```
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
```

```
# Gradient descent function
def gradient(xtrain, ytrain, iter, alpha, w, b):
    m = len(xtrain)
    for i in range(iter):
        prediction = w * xtrain + b
        error = prediction - ytrain
        temp_dw = (2/m) * np.dot(error, xtrain)
        temp_db = (2/m) * np.sum(error)
        w -= alpha * temp_dw
        b -= alpha * temp_db
    return w, b
```

Production Data

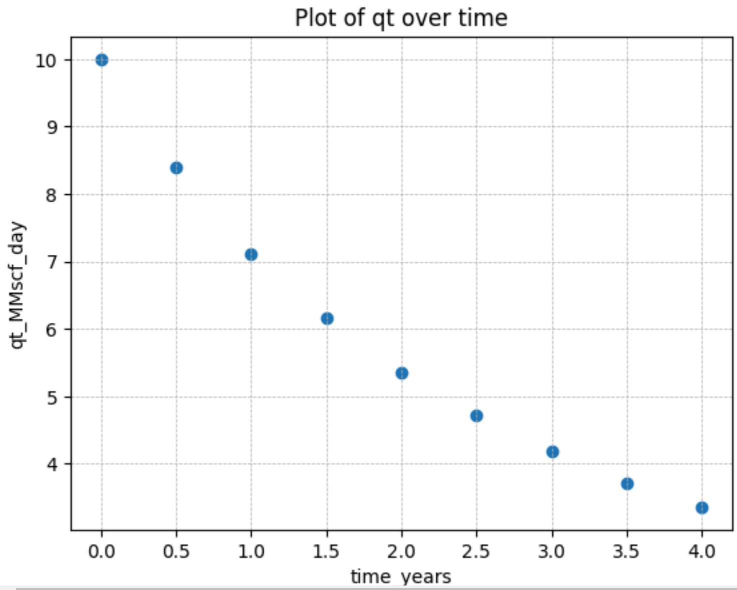
```
time_years = np.array([0.0, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0])
qt_MMscf_day = np.array([10.00, 8.40, 7.12, 6.16, 5.36, 4.72, 4.18, 3.72, 3.36])
Gp_t_MMscf = np.array([0.00 ,1.67 ,3.08 ,4.30 ,5.35 ,6.27 ,7.08 ,7.78, 8.44])
```

Prediction time

```
time_years_new= np.linspace(0,20,41)
```

Visualization of Data

```
plt.scatter(time_years, qt_MMscf_day)
plt.title('Plot of qt over time')
plt.xlabel('time_years')
plt.ylabel('qt_MMscf_day')
plt.grid(True, which='both', ls='--', lw=0.5)
plt.show()
```



Exponential Decline, b = 0

The graphical presentation of this type of decline curve indicates that a plot of q_t versus t on a semi-log scale or a plot of q_t versus $G_{P(t)}$ on a Cartesian scale will produce linear relationships that can be described mathematically by

$$q_t = q_i \exp(-D_i t)$$

Or linearly as

$$\ln(q_t) = \ln(q_i) - D_i t$$

Similarly,

$$G_{p(t)} = \frac{q_i - q_t}{D_i}$$

Or linearly as

$$q_t = q_i - D_i G_{p(t)}$$

```
# Exponential Decline

qt_log=np.log(qt_MMscf_day)

# Normalize the data
qt_log_normalized_exp = (qt_log - np.mean(qt_log)) / np.std(qt_log)
time_years_normalized_exp = (time_years - np.mean(time_years)) / np.std(time_years)

# Run gradient descent with normalized data and a smaller learning rate
w_exp, b_exp = gradient( time_years_normalized_exp,qt_log_normalized_exp, 80000, 0.001, 0, 0)

# Denormalize the data
w_original_exp = w_exp * (np.std(qt_log) / np.std(time_years))
b_original_exp = b_exp* np.std(qt_log) + np.mean(qt_log) - w_original_exp * np.mean(time_years)

# Di,b and qi for exponential decline
Di_exp = -w_original_exp
b_exp = 0
qi_exp=np.exp( w_original_exp *0 + b_original_exp)

# Prediction by Exponential Decline
qt_log_p = w_original_exp * time_years_new + b_original_exp
qt_exp=np.exp(qt_log_p)
Gp_exp=(qi_exp-qt_exp)/(Di_exp/365)
```

In this R2 equation, yi is target value, ybi is model's prediction, and yi is the average of all the target values. R2 values closer to 1 indicat better

$$R^2 = 1 - \frac{\sum_{i=1}^{n}\left(y_i - \widehat{y_i}\right)^2}{\sum_{i=1}^{n}\left(y_i - \overline{y_i}\right)^2}$$

mode performance.

```
Rsq_exp=1-np.sum((qt_MMscf_day-qt_exp[0:len(time_years)])**2)/np.sum((qt_MMscf_day-np.mean(qt_MMscf_day))**2)
print(f'R2 for Exponetial decline equation for production data is {Rsq_exp}')
```

```
R2 for Exponetial decline equation for production data is 0.9912357308721692
```

Hyperbolic Decline, $0 < b < 1$

The two governing relationships for a reservoir or a well whose production follows the hyperbolic decline behavior are given by Equations 16-4 and 16-8:

$$q_t = \frac{q_i}{(1 + b\, D_i\, t)^{1/b}}$$

$$G_{p(t)} = \left[\frac{q_i}{D_i(1-b)}\right]\left[1 - \left(\frac{q_t}{q_i}\right)^{1-b}\right]$$

```
# Hyperbolic Decline

# Normalize the data
qt_loglog_normalized_hp = (np.log(np.log(qt_MMscf_day)) - np.mean(np.log(np.log(qt_MMscf_day)))) / np.std(np.log(np.log(qt_MMscf_day)))
time_years_normalized_hp = (time_years - np.mean(time_years)) / np.std(time_years)

w_hp, b_hp = gradient( time_years_normalized_hp,qt_loglog_normalized_hp, 80000, 0.001, 0, 0)

# Denormalize the data
w_original_hp = w_hp * (np.std(np.log(np.log(qt_MMscf_day))) / np.std(time_years))
b_original_hp = b_hp* np.std(np.log(np.log(qt_MMscf_day))) + np.mean(np.log(np.log(qt_MMscf_day))) - w_original_hp * np.mean(time_years)

# Extend the curve to intercept the y-axis at t ¼ 0 and read qi.
t_hp=0
qi_hp=np.exp((np.exp(b_original_hp+w_original_hp*t_hp)))

# Select the other end point of the smooth curve, record the coordinates of the point, and refer to it as (t2, q2).
t2_hp=time_years[np.size(time_years)-1]
q2_hp=np.exp((np.exp(b_original_hp+w_original_hp*t2_hp)))

# Determine the coordinate of the middle point on the smooth curve that corresponds to (t1, q1) with the value of q1, as obtained from i
q1_hp=(qi_hp*q2_hp)**0.5
t1_hp=(np.log(np.log(q1_hp))-b_original_hp)/w_original_hp

# Solve the following equation iteratively for b.
def nr(q1,t1,q2,t2,qi,bi):
    b_k=bi
    i=1
    b_k1=0.0
    while i>0.000001:
        b_k1 = b_k - (t2*(qi/q1)**b_k - t1*(qi/q2)**b_k - (t2-t1) ) / (t2*(qi/q1)**b_k*np.log(qi/q1) - t1*(qi/q2)**b_k*np.log(qi/q2) )
        i=b_k1-b_k
        b_k=b_k1
    return b_k1
bi_hp=0.5
b_hp=nr(q1_hp,t1_hp,q2_hp,t2_hp,qi_hp,bi_hp)

# Solve for Di, by using the calculated value of b from Step 4 and the coordinate of a point on the smooth graph
Di_hp=((qi_hp/q2_hp)**b_hp - 1)/(b_hp*t2_hp)
Did_hp=Di_hp/365

# Prediction by Hyperbolic Decline
qt_hp = qi_hp/(1 + b_hp*Di_hp*time_years_new)**(1/b_hp)
Gp_hp = (qi_hp/(Did_hp*(1-b_hp)))*(1-(qt_hp*10**6/(qi_hp*10**6))**(1-b_hp))
```

In this R2 equation, yi is target value, ybi is model’s prediction, and yi is the average of all the target values. R2 values closer to 1 indicat better

$$R^2 = 1 - \frac{\sum_{i=1}^{n}\left(y_i - \widehat{y_i}\right)^2}{\sum_{i=1}^{n}\left(y_i - \overline{y_i}\right)^2}$$

mode performance.

```
Rsq_hp=1-np.sum((qt_MMscf_day-qt_hp[0:len(time_years)])**2)/np.sum((qt_MMscf_day-np.mean(qt_MMscf_day))**2)
print(f'R2 for Hyperbolic decline equation for production data is {Rsq_hp}')
```

```
R2 for Hyperbolic decline equation for production data is 0.9998893828088681
```

Harmonic Decline, b = 1

The production-recovery performance of a hydrocarbon system that follows a harmonic decline (i.e., b = 1 in Equation 16-1) is described by Equations 16-5 and 16-9.

$$q_t = \frac{q_i}{1 + D_i t}$$

$$G_{p(t)} = \left(\frac{q_i}{D_i}\right) \ln\left(\frac{q_i}{q_t}\right)$$

These two expressions can be rearranged and expressed as follows:

$$\frac{1}{q_t} = \frac{1}{q_i} + \left(\frac{D_i}{q_i}\right) t \qquad (16\text{-}14)$$

$$\ln(q_t) = \ln(q_i) - \left(\frac{D_i}{q_i}\right) G_{p(t)} \qquad (16\text{-}15)$$

```
# Harmonic Decline

qt_inverse=1/qt_MMscf_day

# Normalizing the data
qt_inverse_normalized = (qt_inverse - np.mean(qt_inverse)) / np.std(qt_inverse)
time_normalized_ha = (time_years - np.mean(time_years)) / np.std(time_years)

# Run gradient descent with normalized data
w_ha, b_ha = gradient( time_normalized_ha,qt_inverse_normalized, 80000, 0.001, 0, 0)

# Denormalize data
w_original_ha = w_ha * (np.std(qt_inverse) / np.std(time_years))
b_original_ha = b_ha* np.std(qt_inverse) + np.mean(qt_inverse) - w_original_ha * np.mean(time_years)

# Di,b and qi for harmonic decline
Di_ha_years=w_original_ha/b_original_ha
qi_ha=1/b_original_ha
Di_ha_day=Di_ha_years/365

# Prediction by Harmonic Decline
qt_line1 = np.array(w_original_ha *time_years_new + b_original_ha)
qt_ha=1/qt_line1
Gp_ha = np.log(qi_ha/qt_ha)/(Di_ha_day/qi_ha)
```

In this R2 equation, yi is target value, ybi is model's prediction, and yi is the average of all the target values. R2 values closer to 1 indicat better

$$R^2 = 1 - \frac{\sum_{i=1}^{n}\left(y_i - \widehat{y_i}\right)^2}{\sum_{i=1}^{n}\left(y_i - \overline{y_i}\right)^2}$$

mode performance.

```
Rsq_ha=1-np.sum((qt_MMscf_day-qt_ha[0:len(time_years)])**2)/np.sum((qt_MMscf_day-np.mean(qt_MMscf_day))**2)
print(f'R2 for Harmonic decline equation for production data is {Rsq_ha}')
```

```
R2 for Harmonic decline equation for production data is 0.9821753450639774
```

Determining the best fitting Decline Equation

```
Rsq=max(Rsq_exp,Rsq_hp,Rsq_ha)
if Rsq==Rsq_exp:
    print(f'Exponential Decline is best fitting the Production Data with a Rsq value of {Rsq}, So exponential decline can be used for pr
    qt=qt_exp
    Gp=Gp_exp
    # Create a DataFrame
    df = pd.DataFrame({
        'Time (Years)': time_years_new,
        'Actual Flow rate (MMscf/day)': np.concatenate([qt_MMscf_day, np.array(['No Data'] * (len(time_years_new) - len(time_years)))]),
        'Calculated Flow rate (MMscf/day)': qt,
        'Actual Cumulative Production (MMMscf)': np.concatenate([Gp_t_MMscf , np.array(['No Data'] * (len(time_years_new) - len(time_yea
        'Calculated Cumulative Production (MMMscf)': Gp/1000 ,
                    })
    # Set index name
    df.index.name = "S.No."

elif Rsq==Rsq_hp:
    print(f'Hyperbolic Decline is best fitting the Production Data with a Rsq value of {Rsq}, So hyperbolic decline can be used for pred
    qt=qt_hp
    Gp=Gp_hp
    # Create a DataFrame
    df = pd.DataFrame({
        'Time (Years)': time_years_new,
        'Actual Flow rate (MMscf/day)': np.concatenate([qt_MMscf_day, np.array(['No Data'] * (len(time_years_new) - len(time_years)))]),
        'Calculated Flow rate (MMscf/day)': qt,
        'Actual Cumulative Production (MMMscf)': np.concatenate([Gp_t_MMscf , np.array(['No Data'] * (len(time_years_new) - len(time_yea
        'Calculated Cumulative Production (MMMscf)': Gp/1000 ,
                    })
    # Set index name
    df.index.name = "S.No."

else:
    print(f'Harmonic Decline is best fitting the Production Data with a Rsq value of {Rsq}, So harmonic decline can be used for predicti
    qt=qt_ha
    Gp=Gp_ha
    # Create a DataFrame
    df = pd.DataFrame({
        'Time (Years)': time_years_new,
        'Actual Flow rate (MMscf/day)': np.concatenate([qt_MMscf_day, np.array(['No Data'] * (len(time_years_new) - len(time_years)))]),
        'Calculated Flow rate (MMscf/day)': qt,
        'Actual Cumulative Production (MMMscf)': np.concatenate([Gp_t_MMscf , np.array(['No Data'] * (len(time_years_new) - len(time_yea
        'Calculated Cumulative Production (MMMscf)': Gp/1000 ,
                    })
    # Set index name
    df.index.name = "S.No."
```

```
Hyperbolic Decline is best fitting the Production Data with a Rsq value of 0.9998893828088681, So hyperbolic decline can be used for
```

Table to show calculated data by model and actual data

```
print(df)
```

```
        Time (Years) Actual Flow rate (MMscf/day)  \
S.No.
0                0.0                         10.0
1                0.5                          8.4
2                1.0                         7.12
3                1.5                         6.16
4                2.0                         5.36
5                2.5                         4.72
6                3.0                         4.18
7                3.5                         3.72
8                4.0                         3.36
9                4.5                      No Data
10               5.0                      No Data
11               5.5                      No Data
12               6.0                      No Data
13               6.5                      No Data
14               7.0                      No Data
15               7.5                      No Data
16               8.0                      No Data
17               8.5                      No Data
18               9.0                      No Data
19               9.5                      No Data
20              10.0                      No Data
21              10.5                      No Data
22              11.0                      No Data
23              11.5                      No Data
24              12.0                      No Data
25              12.5                      No Data
26              13.0                      No Data
```

```
27            13.5                    No Data
28            14.0                    No Data
29            14.5                    No Data
30            15.0                    No Data
31            15.5                    No Data
32            16.0                    No Data
33            16.5                    No Data
34            17.0                    No Data
35            17.5                    No Data
36            18.0                    No Data
37            18.5                    No Data
38            19.0                    No Data
39            19.5                    No Data
40            20.0                    No Data

       Calculated Flow rate (MMscf/day) Actual Cumulative Production (MMMscf)  \
S.No.
0                             10.049765                                   0.0
1                              8.386661                                  1.67
2                              7.120718                                  3.08
3                              6.132698                                   4.3
4                              5.345385                                  5.35
5                              4.706891                                  6.27
6                              4.181238                                  7.08
7                              3.742814                                  7.78
8                              3.372956                                  8.44
9                              3.057794                               No Data
10                             2.786833                               No Data
11                             2.552009                               No Data
```

Plotting the Model

```
fig, ax1 = plt.subplots()

# Plot 'qt' on the left y-axis
ax1.plot(time_years_new, qt, color='b', marker='o', label='Flow rate (MMscf/day)')
ax1.set_xlabel('Time (Years)')
ax1.set_ylabel('Flow rate (MMscf/day)', color='b')
ax1.tick_params(axis='y', labelcolor='b')

# Create a second y-axis for 'Gp'
ax2 = ax1.twinx()
ax2.plot(time_years_new, Gp/1000, color='r', marker='s', label='Cumulative Production (MMMscf)')
ax2.set_ylabel('Cumulative Production (MMMscf)', color='r')
ax2.tick_params(axis='y', labelcolor='r')

# Add legends
lines, labels = ax1.get_legend_handles_labels()
lines2, labels2 = ax2.get_legend_handles_labels()
ax1.legend(lines + lines2, labels + labels2, loc='upper left')

# Show the plot
plt.title('Dual-Axis Plot: Flow Rate vs. Cumulative Production')
plt.grid(True)
plt.show()
```