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where:

- y_{i-1}, y_i, and y_{i+1} are the values of y at the points x_{i-1}, x_i, and x_{i+1}, respectively.
- h is the step size between consecutive points.

For the first derivative y', a centered difference approximation could be used as:

$$y'(x_i) \approx \frac{y_{i+1} - y_{i-1}}{2h}$$

However, in your code, asymmetric forward and backward differences are used to approximate the first derivative at internal points. This results in the following coefficients in the matrix A:

- **Left neighbor (i - 1)**: $\frac{1}{h^2} - \frac{1}{2h}$
- **Center (i)**: $-\frac{2}{h^2} + 10$
- **Right neighbor (i + 1)**: $\frac{1}{h^2} + \frac{1}{2h}$

Construction of the Matrix Equation

For the finite difference method, the ODE at each internal point x_i is replaced by an equation of the form:

$$\left(\frac{1}{h^2} - \frac{1}{2h}\right) y_{i-1} + \left(-\frac{2}{h^2} + 10\right) y_i + \left(\frac{1}{h^2} + \frac{1}{2h}\right) y_{i+1} = 0$$

Boundary conditions are imposed directly on y_1 and y_N by setting the first and last rows of A to enforce:

$$y(0) = 1 \quad \text{and} \quad y(1) = 2$$

This creates a linear system $A\mathbf{y} = \mathbf{b}$, which is solved for y. The matrix A represents the coefficients from the finite difference approximations, and the vector b contains the values from the boundary conditions.

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