Assignment (1)

CS754 - AIP

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Q.3 $\mu = \sqrt{n} \max_{i,j\{0 \ldots n-1\}} |\phi_i^{t} \psi_j|$

-> all rows of ϕ are unit normalized

-> ϕ^{i^t} is a unit vector

-> ψ_j is orthonormal matrix

-> ψ_j is a unit vector.

$\therefore$ Maximum of dot product of 2 unit vectors - by Cauchy Schwartz

$|\phi^{i^t} \psi_j| < \|\phi^{i^t}\| \|\psi_j\| \leq 1$

$\therefore \max(\mu(\phi, \psi)) = \sqrt{n}$

(ii) ~~$g = \sum_{k=1}^{n} \alpha_k \psi_k$ unit vector~~

~~$\tilde{g} \rightarrow$ normalized vector g (unit)~~ $= \dfrac{\sum_{k=1}^{n} \alpha_k \psi_k}{\sqrt{\sum_k \alpha_k^2}}$

~~$\therefore \mu(g, \psi) = \sqrt{n} \max_{i\{0, n-1\}} |\tilde{g} \cdot \psi_i|$~~

21·04
20·08
20·13

(b) $g = \sum_{k=1}^{n} \alpha_k \psi_k$

$$\tilde{g} = \text{unit } g = \frac{\sum_{k=1}^{n} \alpha_k}{\sqrt{\sum_{j=1}^{n} \alpha_j^2}} \psi_k$$

$$\mu(\tilde{g}, \psi) = \sqrt{n} \max_{i \in \{0, n-1\}} \left| \frac{\sum_{k=1}^{n} \alpha_k}{\sqrt{\sum_{j=1}^{n} \alpha_j^2}} \psi_k^t \psi_i \right|$$

Since ψ is orthonormal, $\psi_k^t \psi_i = 0 \quad k \neq i$

$\psi_k^t \psi_i = 1 \quad k = i$

$\cancel{\therefore \sum_{k=1}^{n} \alpha_k \psi_k^t \psi_i =}$

$$\therefore \mu(\tilde{g}, \psi) = \sqrt{n} \max_{i \in \{0, n-1\}} \frac{|\alpha_i|}{\sqrt{\sum_{j=1}^{n} \alpha_j^2}}$$

~~mum~~ minimum when ?

$$\sum_{j=1}^{n} \alpha_j^2 \leq n \, |\max(\alpha_i)|^2$$

$$\frac{1}{n} \leq \frac{|\max \alpha_i|^2}{\sum_{j=1}^{n} \alpha_j^2}$$

$$\frac{1}{\sqrt{n}} \leq \max \frac{|\alpha_i|}{\sqrt{\sum \alpha_j^2}}$$

$\therefore$ minimal $\mu(\tilde{g}, \psi) = \sqrt{n} \times \frac{1}{\sqrt{n}} = 1$

since $\tilde{g}$ corresponds to unit normalized rows of ϕ

~~min ϕ~~ = $\boxed{\min \mu(\phi, \psi) = 1}$

Hence Proved

$\boxed{Q\cdot 5}$ As taught in the class

$$\delta_s = \max\{1-\lambda_{min},\ \lambda_{max}-1\}$$

where λ_{max} λ_{min} are the eigen values of matrix $(A_\Gamma^T)(A_\Gamma)$.

- using Greshgorin's disc theorem-
  - $A_\Gamma^T A_\Gamma$ is a square matrix
  - every eigen value will lie in disc $D(B_{ii}, r_i)$
  - Since columns of A ~~are~~ are unit normalized

$\rightarrow B_{ii} = 1$

$\rightarrow$ also $\mu = \max_{i,j\ i\neq j} \dfrac{|A_i \cdot A_j|}{\|A_i\| \|A_j\|}$

$\Rightarrow$ all the of diagonal elements in $A_\Gamma^T A_\Gamma$ will be less then $|\mu|$

$\Rightarrow$ also maximum size of Γ (support) can be "S" for a S sparse signal of

$\therefore A_\Gamma^T A_\Gamma \sim \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix}$

$$A_T^T A_T = \begin{array}{c} s \\ \begin{bmatrix} 1 & <\mu & <\mu & <\mu \\ <\mu & 1 & & \\ & & \ddots & \\ & & & 1 \end{bmatrix} \end{array}$$

$\rightarrow r_i$ = sum of off diagonal element

$\rightarrow r_i < \mu(s-1)$

$\therefore \quad 1 - r_i < \lambda < 1 + r_i$

$\therefore \quad 1 - \mu(s-1) < \lambda \leq 1 + \mu(s-1)$

$\therefore 1 - \lambda_{min} = 1 - (1 - \mu(s-1))$

$= \mu(s-1)$

$\lambda_{max} - 1 = 1 + \mu(s-1) - 1$

$= \mu(s-1)$

$$\boxed{\delta_s \leq \mu(s-1)}$$

Q6 $(1-\delta_{s_1})\|\theta\|^2 \leq \|A\theta\|^2 \leq (1+\delta_{s_1})\|\theta\|^2$ — (1)

$s_1 < s_2$

→ if θ is s_1 sparse, then it is s_2 sparse also

→ θ will follow ~~sparsest~~ eq^n (1) for δ_{s_2} also

$\Rightarrow (1-\delta_{s_2})\|\theta_{s_1}\|^2 \leq \|A\theta_{s_1}\|^2 \leq (1+\delta_{s_2})\|\theta_{s_1}\|^2$

- Consider $\delta_{s_1} > \delta_{s_2}$ — (2)

$\Rightarrow \quad -\delta_{s_1} < -\delta_{s_2}$

$\Rightarrow \quad 1-\delta_{s_1} < 1-\delta_{s_2}$

& $\quad 1+\delta_{s_1} > 1+\delta_{s_2}$

⇒ Range of values of $\|A\theta\|^2$ values of s_1 sparse θ is larger & superset ~~for~~ of those for s_2 sparse θ where $s_2 > s_1$

→ But since all s_1 sparse θ are s_2 sparse θ $(s_2 > s_1)$ the range should be a subset.

∴ Our assumption (eq^n 2) is wrong

$$\boxed{\delta_{s_1} < \delta_{s_2}}$$

→ This is intuitive also, because the reverse would then ~~as~~ mean that more populated signals (not obeying sparsity) will give better bounds for CS theorems.

$\boxed{Q1}$

(a) if basis pursuit is used

$$m > C \log\left(\frac{n}{\delta}\right) \|\theta\|_0 \, \mu^2(\Psi, \Phi)$$

C is a constant (function of RIC of $A = \Psi\Phi$)
& δ is used to factor in a small number of corner cases.

→ if its s-sparse in some other Ψ basis, minimum samples req. would decrease if the other basis is more incoherent with our sensing matrix.

→ error will also decrease as per Theorem 3, if RIC δ_S of $A_{new} = \Phi\Psi_{new}$, ~~the new A~~ is less because the constant C_0 in theorem 3 is increasing function of δ_S.

- $\delta_s < \mu(s-1)$ always.

also for exact reconstruction, $\delta_{2s} < \sqrt{2}-1$
i.e. the matrix A should follow RIP ↗.

- also δ_s is of the order $O(s \log^4 m)$ or so.

(i) for P0 problem

$$m > 2s$$

because the A matrix should have atleast 2s independent columns.

Q2

Q2

$$E_u = \sum_{t=1}^{T} C_t \cdot F_t$$

E_u acts as our b, the compressed measurements

now, let F_t be the patch of size $n \times n$.
C_t will also be of size $n \times n$.

→ In $b = Ax$

x is the vector of T F_t's which we want to reconstruct.

★ $x = \begin{bmatrix} F_1 \\ F_2 \\ \vdots \\ F_T \end{bmatrix}$ vectorized & concatenated images to be constructed.

★ $b = \begin{bmatrix} E_u \end{bmatrix}$ vectorized coded snapshot

$$A = \begin{bmatrix} \underbrace{\ddots}_{C_1} & \underbrace{\ddots}_{C_2} & \cdots & \underbrace{\ddots}_{C_T} \end{bmatrix}$$

→ elements of generated coded snapshot are arranged in the diagonal form shown above

(d) A & b are the corresponding coded pattern patches & E_u image patches respectively (corresponding to the 8×8 patch of video)

(e) the (0,2) noise was on 0-255 image scale. I worked on 0-1 image scale, so took variance as 0.008 (since 2 was 5% of variance of video of 255 scale, i took 5% of variance of video on 0-1 scale)

$$\varepsilon > \sigma\sqrt{m}$$

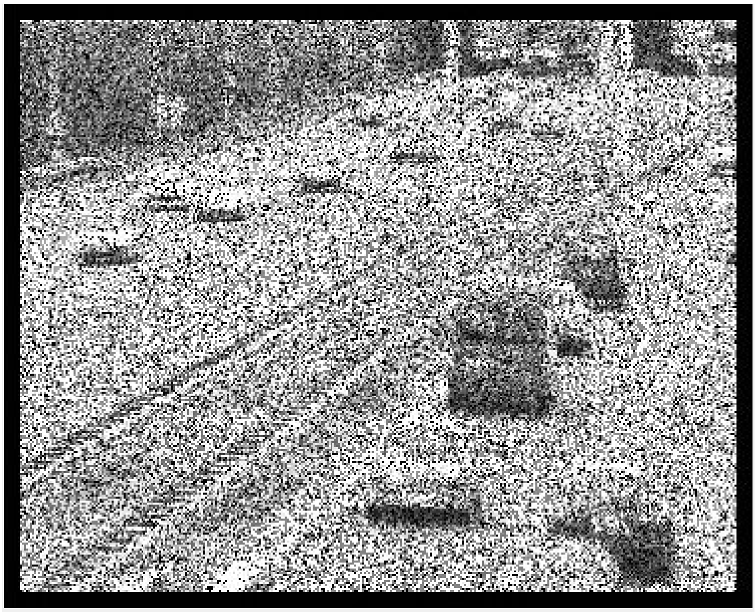
$$\varepsilon \geq 0.008 \times \sqrt{64}$$

$$\varepsilon \geq 0.064$$

I took ε as 0.2.

Question 2) Images

T = 3

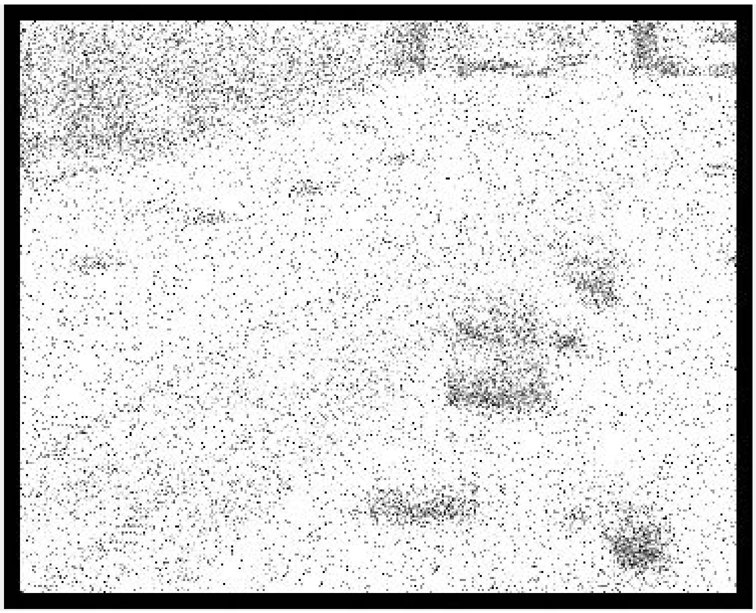


CODED SNAPSHOT

T = 5

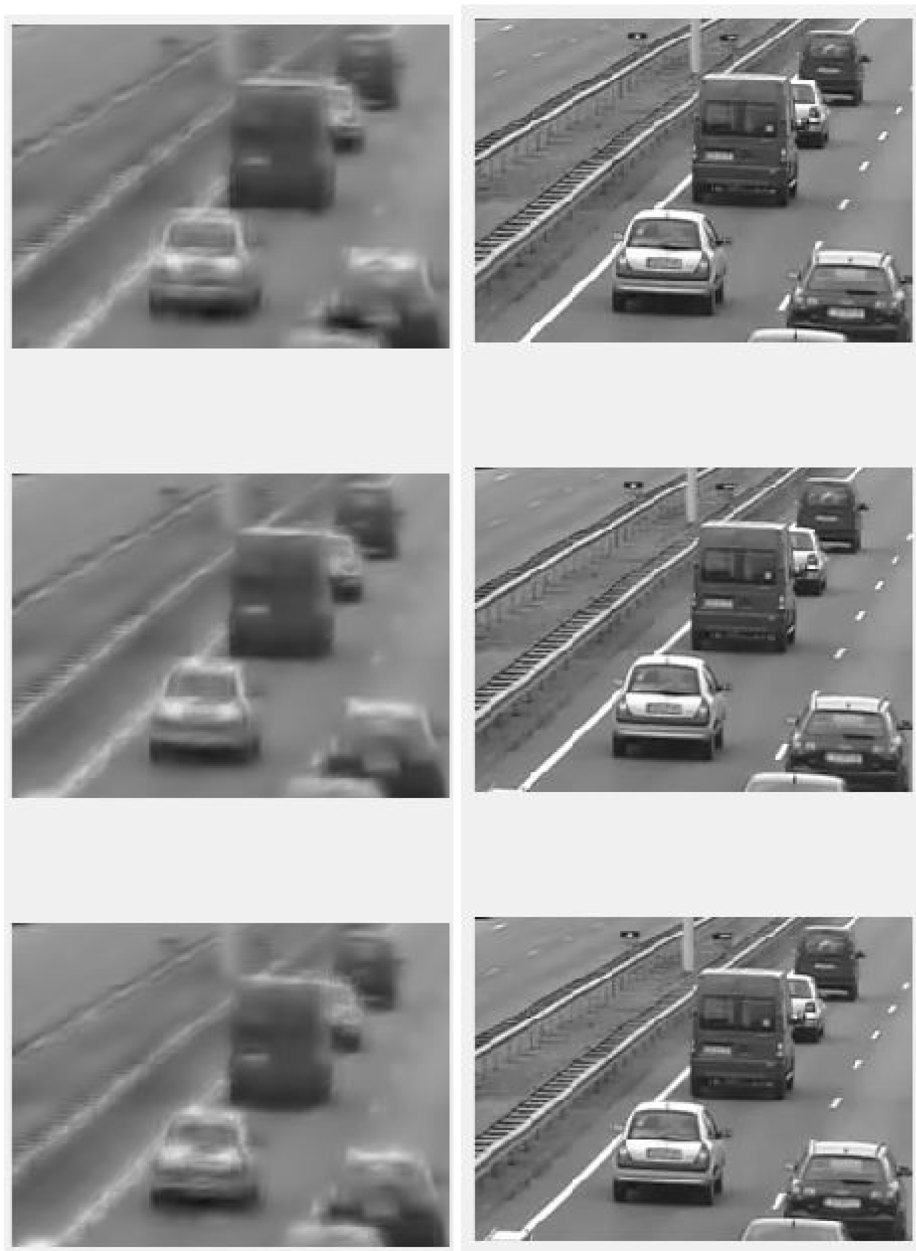
CODED SNAPSHOT

T = 7



CODED SNAPSHOT

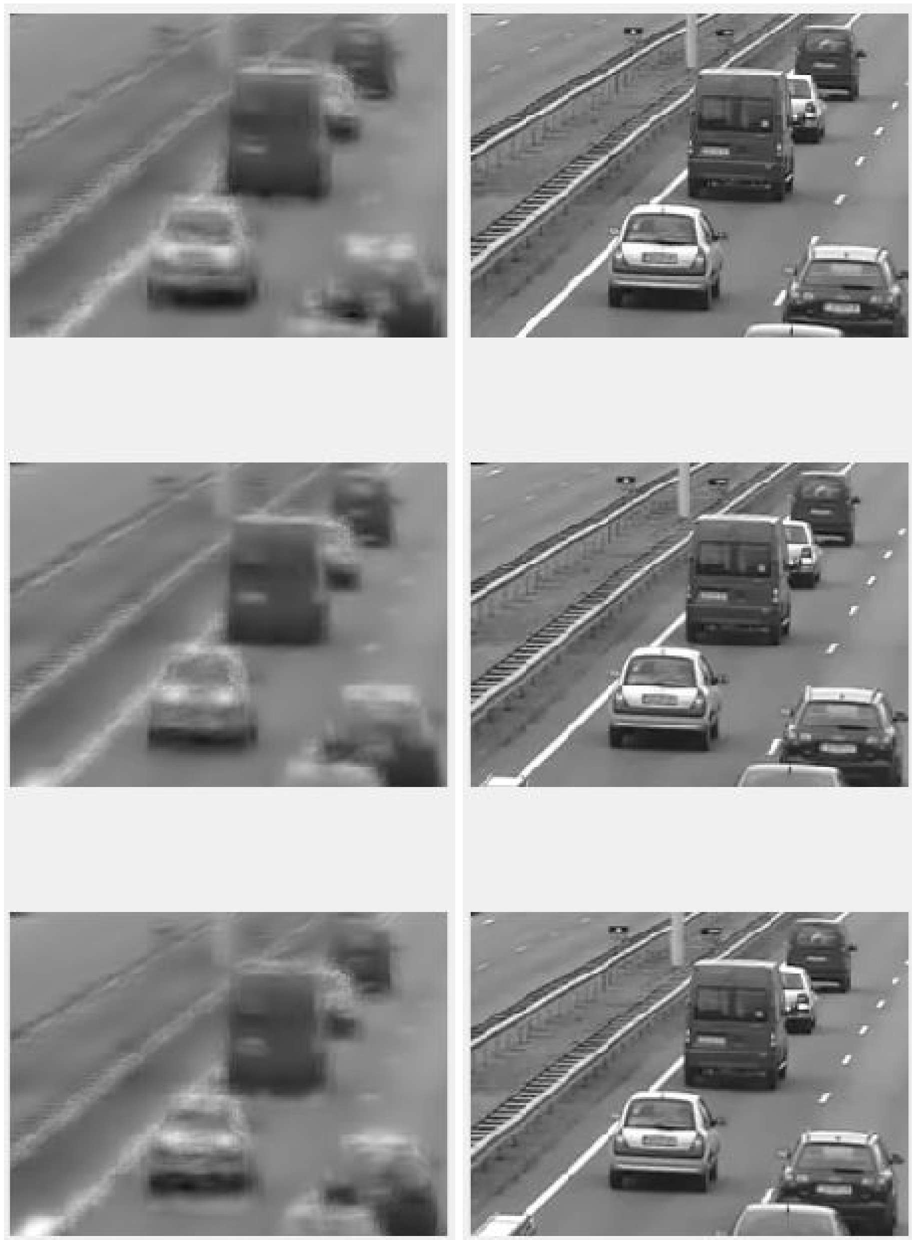
T = 3



Showing the Reconstruction Results for T=3, frames 1 2 3 respectively

Mean Squared Error Calculated using immse function matlab = 0.0078
PSNR Value = 21.05

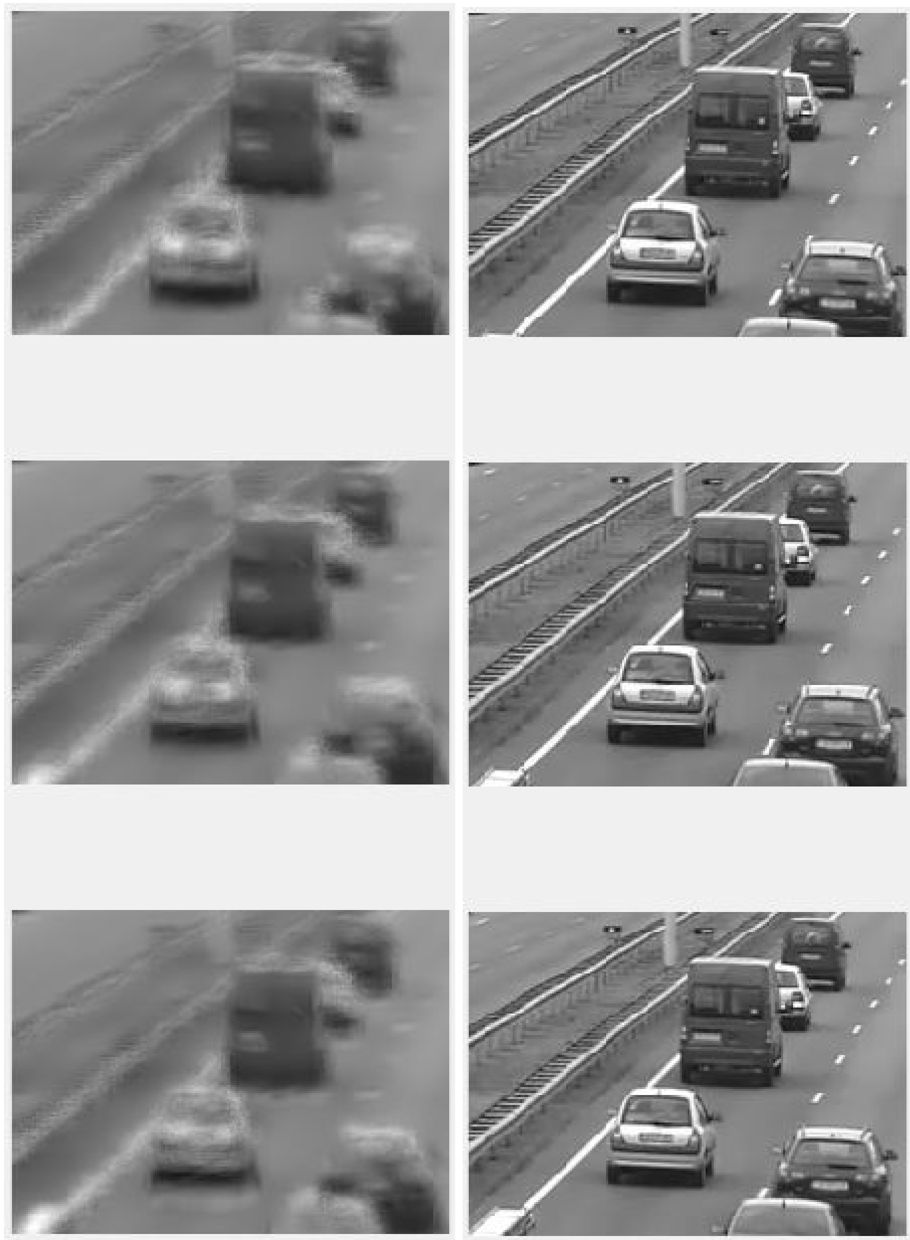
T = 5



Showing the Reconstruction Results for T=5, frames 1 3 5 respectively

Mean Squared Error Calculated using immse function matlab = 0.0087
PSNR Value = 20.613

T = 7



Showing the Reconstruction Results for T=7, frames 2 4 6 respectively

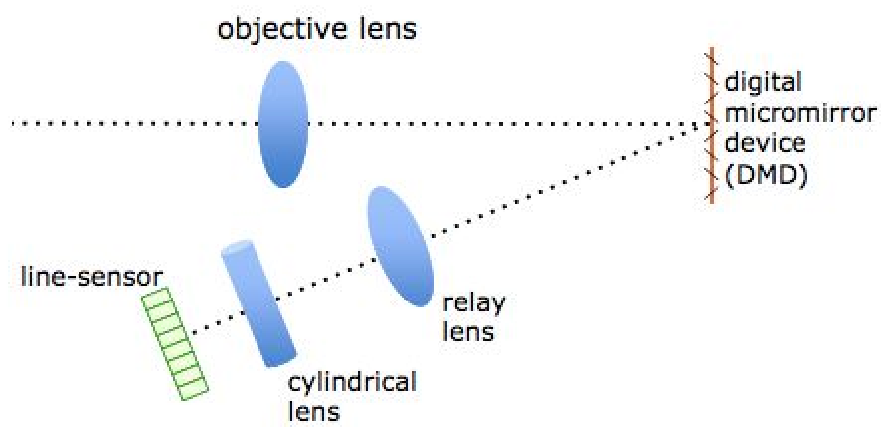
Mean Squared Error Calculated using immse function matlab = 0.0097
PSNR Value = 20.13

Qn4)

LiSens - Line Sensor Based Compressive Camera

A)
In the Rice Single Pixel Camera studied in the class, there was a DMD (Digital Micromirror Device) which was used to calculate the dot product between image and the mirror array on it. The mirror on the DMD reflect the light from the scene falling on them to the photodiode.
The measurement speed of such Single Pixel Cameras are limited to kHZ due to speeds of DMD and photodiodes.

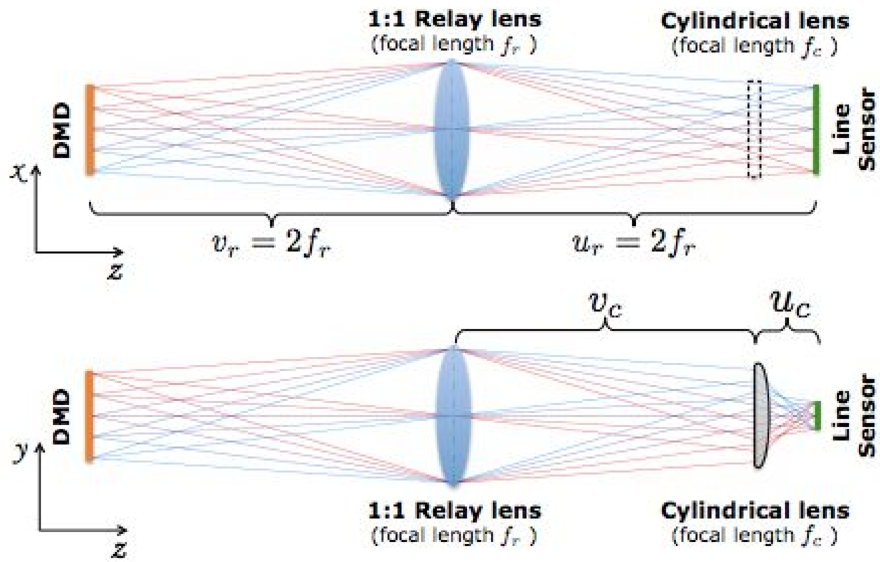
In LiSens (Line Sensor Based Compressive Camera), instead of a single pixel, a 1D array of line-sensors is used. The 2D image is mapped to this 1D sensor. Each pixel on that 1D array corresponds to the dot product of a row of scene with row of the DMD. This unique setup of a row being mapped to a single pixel is achieved with the help of a unique setup of relay cameras and cylindrical lenses.



Wd - DMD width
WL - Line sensor width
Magnification to be produced by the relay lens - WL/Wd
Placed at twice the focal length distance from both the sensor and the DMD.
A cylindrical lens is used to shrunk the aperture of the relay lens output.
After some calculations -

$$f_c = 2f_r \frac{h_L}{d_r + h_L} \frac{d_r}{d_r + h_L} \approx 2f_r \frac{h_L}{d_r}$$

Where f_c corresponds to the focal length of cylindrical lens, d_r is the diameter of the relay lens, f_r is the focal length of relay.



Major Advantages

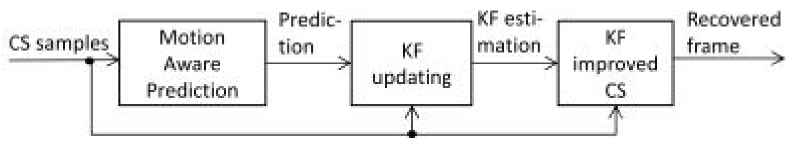
1) There could have been many designs possible for a Spatial Multipixel Cameras and not necessarily a line. But the Line Sensors enables to implement frame transfer (a technology for simultaneous exposure and readout in a sensor) in a cheap and effective way. Increased temporal frame rate.
2) You can use separate ADC for each pixel and hence enabling higher temporal resolution.

Reconstruction

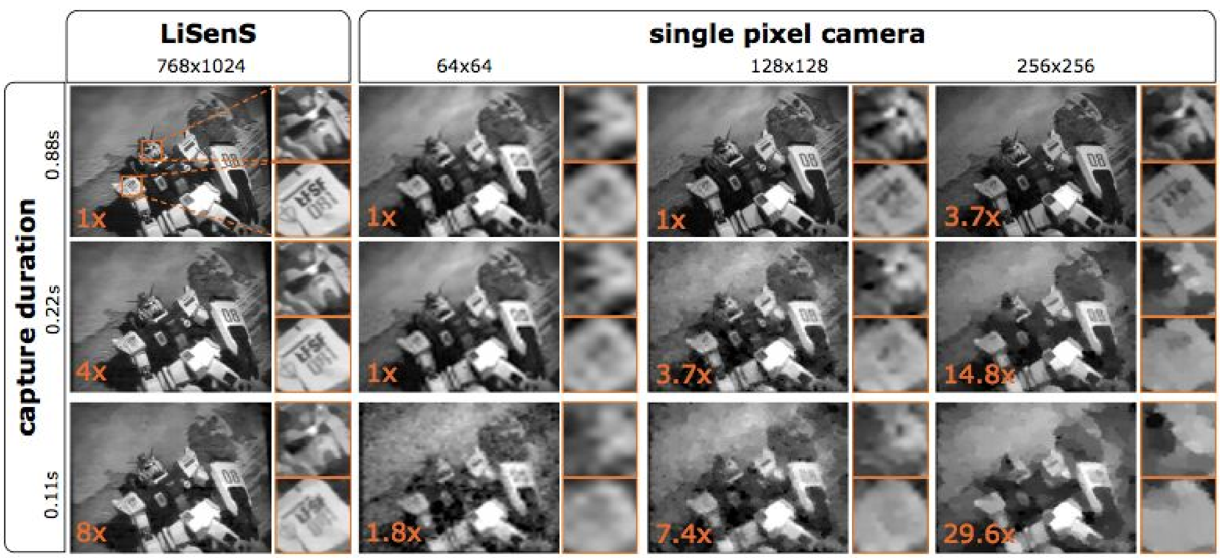
The reconstruction algorithm is the same as the one discussed in the class, i.e. minimize total variance (TV) as a constraint to the objective function of minimizing || Y − XΦ|| ≤ epsilon.

$$\min_{\mathbf{X}} TV(\mathbf{X}), \quad \text{s.t.} \quad \|\mathbf{Y} - \mathbf{X}\Phi\|_F \leq \epsilon,$$

These days sophisticated algorithms for video reconstruction are being used which involve use of Motion Estimation and Kalman Filters along with CS algorithms to reconstruct the video. Motion Estimation and kalman Filters prove very helpful in taking advantage of inter frame dependencies in the videos.



New Adaptive Reconstruction Algorithms



Sample results and comparison for LiSens