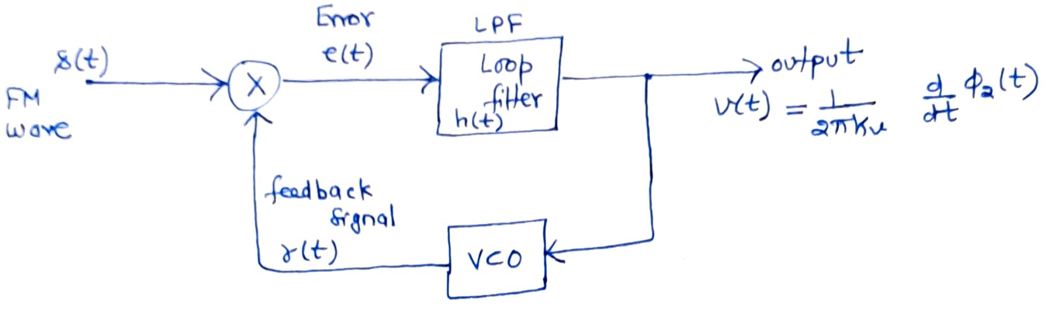
FM Demodulation using Phase Locked Loop (PLL):

PLL consists of three major components:

1. multiplier
2. Loop filter
3. VCO



We assume that initially we have adjusted the VCO so that when the control voltage is zero, two conditions are satisfied:

1. The frequency of the VCO is precisely set at f_c.
2. The VCO o/p has a 90° phase-shift w.r.t. the unmodulated carrier wave.

$$s(t) = A_c \sin\left(2\pi f_c t + \phi_1(t)\right) \quad \text{... i/p to the PLL (FM wave)}$$

$$s(t) = A_c \sin\left(2\pi f_c t + 2\pi k_f \int m(t)\,dt\right)$$

where $\phi_1(t) = 2\pi k_f \int m(t)\,dt$

$$r(t) = A_v \cos\left(2\pi f_c t + \phi_2(t)\right) \quad \text{... o/p of VCO}$$

where $\phi_2(t) = 2\pi k_v \int v(t)\,dt$

Instantaneous freq. of VCO $= \omega_c + \frac{d}{dt}\phi_2(t) = \omega_c + 2\pi k_v v(t)$

where $\frac{d}{dt}\phi_2(t) = \dot{\phi}_2(t) = 2\pi k_v v(t)$

$$v(t) = \frac{1}{2\pi k_v} \frac{d}{dt}\phi_2(t)$$

- The object of the PLL is to generate a VCO output $r(t)$ that has the same phase angle as the input FM signal $s(t)$.

- $\phi_1(t) = 2\pi k_f \int m(t)\,dt$

The time-varying phase $\phi_1(t)$ is due to modulation by a message signal $m(t)$, in which case we wish to recover $\phi_1(t)$ and thereby produce an estimate of $m(t)$.

$$o/p \text{ of multiplier} = A_c \sin(w_c t + \phi_1(t))\, A_v \cos(w_c t + \phi_2(t))$$

$$= \frac{A_c A_v}{2} \sin(\phi_1(t) - \phi_2(t)) + \frac{A_c A_v}{2} \underbrace{\sin(2w_c t + \phi_1(t) + \phi_2(t))}_{\text{high freq. component suppressed by the Loop filter}}$$

Effective input to the Loop filter:

$$e(t) = \frac{A_c A_v}{2} \sin(\phi_1(t) - \phi_2(t))$$

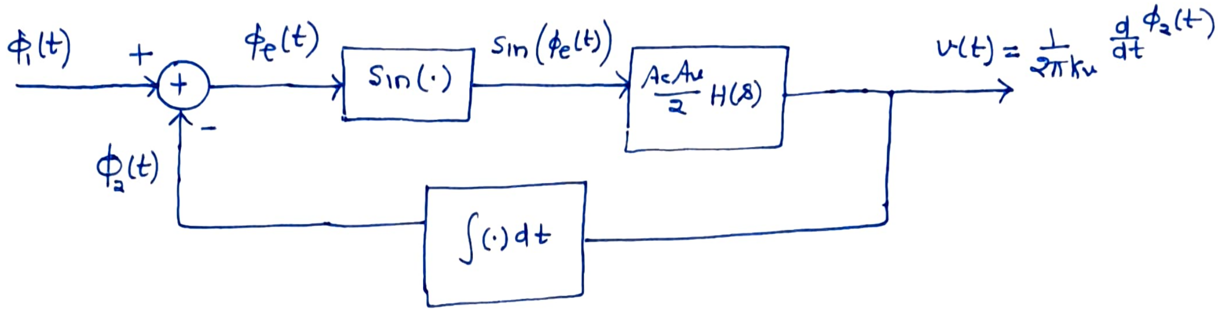
$$e(t) = \frac{A_c A_v}{2} \sin(\phi_e(t)) \qquad \text{where } \phi_e(t) = \phi_1(t) - \phi_2(t)$$

Let $h(t)$ ... impulse response of the loop filter.

$$v(t) = e(t) * h(t)$$

$$= \int_{-\infty}^{\infty} e(\tau)\, h(t-\tau)\, d\tau$$

$$v(t) = \frac{A_c A_v}{2} \int_{-\infty}^{\infty} \sin(\phi_e(\tau))\, h(t-\tau)\, d\tau$$



$\Rightarrow$ when the phase error $\phi_e(t)$ is zero, the PLL is said to be in phase-lock.

$$\phi_1(t) = 2\pi k_f \int m(t)\,dt$$

$$\phi_2(t) = \phi_1(t) - \phi_e(t)$$

$\approx$ when $\phi_e(t)$ is small

$$\phi_2(t) \approx \phi_1(t) = 2\pi k_f \int m(t)\,dt$$

$$v(t) = \frac{1}{2\pi k_v} \frac{d}{dt}\phi_2(t)$$

$$v(t) = \frac{k_f}{k_v} m(t)$$

$$v(t) \propto m(t)$$