}} \quad \& \quad \frac{T_{t5}}{T_{9'}} = \left(\frac{P_{t5}}{P_{9'}}\right)^{\frac{\gamma-1}{\gamma}}$$

Using above, $T_{9'} = 772.54$ K

& $T_9 = 778.9$ K

$\therefore$ Nozzle (core) exit conditions,

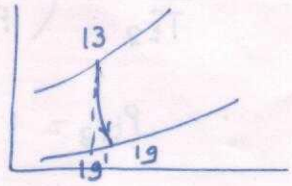
$P_9 = 1.126$ bar, $T_9 = 778.9$ K

$M_9 = 1$

Bypass nozzle

$\eta_n = 0.95$

For fan



Since $P_{t13} = 1.65$ bar, let $M_{19} = 1$

$$\frac{P_{t13}}{P_{t19}^*} = \left(1 + \frac{\gamma-1}{2} M_{19}^2\right)^{\frac{\gamma}{\gamma-1}} \quad \text{where } \gamma = 1.4$$

$$P_{t19}^* = 0.871 < P_0$$

$\therefore$ Nozzle (fan) is not choked.

Now use, $P_{19} = 1$ bar

$\Rightarrow \cancel{M_{19} = 0.877}$

Using $\eta_n = \dfrac{T_{t13} - T_{19}}{T_{t13} - T_{19'}} \Rightarrow \underline{T_{19} = 294.85 \text{ K}}$

Core nozzle:

$$\rho_g = \frac{P_g}{R_g T_g} \quad \text{where} \quad C_{p_g} = R_g \cdot \frac{\gamma}{\gamma - 1}$$

$$\& \ \gamma = 1.33$$

$$\therefore R_g = 287 \ J/kg\,K$$

$$\& \ \rho_g = 0.503 \ kg/m^3$$

$$M_g = 1 \Rightarrow v_g = \sqrt{\gamma R_g T_g}$$

$$v_g = 545.26 \ m/s$$

$$(1+f)\,\dot m_c = \rho_g v_g A_g = 35.83\,(1+0.0237)$$

$$\therefore \underline{A_g = 0.133 \ m^2}$$

$$F_c = \left[(\dot m_c + \dot m_f)\,v_g - \dot m_c v_0\right] + (P_g - P_0)\,A_g$$

$$F_c = (19999.68 + 1675.8) \ N$$

$$\underline{F_c = 21.675 \ kN}$$

Fan section nozzle:

$$\cancel{V_{1g} = M_{1g} \cdot \sqrt{\gamma R T_{1g}}}$$

$$\cancel{\text{where } \gamma = 1.4 \ \& \ R = 287.14 \ J/kg\,K}$$

$$\cancel{\therefore V_{1g} = 344.28 \ m/s}$$

$$\cancel{\therefore F_F = \dot m_F (V_{1g} - V_0)}$$

$$\cancel{\therefore F_F = 61.684 \ kN}$$

$$T_{t13} = T_{1g} + \frac{V_{1g}^2}{2C_p} \quad \Big\} \ \text{where } C_p = 1.005 \ kJ/kg\,K$$

$$\therefore V_{1g} = 293.13 \ m/s$$

$$\therefore F_F = \dot m_F (V_{1g} - V_0)$$

$$\underline{F_F = 52.52 \ kN}$$

Total thrust, $F = F_c + F_F$

$$\therefore \boxed{F = 74.195 \ kN}$$

$$\text{SFC} \Rightarrow \quad \dot m_f = f \cdot \dot m_c = 0.8491 \ kg/s$$

$$\text{SFC} = \frac{\dot m_f}{F} = \boxed{0.01144 \ \frac{kg}{kN \cdot s}}$$