Computed Tomography Reconstruction : Compressed Sensing Approach

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Aim of CT

Task of reconstructing a 2D image (object) from its 1D projections, or a 3D image (object) from its 2D projections

Mathematical Model for CT Projection data

Radon Transform of image f(x,y) is given by

$$R(f) = g(\rho, \theta) = \sum_{x=0}^{M-1} \sum_{x=0}^{N-1} f(x, y)\delta(xcos\theta + ysin\theta - \rho) \tag{1}$$

Algorithms for Image Reconstruction from projections can be divided into two classes

- Analytical Method
- Algebraic Method (Iterative Approach)

Analytical Methods

- High Computational Speed or short computation time
- If Projection data is densely sampled $\Longrightarrow$ accurate reconstruction
- If Projection data sparsely sampled over angular range $\Longrightarrow$ reconstruction images will have aliasing artifacts such as sharp streaks
- Examples are FBP method

Algebraic Methods

- can reconstruct images with relatively less projection data
- Longer Computational Time compared to Analytical Methods
- Examples are ART method

Aim of the project

Reduce Scanning Time and Radiation Dose without reducing image quality

1st Approach

Decrease exposure time at each projection $\Longrightarrow$ Lower SNR of projection data $\Longrightarrow$ Lower Reconstructed image quality

2nd Approach

Reduce the number of projections

CS-based algorithm may be used to reconstruct images from substantially reduced projection data.

Overview of CS

Image should have a sparse representation in known transform domain

$$f = \psi x \tag{2}$$

where $f \in R^{Nx1}$ is sparse in the $\psi \in R^{NxN}$ domain and $x \in R^{Nx1}$ is its sparse representation

In our case, the sparse representation of f can be the gradient image ∇f

$$f = \psi \nabla f \tag{3}$$

Projection-data of image f are modeled by a discrete linear system

$$g = \phi f = \phi \psi x = \phi^{'} x \tag{4}$$

where $g \in R^{Mx1}$, $\phi \in R^{MxN}$ and since $M < N \implies infinitely many x that satisfy g = \phi^{'} x$

Image Reconstruction is aimed at finding the vector x in transform domain

$$x = \text{argmin } \|\tilde{x}\|_1 \quad \text{subject to} \quad \|g - \phi^{'}\tilde{x}\| < \epsilon \tag{5}$$

Reconstruction Algorithm

Step 1 : Initialization of image f

$$f^{(0)} = 0 \tag{6}$$

Step 2 : Algebraic Reconstruction Technique

$$f^{(k)} = f^{(k-1)} + \lambda \frac{g_i - \phi_i \cdot f^{(k-1)}}{\phi_i \cdot \phi_i} \phi_i \tag{7}$$

where $\lambda \in R^+$, k = 1 to M

Step 3 : Gradient Descent Iteration

Initialization of gradient descent image $\hat{f}^{(0)} = f^{(M)}$

$$\text{Gradient Descent Iteration} \quad \hat{f}^{(l)} = \hat{f}^{(l-1)} - \alpha \vec{\Delta} \tag{8}$$

Finally after the iteration, $f^{(0)} = \hat{f}^{(end)}$ then go to STEP 1 until some stopping criteria

Experimental Observation and Results

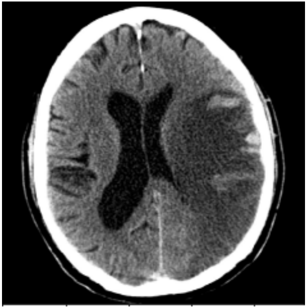

Figure: Original Image

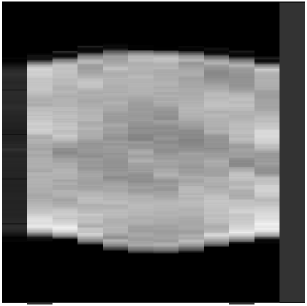

Figure: Radon Transform

Figure: FBP Reconstruction

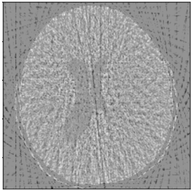

Figure: original - fbp

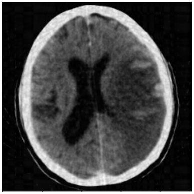

Figure: ART Reconstructed Image

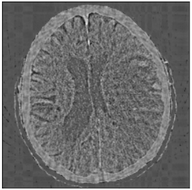

Figure: original - art

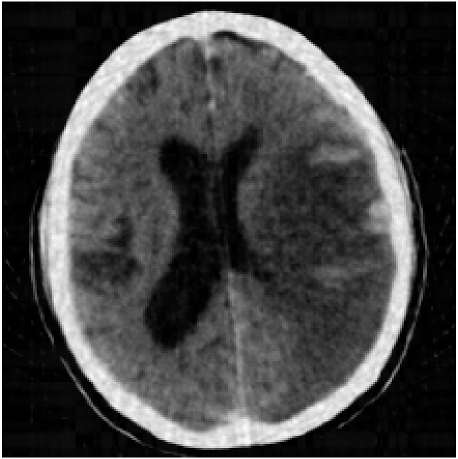

Figure: Compressed Sensing Reconstruction

Experimental Observation and Results

FBP

RMSE: 29.2
SSIM: 0.3977
PSNR: 18.82
UQI: 0.5771
SCC: 0.243

ART

RMSE: 21.31
SSIM: 0.4887
PSNR: 21.56
UQI: 0.6436
SCC: 0.2197

CS

RMSE: 20.38

Figure: Original Image

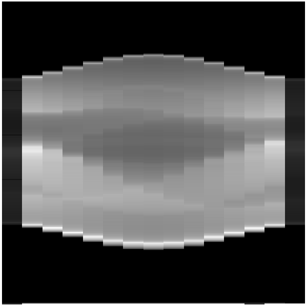

Figure: Radon Transform

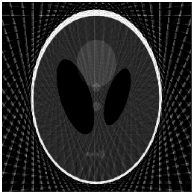

Figure: FBP Reconstruction

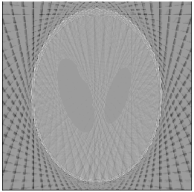

Figure: original - fbp

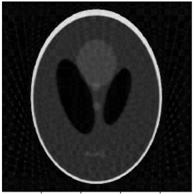

Figure: ART Reconstructed Image

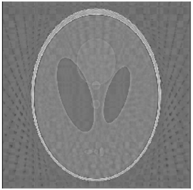

Figure: original - art

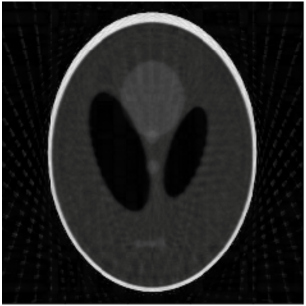

Figure: Compressed Sensing Reconstruction

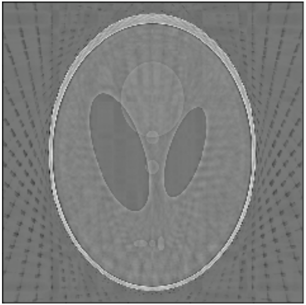

Figure: original - cs

Experimental Observation and Results

FBP

RMSE: 25.51
SSIM: 0.4079
PSNR: 20
UQI: 0.4509
SCC: 0.09833

ART

RMSE: 14.19
SSIM: 0.4623
PSNR: 25.09
UQI: 0.4557
SCC: 0.08446

CS

RMSE: 14.08

Experimental Observation and Results

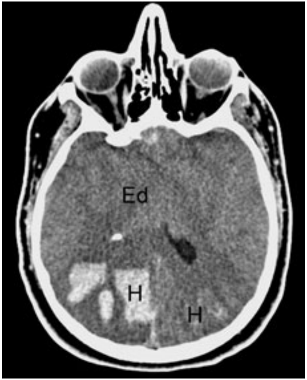

Figure: Original Image

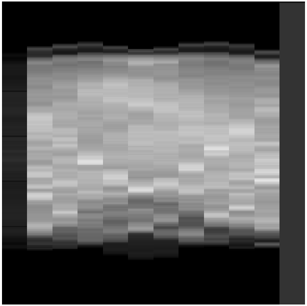

Figure: Radon Transform

Experimental Observation and Results

Figure: FBP Reconstruction

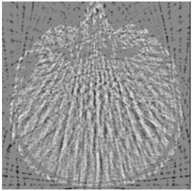

Figure: original - fbp

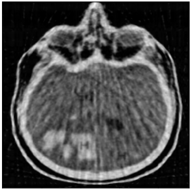

Figure: ART Reconstructed Image

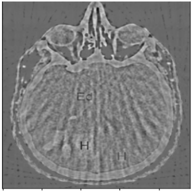

Figure: original - art

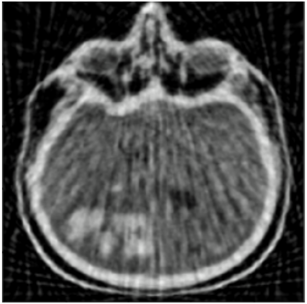

Figure: Compressed Sensing Reconstruction

Experimental Observation and Results

FBP

RMSE: 42.76
SSIM: 0.3756
PSNR: 15.51
UQI: 0.5941
SCC: 0.211

ART

RMSE: 32.1
SSIM: 0.4883
PSNR: 18
UQI: 0.6476
SCC: 0.2386

CS

RMSE: 31.27

References

Xueli Li and Shuqian Luo
A compressed sensing-based iterative algorithm for CT reconstruction and its possible application to phase contrast imaging

Algebraic Reconstruction Technique - ART

Amit Sethi
EE 610 : Tomographic Reconstruction Slides

Ajit Rajwade
CS 754 : Tomographic Reconstruction Slides

Ajit Rajwade
CS 754 : Compressive Sensing Theory

Thank You :)