

Determine the value of the variables that will obtain constrained local minima of the following functions using the appropriate penalty functions using the analytical method

1.  $f(x_1, x_2) = 4x_1^2 + 3x_2^2 - 5x_1x_2 - 8x_1$

Subject to  $x_1 + x_2 = 4$

2.  $f(x_1, x_2) = 9x_1^2 + 18x_1x_2 + 13x_2^2 - 4$

Subject to  $x_1^2 + x_2^2 + 2x_1 = 16$

3.  $f(x_1, x_2) = (x_1 - 1)^2 + (x_2 - 1)^2$

Subject to  $x_1 + x_2 - 4 = 0$

4.  $f(x_1, x_2) = 2x_1 + 3x_2 - x_1^3 - 2x_2^2$

Subject to  $x_1 + 3x_2 \leq 6$   
 $5x_1 + 2x_2 \leq 10$

5.  $f(x_1, x_2) = 4x_1^2 + 3x_2^2 - 5x_1x_2 - 8x_1$

Subject to  $x_1 + x_2 \leq 4$

6.  $f(x_1, x_2) = x_1^2 + x_2^2 - 4x_1 - 2x_2 + 6$

Subject to  $4 - x_1 - x_2 \leq 0$

7.  $f(x_1, x_2) = 2x_1^2 - 6x_1x_2 + 9x_2^2 - 18x_1 + 9x_2$

Subject to  $x_1 + 2x_2 \leq 10$   
 $4x_1 - 3x_2 \leq 20$

8.  $f(x_1, x_2) = 9x_1^2 - 18x_1x_2 + 13x_2^2 - 4$

Subject to  $16 - x_1^2 - x_2^2 - 2x_1 \leq 0$

9.  $f(x_1, x_2) = (x_1 - 3)^2 + (x_2 - 3)^2$

Subject to  $x_1 + x_2 \leq 4$   
 $x_1 - 3x_2 = 1$

10.  $f(x_1, x_2) = x_1^3 - 16x_1 + 2x_2 - 3x_2^2$

Subject to  $x_1 + x_2 \leq 3$