(1)

Passband Data Transmission

(Digital Modulation Techniques)

ASK: Amplitude-shift keying
PSK: Phase-shift keying
FSK: Frequency-shift keying

BASK:
BPSK:
BFSK:

$0, 1.$

$T = T_b.$

M-ary: $M = 2^n \Rightarrow n = \log_2 M.$

M-ary PSK:
M-ary FSK:

$T = nT_b$

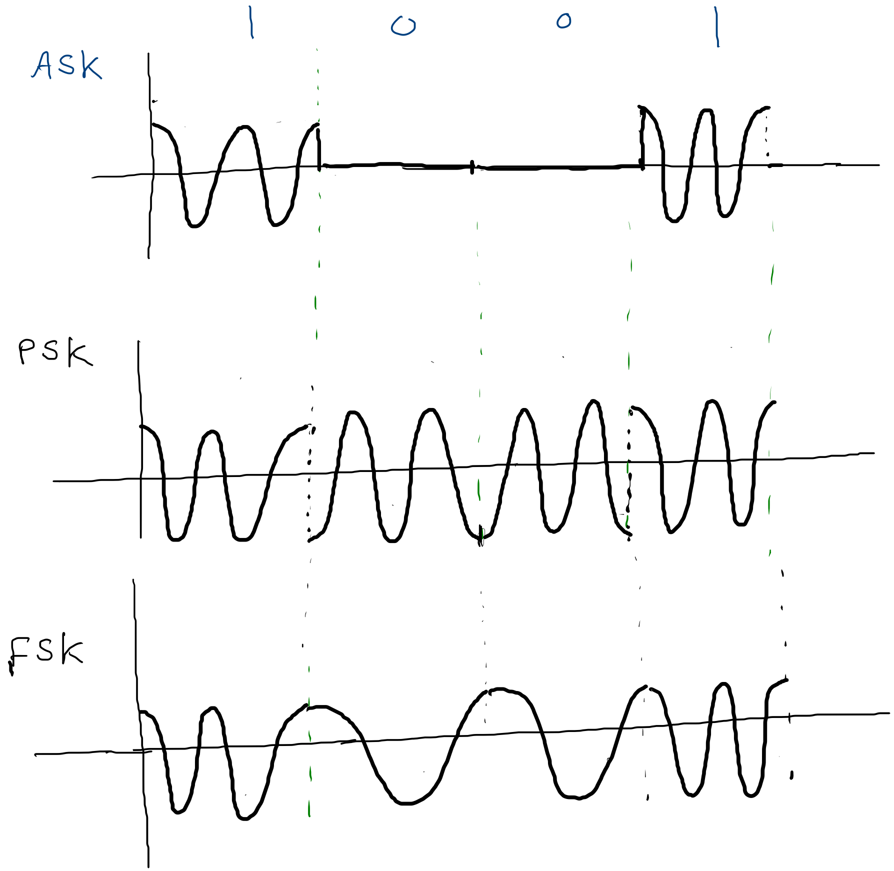
$R = \frac{R_b}{n}$

e.g. BPSK: $1 \Rightarrow 0$
$0 \Rightarrow \pi$

e.g. 4-ary: $2^2 = 4$

4-ary-PSK:

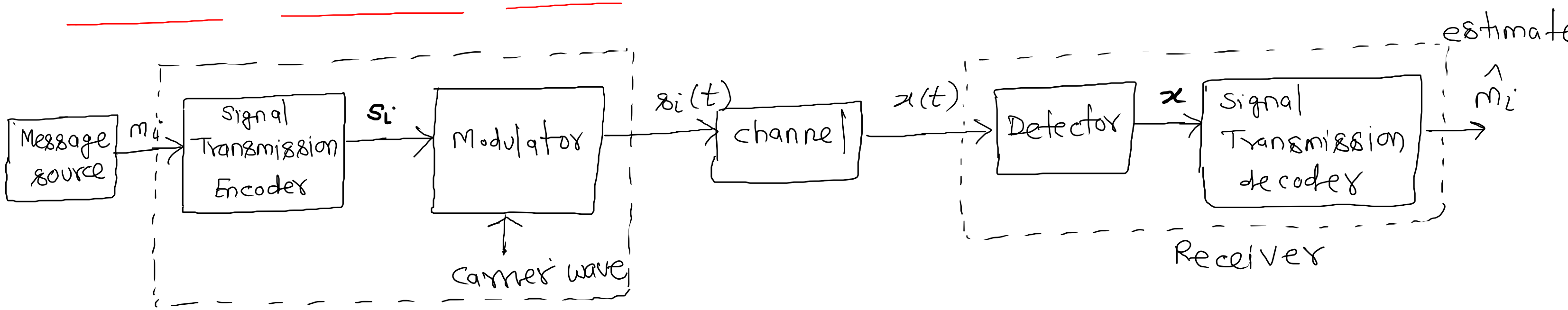
$$\begin{cases} 00 \Rightarrow 0 \\ 01 \Rightarrow \pi/2 \\ 10 \Rightarrow \pi \\ 11 \Rightarrow 3\pi/2 \end{cases}$$



$$f_c = \frac{n_c}{T_b}$$

n_c = +ve integer

Passband Transmission Model:



M

T = symbol duration. Transmitter.

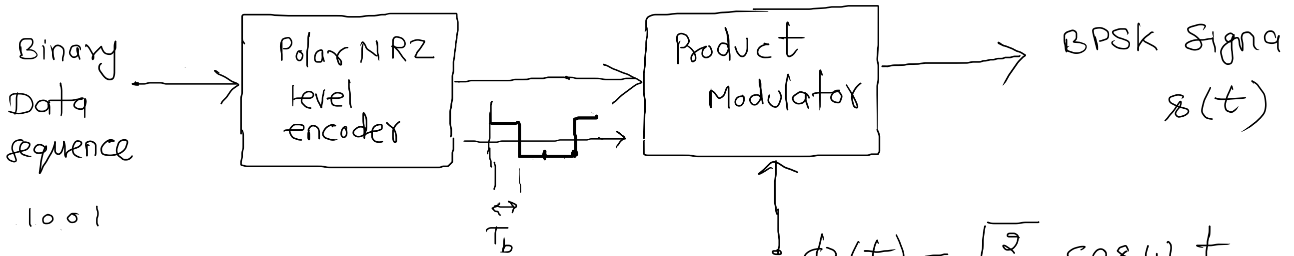
$x(t) = s_i(t) + w(t)$

m_i .. equiprobable

$$P(m_1) = P(m_2) = \cdots = P(m_M) = \frac{1}{M}$$

$$E_i = \int_0^T s_i^2(t)\,dt \qquad i = 1, 2, \cdots, M.$$

Binary phase-shift keying (BPSK)



$$\phi_1(t) = \sqrt{\frac{2}{T_b}} \cos w_c t \qquad \int_0^{T_b} \phi_1^2(t)\, dt = 1$$

1 0 0 1

$1 \Rightarrow +1$
$0 \Rightarrow -1$

antipodal signalling.

E_b .. Transmitted signal energy per bit.

$$s(t) = \begin{cases} s_1(t) & \cdots \text{"1"} \\ s_2(t) & \cdots \text{"0"} \end{cases}$$

$$s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos w_c t \quad \cdots \text{"1"} \qquad 0 \le t \le T_b$$

$$s_2(t) = -\sqrt{\frac{2E_b}{T_b}} \cos w_c t = \sqrt{\frac{2E_b}{T_b}} \cos(w_c t + \pi) \quad \cdots \text{"0"}$$

$$\phi_1(t) = \sqrt{\frac{2}{T_b}} \cos \omega_c t$$

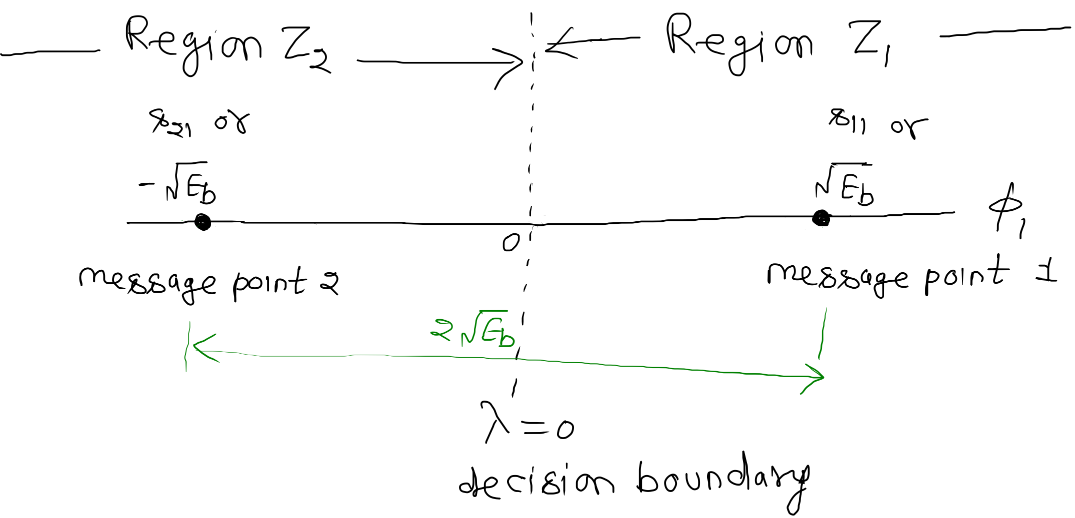
$$s_1(t) = \sqrt{E_b} \cdot \phi_1(t) = s_{11} \, \phi_1(t)$$

$0 \le t \le T_b$

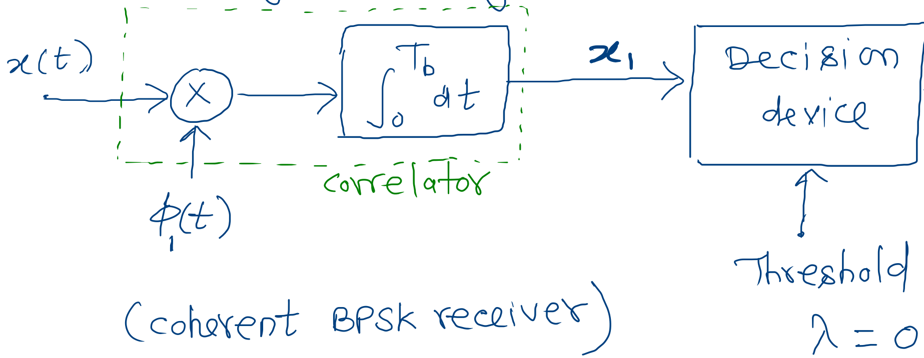
$$s_2(t) = -\sqrt{E_b} \cdot \phi_1(t) = s_{21} \, \phi_1(t)$$

$$s_{11} = \int_0^{T_b} s_1(t) \, \phi_1(t) \, dt = \sqrt{E_b}$$

$$s_{21} = \int_0^{T_b} s_2(t) \, \phi_1(t) \, dt = -\sqrt{E_b}$$



Error Probability of binary PSK:



$$\begin{cases} \text{choose } 1 \text{ if } x_1 > 0 \\ \text{choose } 0 \text{ if } x_1 < 0 \end{cases}$$

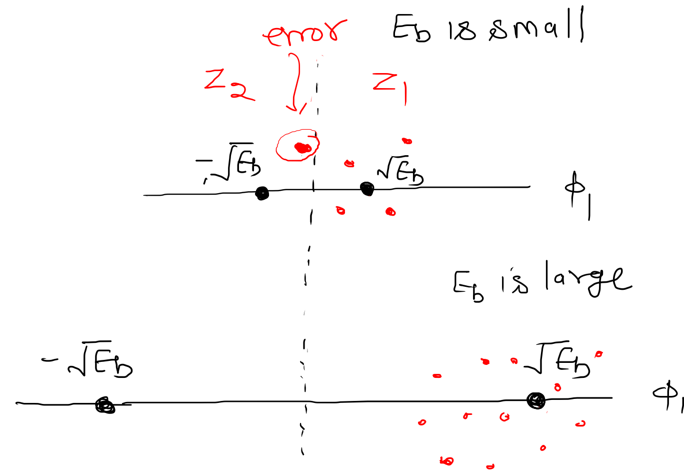
(coherent BPSK receiver)

$$x(t) = s_1(t) + w(t)$$

$$x_1 = \int_0^{T_b} x(t)\,\phi_1(t)\,dt$$

$$= \int_0^{T_b} [s_1(t) + w(t)]\,\phi_1(t)\,dt$$

$$x_1 = \underbrace{\int_0^{T_b} s_1(t)\,\phi_1(t)\,dt}_{s_{11}} + \underbrace{\int_0^{T_b} w(t)\,\phi_1(t)\,dt}_{w_1}$$



$$x_1 = s_{11} + w_1$$

$$x_1 = \sqrt{E_b} + w_1$$

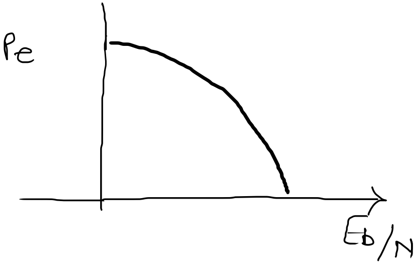
or if $x(t) = s_2(t) + w(t)$

$$x_1 = s_{21} + w_1$$

$$x_1 = -\sqrt{E_b} + w_1$$

$$\boxed{P_e = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)}$$

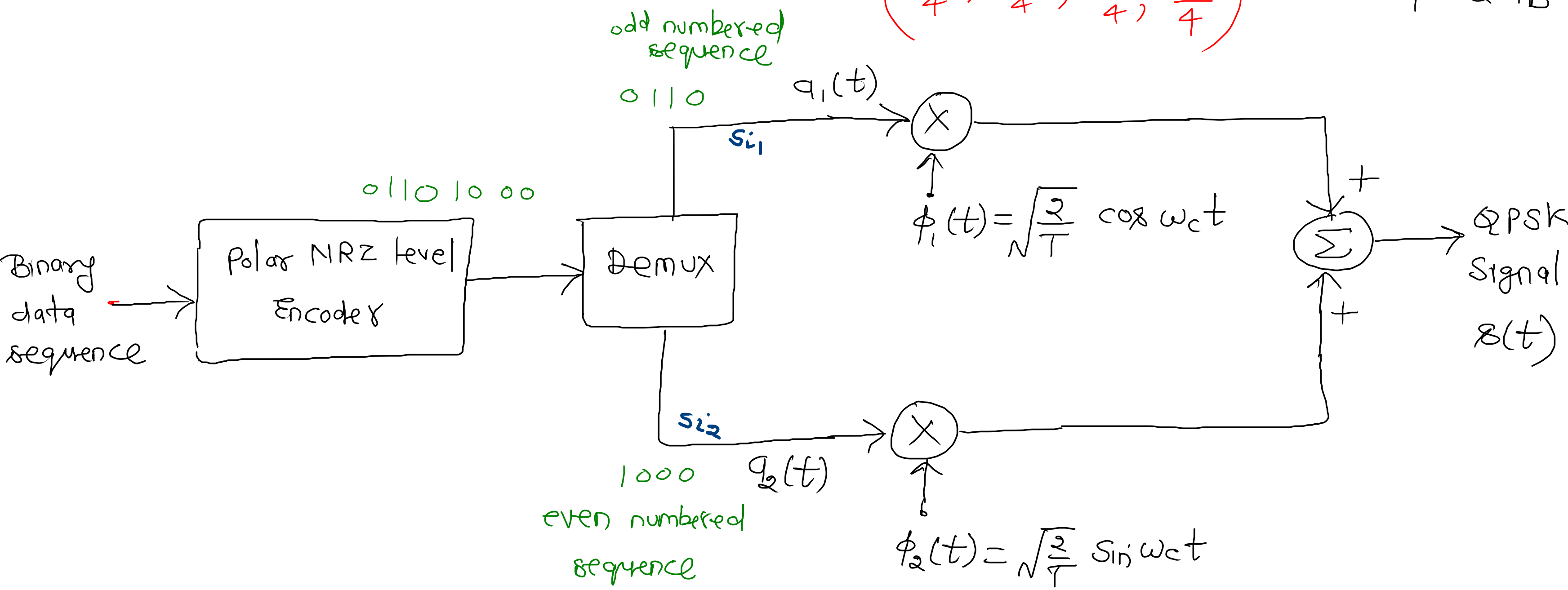
∴ Error probability.



Quadriphase-shift keying: (QPSK)

$$\left(\overset{10}{\frac{\pi}{4}},\ \overset{00}{\frac{3\pi}{4}},\ \overset{01}{\frac{5\pi}{4}},\ \overset{11}{\frac{7\pi}{4}}\right)$$

$T = 2T_b$

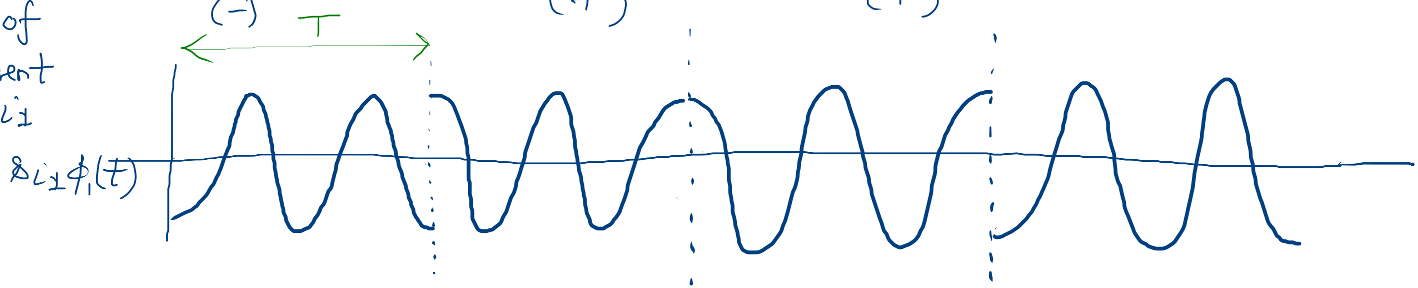


Input binary sequence: 0 1 1 0 1 0 0 0

$\underbrace{0\quad 1}_{\text{dibit }01}$ $\underbrace{1\quad 0}_{\text{dibit }10}$ $\underbrace{1\quad 0}_{\text{dibit }10}$ $\underbrace{0\quad 0}_{\text{dibit }00}$

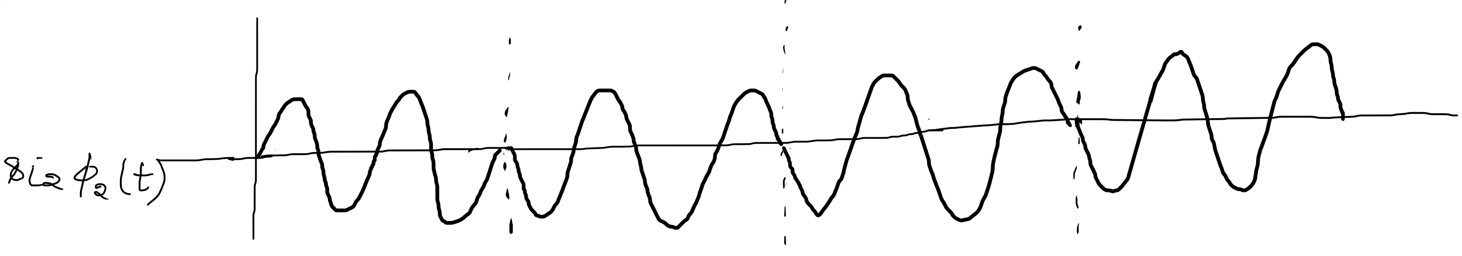
odd numbered sequence polarity of coefficient s_{i1}

0	1	1	0
(−)	(+)	(+)	(−)

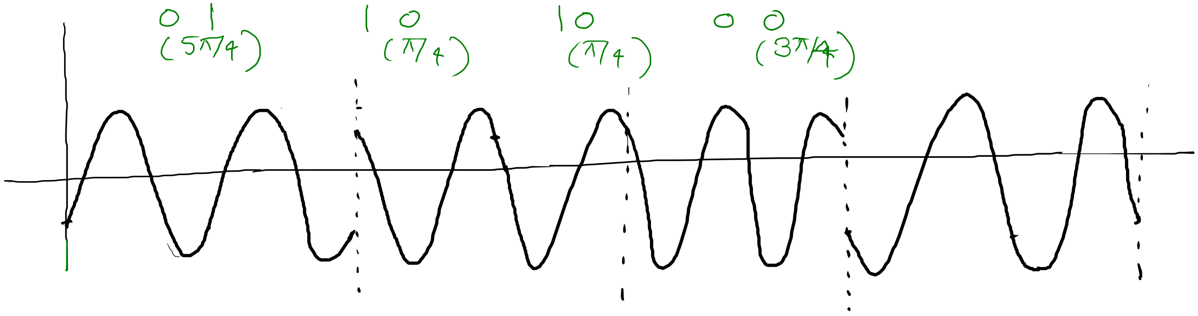


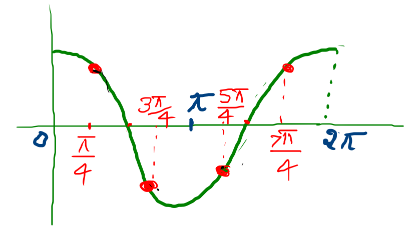
Even-numbered sequence polarity of coefficient s_{i2}

1	0	0	0
(+)	(−)	(−)	(−)



QPSK Signal





$$s_i(t) = \begin{cases} \sqrt{\frac{2E}{T}} \cos\left[w_c t + (2i-1)\frac{\pi}{4} \right] & 0 \le t \le T \\ 0 & \text{otherwise} \end{cases}$$

$i = 1, 2, 3, 4$

$$f_c = \frac{n_c}{T}$$

gray-encoded sequence

1 0
0 0
0 1
1 1

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos\left[(2i-1)\frac{\pi}{4}\right] \cos w_c t - \sqrt{\frac{2E}{T}} \sin\left[(2i-1)\frac{\pi}{4}\right] \sin w_c t$$

$$s_i(t) = \sqrt{E}\cos\left[(2i-1)\frac{\pi}{4}\right]\phi_1(t) - \sqrt{E}\sin\left[(2i-1)\frac{\pi}{4}\right]\phi_2(t)$$

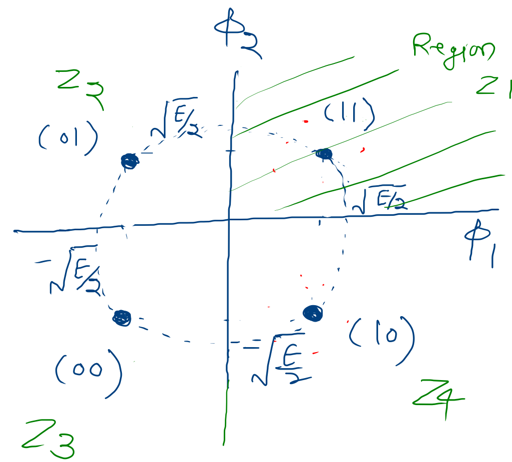
$$s(t) = s_{i1}\,\phi_1(t) + s_{i2}\,\phi_2(t)$$

where
$$\begin{cases} s_{i1} = \sqrt{E}\cos\left[(2i-1)\frac{\pi}{4}\right] \\ s_{i2} = -\sqrt{E}\sin\left[(2i-1)\frac{\pi}{4}\right] \end{cases}$$

$$\phi_1(t) = \sqrt{\frac{2}{T}}\cos w_c t$$

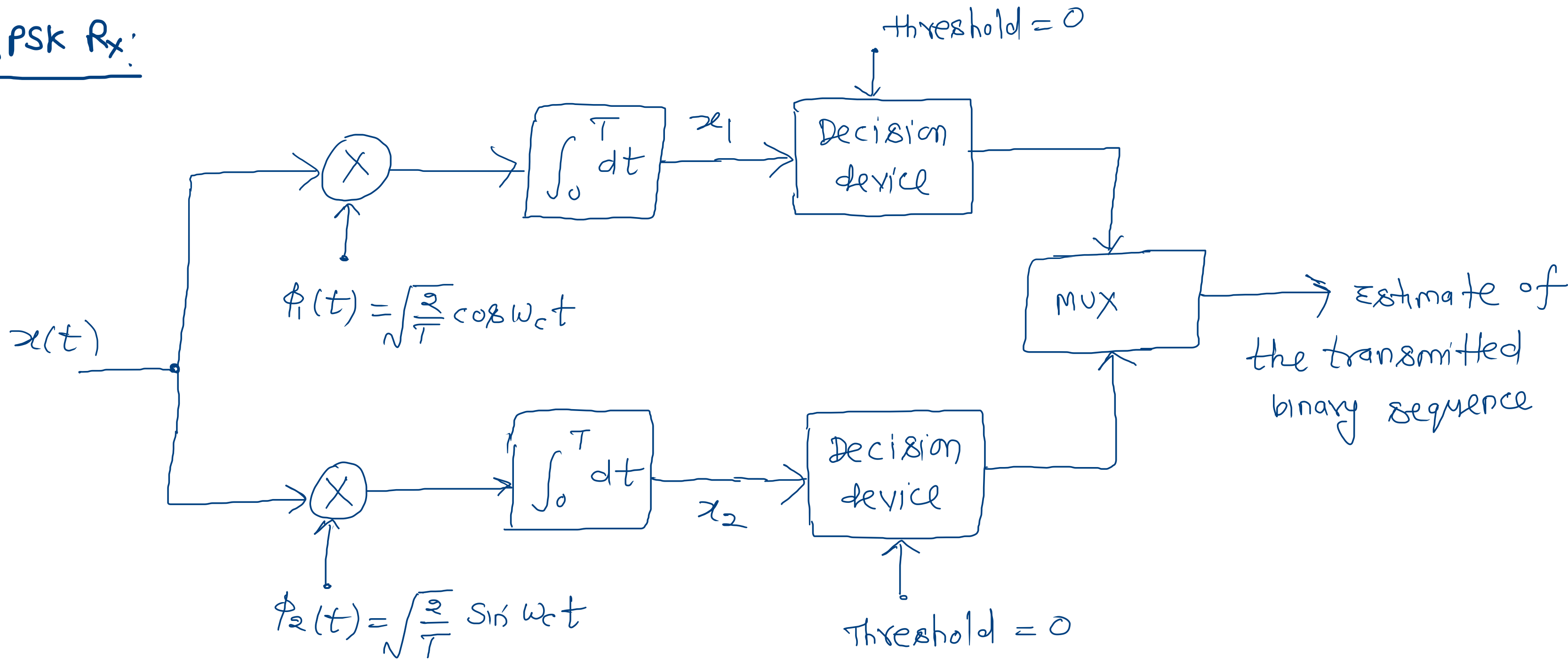
$$\phi_2(t) = \sqrt{\frac{2}{T}}\sin w_c t$$

Gray encoded input dibit	phase of QPSK signal	coordinates of message points s_{i1}	s_{i2}
1 0	$\pi/4$	$+\sqrt{\frac{E}{2}}$	$-\sqrt{\frac{E}{2}}$
0 0	$3\pi/4$	$-\sqrt{\frac{E}{2}}$	$-\sqrt{\frac{E}{2}}$
0 1	$5\pi/4$	$-\sqrt{\frac{E}{2}}$	$+\sqrt{\frac{E}{2}}$
1 1	$7\pi/4$	$+\sqrt{\frac{E}{2}}$	$+\sqrt{\frac{E}{2}}$



$$s_i = \begin{bmatrix} \sqrt{E}\cos\left[(2i-1)\frac{\pi}{4}\right] \\ -\sqrt{E}\sin\left[(2i-1)\frac{\pi}{4}\right] \end{bmatrix} \qquad i = 1, 2, 3, 4$$

QPSK Rx:



$$x(t) = s_i(t) + w(t) \qquad 0 \le t \le T, \qquad i = 1, 2, 3, 4$$

$$x_1 = \int_0^T x(t)\,\phi_1(t)\,dt = \int_0^T s_i(t)\,\phi_1(t)\,dt + \int_0^T w(t)\,\phi_1(t)\,dt$$

$$x_1 = s_{i1} + w_1$$

$$x_1 = \sqrt{E}\cos\left[(2i-1)\frac{\pi}{4}\right] + w_1$$

$$x_1 = \pm\sqrt{\frac{E}{2}} + w_1$$

Similarly $\quad x_2 = \mp\sqrt{\frac{E}{2}} + w_2$

$$BER = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$ ... bit error rate.

$$T = 2T_b$$

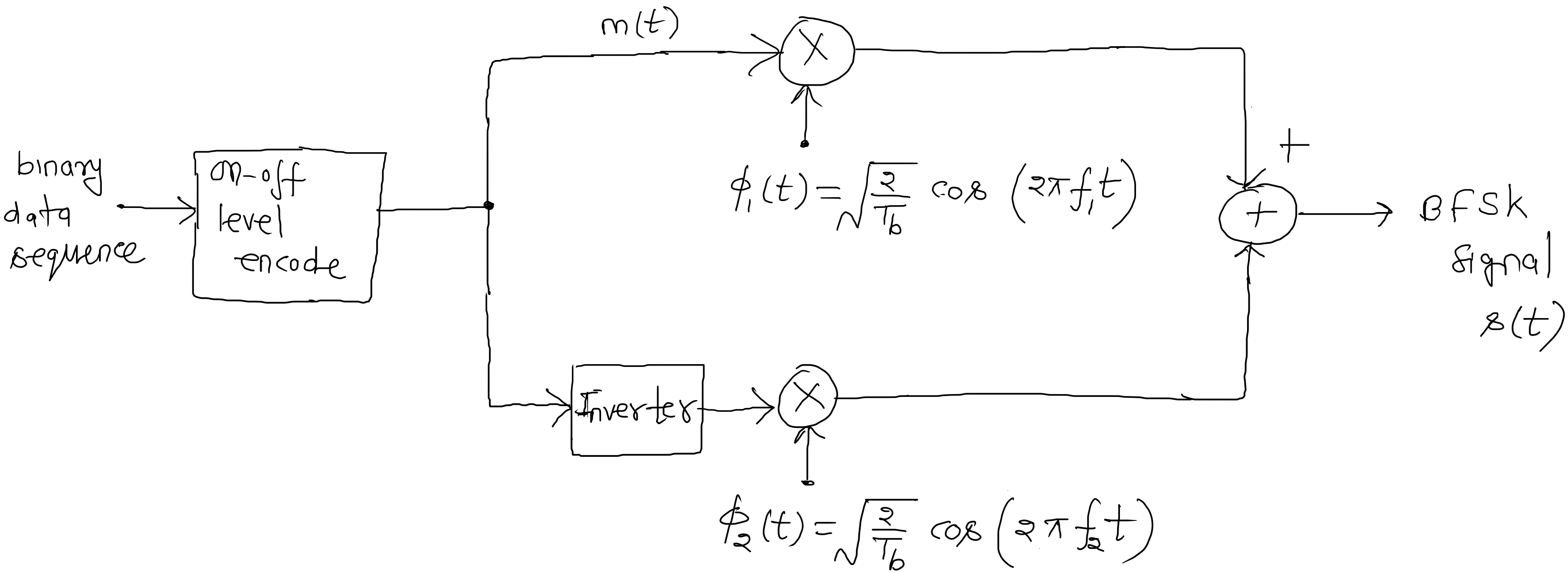
$$E = 2E_b$$ ... symbol energy

$$R = \frac{R_b}{2}$$

Coherent Binary Frequency-shift keying (BFSK):

"1" $\Rightarrow f_1$

"0" $\Rightarrow f_2$



$$s_i(t) = \begin{cases} \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_i t) & 0 \le t \le T_b \\ 0 & \text{otherwise} \end{cases} \qquad i = 1, 2$$

$$f_i = \frac{n_c + i}{T_b}, \qquad i = 1, 2$$

"1" $\Rightarrow$ $s_1(t) \Rightarrow f_1$

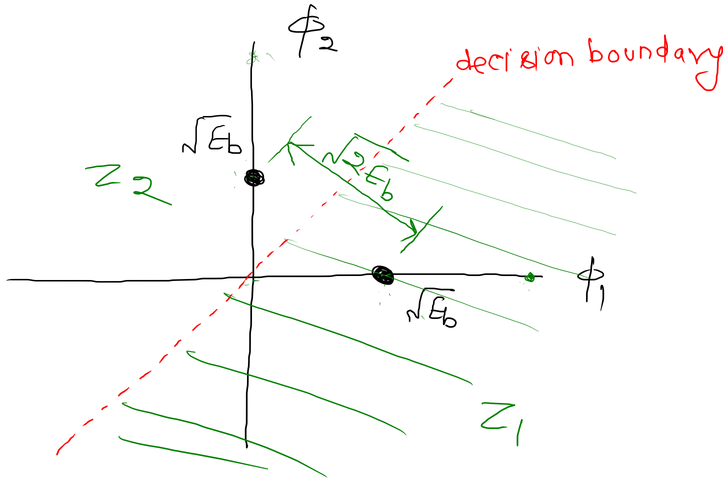
"0" $\Rightarrow$ $s_2(t) \Rightarrow f_2$

$$\phi_i(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_i t) \qquad i = 1, 2$$

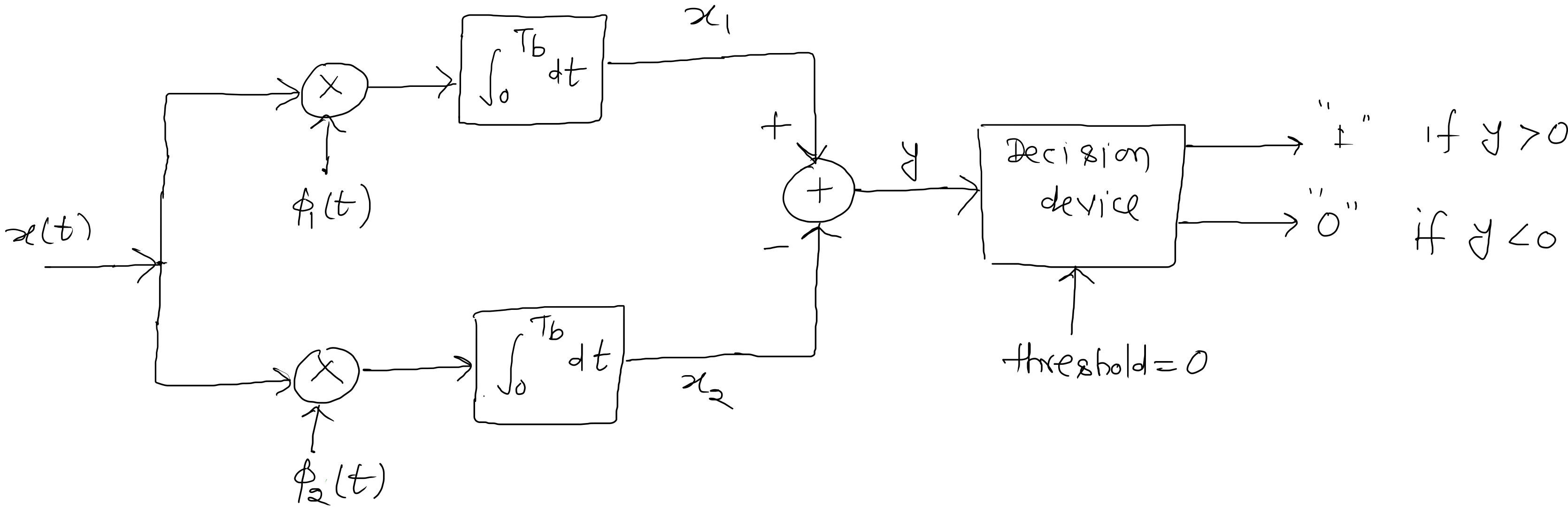
.. orthonormal basis functions

$$s_{ij} = \int_0^{T_b} s_i(t)\,\phi_j(t)\,dt = \begin{cases} \sqrt{E_b} & i = j \\ 0 & i \neq j \end{cases}$$

$$s_1 = \begin{bmatrix} \sqrt{E_b} \\ 0 \end{bmatrix}, \qquad s_2 = \begin{bmatrix} 0 \\ \sqrt{E_b} \end{bmatrix}$$

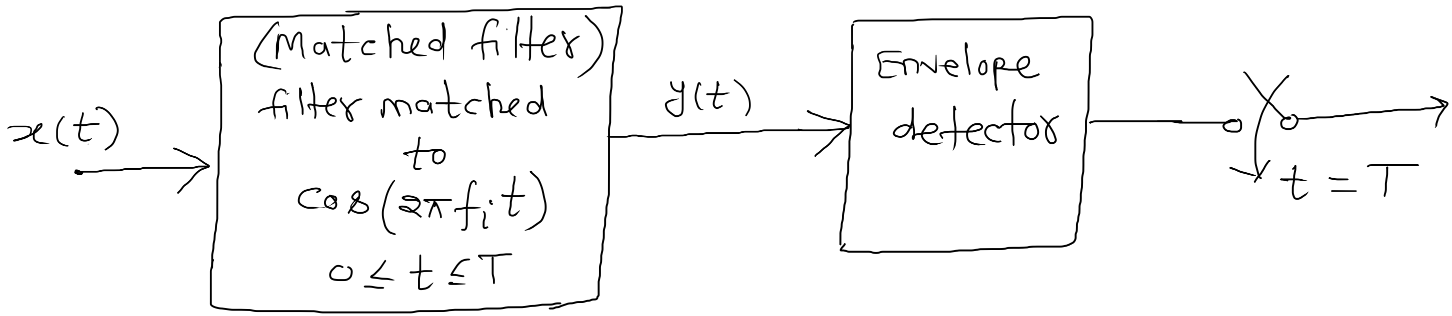


Coherent Rx for BFSK:

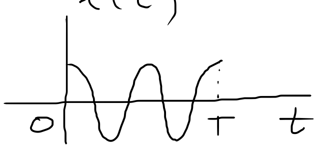


$$P_e = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)$$

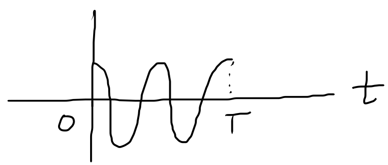
Noncoherent Rx:



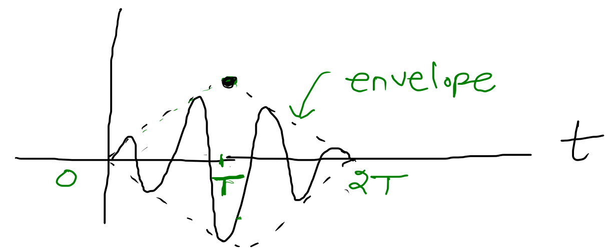
$x(t)$

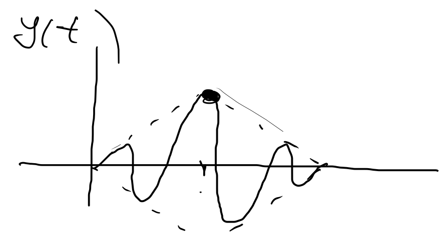


$h(t) = x(T-t)$



$y(t) = x(t) * h(t)$

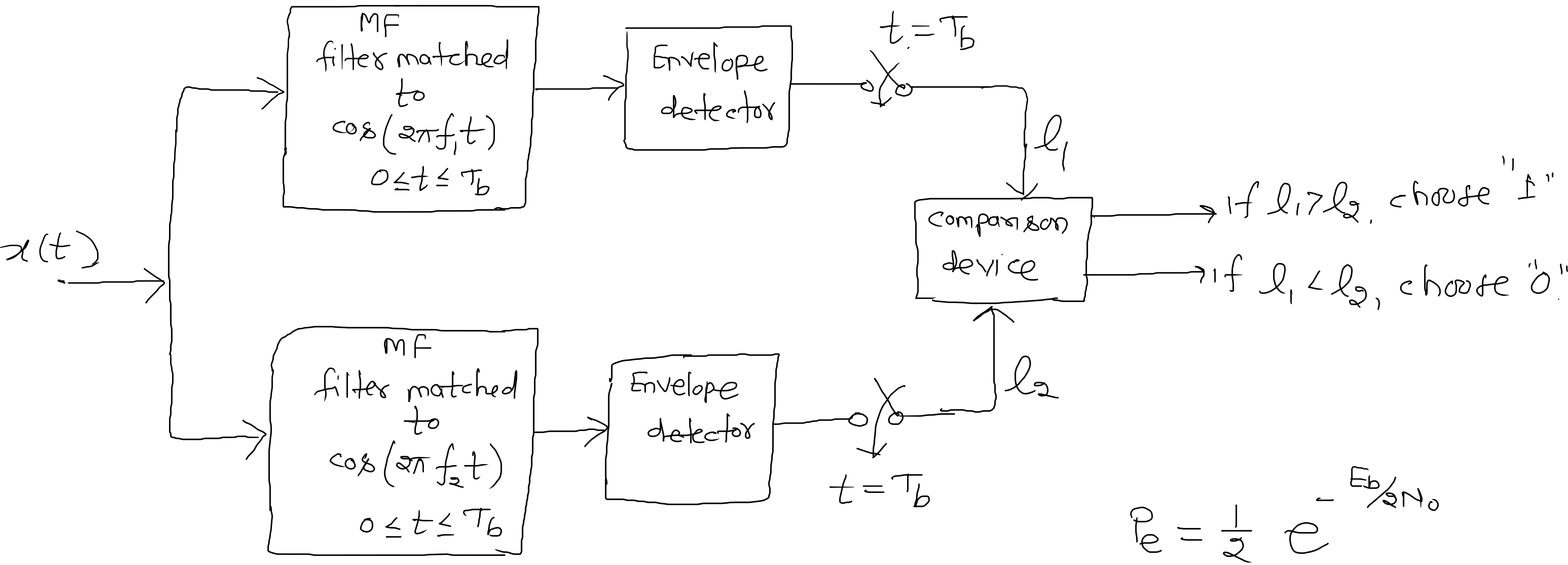




Noncoherent BFSK:

"1" $\Rightarrow s_1(t)$, "0" $\Rightarrow s_2(t)$

$$s_i(t) = \begin{cases} \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_i t) & 0 \le t \le T_b \\ 0 & \text{otherwise} \end{cases} \qquad i = 1, 2$$



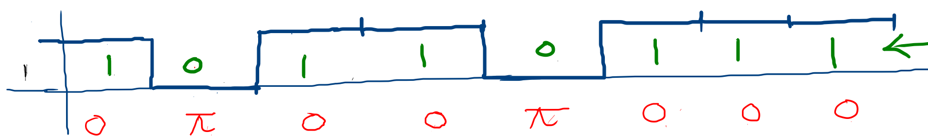
$$P_e = \frac{1}{2} e^{-E_b/2N_0}$$

Differential Phase Shift Keying (DPSK)

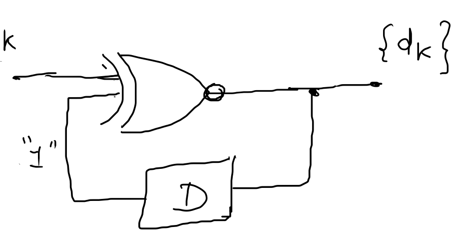
DPSK is noncoherent version of PSK.

(Differential encoding + PSK)

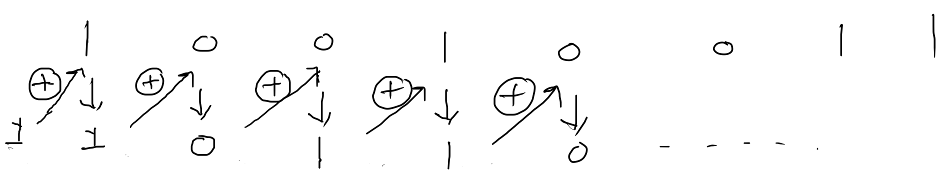
$\{b_k\}$ 1 0 0 1 0 0 1 1

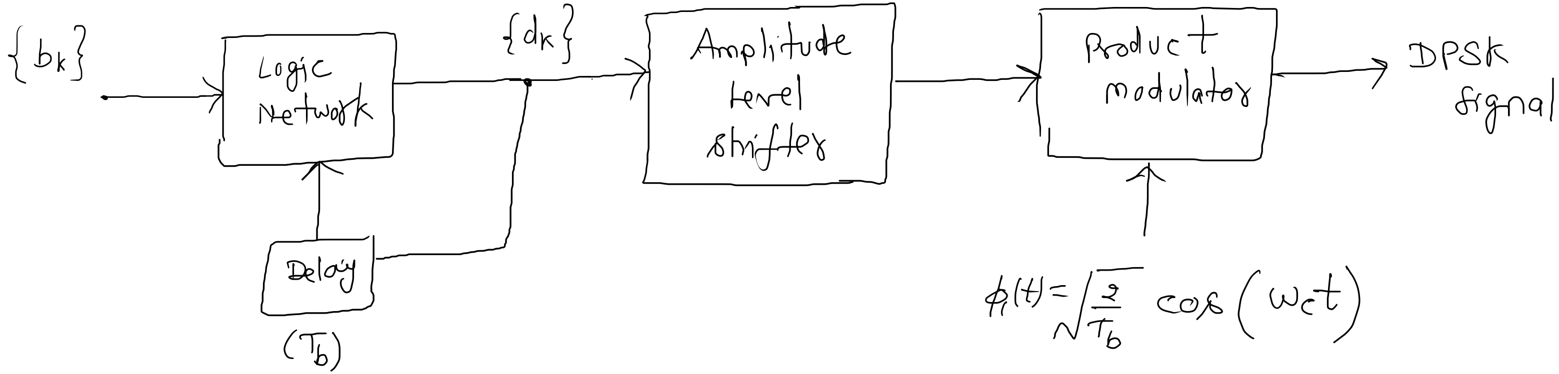


← differentially encoded bits

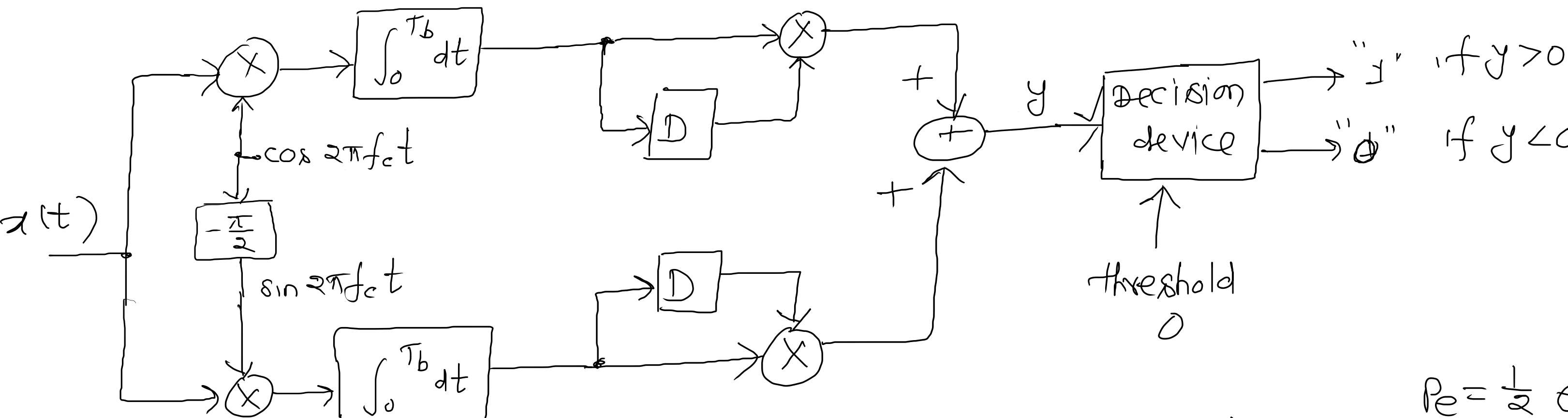


differential encoder





(DPSK transmitter)



$$P_e = \frac{1}{2} e^{-E_b/N_0}$$

(DPSK Rx)

$$s_1(t) = \begin{cases} \sqrt{\dfrac{E_b}{2T_b}} \cos(2\pi f_c t) & 0 \le t \le T_b \\ \sqrt{\dfrac{E_b}{2T_b}} \cos(2\pi f_c t) & T_b \le t \le 2T_b \end{cases}$$

$$s_2(t) = \begin{cases} \sqrt{\dfrac{E_b}{2T_b}} \cos(2\pi f_c t) & 0 \le t \le T_b \\ \sqrt{\dfrac{E_b}{2T_b}} \cos(2\pi f_c t + \pi) & T_b \le t \le 2T_b \end{cases}$$