Department of Mechanical Engineering, IIT Bombay

ME 782 Design Optimization Quiz 1 Augst, 2024 Maximum marks: **12**

Part 2 (Paper and pen) **SOLUTION**

1. Determine all roots of the following polynomial using the Newton-Raphson's method along with synthetic division.

   $f(x) = x^4 - 8.8x^3 + 25.17x^2 - 26.8380x + 9.5256 = 0$

   [8 marks]

$$f(x) = x^4 - 8.8x^3 + 25.17x^2 - 26.8380x + 9.5256 = 0 \tag{1}$$

Derivative of equation (1) w.r.t x is given by

$$f^{'}(x) = 4x^3 - 26.4x^2 + 50.34x - 26.838 = 0 \tag{2}$$

The Newton-Raphson's equation to determine the roots of equation (1) is given by

$$x_{n+1} = x_n - \frac{f(x)}{f^{'}(x)} \tag{3}$$

Results obtained from equations (1), (2) and (3) are presented in Table 1

Table 1 Iterations based on equations (1), (2) and (3)

No.	x_n	$f(x)$	$f^{'}(x)$	x_{n+1}
1	8.8	1722.5	1097.6	7.2307
2	7.2307	538.16	469.05	6.0833
3	6.0833	166.14	202.92	5.2646
4	5.2646	49.98	90.13	4.7101
5	4.7101	14.14	42.56	4.3777
6	4.3777	3.39	23.18	4.2314
7	4.2314	0.5	16.54	4.2012
8	4.2012	0.0188	15.3	4.2
9	4.2	≈ 0		

Hence, the first root is given by $r_1 = 4.2$

Using the above root, synthetic division is carried out using the coefficients of equation (1) as follows

Table 2 Synthetic division

1	-8.8	25.17	-26.8380	9.5256
	4.2	-19.32	24.57	-9.5256
1	-4.6	5.85	-2.268	0

From the synthetic division above , the reduced order polynomial is given by

$$f(x) = x^3 - 4.6x^2 + 5.85x - 2.268 = 0 \tag{4}$$

Derivative of equation (4) w.r.t x is given by

$$f^{'}(x) = 3x^2 - 9.2x + 5.85 \tag{5}$$

Results obtained from equations (4), (5) and (3) are presented in Table 3.

Table 3 Iterations based on equations (4), (5) and (3)

No.	x_n	$f(x)$	$f^{'}(x)$	x_{n+1}
1	4.6	24.64	27.01	3.6877
2	3.6877	6.9	12.72	3.1454
3	3.1454	1.74	6.59	2.8813
4	2.8813	0.32	4.25	2.8062
5	2.8062	0.022	3.67	2.8
6	2.8	≈ 0		

Hence, the second root is given by $r_2 = 2.8$

Using the above root, synthetic division is carried out using the coefficients of equation (4) as follows

Table 4 Synthetic division

1	-4.6	5.85	-2.268
	2.8	-5.04	2.268
1	-1.8	0.81	0

From the synthetic division above, the reduced order polynomial is given by

$$f(x) = x^2 - 1.8x + 0.81 = 0 \tag{6}$$

The quadratic equation of (6) can be solved to obtain

$$r_3 = \frac{1.8 \pm \sqrt{1.8^2 - 4 \times 0.81}}{2} = 0.9 \tag{7}$$

Hence, the third root is given by $r_3 = 0.9$

Using the above root, synthetic division is carried out using the coefficients of equation (6) as follows

Table 5 Synthetic division

1	-1.8	0.81
	0.9	-0.81
1	-0.9	0

From the synthetic division above, the reduced order polynomial is given by

$$f(x) = x - 0.9 = 0 \tag{8}$$

Hence, the fourth root is given by $r_4 = 0.9$

Hence, the four roots are: 4.2, 2.8, 0.9 and 0.9

2. Consider the following matrix

$$\mathbf{H} = \begin{bmatrix} -7.56 & 2 \\ 2 & -3.4392 \end{bmatrix}$$

Prove that the above matrix is positive or negative definite using the following methods

(a) Using eigenvalues

Eigenvalues of the above matrix are given by

$$\begin{vmatrix} -7.56-\lambda & 2 \\ 2 & -3.4392-\lambda \end{vmatrix} = 0 \qquad (1)$$

Expanding equation (1)

$$\lambda^2 + 10.9992\lambda + 22.0004 = 0 \qquad (2)$$

Solving the quadratic equation (2), the eigenvalues are given by

$$\lambda_{1,2} = -8.37, -2.63$$

Since the eigenvalues are negative, the above matrix is a negative definite matrix

(b) Using principal minors

$$A_1 = -7.56 < 0$$

$$A_2 = \begin{vmatrix} -7.56 & 2 \\ 2 & -3.4392 \end{vmatrix} = 22.0004 > 0$$

A matrix is negative definite if the signs of the minors is $(-1)^j$, where j is the size of the principal minors. Since the above principal minors are respectively having negative and positive signs, the given Hessian matrix is positive definite.

© Using the vector $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$$\mathbf{x}^T\mathbf{H}\mathbf{x} = \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} -7.56 & 2 \\ 2 & -3.4392 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = -13.32 < 0$$

Therefore, the matrix is negative definite