Applications of Physics in Computing

Physics of Animation: Taxonomy of physics based animation methods, Frames, Frames per Second, Size and Scale, Weight and Strength, Motion and Timing in Animations, Constant Force and Acceleration, The Odd rule, Odd-rule Scenarios, Motion Graphs, Examples of Character Animation: Jumping, Parts of Jump, Jump Magnification, Stop Time, Walking: Strides and Steps, Walk Timing. Numerical Problems.

Statistical Physics for Computing: Descriptive statistics and inferential statistics, Poisson distribution and modelling the probability of proton decay, Normal Distributions (Bell Curves), Monte Carlo Method: Determination of Value of π. Numerical Problems.

Prerequisites: Motion in one dimension, Probability
Self-learning: Frames, Frames per Second

08 Hours

Introduction

Physics-based animation has emerged as a core area of computer graphics finding widespread application in the film and video game industries as well as in areas such as virtual surgery, virtual reality, and training simulations.

When an animator creates a character it must be life-like as possible. It does not mean that the character should look like an actual human. A life-like character is the one that exhibits human-like emotions, movements, and behaviour patterns.

In order to achieve this, **an animator must have an outstanding knowledge of physics, especially when it comes to Newton's laws, gravity, concepts of energy, and motion.** Possessing the knowledge of these concepts animators can create projects where the characters look like they are a part of the real world.

Animation

Animation is a method of photographing successive drawings, models, or even puppets, to create an illusion of movement in a sequence. Because our eyes can only retain an image for approximately 1/10th of a second, when multiple images appear in fast succession, the brain blends them into a single moving image. Animation is the process of displaying still images in a rapid sequence to create the illusion of movement.

The taxonomy of physics based animations

The field of physics-based animation and simulation can roughly be subdivided into two large groups

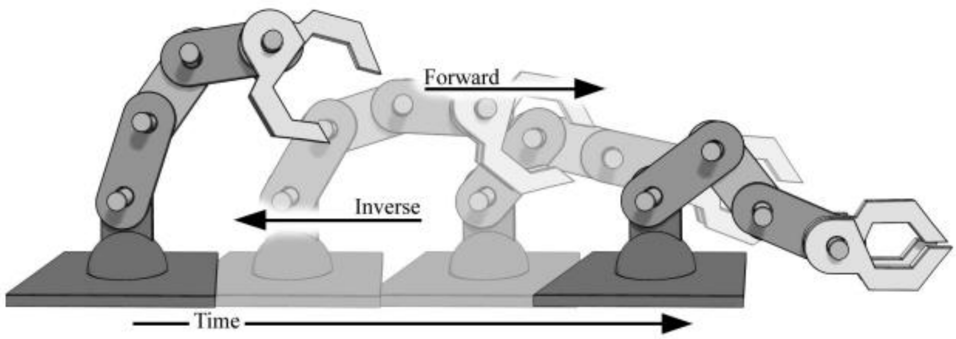
Kinematics - It is the study of motion without consideration of mass or forces.

Dynamics - It is the study of motion taking mass and forces into consideration.

The kinematics and dynamics again come in two subgroups

Inverse is the study of motion knowing the starting and ending points.

Forward is the study of motion solely given the starting point.



The above picture showing the difference in forward and inverse kinematics and dynamics. In the case of inverse motion, the starting and ending positions of the robot gripping tool is given, whereas in the case of forward motion, only the starting position is given,

There are numerous techniques and methods in physics-based animation and simulation some of the most popular techniques and methods according to the four subgroups are listed in the below table.

Group	Inverse	Forward
Kinematics	• Cyclic Coordinate • Descent Jacobian Method	• Spline Driven Animation • Key Frame Animation (Interpolation) • Closed-Form Solutions • Free-Form Deformation
Dynamics	• Recursive Newton Euler Method • Optimization Problems	• Featherstone's Method (The Articulated Body Method) • Composite-Rigid-Body Method • Particle Systems, Mass-Spring Systems • Finite Element Method • Constraint-Based Simulation

Frame

The concept of a Frame is of fundamental importance to animation. A frame is a single image in a sequence of pictures. A frame contains the image to be displayed at a unique time in the animation. In general, one second of a video is comprised of 24 or 30 frames per second also known as FPS. The frame is a combination of the image and the time of the image when exposed to the view. An extract of frames in a row makesthe animation.

Every frame is displayed on the screen until the next frame overwrites it. Since each frame remains displayed on the screen for a tiny but finite time period, one can think of an animation as frames displayed at discreet intervals of time in a continued sequence. Any scene change happens by drawing a whole new frame. Even changing a single pixel requires drawing the next frame in its entirety. Furthermore, even if the image of the scene remains constant, identical content must be drawn in subsequent frames as long as the still scene is needed.

Frame per second (Frame rate)

The frames per second (FPS) is also called the frame rate and refers to the number of images one can see in every second of an animated sequence or movie.

Frame rate greatly impacts the style and viewing experience of a video. Different frame rates yield different viewing experiences, and choosing a frame rate often means thinking about multiple factors, such as how realistic the video is to look or whether or not one can plan to use techniques like slow motion or motion-blur effects.

Animation shot on film and projected is played at 24 frames per second. Animation for television in Europe, Africa, the Middle East and Australia is played at 25 frames per second. These countries use a television system called Phase Alternate Line (PAL).

The Americas, the West Indies and the Pacific Rim countries use National Television Standard Committee (NTSC) that should be animating at 30 frames per second.

Size and Scale

The size and scale of characters often play a central role in a story's plot. What would Superman be without his height and bulging biceps? Some characters, like the Incredible Hulk, are even named after their body types.

We often equate large characters with weight and strength, and smaller characters with quickness and speed. There is a reason for this. In real life, larger people and animals do have a larger capacity for strength, while smaller animals can move faster than their large counterparts. When designing characters, you can run into different situations having to do with size and scale, such as:

- Human or animal-based characters that are much larger than we see in our everyday experience like Superheroes, Greek gods, Monsters etc.
- Human or animal-based characters that are much smaller than we are familiar, such as fairies and elves.
- Characters that need to be noticeably larger, smaller, older, heavier, lighter, or more energetic than other characters.
- Characters that are child versions of older characters. An example would be an animation featuring a mother cat and her kittens. If the kittens are created and animated with the same proportions and timing as the mother cat, they won't look like kittens; they'll just look like very small adult cats.

Proportion and Scale

Creating a larger or smaller character is not just a matter of scaling everything about the character uniformly. To understand this, let's look at a simple cube. When you scale a cube, its volume changes much more dramatically than its surface area. Let us say each edge of the cube is 1 unit length. The area of one side of the cube is 1 square unit, and the volume of the cube is 1 cubed unit. If you double the size of the cube along each dimension, its height increases by 2 times, the surface area increases by 4 times, and its volume increases by 8 times. While the area increases by squares as you scale the object, the volume changes by cubes.

Weight and Strength

Body weight is proportional to volume. The abilities of your muscles and bones, however, increase by area because their abilities depend more on cross-sectional area than volume. To increase a muscle or bone's strength, you need to increase its cross- sectional area. To double a muscle's strength, for example, you would multiply its width by $\sqrt{2}$. To triple the strength, multiply the width by $\sqrt{3}$. Since strength increases by squares and weight increases by cubes, the proportion of a character's weight that it can lift does not scale proportionally to its size.

Let us look at an example of a somewhat average human man at 6 feet tall, he weighs 180 pounds and can lift 90 pounds. In other words, he can lift half his body weight. If you scale up the body size by a factor of 2, the weight increases by a factor of 8. Such a character could then lift more weight. But since he weighs more than 8 times more than he did before, he cannot lift his arms and legs as easily as a normal man. Such a giant gains strength but loses agility.

Motion and Timing in Animations

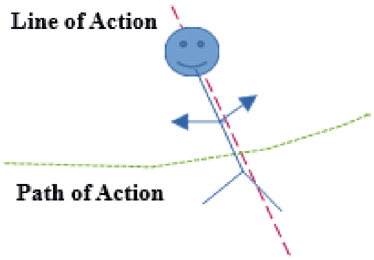
Motion is an essential component in games and animations. The motion is governed by the Newton's laws and kinematic equations. When animating a scene, there are several types of motion to consider. Some of the most common types of motion are

- Linear
- Parabolic
- Circular
- Wave

Motion and timing go hand in hand in animation.

Motion lines and Paths

Individual drawings or poses have a line of action, which indicates the visual flow of action at that single image. Motion has a path of action, which indicates the path along which the object or character moves. The path of action refers to the object's motion in space. While it can help show timing, its primary function is to see the direction and path of the motion, and not necessarily its timing.



Timing

Timing refers to the time it takes for an action to complete from the starting point to the end. It is used to add movement (while obeying the laws of physics) and interest to the animations. This can be done with the help of weight, scaling properties, and the personality of the movements of the character.

Timing has a huge role to play in how animation will look like. If **timing** is really fast or slow, too linear or too long, then the animation will look like unrealistic. In the entertainment industry now, films are played at 24 frames per second (FPS). This means that for an object that is in motion from one point say A to another point B at 24 FPS, it will take one second to complete the journey.

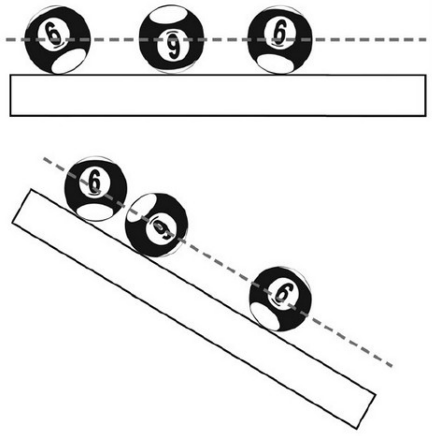
In animation, **timing** is used to show three things, these are weight, scaling properties, and the emotions of a character. The weight of different objects can be shown by altering their timings. On the other hand, the scaling properties of objects can be shown by how fast or slow they move. E.g., heavier objects will move slower on-screen, and lighter objects will move faster. Lastly, emotion can be displayed by the speed at which a character's movements take place. An excited character will move faster whereas a character that is lazy or sad will tend to move slower. The speed at which the characters move will also decide how well the audience interprets and understands the action.

Timing tools

In animation, timing of action consists of placing objects or characters in particular locations at specific frames to give the illusion of motion. Animators work with very small intervals of time; most motion sequences can be measured in seconds or fractions of seconds. Frame intervals between keys are usually smaller than one second.

Linear motion timing

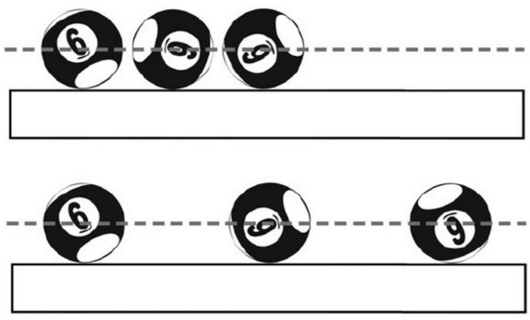
Linear motion refers to motion in a straight line, always in the same direction. An object moving with linear motion might speed up or slow down as it follows a linear path. A heavy ball rolling on a table or incline is an example of linear motion. The ball is rotating, but its centre of gravity follows a linear path. A heavy ball rolling on a table or incline is an example of linear motion. The ball is rotating, but its centre of gravity follows a linear path.



Linear motion.

Uniform motion timing

When uniform motion occurs, the net force on the object is zero. Net force is the total of all forces added up. There might be several forces acting on the object, but when both the magnitude and direction of the forces are added up, they add up to zero. Uniform motion is the easiest to animate because the distance the object travels between frames is always the same. Uniform motion is a type of linear motion with constant speed and no acceleration or deceleration. The object moves the same distance between consecutive frames. The longer the distance between frames, the higher the speed.



Uniform motion.

Slow in and slow out

When motion is accelerating or decelerating, we refer to this type of motion as a slow in or slow out. This type of motion is also called as **ease in** or **ease out**.

- Slow in (ease in) — The object is slowing down, often in preparation for stopping.
- Slow out, (ease out) — The object is speeding up, often from a still position.

The term slow out can be confusing, since it essentially means "speed up." one can think of slow out as the same as ease out, as in easing out of a still position and speeding up to full speed. For example, a ball rolling down an incline or dropping straight down is slowing out, as it goes from a still position or slow speed to a fast speed. A ball rolling up an incline is slowing in.

Slowing out: speeding up. Slowing in: slowing down.

Acceleration timing

Timing for acceleration can be calculated very accurately when the net force being exerted is constant. Let's take a look at the forces and how they can be used to calculate the animation's timing.

Constant forces

A constant force is a force that doesn't vary over time. Examples of constant forces include:

- Gravity pulling an object to the ground
- Friction bringing an object to a stop

Constant force and Acceleration

Constant forces result in constant acceleration. Because the acceleration is constant, we can figure out the timing for such sequences using a few principles of physics.

The resulting acceleration depends on the direction of the force and motion, if there is any motion at all to begin with.

- When constant net force is applied to an unmoving object, the result is acceleration.
- When constant net force is applied to a moving object in the same direction as the motion, the result is acceleration.

- When constant net force is applied in the direction opposite the existing motion, the result is deceleration (acceleration in the opposite direction).

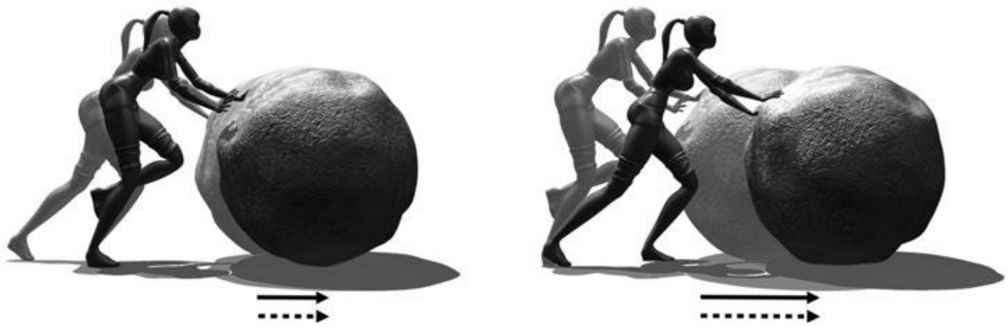
Note that constant acceleration doesn't mean constant speed. It's quite the opposite! Constant acceleration means the object is changing speed constantly.

Forces exerted by characters

Forces exerted by people's bodies are rarely constant throughout an entire motion. For the purposes of animation, however, one can break the character motion into short time segments and consider each of these segments to be responding to constant net force. This will make it easier for one to calculate the timing for each individual segment.

For example, let's look at the push for a jump. The force a character exerts during the push is somewhat constant, and the timing is very short (less than half a second). In such a case the timing for a constant force is an excellent starting point, and in most cases will do the job as is.

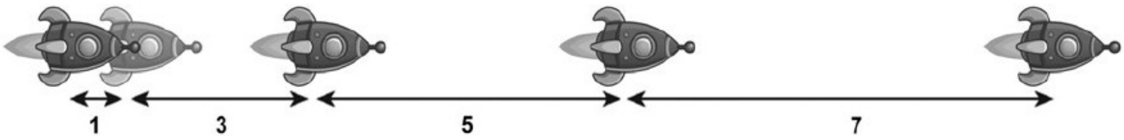
A character walking and pushing a rock is not exerting a constant force throughout the entire sequence, but during each short part of the walk cycle the net force could be considered to be a different constant value.



Arrows show relative force and speed between different parts of walk cycle.

The Odd rule

When acceleration is constant, one can use the Odd Rule to time the frames. With this method, one can calculate the distance the object moves between frames using a simple pattern of odd numbers. Between consecutive frames, the distance the object moves is a multiple of an odd number. For acceleration, the distance between frames increases by multiples of 1, 3, 5, 7, etc.



Rocket speeding up using the Odd Rule.

For deceleration, the multiples start at a higher odd number and decrease, for example 7, 5, 3, 1.

Sled coming to a stop using the Odd Rule.

The Odd Rule is a multiplying system based on the smallest distance travelled between two frames in the sequence. For a slow-out, this is the distance between the first two frames: for a slow-in, it’s the distance between the last two frames. This distance, the base distance, is used in all Odd Rule calculations.

Odd rule multipliers

The Odd Rule in its simplest form, as described above, is just one way to use it. For example, one can instead calculate the distance from the first frame to the current frame and use these distances to place the object on specific frames.

If you add up all the consecutive frame multipliers up to a particular frame, you get the multiple for the entire distance. For example, on frame 4, the consecutive multiples thus far are 1, 3, and 5. If you add up these numbers you get 9, which is the multiplier for the entire distance up to frame 4.

Frame #	**Multiply by base distance to get distance between:**	
	Consecutive frames	**First frame and this frame**
1	NA	0
2	**1**	1
3	**3**	4
4	**5**	**9**
5	7	16
6	9	25
7	11	36
8	13	49

Calculating the distance for a large number of frames and a chart like this isn't practical, one can figure out the oddnumber multiplier for consecutive frames with this formula:

Odd number multiplier for consecutive frames = ((frame # – 1) * 2) – 1

In the charts above, note that the distances in the last column are squared numbers: $2^2 = 4$, $3^2 = 9$, $4^2 = 16$, and so on. One of the benefits of the Odd Rule is one can calculate the total distance travelled from the start point to the current frame with the following formula:

Multiplier for distance from first frame to current frame = (current frame # – 1)^2

When setting the keys, one can use either the consecutive key multipliers or total distance multipliers but need to choose the one that's easiest to use for the animated sequence.

Odd-rule Scenarios

There are a few different scenarios for calculating the distance an object travels between keys in a slow-in or slow-out.

i) Base distance known, speeding up

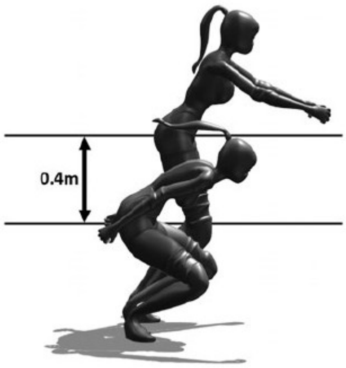
If the object is speeding up, the first frame distance is the base distance. If one knows the base distance, figuring out the distance the object travels at each frame is pretty straightforward. Just multiply the base distance by 3, 5, 7, etc. to get the distances between consecutive frames, or use squares to multiply the base distance to get the total distance travelled on each frame.

ii) Base distance known, slowing down

Suppose one wants an object to slow down, and one knows the distance between the last two frames before it stops. For slow ins, the base distance is the distance between the last two frames. The solution is to work backward, as if the object were speeding up in the opposite direction. Working backward, multiply the base distance by 3, 5, 7, etc. to get the distances between each previous frame in the sequence.

iii)Total distance and number of frames known, speeding up

If one need to know the total distance and the total number of frames, can find the base distance with this formula:



$$\text{Base distance} = \text{Total distance}/ (\text{Last frame number} - 1)^2$$

Suppose there is a jump push (take off) with constant acceleration over 5 frames, and the total distance travelled is 0.4m. Using the above formula we find the base distance.

$$\text{Base distance} = 0.4\text{m} / (5 - 1)^2 = 0.4\text{m} / 16 = 0.025\text{m}$$

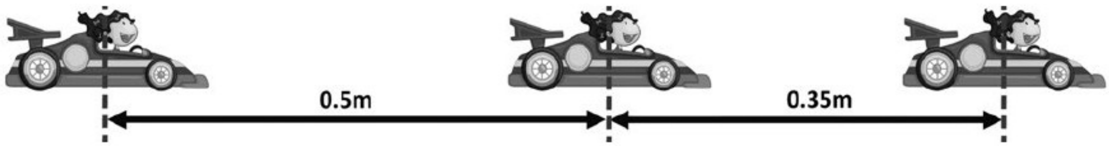
Using the base distance one can calculate the distances between each frame.

Frame #	Consecutive frame multiplier	Distance from previous frame
1	NA	0
2	1	1×0.025 = 0.025 m
3	3	3×0.025 = 0.075 m
4	5	5×0.025 = 0.125 m
5	7	7×0.025 = 0.175 m

If the above distances are added, one can find that they add up to exactly 0.4 m

iv) First key distance known, slowing down

Suppose one has a moving object and wants to slow down and set the first frame of the slow-in to give an idea of the pacing for the sequence. In this case, one can consider that the distance the object moved between the last two frames before the slow-in is part of the calculation — the distance between them becomes the first frame distance, and the first slow-in frame becomes the second frame in the sequence.



One feature of the Odd Rule is that the base distance is always half the difference between any two adjacent distances.To find the base distance, one can simply calculate:

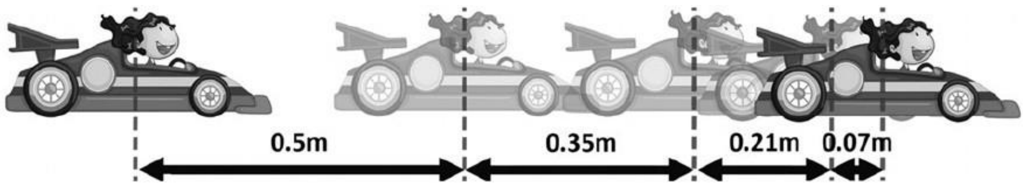
$$(0.5m - 0.35m)/2 = 0.07m$$

To figure out how many frames are in the slow-in, divide the first distance by the base distance to find out which oddnumber it corresponds to.

$$0.5/0.07 = 7$$

This means the first distance corresponds to 7 in the 7, 5, 3, 1 sequence, making the sequence four frames long. Now one can work back the other way, multiplying the base distance by odd numbers to get the distances for the rest of the slow- in frames.

Frame #	Consecutive frame multiplier	Distance from previous frame
1	7	7×0.07 = 0.5 m
2	5	5×0.07 = 0.35 m
3	3	3×0.07 = 0.21 m
4	1	1×0.07 = 0.07 m



Motion graphs

A motion graph plots an object's position against time. For an animation software, understanding, and using motion graphs is a key skill in animating anything beyond the simplest of motions. If one is drawing the animation, drawing motion graphs before animating can help to visualize the motion.

On a motion graph, the time goes from left to right across the bottom of the graph, while the object's position is plotted vertically against the time. Each axis in 3D space (X, Y, Z) has its own line showing the object's position along that axis. At the very least, one will need to understand the types of lines in a motion graph and what they represent in terms of visible motion. one can also look at motion graphs to get a better understanding of any difficulties with the timing or action.

Character animation

Character animation is a specialised component of the animation process in which animators bring still character designs to life through expressive movements and relatable personalities, providing viewers with memorable viewing experiences.

Character animation is a type of animation that uses movement, speech and tone to bring a character to life. Animators can shape characters to take on a desired personality, experience specific emotions, or embark on a physical or mental journey.
Let us discuss the Jumping and Walking examples.

Jumping

A jump is an action where the character's entire body is in the air, and both the character's feet leave the ground at roughly the same time. A jump action includes a take-off, free movement through the air, and a landing.

Parts of Jump

A jump can be divided into several distinct parts:

- **Crouch –** A squatting pose taken as preparation for jumping.
- **Take-off** – Character pushes up fast and straightens legs with feet still on the ground. The distance from the character's centre of gravity (CG) in the crouch to the CG when the character's feet are just about to leave the ground is called the push height. The amount of time (or number of frames) needed for the push is called the push time.
- **In the air** – Both the character's feet are off the ground, and the character's centre of gravity (CG) moves in a parabolic arc as any free-falling body would. First it reaches an apex, and then falls back to the ground at the same rate at which it rose. The height to which the character jumps, called the jump height, is measured from the CG at take-off to the CG at the apex of the jump. The amount of time the character is in the air from take-off to apex is called the jump time. If the take-off pose and the landing pose are similar, then the jump height and jump time are about the same going up as they are going down.
- **Landing** – Character touches the ground and bends knees to return to a crouch. The

distance from the character's CG when her feet hit to the ground to the point where the character stops crouching is called the stop height. The stop height is not always exactly the same as the push height.

Calculating Jump actions

When working out the timing for a jump, one will need to first decide on:

- Jump height or jump time
- Push height
- Stop height
- Horizontal distance the character will travel during the jump

From these factors, one can calculate the timing for the jump sequence.

Calculating jump timing

When planning the jump animation, the most likely scenario is that one must know the jump height, expressed in the units using for the animation (e.g., inch or cm).

Placement and timing for frames while the character is in the air follow the same rules as any object thrown into the air against gravity. Using the tables from the gravity concept (or an online calculator), one can figure out the jump time for each frame. Look up the amount of time it takes an object to fall that distance due to gravity and express the jump time in frames based on the frames per second.

Example:

Jump height = 1.2m

Jump time for 1.2m = 0. 5 seconds

Jump time at 30fps = 0. 5 × 30 = 15 frames

Jump magnification

When calculating the remainder of the timing for the entire jump action, you can use a factor called jump magnification(JM). It can be used to calculate the push timing and stop timing.

The jump magnification is the ratio of the jump height to the push height.

$$Jump\ magnification\ (JM) = \frac{Jump\ height}{Push\ height}$$

If jump height and push height are known then JM can be easily calculated, and it can be used to calculate other aspects of the jump.

Example:

Jump Height = 1m

Push Height = 0.33m

$$Jump\ magnification\ (JM) = \frac{Jump\ height}{Push\ height} = \frac{1}{0.33} = 3.03$$

Jump magnification and acceleration

Jump Magnification is in fact an exact ratio that tells how much the character has to accelerate against gravity to get into the air. The JM, besides being the ratio of jump-to-push vertical height and time, is also the ratio of push-to-jump vertical acceleration. Note that this is opposite the other ratios: while a longer jump time means a shorter push time, a higher jump acceleration means a much, much higher push acceleration. Knowing about this can help to make more informed decisions about push timing.

To see how this works, let's look at the formula for JM and relate it to acceleration:

$$(JM) = \frac{Jump\ time}{Push\ time} = \frac{Jump\ height}{Push\ height} = \frac{Push\ acceleration}{Jump\ acceleration}$$

The magnitude of jump acceleration is always equal to gravitational acceleration, with deceleration as the characterrises and acceleration as it falls.

$$JM = \frac{Push\ acceleration}{Jump\ acceleration} = \frac{Push\ acceleration}{Gravitational\ acceleration}$$

If the landing speed is the same as the velocity of any falling object, so one can easily calculate from the free fall time. Since acceleration due to gravity is 10 m/sec^2, this means that after one second a falling object is traveling at 10 m/sec, after two seconds at 20 m/sec, after three seconds at 30 m/sec, and so on. Since take-off speed is the same as landing speed, one needs to get up to that same speed when taking off for a jump. If a person's landing speed is 10 m/sec, then during his take-off he need to get up to a speed of 10 m/sec in that little bit of push time.

The general formula for calculating the velocity of an accelerating object is:

$$velocity = acceleration\ \times time$$

$$v = a \times t$$

Let's relate this back to jump. If the landing velocity is same as push velocity, then we have

$$v = Jump\ acceleration\ \times Jump\ time$$

Thus,

$$Jump\ acceleration\ \times Jump\ time = Push\ acceleration \times Push\ time$$

Moving things around with a bit of algebra, we arrive at this equation:

$$\frac{Jump\ time}{Push\ time} = \frac{Push\ acceleration}{Jump\ acceleration\ (Gravity)}$$

Hence it is JM and is equal to the ratio of the push acceleration to gravity. If jump time increases the push acceleration goes up and if push time decreases, the push acceleration goes up.

Distance (or in this case, jump or push height) is also related to velocity by the relation:

$$Distance = average\ velocity \times time$$

$$Or\ \ d = v \times t$$

Therefore average velocity is given by,

$$v = \frac{d}{t}$$

Since the average velocity is the same for both the push and jump, we can say that d/t is the same for both jump and push,

$$\frac{Jump\ height}{Jump\ time} = \frac{Push\ height}{Push\ time}$$

Push time

We know that the JM is the ratio of jump time to the push time i.e.,

$$JM = \frac{Jump\ time}{Push\ time}$$

So we can express the push time as

$$Push\ time = \frac{Jump\ time}{JM}$$

Example:

JM = 3

Jump Time: 15 frames

Push Time = 15 / 3 = 5 frames

Landing

The forces on landing are similar to take-off. If the landing has faster timing, the forces will be larger than for a longer timing.

Stop time

The stop height is often a bit larger than the push height, but the timing of the push and stop are the same in the sense that the centre of gravity moves the same distance per frame in the push and stop. If the stop height is larger than the push height, you'll just need more frames for the stop than the push.

$$\frac{Push\ height}{Push\ frames} = \frac{Stop\ height}{Stop\ frames}$$

This can also be expressed as,

$$\frac{Push\ height}{Push\ time} = \frac{Stop\ distance}{Stop\ time}$$

Thus, stop time can be written as

$$Stop\ time = \frac{Push\ time \times Stop\ distance}{Push\ height}$$

Example:

Push Time: 5 frames

Push Height: 0.4m

Stop Height: 0.5m

Stop Time = (5 * 0.5) / 0.4 = 6 frames

Walking

In animation, a **walk cycle** is a series of frames or illustrations drawn in sequence that loop to create an animation of a walking character. The walk cycle is looped over and over, thus having to avoid animating each step again.

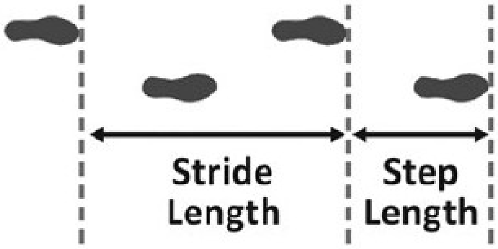
Walk cycle is the best way to visualize the movement of a character. But, how the character walks can define many things other than simply moving it from one point to the other. Simple things such as body language, walking pace, and pose can refine the character's movement and make it not walk like a glitchy video-game character. The walk cycle can be applied to any character and creating a series of walk cycles for different scenes can bring a life to the animation and make it even more realistic.

Poses

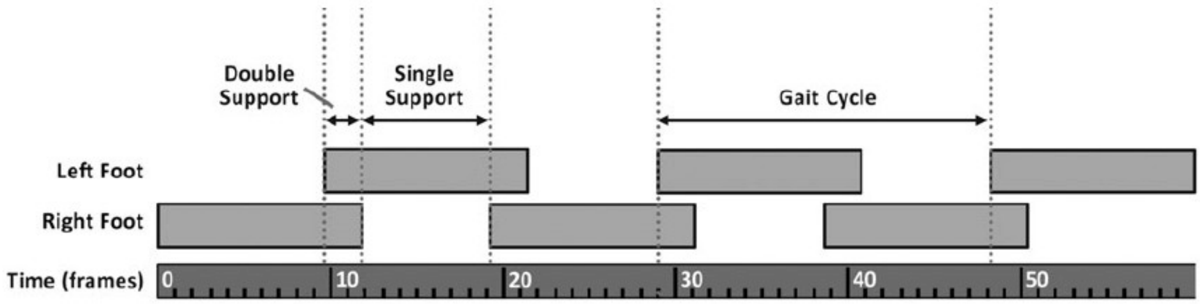
Walking consists of a series of poses. The four basic poses for a single step are *passing*, *step*, *contact*, and *lift*. In the passing pose, the free foot is passing by the opposite leg, and the body is at its most upright. In the contact pose, the free foot has come forward just enough to contact the ground. Passing and contact are the two that are the most important to get right, as these poses include the most dynamic shifts for centre of gravity, limbs, and secondary motion.

Strides and Steps

A step is one step with one foot. A stride is two steps, one with each foot. Stride length is the distance the character travels in a stride, measured from the same part of the foot. Step and stride length indicate lengthwise spacing for the feet during a walk.



Gait is the timing of the motion for each foot, including how long each foot is on the ground or in the air. During a walk, the number of feet the character has on the ground changes from one foot (single support) to two feet (double support) and then back to one foot. You can plot the time each foot is on the ground to see the single and double support times over time. A normal walking gait ranges from 1/3 to 2/3 of a second per step, with 1/2 second being average.



Walk timing

Walking is sometimes called "controlled falling." Right after you move past the passing position, your body's centre of gravity is no longer over your base of support, and you begin to tip. Your passing leg moves forward to stop the fall, creating your next step. Then the cycle begins again.

The horizontal timing for between the four walk poses is not uniform. The centre of gravity slows in going from the contact to passing position, then slows out from passing to contact. The centre of gravity also rises and falls, rising to the highest position during passing and the lowest during contact. The head is in the highest position during passing.

To walk faster, both stride length and rate has to be increased, and decrease the time of double support. A fast walk has a stride rate of about 4 feet/sec. At 6-7 feet per second, the movement transitions into a run.

Statistical physics for computing

Statistical physics is a branch of physics that evolved from a foundation of statistical mechanics, which uses methods of probability theory and statistics, and particularly the mathematical tools for dealing with large populations and approximations, in solving physical problems. Its applications include many problems in the fields of physics, biology, chemistry, and neuroscience etc.

Descriptive and Inferential statistics

Descriptive Statistics describes the characteristics of a data set. It is a simple technique to describe, show and summarize data in a meaningful way. It helps analysts to understand the data better. It represent the available data sample and do not include theories, inferences, probabilities, or conclusions.

In Inferential Statistics, the focus is on making predictions about a large group of data based on a representative sample of the population. A random sample of data is considered from a population to describe and make inferences about the population. This technique allows one to work with a small sample rather than the whole population. Since inferential statistics make predictions rather than stating facts, the results are often in the form of probability.

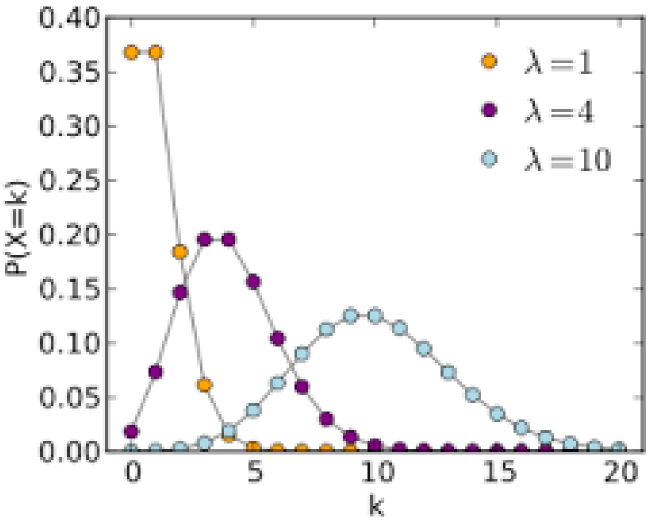
Difference between Descriptive and Inferential statistics

Descriptive statistics	Inferential statistics
Descriptive Statistics gives information about raw data regarding its description or features.	Inferential statistics draw inferences about the population by using data extracted from the population.
It is used to describe a situation.	It is used to explain the probability of occurrence of an event.
It helps to organize, analyse and present data in a meaningful manner.	It helps to compare data, make hypotheses and predictions.
Descriptive statistics explains already known data related to a particular sample or population of a small size.	Inferential statistics aims to draw inferences or conclusions about a whole population.
Descriptive statistics can be represented with charts, graphs, and tables.	Inferential statistics can be represented by probability methods.

Poisson’s distribution

A Poisson distribution is a discrete probability distribution, meaning that it gives the probability of a discrete (i.e., countable) outcome. For Poisson distributions, the discrete outcome is the number of times an event occurs, represented by **k.** It was named after French mathematician Siméon Denis Poisson.

The Poisson distribution has only one parameter, **λ** (lambda), which is the mean number of events. The graph below shows examples of Poisson distributions with different values of λ.



One can use a Poisson distribution to predict or explain the number of events occurring within a given interval of time or space. "Events" could be anything from disease cases to customer purchases to meteor strikes. The interval can be any specific amount of time or space, such as 10 days or 5 square inches.

In general, Poisson distributions are often appropriate for **count data**. Count data is composed of observations that are non-negative integers (i.e., numbers that are used for counting, such as 0, 1, 2, 3, 4, and so on). Poisson distribution could be used to explain or predict many things for example: Text messages per hour, Machine malfunctions per year, Website visitors per month etc.

Probability mass function

A Poisson distribution can be represented visually as a graph of the probability mass function. A probability mass function is the probability distribution of a discrete random variable and provides the possible values and their associated probabilities.

A discrete random variable *X* is said to have a Poisson distribution, with parameter $\lambda > 0$ if it has a probability mass function given by,

$$f(k;\lambda) = \Pr(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

Where,

k – Is the number of times an event occurs

X – Is a random variable following a Poisson distribution

Pr (X=k) – Is the probability that an event will occur k times

e – Is the Euler's number (e = 2.71828)

λ – Is the average number of times an event occurs

! – Is the factorial function

The positive real number λ is equal to the expected value of X and also to its Variance.

The Poisson distribution may be useful to model events such as:

- The number of meteorites greater than 1-meter diameter that strike Earth in a year.
- The number of laser photons hitting a detector in a particular time interval.
- The number of students achieving a low and high mark in an exam.

Examples of probability for Poisson distributions

1. On a particular river, overflow floods occur once every 100 years on average. Calculate the probability of k = 0, 1, 2, 3, 4, 5, or 6 overflow floods in a 100 year interval, assuming the Poisson model is appropriate. Because the average event rate is one overflow flood per 100 years, λ = 1

Solution: We have,

$$f(k;\lambda) = \Pr(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

$$P(k\ overflow\ floods\ in\ 100\ years) = \frac{\lambda^k e^{-\lambda}}{k!} = \frac{\lambda^1 e^{-1}}{k!}$$

$$P(k = 0\ overflow\ floods\ in\ 100\ years) = \frac{1^0 e^{-1}}{0!} = \frac{e^{-1}}{1} = 0.3678 \approx 0.368$$

$$P(k = 1\ overflow\ floods\ in\ 100\ years) = \frac{1^1 e^{-1}}{1!} = \frac{e^{-1}}{1} = 0.3678 \approx 0.368$$

$$P(k = 2\ overflow\ floods\ in\ 100\ years) = \frac{1^2 e^{-1}}{2!} = \frac{e^{-1}}{2} = 0.1839 \approx 0.184$$

$$P(k = 3\ overflow\ floods\ in\ 100\ years) = \frac{1^3 e^{-1}}{3!} = \frac{e^{-1}}{6} = 0.061$$

$$P(k = 4\ overflow\ floods\ in\ 100\ years) = \frac{1^4 e^{-1}}{4!} = \frac{e^{-1}}{24} = 0.0153$$

$$P(k = 5\ overflow\ floods\ in\ 100\ years) = \frac{1^5 e^{-1}}{5!} = \frac{e^{-1}}{120} = 0.003$$

$$P(k = 6\ overflow\ floods\ in\ 100\ years) = \frac{1^6 e^{-1}}{6!} = \frac{e^{-1}}{120} = 0.00051$$

Modelling the probability for Proton decay

The experimental search for proton decay was undertaken because of the implications of the Grand unification theories. The lower bound for the lifetime is now projected to be on the order of $\tau = 10^{33}$ years. The probability for observing a proton decay can be estimated from the nature of particle decay and the application of Poisson Statistics. The number of protons **N** can be modelled by the decay equation,

$$N = N_0 e^{-\lambda t}$$

Where $\lambda = 1/\tau = 10^{-33}$ per year is the probability that any given proton will decay in a year. Since the decay constant λ is so small, the exponential can be represented by the first two terms of the exponential series.

$$e^{-\lambda t} = 1 - \lambda t$$

$$\text{Thus,} \qquad N \approx (1 - \lambda t)$$

For a small sample, the observation of a proton decay is infinitesimal, but suppose we consider the volume of protons represented by the Super Kameokande neutrino detector in Japan. The number of protons in the detector volume is reported by Ed Kearns of Boston University to be 7.5×10^{33} protons. For one year of observation, the number of expected proton decays is then,

$$N - N_0 = N_0 \lambda t = (7.5 \times 10^{33}\ protons)\left(\frac{10^{-33}}{year}\right)$$

$$\therefore (1\ year) = 7.5$$

About 40% of the area around the detector tank is covered by photo-detector tubes, and if we take that to be the nominal efficiency of detection, we expect about three observations of proton decay events per year based on a 10^{33} year lifetime.

So far, no convincing proton decay events have been seen. Poisson statistics provides a convenient means for assessing the implications of the absence of these observations. If we presume that $\lambda = 3$ observed decays per year is the mean, then the Poisson distribution function tells us that the probability for zero observations of a decay is

$$P(k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

$$P(k) = \frac{3^0 e^{-3}}{0!} = 0.05$$

This low probability for a null result suggests that the proposed lifetime of 1033 years is too short. While this is not a realistic assessment of the probability of observations because there are a number of possible pathways for decay, it serves to illustrate in principle how even a non-observation can be used to refine a proposed lifetime.

Normal distribution and Bell curves

The normal distribution, also known as the Gaussian distribution, is the most important probability distribution in statistics for independent, random variables. The normal distribution is often called the bell curve because the graph of its probability density looks like a bell. It is also known as called Gaussian distribution, after the German mathematician Carl Gauss who first described it.

The normal distribution is a continuous probability distribution that is symmetrical on both sides of the mean, so the right side of the centre is a mirror image of the left side. In graphical form, the normal distribution appears as a bell curve.

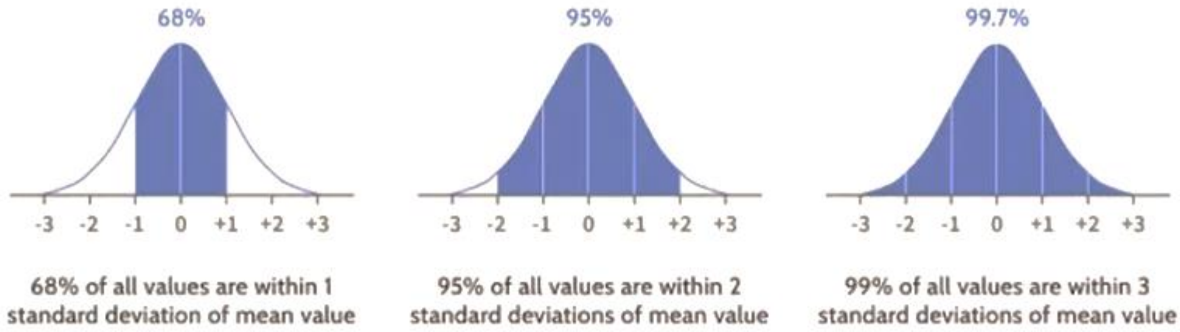
The normal distribution has several key features and properties that define it. First, its mean (average), median (midpoint), and mode (most frequent observation) are all equal to one another. Moreover, these values all represent the peak, or highest point, of the distribution. The distribution then falls symmetrically around the mean, the width of which is defined by the standard deviation.

As with any probability distribution, the normal distribution describes how the values of a variable are distributed. It is the most important probability distribution in statistics because it accurately describes the distribution of values for many natural phenomena. Characteristics that are the sum of many independent processes frequently follow normal distributions. For example, heights, blood pressure, measurement error, and IQ scores follow the normal distribution.

The empirical rule

For all normal distributions, 68.2% of the observations will appear within plus or minus one standard deviation of the mean; 95.4% of the observations will fall within +/- two standard deviations; and 99.7% within +/- three standard deviations. This fact is sometimes referred to as the "empirical rule," a heuristic that describes where most of the data in a normal distribution will appear.

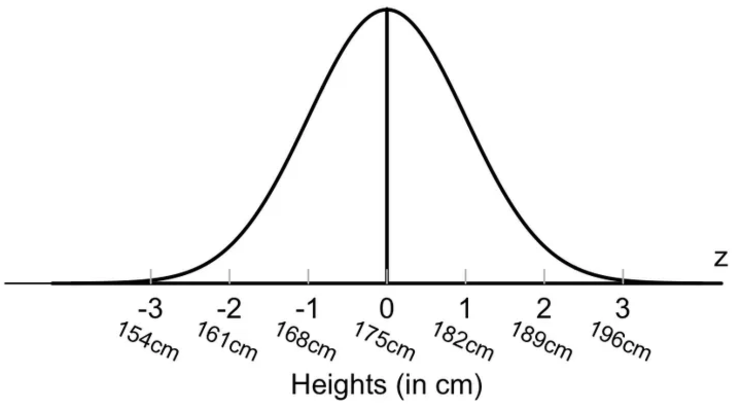
This means that data falling outside of three standard deviations ("3-sigma") would signify rare occurrences.



Example of a Normal distribution

Many naturally occurring phenomena appear to be normally distributed. Take, for example, the distribution of the heights of human beings. The average height is found to be roughly 175 cm (5' 9"), counting both males and females.

As the chart below shows, most people conform to that average. Meanwhile, taller and shorter people exist, but with decreasing frequency in the population. According to the empirical rule, 99.7% of all people will fall with +/- three standard deviations of the mean, or between 154 cm (5' 0") and 196 cm (6' 5"). Those taller and shorter than this would be quite rare (just 0.15% of the population each).



Monte Carlo method

Monte Carlo methods, or Monte **Carlo experiments**, are a broad class of computational algorithms that rely on repeated random sampling to obtain numerical results. The underlying concept is to use randomness to solve problems that might be deterministic in principle. They are often used in physical and mathematical problems and are most useful when it is difficult or impossible to use other approaches. Monte Carlo methods are mainly used in three problem classes: optimization, numerical integration, and generating draws from a probability distribution.

Monte Carlo method is a technique that can be used to solve a mathematical or statistical problem, and a Monte Carlo simulation uses repeated sampling to obtain the statistical properties of some phenomenon or behaviour.

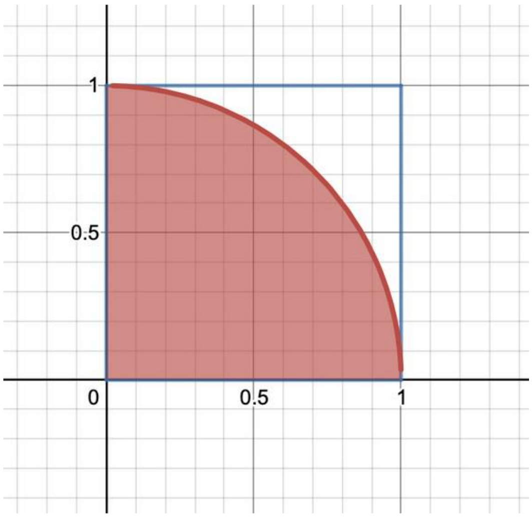
For example: Pouring out a box of coins on a table, and then computing the ratio of coins that land heads versus tails is a Monte Carlo method of determining the behaviour of repeated coin tosses, but it is not a simulation.

Monte Carlo methods are very important in computational physics, physical chemistry, and related applied fields, and have diverse applications from complicated quantum chromodynamics calculations to designing heat shields and aerodynamic forms as well as in modelling radiation transport for radiation dosimetry calculations. Quantum Monte Carlo methods solve the many-body problem for quantum systems. In experimental particle physics, Monte Carlo methods are used for designing detectors, understanding their behaviour and comparing experimental data to theory.

Monte Carlo methods vary, but tend to follow a particular pattern:

- Define a domain of possible inputs
- Generate inputs randomly from a probability distribution over the domain
- Perform a deterministic computation on the inputs
- Aggregate the results

For example, consider a quadrant (circular sector) inscribed in a unit square. Given that the ratio of their areas is $\pi/4$ the value of π can be approximated using a Monte Carlo method:



- Draw a square, then inscribe a quadrant within it.
- Uniformly scatter a given number of points over the square.
- Count the number of points inside the quadrant, i.e. having a distance from the origin of less than 1.
- The ratio of the inside-count and the total-sample-count is an estimate of the ratio of the two areas, $\pi/4$. Multiply the result by 4 to estimate π.

In this procedure the domain of inputs is the square that circumscribes the quadrant. We generate random inputs by scattering grains over the square then perform a computation on each input (test whether it falls within the quadrant). Aggregating the results yields our final result, the approximation of π.

There are two important considerations:

- If the points are not uniformly distributed, then the approximation will be poor.
- The approximation is generally poor if only a few points are randomly placed in the whole square. On average, the approximation improves as more points are placed.

Uses of Monte Carlo methods require large amounts of random numbers, and their use benefitted greatly from pseudorandom number generators, which were far quicker to use than the tables of random numbers that had been previously used for statistical sampling.

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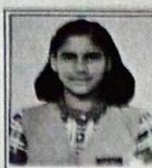

RINKU DUHAN
08/10/20

यह प्रमाणित किया जाता है कि
This is to certify that **RINKU DUHAN**

अनुक्रमांक
Roll No. **18606952**

माता का नाम
Mother's Name **SUSHILA DEVI**

पिता/संरक्षक का नाम
Father's / Guardian's Name **JAIDEV SINGH**

विद्यालय
School **49011 KENDRIYA VIDYALAYA AFS YELAHANKA BANGALORE KK**

की शैक्षणिक उपलब्धियां निम्नानुसार हैं has achieved Scholastic Achievements as under :

विषय कोड SUB. CODE	विषय SUBJECT	प्राप्तांक MARKS OBTAINED				स्थितीय ग्रेड POSITIONAL GRADE
		लिखित THEORY	प्रा./PR. आं.मू./IA	योग TOTAL	योग (शब्दों में) TOTAL (IN WORDS)	
301	ENGLISH CORE	069	020	089	EIGHTY NINE	A2
041	MATHEMATICS	048	020	068	SIXTY EIGHT	B2
042	PHYSICS	049	029	078	SEVENTY EIGHT	B1
043	CHEMISTRY	048	030	078	SEVENTY EIGHT	B1
044	BIOLOGY	051	030	081	EIGHTY ONE	B1
500	WORK EXPERIENCE					A1
502	HEALTH & PHYSICAL EDUCATION					A1
503	GENERAL STUDIES					A1

परिणाम Result **PASS**

दिल्ली Delhi
दिनांक Dated : 22-07-2022

परीक्षा नियंत्रक
Controller of Examinations

सह-शैक्षिक उपलब्धियां : सह-शैक्षिक एवम् अनुशासन क्षेत्र में ग्रेडिंग विद्यालय द्वारा अपने स्तर पर बोर्ड द्वारा जारी प्रारूपनुसार प्रदान की जाती है।
Co-Scholastic achievements : Grading for Co-Scholastic and Discipline area is being issued by the school as per format prescribed by the Board.
परिवर्तित विषय कोड में, परीक्षा परिणाम 100 अंकों के अंतर्गत दिया गया है।
In the changed subject code, the test result is given under 100 marks.