Computed Tomography Reconstruction : Compressed Sensing Approach

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***Abstract*— The aim of computed tomography (CT) is to reconstruct a 2D image (object) from its 1D projections, or a 3D image (object) from its 2D projections. In most medical applications of CT, the aim is to reduce scanning time and radiation dose without reducing image quality. The widely used Filtered Back-Projection (FBP) method has been used for a long time but with less projection data, this algorithm fails to provide good results. In this project, we have tried to approach the CT reconstruction problem using Algebraic Reconstruction Technique (ART) and Compressed Sensing (CS) with an aim to produce better results with less projection data. We have compared the reconstruction results of FBP, ART and CS using popular metrics such as MSE, SSIM etc.**

I. INTRODUCTION

Tomographic reconstruction is one of the most critical technologies in medical imaging. It is the base for diagnosis like CT scan and MRI. The patients region of interest is subjected to a beam of X rays from different angles which scan the body and give the projections as output. But prolonged exposure to X rays has been shown to cause cancer. Today, most of the research in CT is aiming to reduce the scan time since the radiation of X-rays produces harmful effect on the body if exposed for longer duration. There are two ways to reduce the radiation effect. Firstly, by reducing the scan time at each projection angle and secondly, by reducing the number of angles at which the projection are taken keeping the exposure time of each projection the same. The first method that is reducing the exposure time can lead to decrease in the signal-to-noise ratio of the acquired signal and thus leading to poor reconstruction quality. The second approach to reduce the number of projection angles is a feasible approach and many new methods have developed over the years which are able to achieve good reconstruction results. Hence there has been a constant need for algorithms which can produce better reconstruction with less number of samples as well. Compared with the common reconstruction from 180 projection images, in our approach we used only 60 projection images, which cuts the scan time, and maintains the acceptable quality of the reconstructed images.

The traditional FBP method fails and performs poorly in case of lesser projection data. The CS-based approach, however, produces significantly better compared to FBP. Just to give a brief idea of CS, Compressive sensing is a new technology where the data are acquired/measured in a compressed format. These compressed measurements are then fed to some optimization algorithm (also called inversion algorithm) to produce the complete signal.

II. BACKGROUND

A. Mathematical Model for CT

The Radon transform of $f(x, y)$ is given by,

$$R(f) = g(\rho, \theta) = \sum_{x=0}^{M-1} \sum_{x=0}^{N-1} f(x, y)\delta(xcos\theta + ysin\theta - \rho) \tag{1}$$

where f(x,y) is the original image, ρ and θ are variables in the transform domain.

Algorithms for Image Reconstruction from projections can be divided into two classes

- Analytical Method (e.g, FBP)
- Algebraic Method (Iterative Approach) (e.g, ART)

B. Simple Back-projection for Reconstruction

For this, we fix the angle θ_k and for all x and y, compute the value of ρ. Then copy $g(\rho, \theta_k)$ to $\hat{f}_{\theta_k}(x, y)$, which is the image obtained when you back-project along angle θ_k.

$$\hat{f}_{\theta_k}(x, y) = g(\rho, \theta_k) = g(xcos\theta_k + ysin\theta_k, \theta_k) \tag{2}$$

$$\hat{f}(x, y) = \int_0^{\pi} g(xcos\theta + ysin\theta, \theta)d\theta \simeq \sum_{\theta=0}^{\pi} \hat{f}_\theta(x, y) \tag{3}$$

The back-projection operator is not the same as the inverse of the Radon transform. So this does not yield back the true signal f(x,y), but the signal f(x,y) blurred with the kernel $(x^2 + y^2)^{-0.5}$. This limitation can be mitigated by using FBP.

C. Filtered Back-Projection

The widely used Filtered Back-Projection (FBP) method for CT reconstruction has high computational time and gives accurate reconstruction if the projection data is densely sampled but if the projection data is sparsely sampled over the angular range, the reconstruction has aliasing artifacts such as sharp streaks.

The simple back-projection and FBP described above comes under the analytical methods where we have a closed form solution. On the other hand, the algebraic methods can reconstruct images with relatively less projection data but take longer reconstruction time. The approach which we have followed in this project is based on iterative method in which we use a combination of ART and CS to generate better results compared to FBP as well as simple back-projection.

D. Compressed Sensing

Compressed Sensing (CS) is based on the principle that by exploiting the sparsity of a signal in a particular domain or transformed space, the signal can be reconstructed with far more fewer samples as required by the nyquist theorem. We plan to explore the use of compressed sensing(CS) for improving the results of reconstruction by exploring the algorithms like orthogonal matching pursuit (OMP) and Iterative Shrinkage and Thresholding Algorithm (ISTA). CS algorithms generally aim to find a solution which are sparse (if not the sparsest) by solving the P0 or P1 optimization problem (assuming the fact that natural signals/images have sparse representation in some convenient basis)

E. Algebraic Reconstruction Technique (ART)

The Algebraic Reconstruction Technique (ART) is aimed at estimating f from $g = \phi f$ without any constraint on f. Let the system matrix columns be represented by hyperplanes. The procedure for locating the solution consists of first starting with an initial guess for f, projecting the first guess on first hyperplane, reprojecting the resulting point on second hyperplane, and so on up to the M^{th} hyperplane. This reconstructed solution can now be improvised by reducing its sparsity using gradient descent on the chosen sparsity metric. People have used the gradient image as one of the metric. We plan to explore upon the same.
Here, the solution given by ART is not unique because $M < N$, hence the system is inherently under defined. Therefore there is always a scope for improvement in the solution predicted by ART and hence we exploit sparsity for the same

III. Approach

We have described here CS-based iterative algorithm for image reconstruction in CT. A successful application of CS requires that the desired image should have a sparse representation in a known transform domain. CS aims at having a better reconstruction even with small values of observations. Consider an image f, which can be viewed as a column vector in R^N.

$$f = \psi x \tag{4}$$

where ψ is the NxN orthonormal basis and x is the sparse representation of the image in the ψ basis.
In CT imaging, suppose the sampled parallel-beam projection-data of image f are modeled by a discrete linear system

$$g = \phi f = \phi\psi x = \phi^{'} x \tag{5}$$

where $g \in R^M$, ϕ is MxN system matrix.
For a sparse image, since $M < N$ in the above equation there are infinitely many $\tilde{x}$ that satisfy $g = \phi^{'}\tilde{x}$. Therefore, the image reconstruction is aimed at finding the vector x in the transform domain by solving the linear program.

$$x = \text{argmin } \|\tilde{x}\|_1 \quad \text{subject to} \quad \|g - \phi^{'}\tilde{x}\|_2^2 \leq \epsilon \tag{6}$$

Pseudo Code

1) Step 1 : Initialization of image f

$$f^{(0)} = 0 \tag{7}$$

2) Step 2 : Algebraic Reconstruction Technique

$$f^{(k)} = f^{(k-1)} + \lambda \frac{g_i - \phi_i \cdot f^{(k-1)}}{\phi_i \cdot \phi_i} \phi_i \tag{8}$$

where $\lambda \in R^+$, k = 1 to M

3) Step 3 : Gradient Descent Iteration
Initialization of gradient descent image $\hat{f}^{(0)} = f^{(M)}$

$$\text{Gradient Descent Iteration} \quad \hat{f}^{(l)} = \hat{f}^{(l-1)} - \alpha\vec{\Delta} \tag{9}$$

where $\vec{\Delta} = \|\hat{f}^{(0)} - f^{(0)}\| \frac{v_{\gamma,x}}{\|v_{\gamma,x}\|}$
Finally after the iteration, $f^{(0)} = \hat{f}^{(end)}$ then go to STEP 1 until some stopping criteria is satisfied.

IV. Experimental Results and Observations

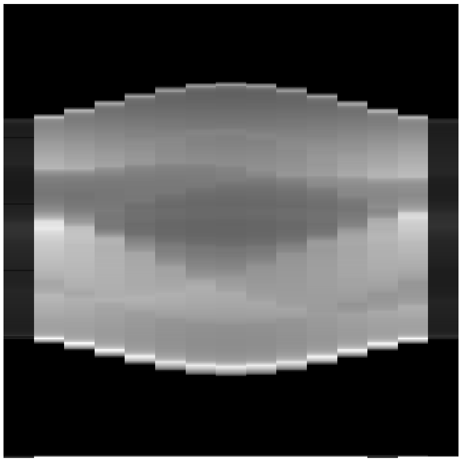
We tested the described approach on multiple images whose results are shown below.

A. Phantom Image

Metrics	FBP	ART	CS
RMSE	25.51	14.19	14.08
SSIM	0.4079	0.4623	0.3954
PSNR	20	25.09	28.67
UQI	0.4509	0.4557	0.4901
SSC	0.09833	0.08446	0.0754

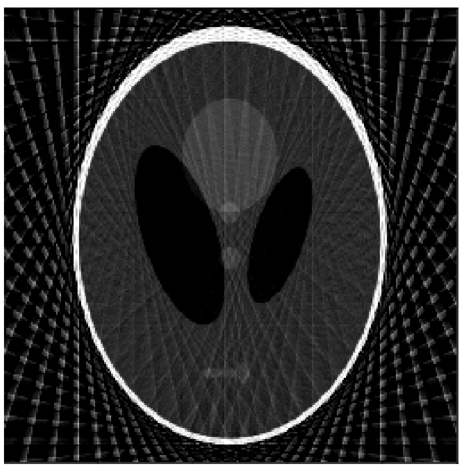
(a)

Fig. 1: Original Phantom Image



(a)

Fig. 2: Radon Transform of Phantom



(a)

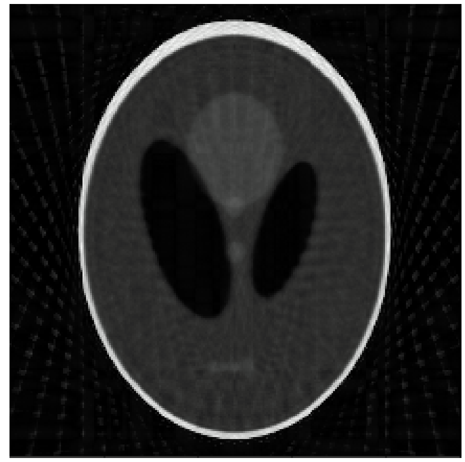
Fig. 3: Reconstruction using FBP

(a)

Fig. 4: Reconstruction using only ART

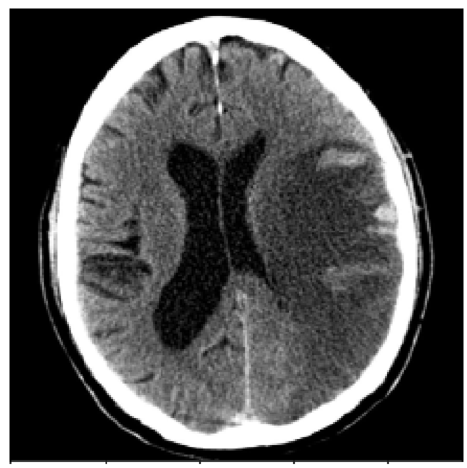
B. Second Image

Metrics	FBP	ART	CS
RMSE	29.2	21.31	20.38
SSIM	0.3977	0.4887	0.5632
PSNR	18.82	21.56	21.98
UQI	0.5771	0.6436	0.7012
SSC	0.243	0.2197	0.2097



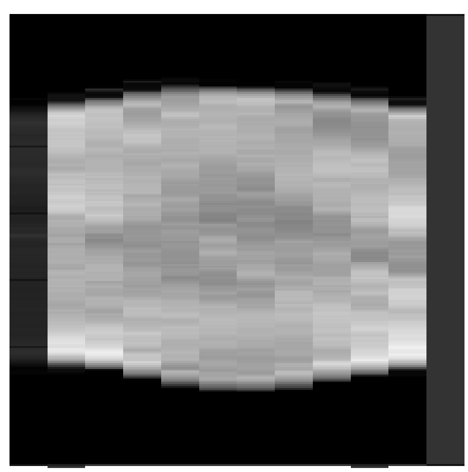
(a)

Fig. 5: Reconstruction using CS Approach



(a)

Fig. 6: Original Image

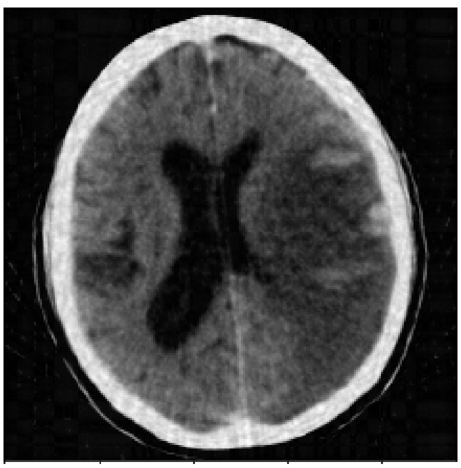


(a)

Fig. 7: Radon Transform

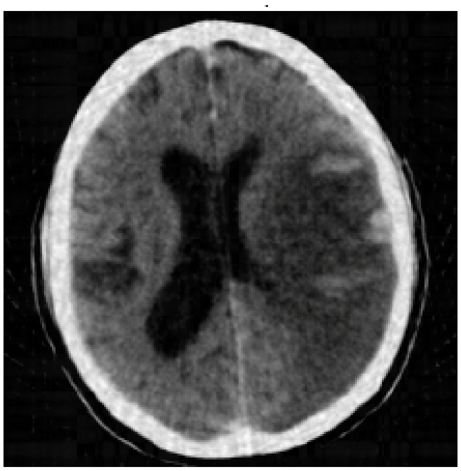
(a)

Fig. 8: Reconstruction using FBP



(a)

Fig. 9: Reconstruction using only ART

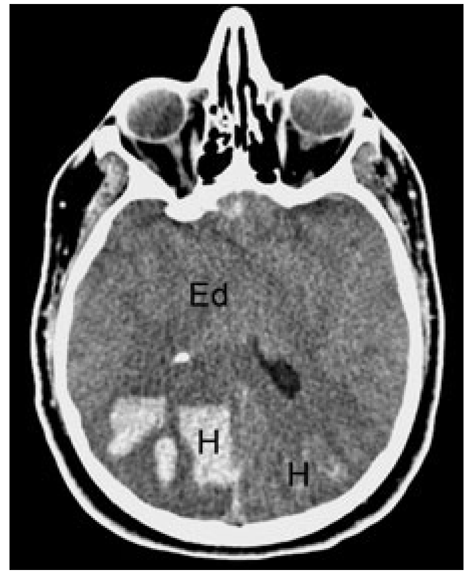


(a)

Fig. 10: Reconstruction using CS Approach

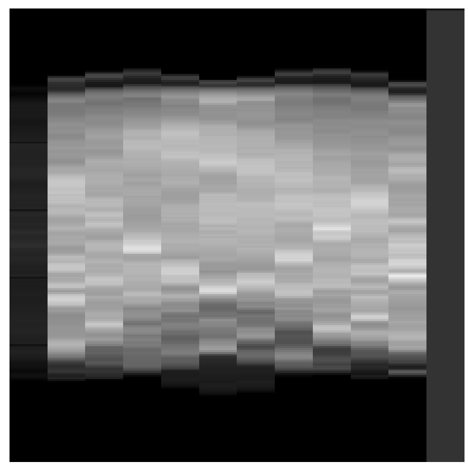
C. *Brain Image*

Metrics	FBP	ART	CS
RMSE	42.76	32.1	31.27
SSIM	0.3756	0.4883	0.5998
PSNR	15.51	18	23.54
UQI	0.5941	0.6476	0.7529
SSC	0.211	0.2386	0.2490



(a)

Fig. 11: Original Image



(a)

Fig. 12: Radon Transform

V. Conclusion

It can be seen that analytical approaches give a closed form solution and have faster computational time however they don't give desired results when the number of angles decrease. Iterative approaches are better suited in such cases. In this project we applied a combination of iterative and compressed sensing based approach to get better result than ART and FBP.

References

[1] Xueli Li, Shuqian Luo, A compressed sensing-based iterative algorithm for CT reconstruction and its possible application to phase contrast imaging

[2] Algebraic Reconstruction Technique, https://engineering.purdue.edu/ malcolm/pct/CTI_Ch07.pdf

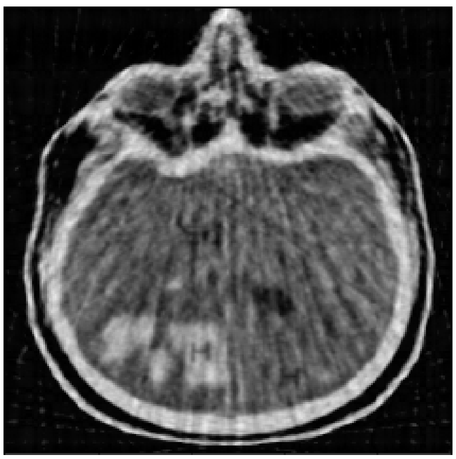
[3] A.Sethi, EE 610 : Tomographic Reconstruction Slides

[4] Ajit Rajwade, CS 754 - Tomographic Reconstruction Slides

[5] Ajit Rajwade, CS 754 - Compressive Sensing Theory

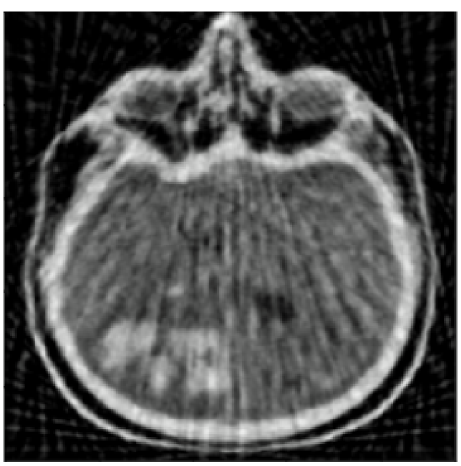
(a)

Fig. 13: Reconstruction using FBP



(a)

Fig. 14: Reconstruction using only ART



(a)

Fig. 15: Reconstruction using CS Approach

[6] **Github Link to code** - https://github.com/darshan3/ct-image-reconstruction